5th International Conference on Perspectives in Hadronic Physics
Particle-Nucleus and Nucleus-Nucleus Scattering at Relativistic Energies
ICTP, May 22-26, 2006

Structure functions at low Q^2

- higher twists and target mass effects

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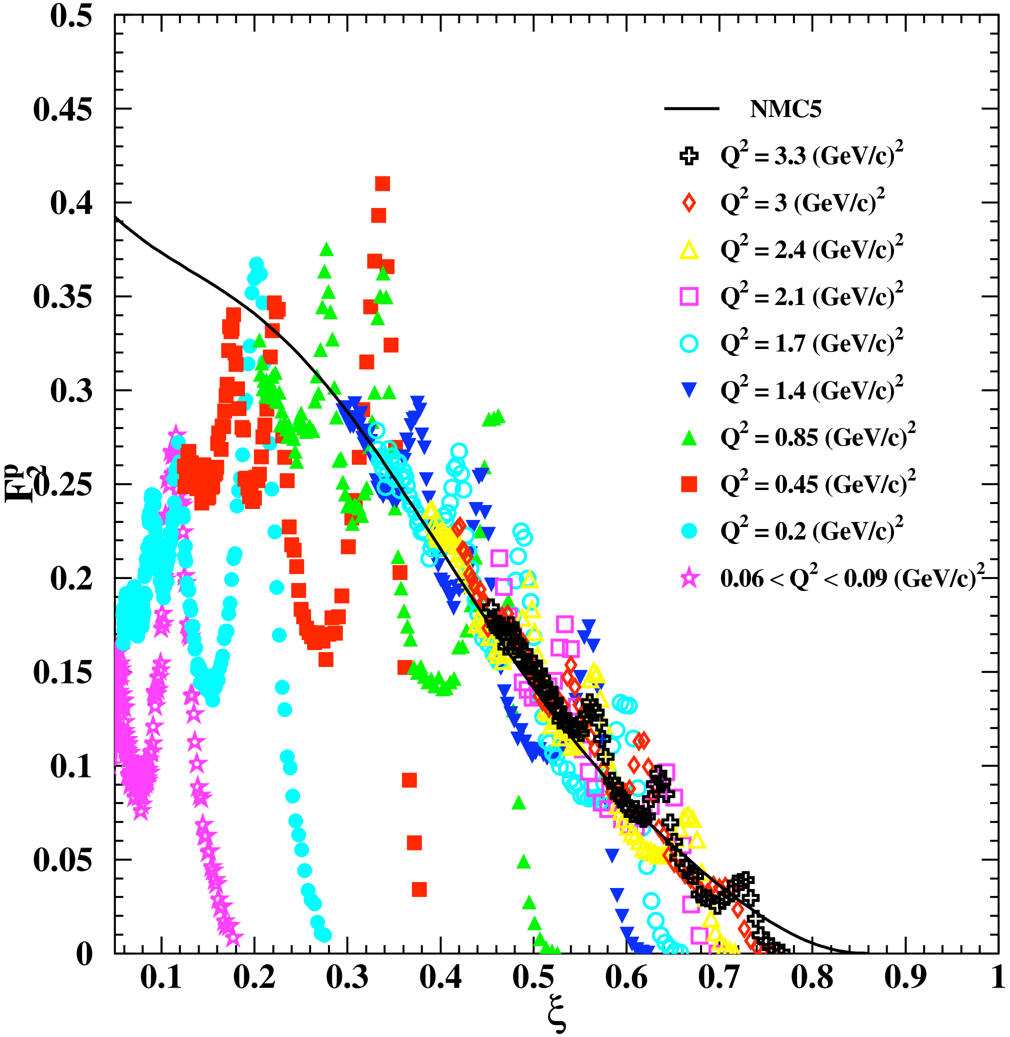
Outline

- Quark-hadron duality
  - $\longrightarrow$ resonances and higher twists
  - $\longrightarrow$ quark models of local duality
- Target mass corrections
  - $\longrightarrow$ kinematical $1/Q^2$ corrections
  - $\longrightarrow$ new formulation without "threshold problem"

1.
Quark-hadron duality

Bloom-Gilman duality



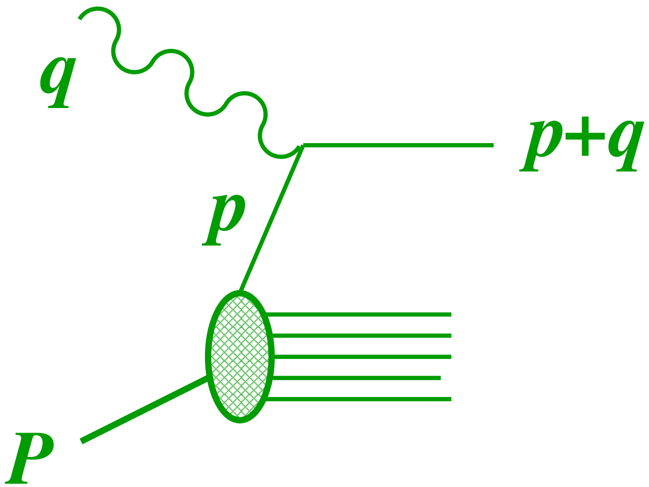
Jefferson Lab (Hall C)

Niculescu et al., Phys. Rev. Lett. 85 (2000) 1182

Average over (strongly Q^2 dependent) resonances

$\approx$ Q^2 independent scaling function

Kinematics



$$m_q = 0$$

$$p_T = 0$$

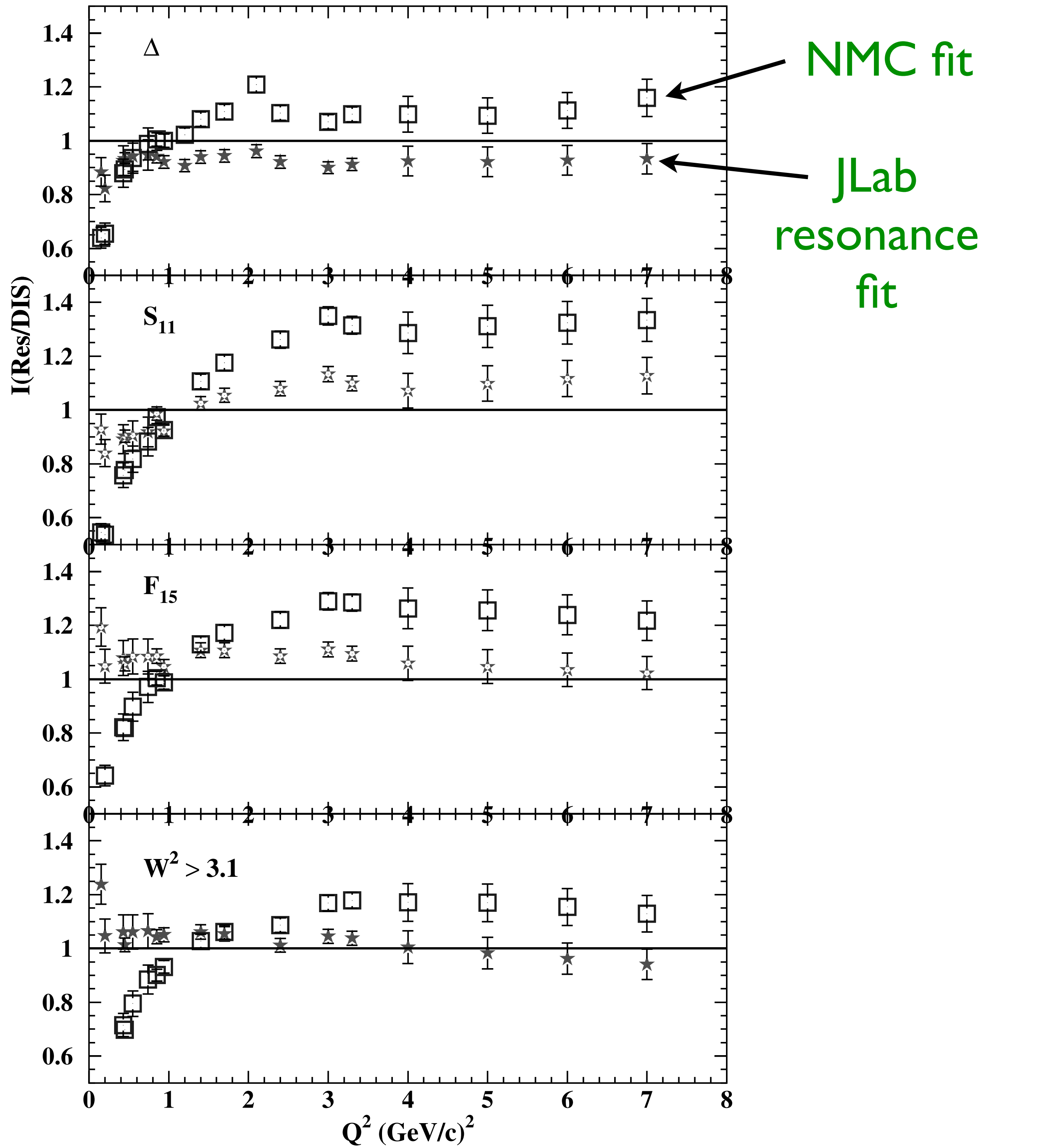
$$\xi = \frac{p^+}{P^+} = \frac{p^0 + p^z}{M}$$

⇒ Nachtmann scaling variable

$$\xi = \frac{2x}{1 + \sqrt{1 + 4x^2 M^2 / Q^2}}$$

target mass dependence

Integrated strength



~10% agreement for $Q^2 > 1$ GeV^2

Niculescu et al., Phys. Rev. Lett. 85 (2000) 1186

Duality and the OPE

Operator product expansion

→ expand moments of structure functions in powers of $1/Q^2$

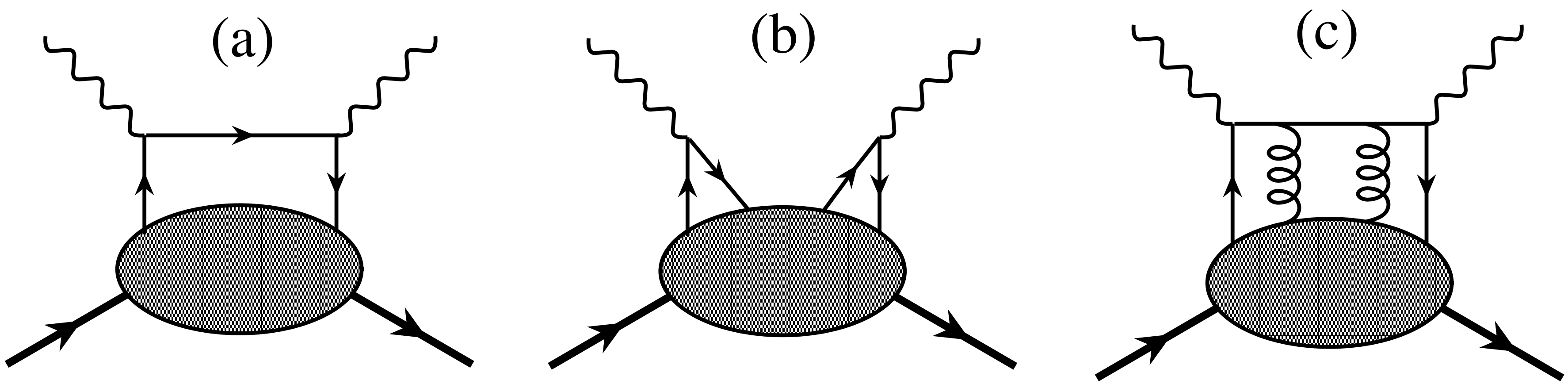
$$M_n(Q^2) = \int_0^1 dx\; x^{n-2}\; F_2(x,Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

matrix elements of operators with specific "twist" τ

$$\tau = \text{dimension} - \text{spin}$$

Higher twists



$\tau = 2$

single quark scattering

$\tau > 2$

qq and qg correlations

Duality and the OPE

Operator product expansion

⇒ expand moments of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\; x^{n-2}\; F_2(x,Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

If moment $\approx$ independent of Q^2

⇒ higher twist terms $A_n^{(\tau>2)}$ small

Duality and the OPE

Operator product expansion

→ expand moments of structure functions in powers of $1/Q^2$

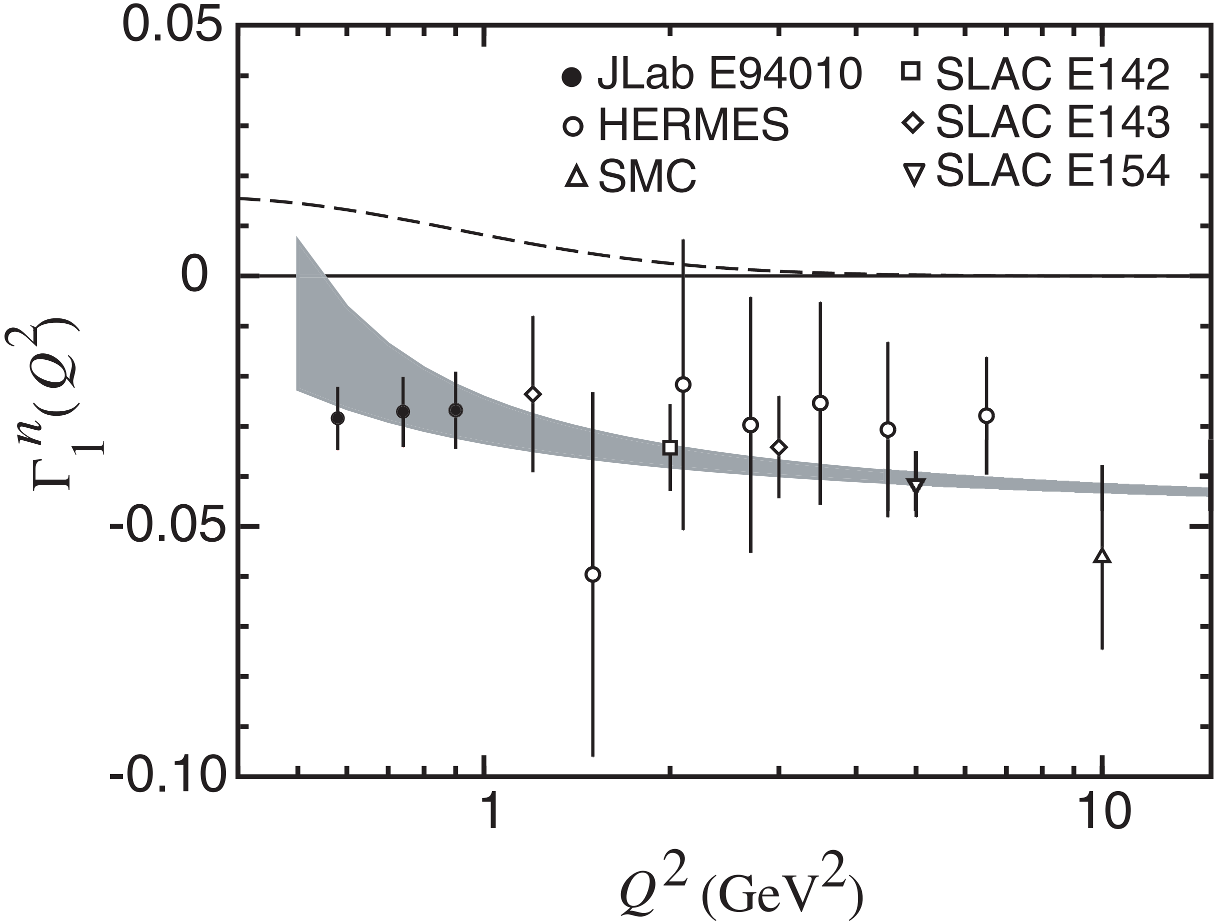
$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x,Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

Duality $\Longleftrightarrow$ suppression of higher twists

de Rujula, Georgi, Politzer,
Ann. Phys. 103 (1975) 315

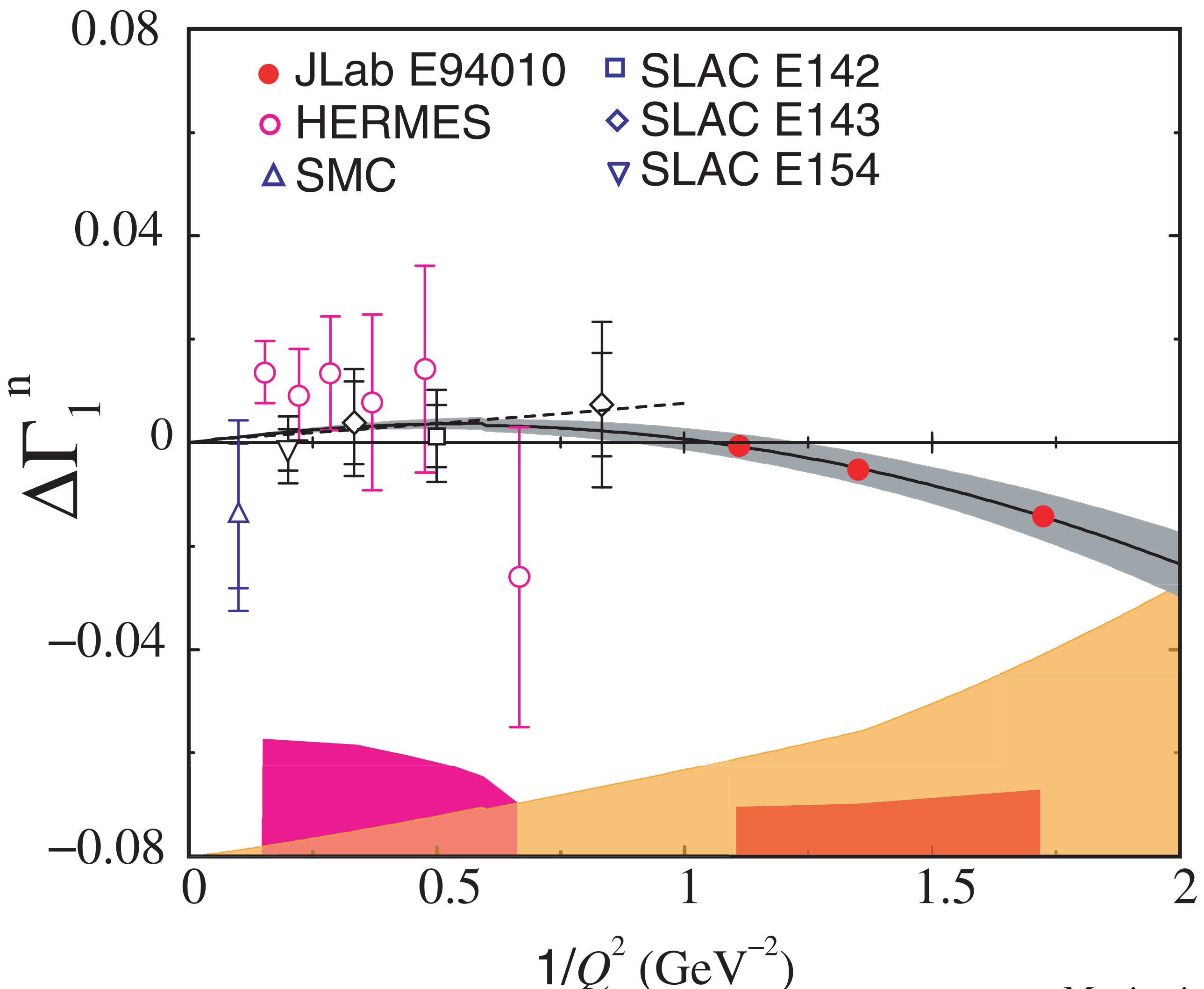
Neutron g_1 moment



Γ_1^n extracted from $\Gamma_1^{^3\mathrm{He}}$ data correcting for nuclear effects

Neutron g_1 moment

→ higher twist contribution



Meziani, WM et al.,
Phys. Lett. B613 (2005) 148

Total higher twist $\sim zero$ at $Q^2 \sim 1 - 2\ \mathrm{GeV}^2$

⟹ nonperturbative interactions between quarks and gluons not dominant at these scales

⟹ suggests *strong cancellations* between resonances, resulting in dominance of *leading twist*

⟹ OPE does not tell us *why* higher twists are small !

Can we understand this behavior dynamically?

How do cancellations between *coherent* resonances produce *incoherent* scaling function?

Coherence vs. incoherence

Exclusive form factors

⟹ *coherent* scattering from quarks

$$d\sigma \sim \left(\sum_i e_i \right)^2$$

Inclusive structure functions

⟹ *incoherent* scattering from quarks

$$d\sigma \sim \sum_i e_i^2$$

⟹ How can *square of a sum* $\approx$ *sum of squares* ?

Pedagogical model

Two quarks bound in a harmonic oscillator potential

⟹ exactly solvable spectrum

Structure function given by sum of squares of transition form factors

$$F(\nu, \mathbf{q}^2) \;\sim\; \sum_n \left| G_{0,n}(\mathbf{q}^2) \right|^2 \; \delta(E_n - E_0 - \nu)$$

Charge operator $\Sigma_i \; e_i \exp(i\mathbf{q} \cdot \mathbf{r}_i)$ excites

even partial waves with strength $\propto (e_1 + e_2)^2$

odd partial waves with strength $\propto (e_1 - e_2)^2$

Pedagogical model

Resulting structure function

$$F(\nu, \mathbf{q}^2) \;\sim\; \sum_n \left\{ (e_1 + e_2)^2 \; G^2_{0,2n} + (e_1 - e_2)^2 \; G^2_{0,2n+1} \right\}$$

If states degenerate, cross terms $(\sim e_1 e_2)$ cancel when averaged over nearby even and odd parity states

Minimum condition for duality:

⟹ *at least one complete set of <u>even</u> and <u>odd</u> parity resonances must be summed over*

Close, Isgur, Phys. Lett. B509 (2001) 81

Quark model

Even and odd parity states generalize to $\mathbf{56}^+$ ($L=0$) and $\mathbf{70}^-$ ($L=1$) multiplets of spin-flavor SU(6)

⟹ scaling occurs if contributions from $\mathbf{56}^+$ and $\mathbf{70}^-$ have equal overall strengths

representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
F_1^p	$9\rho^2$	$8\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2+9\lambda^2$
F_1^n	$(3\rho+\lambda)^2/4$	$8\lambda^2$	$(3\rho-\lambda)^2/4$	$4\lambda^2$	λ^2	$(9\rho^2+27\lambda^2)/2$
g_1^p	$9\rho^2$	$-4\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2-3\lambda^2$
g_1^n	$(3\rho+\lambda)^2/4$	$-4\lambda^2$	$(3\rho-\lambda)^2/4$	$-2\lambda^2$	λ^2	$(9\rho^2-9\lambda^2)/2$

$\lambda\ (\rho) =$ (anti) symmetric component of ground state wfn.

Quark model

SU(6) limit ⟹ $\lambda = \rho$

$SU(6):$	$[\mathbf{56},\mathbf{0}^+]^\mathbf{2}\mathbf{8}$	$[\mathbf{56},\mathbf{0}^+]^\mathbf{4}\mathbf{10}$	$[\mathbf{70},\mathbf{1}^-]^\mathbf{2}\mathbf{8}$	$[\mathbf{70},\mathbf{1}^-]^\mathbf{4}\mathbf{8}$	$[\mathbf{70},\mathbf{1}^-]^\mathbf{2}\mathbf{10}$	*total*
F_1^p	9	8	9	0	1	27
F_1^n	4	8	1	4	1	18
g_1^p	9	-4	9	0	1	15
g_1^n	4	-4	1	-2	1	0

Summing over all resonances in $\mathbf{56}^+$ and $\mathbf{70}^-$ multiplets

⟹ $R^{np} = \frac{F_1^n}{F_1^p} = \frac{2}{3}$ $A_1^p = \frac{g_1^p}{F_1^p} = \frac{5}{9}$ $A_1^n = \frac{g_1^n}{F_1^n} = 0$

⟹ as in quark-parton model !

Quark model

SU(6) may be $\approx$ valid at $x \sim 1/3$

But significant deviations at large x

→ which combinations of resonances reproduce behavior of structure functions at large x?

Model	SU(6)	No $^4\mathbf{10}$	No $^2\mathbf{10}$, $^4\mathbf{10}$	No $S_{3/2}$	No $\sigma_{3/2}$	No ψ_λ
R^{np}	2/3	10/19	1/2	6/19	3/7	1/4
A_1^p	5/9	1	1	1	1	1
A_1^n	0	2/5	1/3	1	1	1

gives $\Delta u/u > 1$ → *inconsistent with duality*

Quark model

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A_1^p	5/9	1	1	1	1	1
A_1^n	0	2/5	1/3	1	1	1

$^4\mathbf{10}\ [\mathbf{56}^+]$ and $^4\mathbf{8}\ [\mathbf{70}^-]$ suppressed

Quark model

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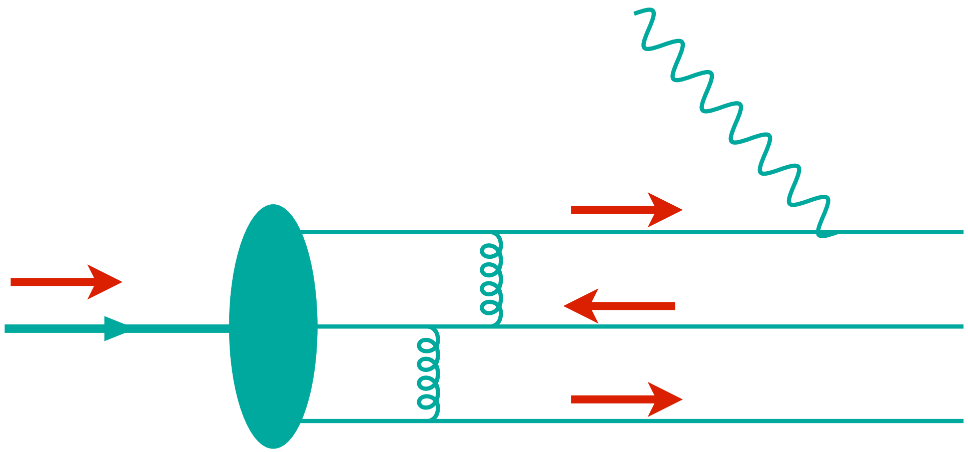
A_1^n	0	2/5	1/3	1	1	1

helicity 3/2 suppression

hard gluon exchange

at large x, helicity of struck quark = helicity of hadron



$$q^{\uparrow} \gg q^{\downarrow}$$

$\Longrightarrow$ helicity-zero diquark dominant in $x \to 1$ limit

$\longrightarrow$ $$\frac{d}{u} \to \frac{1}{5}$$

$\longrightarrow$ $$\frac{F_2^n}{F_2^p} \to \frac{3}{7}$$

Farrar, Jackson 1975

$N \to N^*$ transitions for helicity-1/2 dominance

SU(6) representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
$F_1^p = g_1^p$	9	2	9	0	1	21
$F_1^n = g_1^n$	4	2	1	1	1	9

polarization asymmetries $A_1^N \to 1$

⟹ cf. pQCD "counting rules"

⟹ hard gluon exchange between quarks

neutron to proton ratio $F_2^n / F_2^p \to 3/7$

⟹ cf. "helicity retention" model

Farrar, Jackson, Phys. Rev. Lett. 35 (1975) 1416

Quark model

SU(6) may be $\approx$ valid at $x \sim 1/3$

But significant deviations at large x

$\Longrightarrow$ which combinations of resonances reproduce behavior of structure functions at large x?

Model	SU(6)	No $^4\mathbf{10}$	No $^2\mathbf{10}$, $^4\mathbf{10}$	No $S_{3/2}$	No $\sigma_{3/2}$	No ψ_λ
R^{np}	2/3	10/19	1/2	6/19	3/7	1/4
A_1^p	5/9	1	1	1	1	1
A_1^n	0	2/5	1/3	1	1	1

e.g. through $\vec{S}_i \cdot \vec{S}_j$ interaction between quarks $\Longleftarrow$ suppression of symmetric part of spin-flavor wfn.

Valence quarks

- scalar diquark dominance

$M_\Delta > M_N \implies (qq)_1$ has larger energy than $(qq)_0$

$\implies$ scalar diquark dominant in $x \to 1$ limit

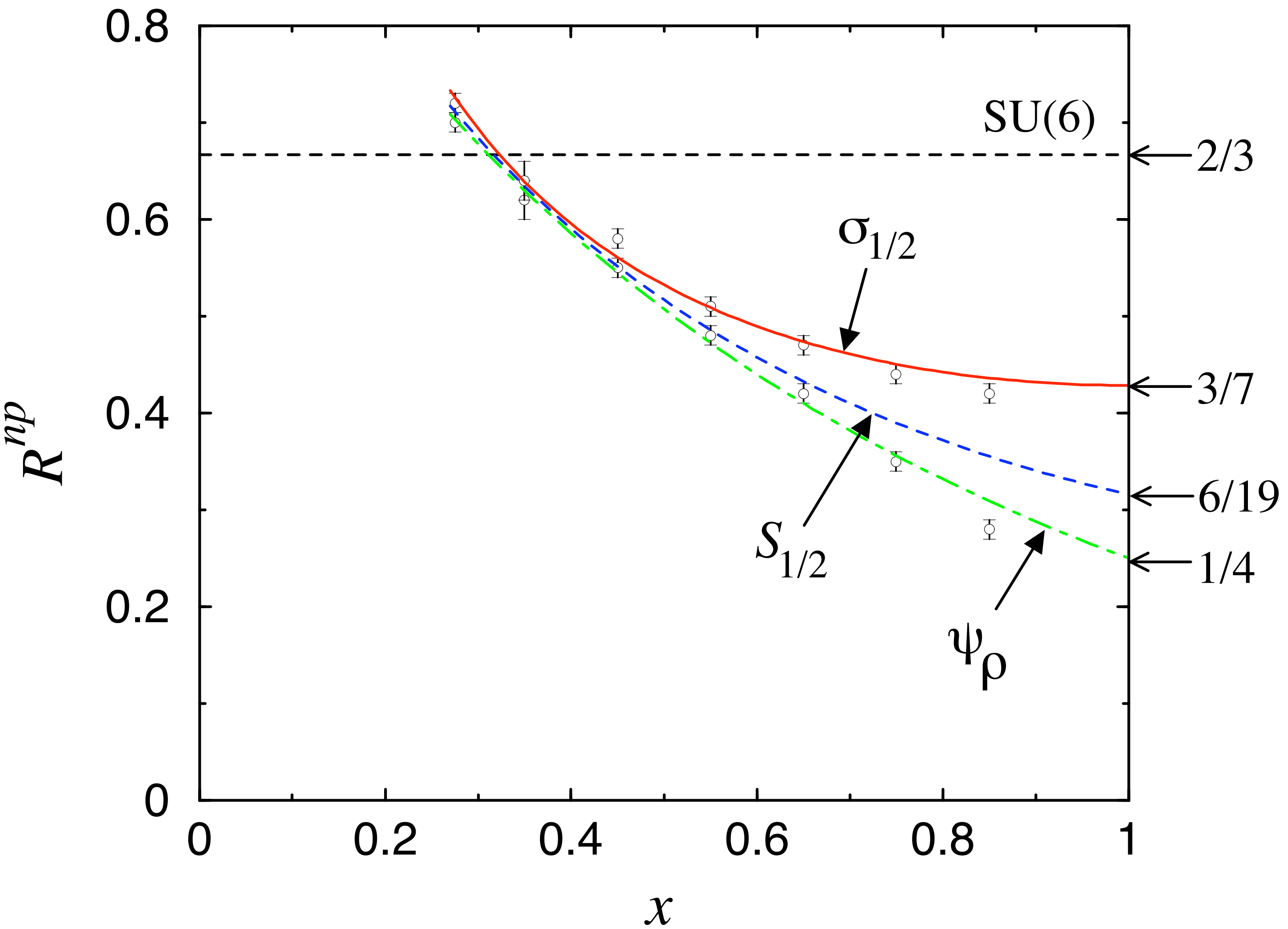
since only u quarks couple to scalar diquarks

$\longrightarrow \quad \frac{d}{u} \to 0$

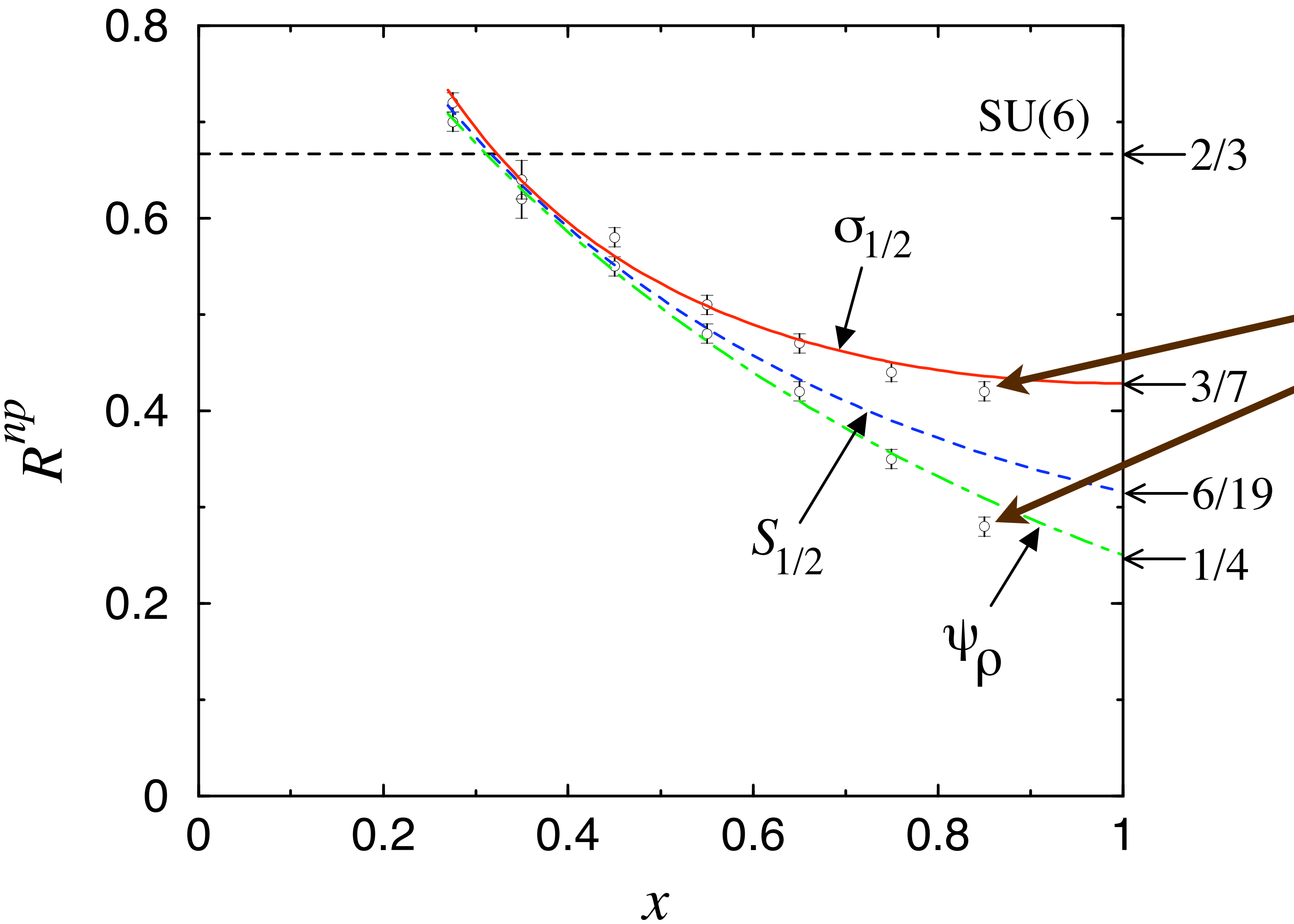
$\longrightarrow \quad \frac{F_2^n}{F_2^p} \to \frac{1}{4}$

Feynman 1972, Close 1973, Close/Thomas 1988

Fit to $\begin{cases} \text{SU(6) symmetry at } x \sim 1/3 \\ \text{SU(6) breaking at } x \sim 1 \end{cases}$

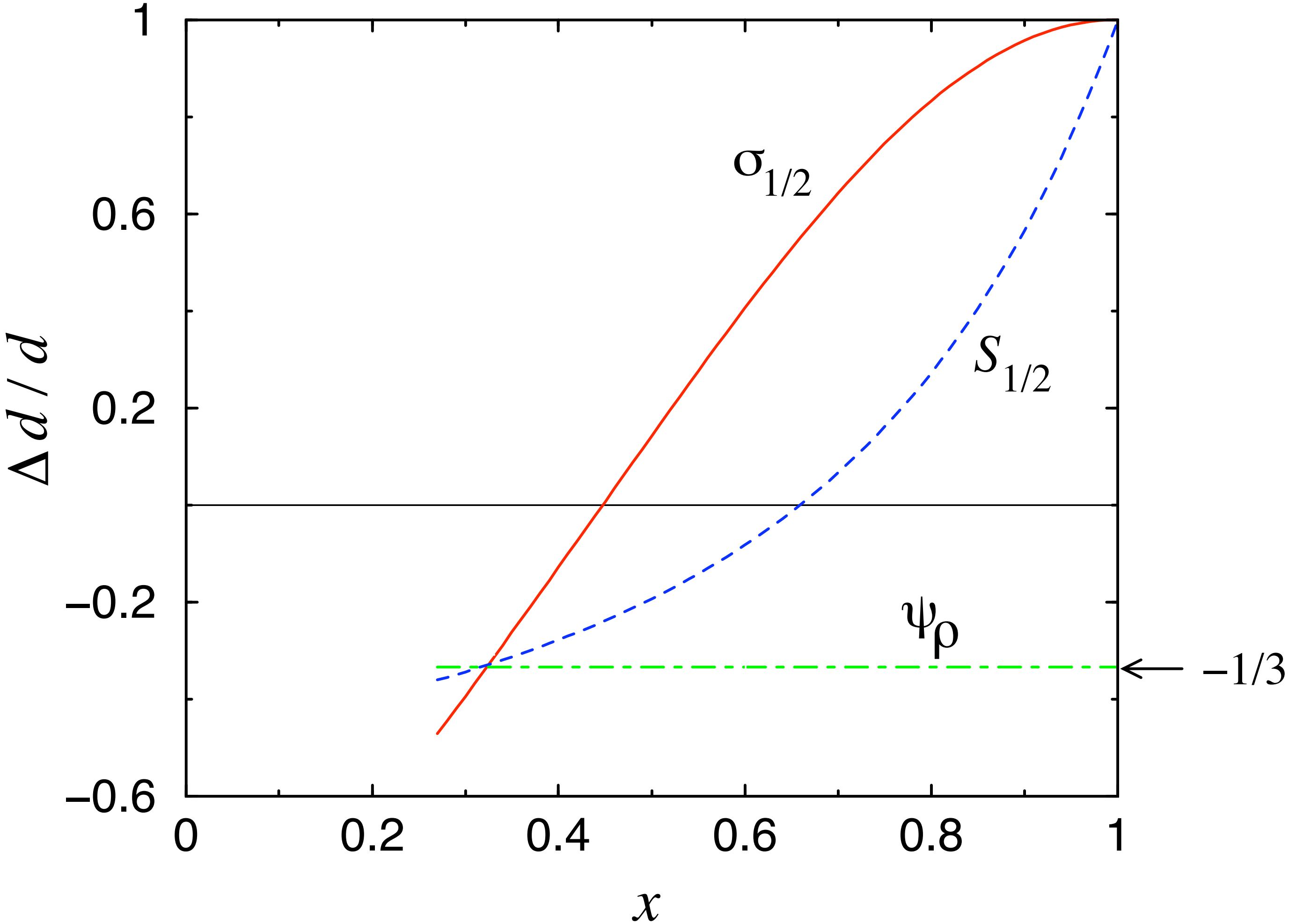


Fit to $\begin{cases} \text{SU(6) symmetry at } x \sim 1/3 \\ \text{SU(6) breaking at } x \sim 1 \end{cases}$



uncertainty in F_2^n due to nuclear effects in deuteron

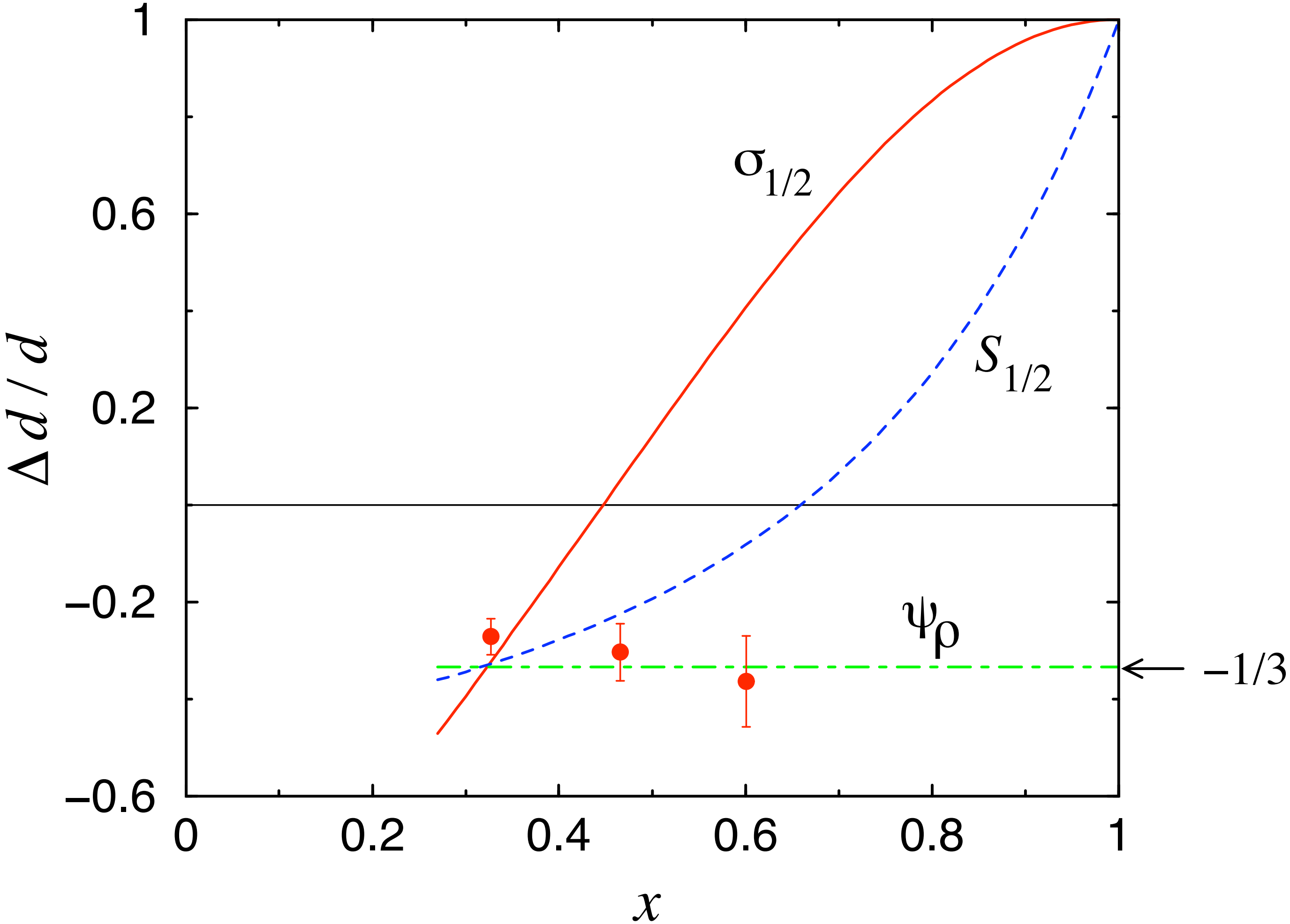
$$\frac{\Delta d}{d} = \frac{4}{15} A_1^n \left(4 + \frac{u}{d}\right) - \frac{1}{15} A_1^p \left(1 + 4\frac{u}{d}\right)$$



$$\frac{u}{d} = \frac{4 - R^{np}}{4R^{np} - 1}$$

Close, WM
PRC68 (2003) 035210

$$\frac{\Delta d}{d} = \frac{4}{15} A_1^n \left(4 + \frac{u}{d}\right) - \frac{1}{15} A_1^p \left(1 + 4\frac{u}{d}\right)$$



$$\frac{u}{d} = \frac{4 - R^{np}}{4R^{np} - 1}$$

Zheng et al. (JLab Hall A)
PRL (2004) 012004

→ nonperturbative physics still dominant at $x \sim 0.6$!

2.
Target mass corrections

Operator Product Expansion

Georgi, Politzer (1976)

$$\int d^4x \; e^{iq\cdot x} \langle N | T(J^\mu(x) J^\nu(0)) | N \rangle$$

$$= \sum_k \left(-g^{\mu\nu} q^{\mu_1} q^{\mu_2} + g^{\mu\mu_1} q^\nu q^{\mu_2} + q^\mu q^{\mu_1} g^{\nu\mu_2} + g^{\mu\mu_1} g^{\nu\mu_2} Q^2 \right)$$

$$\times q^{\mu_3} \cdots q^{\mu_{2k}} \frac{2^{2k}}{Q^{4k}} \underbrace{A_{2k} \Pi_{\mu_1 \cdots \mu_{2k}}}_{\langle N | \mathcal{O}_{\mu_1 \cdots \mu_{2k}} | N \rangle}$$

$$\Pi_{\mu_1 \cdots \mu_{2k}} = p_{\mu_1} \cdots p_{\mu_{2k}} - (g_{\mu_i \mu_j} \text{ terms})$$

$$= \sum_{j=0}^{k} (-1)^j \frac{(2k-j)!}{2^j (2k)^j} g \cdots g \; p \cdots p$$

traceless, symmetric rank-$2k$ tensor

n-th moment of F_2 structure function

$$M_2^n(Q^2) = \int dx \; x^{n-2} \; F_2(x, Q^2)$$

$$= \sum_{j=0}^{\infty} \left(\frac{M^2}{Q^2}\right)^j \frac{(n+j)!}{j!(n-2)!} \frac{A_{n+2j}}{(n+2j)(n+2j-1)}$$

$$\Rightarrow \quad A_n = \int_0^1 dy \; y^n \; F(y)$$

"quark distribution function"

$$F(y) = \frac{F_2(y)}{y^2}$$

- inverse Mellin transform (+ tedious manipulations)

$$F_2^{\mathrm{GP}}(x,Q^2) = \frac{x^2}{r^3} F(\xi) \; + \; 6\frac{M^2}{Q^2}\frac{x^3}{r^4}\int_\xi^1 d\xi' \; F(\xi')$$

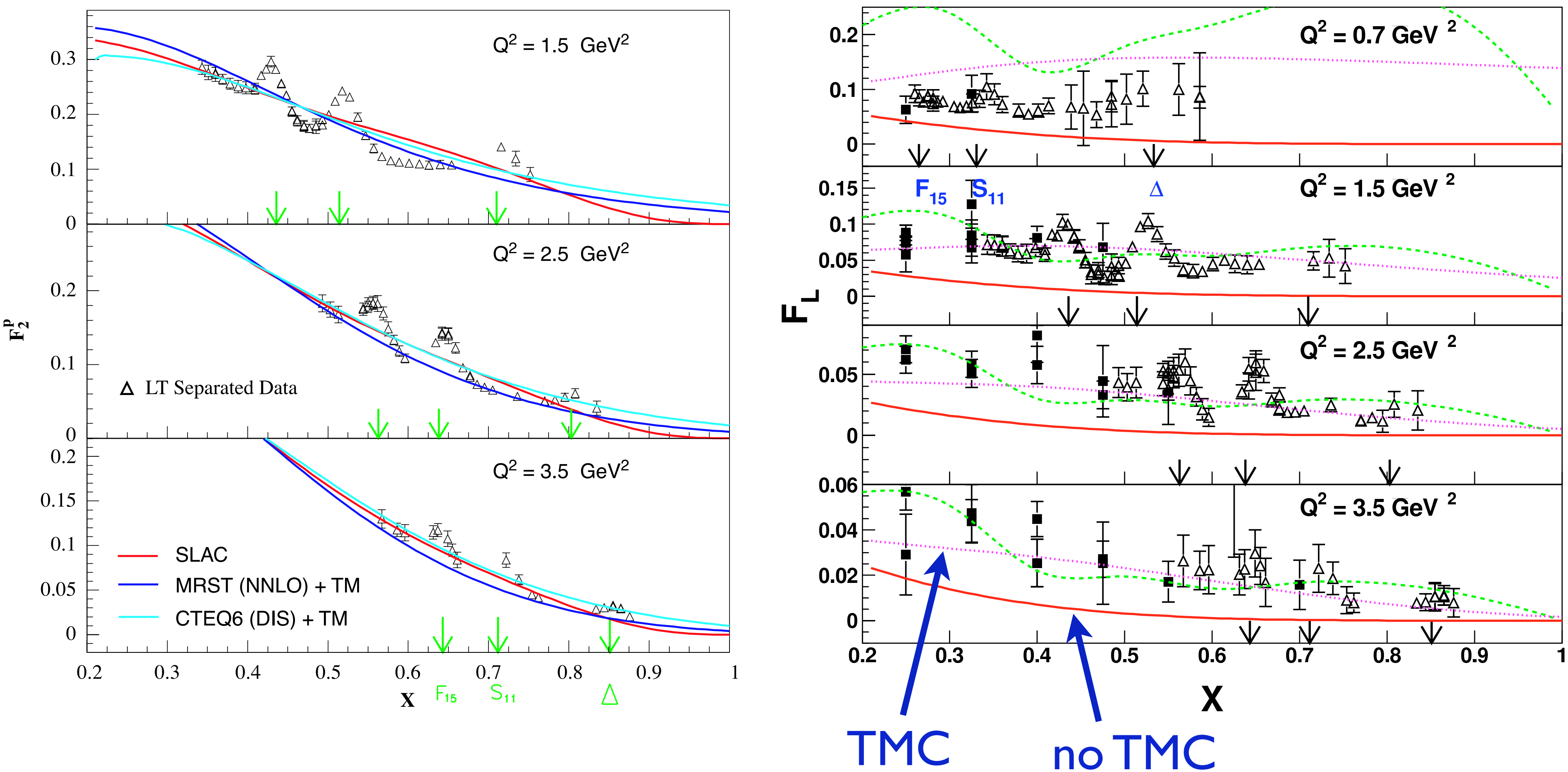
$$+ \; 12\frac{M^4}{Q^4}\frac{x^4}{r^5}\int_\xi^1 d\xi' \; \int_{\xi'}^1 d\xi'' \; F(\xi'')$$

$$\xi = \frac{2x}{1+r} \qquad\qquad r = \sqrt{1+4x^2M^2/Q^2}$$

... similarly for other structure functions F_1, F_L

duality in F_2 and F_L structure functions

Christy et al. (2005)



TMCs significant at large x^2/Q^2, especially for F_L

Threshold problem

- if $F(y) \sim (1-y)^{\beta}$ at large y

  then since $\xi_0 \equiv \xi(x=1) < 1$

  ⇒ $F(\xi_0) > 0$

  ⇒ $F_i^{\rm TMC}(x=1, Q^2) > 0$

  is this physical?

  ⇒ problem with GP formulation?

Possible solutions

- Johnson/Tung - modified threshold factor

Nachtmann moment

$$\mu_2^n(Q^2) = \int_0^1 dx \frac{\xi^{n+1}}{x^3} \left(\frac{3 + 3(n+1)r + n(n+2)r^2}{(n+2)(n+3)} \right) F_2(x, Q^2)$$

→ supposed to remove TMCs explicitly from SF moment

Possible solutions

- Johnson/Tung - modified threshold factor

Nachtmann moment

$$\mu_2^n(Q^2) = \int_0^1 dx \frac{\xi^{n+1}}{x^3} \left(\frac{3 + 3(n+1)r + n(n+2)r^2}{(n+2)(n+3)} \right) F_2(x, Q^2)$$

➡ n fixed, $Q^2 \to \infty$

$$\mu_2^n(Q^2) \to (\ln Q^2/\Lambda^2)^{-\lambda_n} \; A_n$$

➡ $n \to \infty$, Q^2 fixed

$$\mu_2^n(Q^2) \to \xi_0^n(Q^2) \; \widetilde{\mu}_2^n(Q^2)$$

"regularized" amplitudes
(threshold-independent)

Possible solutions

- Johnson/Tung - modified threshold factor

Nachtmann moment

$$\mu_2^n(Q^2) = \int_0^1 dx \frac{\xi^{n+1}}{x^3} \left(\frac{3 + 3(n+1)r + n(n+2)r^2}{(n+2)(n+3)} \right) F_2(x, Q^2)$$

ansatz $\mu_2^n(Q^2) = \xi_0^n(Q^2)\ (\ln Q^2/\Lambda^2)^{-\lambda_n}\ A_n$

➡ consistent with asymptotic pQCD behavior

➡ not unique!

Bitar, Johnson, Tung
PLB 83B (1979) 114

Possible solutions

- Johnson/Tung - modified threshold factor

moreover, if identify A_n with $M_2^n = \int_0^1 dx\ x^{n-2}\ F_2(x)$

$$\mu_2^n(Q^2) = \xi_0^n(Q^2)\ M_2^n(Q^2)$$

$$\Rightarrow \quad M_2^n(Q^2) = \mu_2^n(Q^2) + \frac{nM^2}{Q^2} M_2^n + \cdots$$

cf. exact expression

$$M_2^n(Q^2) = \mu_2^n(Q^2) + \frac{n(n-1)}{n+2} \frac{M^2}{Q^2} M_2^{n+2} + \cdots$$

→ inconsistency at low Q^2 ?

Alternative solution

- work with ξ_0 dependent PDFs

→ n-th moment A_n of distribution function

$$A_n = \int_0^{\xi_{\max}} d\xi \; \xi^n \; F(\xi)$$

→ what is $\xi_{\max}$?

- GP use $\xi_{\max} = 1, \quad \xi_0 < \xi < 1$ unphysical
- strictly, should use $\xi_{\max} = \xi_0$

Steffens, WM
PRC 73 (2006) 055202

Alternative solution

- what is effect on phenomenology?

  $\longrightarrow$ try several "toy distributions"

standard TMC ("sTMC")

$$q(\xi) = \mathcal{N}\ \xi^{-1/2}\ (1-\xi)^3\ , \qquad \xi_{\max} = 1$$

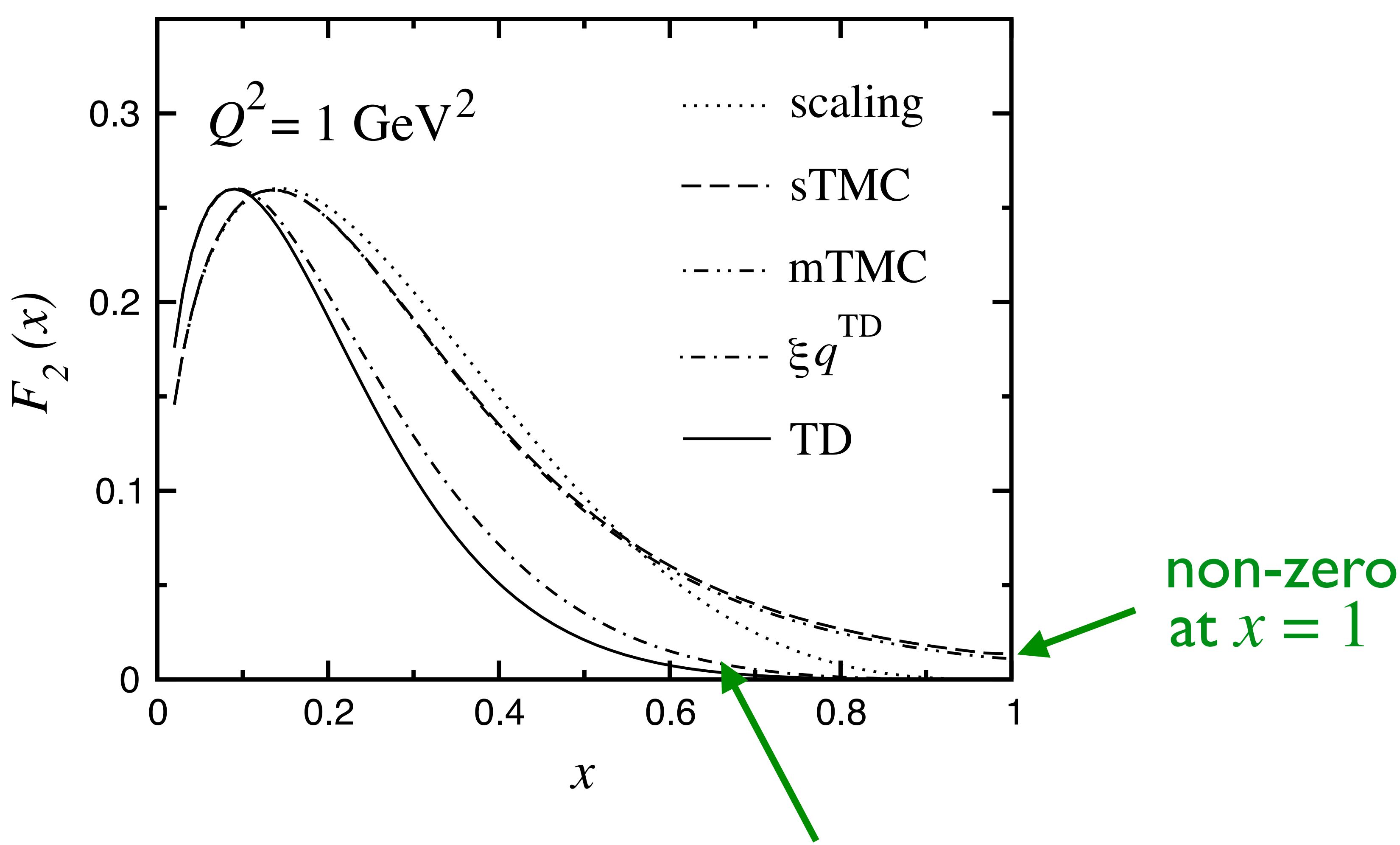
modified TMC ("mTMC")

$$q(\xi) = \mathcal{N}\ \xi^{-1/2}\ (1-\xi)^3\ \Theta(\xi - \xi_0), \qquad \xi_{\max} = \xi_0$$

threshold dependent ("TD")

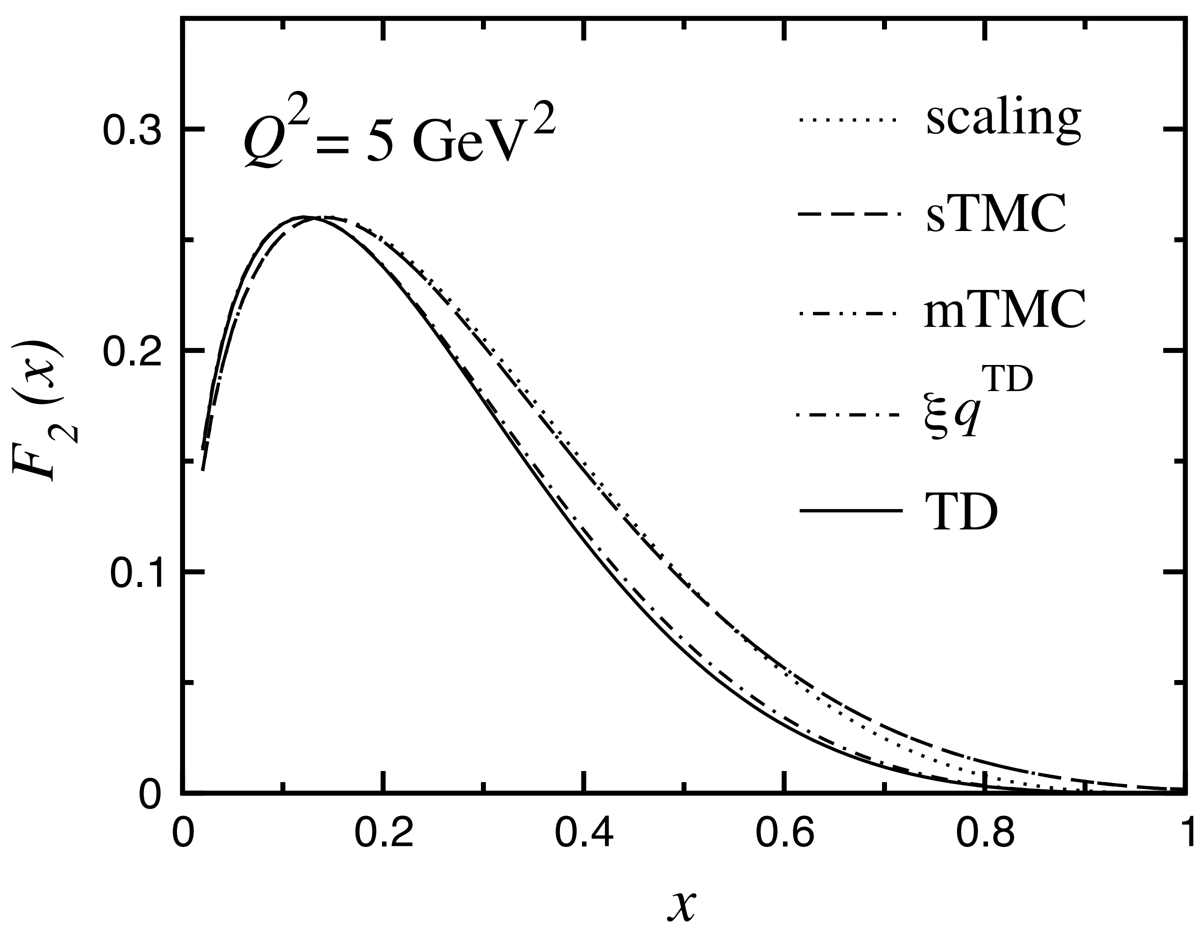
$$q^{\mathrm{TD}}(\xi) = \mathcal{N}\ \xi^{-1/2}\ (\xi_0 - \xi)^3\ , \qquad \xi_{\max} = \xi_0$$

TMCs in F_2



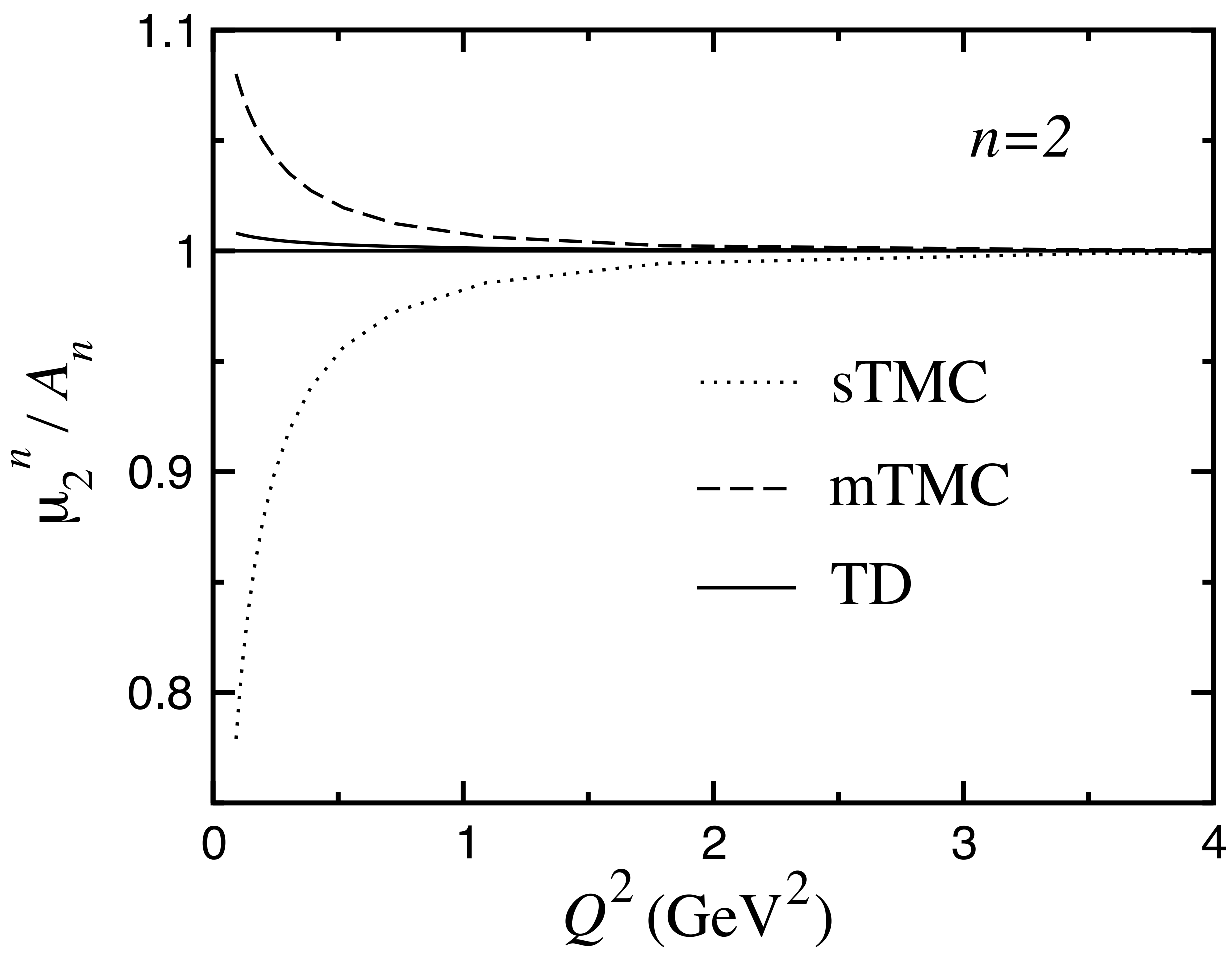
→ correct threshold behavior for "TD" correction

TMCs in F_2



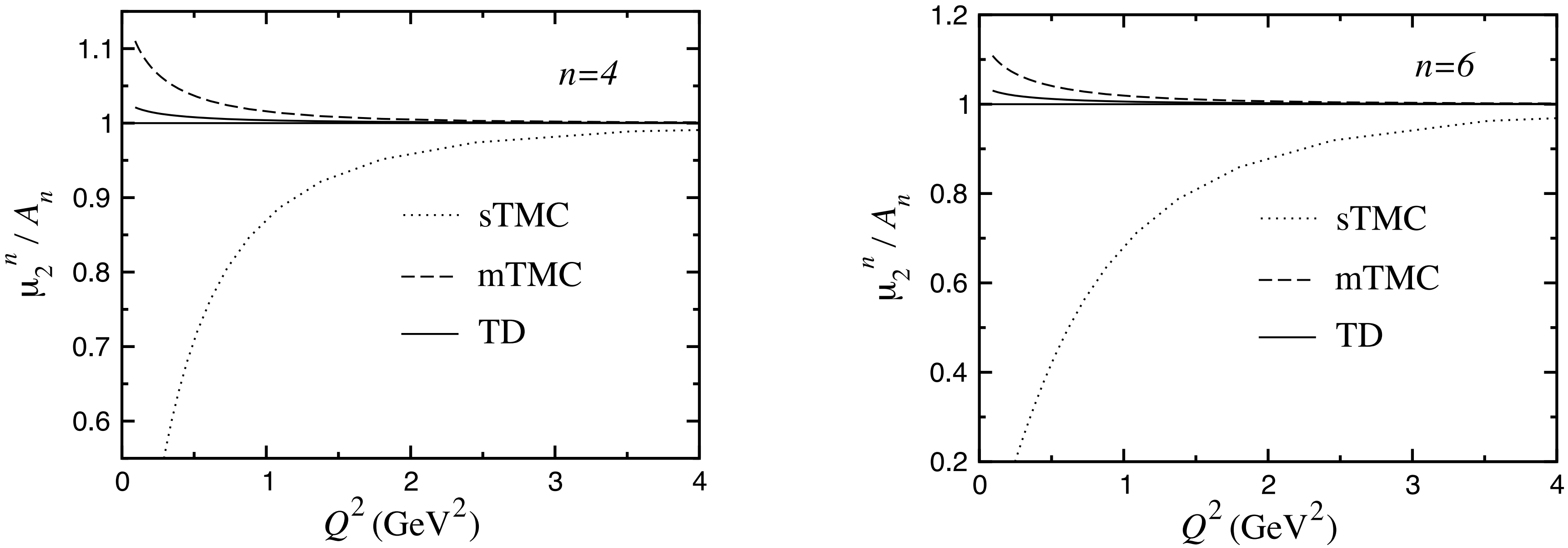
→ effect small at higher Q^2

Nachtmann F_2 moments



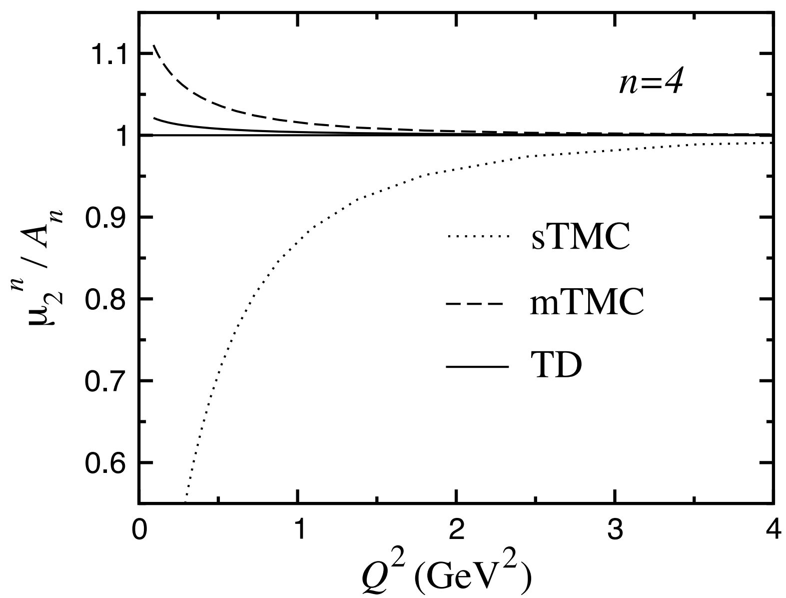
→ moment of structure function agrees with moment of PDF to 1% down to very low Q^2

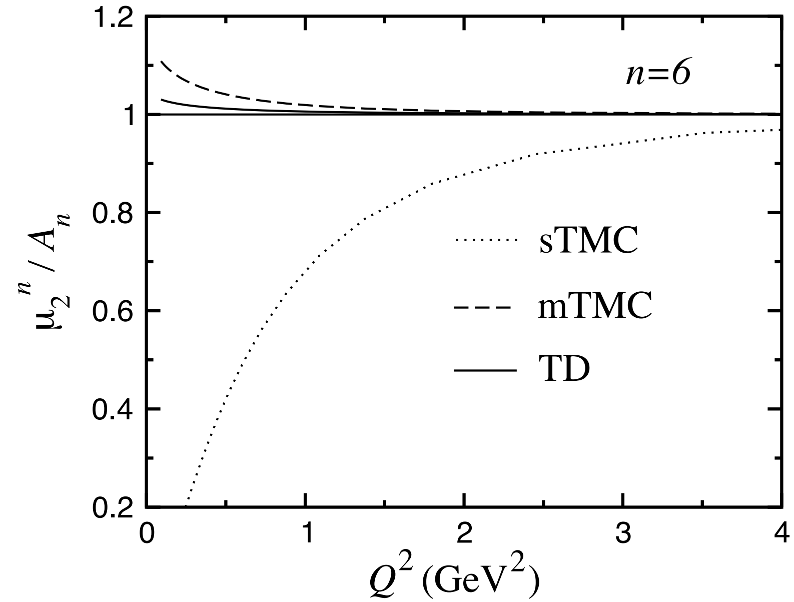
Nachtmann F_2 moments



→ higher moments show much weaker Q^2 dependence than sTMC & mTMC prescriptions

Nachtmann F_2 moments

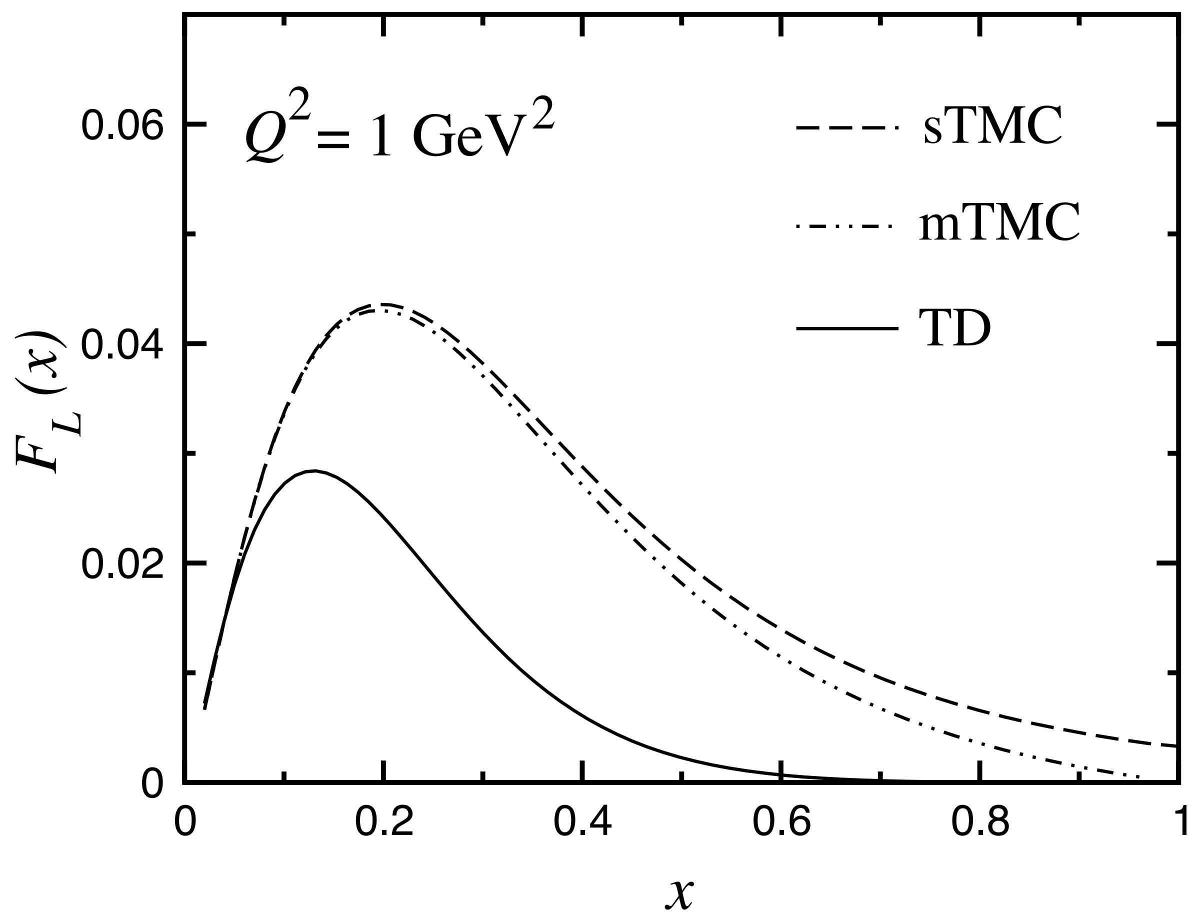




→ $$\frac{\mu_2^n(\text{finite } Q^2)}{A_n(\text{finite } Q^2)} = \frac{\mu_2^n(Q^2 \to \infty)}{A_n(Q^2 \to \infty)}$$

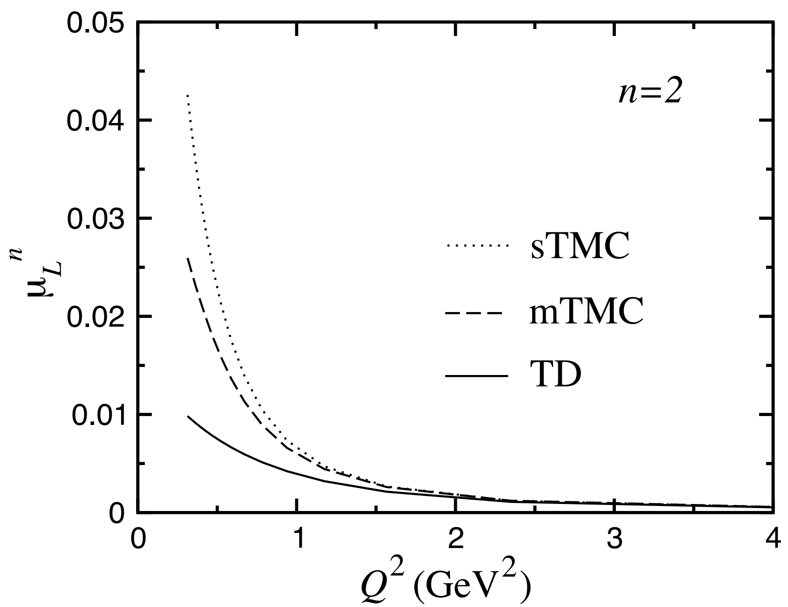
→ extract PDFs from structure function data at lower Q^2

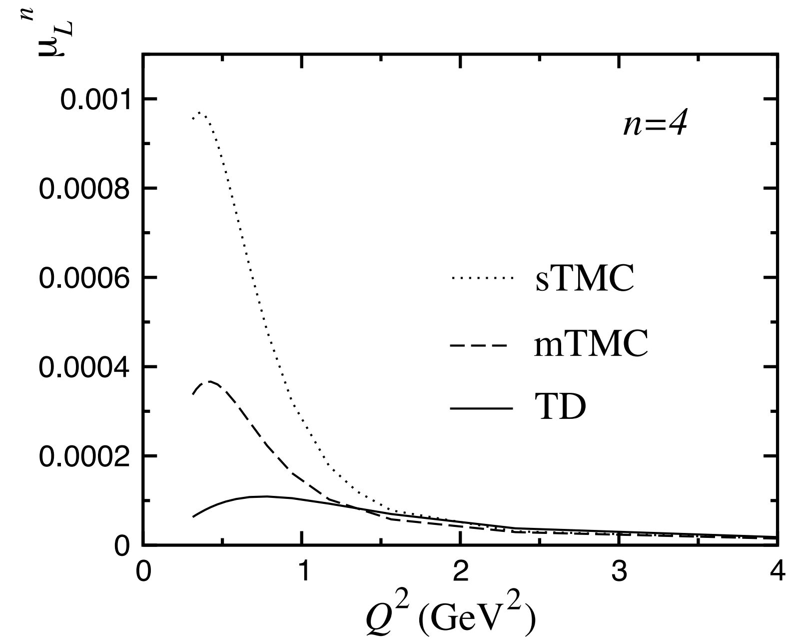
TMCs in F_L



- → correct threshold behavior for "TD" correction
- → reduced TMC effect *cf.* sTMC and mTMC

Nachtmann F_L moments





→ weaker Q^2 dependence for TD prescription

Summary

- Remarkable confirmation of quark-hadron duality in structure functions at low Q^2

  $\longrightarrow$ higher twists "small" down to $Q^2 \sim 1\ \mathrm{GeV}^2$

  $\longrightarrow$ quark models provide clues to origin of resonance cancellations (local duality)

  WM, Ent, Keppel, Phys. Rept. 406 (2005) 127

- Formulation of TMC in GP approach *without* threshold problem

  $\longrightarrow$ much faster approach to scaling for ξ_0 dependent PDF

  $\longrightarrow$ extend to polarized and nuclear structure functions

  $\longrightarrow$ how should $F(\xi, \xi_0)$ be interpreted in partonic language?
  how do evolution equations change?

  Steffens, WM, PRC 73 (2006) 055202

The End