Exotic and excited-state meson spectroscopy from lattice QCD

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With Jo Dudek, Robert Edwards, Mike Peardon,
David Richards and the *Hadron Spectrum Collaboration*

Jefferson Lab

Outline

- Introduction and motivation
- Spectra from LQCD – overview of method
- Isovector and kaon spectra
- Multi-meson states
- Photocouplings
- Summary and outlook

PR D79 094504 (2009)
PRL 103 262001 (2009)
arXiv:1004.4930

Motivation

Renaissance in excited charmonium spectroscopy

BABAR, Belle, BES, CLEO-c

Upcoming experimental efforts, also in the light meson sector

GlueX (JLab), BESIII, PANDA

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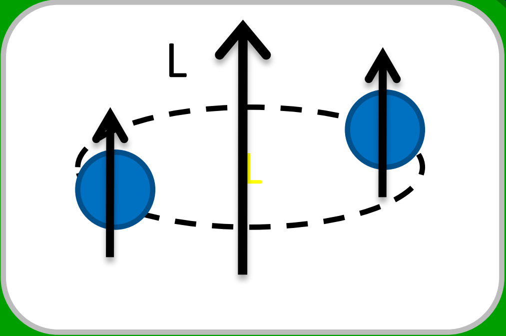
Exotics (J^{PC} = **1**$^{-+}$, **2**$^{+-}$, ...)? – can't just be a $q\bar{q}$ pair

e.g. hybrids, multi-mesons

Two spin-half fermions:

Parity: $P = (-1)^{(L+1)}$

Charge Conj Sym: $C = (-1)^{(L+S)}$



$J^{PC} = 0^{-+}, 0^{++}, 1^{--}, 1^{++}, 1^{+-}, 2^{--}, 2^{++}, 2^{-+}, \ldots$

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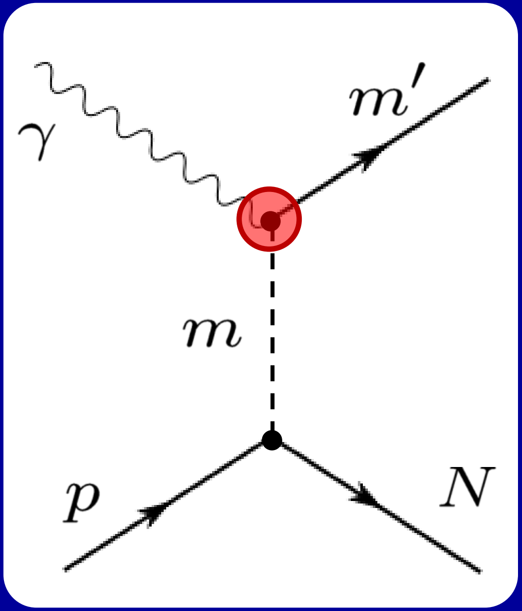

Photoproduction at GlueX (JLab 12 GeV upgrade)

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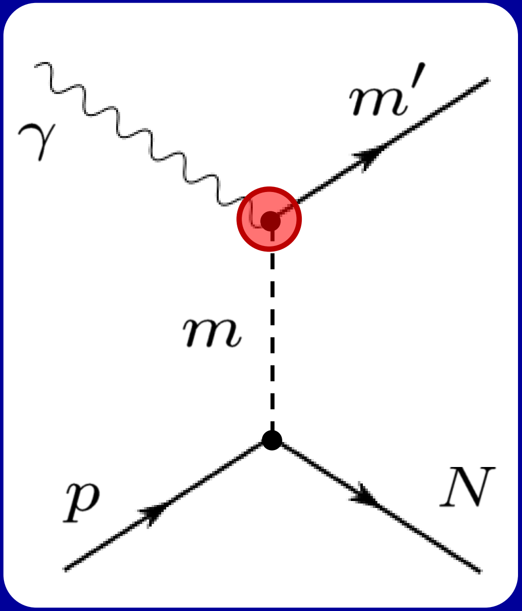

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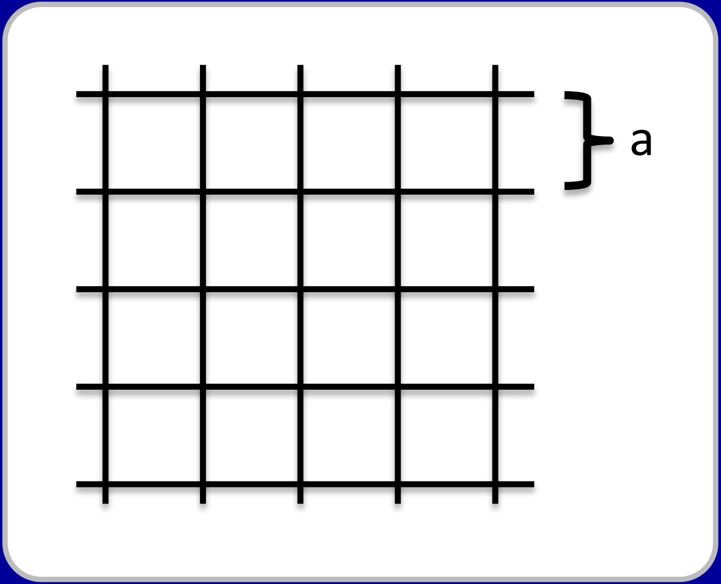
Use Lattice QCD to extract excited spectrum...

... and photocouplings (tested in charmonium)

PR D77 034501 (2008), PR D79 094504 (2009)

QCD on a Lattice



Discretise on a grid (spacing = a) – regulator

Finite volume → finite no. of d.o.f.

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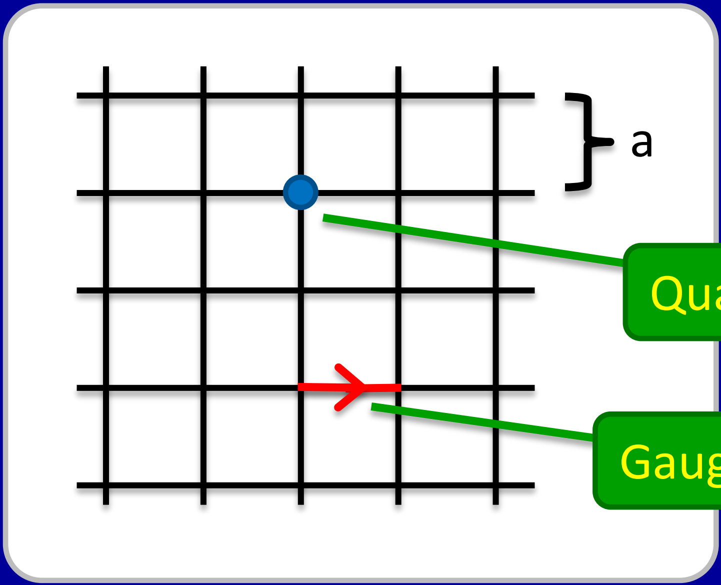

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Quarks fields on lattice sites $\psi(x) \longrightarrow \psi_x$

Gauge fields on links $A_\mu(x) \longrightarrow U_{x,\mu} = e^{-aA_{x,\mu}}$

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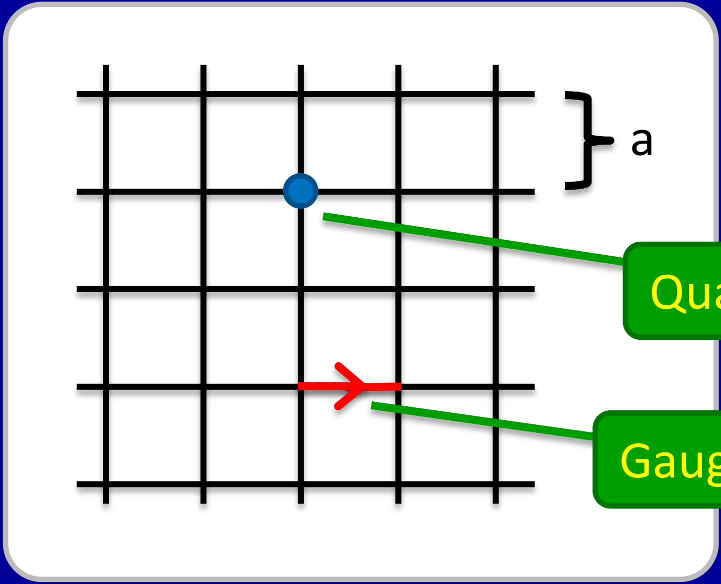

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Path integral formulation $\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U \, f(\psi, \bar{\psi}, U) e^{iS[\psi,\bar{\psi},U]}$

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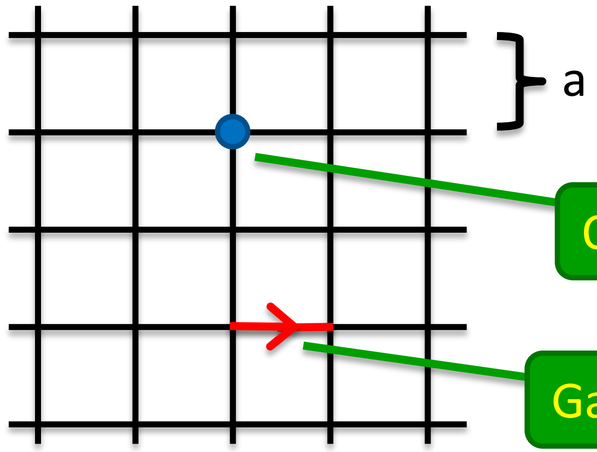

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Euclidean time: t → i t $\int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}U\, f(\psi,\bar{\psi},U)e^{-\tilde{S}[\psi,\bar{\psi},U]}$

Do fermion integral analytically then use importance sampling Monte Carlo

Spectroscopy on the lattice

Calculate energies and matrix elements (“overlaps”, Z’s) from correlation functions of meson interpolating fields

$$C_{ij}(t) = < 0|O_i(t)O_j(0)|0 >$$

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Construct operators which only overlap on to one spin in the continuum limit

$$O(t) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \; \bar{\psi}(x)\Gamma_i \overleftrightarrow{D}_j \overleftrightarrow{D}_k \ldots \psi(x)$$

(p = 0)

definite J^{PC}

'Distillation' technology for constructing on lattice PR D80 054506 (2009)

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(p = 0)

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$$Z_i^{(n)} \equiv < 0|O_i|n >$$

$$C_{ij}(t) = \sum_n \frac{e^{-E_n t}}{2 E_n} < 0|O_i(0)|n >< n|O_j(0)|0 >$$

Variational Method

Large basis of operators → matrix of correlators

$$C_{ij}(t) =< 0|O_i(t)O_j(0)|0 >$$

Generalised eigenvector problem:

$$C_{ij}(t)v_j^{(n)} = \lambda^{(n)}(t)C_{ij}(t_0)v_j^{(n)}$$

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($t >> t_0$)

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$Z^{(n)}$ related to eigenvectors

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Var. method uses orthog of eigenvectors; don't just rely on separating energies

Light Meson Spectroscopy

- Dynamical calculation (unquenched)

- Anisotropic – finer in temporal direction (a_s/a_t = 3.5, $a_s \sim$ 0.12 fm)

- Only connected diagrams – isovectors (I=1) and kaons

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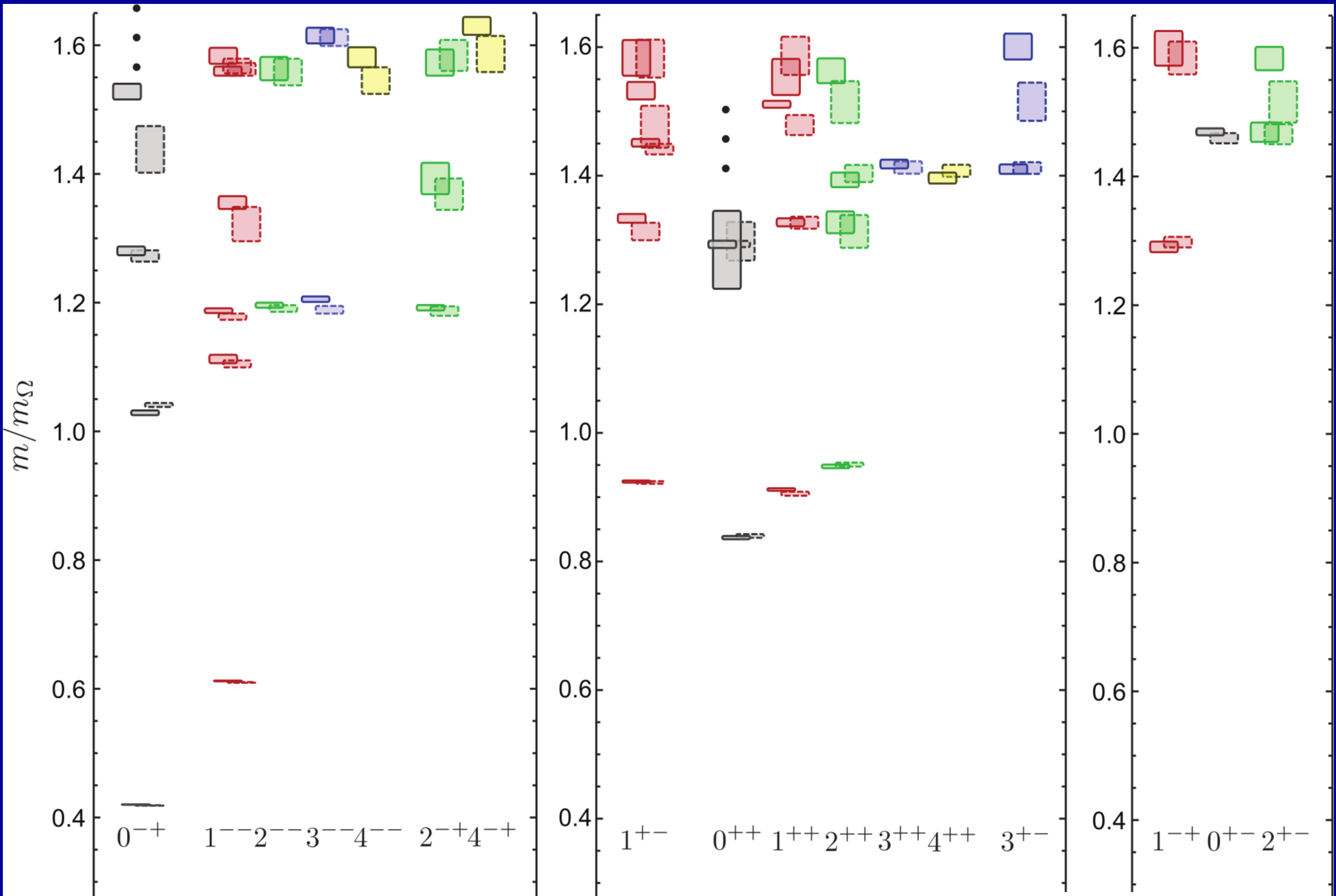
- As an example: three degenerate 'light' quarks (N_f = 3, $M_\pi \approx$ 700 MeV)

SU(3) symmetry

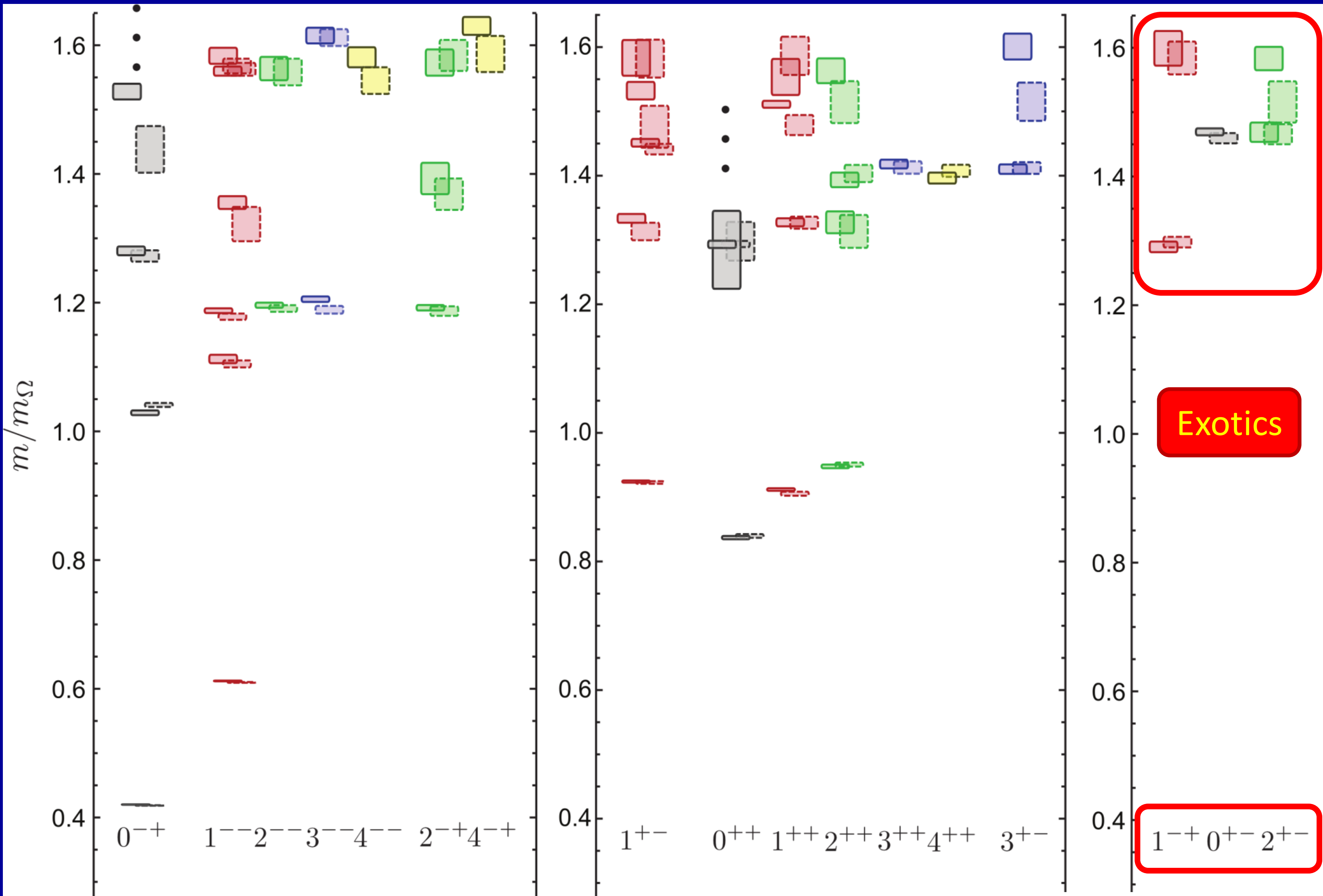
- Also (N_f = 2+1) $M_\pi \approx$ 520, 440, 400 MeV

PRL 103 262001 (2009) and arXiv:1004.4930



$N_f = 3$ isovectors

16^3 (~2 fm) and 20^3 (2.4 fm)



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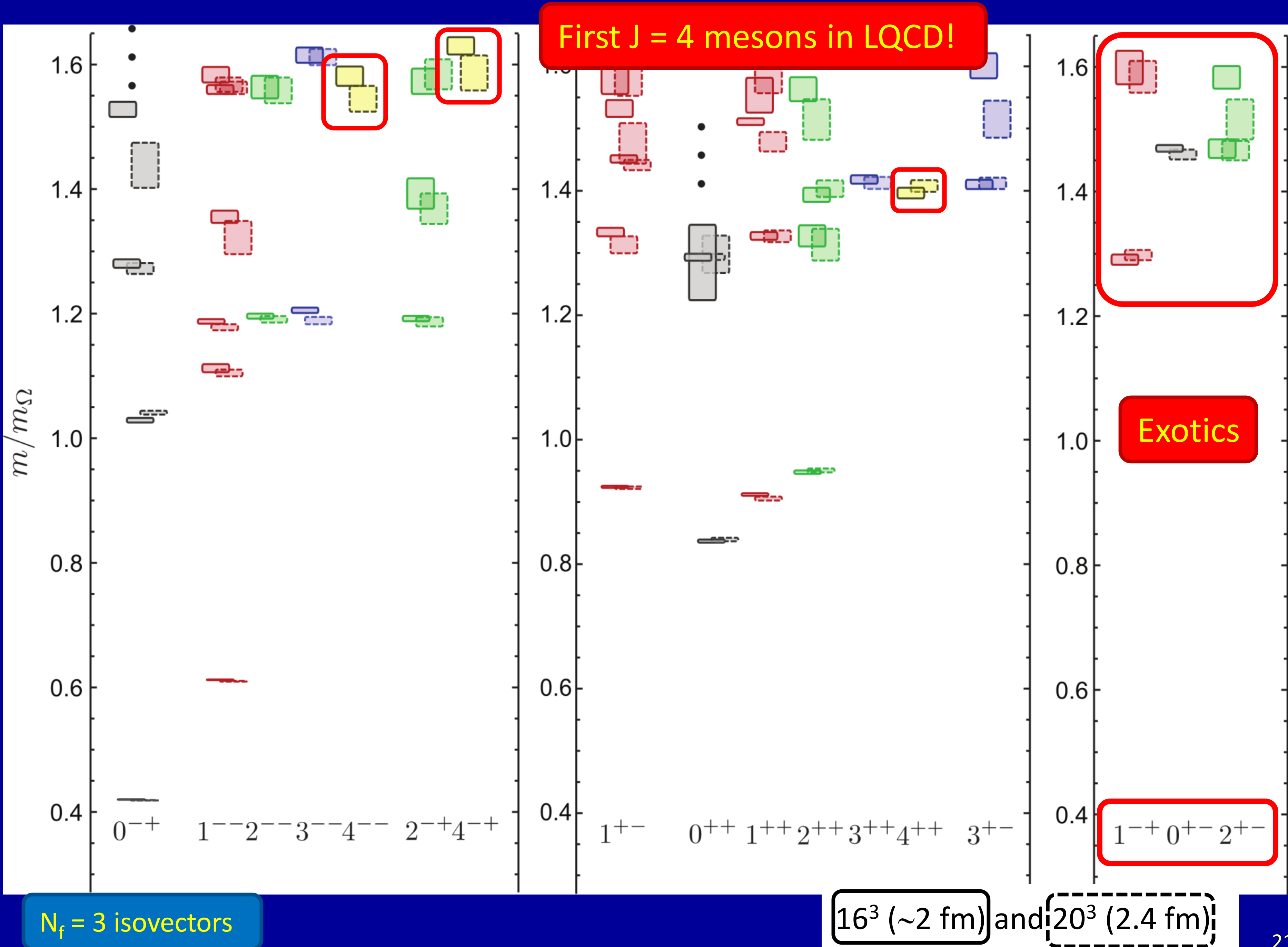
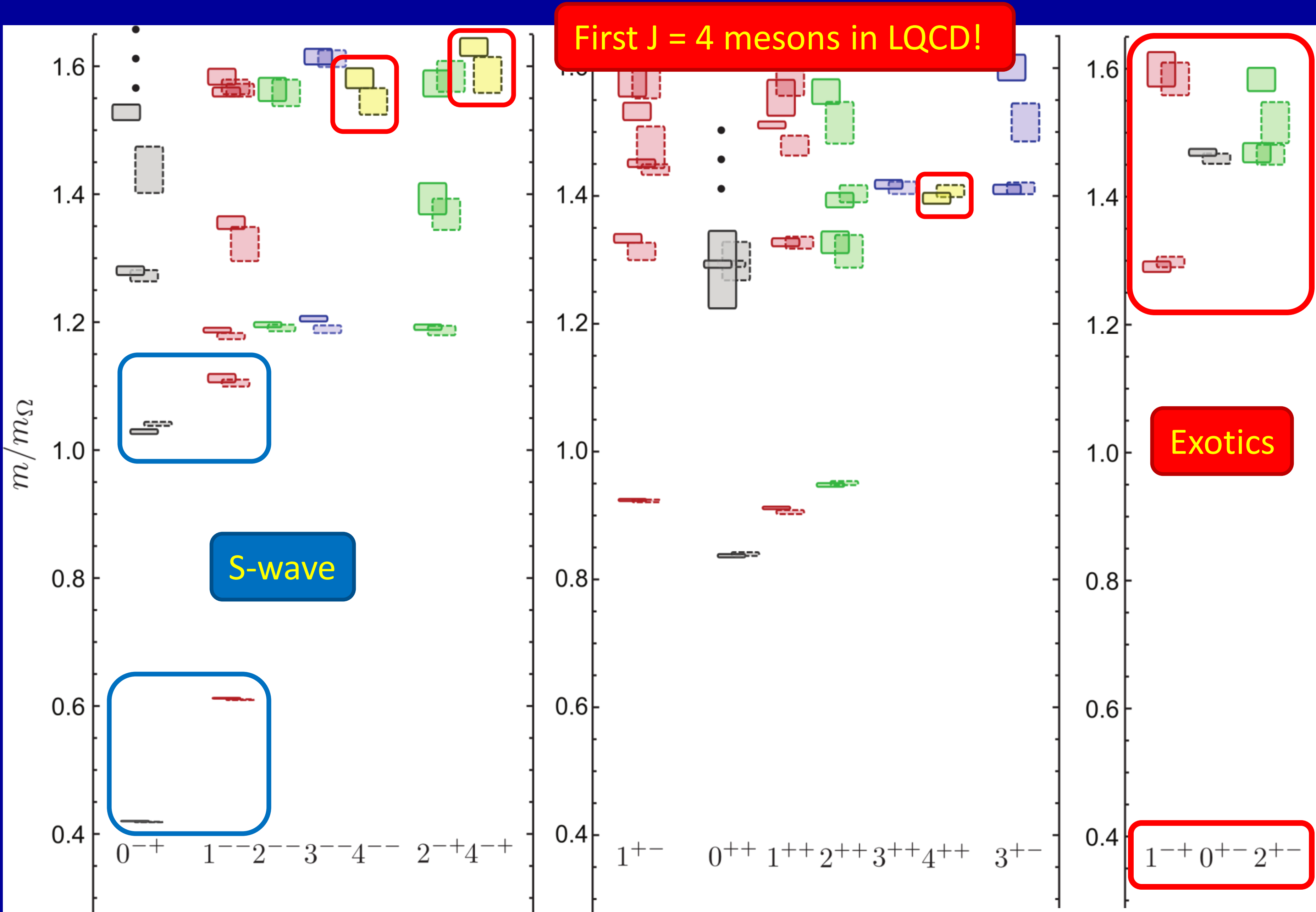


First J = 4 mesons in LQCD!
m/mΩ
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0⁻⁺ 1⁻⁻ 2⁻⁻ 3⁻⁻ 4⁻⁻ 2⁻⁺ 4⁻⁺
1⁺⁻ 0⁺⁺ 1⁺⁺ 2⁺⁺ 3⁺⁺ 4⁺⁺ 3⁺⁻
Exotics
1⁻⁺ 0⁺⁻ 2⁺⁻
Nf = 3 isovectors
16³ (~2 fm) and 20³ (2.4 fm)



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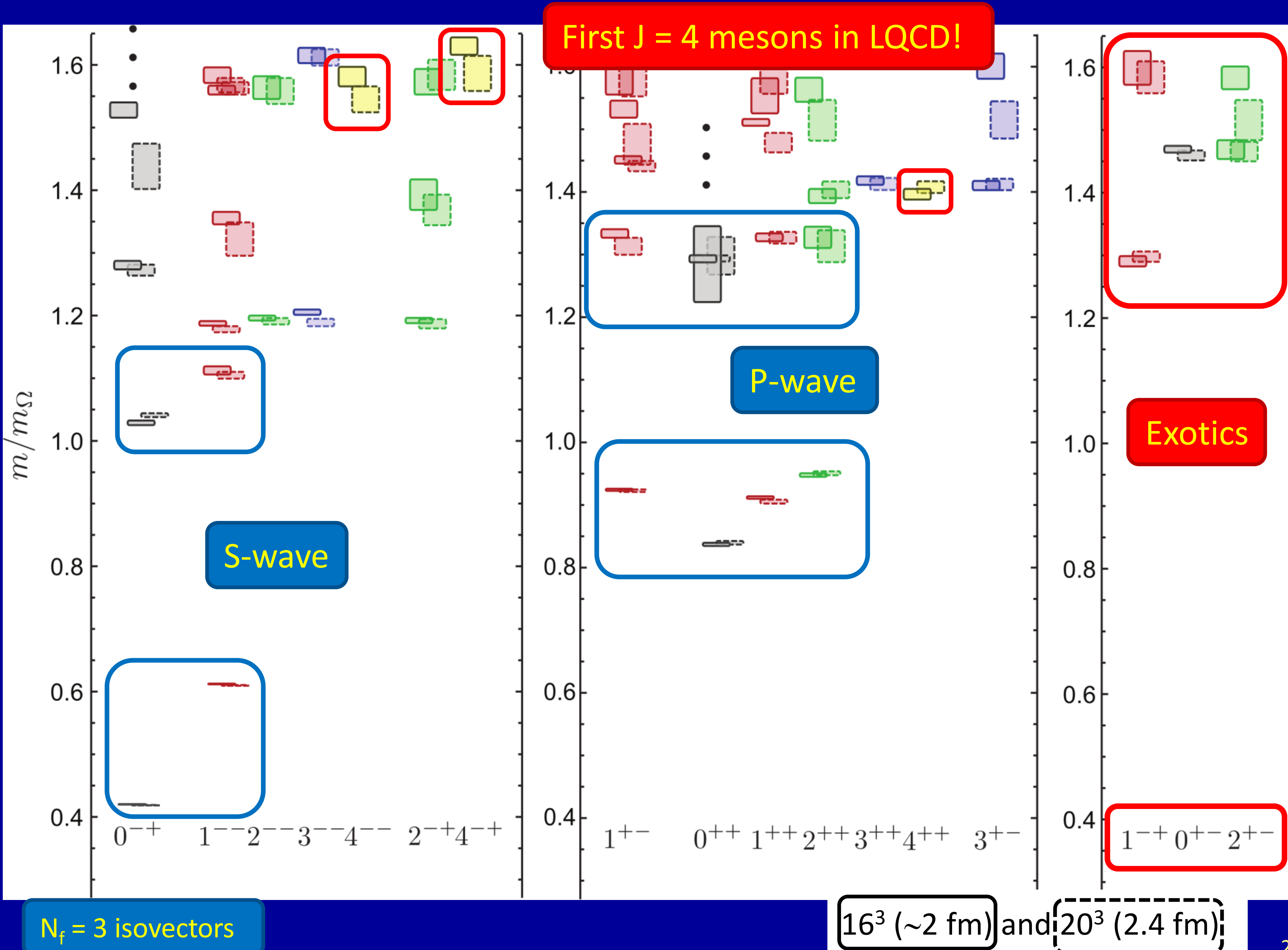

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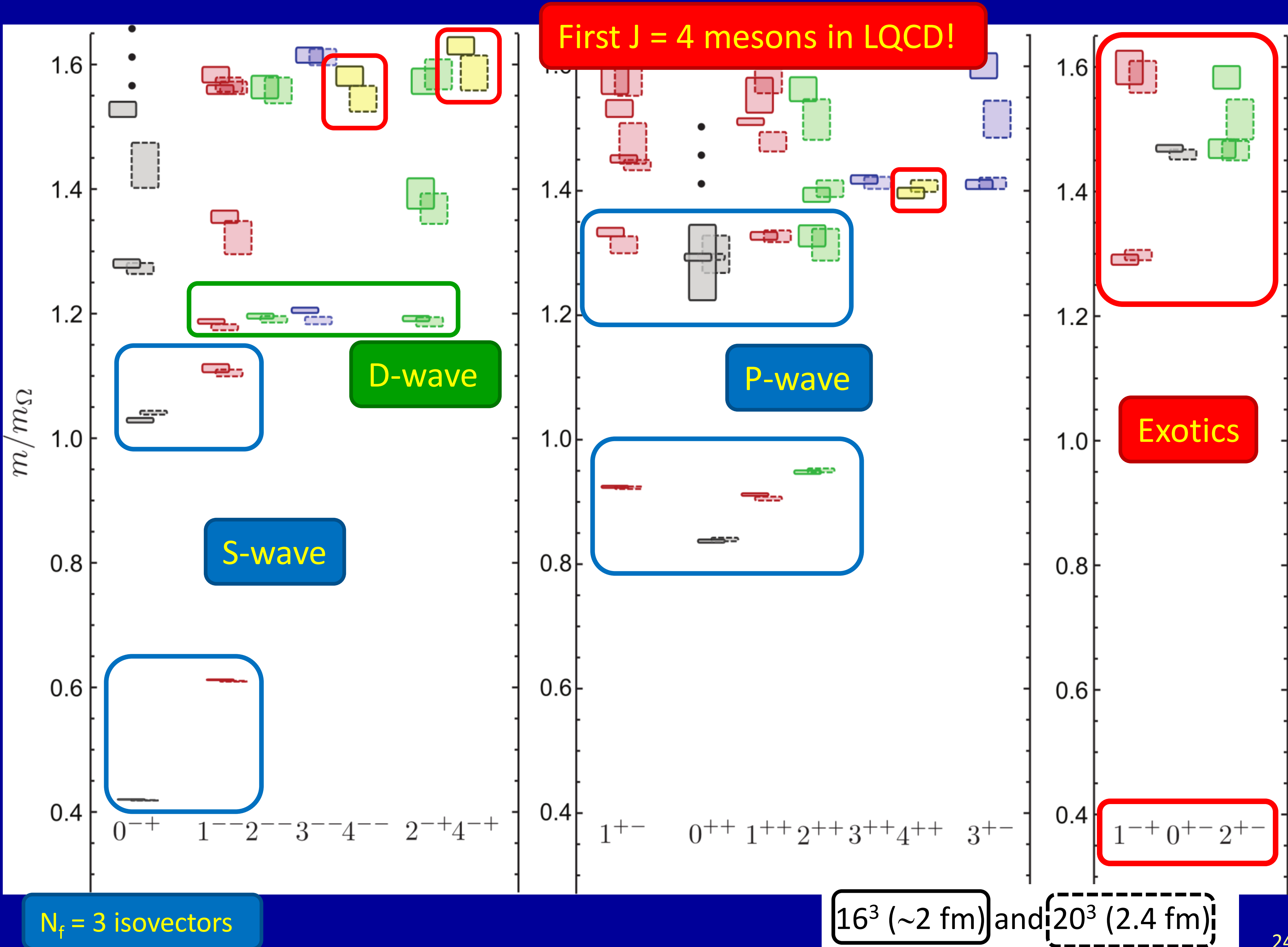

First J = 4 mesons in LQCD!
1.6
1.4
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1.0
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0.6
0.4
m/m_Ω
D-wave
S-wave
P-wave
Exotics
0^{-+} 1^{--} 2^{--} 3^{--} 4^{--} 2^{-+} 4^{-+}
1^{+-} 0^{++} 1^{++} 2^{++} 3^{++} 4^{++} 3^{+-}
1^{-+} 0^{+-} 2^{+-}
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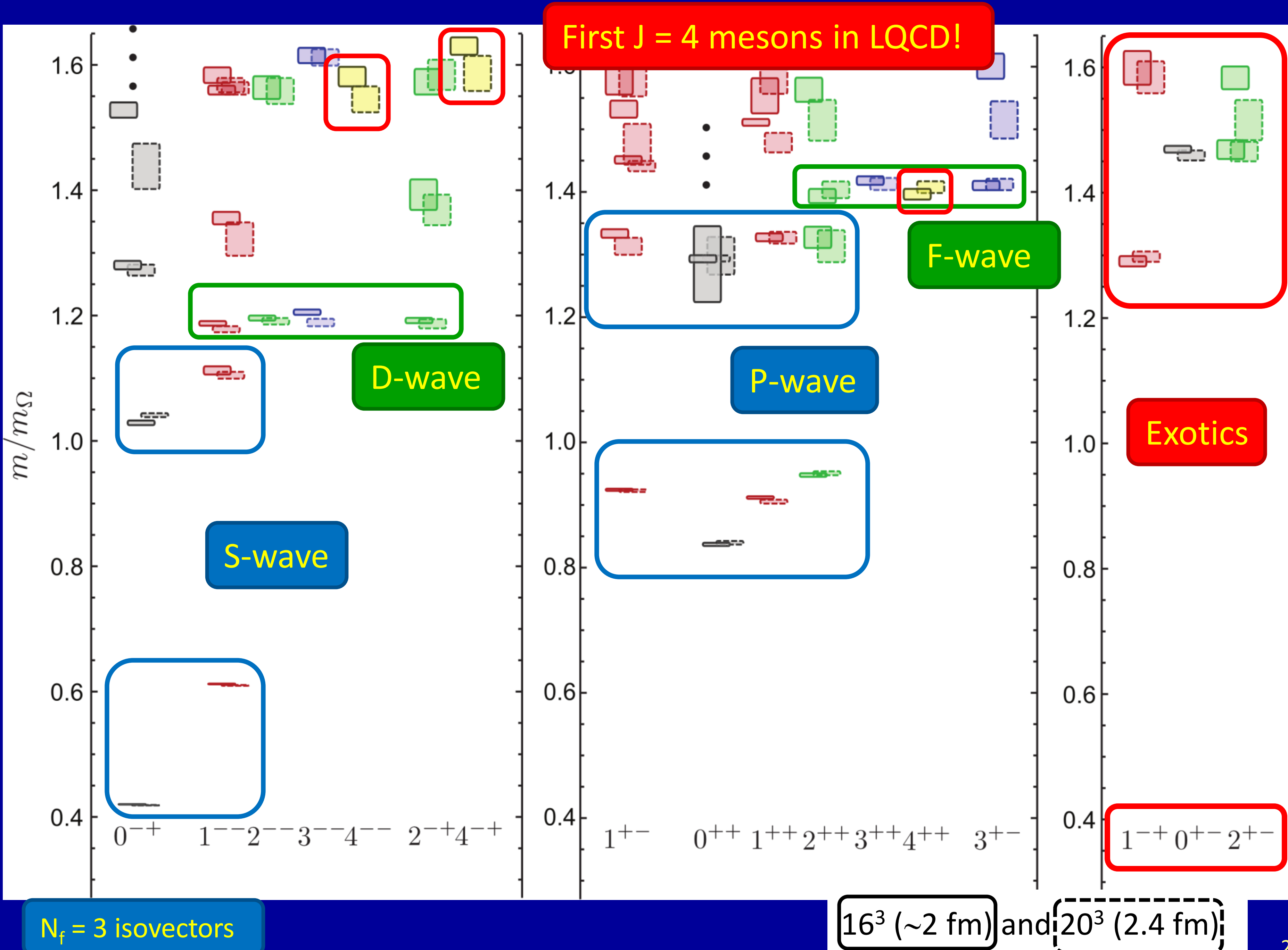
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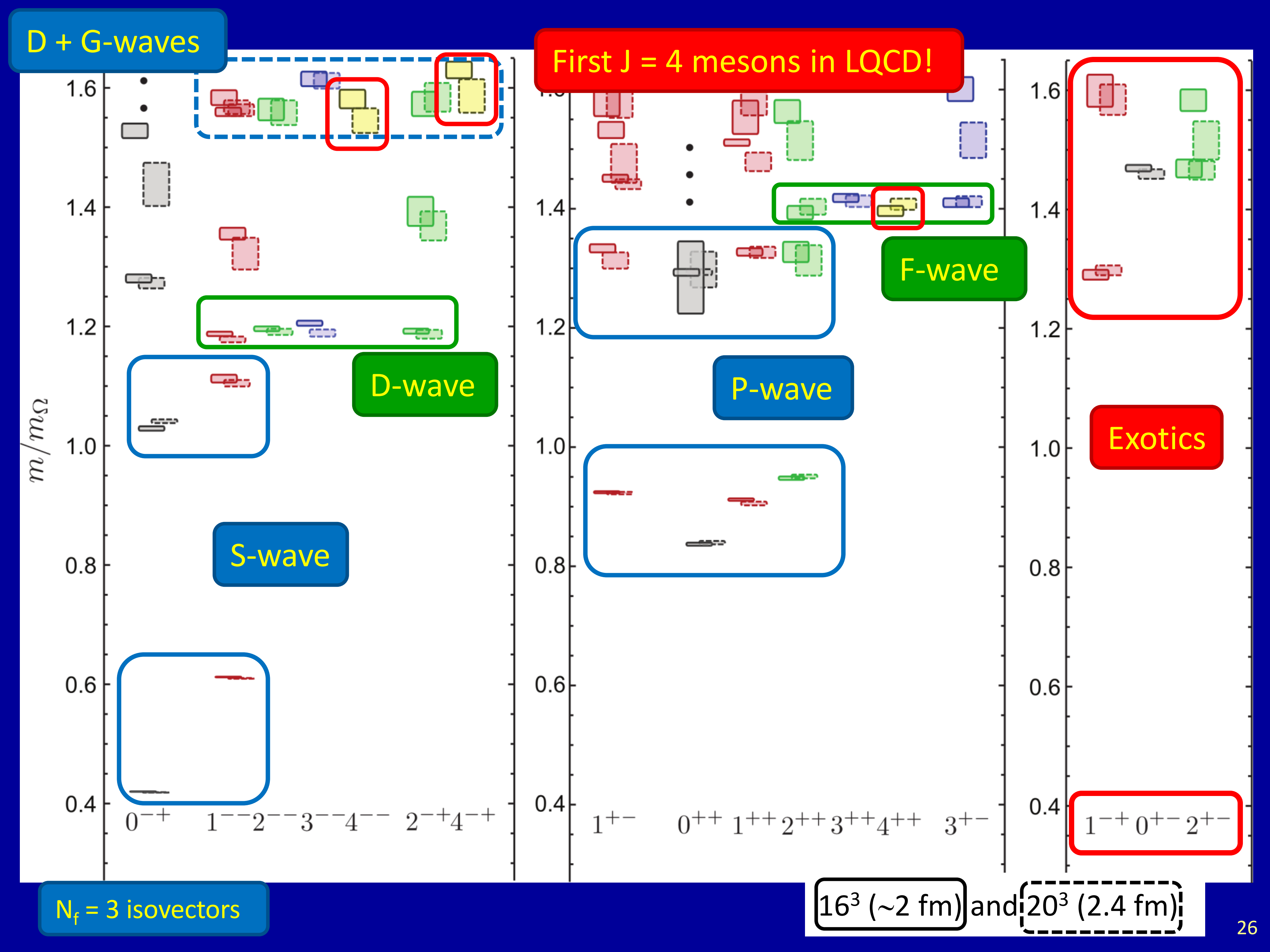
D + G-waves
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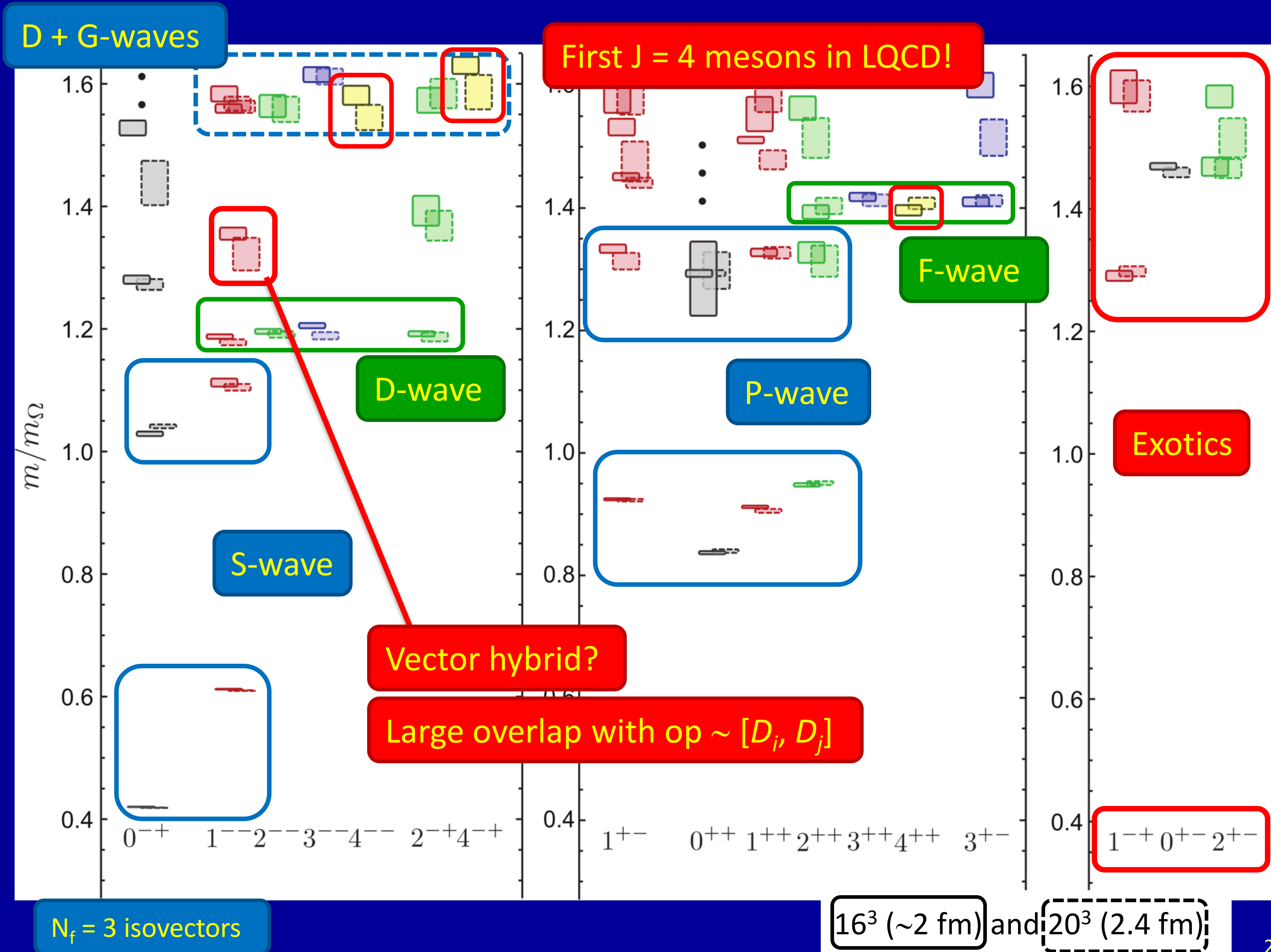
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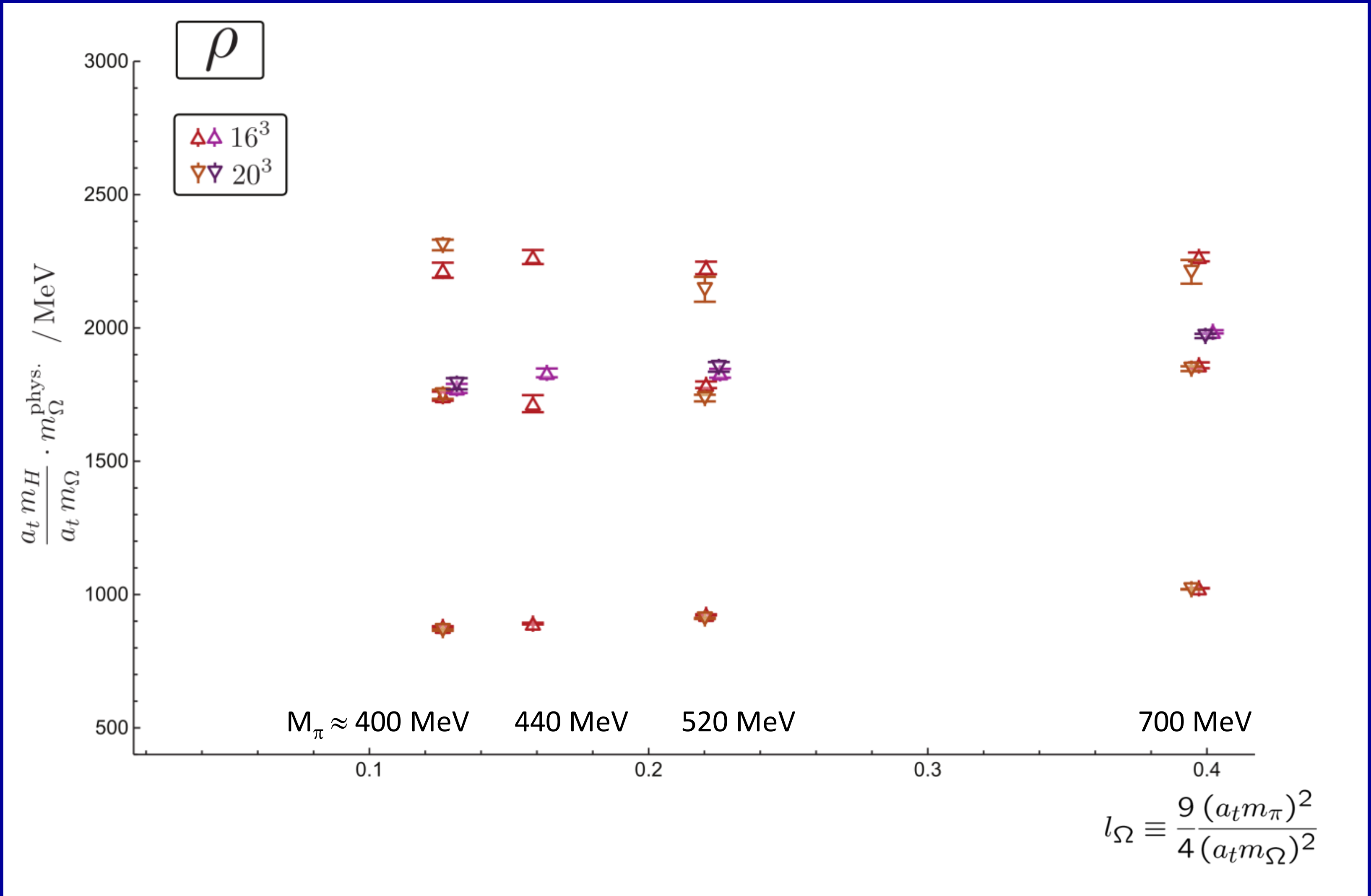
1^{-+} 0^{+-} 2^{+-}
D-wave
P-wave
F-wave
S-wave
Exotics
Vector hybrid?
Large overlap with op ~ $[D_i, D_j]$
N_f = 3 isovectors
16^3 (~2 fm) and 20^3 (2.4 fm)

Lower pion masses



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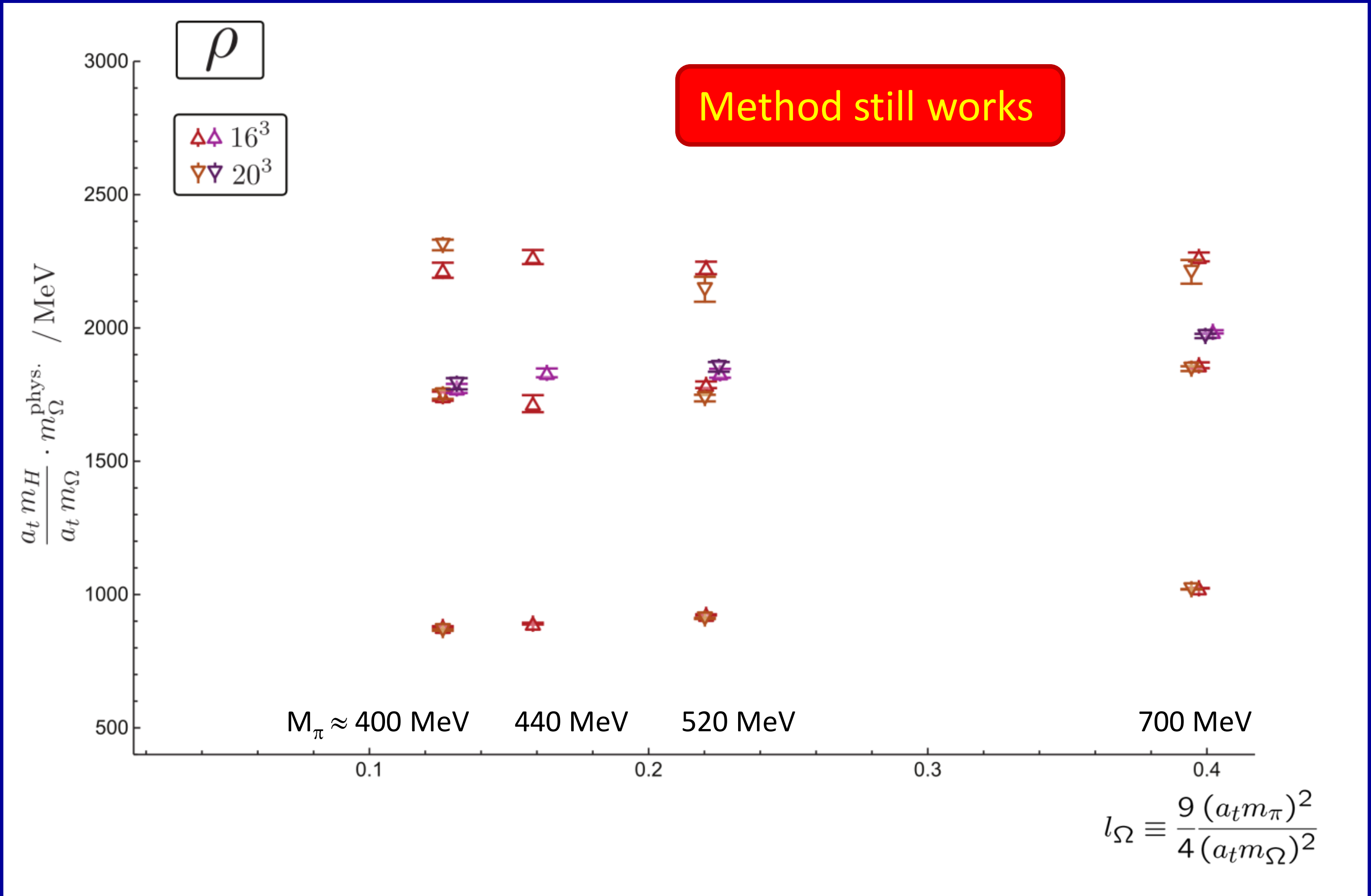
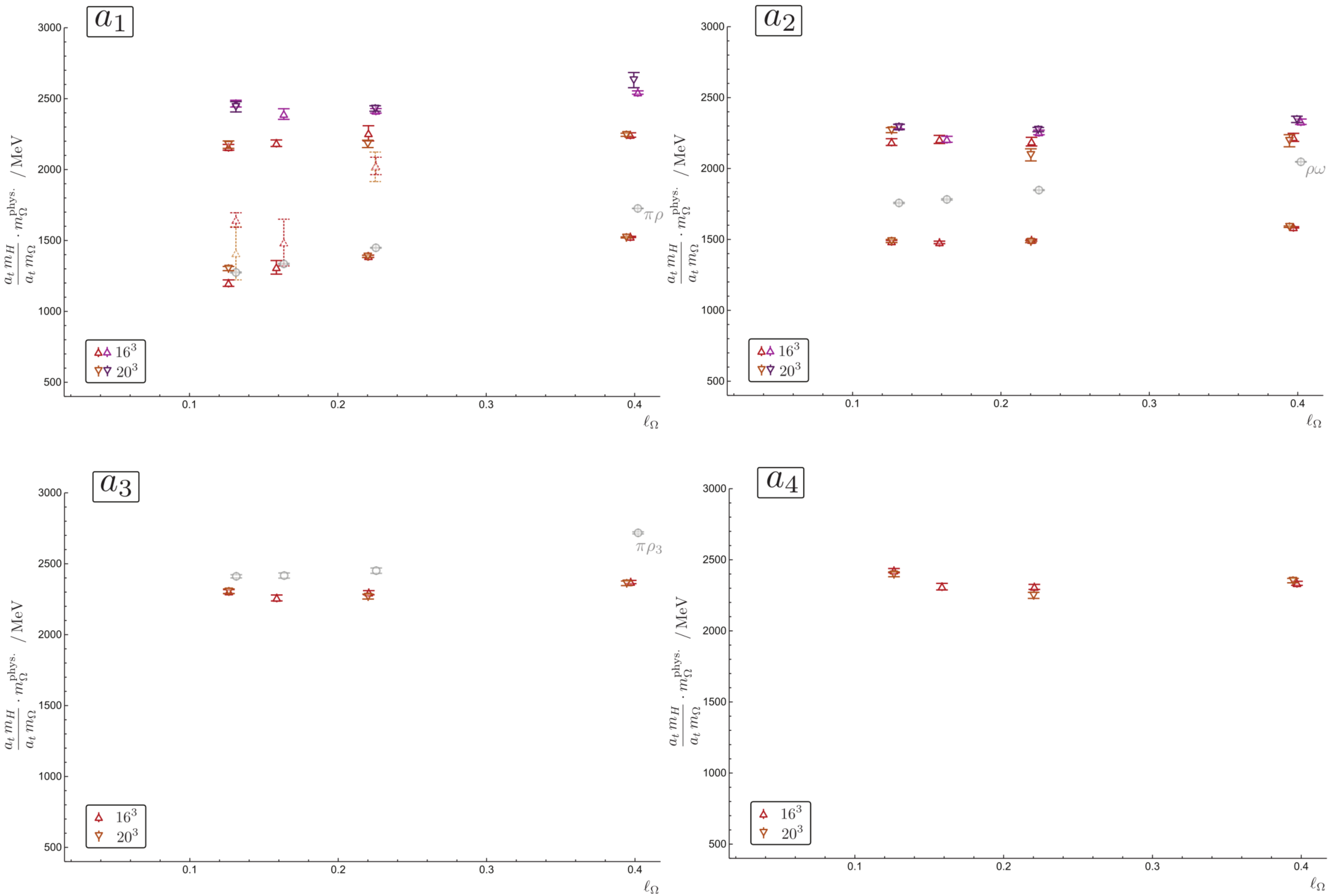
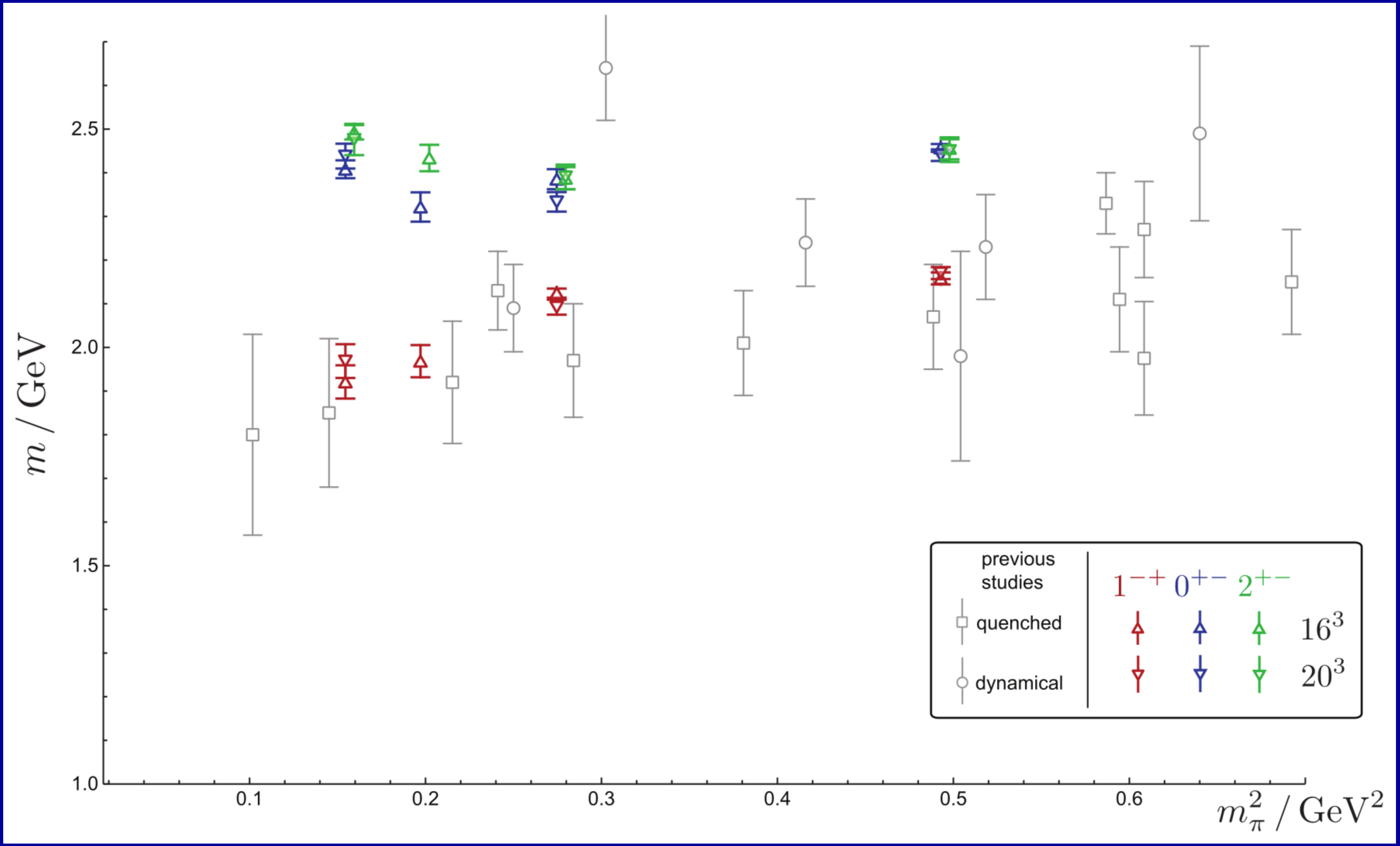



a1
a2
a3
a4
πρ
ρω
πρ3
16^3
20^3
ℓΩ
at mH / at mΩ · mΩ phys. / MeV
3000
2500
2000
1500
1000
500
0.1
0.2
0.3
0.4

Exotics summary



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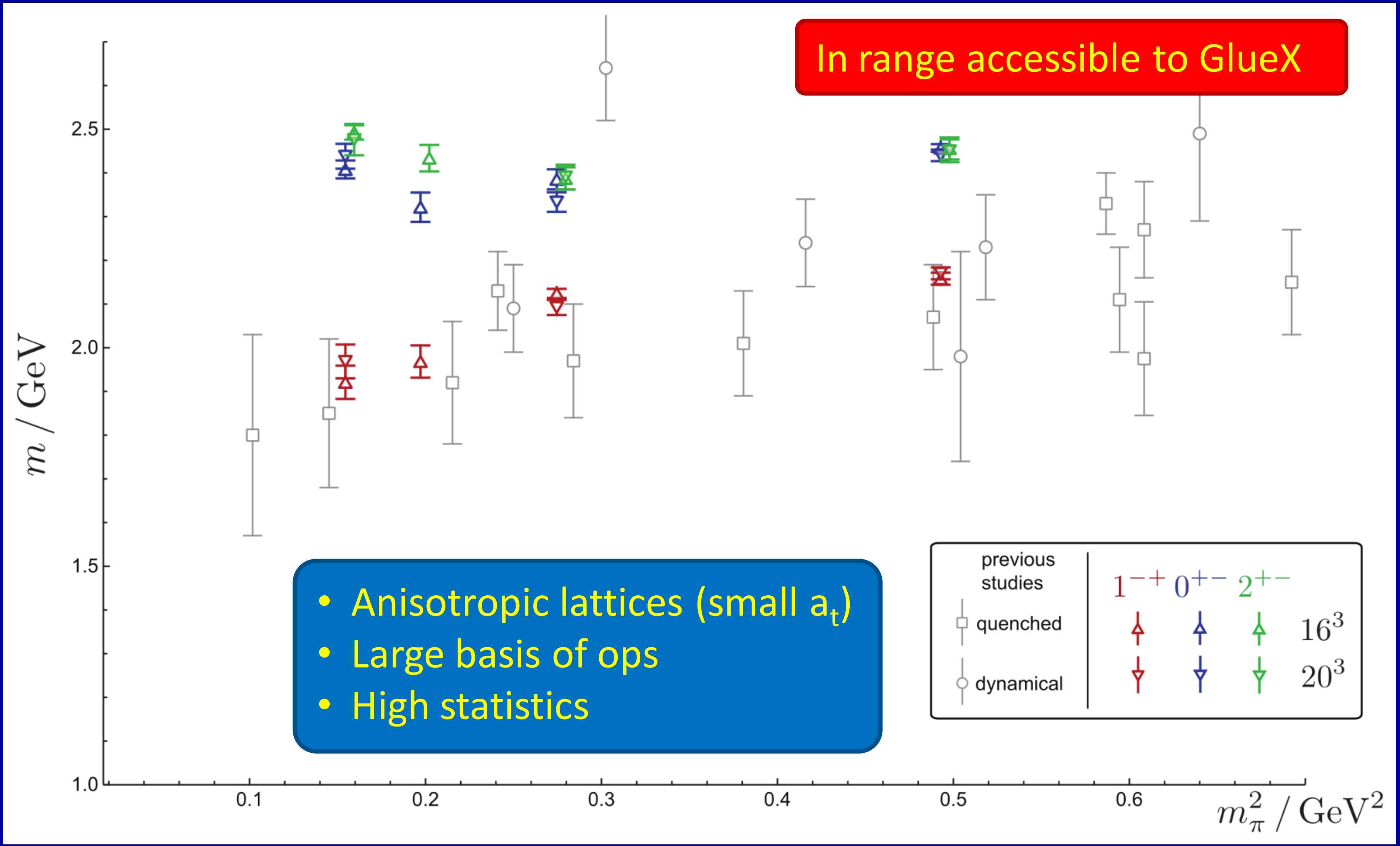

- Anisotropic lattices (small a_t)
- Large basis of ops
- High statistics

Kaons

Lower the light quark mass ($N_f = 2+1$) — SU(3) sym breaking

M_π / MeV	700	520	440	400
M_K / M_π	1	1.2	1.3	1.4

c.f. physical
$M_K / M_\pi = 3.5$

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No longer is C-parity a good quantum number for kaons (or a gen. of C-parity)

Combine J^{P+} and J^{P-} operators

Physically, axial kaons [$K_1(1270)$, $K_1(1400)$] are a mixture
Suggested mixing angle $\approx 45^o$ (combination of exp and models)

But...

Kaons

Lower th

M_π / M

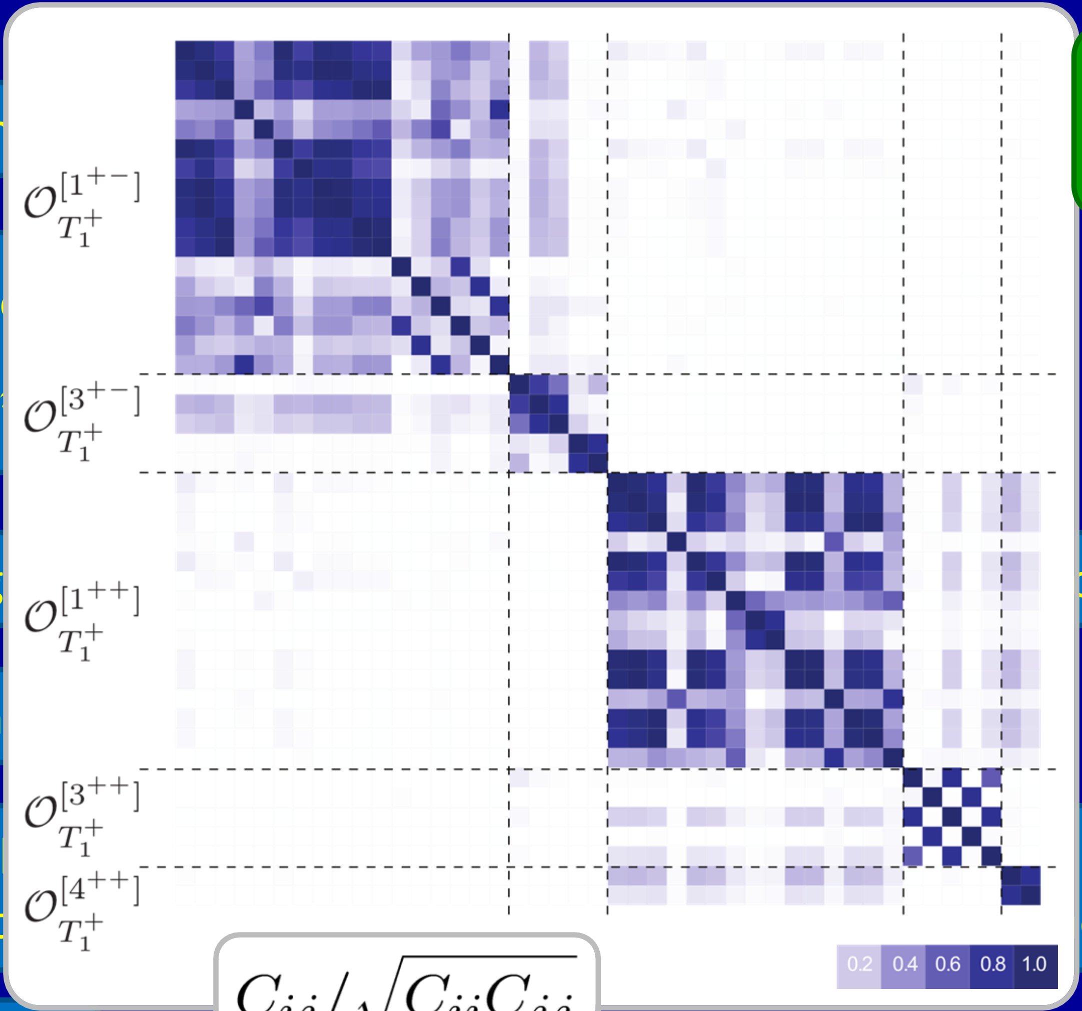
M_K / M

No long

Combin

Physical
Suggest

But...



$$C_{ij}/\sqrt{C_{ii}C_{jj}}$$

16^3
$M_\pi \approx 400$ MeV
$M_K/M_\pi \approx 1.4$

c.f. physical
$M_K/M_\pi = 3.5$

or a gen. of C-parity)

odels)

Kaons – Operator Overlaps

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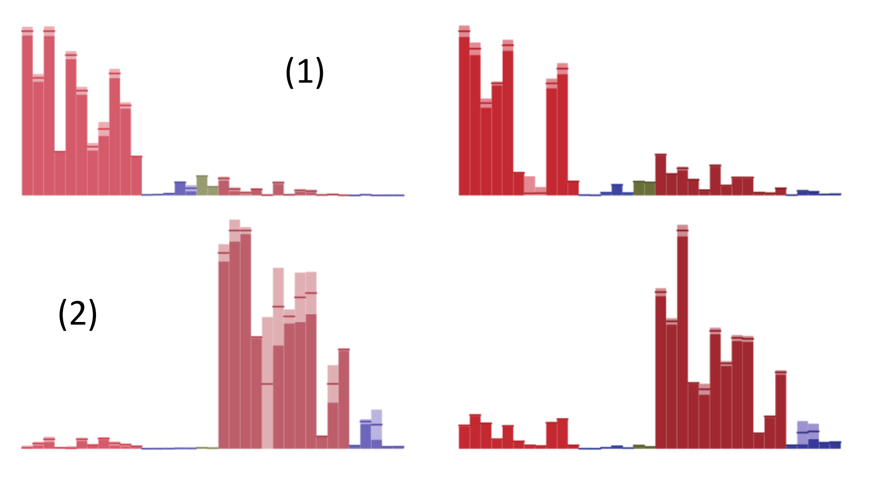

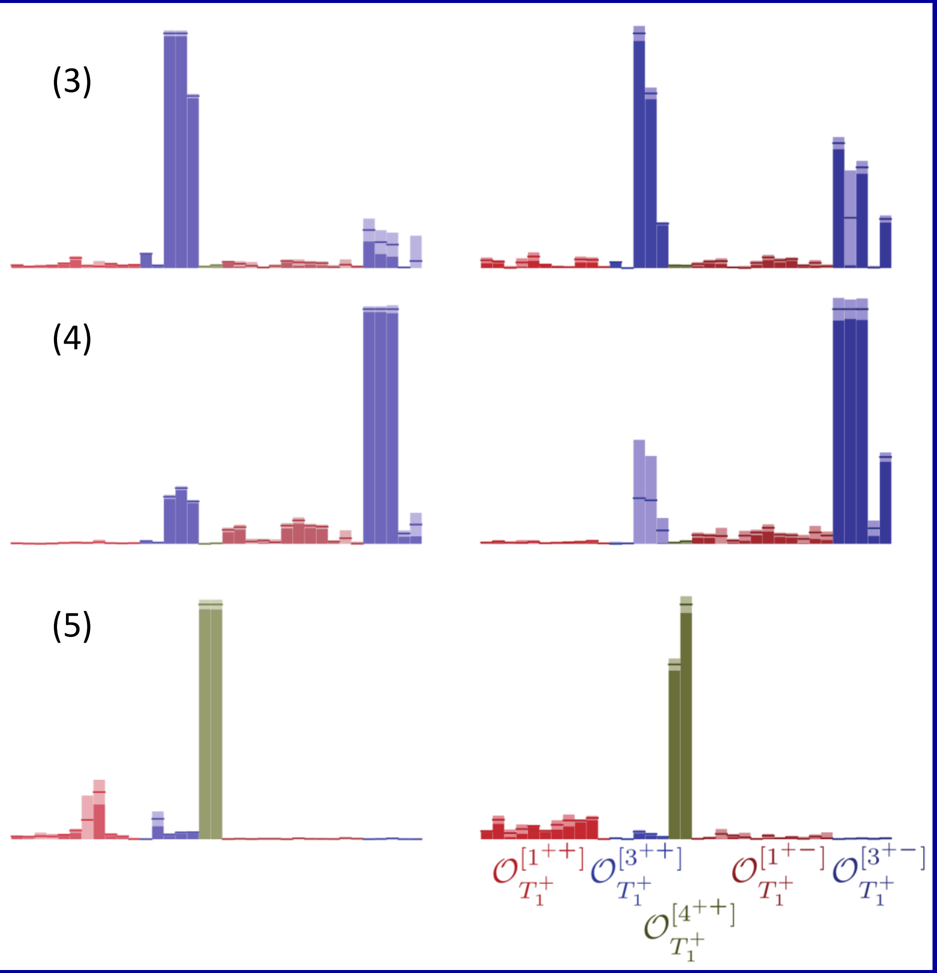

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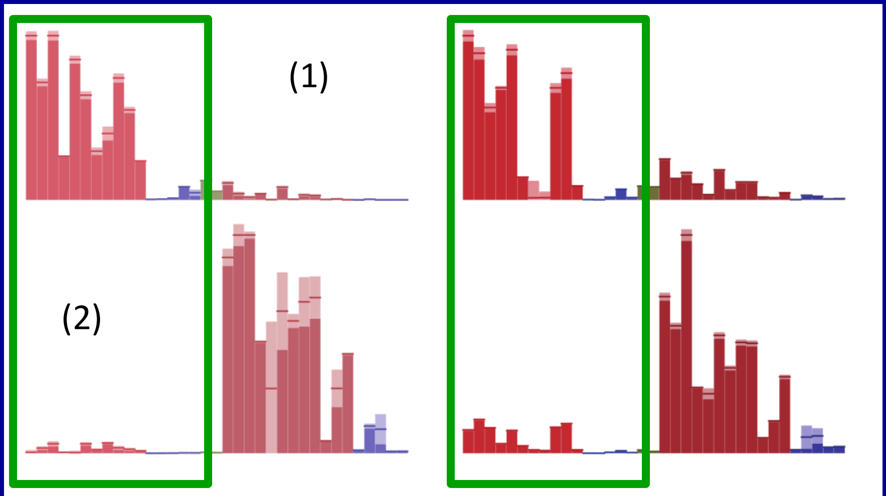

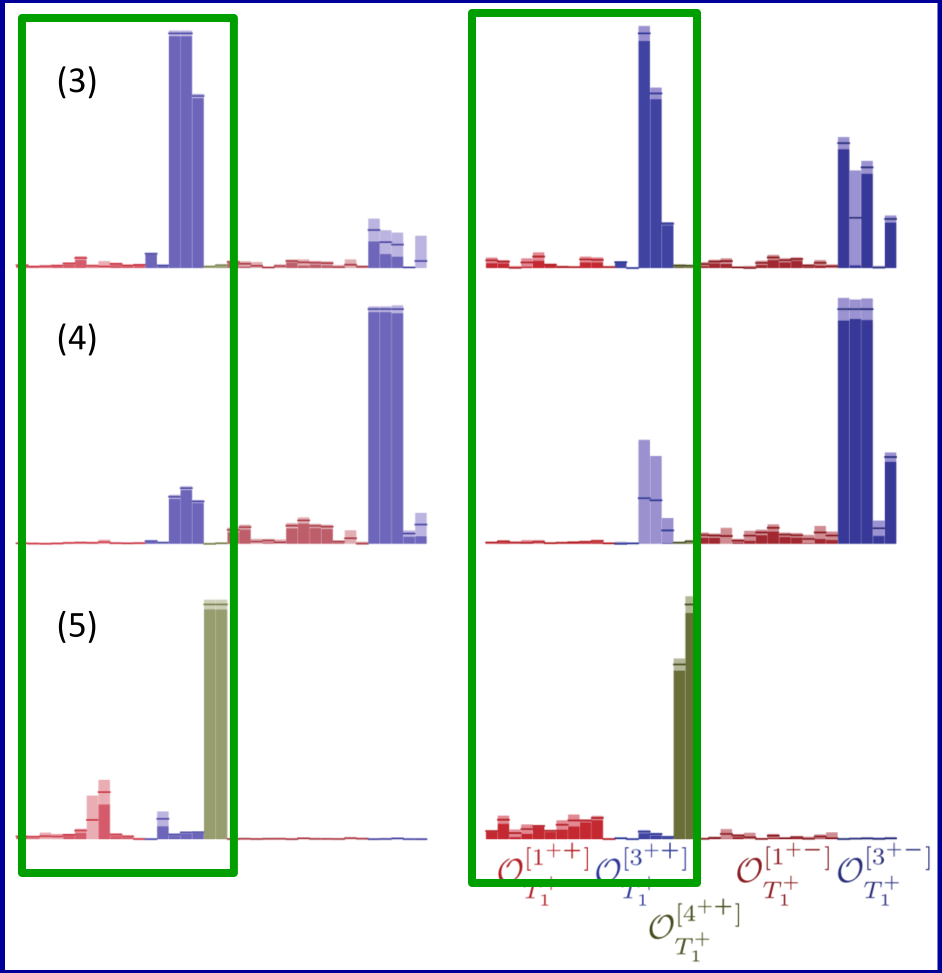

J^{++}

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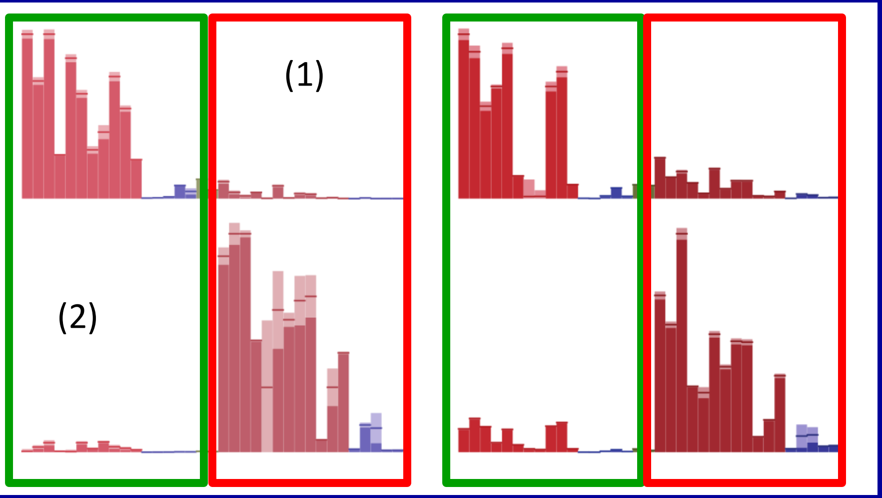
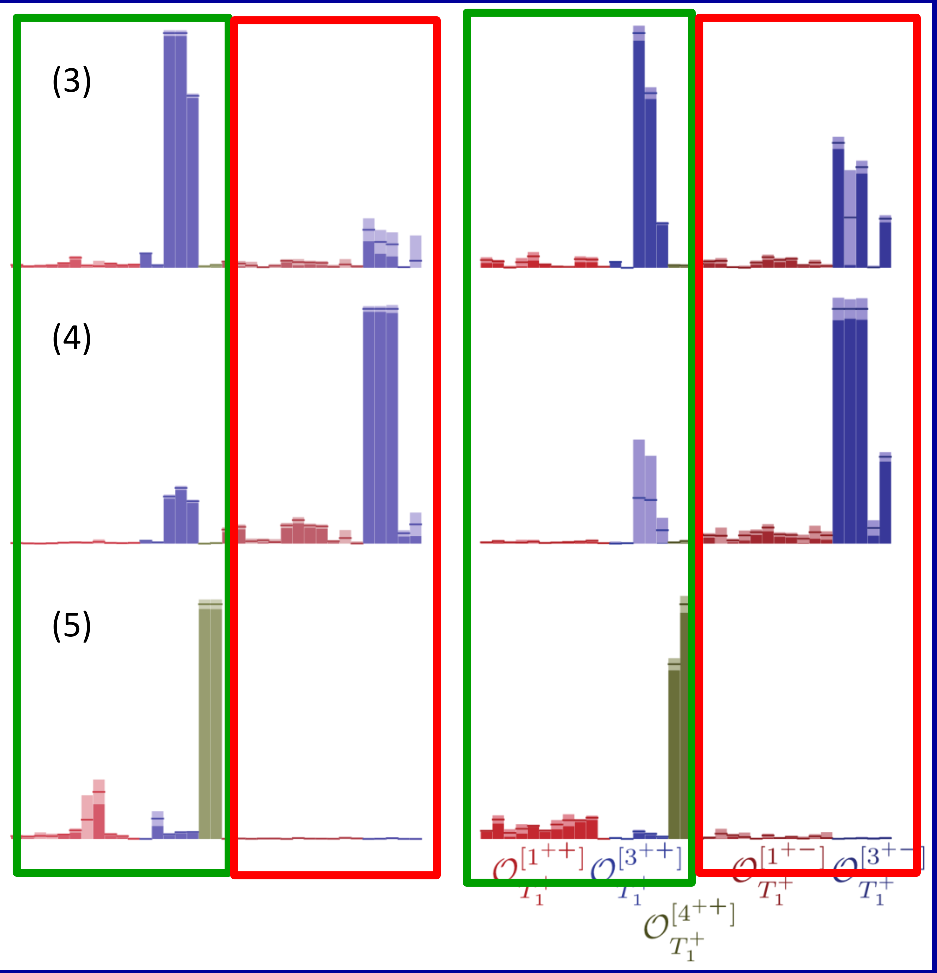
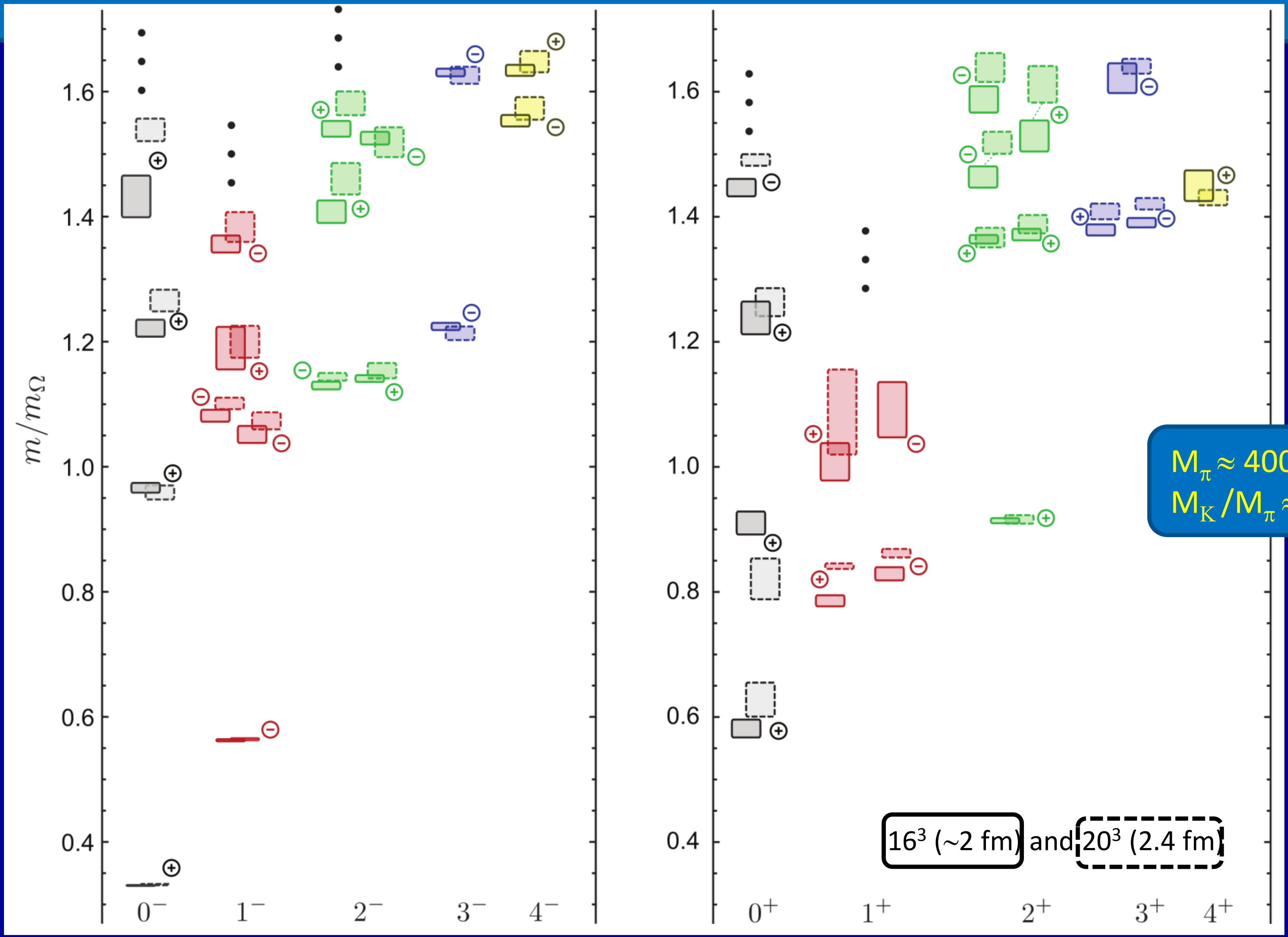




J^{++}

J^{+-}

Kaons - spectrum



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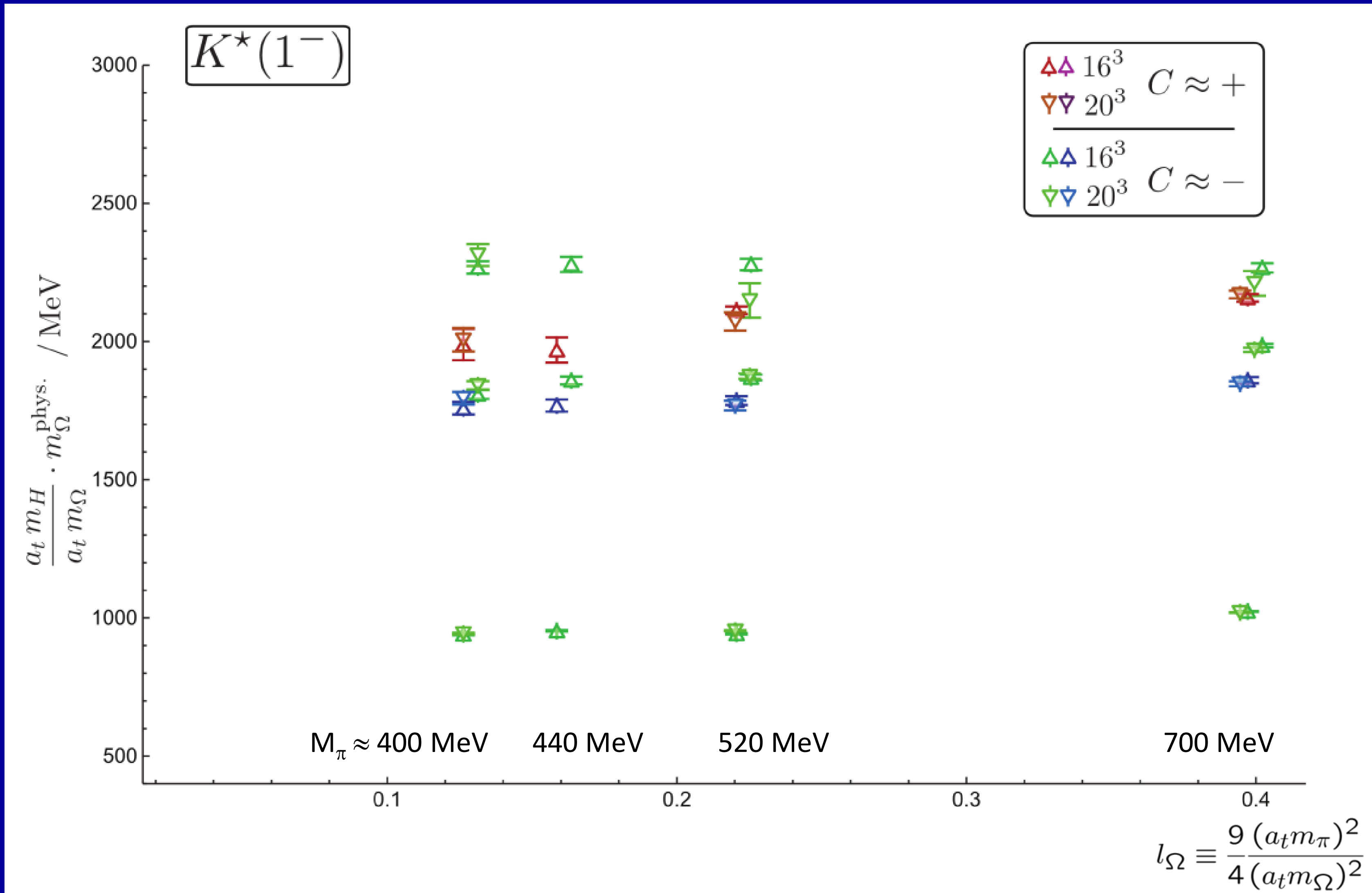
Kaons – Various pion masses



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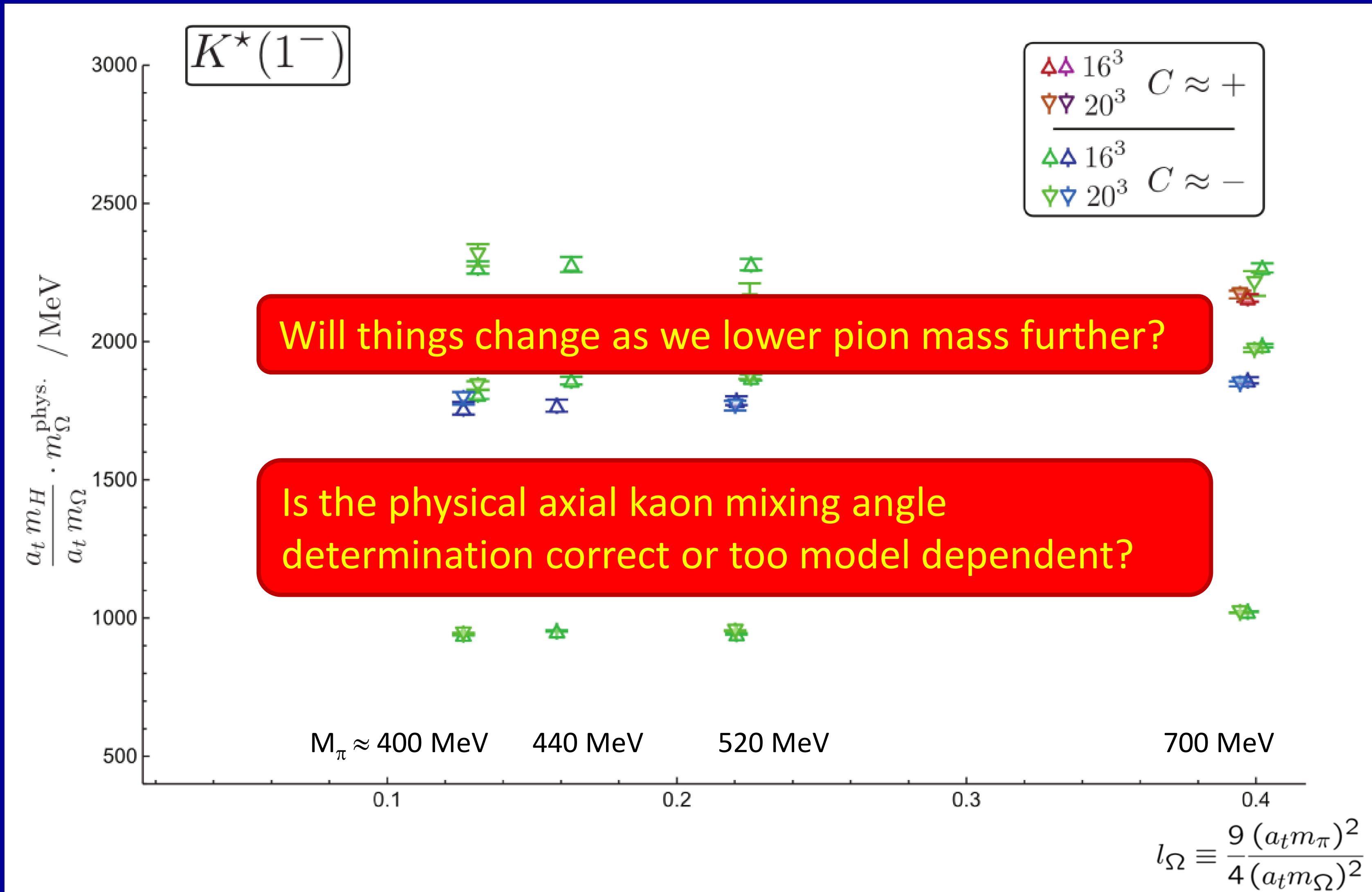
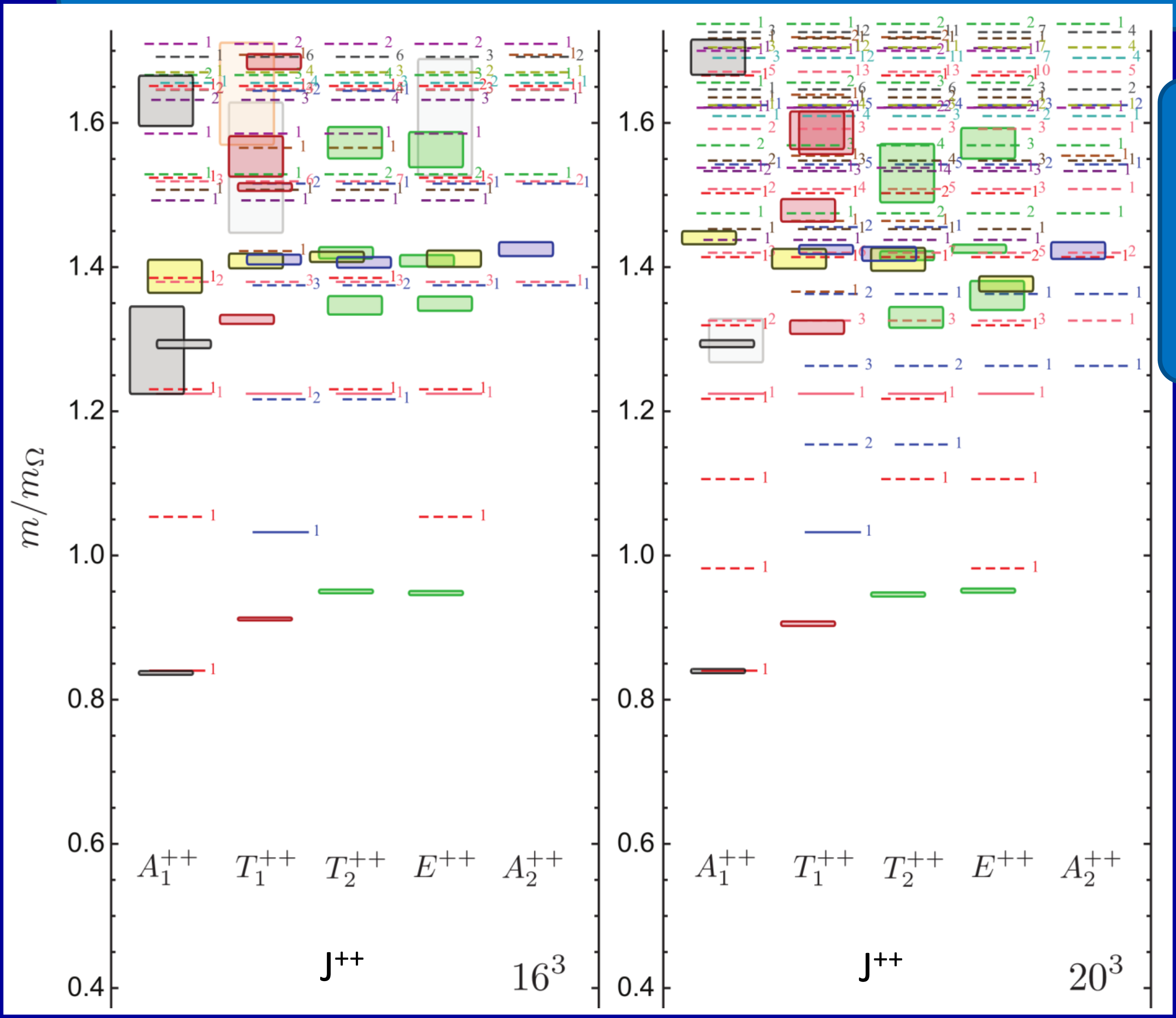


Will things change as we lower pion mass further?

Is the physical axial kaon mixing angle determination correct or too model dependent?

Multi-particle states?



Finite box – discrete allowed momenta → discrete spectrum of multiparticle states

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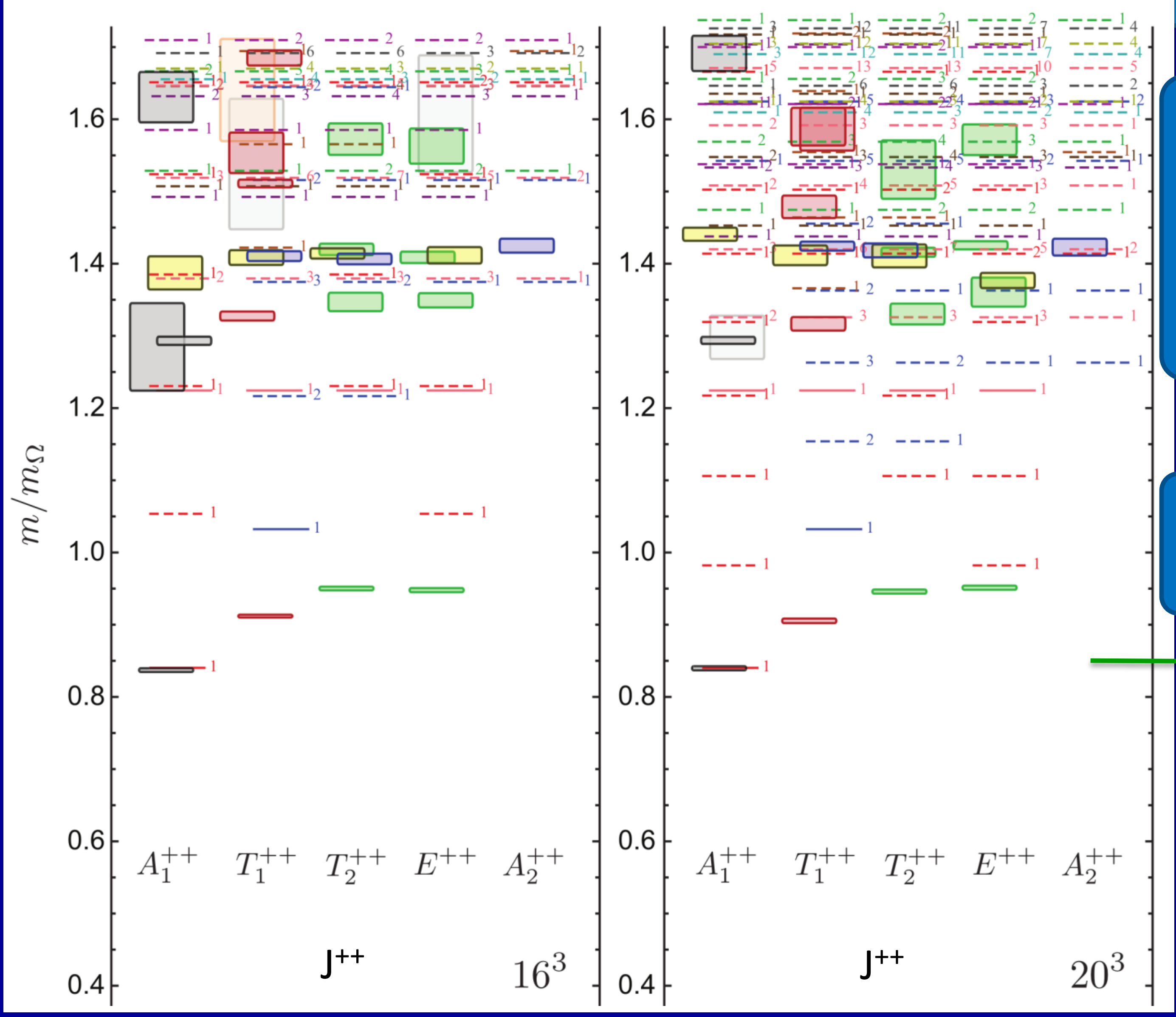


Finite box – discrete allowed momenta → discrete spectrum of multiparticle states

Expect two-meson states above $2m_\pi$

$2m_\pi \sim 0.85\ m_\Omega$

Where are they?

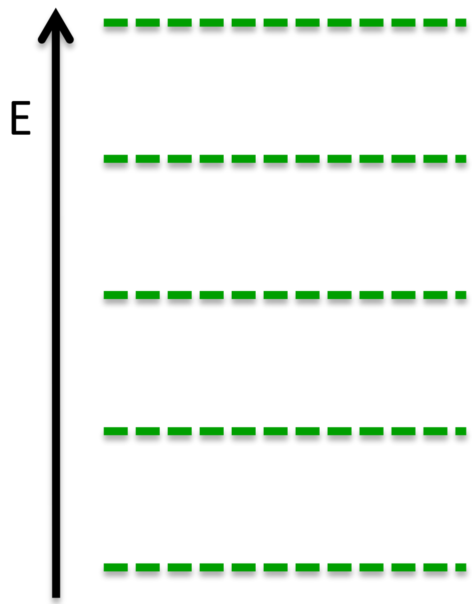
Multi-particle states

Euclidean time: can’t directly study dynamical properties like widths

Lüscher: energy shifts in finite volume → phase shift

Free 2-particle levels

$$\vec{p} = \frac{2\pi}{L_s}(n_x, n_y, n_z)$$



$$E = 2\sqrt{m^2 + \vec{p}^2}$$

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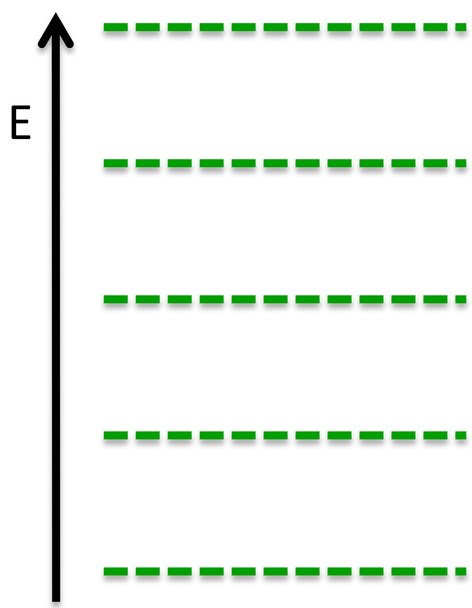
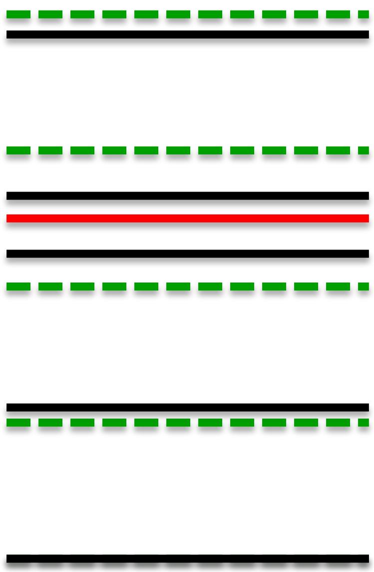
Lüscher: energy shifts in finite volume → phase shift

Free 2-particle levels



$$E = 2\sqrt{m^2 + \vec{p}^2}$$

Measured levels



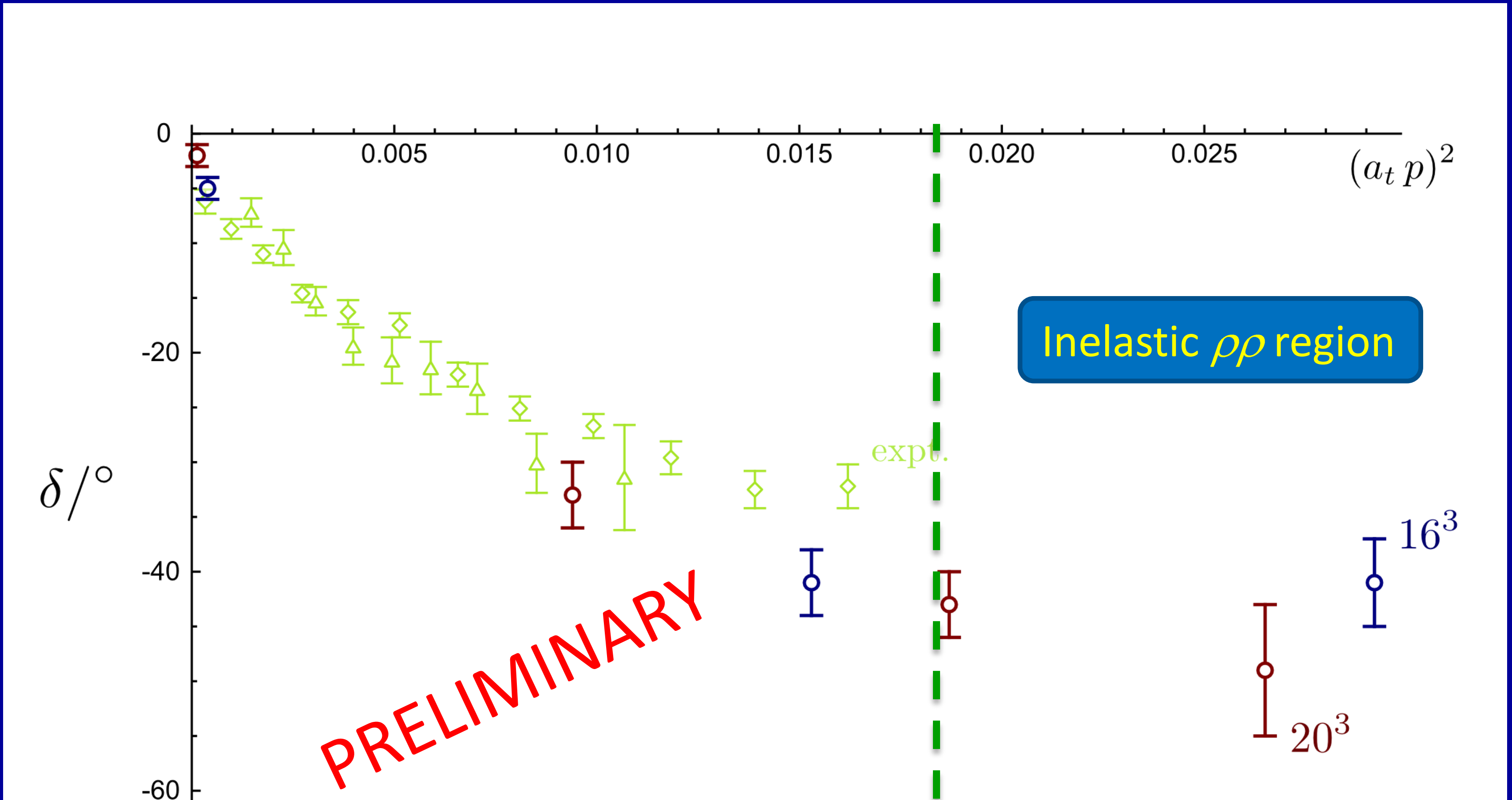
$$\vec{p} = \frac{2\pi}{L_s}(n_x, n_y, n_z)$$

Extract phase shift at discrete E – Lüscher method

$$\Delta E(L_s) \longrightarrow \delta(E, L_s)$$

$\pi\pi$ isospin 2

$M_\pi \approx 400$ MeV

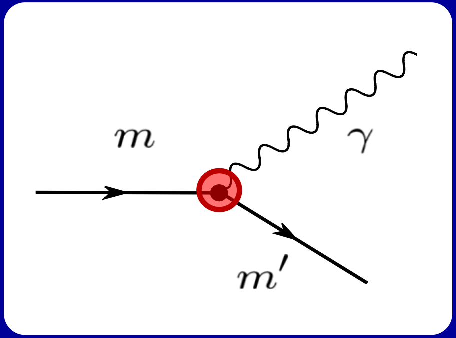


$$\mathcal{O}_{\pi\pi} = \sum_{\Omega_{\vec{p}}} Y_L^M(\Omega_{\vec{p}})\ O_\pi(\vec{p})\ O_\pi(-\vec{p})$$

Photocouplings

Charmonium (quenched) – testing method

$$C_{ij}(t_f,t,t_i) = < 0|O_i(t_f)\ \bar{\psi}(t)\gamma^\mu\psi(t)\ O_j(t_i)|0 >$$



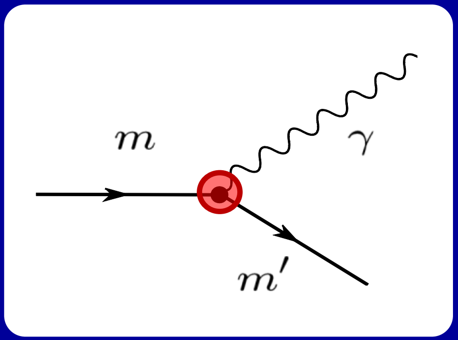
PR D79 094504 (2009)

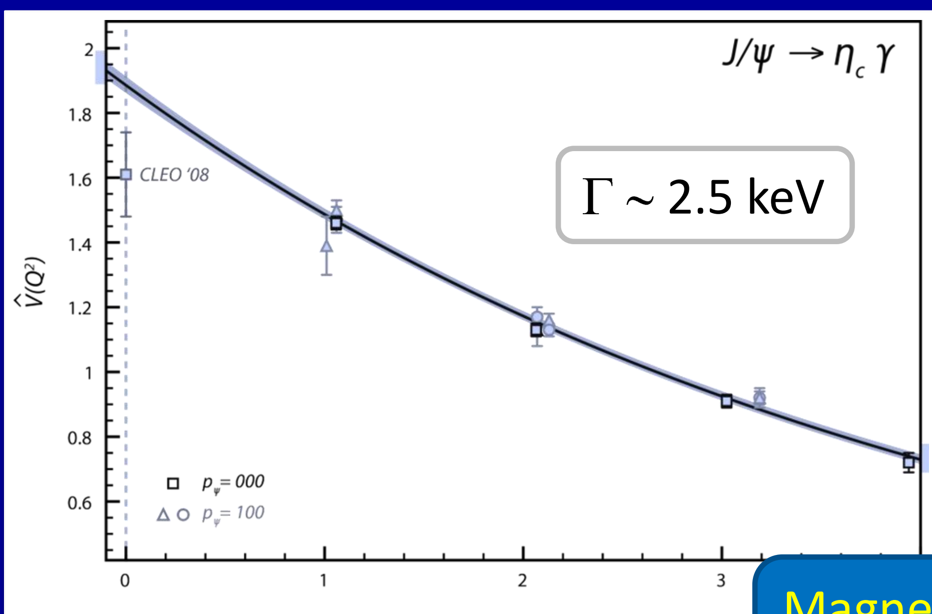
Photocouplings

Charmonium (quenched) – testing method

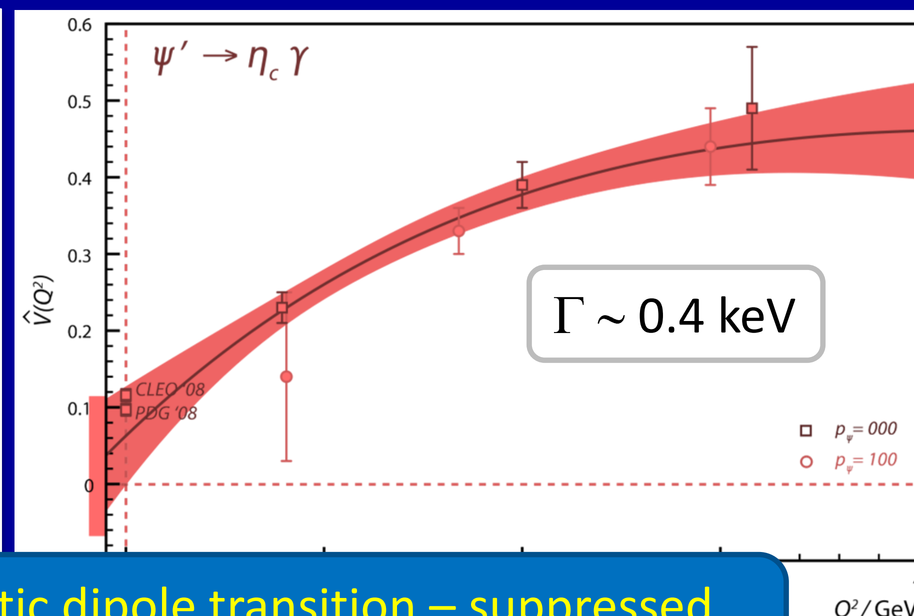
$$C_{ij}(t_f, t, t_i) =< 0|O_i(t_f)\ \bar{\psi}(t)\gamma^\mu\psi(t)\ O_j(t_i)|0 >$$

Conventional vector – pseudoscalar transition





$\Gamma \sim 2.5$ keV



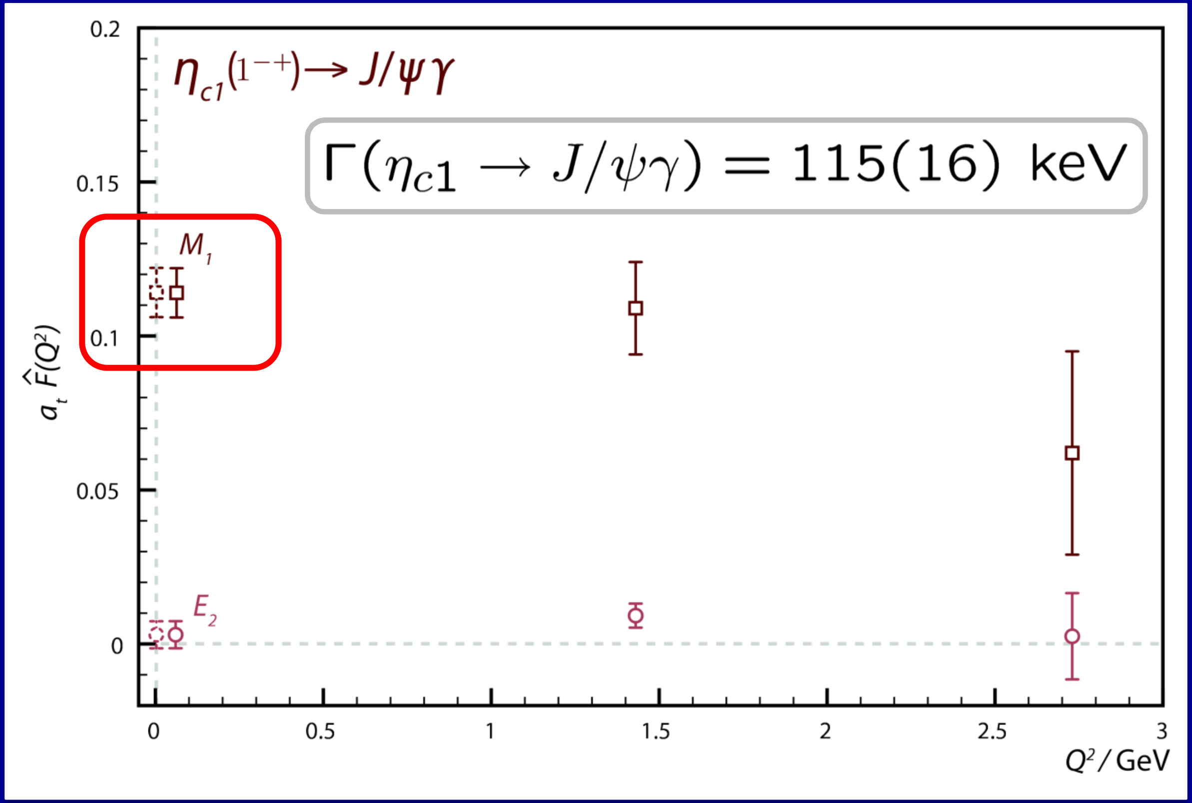
$\Gamma \sim 0.4$ keV

Magnetic dipole transition – suppressed
(in quark model spin flip ~ $1/m_c$)

PR D79 094504 (2009)

Photocouplings

Exotic meson photocoupling



PR D79 094504 (2009)

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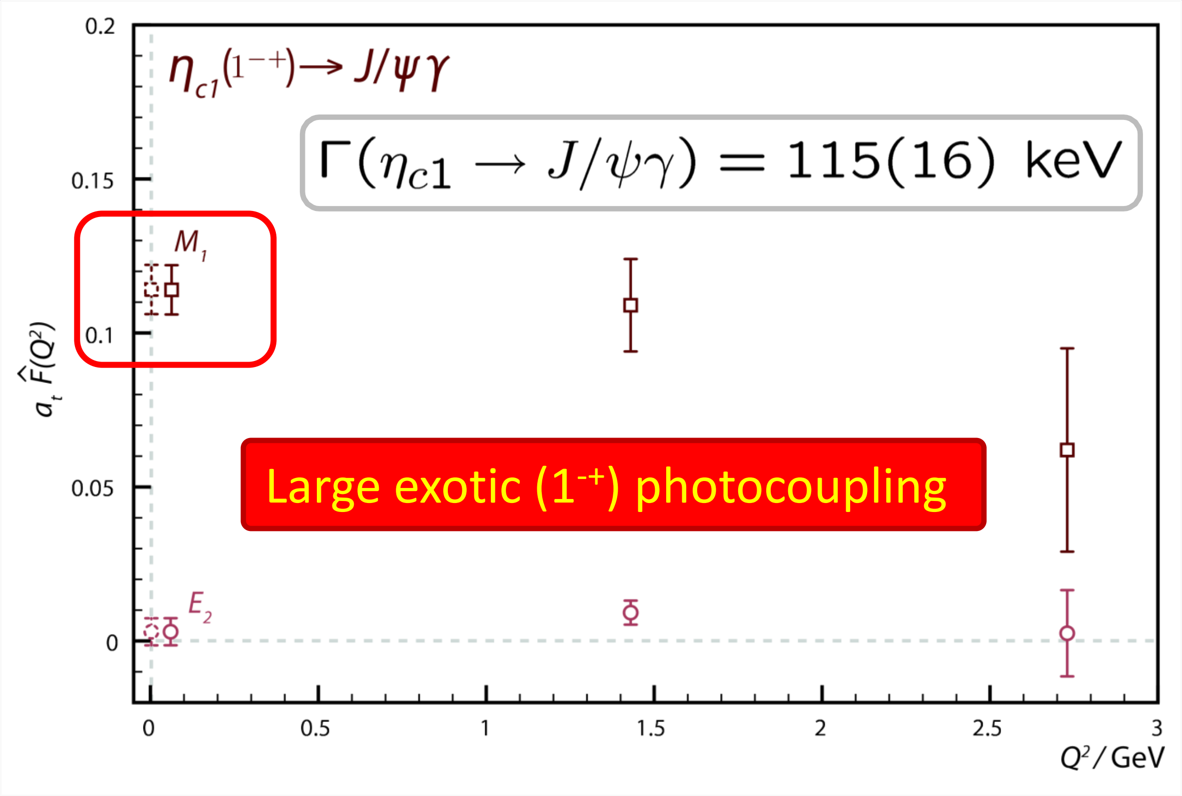

Same scale as many measured conventional charmonium transitions

BUT very large for an M_1 transition
$\Gamma(J/\psi \to \eta_c\gamma) \sim 2$ keV

Suggests a spin-triplet hybrid

PR D79 094504 (2009)

Summary and Outlook

Summary

- Our first results on light mesons – **technology and method work**
- **First spin 4** meson extracted and confidently identified on lattice
- **Exotics** (and non-exotic **hybrid** candidates)
- Isovectors and kaons

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- Our first results on light mesons – **technology and method work**
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Outlook – ongoing work

- Multi-meson operators – resonance physics
- Disconnected diagrams – isoscalars and multi-mesons
- Baryons
- Photocouplings
- Lighter pion masses and larger volumes

Extra Slides

Spin on the lattice

On a lattice, 3D rotation group is broken to Octahedral Group

In continuum:

Infinite number of *irreps*: J = 0, 1, 2, 3, 4, ...

Spin on the lattice

On a lattice, 3D rotation group is broken to Octahedral Group

<u>In continuum:</u>

Infinite number of *irreps*: J = 0, 1, 2, 3, 4, ...

<u>On lattice:</u>

Finite number of *irreps*: A_1, A_2, T_1, T_2, E (and others for half-integer spin)

Irrep	A_1	A_2	T_1	T_2	E
Dim	1	1	3	3	2

Cont. Spin	0	1	2	3	4	...
Irrep(s)	A_1	T_1	T_2 + E	$T_1 + T_2 + A_2$	$A_1 + T_1 + T_2$ + E	...

Spin on the lattice

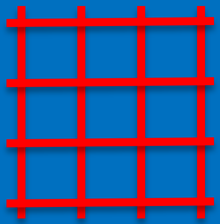
On a lattice, 3D rotation group is broken to Octahedral Group

2D Example

Eigenstates of angular momentum are $e^{iJ\phi}$

On a lattice, the allowed rotations are $\phi \rightarrow \phi + \pi/2$

Can't distinguish e.g. $J = 0$ and $J = 4$



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Irrep	A_1	A_2	T_1	$\mathbf{T_2}$	**E**
dim	1	1	3	3	2
cont. spins	0,4,6, ...	3,6,7, ...	1,3,4, ...	**2**,3,4, ...	**2**,4,5, ...

(and others for half-integer spin)

Spin and operator construction

Construct operators which only overlap on to one spin in the continuum limit

$$\langle 0|\mathcal{O}^{J,M}|J',M'\rangle = Z^{[J]}\delta_{J,J'}\delta_{M,M'}$$

definite J^{PC}

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'Subduce' operators on to lattice irreps (J → Λ):

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Given continuum op →
same Z in each Λ
(ignoring lattice mixing)

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Couple together Γ and many D using SU(2) Clebsch Gordans

E.g. γ_i x D = 1 x 1 → J = 0,1,2

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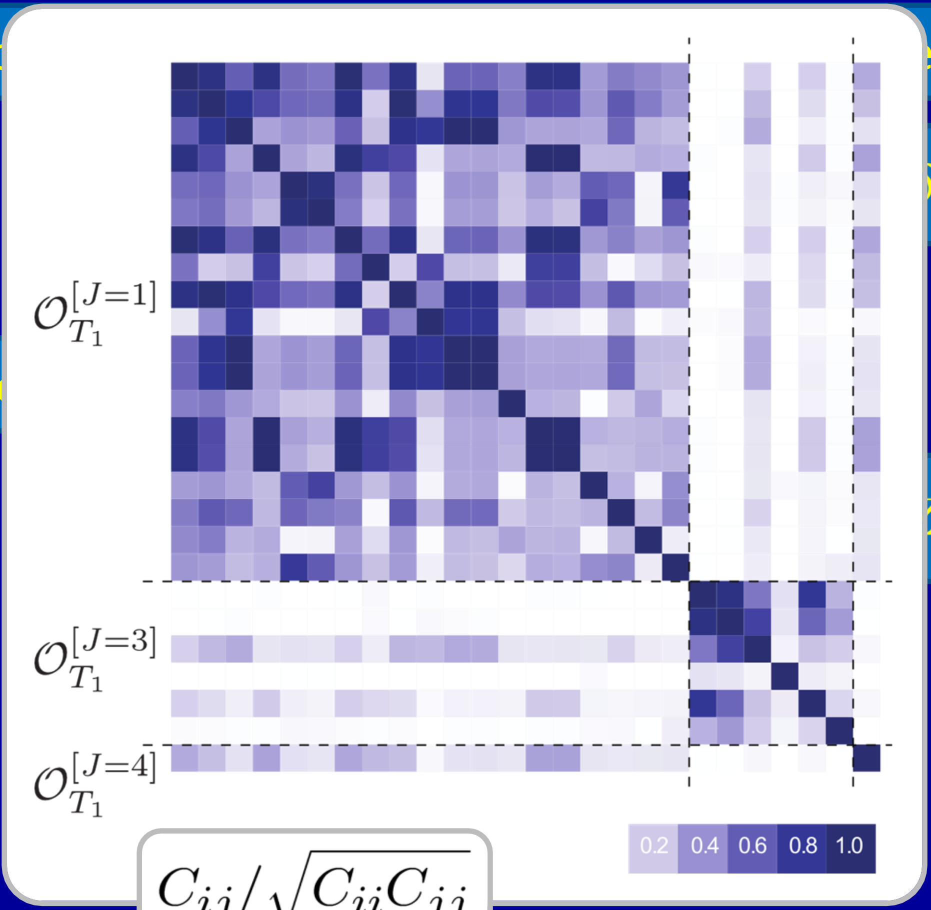
Spin and operator construction

Construct operato … continuum limit

… $\delta_{J,J'}\delta_{M,M'}$

'Subduce' operat…

… $Z^{[J]}\delta_{J,J'}$



$$C_{ij}/\sqrt{C_{ii}C_{jj}}$$

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- As an example: three degenerate 'light' quarks ($N_f = 3$, $M_\pi \approx 700$ MeV)
- Dynamical (unquenched). Only connected diagrams (isovectors and kaons)

PRL 103 262001 (2009) and arXiv:1004.4930

Calculation details

- Dynamical calculation. Clover fermions
- Anisotropic (a_s/a_t = 3.5), $a_s \sim 0.12$ fm, $a_t^{-1} \sim 5.6$ GeV
- Two volumes: 16^3 ($L_s \approx 2.0$ fm) and 20^3 ($L_s \approx 2.4$ fm)

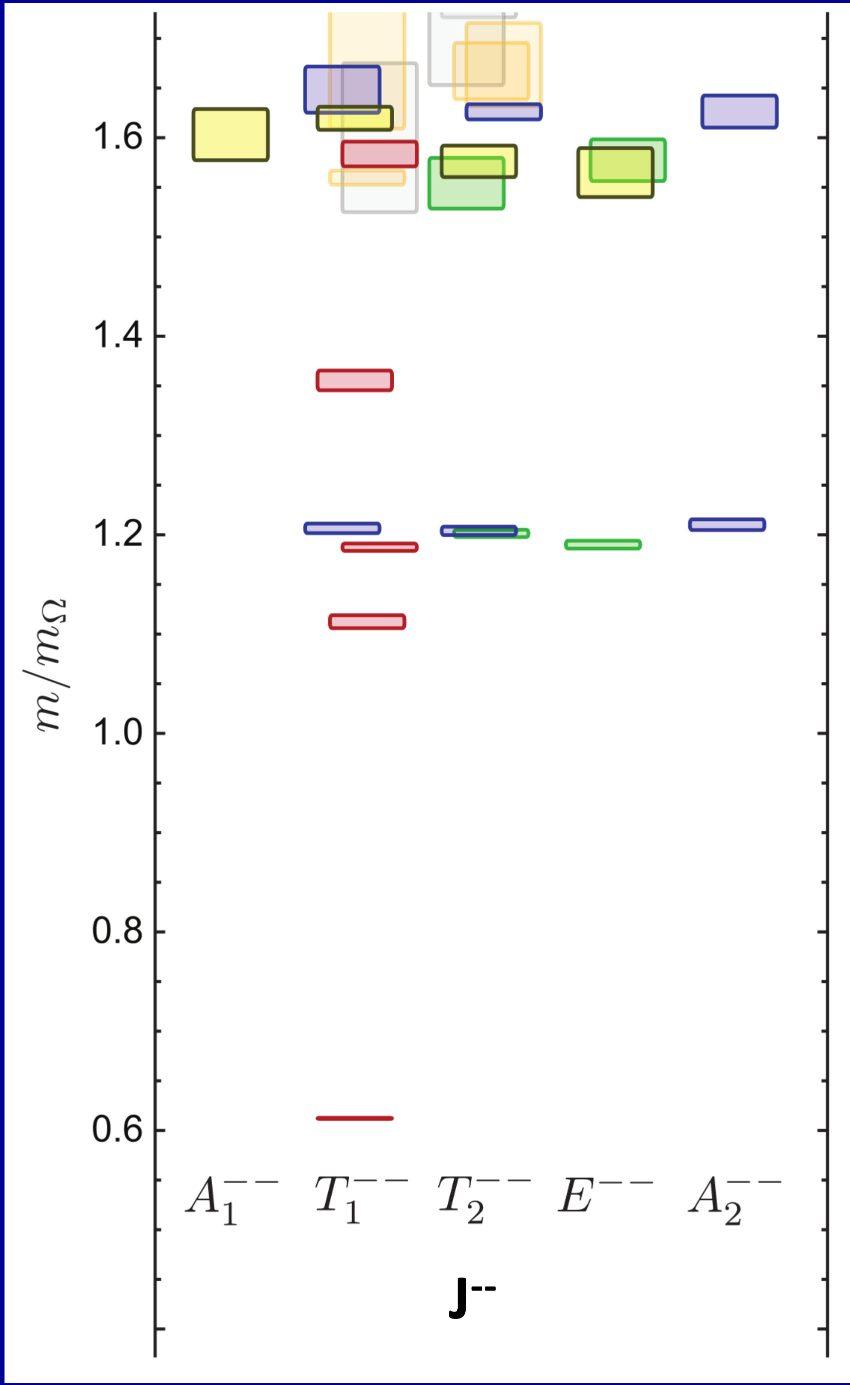
Lattice details in: PR D78 054501, PR D79 034502

- Only connected diagrams – Isovectors (I=1) and kaons
- As an example: three degenerate 'light' quarks (N_f = 3, $M_\pi \approx 700$ MeV)

$\sim$ 500 cfgs x 9 t-sources

- Also (N_f = 2+1) $M_\pi \approx 520, 440, 400$ MeV

Method details and results: PRL 103 262001 (2009) and arXiv:1004.4930

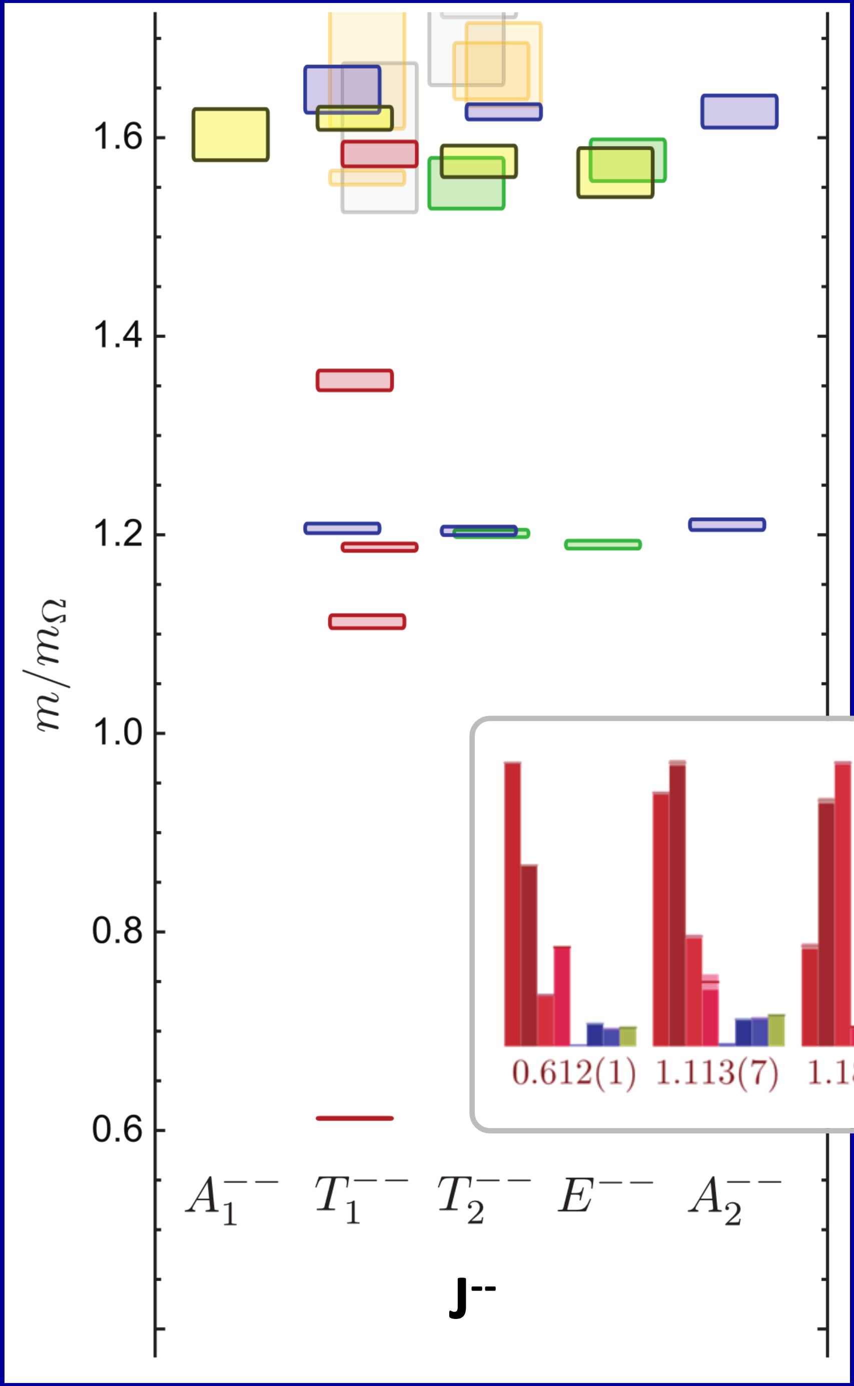


Z values

$$\langle 0|\mathcal{O}^{[J]}_{\Lambda,\lambda}|J',M\rangle = \mathcal{S}^{J,M}_{\Lambda,\lambda} Z^{[J]}\delta_{J,J'}$$

J = continuum spin of op

Z values



$$\langle 0|\mathcal{O}^{[J]}_{\Lambda,\lambda}|J', M\rangle = \mathcal{S}^{J,M}_{\Lambda,\lambda} Z^{[J]} \delta_{J,J'}$$

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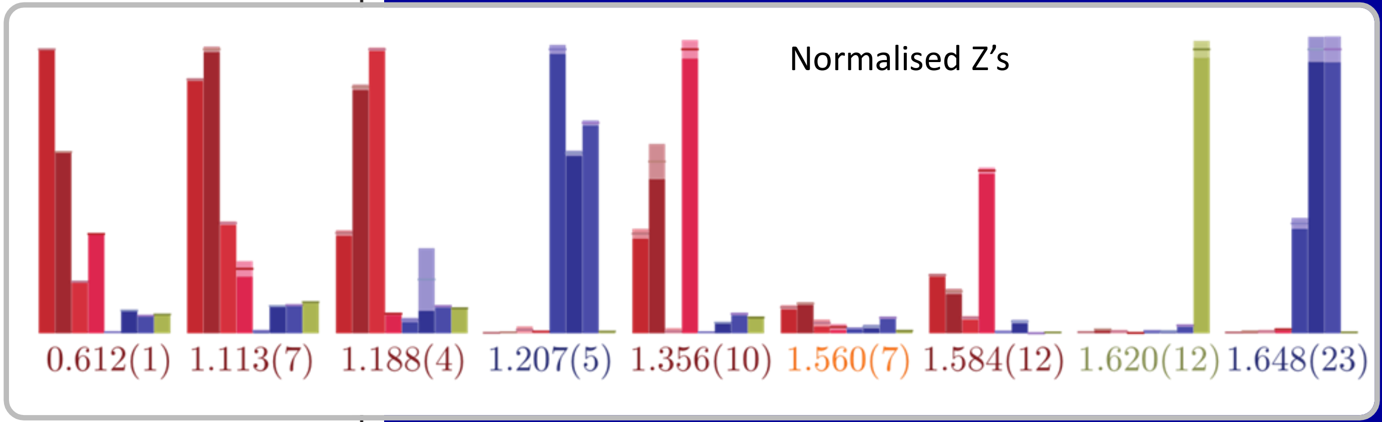

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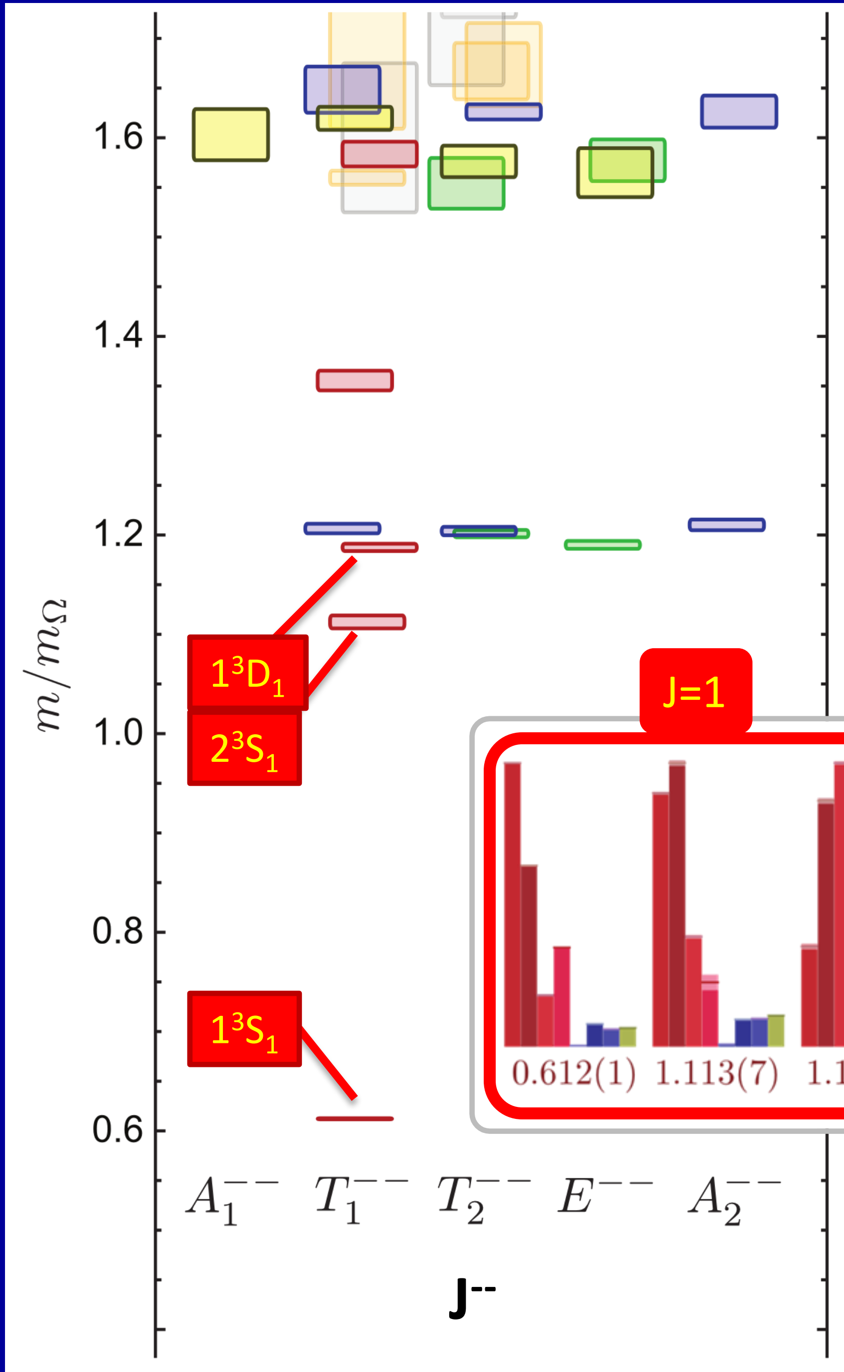

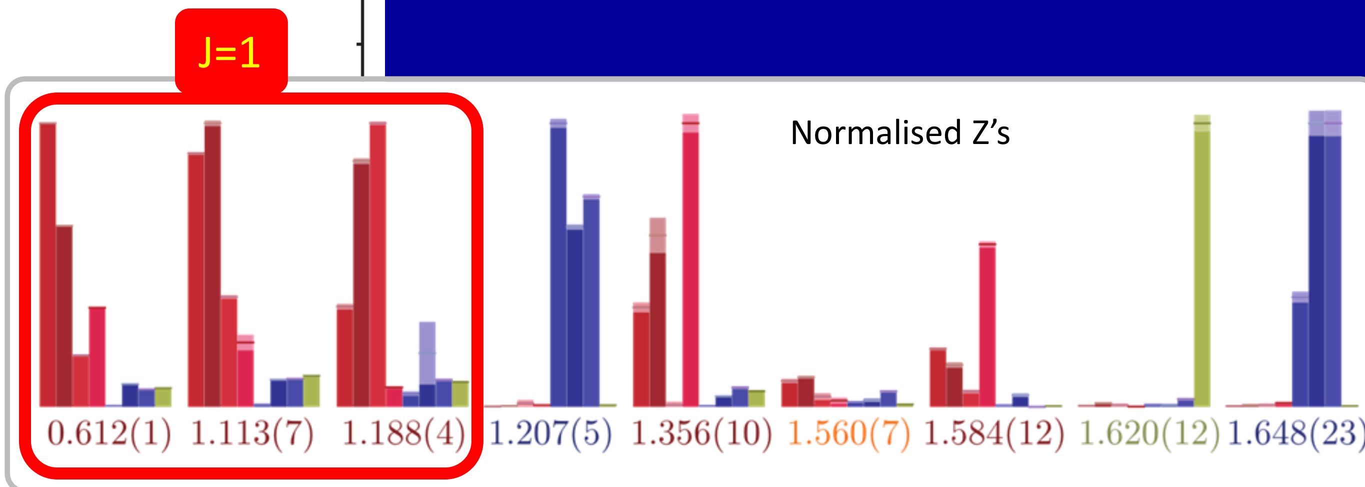

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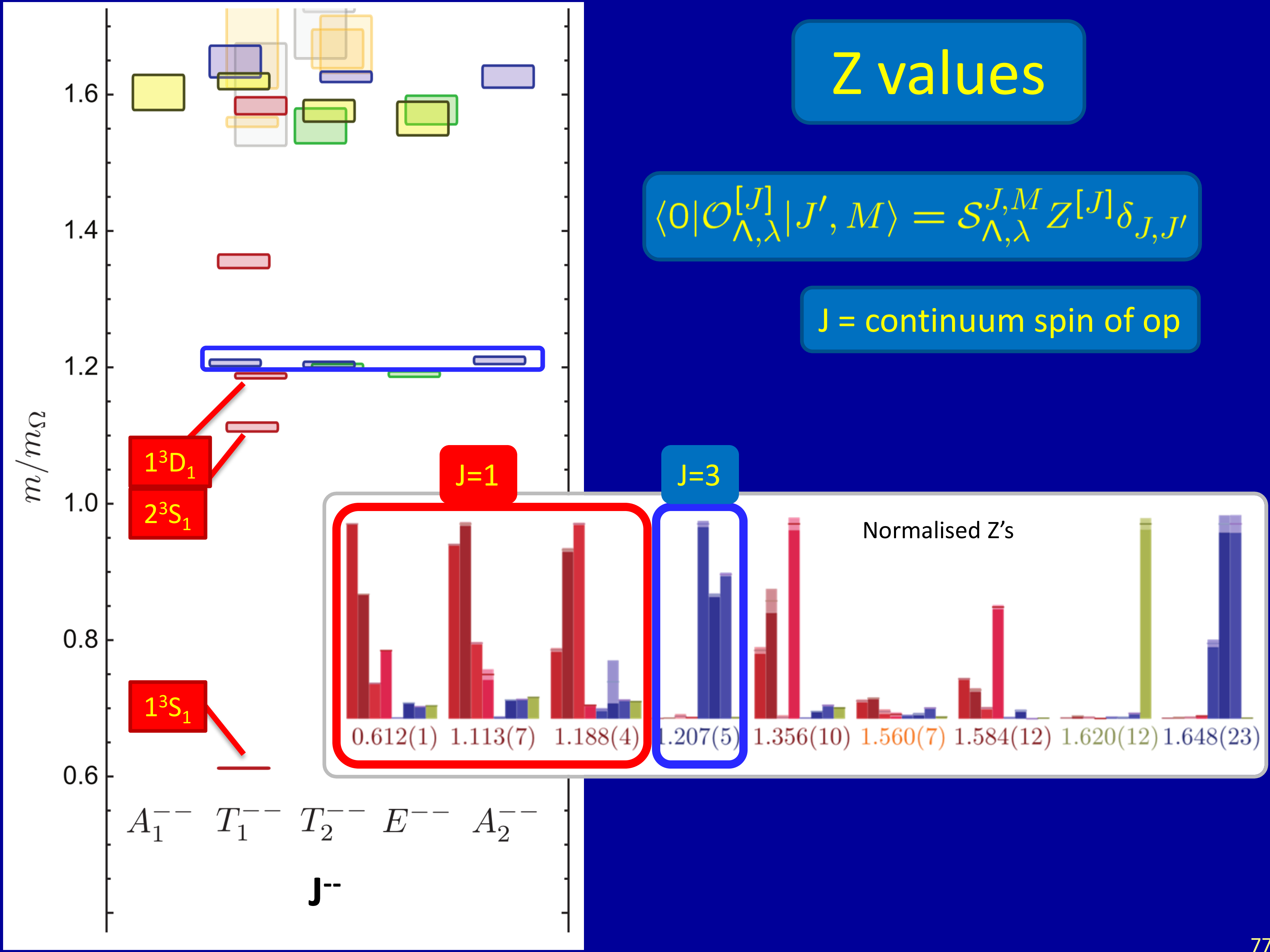

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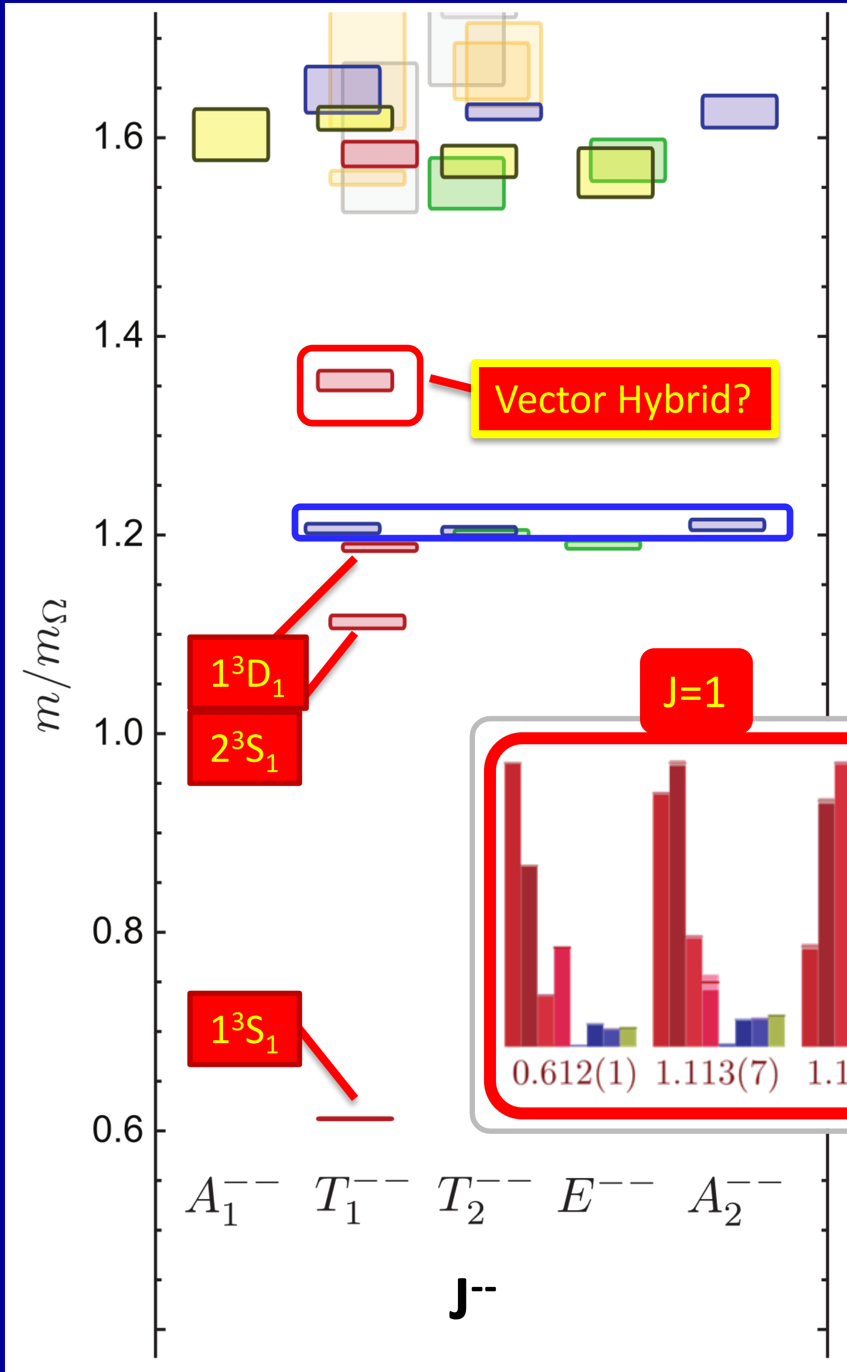

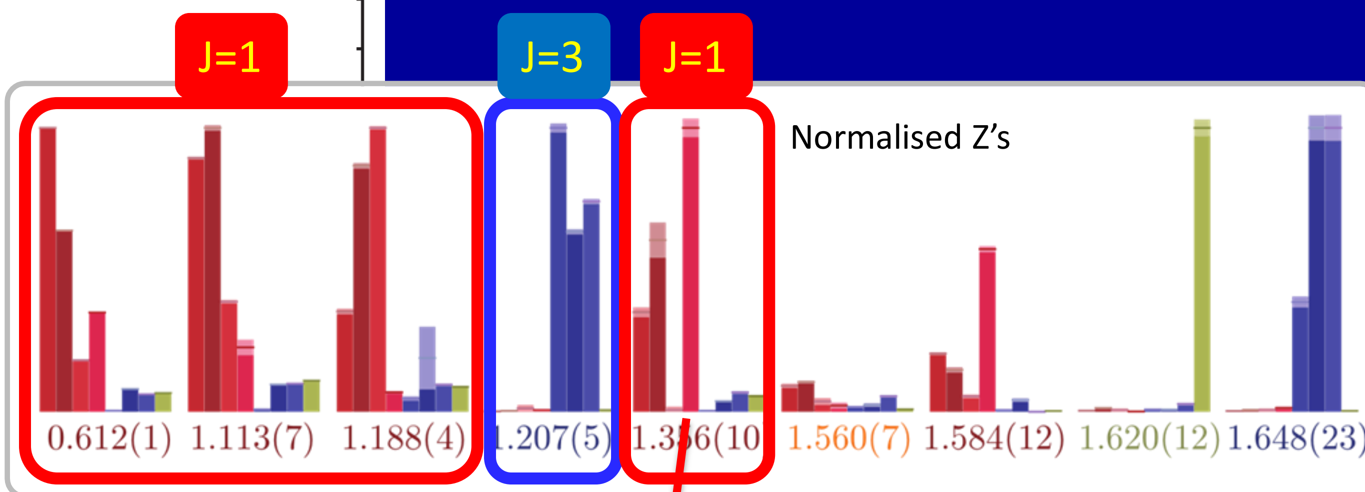

This operator ~ $[D_i, D_j]$

Z values

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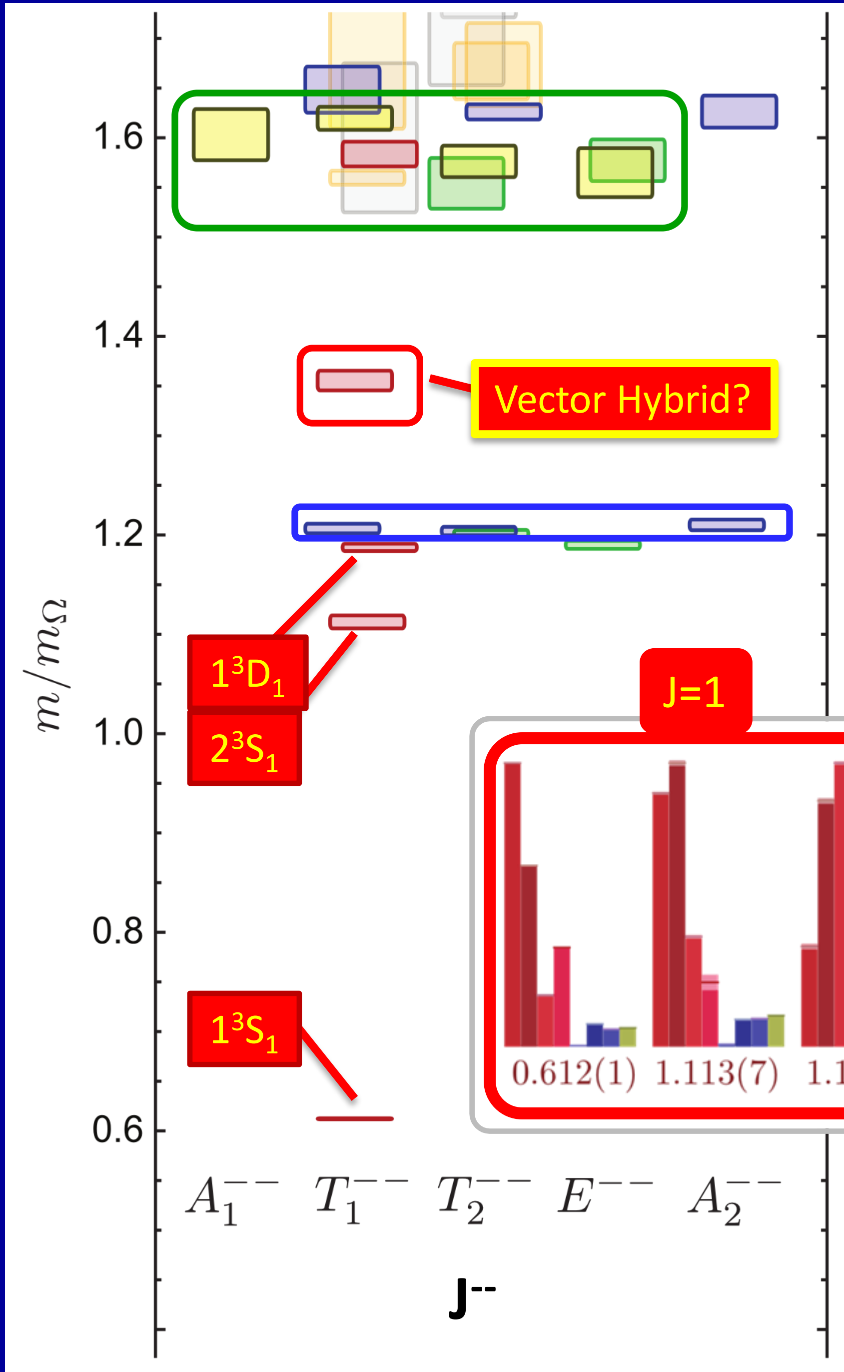

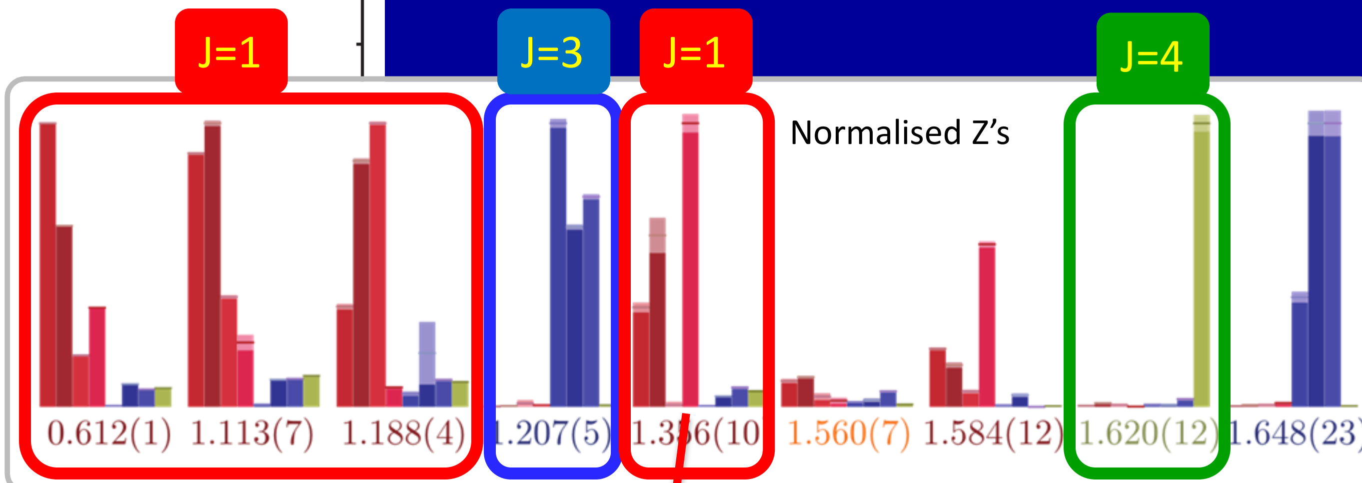

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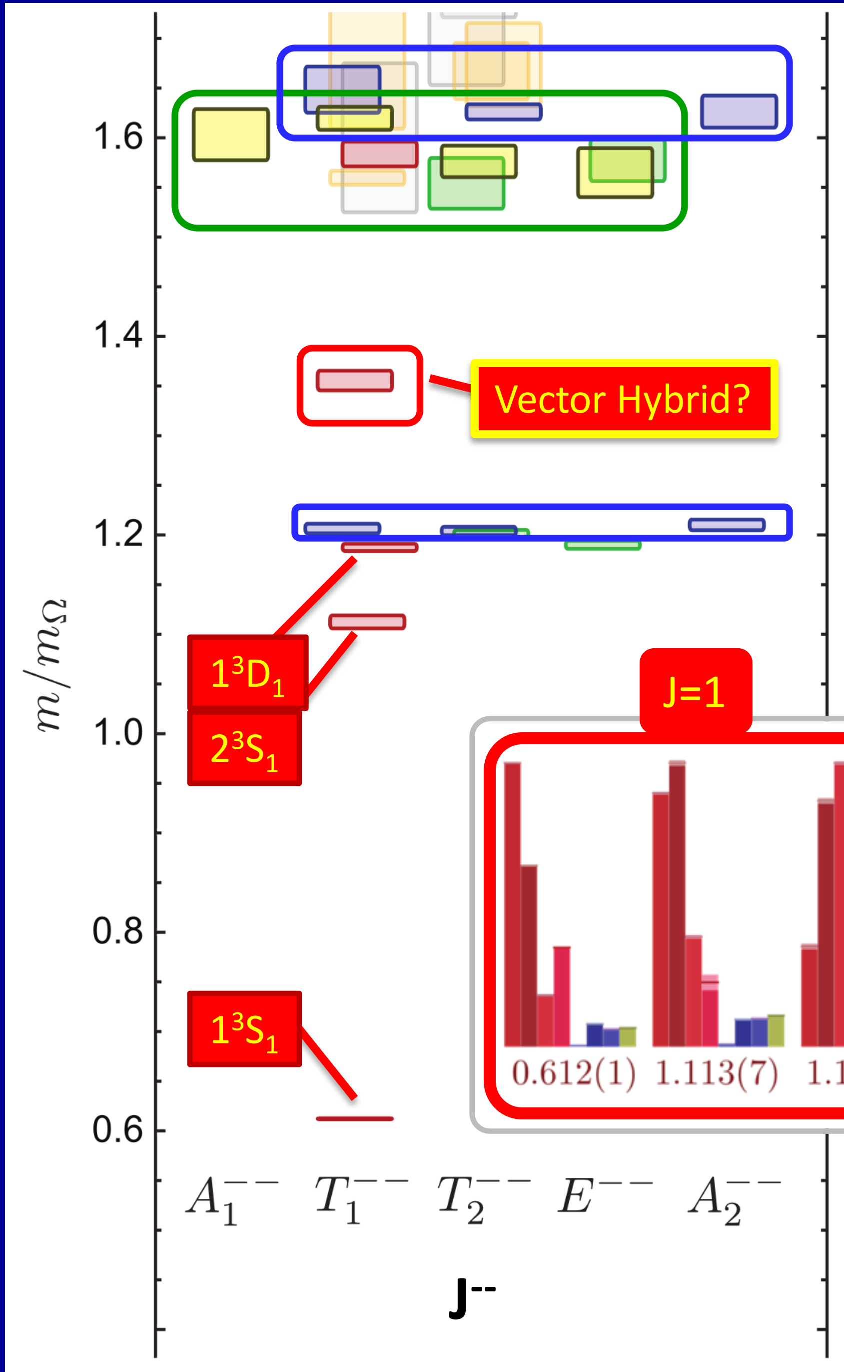
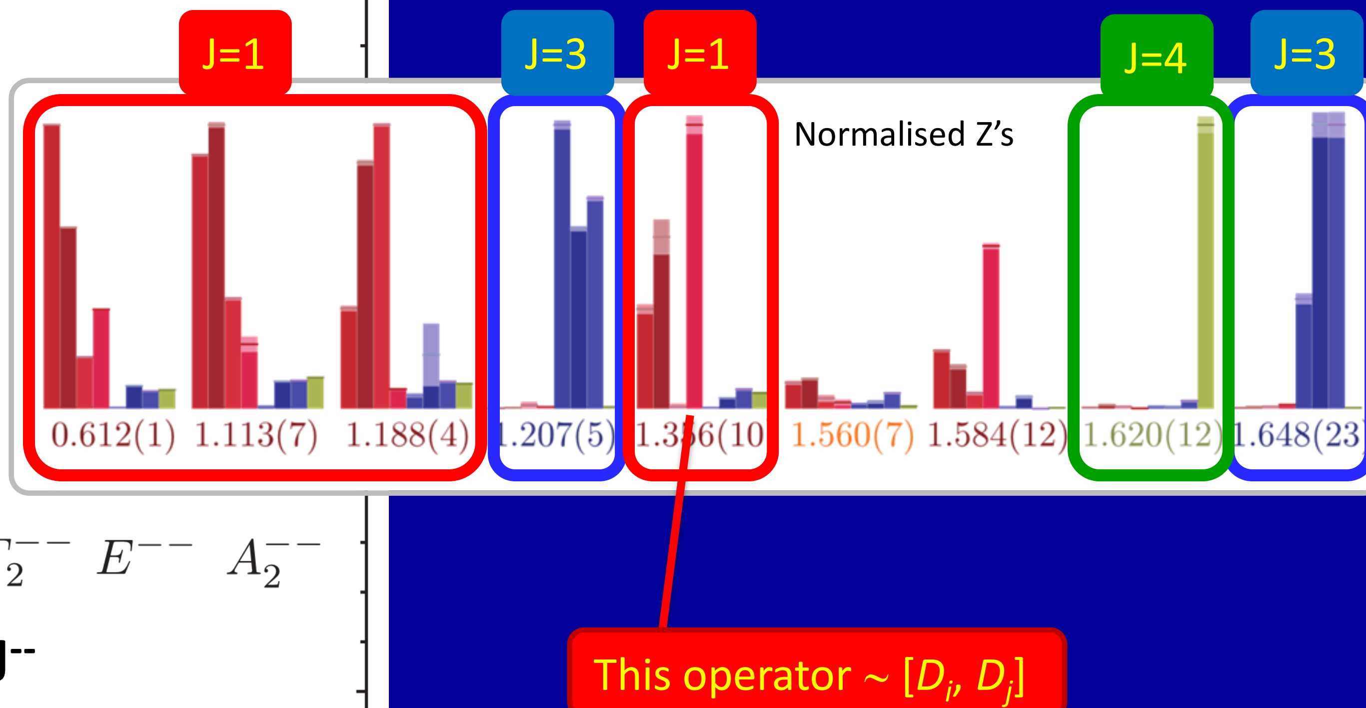
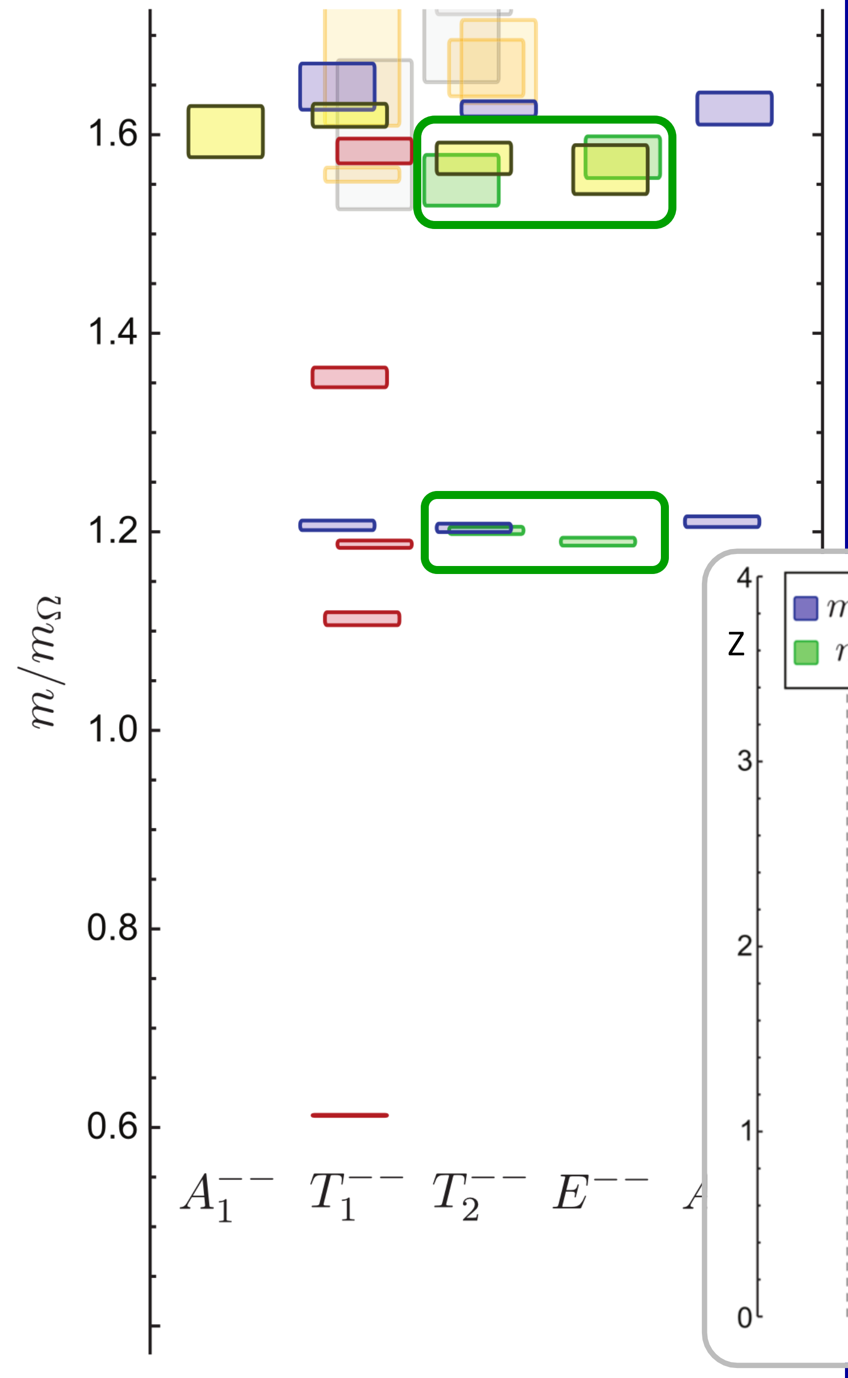


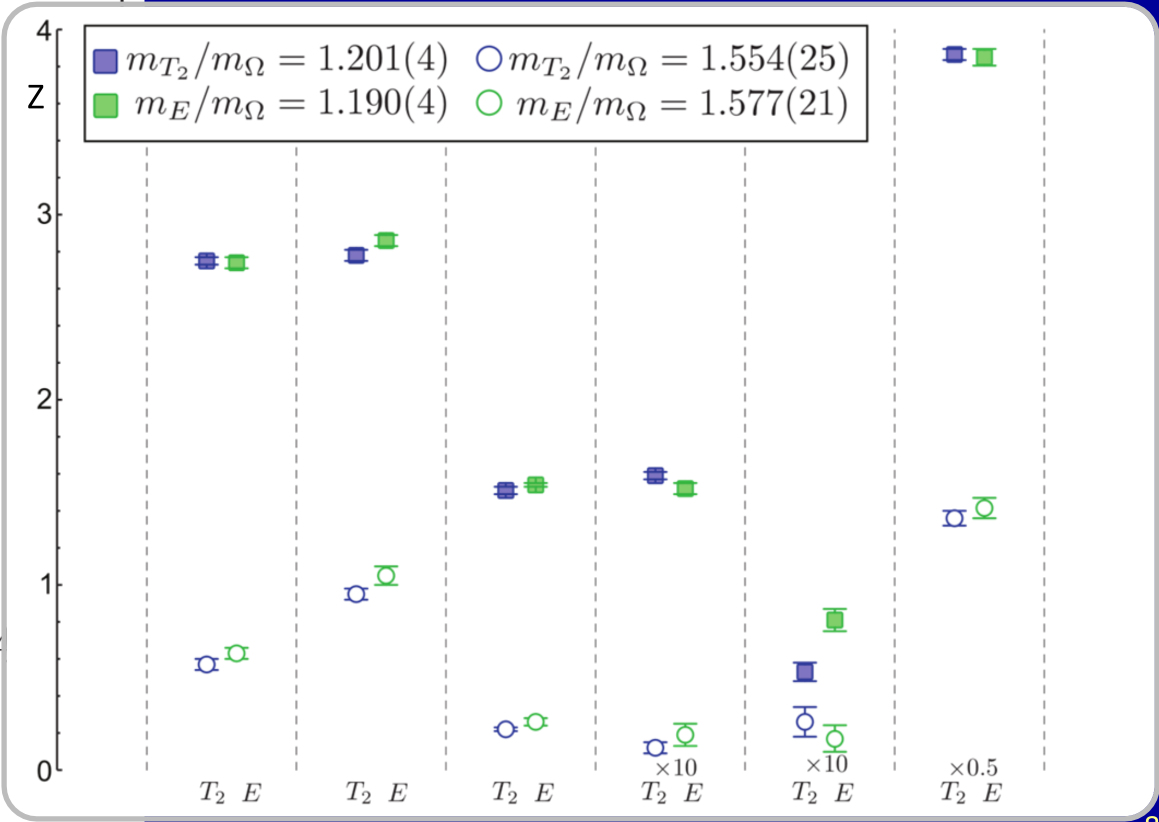


Z values – spin 2

$$\langle 0|\mathcal{O}^{[J]}_{\Lambda,\lambda}|J',M\rangle = \mathcal{S}^{J,M}_{\Lambda,\lambda} Z^{[J]} \delta_{J,J'}$$

Given continuum op →
same Z for each irrep subduced to

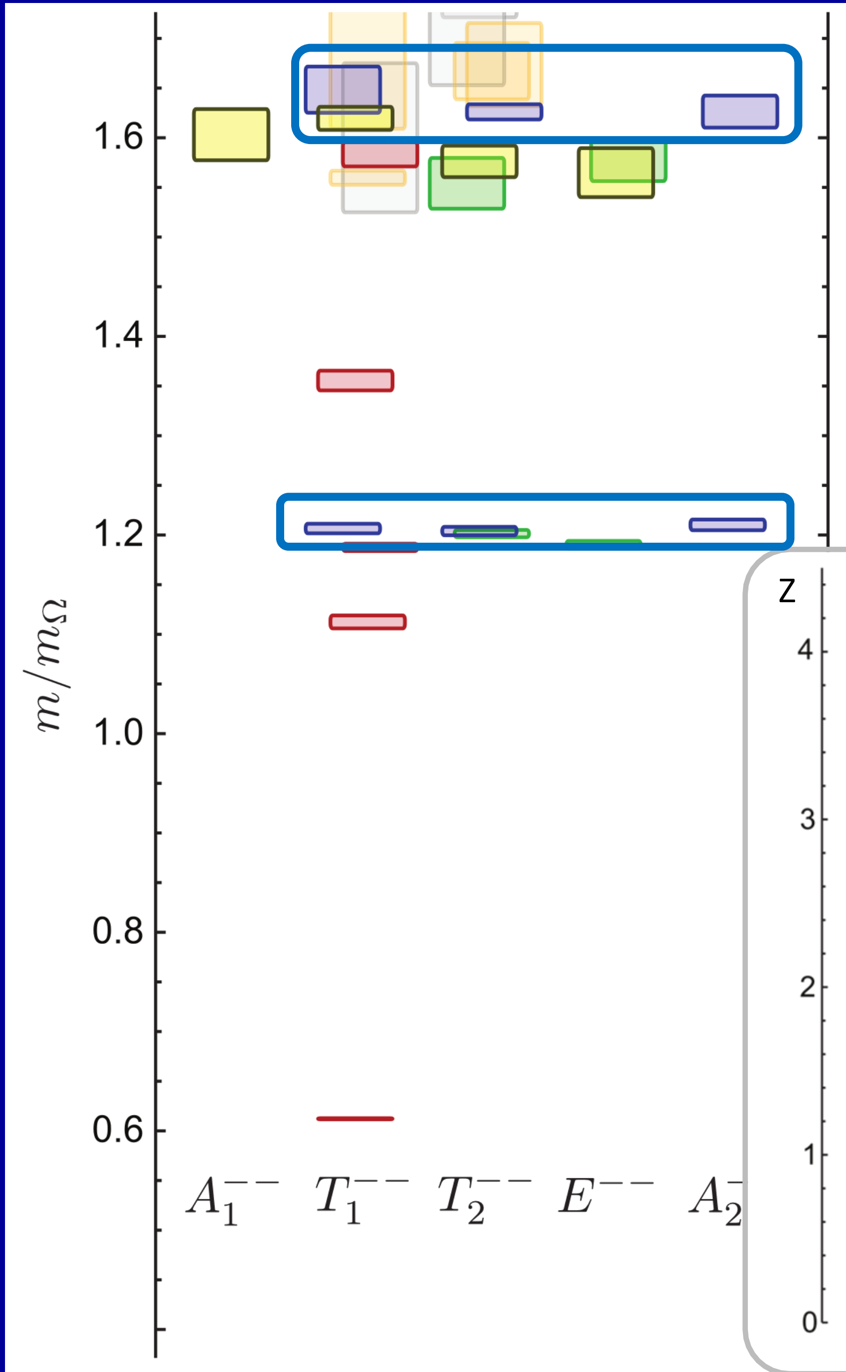


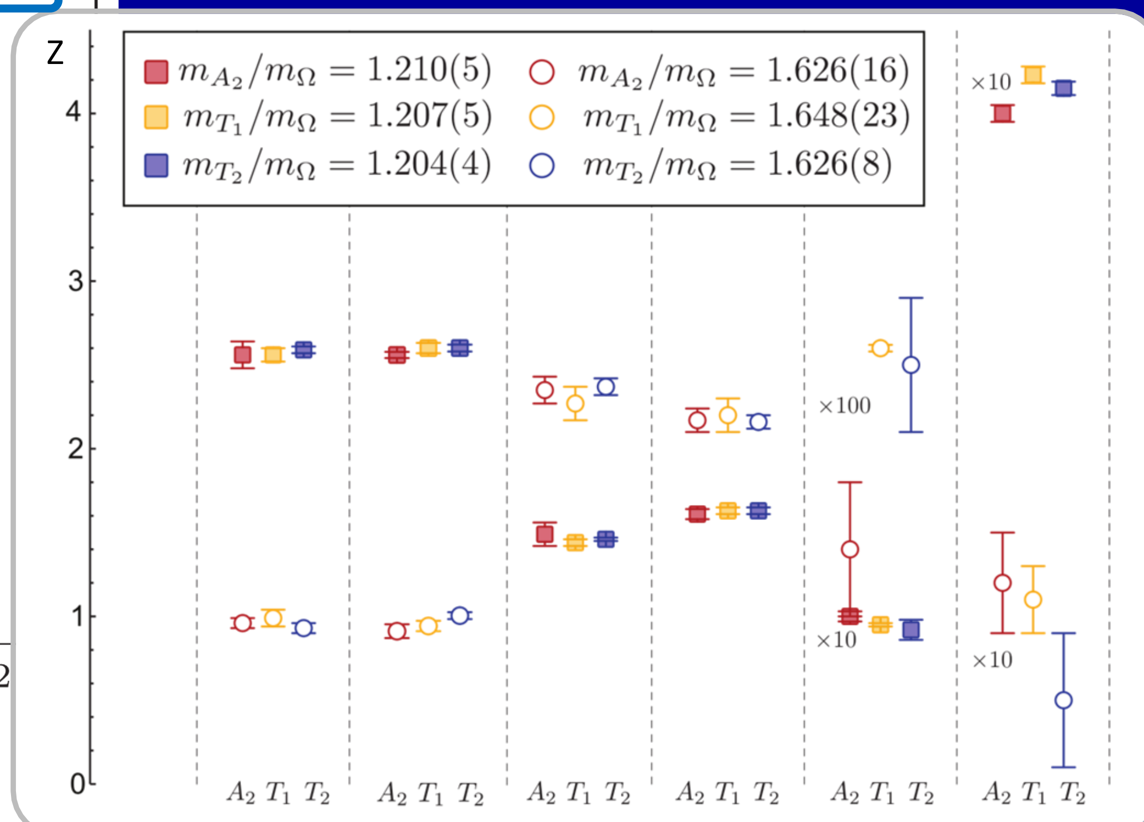


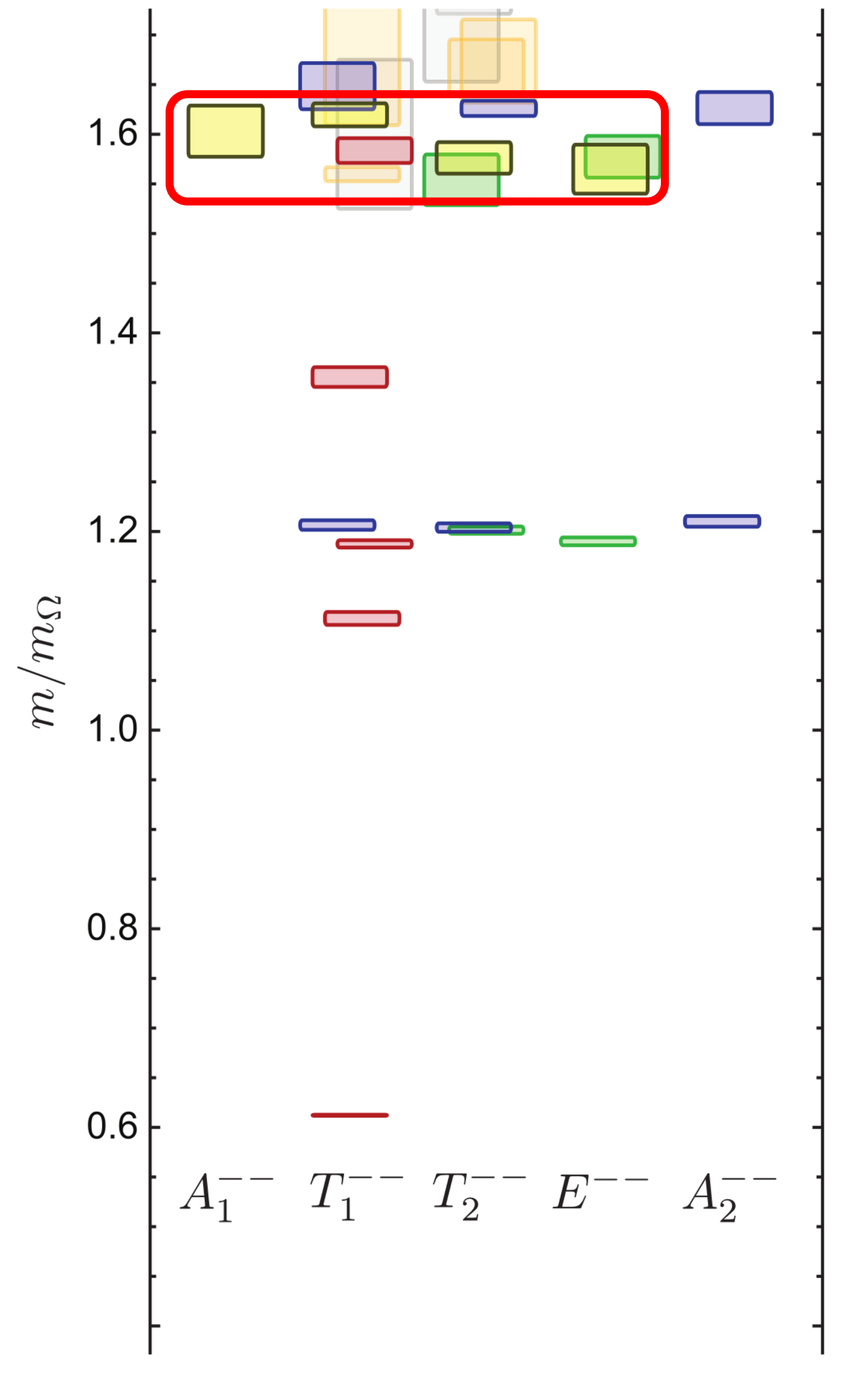
Z values – spin 3

$$\langle 0|\mathcal{O}^{[J]}_{\Lambda,\lambda}|J',M\rangle = \mathcal{S}^{J,M}_{\Lambda,\lambda} Z^{[J]} \delta_{J,J'}$$

Given continuum op →
same Z for each irrep subduced to



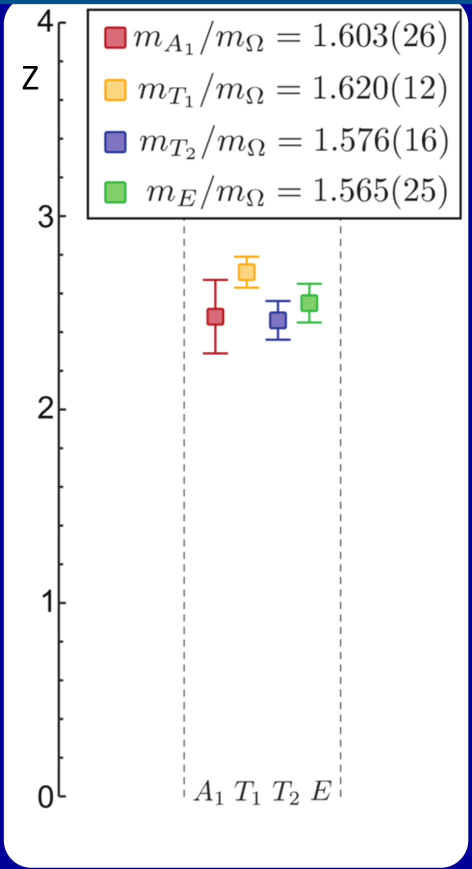


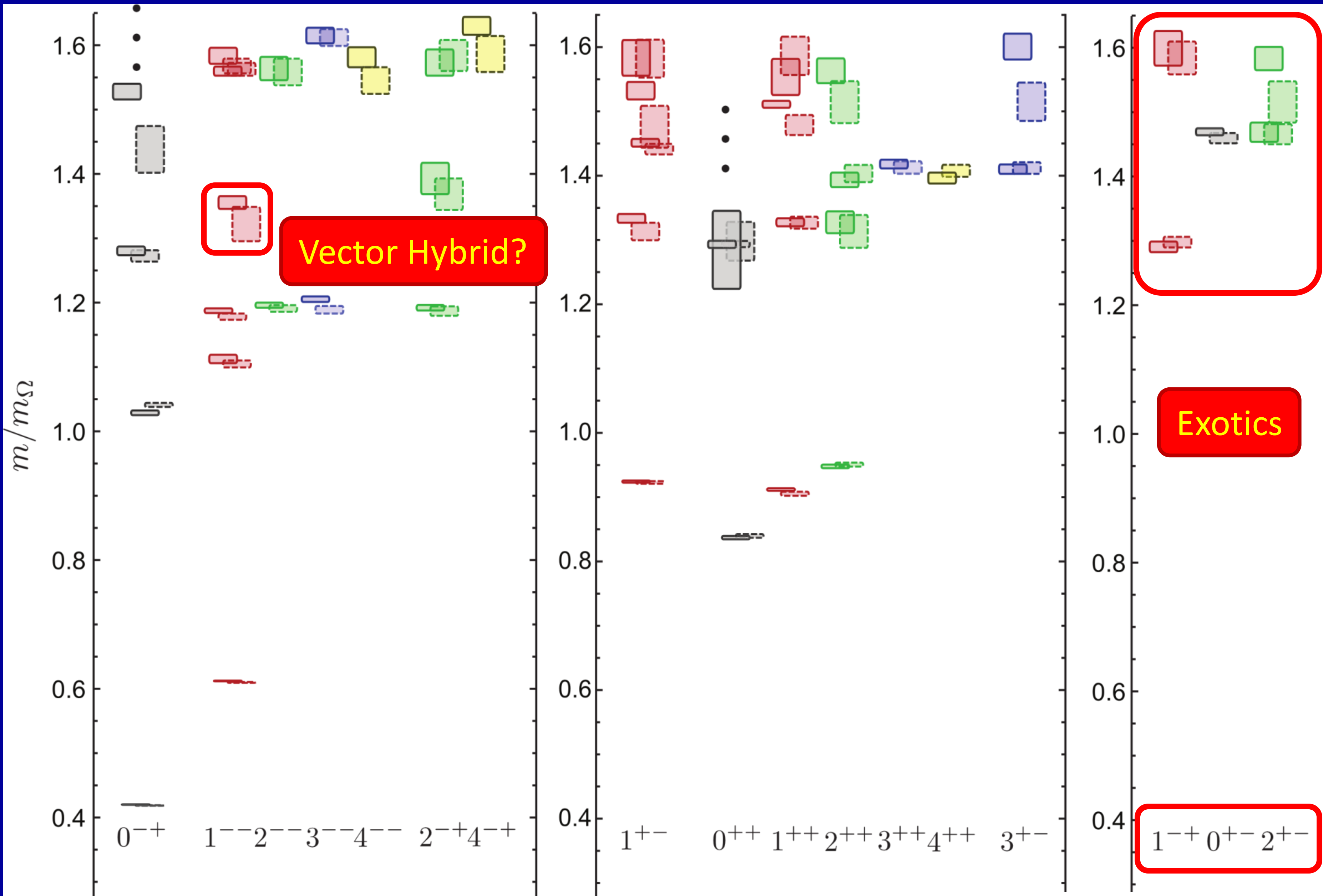


Z values – spin 4

$$\langle 0|\mathcal{O}^{[J]}_{\Lambda,\lambda}|J', M\rangle = \mathcal{S}^{J,M}_{\Lambda,\lambda} Z^{[J]} \delta_{J,J'}$$

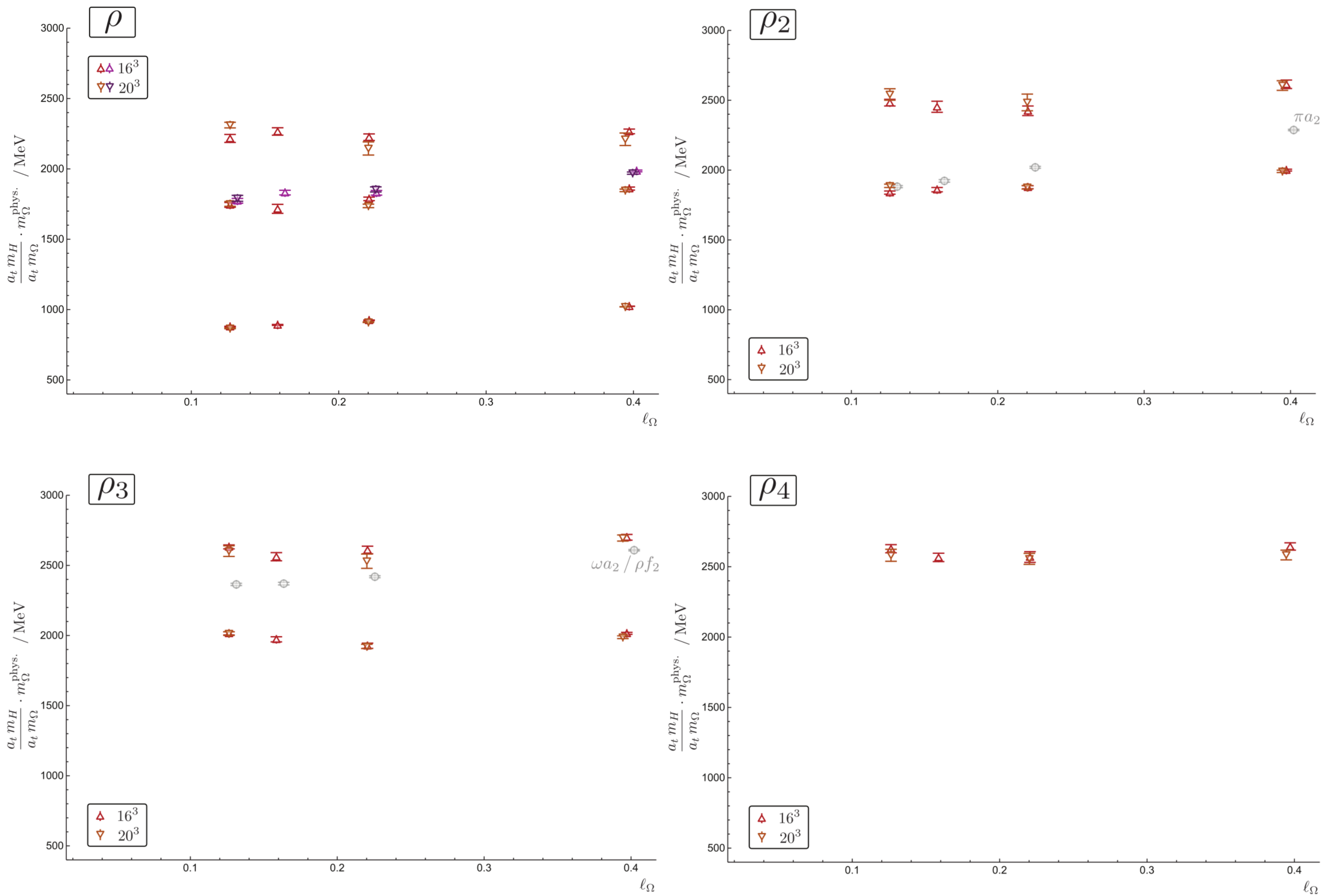
Given continuum op →
same Z for each irrep subduced to



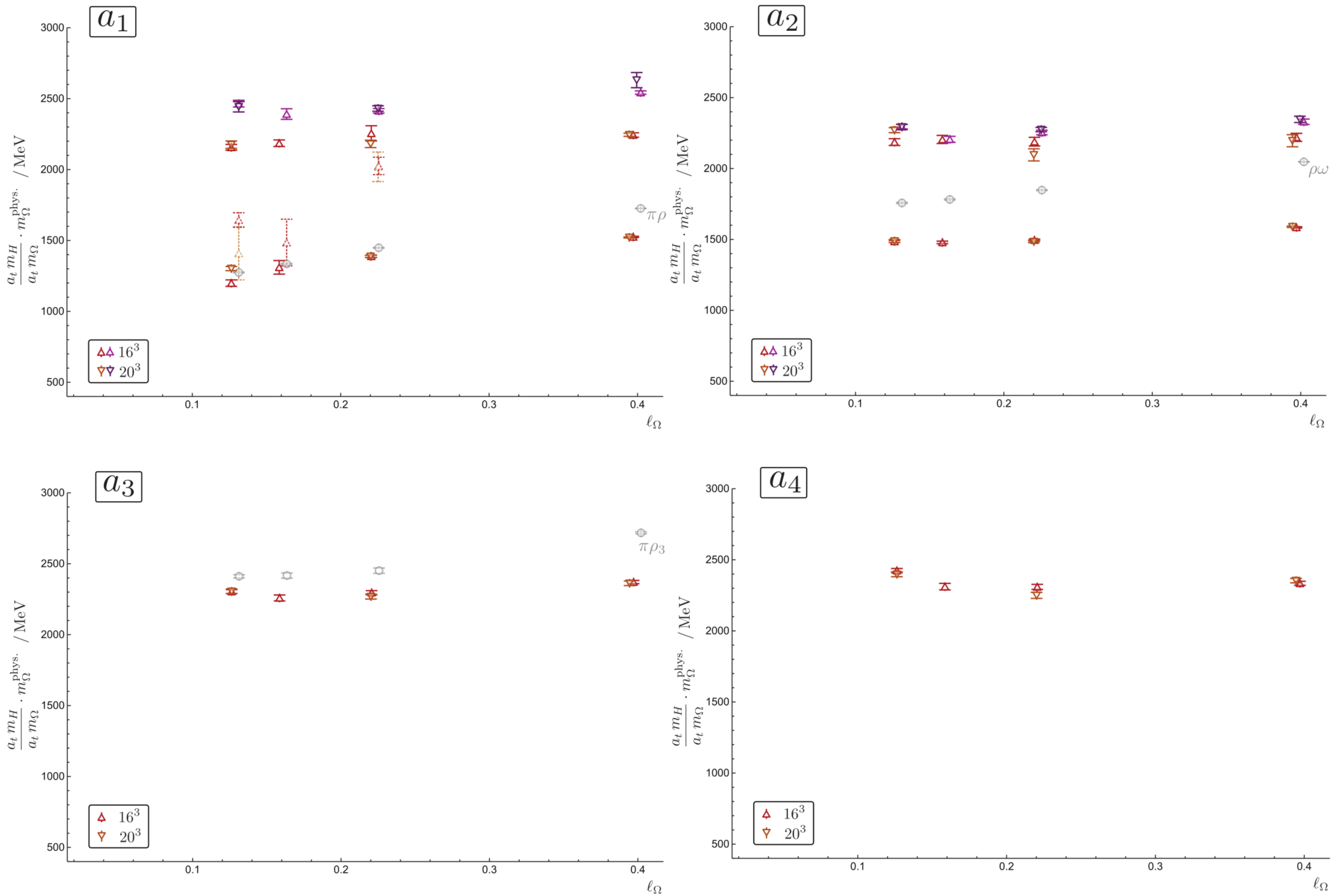


N_f = 3 isovectors

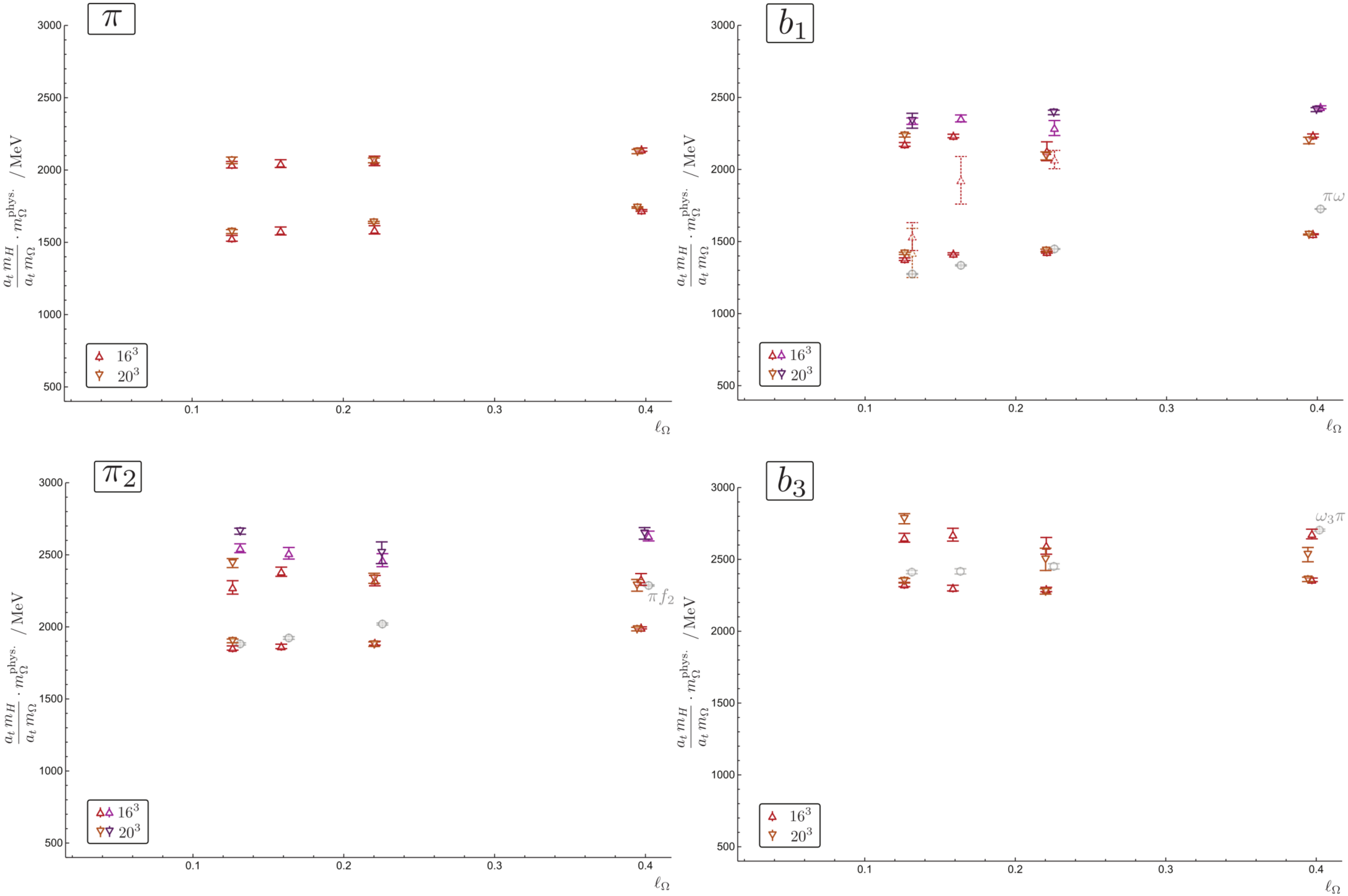
16^3 (~2 fm) and 20^3 (2.4 fm)



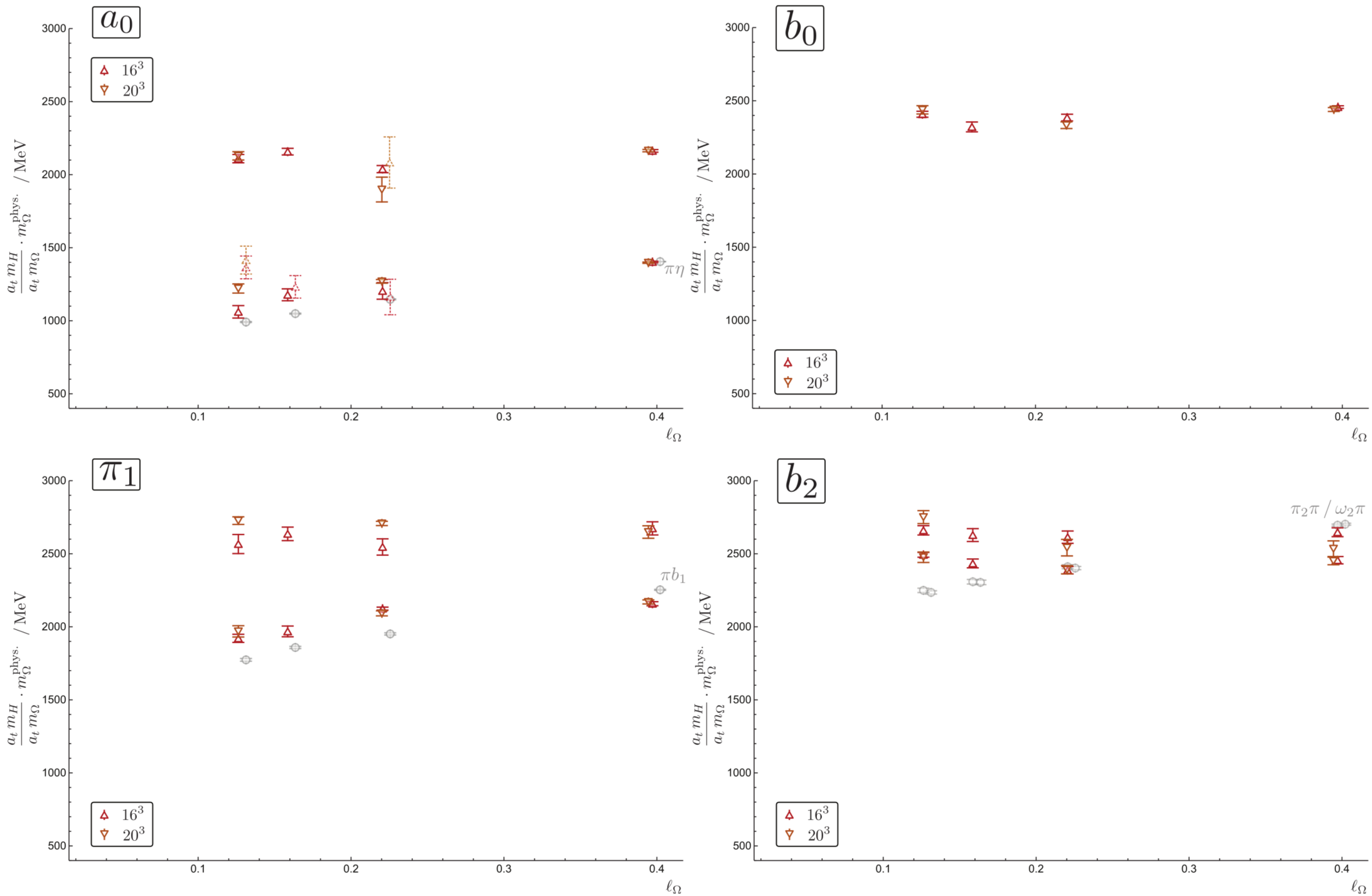
ρ
16^3
20^3
ρ2
πa_2
ρ3
$\omega a_2 / \rho f_2$
ρ4
$\frac{a_t m_H}{a_t m_\Omega} \cdot m_\Omega^{\text{phys.}}$ / MeV
ℓ_Ω



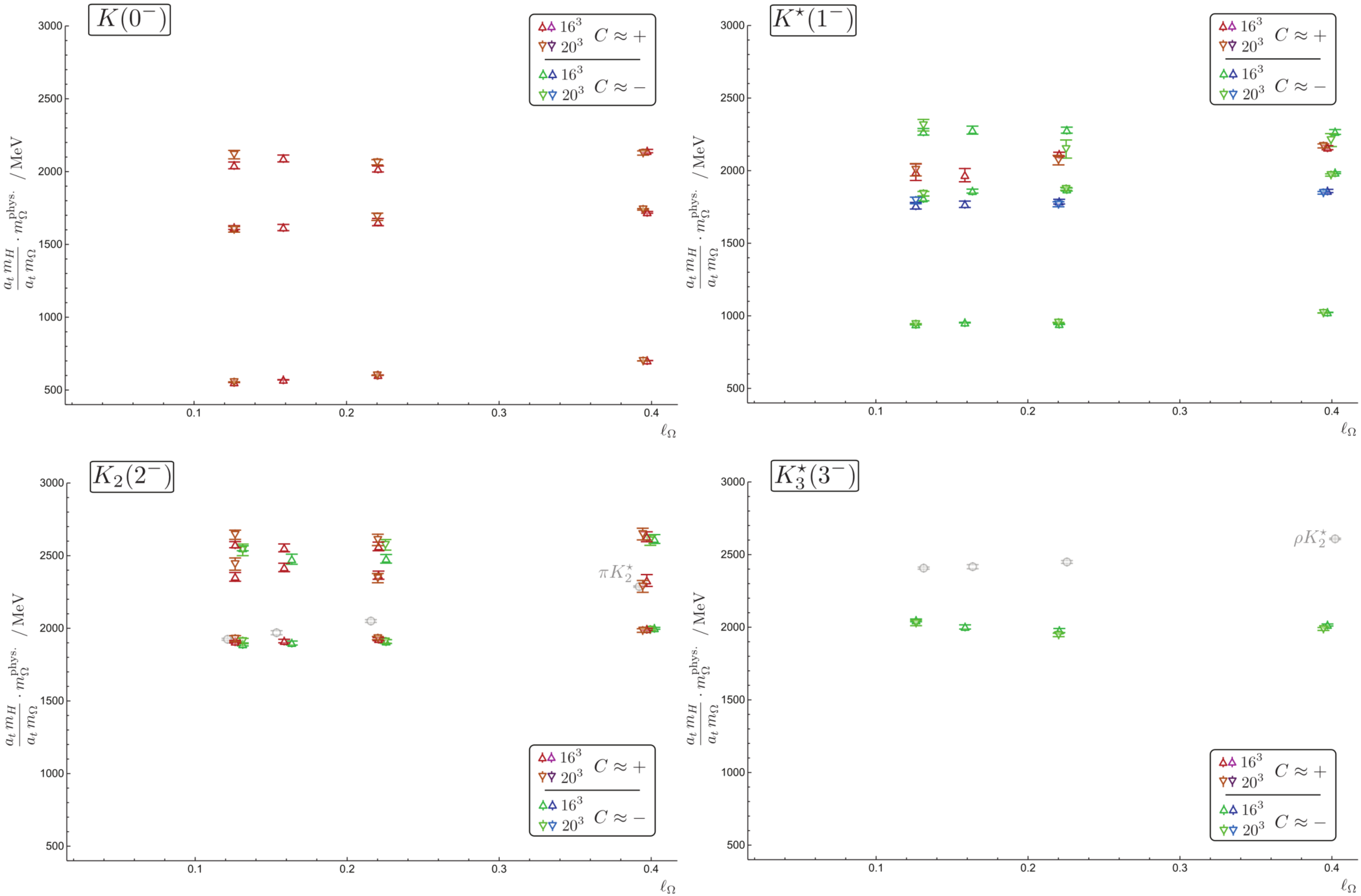
a_1
a_2
a_3
a_4
$\frac{a_t m_H}{a_t m_\Omega} \cdot m_\Omega^{\text{phys.}}$ / MeV
ℓ_Ω
$\pi\rho$
$\rho\omega$
$\pi\rho_3$
16^3
20^3



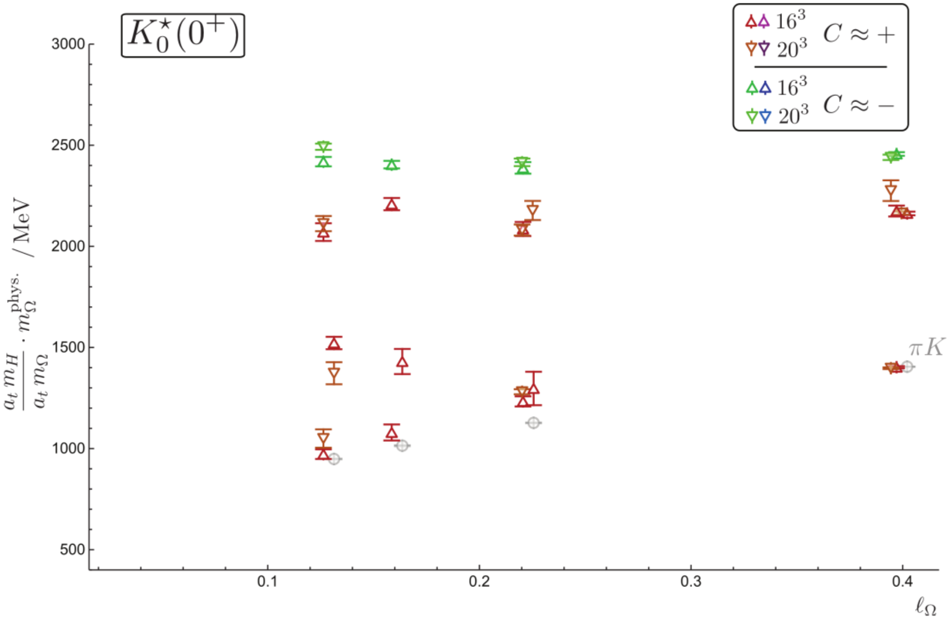
π
b_1
π_2
b_3
$\frac{a_t\, m_H}{a_t\, m_\Omega} \cdot m_\Omega^{\text{phys.}}$ / MeV
ℓ_Ω
16^3
20^3
$\pi\omega$
πf_2
$\omega_3\pi$

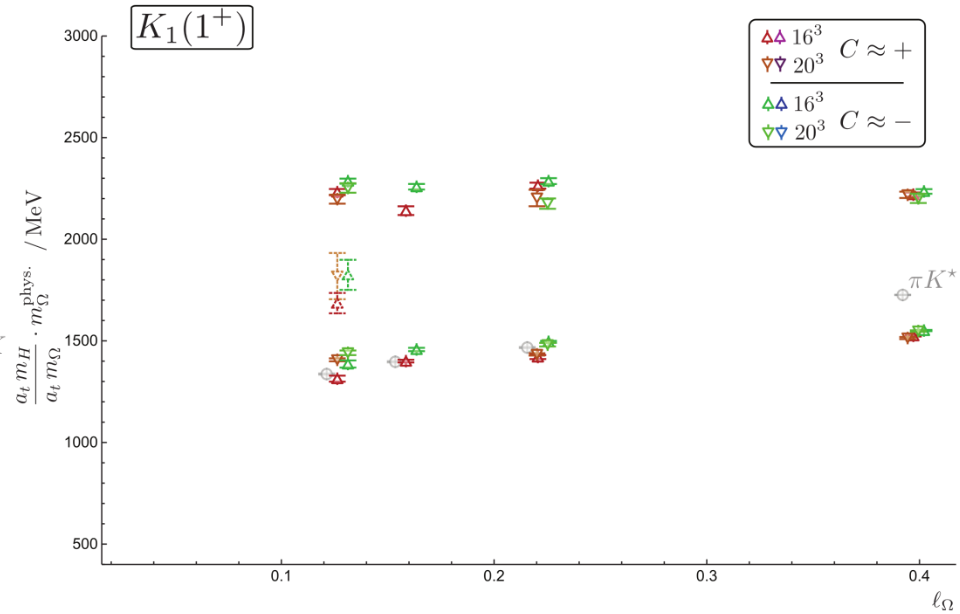


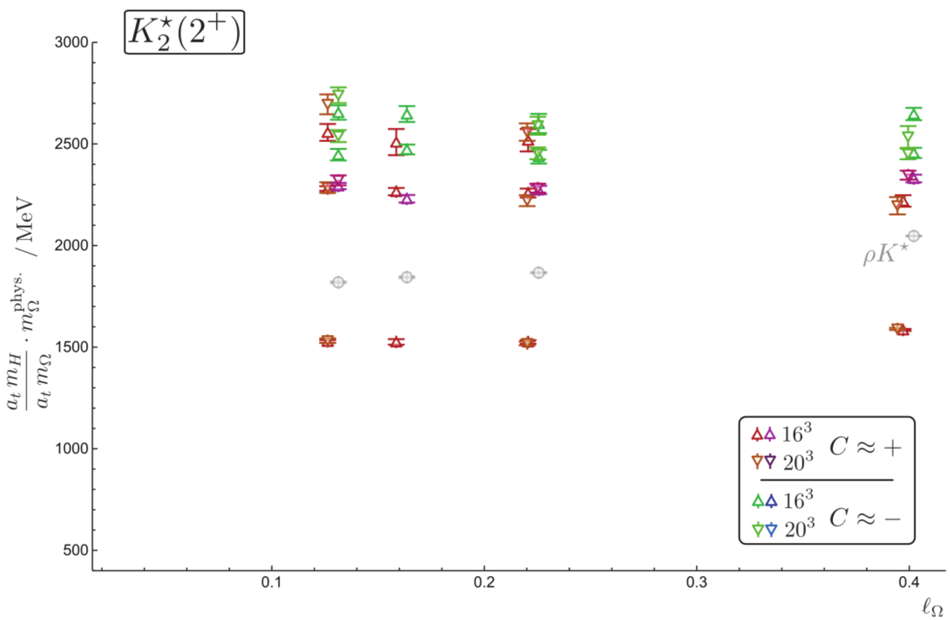
a_0
b_0
π_1
b_2
16^3
20^3
$\pi\eta$
πb_1
$\pi_2\pi \,/\, \omega_2\pi$
$\frac{a_t\, m_H}{a_t\, m_\Omega} \cdot m_\Omega^{\mathrm{phys.}}$ / MeV
ℓ_Ω
500
1000
1500
2000
2500
3000
0.1
0.2
0.3
0.4

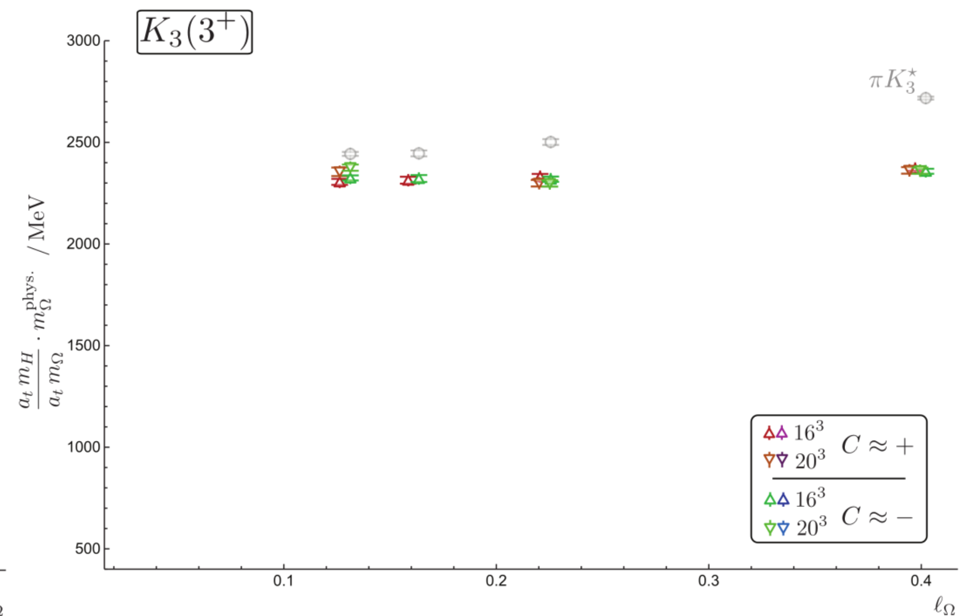


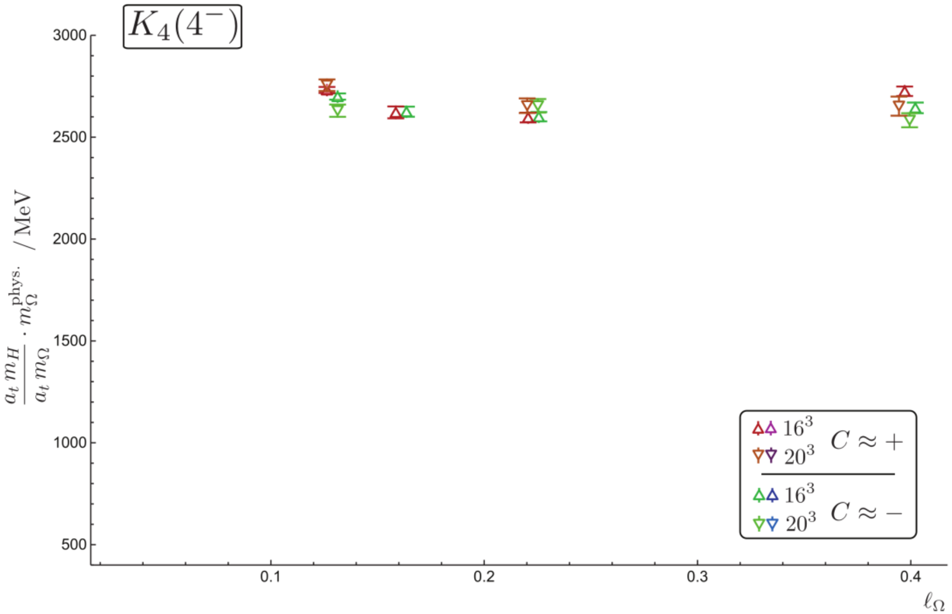
$K(0^-)$
$K^\star(1^-)$
$K_2(2^-)$
$K_3^\star(3^-)$
16^3 $C \approx +$
20^3
16^3 $C \approx -$
20^3
$\frac{a_t\, m_H}{a_t\, m_\Omega} \cdot m_\Omega^{\text{phys.}}$ / MeV
ℓ_Ω
$\pi K_2^\star$
$\rho K_2^\star$

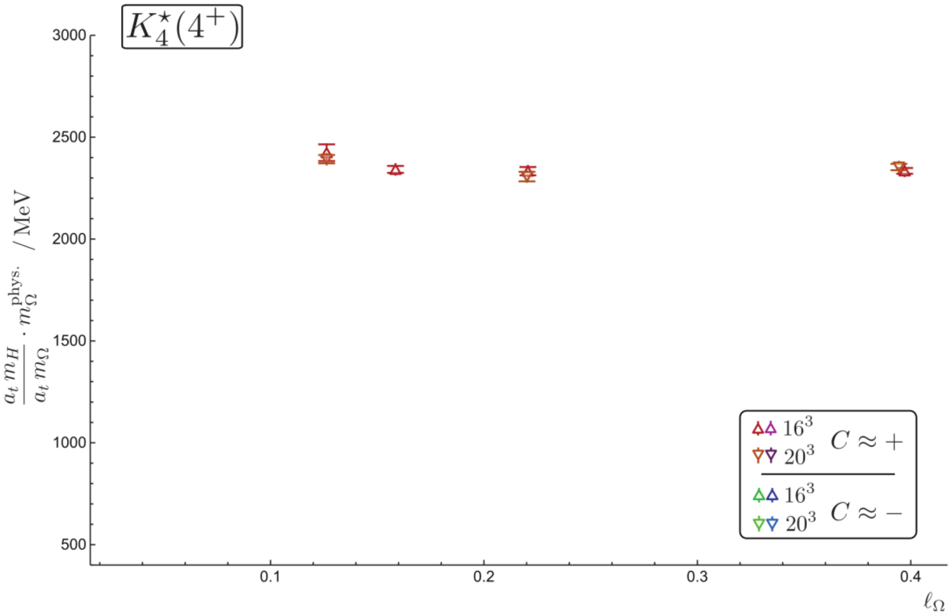

$K_0^\star(0^+)$
16^3 $C \approx +$
20^3
16^3 $C \approx -$
20^3
πK
$\frac{a_t\, m_H}{a_t\, m_\Omega} \cdot m_\Omega^{\text{phys.}}$ / MeV
ℓ_Ω


$K_1(1^+)$
16^3 $C \approx +$
20^3
16^3 $C \approx -$
20^3
$\pi K^\star$
$\frac{a_t\, m_H}{a_t\, m_\Omega} \cdot m_\Omega^{\text{phys.}}$ / MeV
ℓ_Ω


$K_2^\star(2^+)$
$\rho K^\star$
16^3 $C \approx +$
20^3
16^3 $C \approx -$
20^3
$\frac{a_t\, m_H}{a_t\, m_\Omega} \cdot m_\Omega^{\text{phys.}}$ / MeV
ℓ_Ω


$K_3(3^+)$
$\pi K_3^\star$
16^3 $C \approx +$
20^3
16^3 $C \approx -$
20^3
$\frac{a_t\, m_H}{a_t\, m_\Omega} \cdot m_\Omega^{\text{phys.}}$ / MeV
ℓ_Ω


$K_4(4^-)$
3000
2500
2000
1500
1000
500
0.1
0.2
0.3
0.4
ℓ_Ω
$\frac{a_t\, m_H}{a_t\, m_\Omega} \cdot m_\Omega^{\text{phys.}}$ / MeV
16^3
20^3
$C \approx +$
16^3
20^3
$C \approx -$


$K_4^\star(4^+)$
3000
2500
2000
1500
1000
500
0.1
0.2
0.3
0.4
ℓ_Ω
$\frac{a_t\, m_H}{a_t\, m_\Omega} \cdot m_\Omega^{\text{phys.}}$ / MeV
16^3
20^3
$C \approx +$
16^3
20^3
$C \approx -$