Meson Radiative Transitions on the Lattice
hybrids and charmonium

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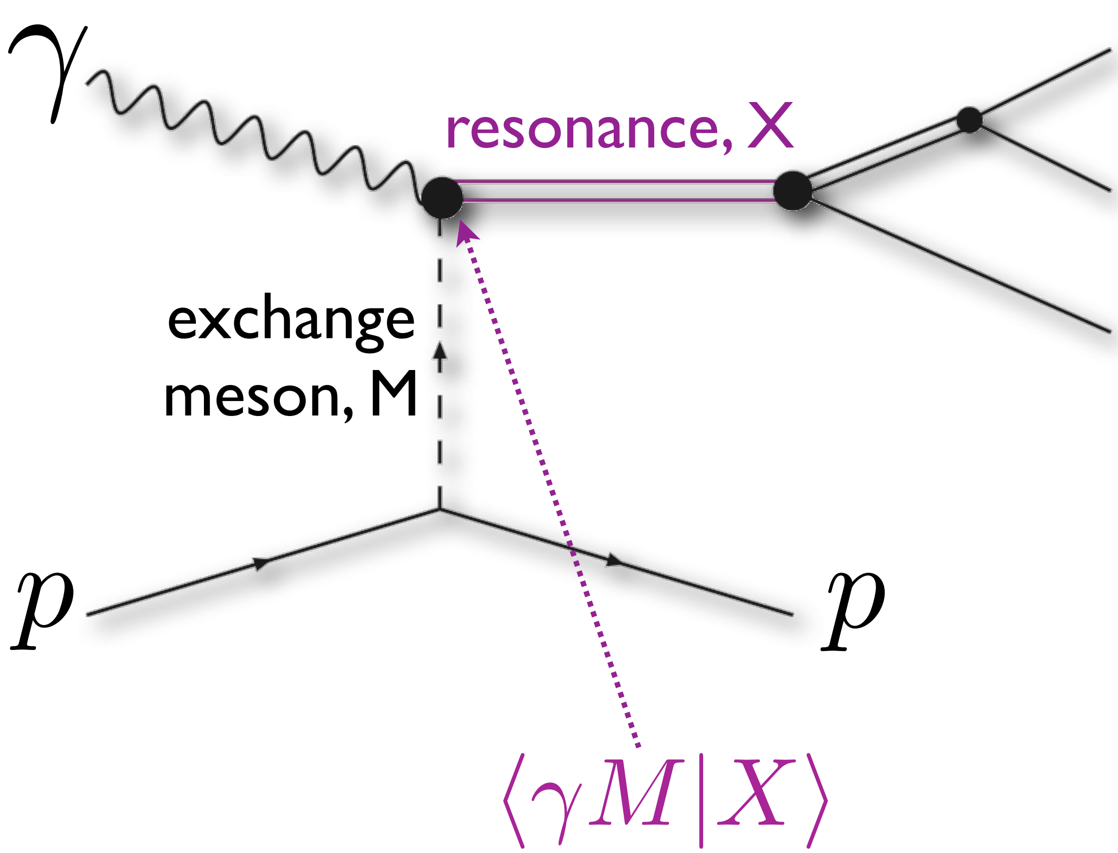
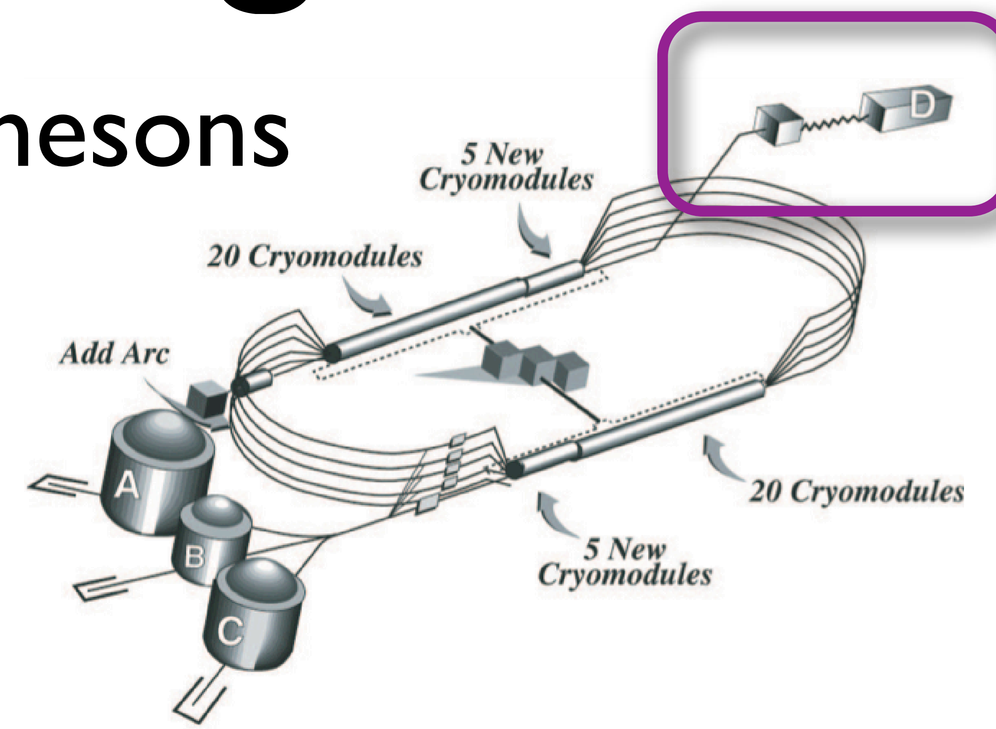
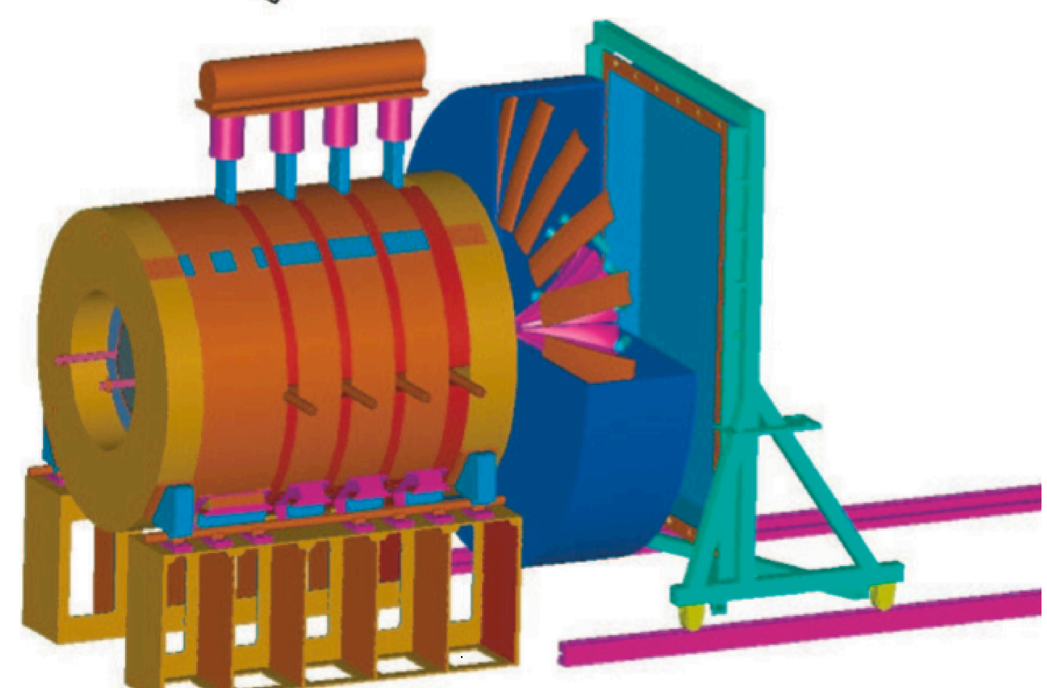
JLab, GlueX and photocouplings

- GlueX plans to photoproduce mesons
- especially exotic J^{PC} mesons







photoproduction of exotics?

- exotic quantum numbers $1^{-+}, 0^{+-}, 2^{+-}$ may be explicable as hybrid meson states
- photoproduction an untested method
- relies upon reasonably large couplings e.g. $\left\langle \gamma\,{}^{\pi}_{\substack{\rho \\ a_2}} \middle| \pi_1 \right\rangle$
- large in some model estimations

Lattice QCD estimation?

- relatively straightforward in principle; evaluate three-point function with a vector current
- in practice, not so easy
  - truly light quarks unfeasible
  - transitions involve unstable states
  - experimental data is limited and imprecise

(even for conventional meson transitions)

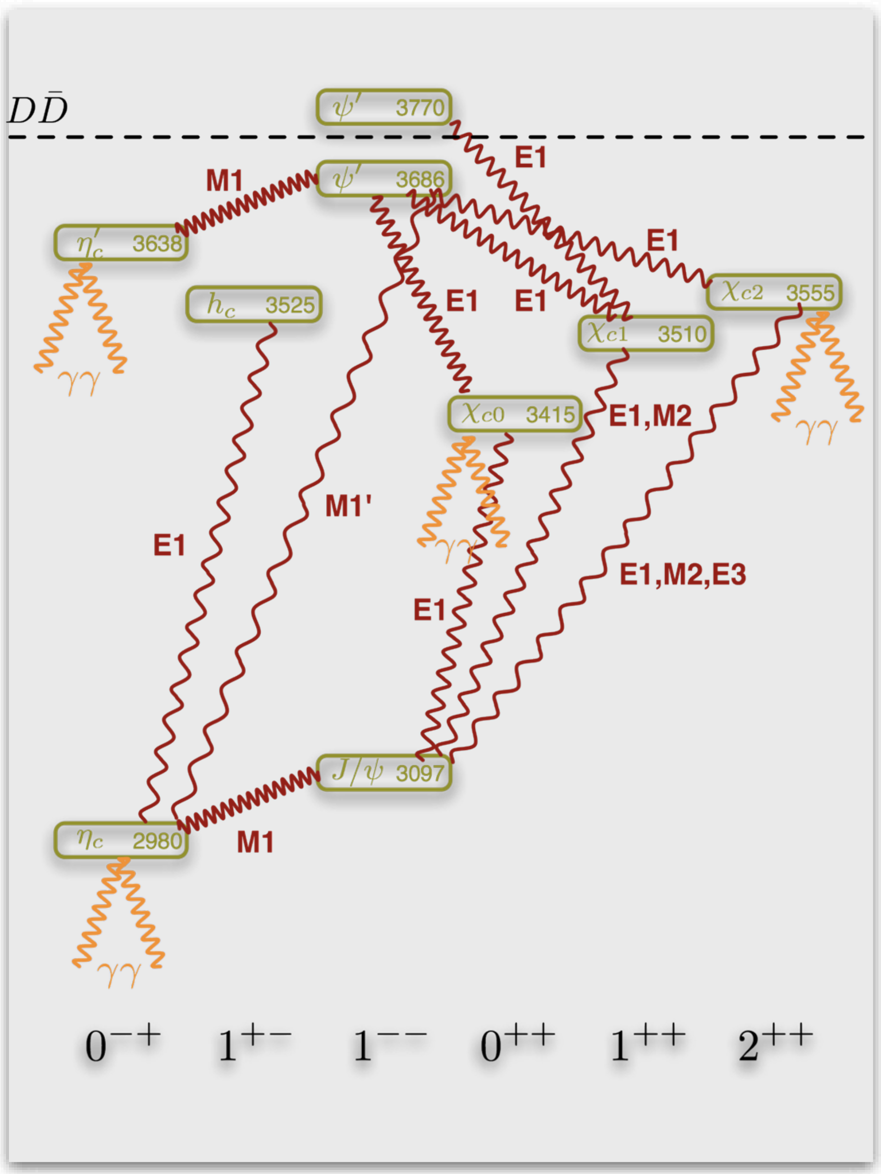
pragmatic approach

- try out an untested method in a region where approximations are controllable
- and where there is good experimental data to compare with

charmonium

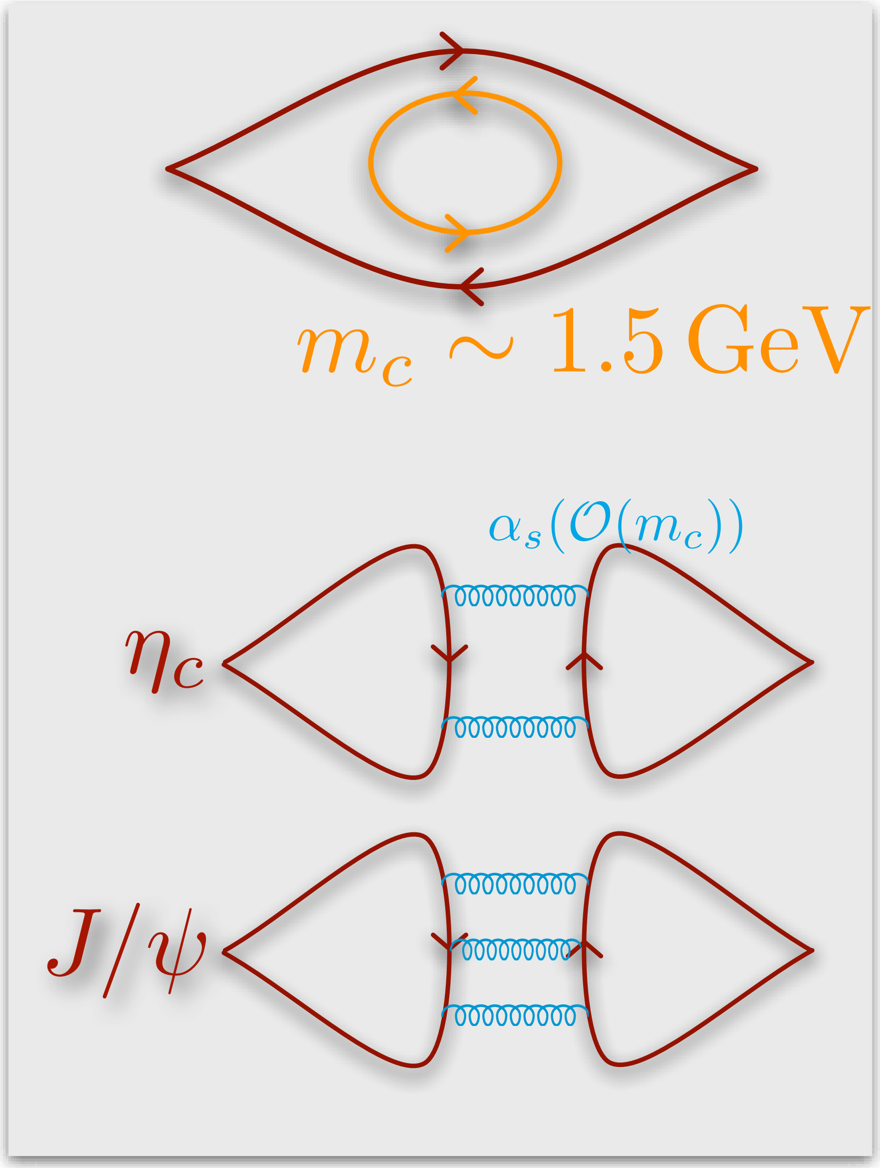
Charmonium - expt.

- multiple states below DD threshold have narrow widths
- radiative transitions are big branching fractions
- precision measurements



Charmonium - lattice

- states are small - small volumes OK
- quenched theory not sick* - just not expt.
- disconnected diagrams perturbatively suppressed
- need a fine lattice spacing $a \gtrsim 3\,\mathrm{GeV}$?



* "one heavy flavour QCD"; will only notice non-unitary up near 6 GeV

anisotropy

- charm quark mass scale requires a fine lattice
- but only in the temporal direction?
- spatial scale $\sim |\vec{p}| \sim 500\,\mathrm{MeV}$
- so space direction can be more coarse
- introduce anisotropy param into fermion action and tune to get meson disp$^{\underline{n}}$ rel$^{\underline{ns}}$ right

our initial simulation

- anisotropic Wilson glue with $\xi = 3$ at $\beta = 6.1$
- 12^3x48 gives a 1.2 fm box
- Domain-Wall fermions (L_5 = 16)
  - Ginsparg-Wilson ensures O(a) improvement
  - vector current only multiplicatively renorm^d

spectroscopic splittings came out reasonably - usual quenched problem of hyperfine too small

scale setting by Sommer parameter, but 1P-1S very similar

three-point functions

$$\Gamma(t_f, t\,; \vec{p}_f, \vec{q}) = \sum_{\vec{x},\vec{y}} e^{-i\vec{p}_f\cdot\vec{x}} e^{i\vec{q}\cdot\vec{y}} \, \langle \varphi_f(\vec{x}, t_f) j^\mu(\vec{y}, t) \varphi_i(\vec{0}, 0) \rangle$$

- we use gaussian smeared fermion bilinears as interpolating fields $\sum_{\vec{z}} F(\vec{z}) \bar{\psi}_{\vec{x}+\vec{z},t} \Gamma \psi_{\vec{x}-\vec{z},t}$
- connected diagram constructed from forward propagator and sequential sink propagator with the simple point-like vector current

new inversion for each change of the sink, but all possible momenta inserted at the current

three-point functions and matrix elements

$$\Gamma(t_f, t; \vec{p}_f, \vec{q}) = \sum_{\vec{x},\vec{y}} e^{-i\vec{p}_f\cdot\vec{x}} e^{i\vec{q}\cdot\vec{y}} \, \langle \varphi_f(\vec{x}, t_f) j^\mu(\vec{y}, t) \varphi_i(\vec{0}, 0) \rangle$$

- inserting two complete sets of states

$$\Gamma(t_f, t; \vec{p}_f, \vec{q}) = \frac{Z_i Z_f}{4E_i E_f} e^{-E_f(t_f - t)} e^{-E_i t} \langle f(\vec{p}_f) | j^\mu(0) | i(\vec{p}_i) \rangle$$

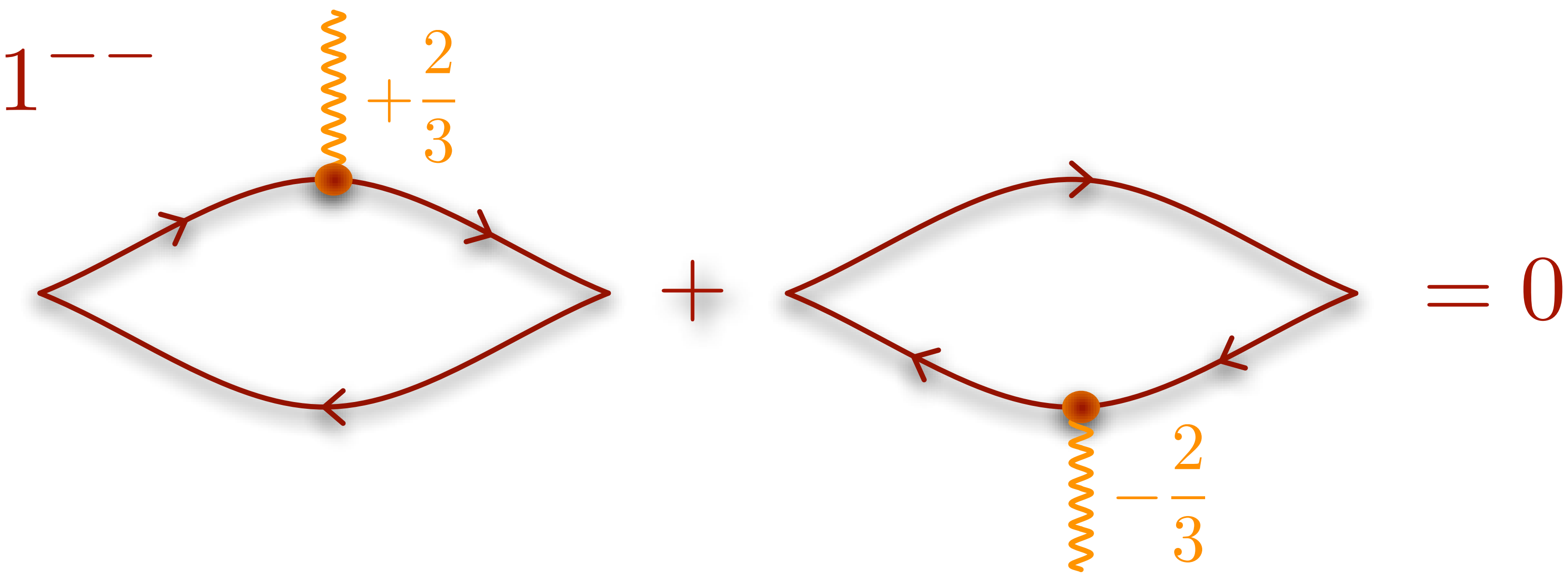
obtained from fits to two-point functions ($\frac{Z_i Z_f}{4E_i E_f}$)

to be extracted ($\langle f(\vec{p}_f) | j^\mu(0) | i(\vec{p}_i) \rangle$)

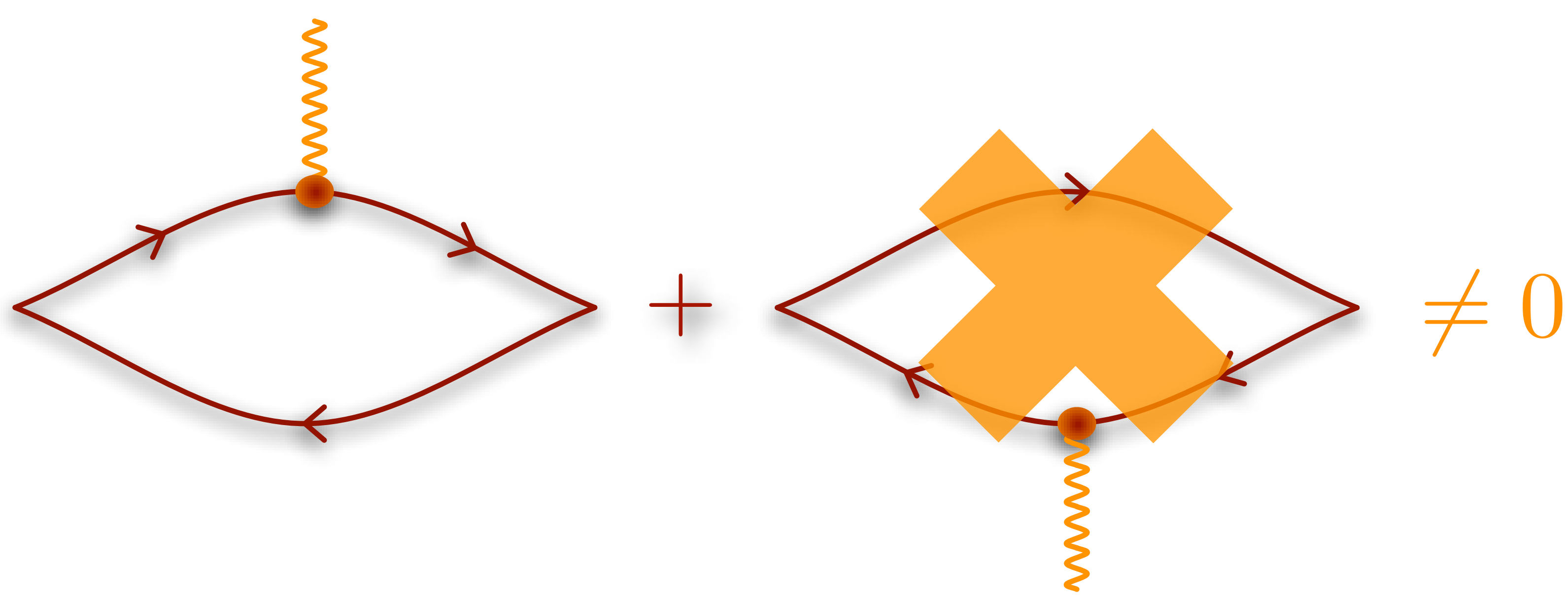
η_c ‘form-factor’

- strictly speaking this does not exist due to charge conjugation invariance

$0^{-+} \not\to 0^{-+}1^{--}$



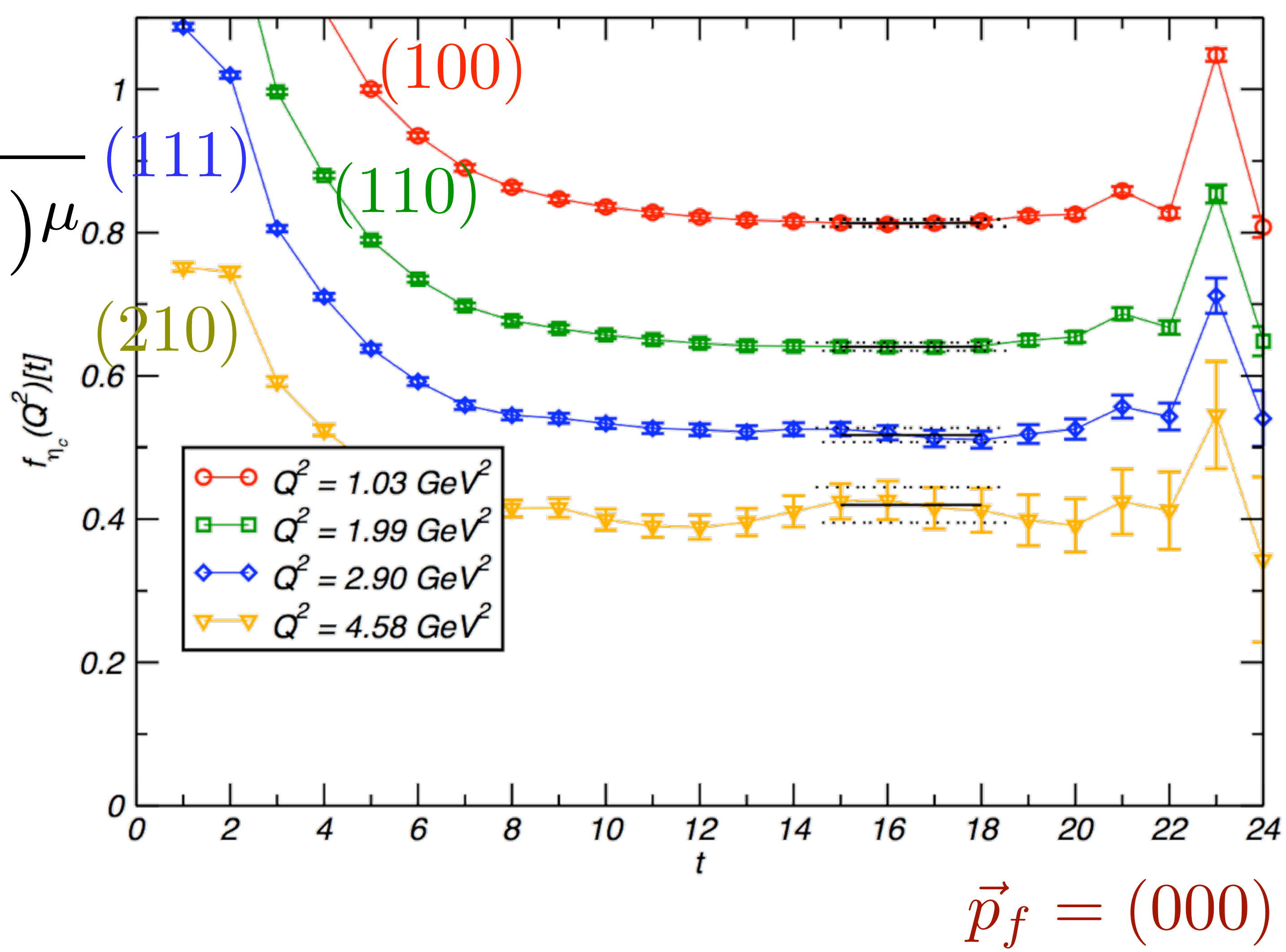
$0^{-} \to 0^{-}1^{--}$



η_c 'form-factor'

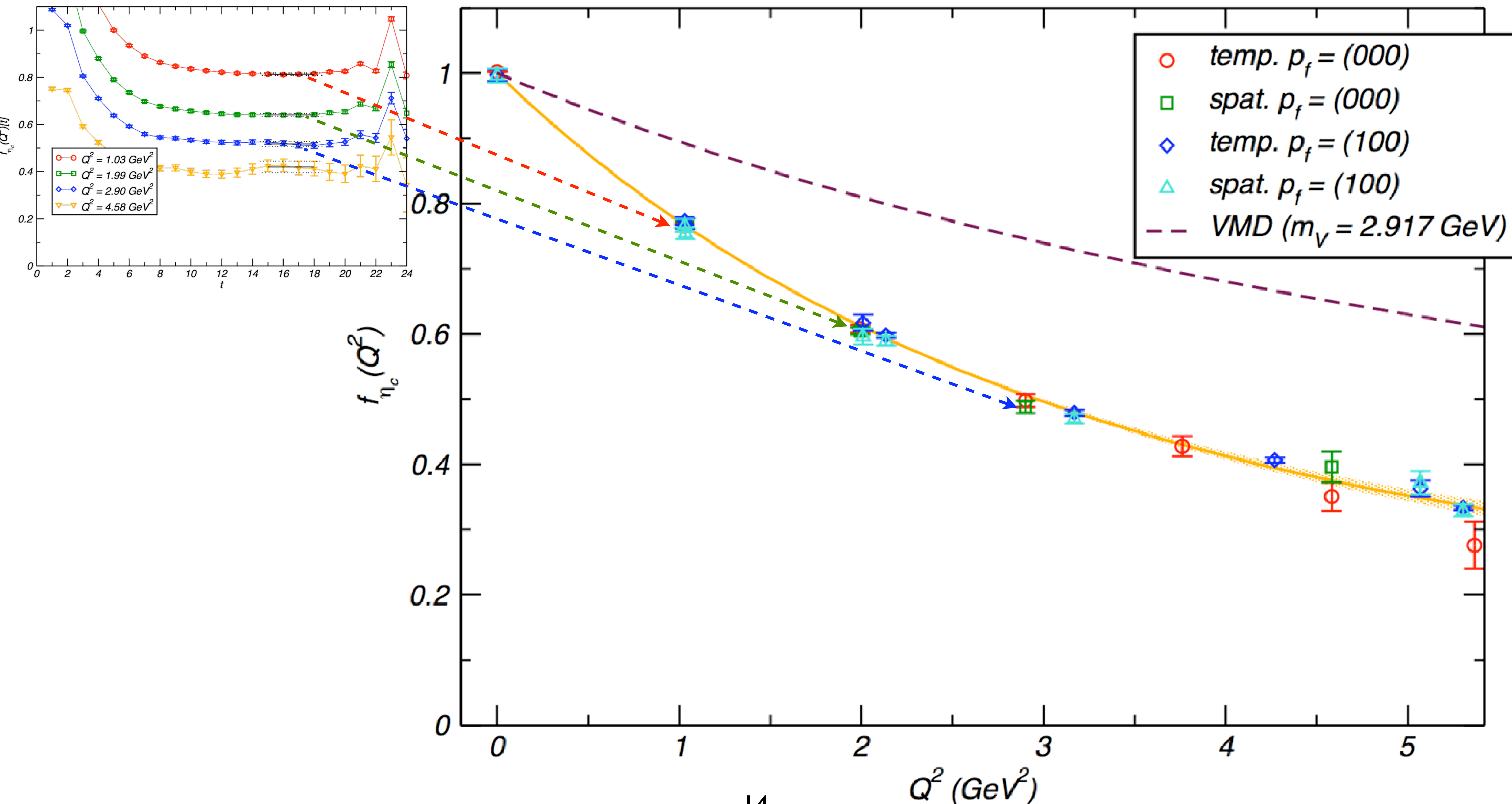
$$\langle \eta_c(\vec{p}_f) | j^\mu(0) | \eta_c(\vec{p}_i) \rangle = f(Q^2)(p_i + p_f)^\mu$$

$$\frac{\Gamma(t_f = 24, t)}{\frac{Z_i Z_f}{4E_i E_f} e^{-E_f(t_f - t) - E_i t}(p_f + p_i)^\mu}$$

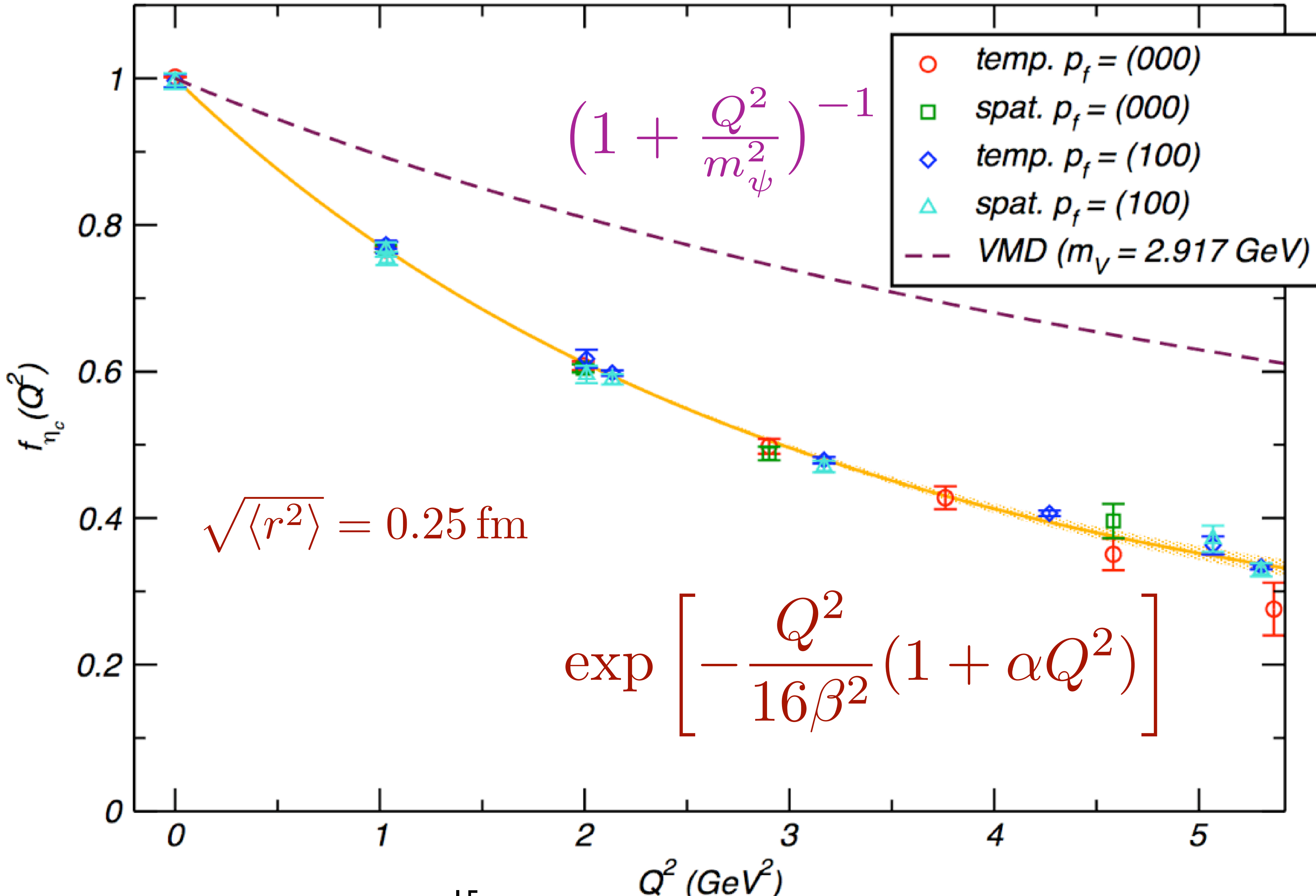


plateaux observed

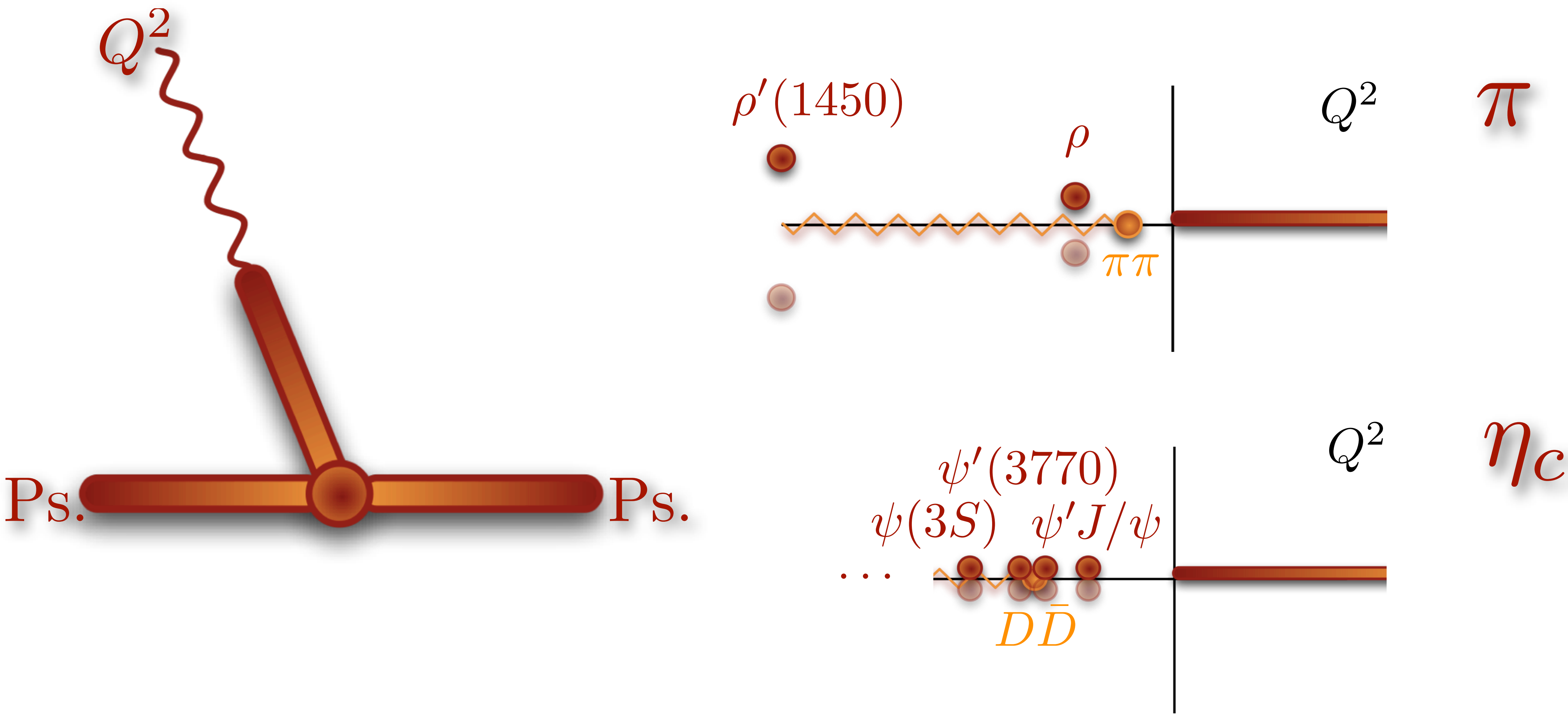
η_c 'form-factor'



η_c 'form-factor'



η_c 'form-factor' - not VMD?



J/ψ 'form-factors'

- vector particle has three form-factors (c.f. deuteron)
  - charge
  - magnetic
  - quadrupole

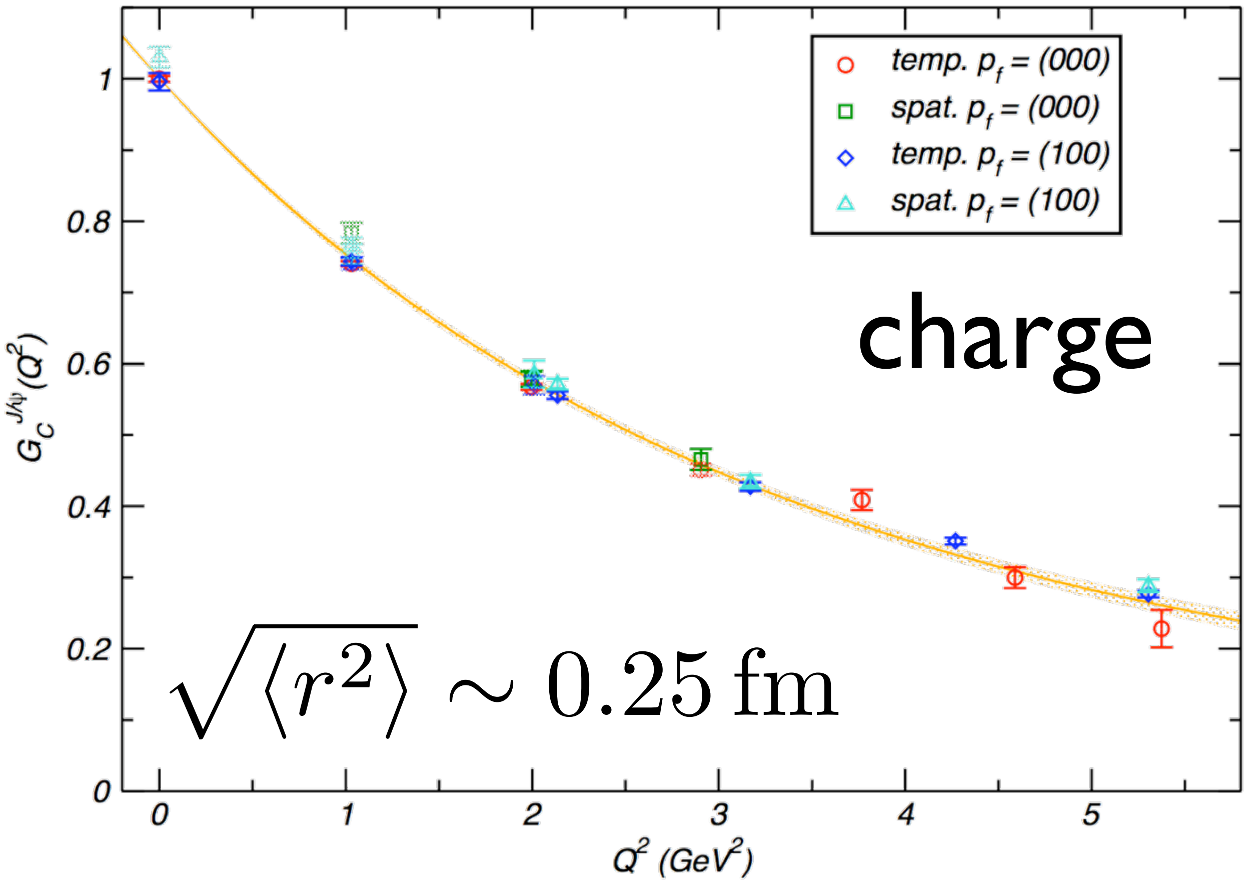
$$\langle V(\vec{p}_f, r_f)|j^\mu(0)|V(\vec{p}_i, r_i)\rangle$$

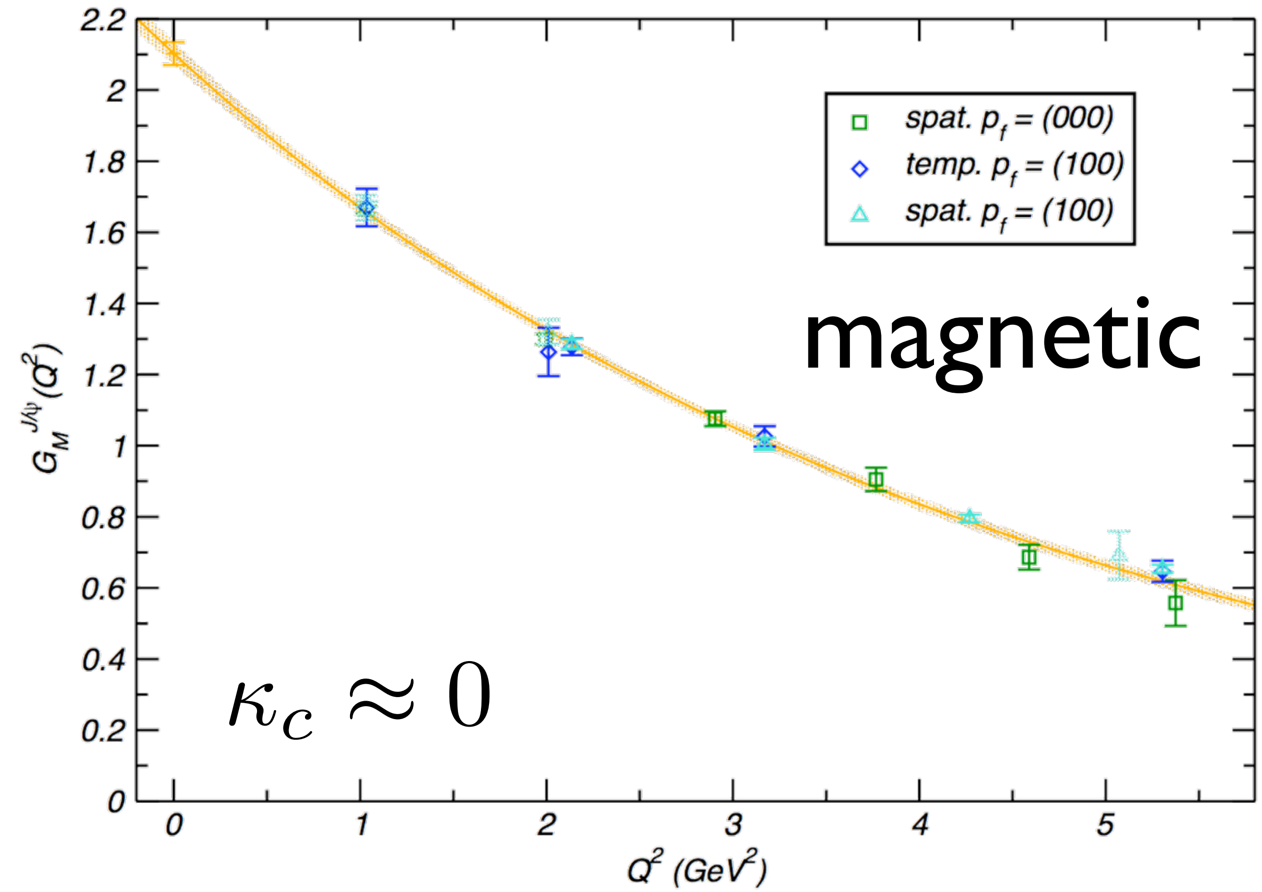
$$= -(p_f + p_i)^\mu \Big[G_1(Q^2) \epsilon^*(\vec{p}_f, r_f) \cdot \epsilon(\vec{p}_i, r_i)$$

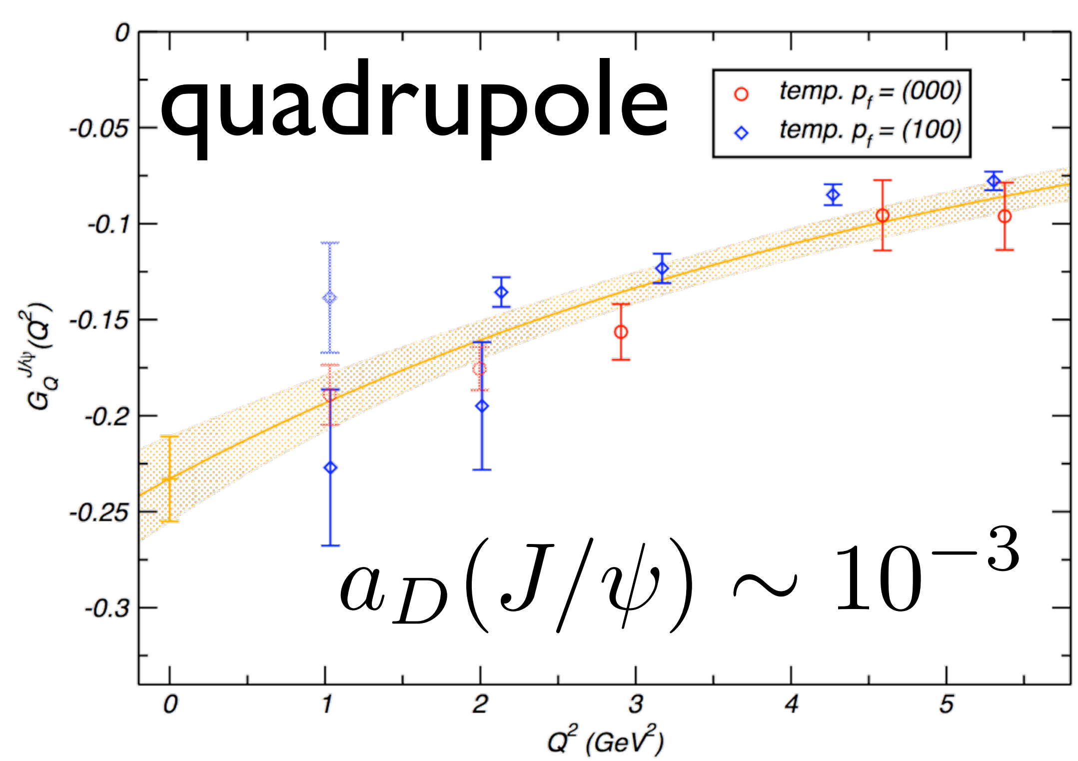
$$+ \frac{G_3(Q^2)}{2m_V^2} \epsilon^*(\vec{p}_f, r_i).p_i \, \epsilon(\vec{p}_i, r_i).p_f \Big]$$

$$+ G_2(Q^2) \Big[\epsilon^\mu(\vec{p}_i, r_i) \epsilon^*(\vec{p}_f, r_f).p_i + \epsilon^{\mu *}(\vec{p}_f, r_f) \epsilon(\vec{p}_i, r_i).p_f \Big]$$

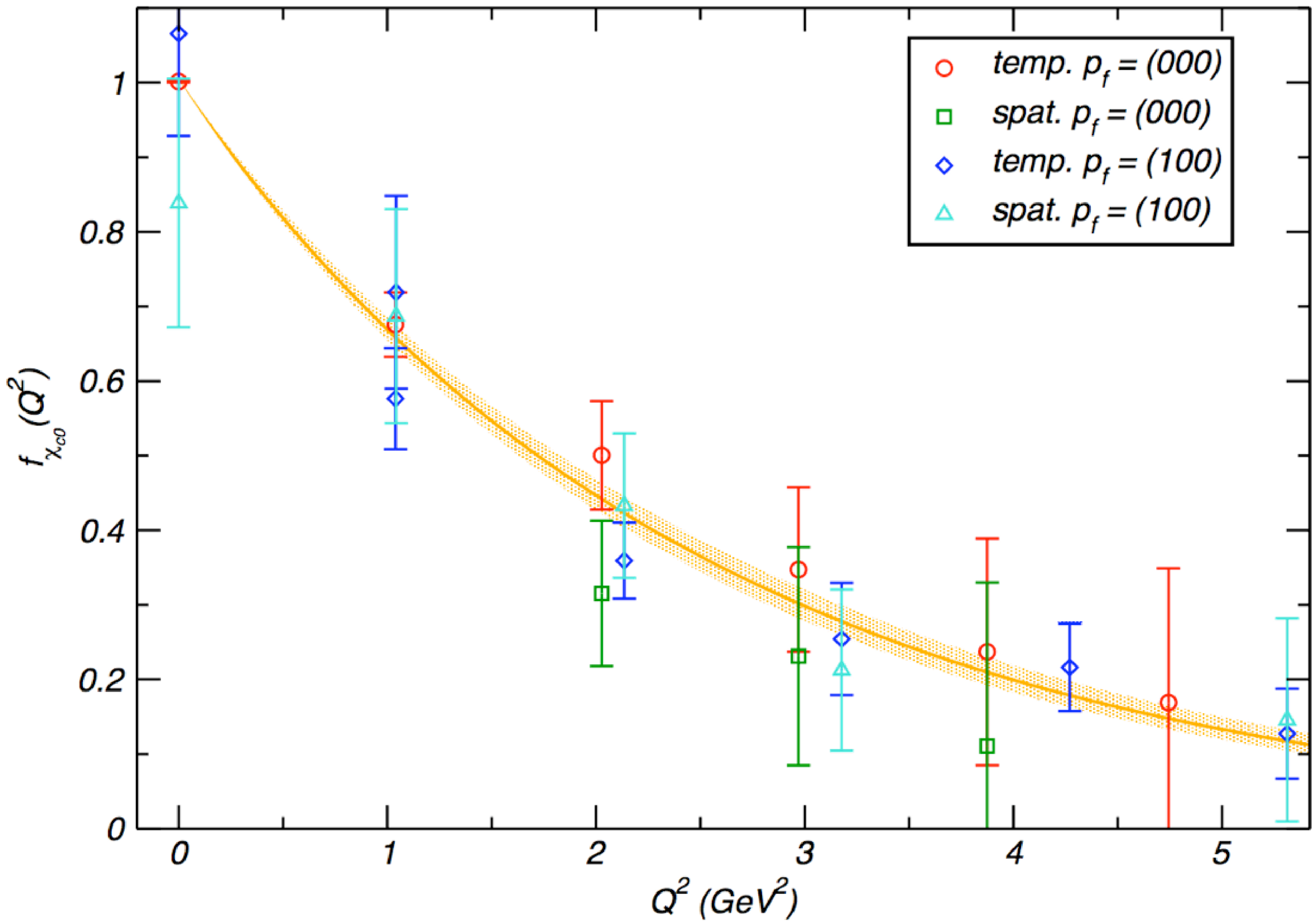
J/ψ ‘form-factors’







χ_{c0} 'form-factors'

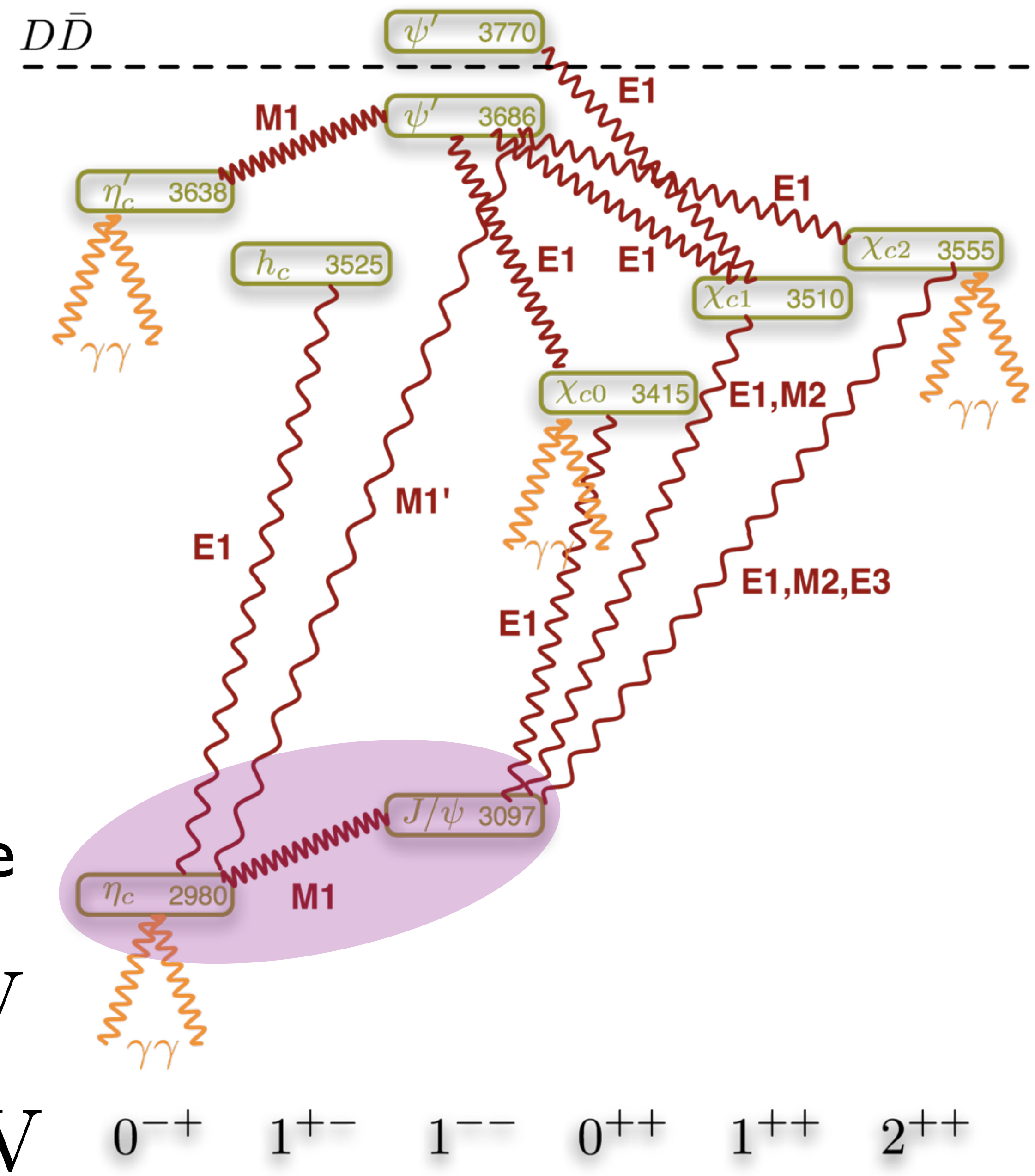


$$\sqrt{\langle r^2 \rangle} \sim 0.3\,\mathrm{fm}$$

larger radius due to centripetal barrier in P-wave meson so much for *un*observables, how about observables?

$J/\psi \to \eta_c \gamma$ transition

$$\langle \eta_c(\vec{p}\,')|j^\mu(0)|J/\psi(\vec{p},r)\rangle = \frac{2V(Q^2)}{m_{\eta_c}+m_\psi}\epsilon^{\mu\alpha\beta\gamma}p'_\alpha p_\beta \epsilon_\gamma(\vec{p},r)$$



very sensitive to hyperfine

$$m_{J/\psi} - m_{\eta_c}|_{\rm expt.} = 117\,{\rm MeV}$$

$$m_{J/\psi} - m_{\eta_c}|_{\rm our\,lat.} \sim 80\,{\rm MeV}$$

$J/\psi \to \eta_c \gamma$ transition

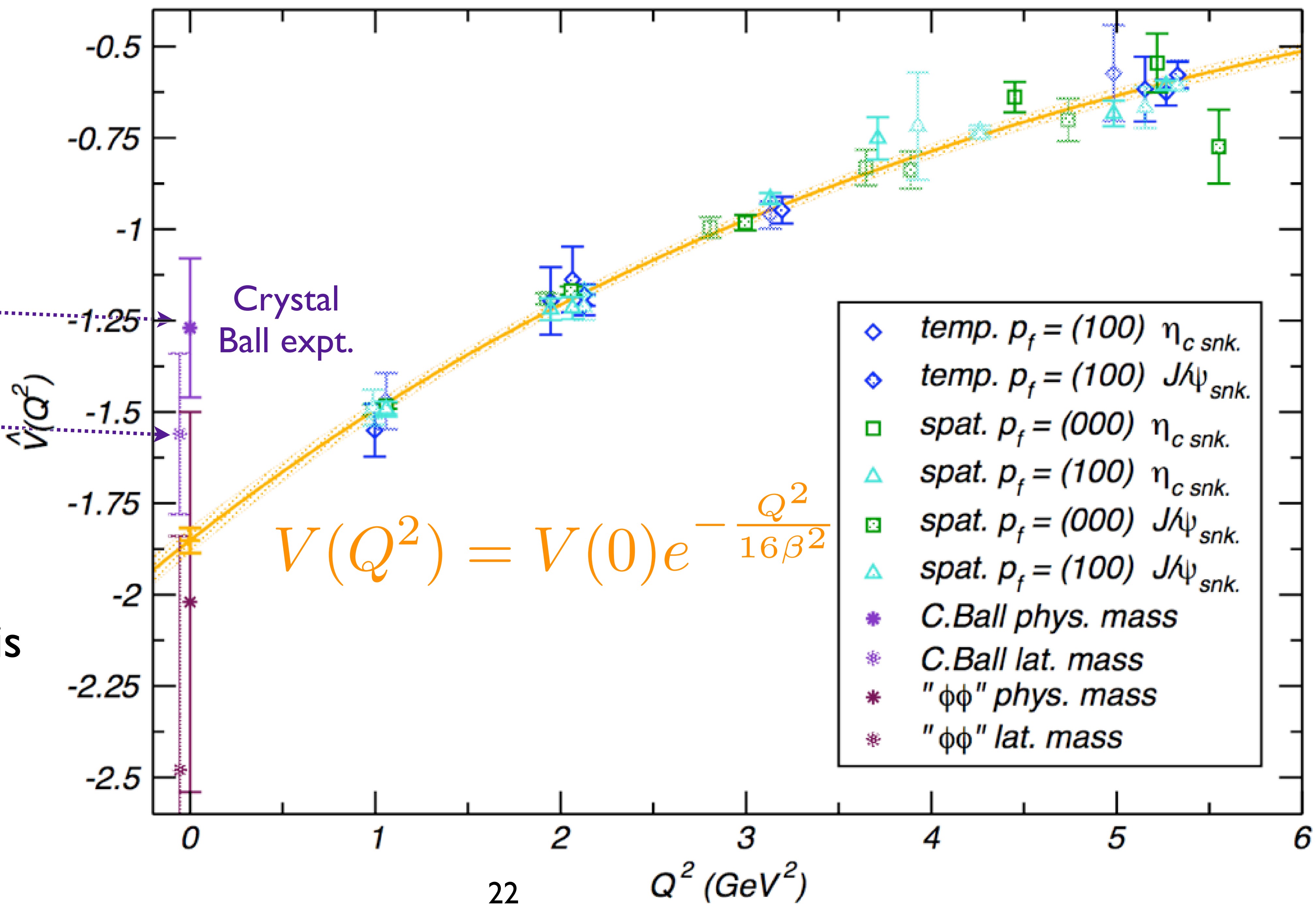
$$\Gamma(J/\psi \to \eta_c\gamma) = \alpha \frac{|\vec{q}|^3}{(m_{\eta_c}+m_\psi)^2}\frac{64}{27}\left|\hat{V}(0)\right|^2.$$

phase space

physical

lattice

Crystal Ball expt.

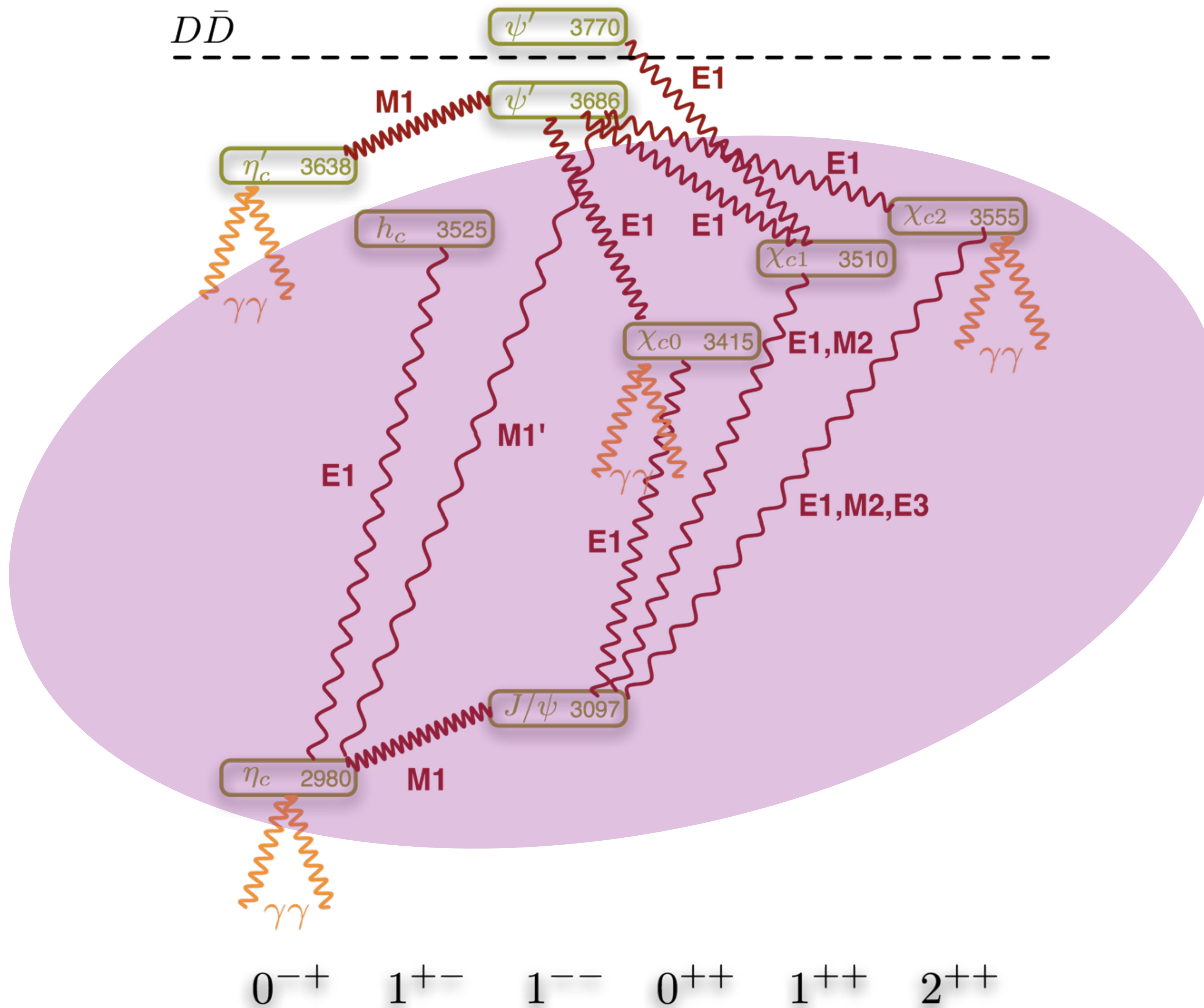


beware -
nobody gets this
number 'right'

new CLEO-c
number soon!

P-wave to S-wave transitions

many good measurements



$\chi_{c0} \to J/\psi\gamma$ transition

our 'poster boy'

covariant multipole decomposition of matrix element

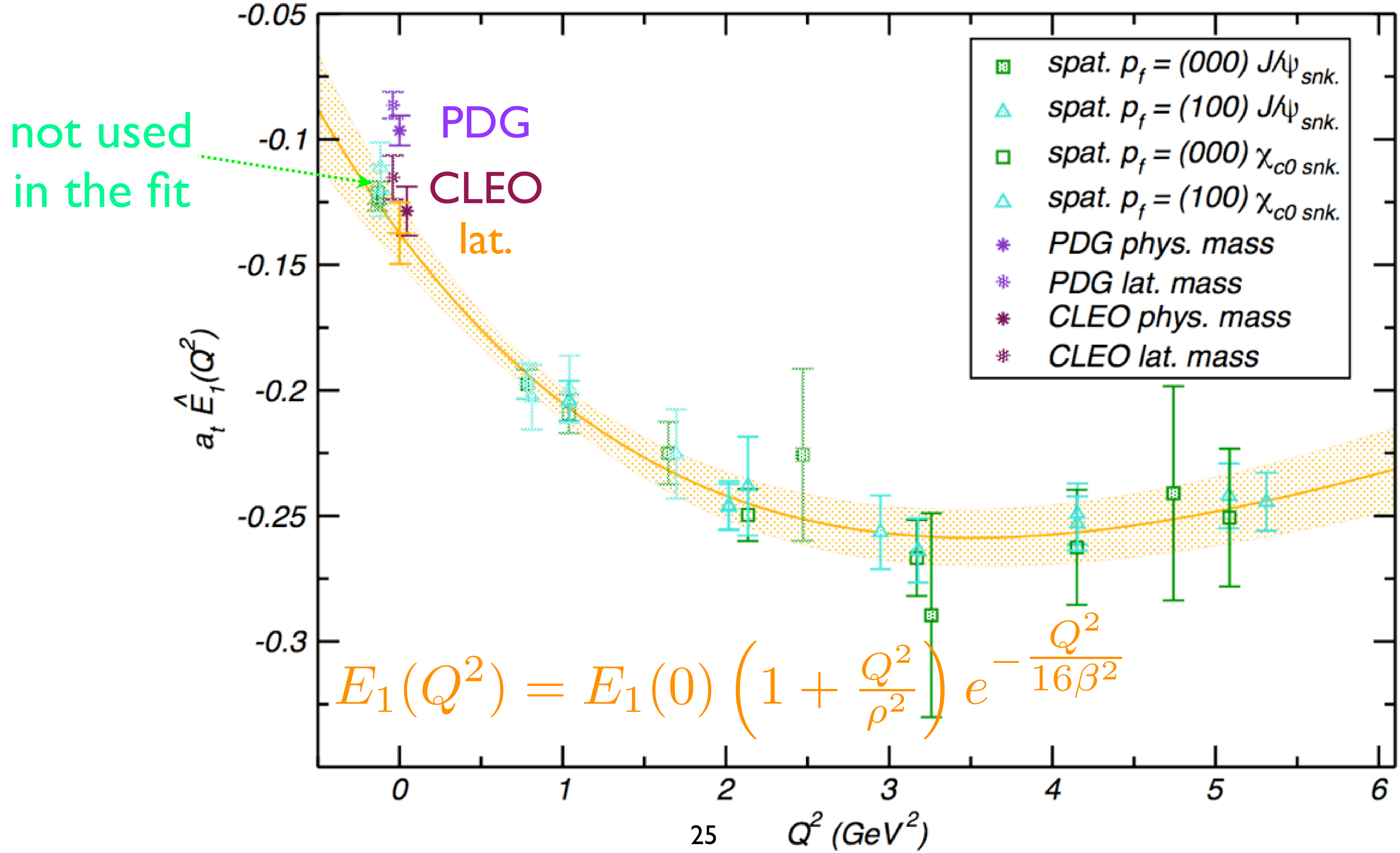
$$\langle S(\vec{p}_S)|j^\mu(0)|V(\vec{p}_V,r)\rangle =$$
$$\Omega^{-1}(Q^2)\Bigg(E_1(Q^2)\Big[\Omega(Q^2)\epsilon^\mu(\vec{p}_V,r) - \epsilon(\vec{p}_V,r).p_S\big(p_V^\mu p_V.p_S - m_V^2 p_S^\mu\big)\Big]$$
$$+\frac{C_1(Q^2)}{\sqrt{q^2}} m_V \epsilon(\vec{p}_V,r).p_S\Big[p_V.p_S(p_V+p_S)^\mu - m_S^2 p_V^\mu - m_V^2 p_S^\mu\Big]\Bigg).$$

E_1 - electric dipole, expt$^{\underline{\text{ally}}}$ measured at $Q^2=0$

C_1 - longitudinal, only non-zero at non-zero Q^2

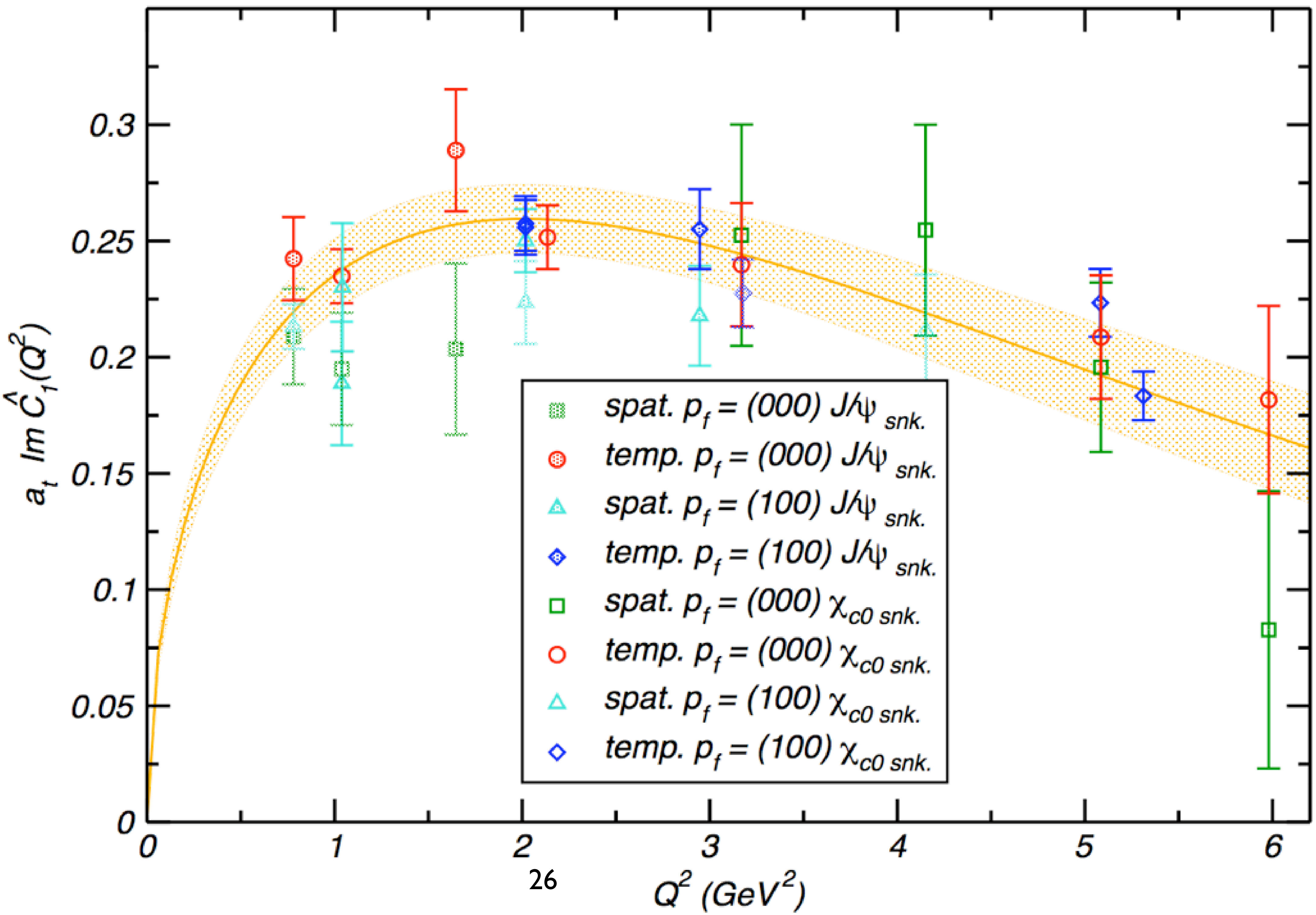
$\chi_{c0} \to J/\psi\gamma$ *E1* transition

our 'poster boy'



$$E_1(Q^2) = E_1(0)\left(1 + \frac{Q^2}{\rho^2}\right) e^{-\frac{Q^2}{16\beta^2}}$$

$\chi_{c0} \to J/\psi\gamma$ C1 transition



$\chi_{c1} \to J/\psi\gamma$ transition

covariant multipole decomposition of matrix element

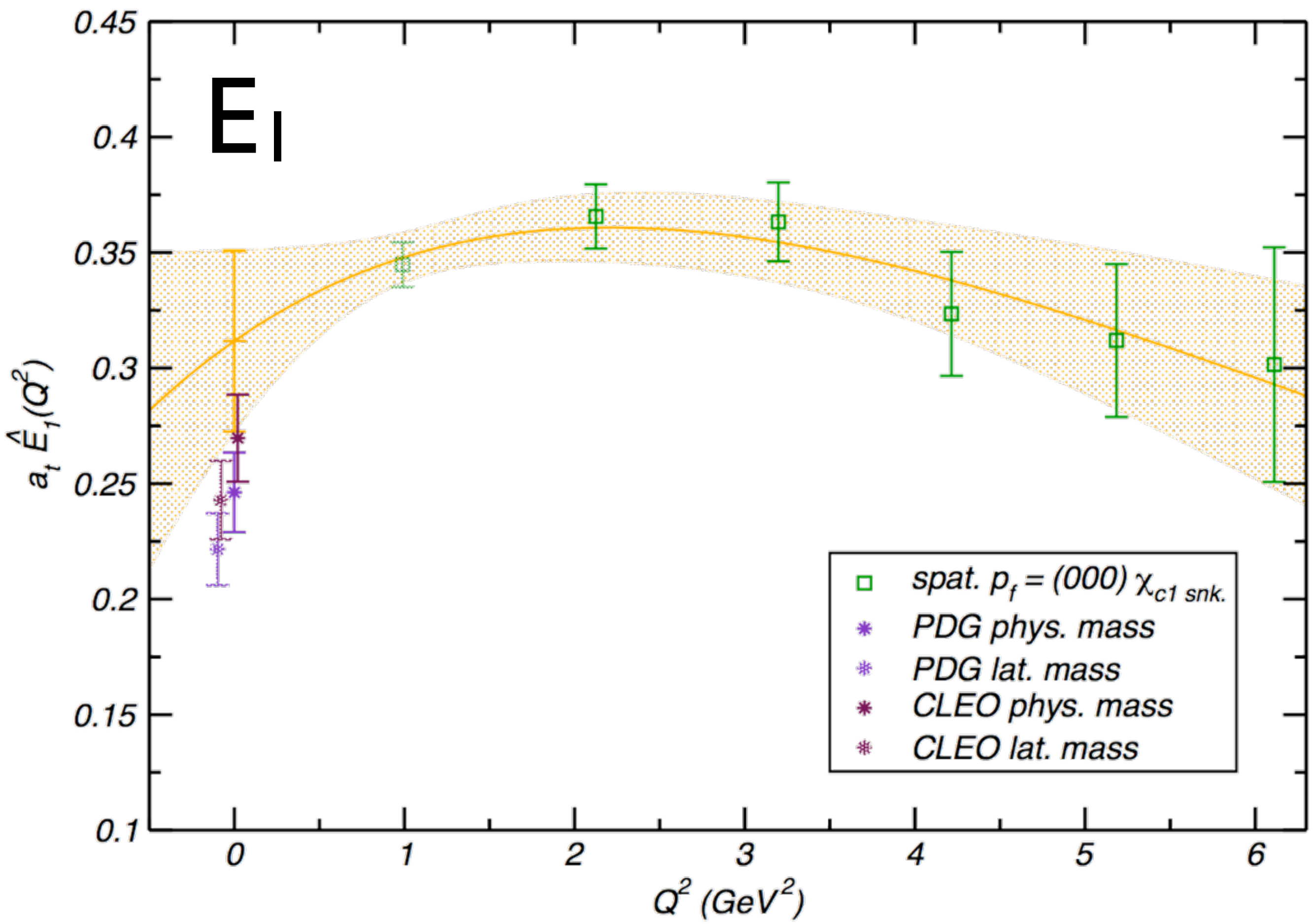
$$\langle A(\vec{p}_A, r_A)|j^\mu(0)|V(\vec{p}_V, r_V)\rangle = \frac{i}{4\sqrt{2}\Omega(Q^2)}\epsilon^{\mu\nu\rho\sigma}(p_A - p_V)_\sigma \times$$
$$\times \Bigg[E_1(Q^2)(p_A + p_V)_\rho\Big(2m_A[\epsilon^*(\vec{p_A}, r_A).p_V]\epsilon_\nu(\vec{p_V}, r_V) + 2m_V[\epsilon(\vec{p}_V, r_V).p_A]\epsilon^*_\nu(\vec{p}_A, r_A)\Big)$$
$$+ M_2(Q^2)(p_A + p_V)_\rho\Big(2m_A[\epsilon^*(\vec{p_A}, r_A).p_V]\epsilon_\nu(\vec{p_V}, r_V) - 2m_V[\epsilon(\vec{p}_V, r_V).p_A]\epsilon^*_\nu(\vec{p}_A, r_A)\Big)$$
$$+ \frac{C1(Q^2)}{\sqrt{q^2}}\Big(- 4\Omega(Q^2)\epsilon^*_\nu(\vec{p}_A, r_A)\epsilon_\rho(\vec{p}_V, r_V)$$
$$+ (p_A + p_V)_\rho\Big[(m_A^2 - m_V^2 + q^2)[\epsilon^*(\vec{p}_A, r_A).p_V]\;\epsilon_\nu(\vec{p}_V, r_V) + (m_A^2 - m_V^2 - q^2)[\epsilon(\vec{p}_V, r_V).p_A]\;\epsilon^*_\nu(\vec{p_A}, r_A)\Big]\Big)\Bigg]$$

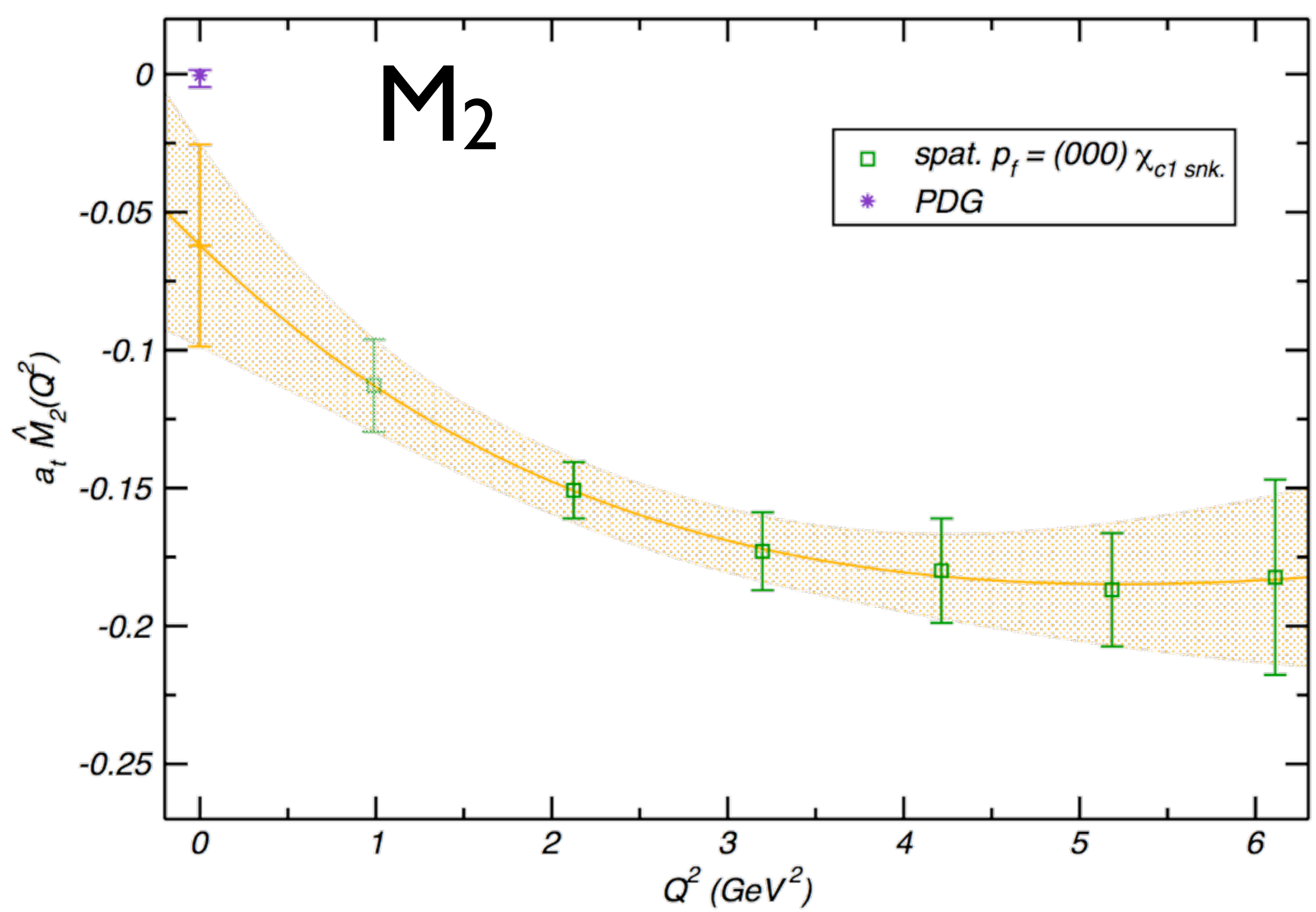
E_1 - electric dipole, expt<u>ally</u> measured at $Q^2 = 0$

M_2 - magnetic quadrupole, expt<u>ally</u> measured at $Q^2 = 0$

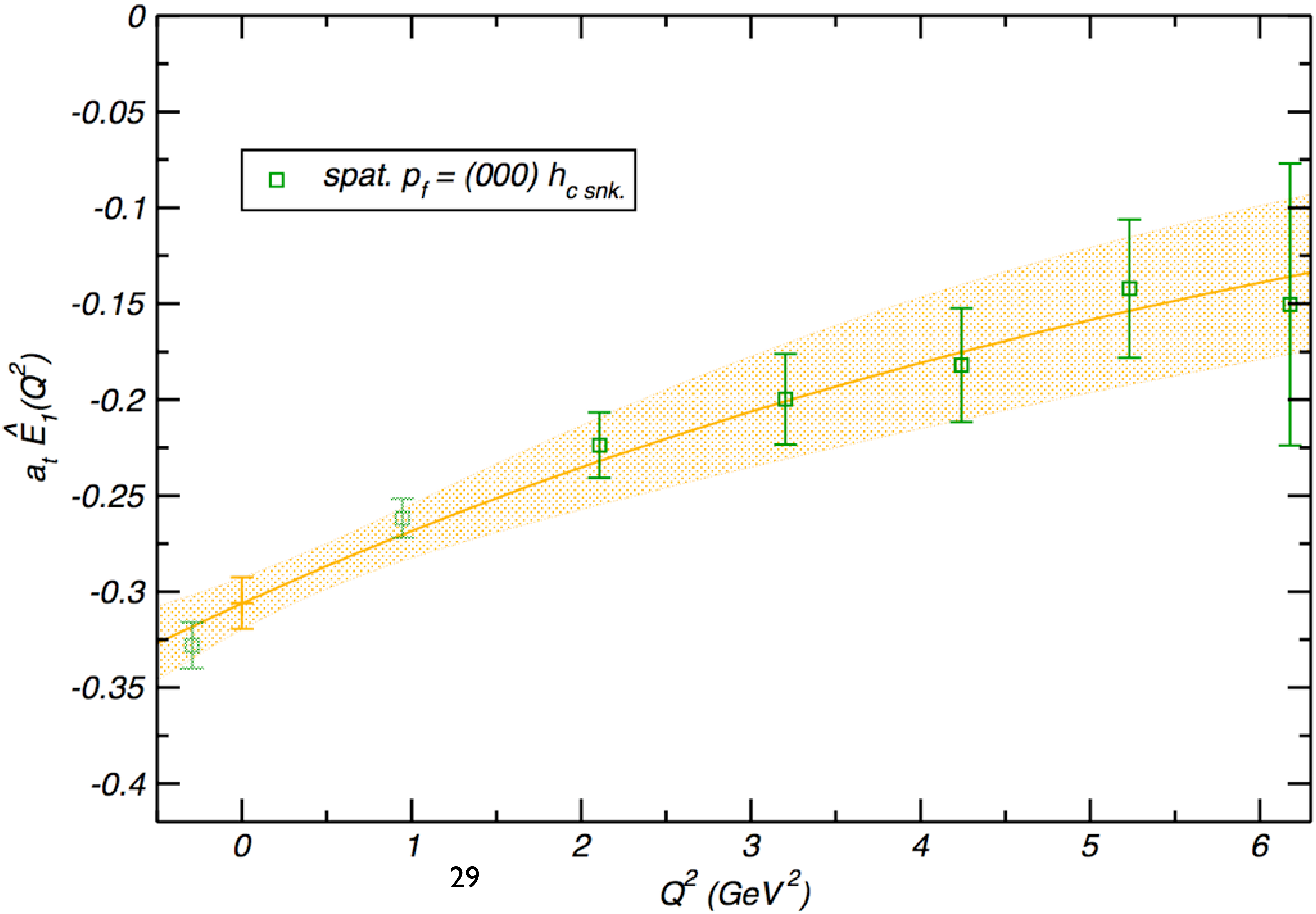
C_1 - longitudinal, only at non-zero Q^2

$\chi_{c1} \to J/\psi\gamma$ transition





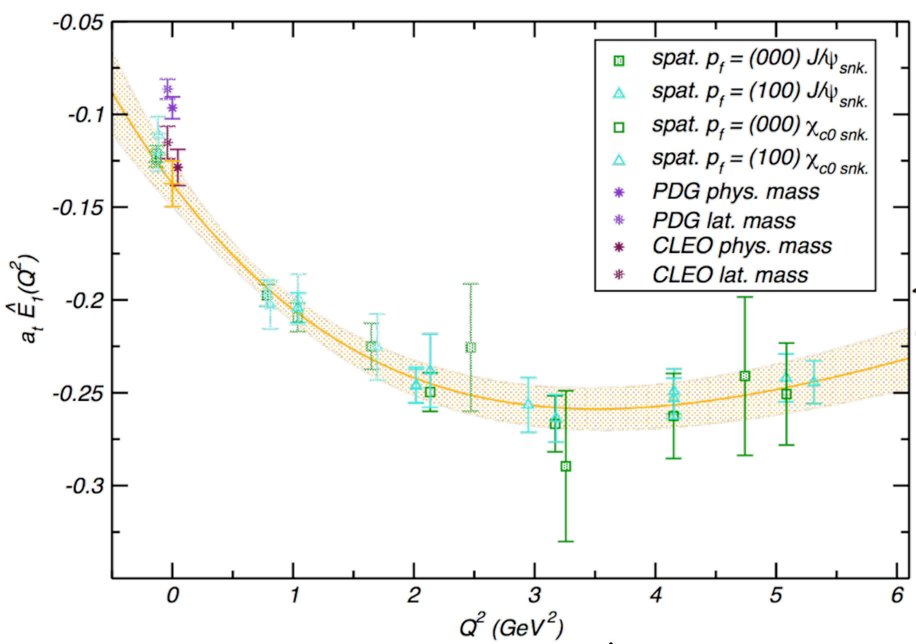
$h_c \rightarrow \eta_c \gamma$ transition



quark potential model?

- our fitting form inspired by NR potential model with rel. corrections:

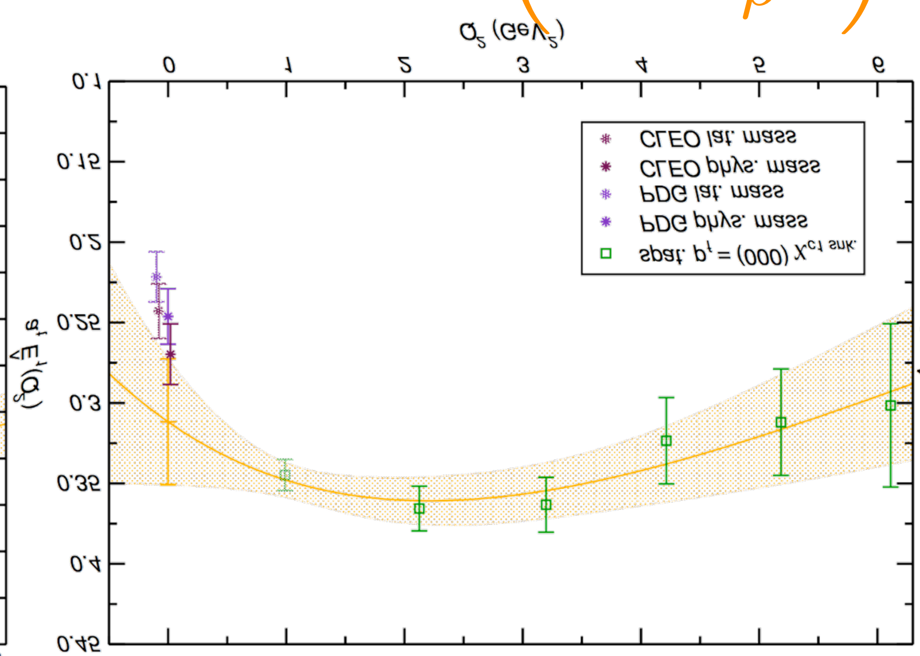
$$E_1(Q^2) = E_1(0)\left(1+\frac{Q^2}{\rho^2}\right)e^{-\frac{Q^2}{16\beta^2}}$$



$\chi_{c0} \to J/\psi\gamma_{E1}$

$\beta = 542(35)$ MeV

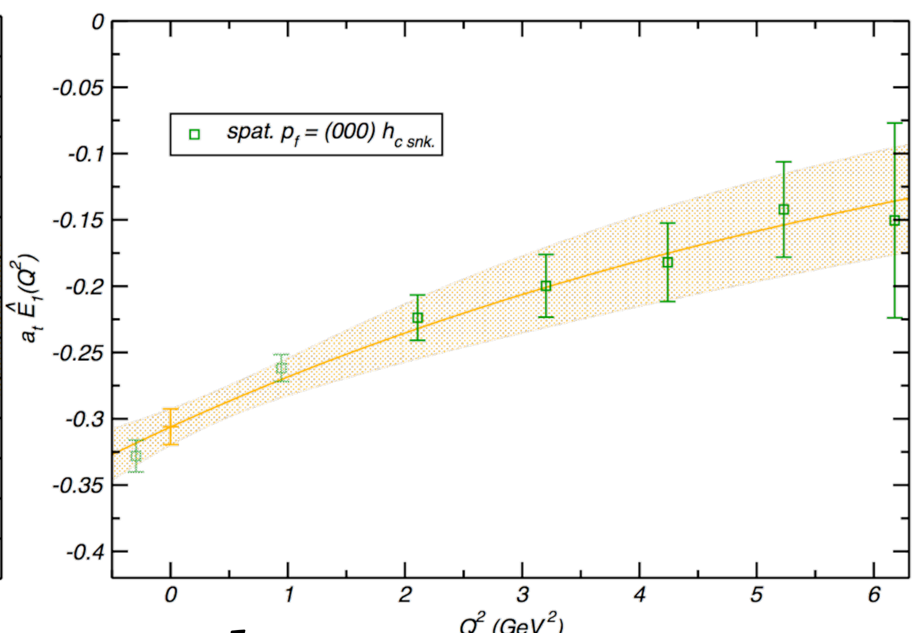
$\rho = 1.08(13)$ GeV



$\chi_{c1} \to J/\psi\gamma_{E1}$

$\beta = 555(113)$ MeV

$\rho = 1.65(59)$ GeV



$h_c \to \eta_c\gamma_{E1}$

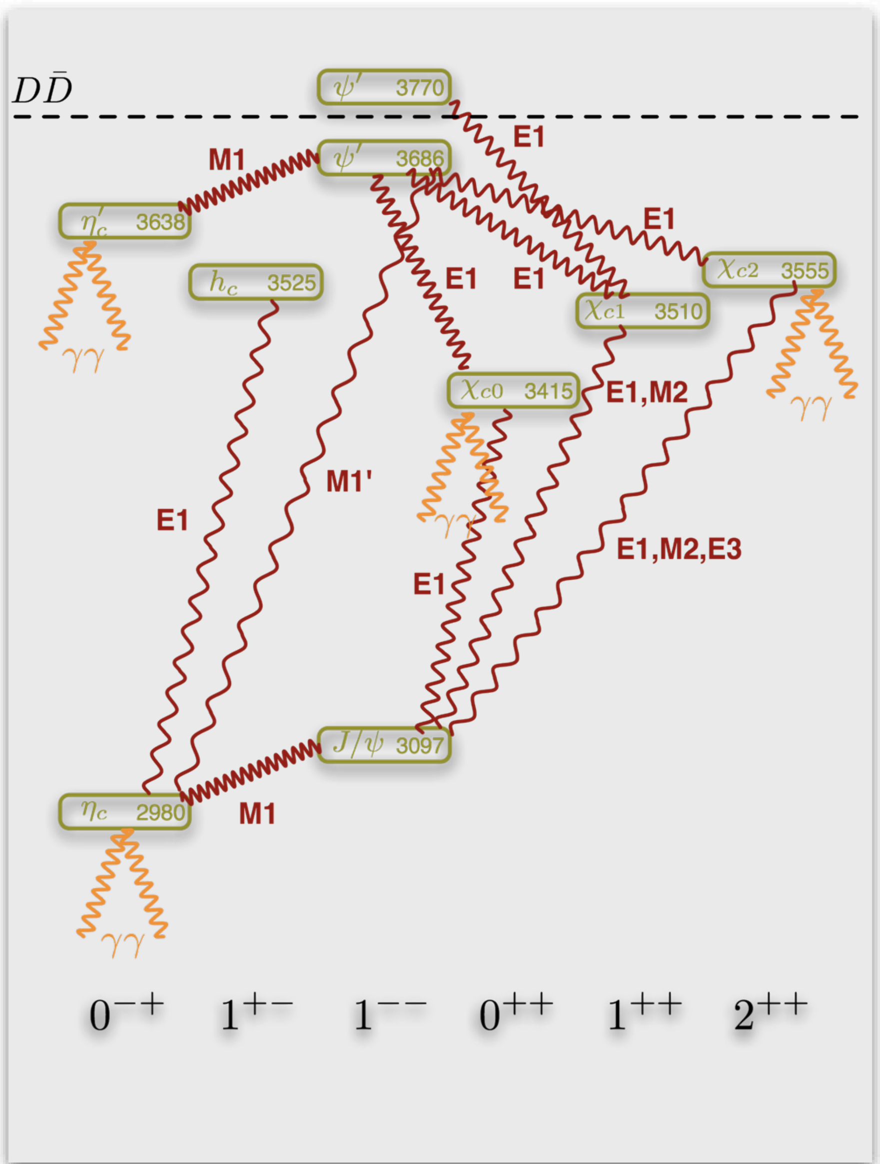
$\beta = 689(133)$ MeV

$\rho \to \infty$

what about $\chi_{c2} \to J/\psi\gamma$?

- can't get at spin 2 with point-like fermion bilinears

- we have to extend our interpolating field set



extended interpolators

Operator	O_h rep.	lowest J^{PC}	name	remark
1	A_1	0^{++}	a_0	${}^3P_0(\chi_{c0})$
γ_5	A_1	0^{-+}	π	${}^1S_0(\eta_c)$
γ_i	T_1	1^{--}	ρ	${}^3S_1(J/\psi)$
$\gamma_5\gamma_i$	T_1	1^{++}	a_1	${}^3P_1(\chi_{c1})$
$\gamma_i\gamma_j$	T_1	1^{+-}	b_1	${}^1P_1(h_c)$
$\gamma_5\nabla_i$	T_1	1^{+-}	$\pi\times\nabla$	
∇_i	T_1	1^{--}	$a_0\times\nabla$	
$\gamma_4\nabla_i$	T_1	1^{-+}	$a_0'\times\nabla$	
$\gamma_i\nabla_i$	A_1	0^{++}	$\rho\times\nabla\_A_1$	${}^3P_0(\chi_{c0})$
$\epsilon_{ijk}\gamma_j\nabla_k$	E	1^{++}	$\rho\times\nabla\_T_1$	${}^3P_1(\chi_{c1})$
$s_{ijk}\gamma_j\nabla_k$	T_2	2^{++}	$\rho\times\nabla\_T_2$	${}^3P_2(\chi_{c2})$
$\gamma_5\gamma_i\nabla_i$	A_1	0^{--}	$a_1\times\nabla\_A_1$	exotic
$\gamma_5 s_{ijk}\gamma_j\nabla_k$	T_2	2^{--}	$a_1\times\nabla\_T_2$	
$\gamma_5 S_{\alpha jk}\gamma_j\nabla_k$	T_2	2^{--}	$a_1\times\nabla\_E$	
$\gamma_4\gamma_5\epsilon_{ijk}\gamma_j\nabla_k$	T_1	1^{-+}	$b_1\times\nabla\_T_1$	exotic
$\gamma_4 s_{ijk}\nabla_j\nabla_k$	T_2	2^{+-}	$a_0'\times D$	exotic
$\gamma_5\gamma_i D_i$	A_2	3^{++}	$a_1\times D\_A_2$	
$\gamma_5 S_{\alpha jk}\gamma_j D_k$	E	2^{++}	$a_1\times D\_E$	
$\gamma_5 s_{ijk}\gamma_j D_k$	T_1	1^{++}	$a_1\times D\_T_1$	
$\gamma_5\epsilon_{ijk}\gamma_j D_k$	T_2	2^{++}	$a_1\times D\_T_2$	
$\gamma_4\gamma_5 s_{ijk}\gamma_i\nabla_j\nabla_k$	A_2	3^{+-}	$b_1\times D\_A_2$	
$\gamma_4\gamma_5 S_{\alpha jk}\gamma_j D_k$	E	2^{+-}	$b_1\times D\_E$	
$\gamma_4\gamma_5 s_{ijk}\gamma_j D_k$	T_1	1^{+-}	$b_1\times D\_T_1$	
$\gamma_4\gamma_5\epsilon_{ijk}\gamma_j D_k$	T_2	3^{+-}	$b_1\times D\_T_2$	
$\gamma_i D_i$	A_2	3^{--}	$\rho\times D\_A_2$	
$s_{ijk}\gamma_j D_k$	T_1	1^{--}	$\rho\times D\_T_1$	
$\epsilon_{ijk}\gamma_j D_k$	T_2	2^{--}	$\rho\times D\_T_2$	
$\gamma_4\gamma_5 s_{ijk}\nabla_j\nabla_k$	T_2	2^{-+}	$\pi\times D\_T_2$	
$\gamma_5 B_i$	T_1	1^{--}	$\pi\times B\_T_1$	
$\epsilon_{ijk}\gamma_j B_k$	T_1	1^{-+}	$\rho\times B\_T_1$	exotic
$s_{ijk}\gamma_j B_k$	T_2	2^{-+}	$\rho\times B\_T_2$	
$\gamma_5\gamma_i B_i$	A_1	0^{+-}	$a_1\times B\_A_1$	exotic
$\gamma_5\epsilon_{ijk}\gamma_j B_k$	T_1	1^{+-}	$a_1\times B\_T_1$	
$\gamma_5 s_{ijk}\gamma_j B_k$	T_2	2^{+-}	$a_1\times B\_T_2$	exotic

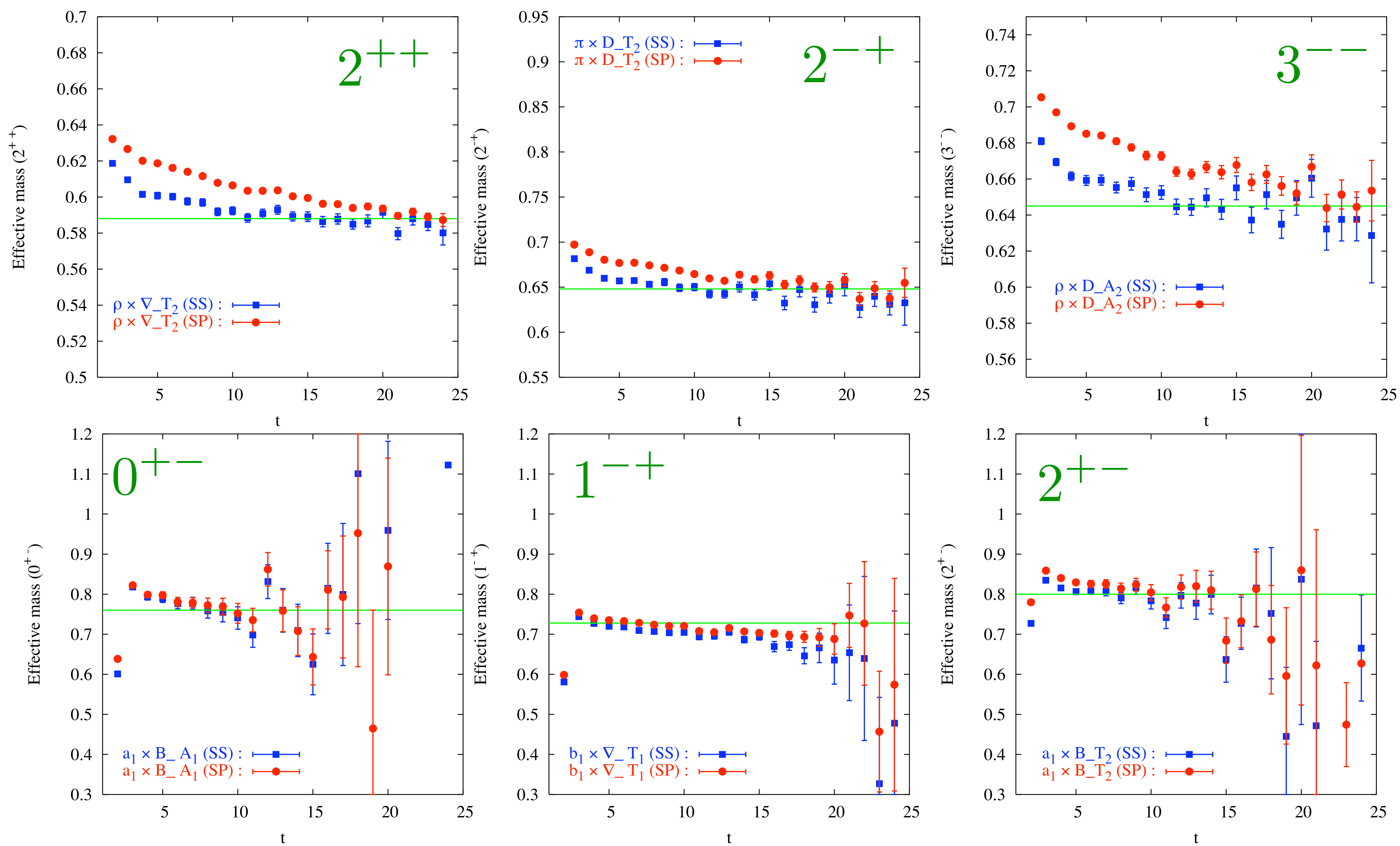
Table 1: Meson operators, names and quantum numbers. $s_{ijk} = |\epsilon_{ijk}|$ and $S_{\alpha jk} = 0(j \neq k), S_{111} = 1, S_{122} = -1, S_{222} = 1, S_{233} = -1. D_i = s_{ijk}\nabla_j\nabla_k, B_i = \epsilon_{ikj}\nabla_j\nabla_k$

higher spins
and the J^{PC} exotics

extended interpolators

non-exotics

exotics

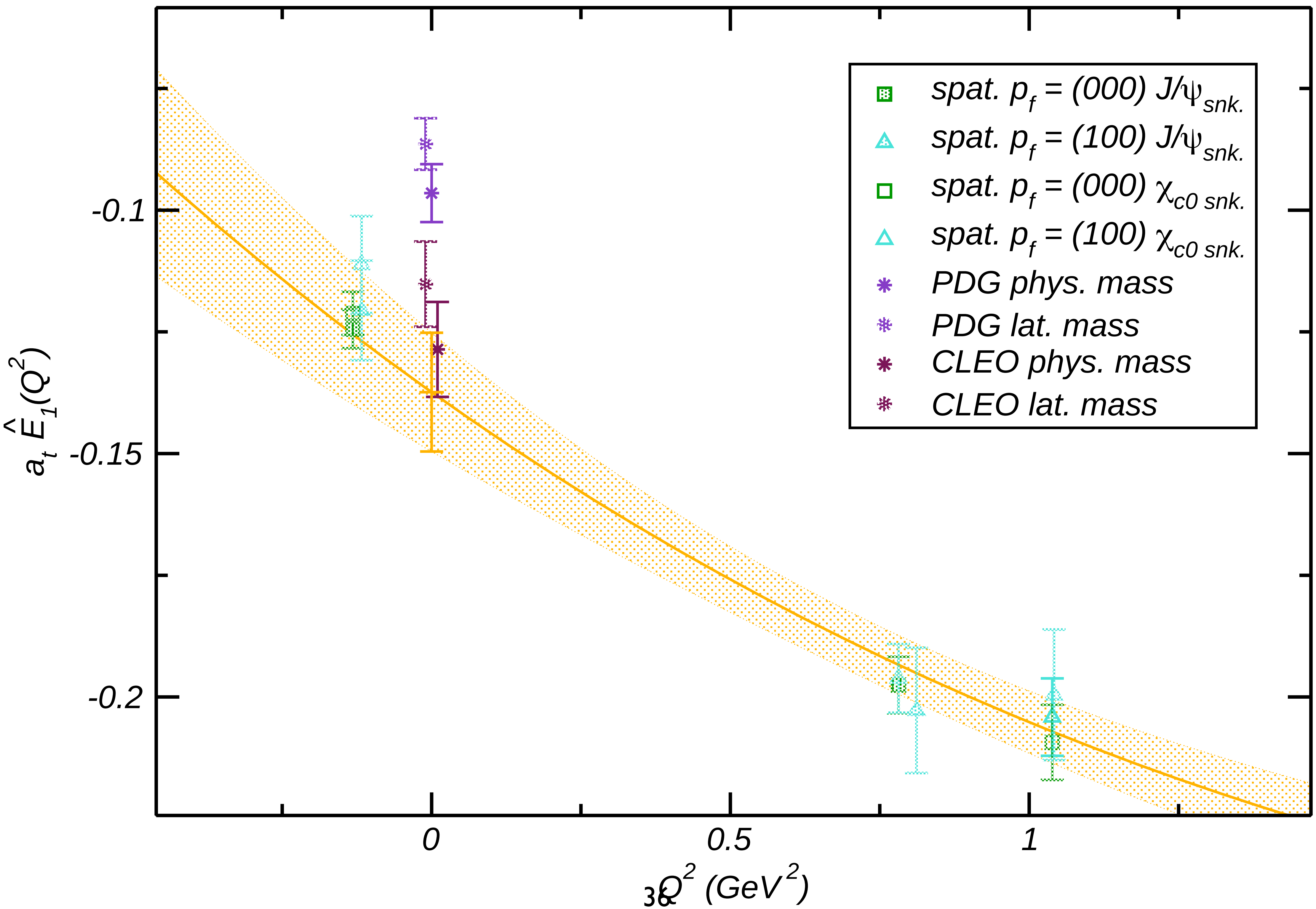


next up?

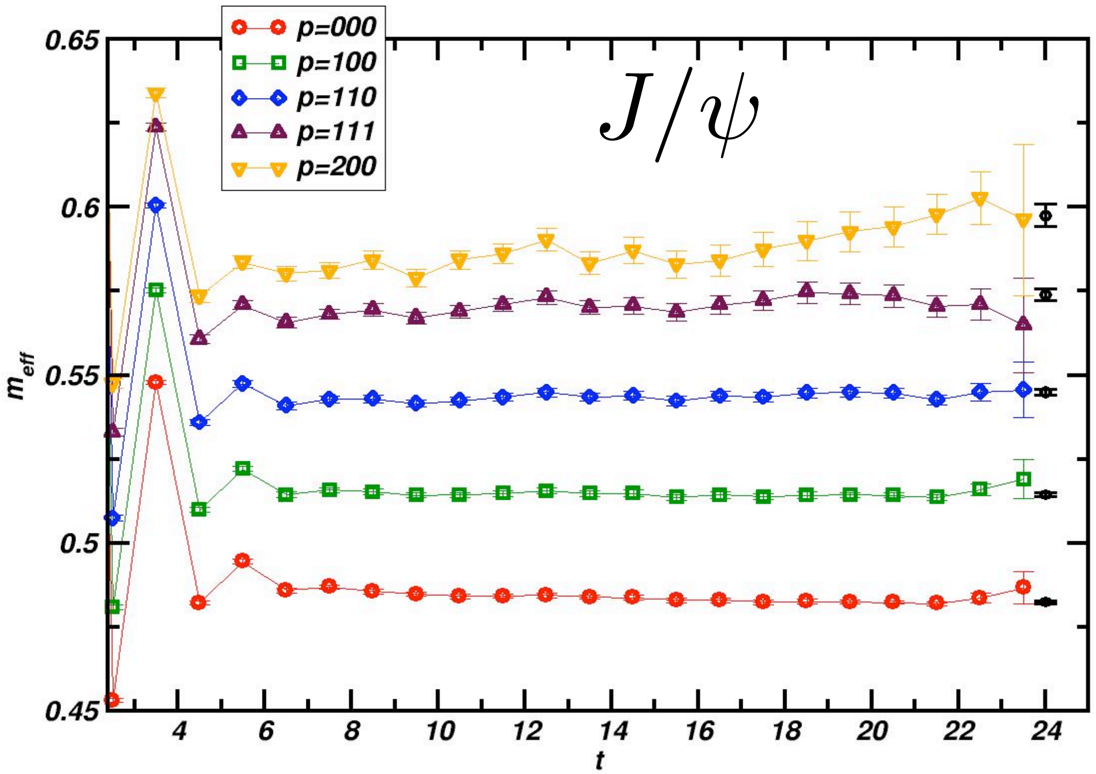
- radiative transitions with this extended set
- think we can do two-photon decays
- charmonium for now
- dynamical lattices for precision & maybe multi-particle (DD) states
- start turning down the quark mass if it all ‘works’ to get at JLab physics

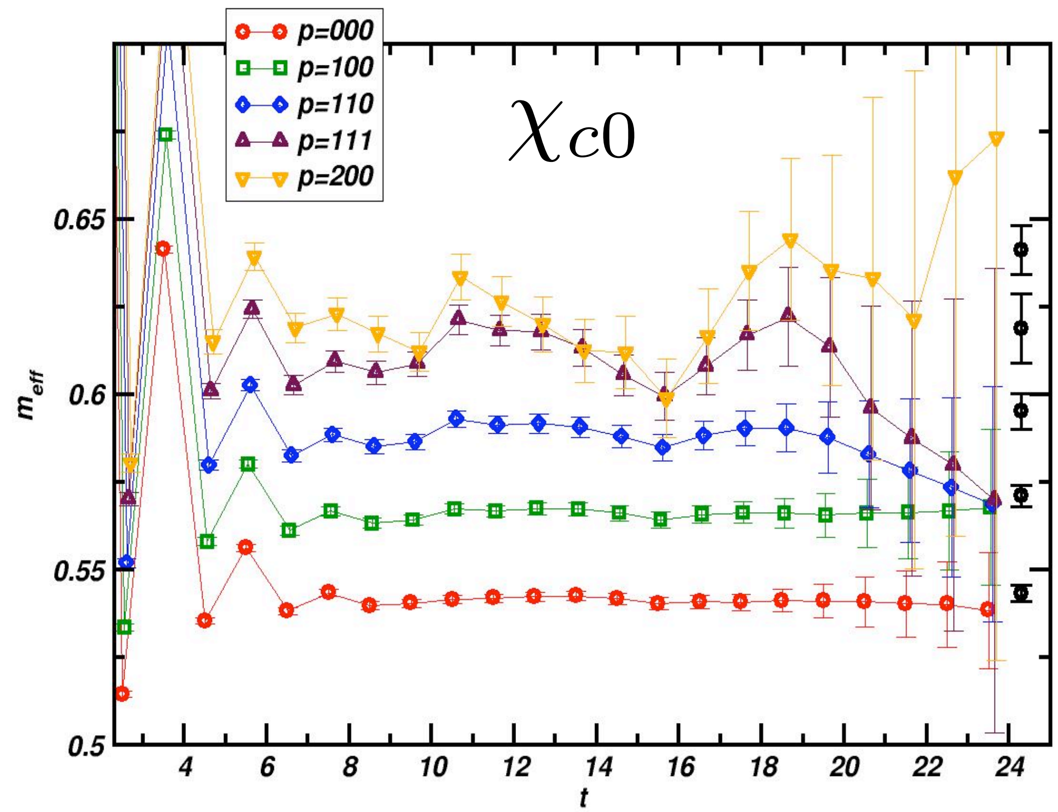
extra slides for the inquisitive

$\chi_{c0} \rightarrow J/\psi\gamma$ E1 transition



some two-point functions





multiple form-factors?

- pick out the three-point functions with the same Q² - various momentum and Lorentz index combinations

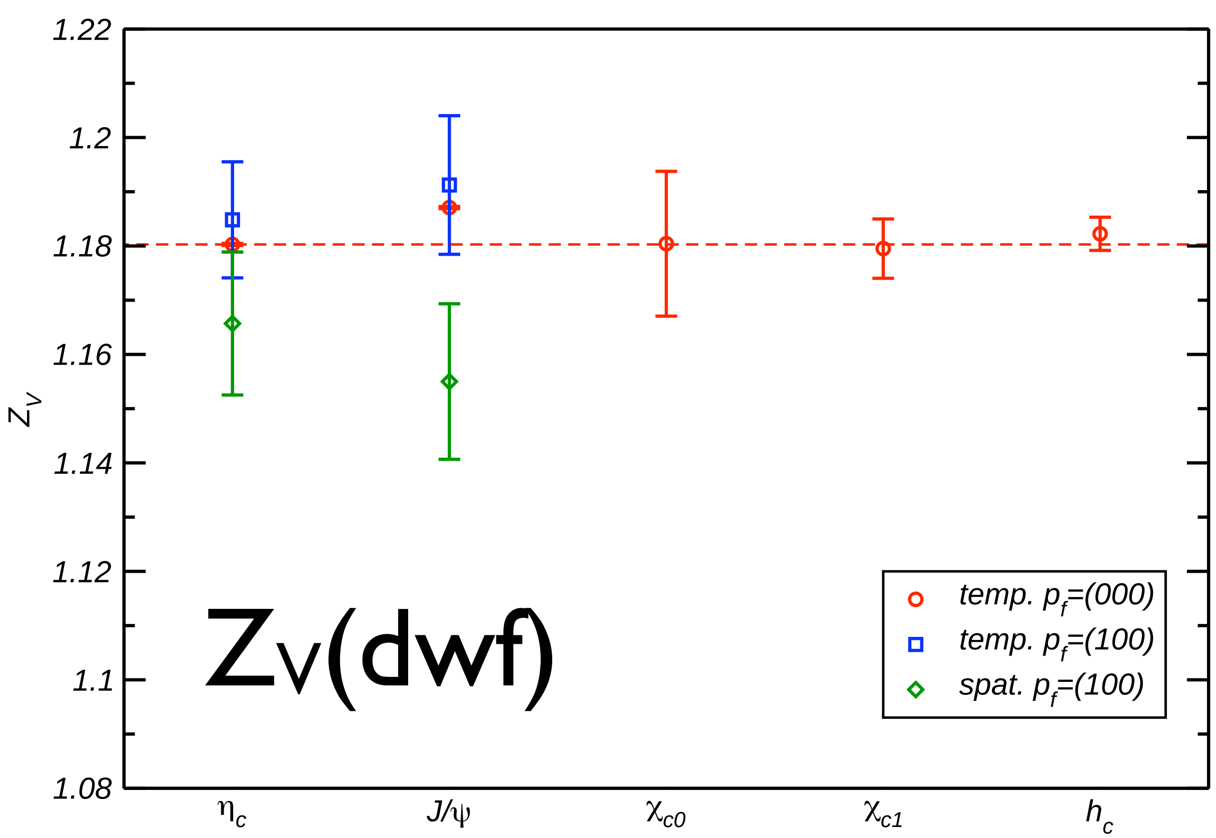
$$\begin{bmatrix} \Gamma(a;t) \\ \Gamma(b;t) \\ \Gamma(c;t) \\ \vdots \end{bmatrix} = \begin{bmatrix} P(a;t)K_1(a) & P(a;t)K_2(a) & \cdots \\ P(b;t)K_1(b) & P(b;t)K_2(b) & \\ P(c;t)K_1(c) & P(c;t)K_2(c) & \\ \vdots & & \ddots \end{bmatrix} \begin{bmatrix} f_1(Q^2)[t] \\ f_2(Q^2)[t] \\ \vdots \end{bmatrix},$$

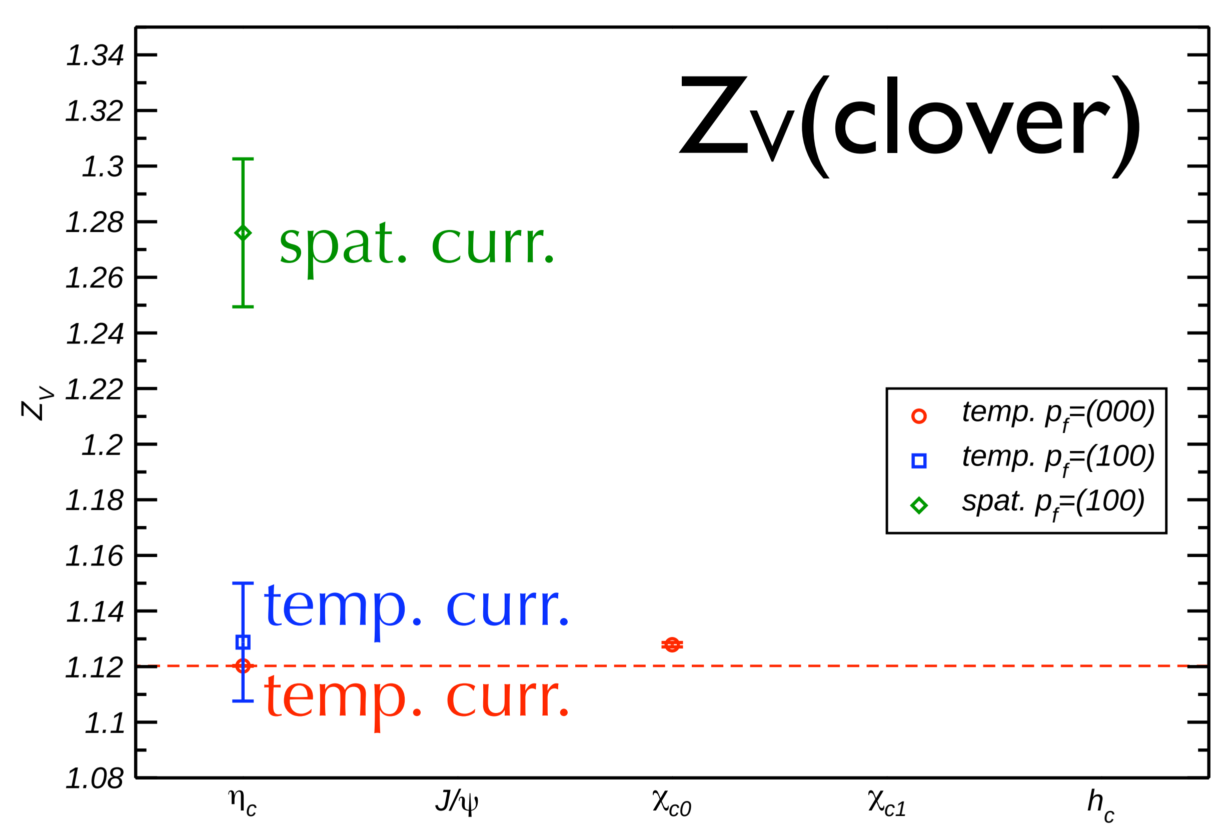
- invert this system with SVD

$P \cdot K$ are known quantities

Z_V

- set using meson form-factors at zero Q^2



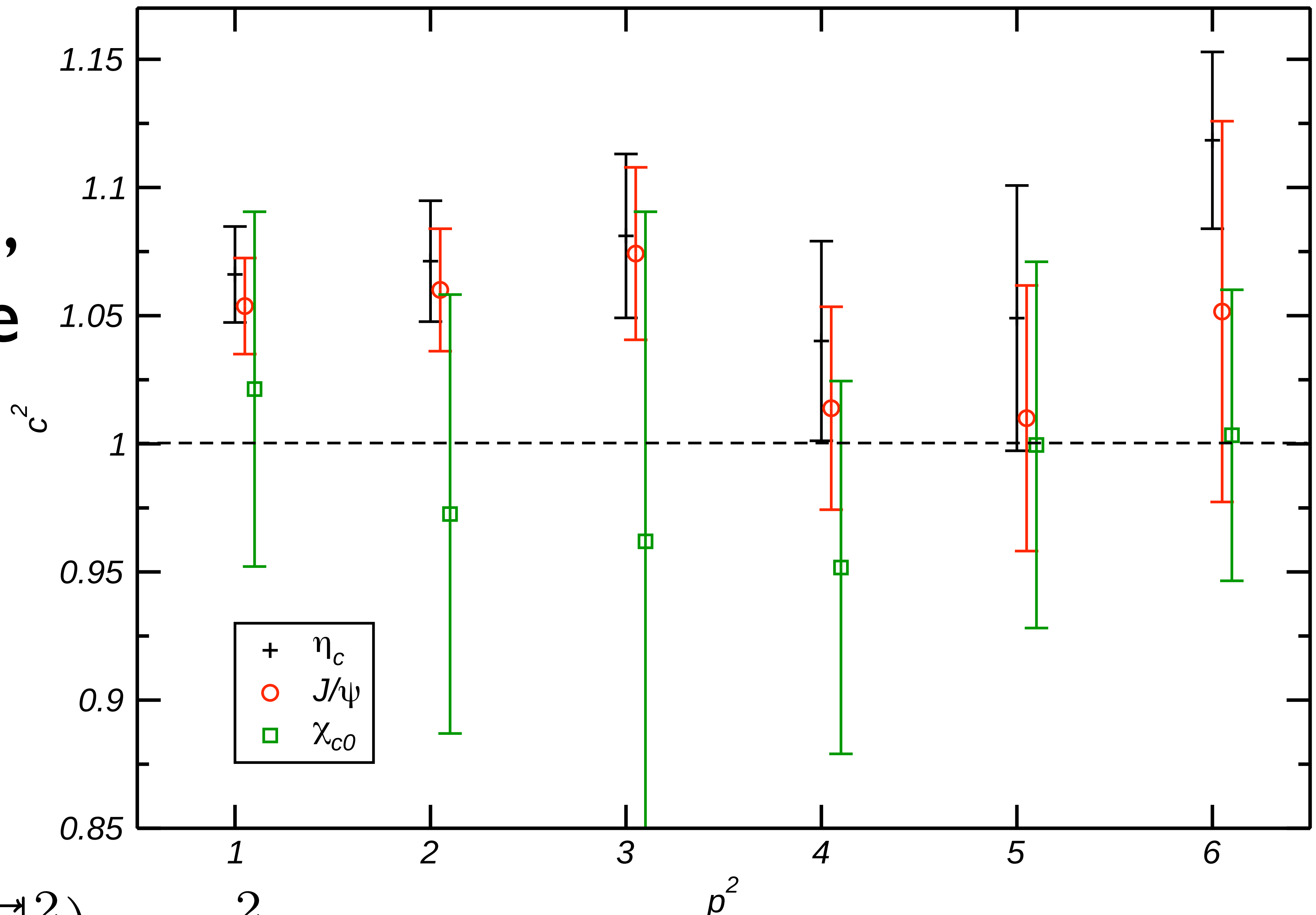


quenched?

- scale setting ambiguity - running coupling
- non-unitarity a negligible issue
- above threshold states rendered stable - they were narrow anyway

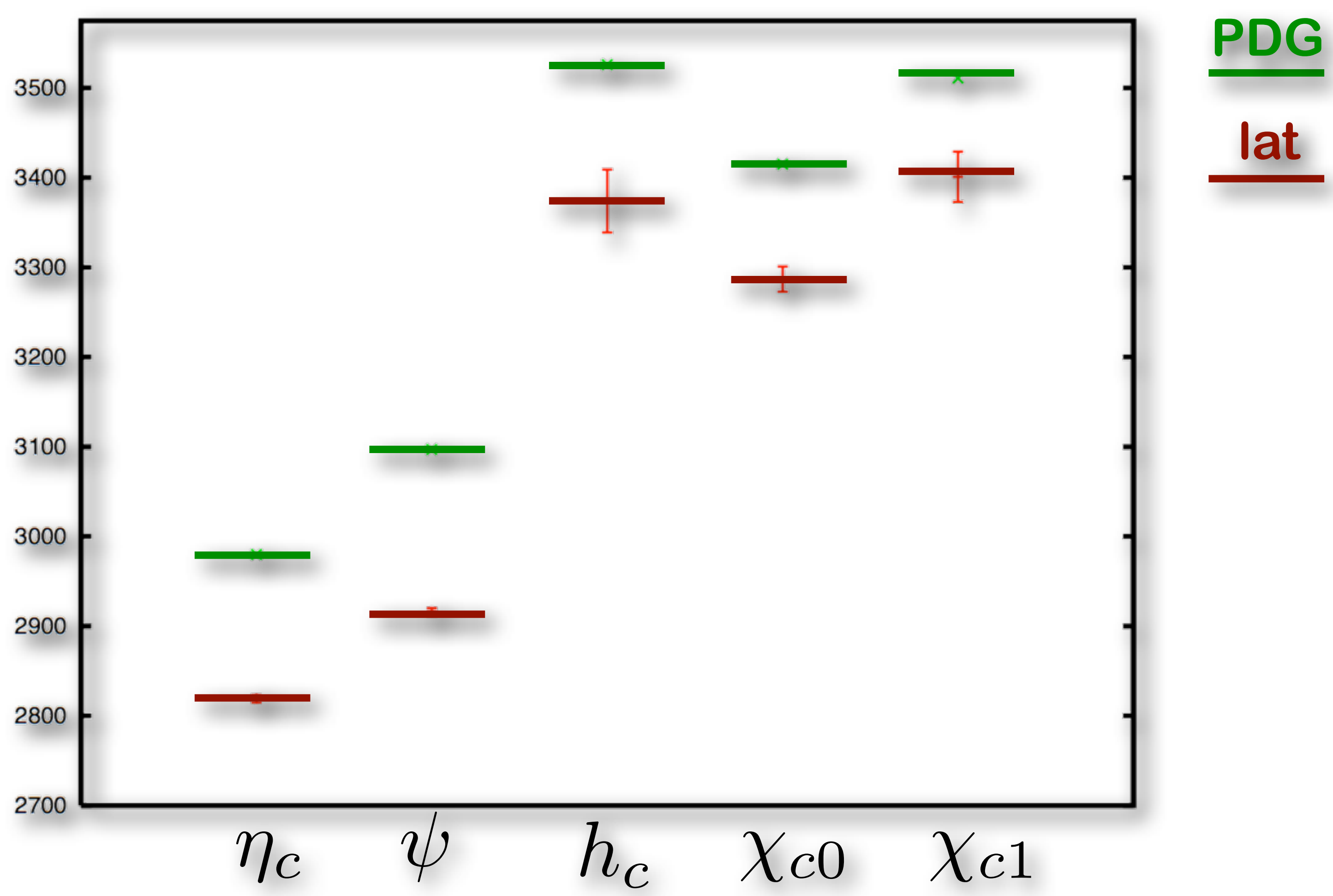
anisotropy - $\text{disp}^{\underline{n}}\ \text{rel}^{\underline{n}}$

~6% deviation, could easily be reduced

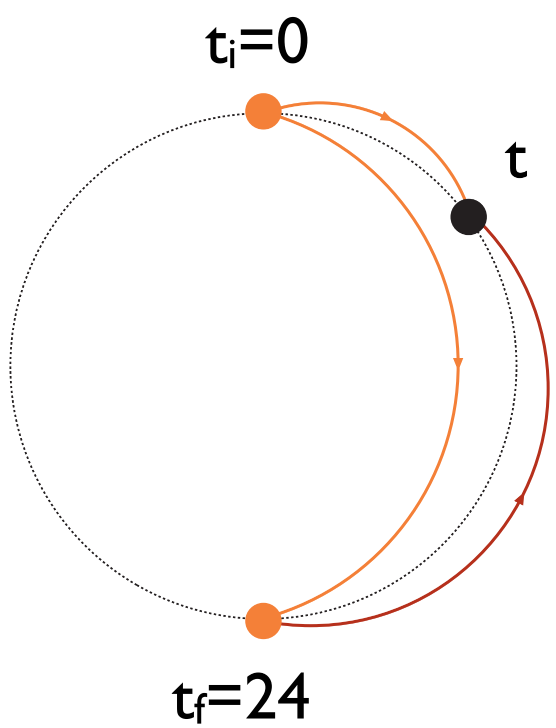


$$c^2(|\vec{p}|^2) \equiv \frac{E^2(|\vec{p}|^2) - m^2}{|\vec{p}|^2}$$

spectrum

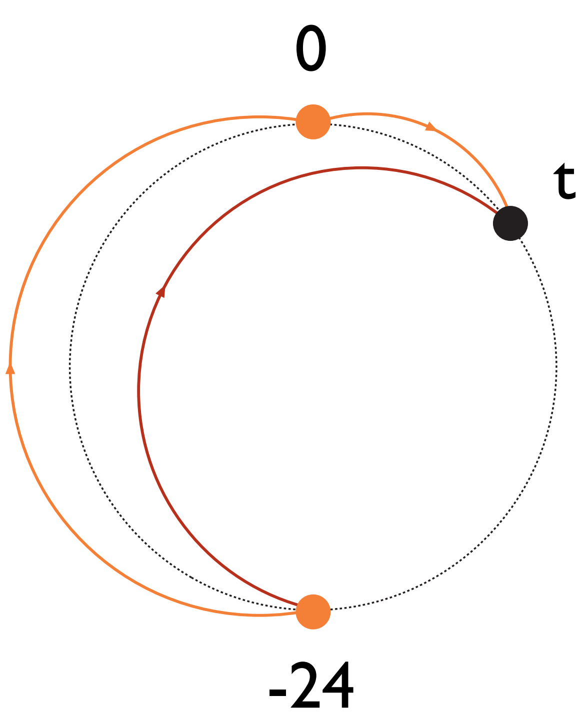

PDG
lat
3500
3400
3300
3200
3100
3000
2900
2800
2700
η_c
ψ
h_c
χ_{c0}
χ_{c1}

‘wrap-around’ pollution



$$\langle 0|\varphi_f(t_f = 24)|f\rangle\langle f|j^\mu(t)|i\rangle\langle i|\varphi_i(t_i = 0)|0\rangle$$

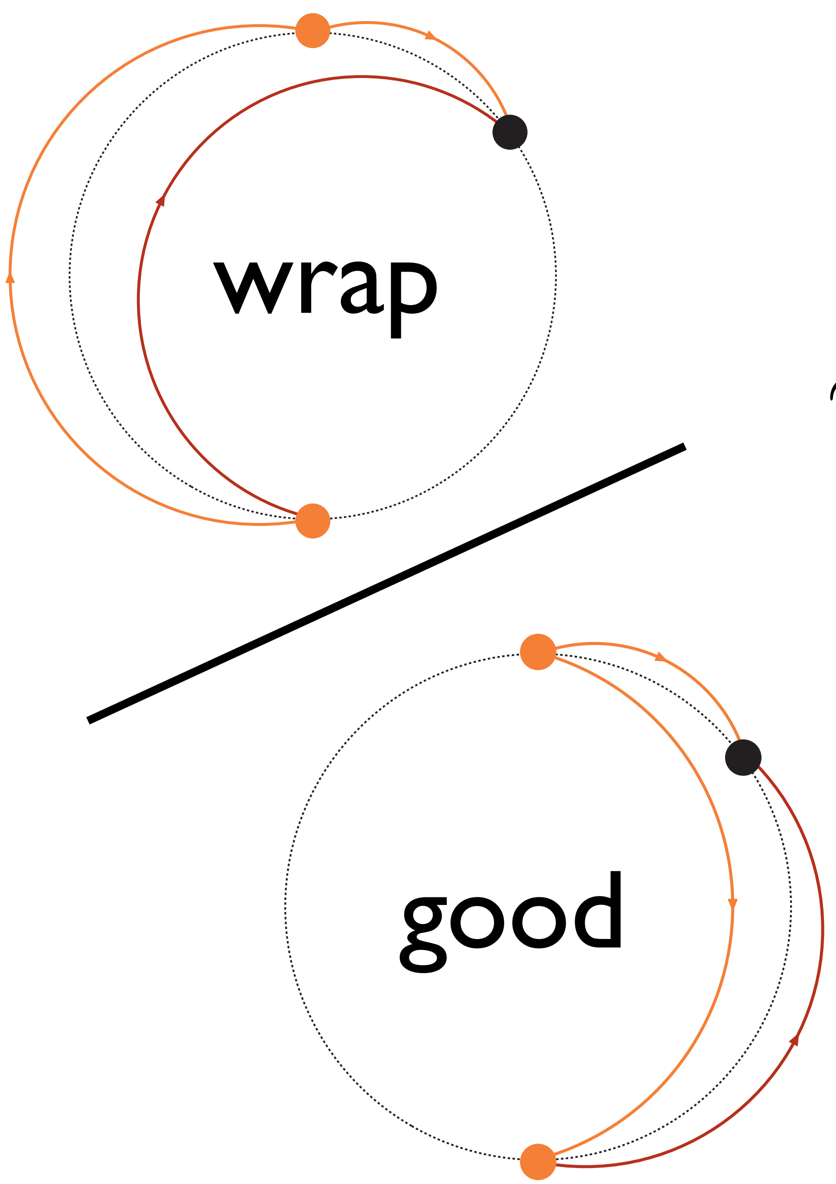
$$\sim Z_i Z_f \langle f|j^\mu(0)|i\rangle\; e^{-E_f(24-t)-E_i t}$$



$$\langle 0|j^\mu(t)|V\rangle\langle V|\varphi_i(0)|f\rangle\langle f|\varphi_f(t_i = -24)|0\rangle$$

$$\sim Z_V Z_f \langle V|\varphi_i(0)|f\rangle\; e^{-E_V t - E_f 24}$$

‘wrap-around’ pollution



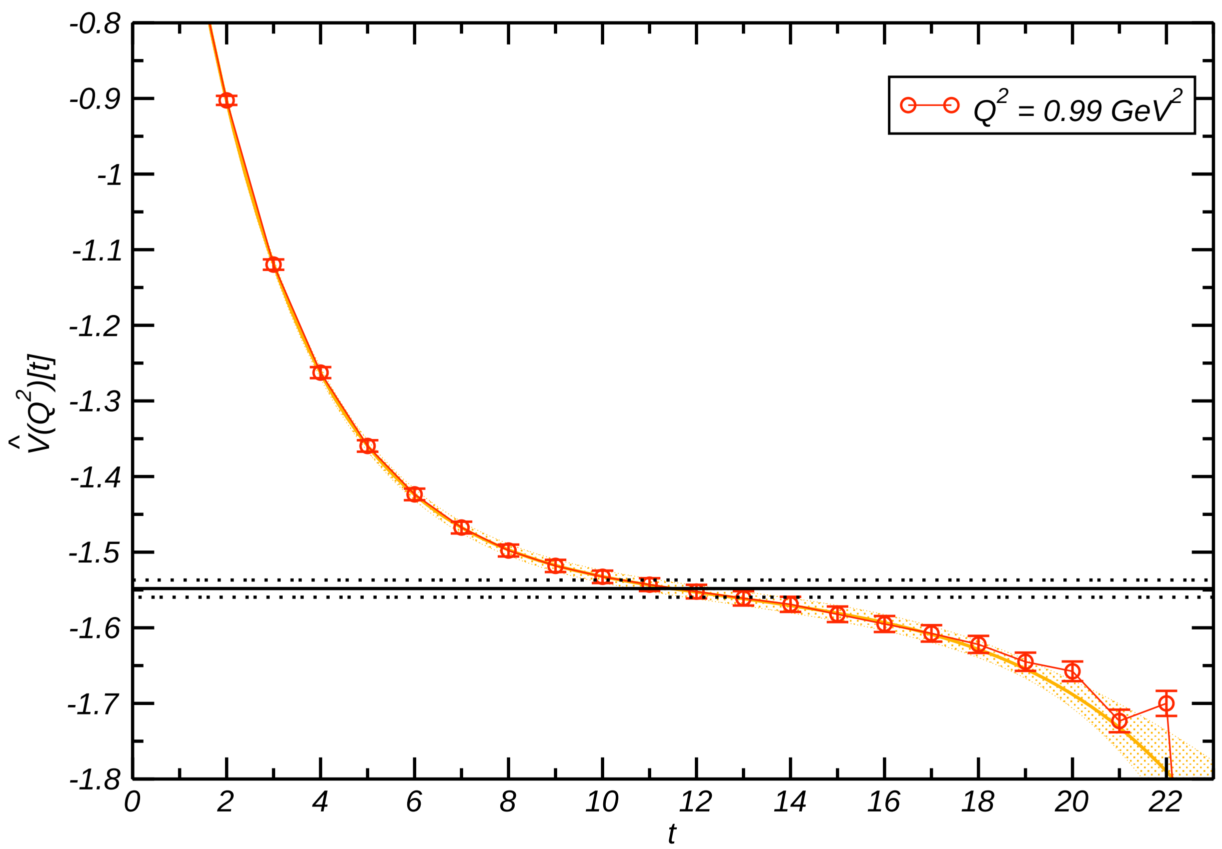
$$\sim \frac{Z_V \langle V|\varphi_i(0)|f\rangle}{Z_i \langle f|j^\mu(0)|i\rangle} \, e^{-(E_V - \delta E_{if})t}$$

$$E_V \sim m_{J/\psi} \sim 3\,\mathrm{GeV}$$

$$\delta E_{if} \sim m_\chi - m_\psi \sim 600\,\mathrm{MeV}$$

so wrap around should fall off relatively sharply.
if amplitude is large this will be a nasty pollution
(prevents excited state extraction)

‘wrap-around’ pollution



rapid fall-off near t=0 indicative of the wrap-around pollution

we resorted to fitting the pollution with a single exponential

$$f_n(Q^2)[t] = f_n(Q^2) + \mathfrak{f}_i e^{-\mathfrak{m}_i t} + \mathfrak{f}_f e^{-\mathfrak{m}_f (24-t)}$$

$\hat{V}(Q^2) = -1.55(1)$, $\mathfrak{f}_i = 1.45(3)$, $\mathfrak{f}_f = -0.42(14)$, $\mathfrak{m}_i = 0.41(1)$, $\mathfrak{m}_f = 0.27(7)$

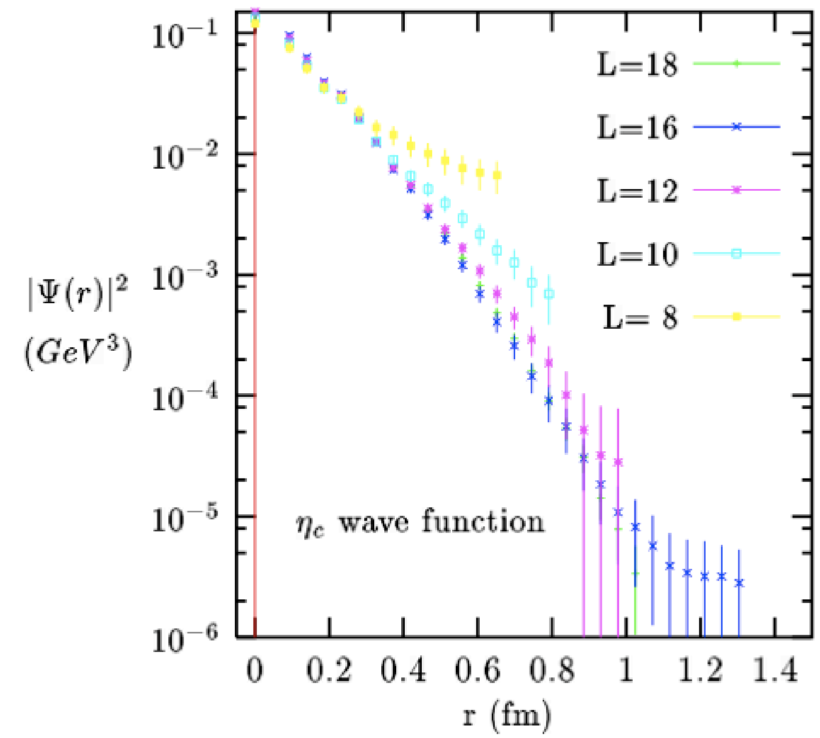
finite-size effects ?

- previous charmonium spectrum studies saw no significant finite volume effects with $L_s \gtrsim 1.1\,\mathrm{fm}$

QCD-TARO collabn.

Table 6: Pseudoscalar mass and hyperfine splitting from non-perturbatively improved clover Dirac operator. The lattice spacing is fixed to 0.093 fm ($\beta = 6.0$) and the number of lattice points L, hence the physical volume La, is varied as indicated in the table. Results, averaged over 100 configurations (190 for $L = 8$), are given in physical units (MeV) with the scale set by r_0.

L	La (fm)	$^1S_0$	$^3S_1$	$^3S_1 - {}^1S_0$
8	0.75	2958(10)	3019(12)	61.4(4.4)
10	0.93	2953(5)	3023(6)	70.6(2.5)
12	1.12	2957(3)	3032(5)	75.4(2.7)
14	1.30	2947(3)	3020(4)	72.6(1.9)
16	1.49	2952(3)	3025(4)	74.9(2.1)
18	1.68	2949(2)	3021(3)	72.5(1.5)



- we extracted from the form-factors that radius of charmonium states is $\sim 0.2 \rightarrow 0.3\,\mathrm{fm}$

finite-size should be no problem for us @ Ls = 1.2 fm