Neutrino Physics

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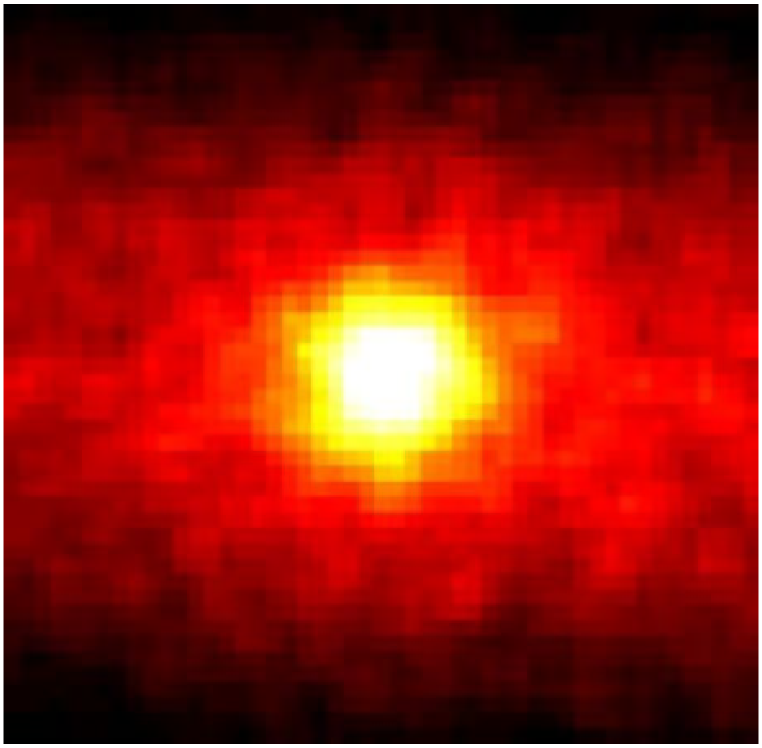


Outline

1. neutrinos and weak interactions
2. massive neutrinos
3. neutrino oscillations: vacuum
4. atmospheric neutrinos
5. neutrino oscillations: matter
6. solar neutrinos
7. other experiments: Kamland, K2K, ...
8. theories of mass: seesaw, extra dimensions, ...
9. neutrinos in astrophysics: supernovae and the R process

Neutrino History

1914	Electron spectrum in β decay is continuous
1930	Pauli postulates that a new particle is emitted
1933	Fermi names the new particle neutrino and introduces four-fermion interaction
1956	Reines and Cowan discover the neutrino
1956	Lee and Yang suggest that parity is violated Wu discovers parity violation 6 months later
1957	V-A theory
1962	At least two neutrinos: $\nu_e \neq \nu_\mu$
1969	Davis begins measurements of solar ν flux, result smaller than solar models
1973	Discovery of neutral currents at CERN
1983	Discovery of the W and Z at CERN
1986	IMB discovers the atmospheric ν deficit
1998	SuperK reports strong evidence for ν oscillations in atmospheric νs
2001	SNO reports strong evidence for ν oscillations in solar νs
2002	τ neutrino discovered

Pauli's letter (1930)

> Dear Radioactive Ladies and Gentlemen,
> As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the "wrong" statistics of the N and ^6Li nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin 1/2 and obey the exclusion principle and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant...
>
> Your humble servant,
> W. Pauli

- Pauli is trying to solve to problems at once, nuclear statistics (solved by the neutron) and energy conservation in β decay (solved by the neutrino)
- in today's notation

$$n \to p + e + \bar{\nu}$$

  which corresponds to a nuclear transition

$$(Z, A) \to (Z+1, A) + e + \bar{\nu}$$

- energy conservation

$$E_e + E_\nu = Q$$

Properties

- mass?
  - $m_{\nu_e} < 2$ eV from tritium β decay
  - $m_{\nu_\mu} < 170$ keV from π decay
  - $m_{\nu_\tau} < 18$ MeV from τ decay
- spin $s = 1/2$
- type?
  - Dirac $\nu \neq \bar{\nu}$
  - Majorana $\nu = \bar{\nu}$
- charge 0
- interactions: weak (and gravitational) only
- flavors
  - 3 active flavors (from Z width)
  - sterile flavors?

Standard Model

- the standard model contains 3 left-handed lepton doublets

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L \quad \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L \quad \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L$$

  and 3 times 3 (color) left handed quark doublets

$$\begin{pmatrix}u\\ d\end{pmatrix}_L \quad \begin{pmatrix}c\\ s\end{pmatrix}_L \quad \begin{pmatrix}t\\ b\end{pmatrix}_L$$

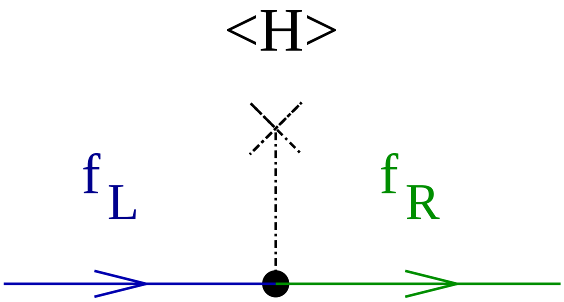
- all these fermions have right handed partners, except neutrinos, which are always left handed

$$e_R,\ \mu_R,\ \tau_R,\ u_R,\ d_R,\ c_R,\ s_R,\ t_R,\ b_R$$

- interactions are mediated by gauge bosons

$$\gamma,\ g(8), W^\pm,\ Z$$

- fermions acquire mass by interacting with the Higgs field

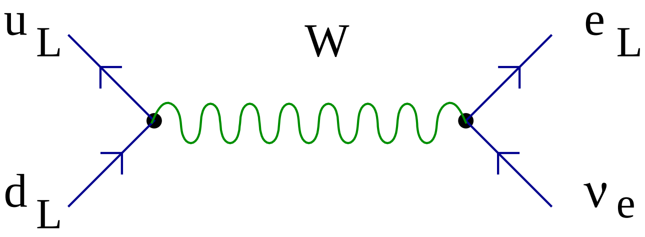


- no right handed neutrinos

  $m_\nu = 0$ in the standard model

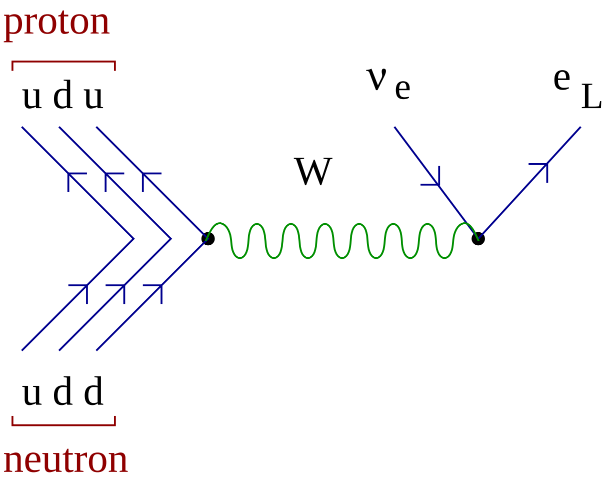
Weak Interactions in the standard model

- the weak gauge bosons $W^{\pm}$ act on left handed doublets



this is called a "charged current interaction"

- the simplest example is β decay



since $m_W = 80.4$ GeV$\gg m_p$ decay is governed by

$$\text{Fermi coupling} \qquad \frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8m_W^2}$$

g_2 is the W gauge coupling. The ratio

$$\frac{e}{g_2} = \sin\theta_W = 0.48$$

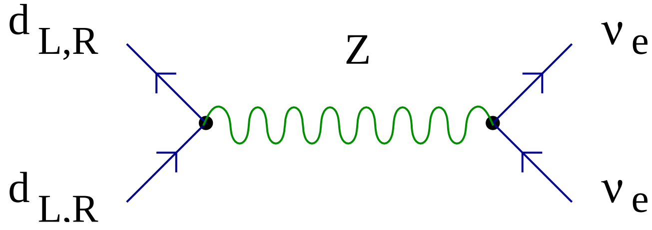
is called the Weinberg angle

Neutral Currents

- the neutral partner of the $W^{\pm}$, the W^0, mixes with the hypercharge gauge boson B to form a massless photon γ and a massive neutral gauge boson Z

$$Z = \cos\theta_W W^0 - \sin\theta_W B$$

- this means that the Z couples to both left and right handed quarks and charged leptons



this is called a “neutral current” interaction

- neutral current couplings

Z couplings	g_L	g_R
ν_e, ν_μ, ν_τ	$\frac{1}{2}$	0
e, μ, τ	$-\frac{1}{2} + \sin^2\theta_W$	$\sin^2\theta_W$
u, c, t	$\frac{1}{2} - \frac{2}{3}\sin^2\theta_W$	$-\frac{2}{3}\sin^2\theta_W$
d, s, c	$-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W$	$\frac{1}{3}\sin^2\theta_W$

The CKM matrix

- The Higgs mechanism that gives masses to the fermions need not be diagonal flavor

mass eigenstates ≠ weak eigenstates

- weak eigenstates are related to mass eigenstates via a 3 × 3 matrix

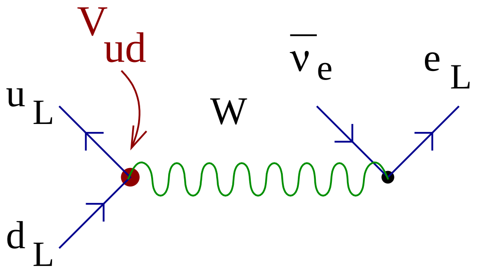
$$\begin{aligned} q'_f &= (S^u)_{fg}\, q_g \quad & f, g &= (u, c, t) \\ q'_f &= (S^d)_{fg}\, q_g \quad & f, g &= (d, s, b) \\ l'_f &= (S^l)_{fg}\, l_g \quad & f, g &= (e, \mu, \tau) \end{aligned}$$

- this does not affect neutral current interactions, because in and out states are rotated in the same way $(SS^\dagger = 1)$

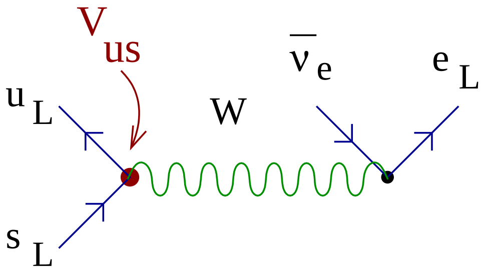
there are no flavor changing neutral currents

- It does affect charge current scattering, but only the relative rotation of up and down type quark matters. This is called the CKM matrix

$$V_{fg} = [(S^u)^\dagger (S^d)]_{fg} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



neutron β decay



hyperon β decay

CKM continued

- remarkably, the CKM matrix is close to diagonal, e.g.

$$V_{ud} \simeq 0.97, \;\; V_{us} \simeq 0.22, \;\; V_{ut} \simeq 0.005$$

nobody knows why

- in the SM model neutrinos are massless

$\Rightarrow$ any linear combinations of $\nu_{e,\mu,\tau}$ are mass eigenstates

$\Rightarrow$ can eliminate lepton matrix by rotating ν

$\Rightarrow$ no CKM matrix for leptons

we now know that this is wrong

- neutrinos have mass: weak eigenstates $\neq$ mass eigenstates

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

the analog of the CKM matrix in the neutrino sector is called the MNS matrix

$$V^{MNS} = [(S^l)^\dagger (S^\nu)]$$

Majorana vs. Dirac

- we can define chirality (L/R) projections

$$\psi_{L,R} = P_{L,R}\psi = \frac{1}{2}(1 \pm \gamma_5)\psi$$

and helicity projections

$$\psi_{\pm} = H_{\pm}\psi = \frac{1}{2}(1 \pm \vec{\Sigma} \cdot \hat{p})\psi$$

helicity = projection of spin on the direction of motion

- for a massless particle

$$\text{helicity} = \text{chirality}$$

- also note that

$$(\psi_L)^{\dagger}\gamma_0 \equiv \bar{\psi}_L = \bar{\psi} P_R$$

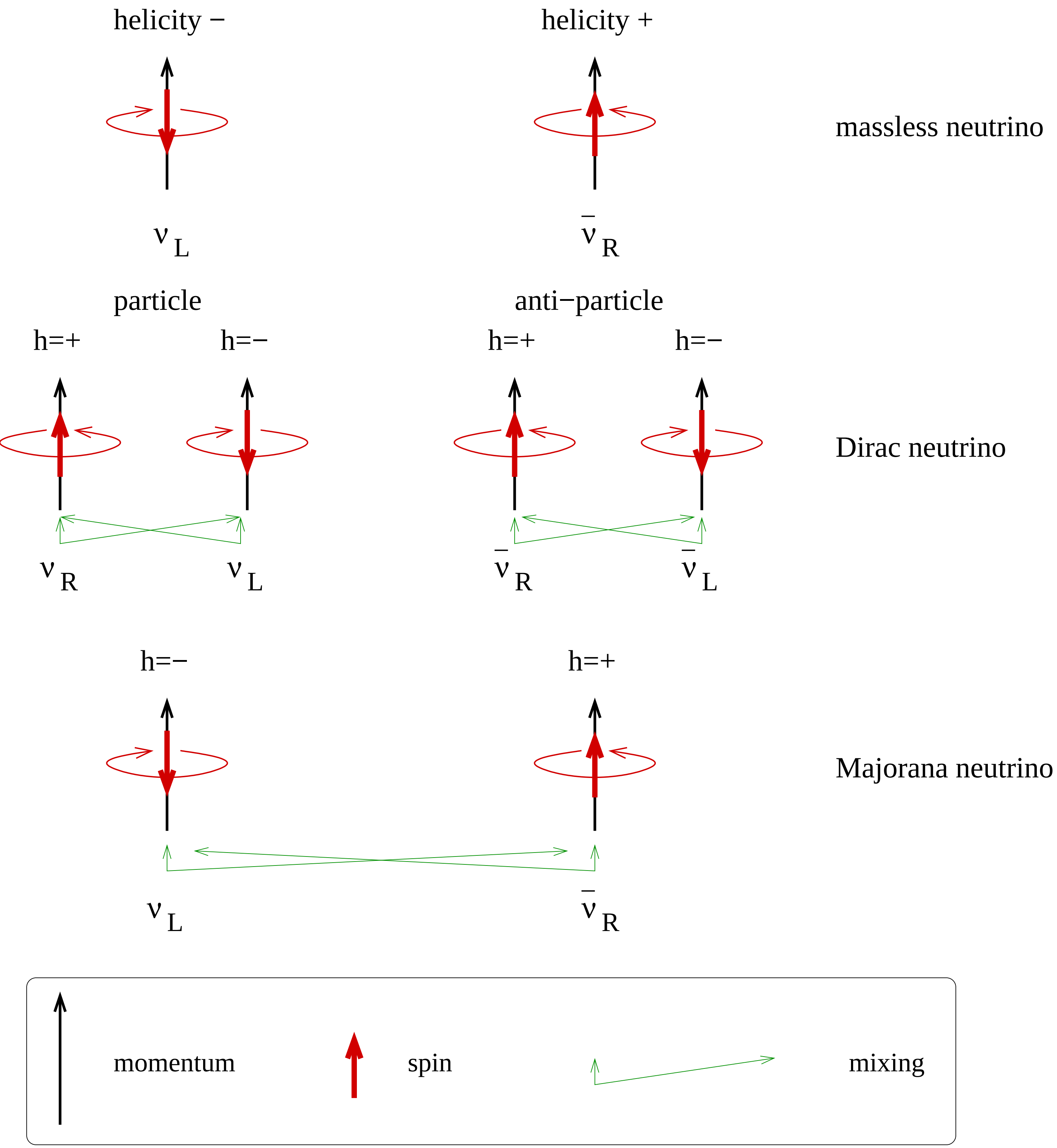
The anti-particle of a left-handed neutrino
ν_L is a right-handed anti-neutrino.

- the W boson only interacts with left handed fields

$$\mathcal{L} = g_2 \vec{W}^{\mu} \bar{\psi}_L \gamma_{\mu} \vec{\tau} \psi_L$$

right handed neutrinos do not couple to W, Z

Majorana vs Dirac



helicity −

helicity +

massless neutrino

ν_L

$\bar{\nu}_R$

particle

anti−particle

h=+

h=−

h=+

h=−

Dirac neutrino

ν_R

ν_L

$\bar{\nu}_R$

$\bar{\nu}_L$

h=−

h=+

Majorana neutrino

ν_L

$\bar{\nu}_R$

momentum

spin

mixing

Majorana vs Dirac

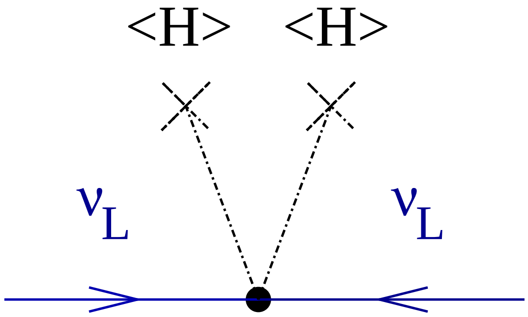
- Dirac mass ($\bar{\psi} = \psi^\dagger \gamma_0$)

$$\mathcal{L} = m\,(\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L)$$

Lepton number conserved

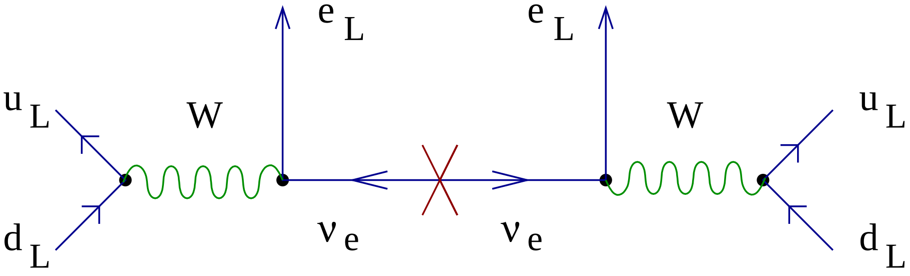
- Majorana mass ($\psi^c = \bar{\psi}^T C$)

$$\mathcal{L} = m(\bar{\nu}^c_L \nu_L + h.c.) = m(\nu_L C \nu_L + h.c.)$$



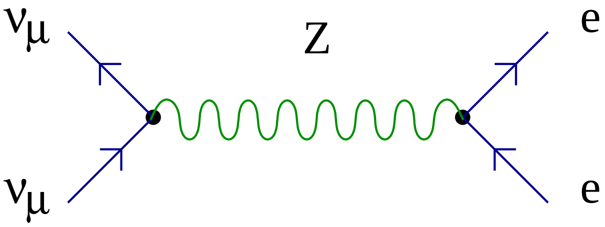
Lepton number violated

- how to tell the difference? $0\nu 2\beta$ decay



Example: Neutrino Electron Scattering

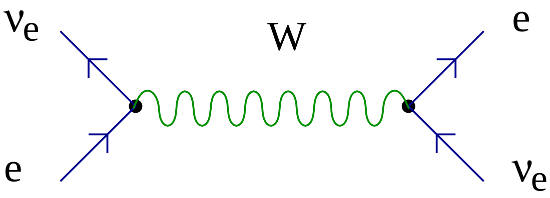
- consider $\nu_\mu e \to \nu_\mu e$



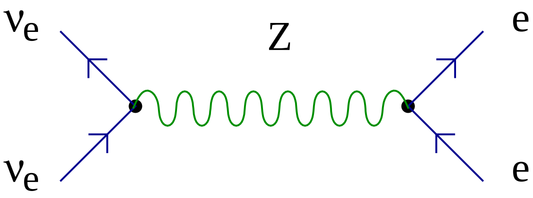
NC

$$\sigma = \frac{G_F^2}{4\pi} 2 m_e E_\nu \left(g_L^2 + \frac{g_R^2}{3} \right) \qquad \begin{aligned} g_L &= -1 + 2\sin^2\Theta_W \\ g_R &= 2\sin^2\Theta_W \end{aligned}$$

and $\nu_e e \to \nu_e e$



CC



NC

$$\sigma = \frac{G_F^2}{4\pi} 2 m_e E_\nu \left((g_L + 2)^2 + \frac{g_R^2}{3} \right)$$

note $\quad \sigma = \sigma^{CC} + \sigma^{NC} + \sigma^{CC-NC}$

- also note $\sigma \sim E_\nu$. Follows from dimensional analysis:

$$\sigma \sim (\text{area}) \sim (\text{mass})^{-2}$$

$$\mathcal{L} \sim \frac{g^2}{m_W^2} (\psi^\dagger \psi)^2 \quad \Rightarrow \quad \sigma \sim s \left(\frac{g^4}{m_W^4} \right)$$

with $s = E_{CM}^2 = 2 m_e E_\nu$

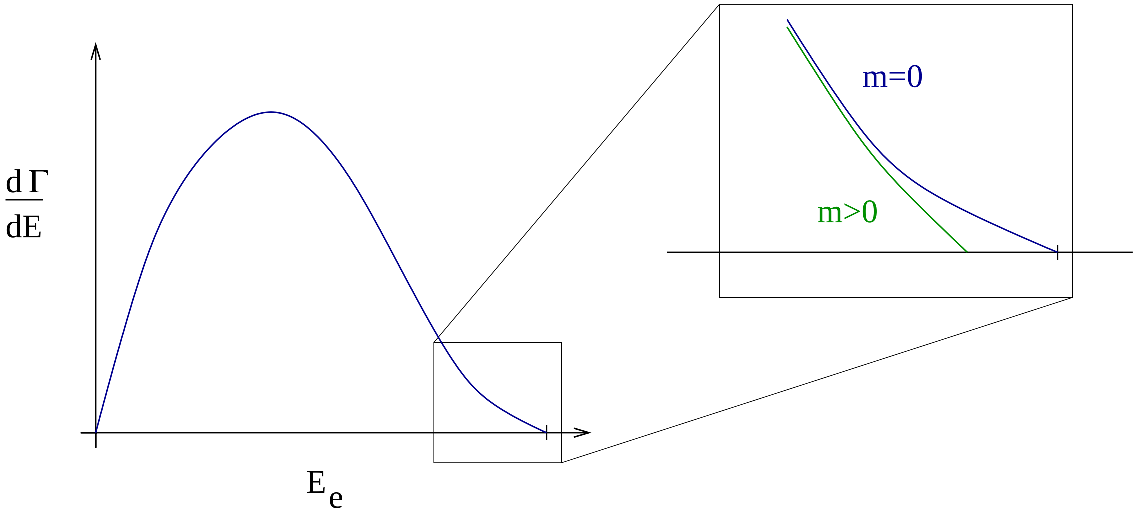
Direct Measurements: Tritium β decay

- tritium beta decay

$$^3H \rightarrow {}^3He + e^- + \bar{\nu}_e$$

$$E_0 = 18.6\,keV, \qquad T_{1/2} = 12.3\,a$$

- neutrino mass modifies electron spectrum near end point

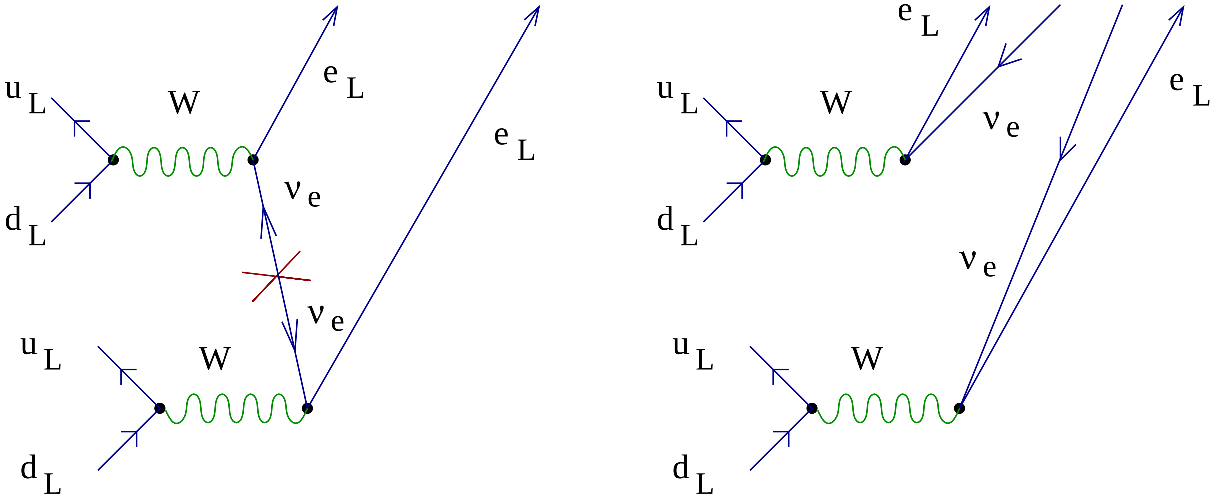


- Mainz neutrino mass experiment (1998-2001)

$$m_{\nu_e} < 2.2\,\mathrm{eV} \quad (95\%\ \mathrm{CL})$$

Double Beta Decay

- $2\nu 2\beta$ vs $0\nu 2\beta$ decay

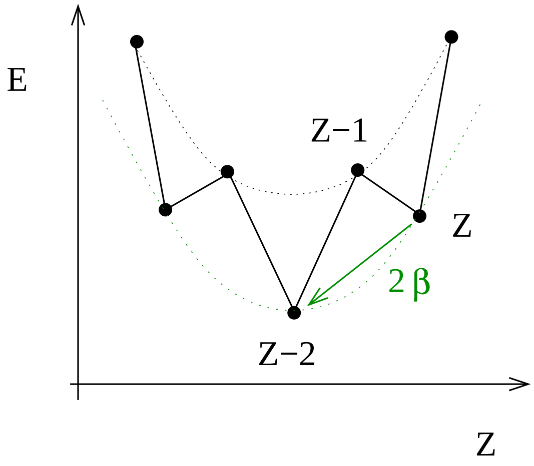


$$(T^{0\nu}_{1/2})^{-1} \sim (\text{phase space}) \times (\text{nuclear m.e.})^2 \times \langle m^M_\nu \rangle^2$$

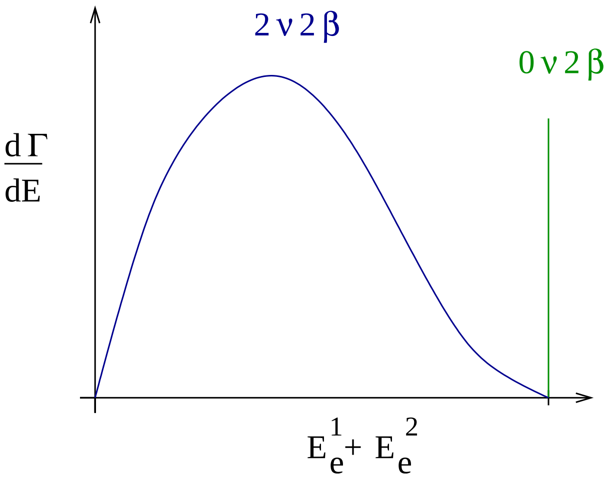
$$\langle m^M_\nu \rangle = \left| \Sigma_k U^2_{ek} m^M_k \right|$$

where U_{ek} is the MNS matrix

- need nucleus which is β stable, but 2β unstable



- look for events with $E_e^1 + E_e^2 = Q$



- Heidelberg-Moscow (1999-2000)

$$^{76}Ge \qquad T_{1/2}^{0\nu} > 1.9 \cdot 10^{25}\,\mathrm{yr} \qquad \langle m_\nu \rangle < 0.35\,\mathrm{eV}$$

Neutrino Oscillations

- neutrinos come in different flavors $\nu_e, \nu_\mu, \nu_\tau, \ldots$

$$\boxed{\text{flavor eigenstates} \neq \text{mass eigenstates}}$$

Pontecorvo (1957)

the rest is just quantum mechanics …

- consider two flavors ν_e, ν_μ

$$\begin{aligned} |\nu_e\rangle &= \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle \\ |\nu_\mu\rangle &= -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle \end{aligned}$$

Flavor eigenstates $|\nu_{e,\mu}\rangle$ ↔ Mass eigenstates $|\nu_{1,2}\rangle$

What does that have to do with oscillations?

- consider Schrödinger equation

$$i\hbar\frac{d}{dt}\psi = H\psi \quad \Rightarrow \quad i\hbar\frac{d}{dt}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = H\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

- what is the Hamiltonian H?

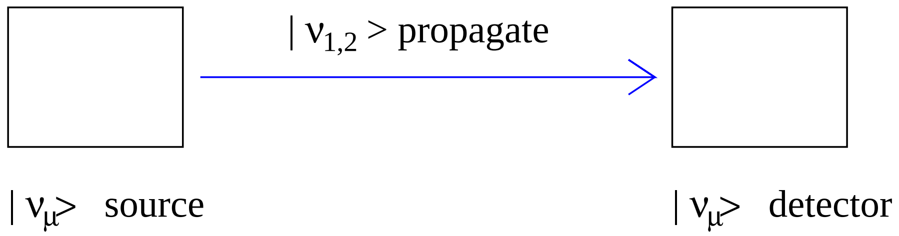
$$\begin{aligned} H|\nu_1\rangle &= (p^2 + m_1^2)^{1/2}|\nu_1\rangle \\ H|\nu_2\rangle &= (p^2 + m_2^2)^{1/2}|\nu_2\rangle \end{aligned}$$

H is diagonal in basis of mass eigenstates

- write $|\nu_e\rangle = \cos\theta|\nu_1\rangle + \ldots$. The rest is algebra.

- wave equation

$$i\hbar c\frac{d}{dr}\begin{pmatrix}|\nu_e\rangle\\|\nu_\mu\rangle\end{pmatrix}=\frac{\delta m^2}{4E}\begin{pmatrix}-\cos 2\theta & \sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{pmatrix}\begin{pmatrix}|\nu_e\rangle\\|\nu_\mu\rangle\end{pmatrix}$$



| $\nu_{1,2}$ > propagate

| ν_μ> source

| ν_μ> detector

$$P(|\nu_\mu\rangle\rightarrow|\nu_\mu\rangle)=1-\sin(2\theta)\sin^2\left(\frac{1.27\delta m^2L}{E}\right)$$

θ	mixing angle
δm^2	mass difference
L	path length
E	neutrino energy

- real world: 3 × 3 (or more) mixing → MNS matrix

$$\begin{pmatrix}|\nu_e\rangle\\|\nu_\mu\rangle\\|\nu_\tau\rangle\end{pmatrix}=\begin{pmatrix}U_{e1} & U_{e2} & U_{e3}\\U_{\mu1} & U_{\mu2} & U_{\mu3}\\U_{\tau1} & U_{\tau2} & U_{\tau3}\end{pmatrix}\begin{pmatrix}|\nu_1\rangle\\|\nu_2\rangle\\|\nu_3\rangle\end{pmatrix}$$

3 mixing angles, CP violating phase

$$\begin{aligned}U_{e1} &= \cos\theta_{12}\cos\theta_{13}\\U_{e2} &= \sin\theta_{12}\cos\theta_{13}\\U_{e3} &= \sin\theta_{13}e^{-i\delta}\\ &\ldots\end{aligned}$$

Atmospheric Neutrinos

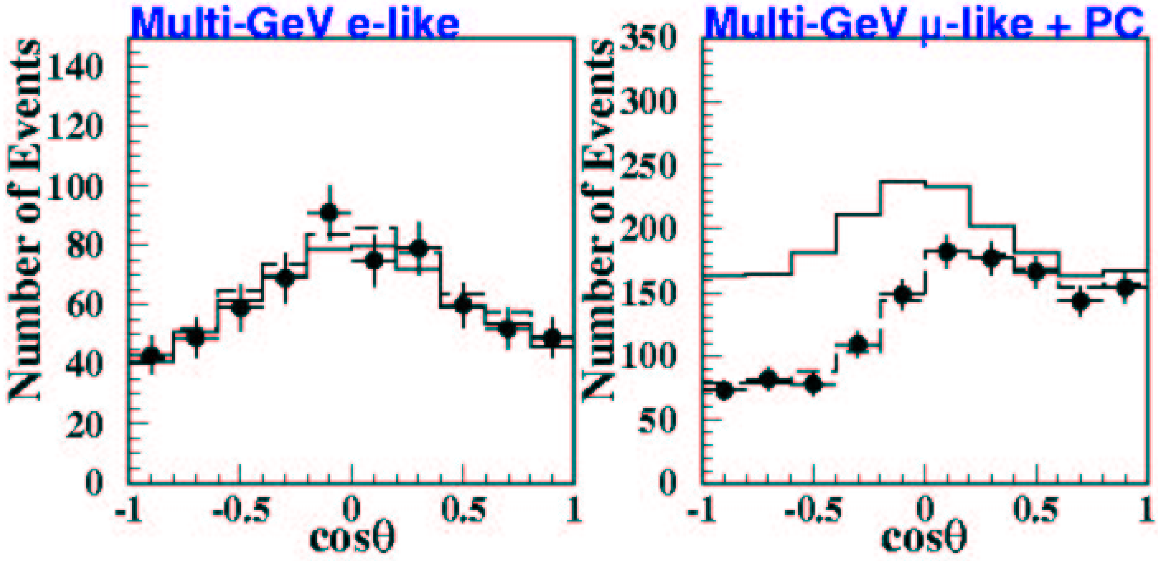
- cosmic rays collide with O, N and produce $\pi, K, \ldots$ which decay to ν_e, ν_μ

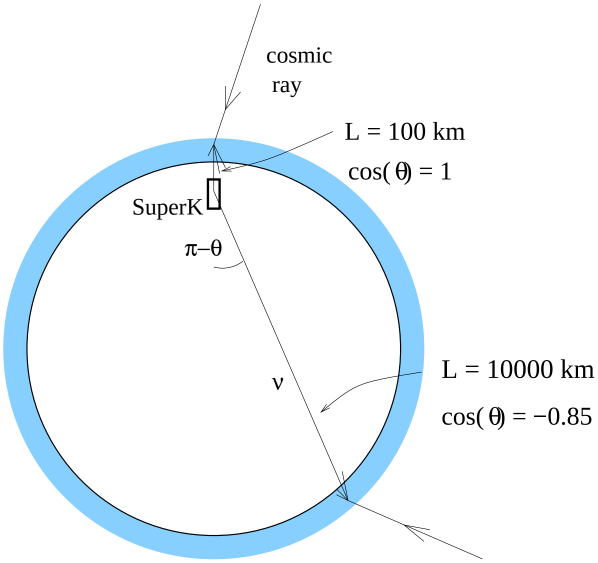
$$\pi \longrightarrow \mu\ \nu_\mu$$

$$\hookrightarrow e\ \nu_e\ \nu_\mu$$

$$N(\nu_\mu):N(\nu_e) \sim 2:1$$

- SuperK finds a deficit of ν_μ, no enhancement of ν_e.
- SuperK also finds azimuthal dependence of ν_μ suppression





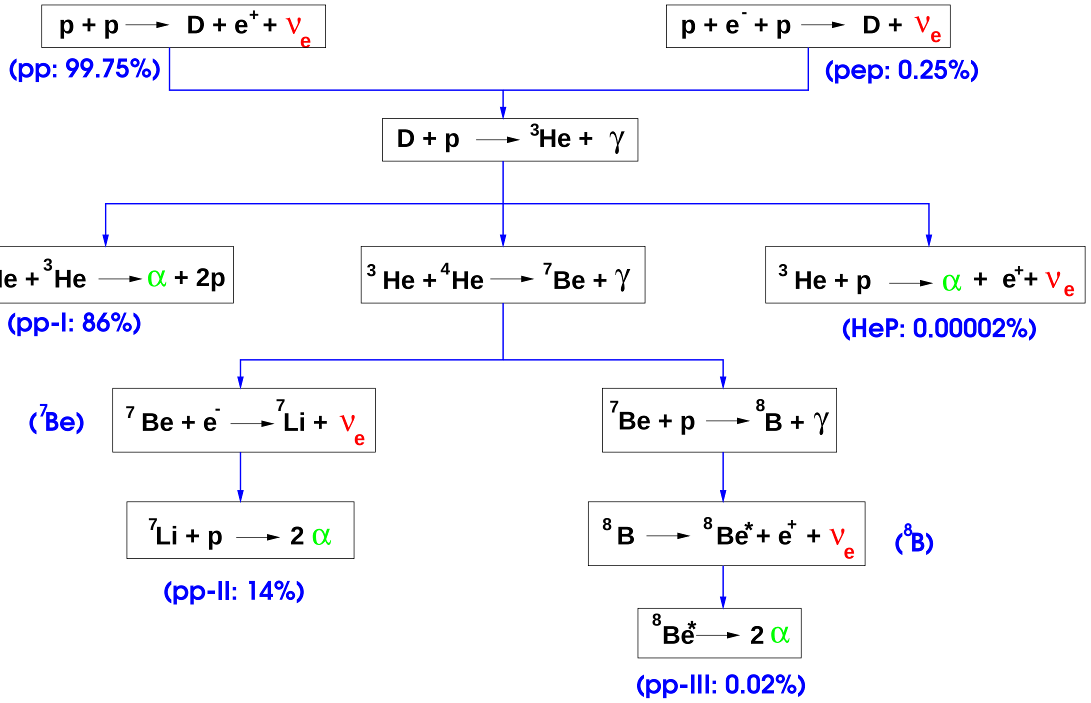
Explanation: $\nu_\mu \rightarrow \nu_\tau$ oscillations

$$\delta m^2_{23} \text{ or } \delta m^2_{32} \simeq 10^{-3}\,\mathrm{eV}^2$$

$$\sin^2 2\theta_{23} \simeq 1$$

Solar Neutrinos

- sun produces ν_e through nuclear burning: pp cycle



$p + p \rightarrow D + e^+ + \nu_e$

(pp: 99.75%)

$p + e^- + p \rightarrow D + \nu_e$

(pep: 0.25%)

$D + p \rightarrow {}^3He + \gamma$

${}^3He + {}^3He \rightarrow \alpha + 2p$

(pp-I: 86%)

${}^3He + {}^4He \rightarrow {}^7Be + \gamma$

${}^3He + p \rightarrow \alpha + e^+ + \nu_e$

(HeP: 0.00002%)

(^{7}Be)

${}^7Be + e^- \rightarrow {}^7Li + \nu_e$

${}^7Li + p \rightarrow 2\,\alpha$

(pp-II: 14%)

${}^7Be + p \rightarrow {}^8B + \gamma$

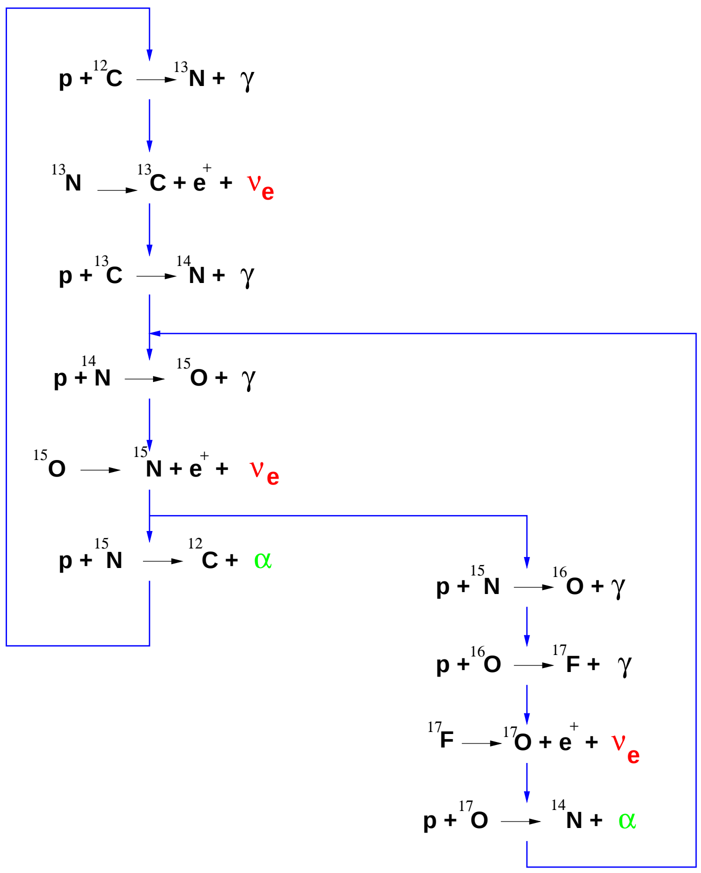
${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$

(^{8}B)

${}^8Be^* \rightarrow 2\,\alpha$

(pp-III: 0.02%)

CNO cycle (small)



$p + {}^{12}C \rightarrow {}^{13}N + \gamma$

${}^{13}N \rightarrow {}^{13}C + e^+ + \nu_e$

$p + {}^{13}C \rightarrow {}^{14}N + \gamma$

$p + {}^{14}N \rightarrow {}^{15}O + \gamma$

${}^{15}O \rightarrow {}^{15}N + e^+ + \nu_e$

$p + {}^{15}N \rightarrow {}^{12}C + \alpha$

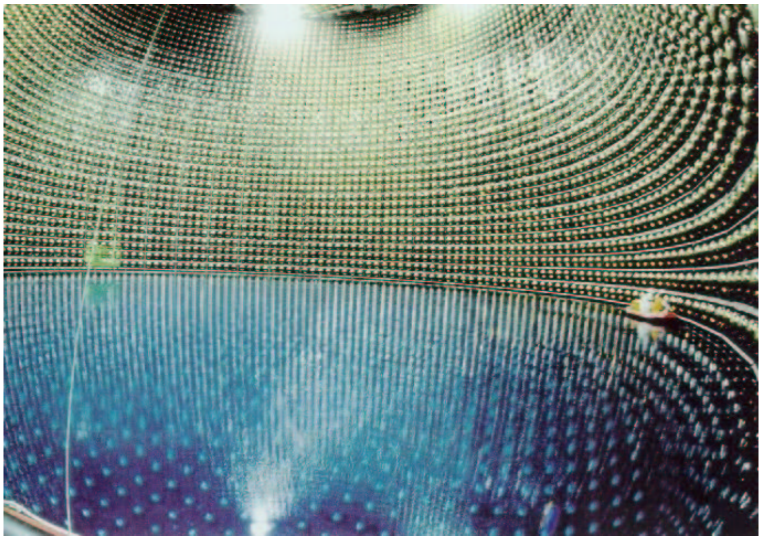
$p + {}^{15}N \rightarrow {}^{16}O + \gamma$

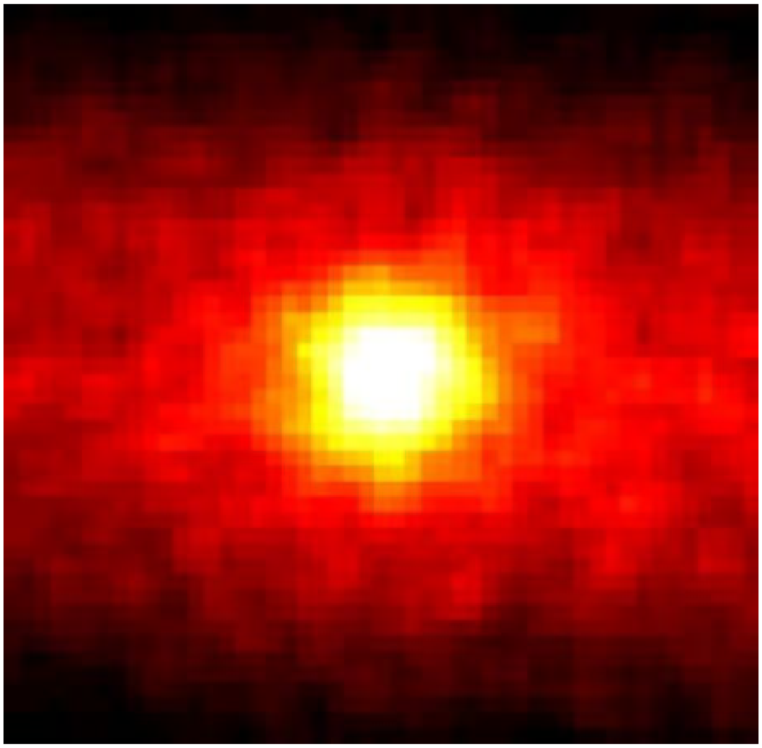
$p + {}^{16}O \rightarrow {}^{17}F + \gamma$

${}^{17}F \rightarrow {}^{17}O + e^+ + \nu_e$

$p + {}^{17}O \rightarrow {}^{14}N + \alpha$

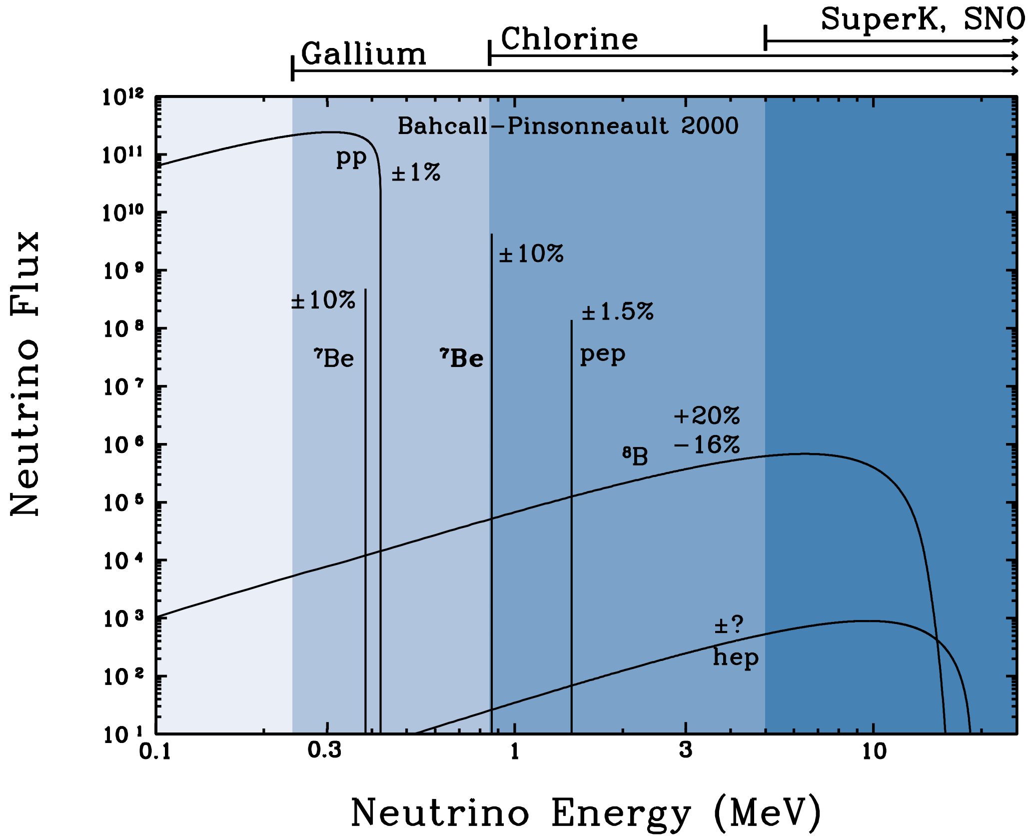
- SuperK can see the sun in neutrinos





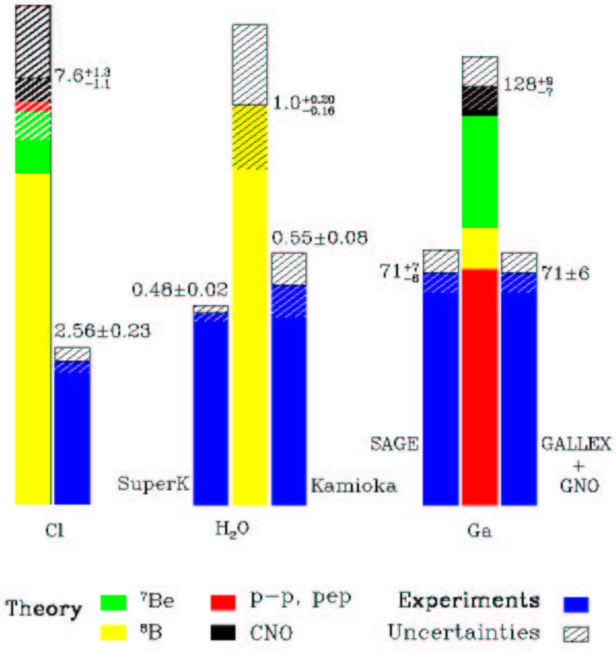
$$\nu_e + e^- \rightarrow \nu_e + e^-$$

- ν_e flux predicted by standard solar model (Bahcall and Pinsonneault, *pp* chain only)



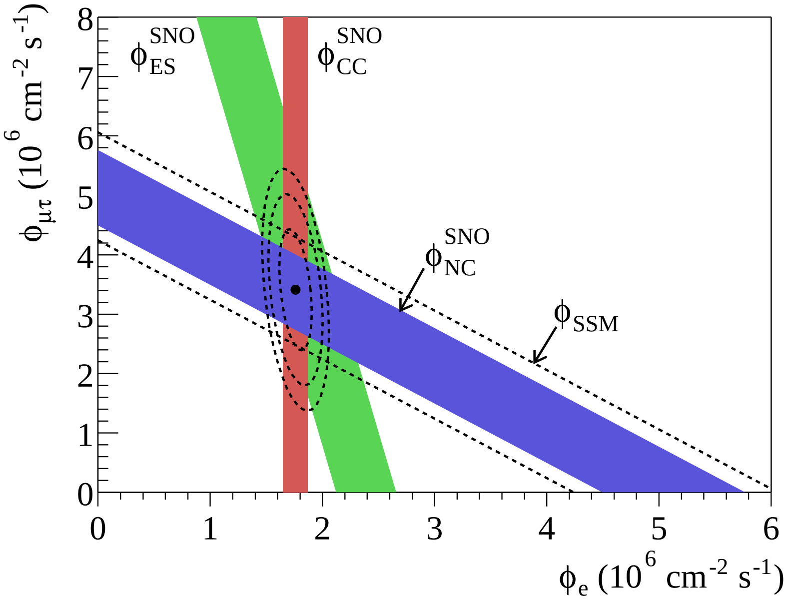
Experiments measure deficit of ν_e

(Homestake, Gallex, Sage, Kamiokande, SuperK, SNO)



Homestake	$\nu_e + Cl \rightarrow e^- + Ar$
SAGE, GALLEX	$\nu_e + Ga \rightarrow e^- + Ge$
SuperK	$\nu_e + e^- \rightarrow \nu_e + e^-$

- SNO can measure both the total $\nu_e + \nu_\mu + \nu_\tau$ flux and the ν_e flux



$$(CC)\ \ \nu_e + D \rightarrow p + p + e^-$$

$$(NC)\ \ \nu_x + D \rightarrow n + p + \nu_x$$

Total flux matches prediction!

- Explanation: $\nu_e \rightarrow \nu_\mu, \nu_\tau$. Survival probability

$$P(\nu_e) = 1 - \sin^2(2\theta)\sin^2\left(\frac{\delta m^2 c^4 L}{4\hbar c E}\right)$$

oscillation length $L_0 = \frac{4\hbar c E}{\delta m^2 c^4} \ll L$ (if $\delta m^2 c^4 > 10^{-9}$ eV)

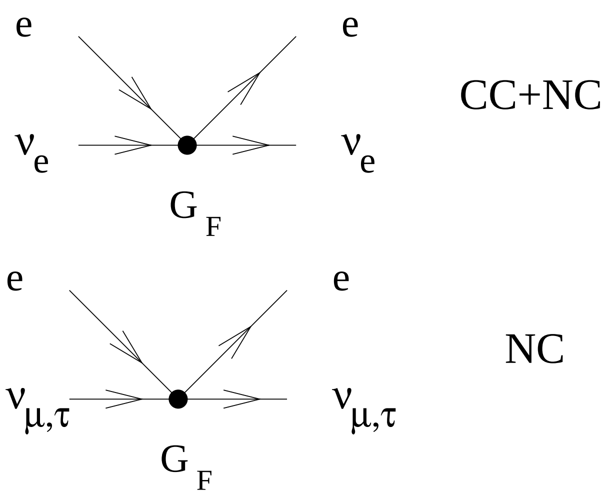
^8B νs have broad spectrum $\rightarrow$ oscillations average to 1/2

$$P(\nu_e) = 1 - \frac{1}{2}\sin^2(2\Theta)$$

But: suppression is factor 3

Matter Enhanced (MSW) Oscillations

- neutrino propagation in matter: forward scattering on electrons leads to effective potential



$$V = \frac{V_e - V_x}{2} = 2\sqrt{2} G_F N_e(r)$$

electron density $N_e(r)$

Wolfenstein (1978)

Mikheyev-Smirnov (1985)

- modified wave equation

$$i\hbar c \frac{d}{dr} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} V - \frac{\delta m^2}{4E}\cos(2\theta) & \frac{\delta m^2}{4E}\sin(2\theta) \\ \frac{\delta m^2}{4E}\sin(2\theta) & -V + \frac{\delta m^2}{4E}\cos(2\theta) \end{pmatrix} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

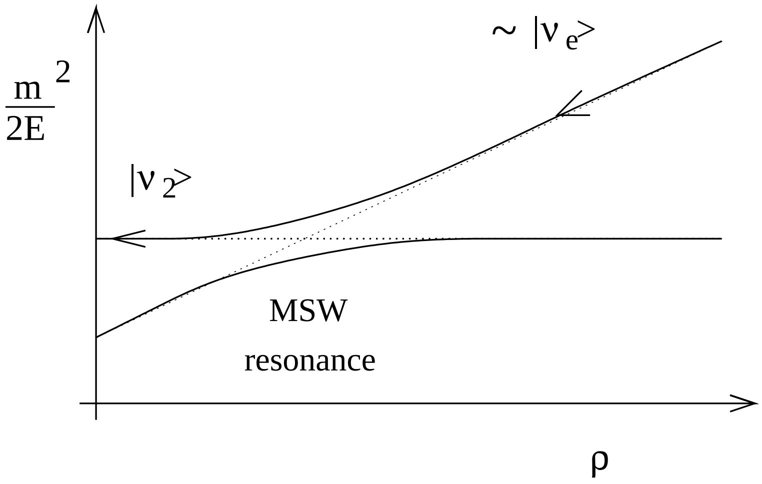
- consider eigenstates of RHS (“matter eigenstates”)
- start at high density

matter eigenstates $\simeq$ flavor eigenstates

- resonance occurs if diagonal element vanishes. Possibilities

  1) $\nu_e \to \nu_1$ non − adiabatic $\quad P(\nu_e) \sim \cos^2\theta, \quad P(\nu_\mu) \sim \sin^2\theta$

  2) $\nu_e \to \nu_2$ adiabatic $\quad P(\nu_e) \sim \sin^2\theta, \quad P(\nu_\mu) \sim \cos^2\theta$



- Landau-Zener method

$$P(\nu_e) = \frac{1}{2} + \frac{1}{2}\cos(2\theta)\cos(2\theta_i)(1 - 2e^{-\pi\gamma_c/2})$$

θ_i local mixing angle at production point

$$\cos(2\theta_i) = -\frac{X}{\sqrt{X^2 + \sin^2(2\theta)}} \qquad X = \frac{VE}{\delta m^2} - \cos(2\theta)$$

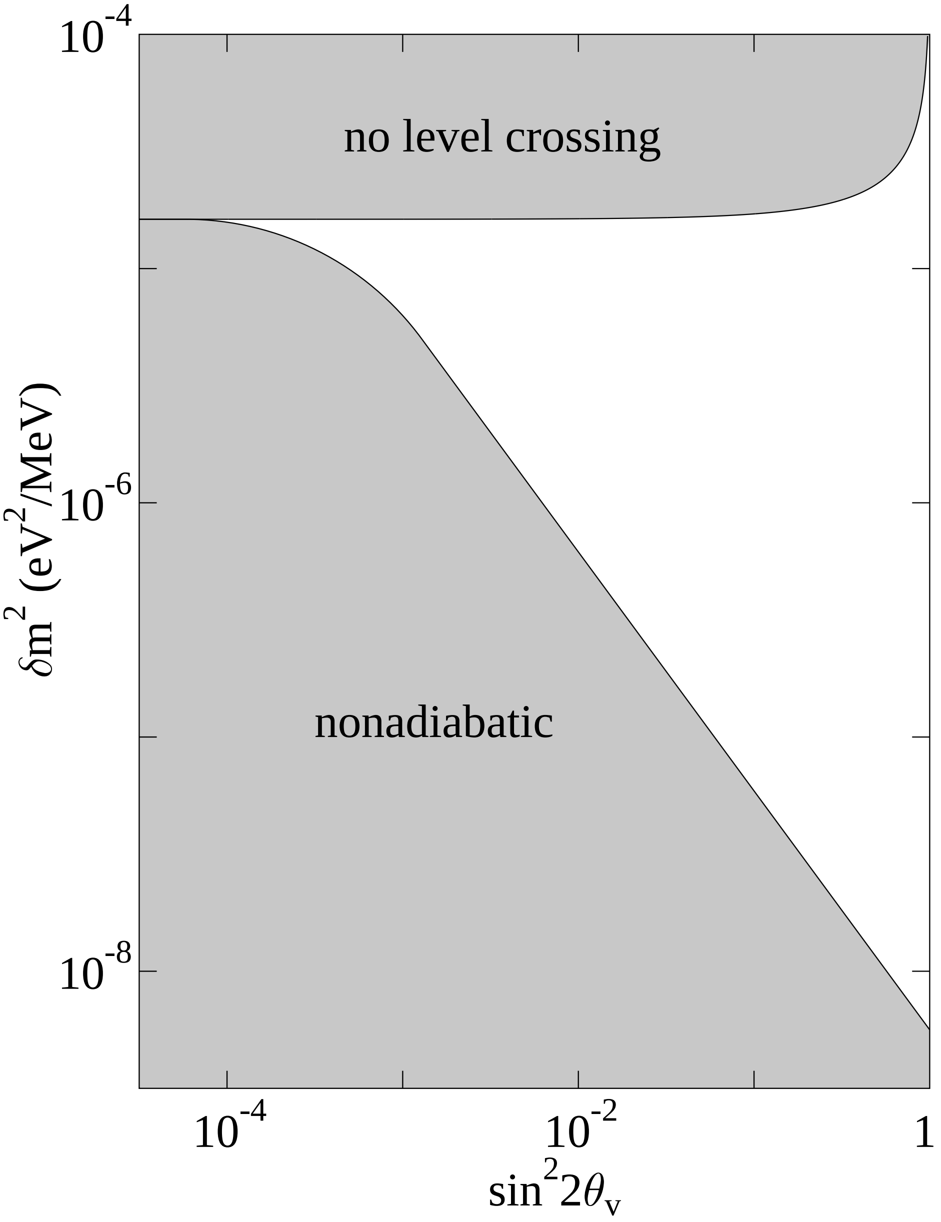
γ_c adiabaticity parameter

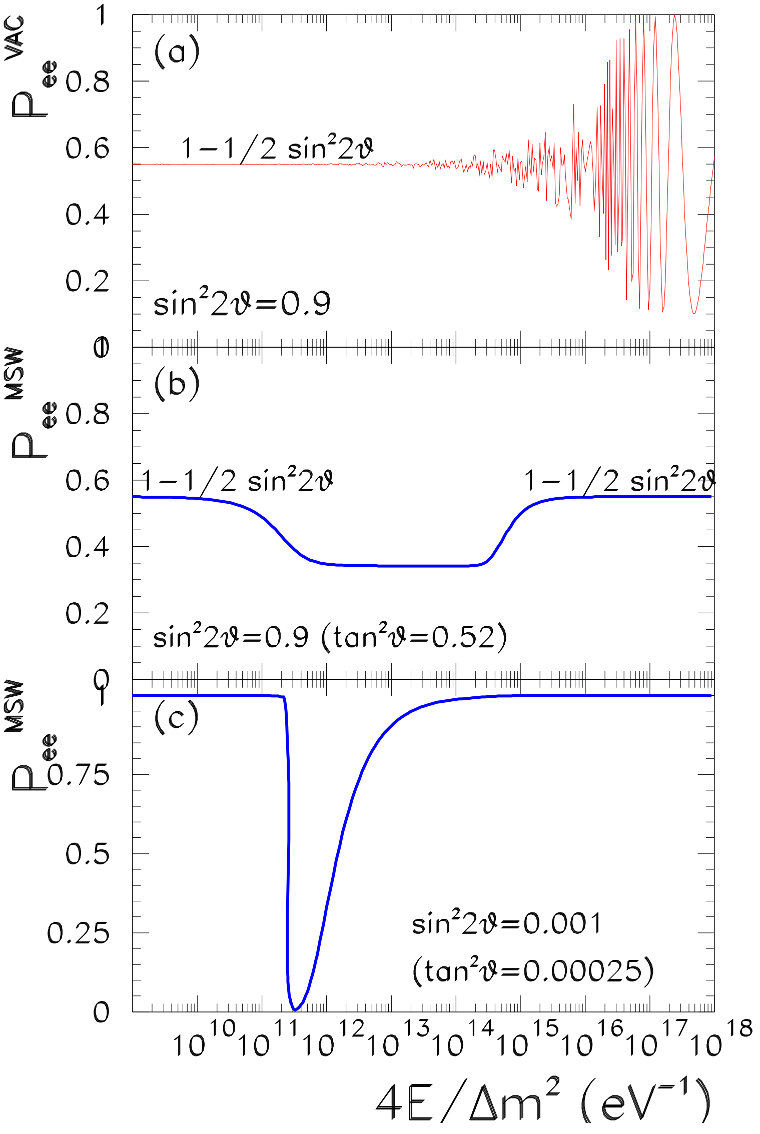
$$\gamma_c = \frac{\sin^2(2\theta)}{\cos(2\theta)}\frac{\delta m^2}{2E}\left[\frac{1}{\rho_c}\left.\frac{d\rho(x)}{dx}\right|_{x_c}\right]^{-1}$$

- resonant conversion requires

  1. resonance (level crossing) exists
  2. adiabatic parameter $\gamma_c > 1$

- resonant conversion of neutrinos in the sun ($\rho_0(\mathrm{sun}) \simeq 150\,\mathrm{gr/cm}^3$)



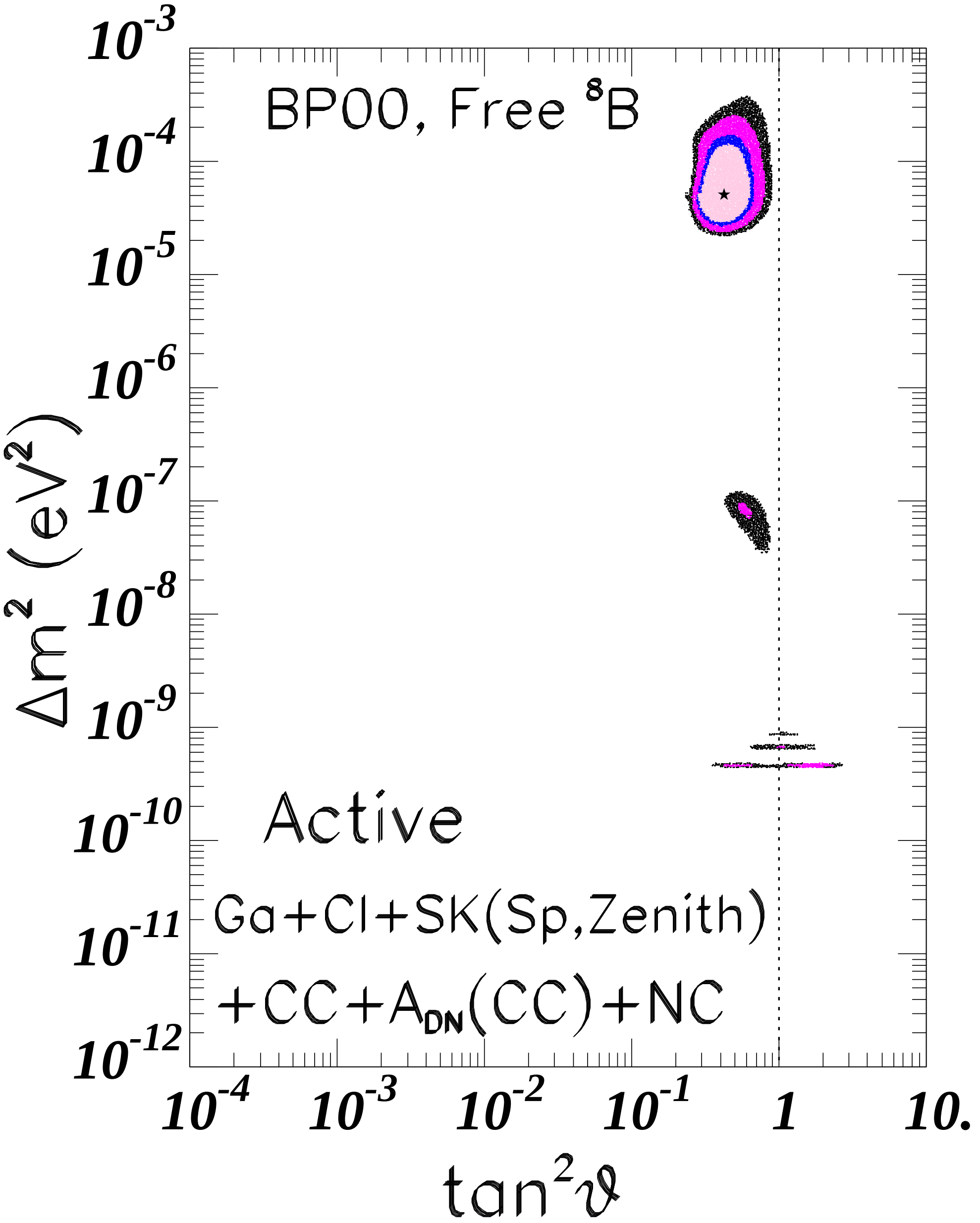


no MSW effect

large mixing angle

small mixing angle

global fit

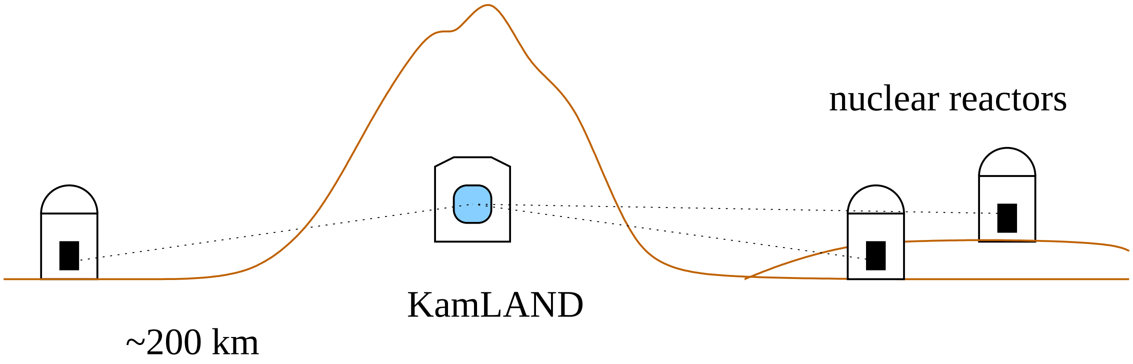


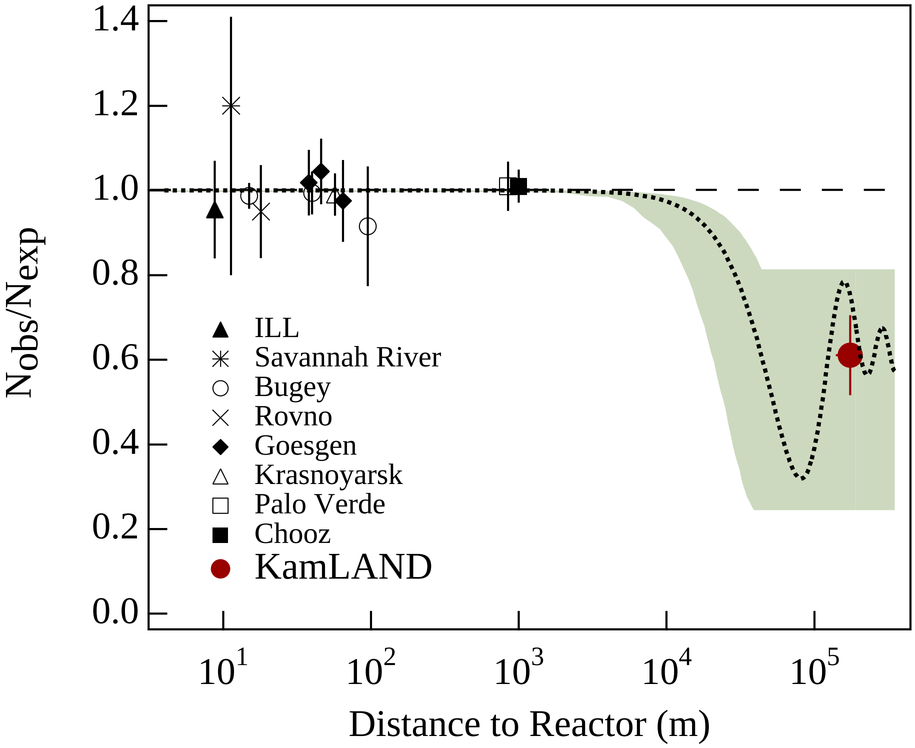
- neutrino fluxes from BP solar model (but: ^8B treated as free parameter)
- Cl and Ga experiments (Homestake, Gallex, SAGE)
- SuperK zenith angle-recoil energy spectra
- CC and NC fluxes from SNO, SNO day-night effect.

J. N. Bahcall, M. C. Gonzalez-Garcia and C. Pena-Garay, JHEP **0207**, 054 (2002)

Checking Solar ν Oscillations at KamLAND

- KamLAND looks for $\bar{\nu}_e$ disappearance





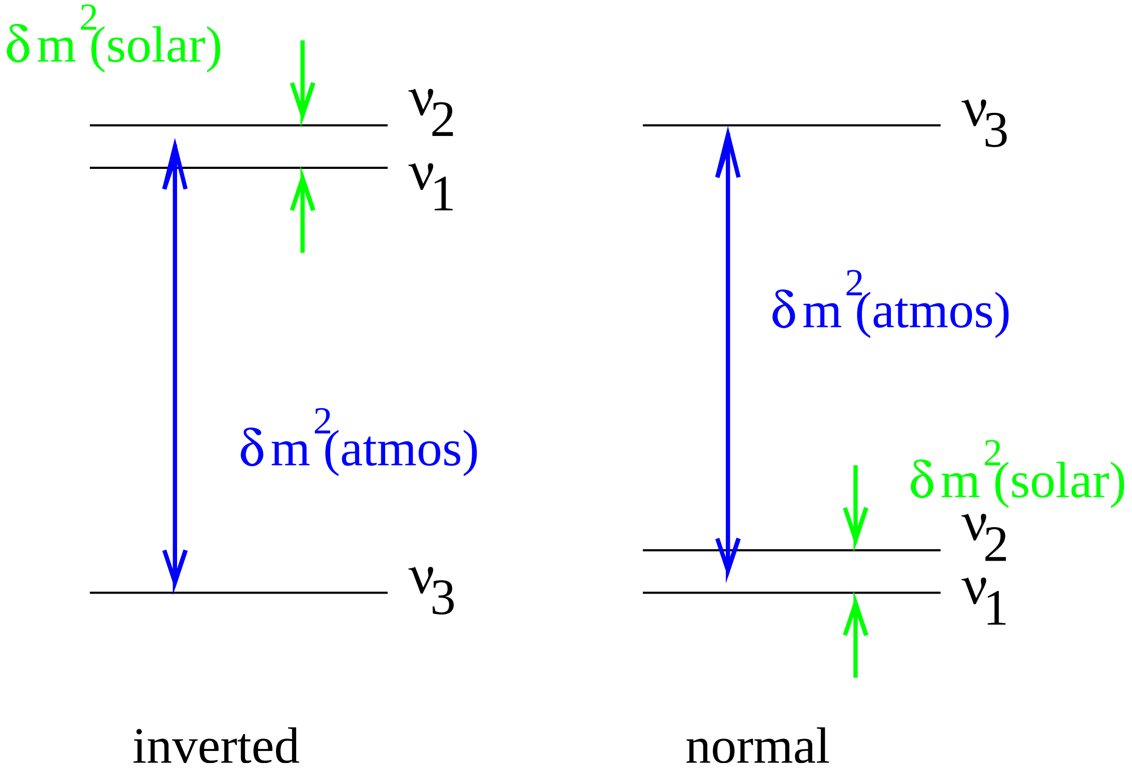
- results favor LMA solution to solar neutrino problem

$$\delta m^2 = 5.5 \cdot 10^{-5}\,\mathrm{eV}^2, \quad \sin^2(2\theta) = 0.833$$

- Checking atmospheric ν oscillations: K2K, MINOS

Global Fit

- Have to account for solar and atmospheric neutrino oscillations (ignoring LSND). Two possible schemes



- experimental results determine

$$\theta_{12}, \quad \theta_{23}, \quad \text{all } |\delta m^2|$$

- unknown parameters

$$\text{hierarchy}, \quad \theta_{13}, \quad \text{phases}$$

More evidence for $\bar{\nu}_e$ oscillations

- LSND

$$\text{beam of } \bar{\nu}_\mu \quad \longrightarrow \quad \text{detect } \bar{\nu}_e$$

being tested by MiniBooNe!

- LSND parameters

$$\delta m^2 = (0.2 - 2)\,\mathrm{eV}^2 \qquad \sin^2 2\theta = 10^{-1} - 10^{-3}$$

- cannot be accommodated in 3×3 mixing scheme

$$\sum \delta m^2 = (m_2^2 - m_1^2) + (m_3^2 - m_2^2) + (m_1^2 - m_3^2) = 0$$

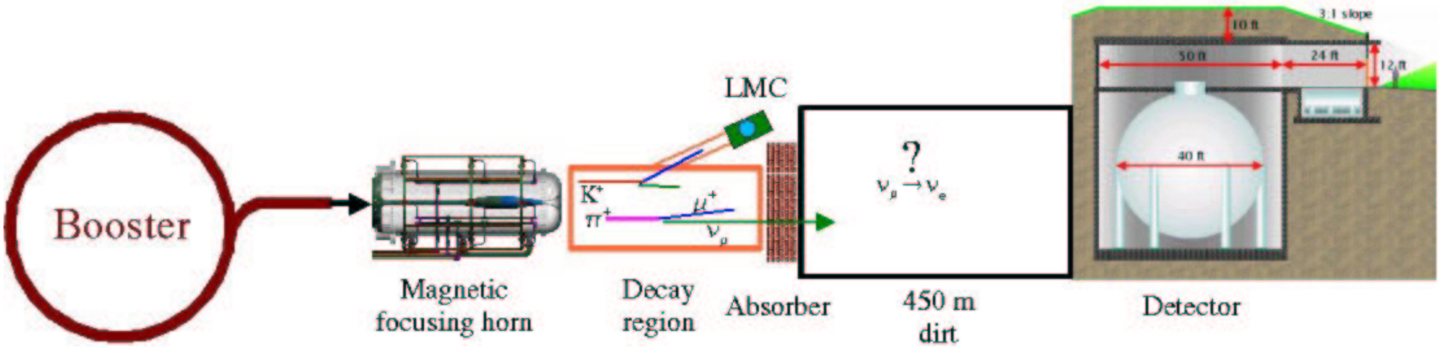
$$\text{but } \; \delta m^2(\text{solar}) + \delta m^2(\text{atmos}) + \delta m^2(\text{LSND}) \neq 0$$

- possibilities

1. experiment(s) not interpreted correctly
2. new neutrino physics (sterile ν, CPT violation, ...)

BooNe

(Booster Neutrino experiment)



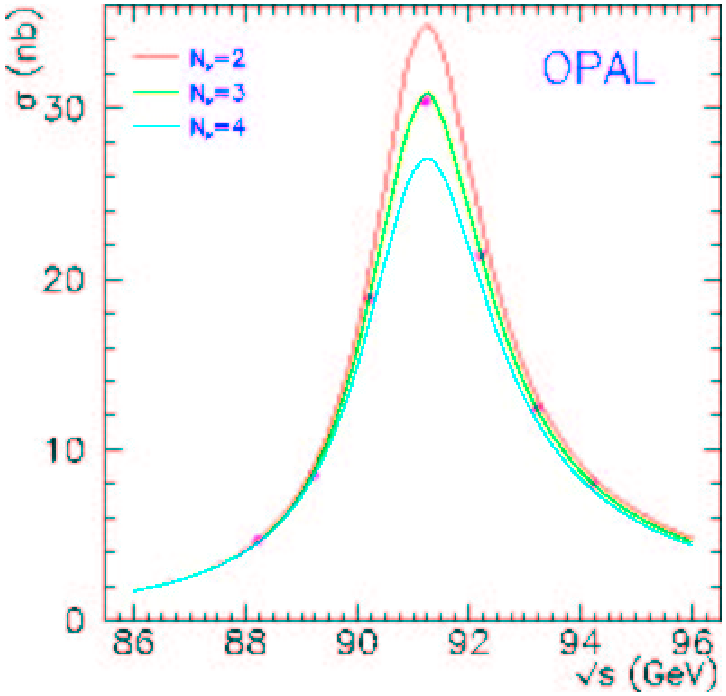
higher statistics

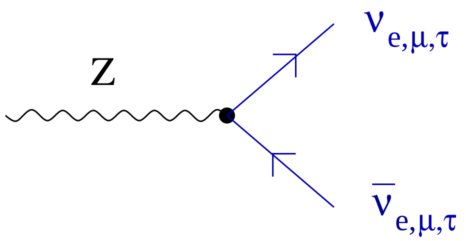
higher energy

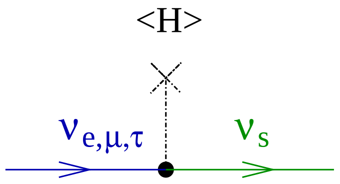
will provide 5σ check on LSND

What is a Sterile Neutrino?

- A sterile neutrino doesn't couple to the W or Z boson, but mixes with the other neutrinos







- Mixing matrix is at least 4x4!

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

- Also, it's not really sterile...

Theories of neutrino mass

- simplest possibility: right handed singlet neutrino

$$\mathcal{L} = y\bar{\nu}_R H \nu_L + h.c.$$

- Higgs expectation value

$$\langle H^0 \rangle = 246\,\mathrm{GeV}$$

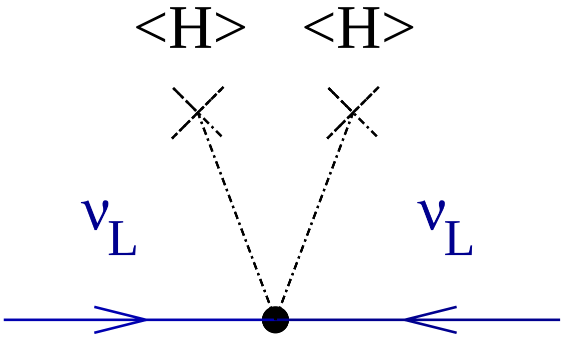
$$m_\nu \sim 10^{-2}\,\mathrm{eV} \qquad \rightarrow \qquad y \sim 10^{-13}$$

why so small?

Theories of neutrino mass

- Majorana mass

$$\mathcal{L} = \frac{g}{\Lambda}(\bar{\nu}_L^c H)(H \nu_L) + h.c.$$



- dim 5 operator characterized by scale of new physics

$$m_\nu \sim \frac{v^2}{\Lambda} \sim 10^{-2}\,\mathrm{eV} \qquad \to \qquad \Lambda \sim 10^{16}\,\mathrm{GeV}$$

unnaturally large scale?

$\Lambda \sim 10^3\,\mathrm{GeV}$ scale of new physics (?)

$\Lambda \sim 10^{16}\,\mathrm{GeV}$ scale of grand unification (?)

$\Lambda \sim 10^{19}\,\mathrm{GeV}$ Planck scale

Seesaw mechanism

- introduce right handed singlet neutrino

$$\mathcal{L} = (\bar{\nu}_L^c \bar{N}_R) \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \begin{pmatrix} \nu_L \\ N_R^c \end{pmatrix}$$

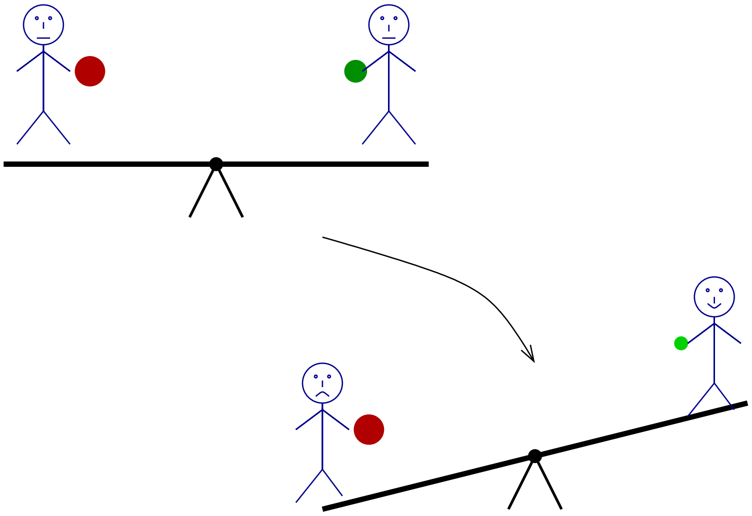
$m = yv \sim m_t \sim 100\,\mathrm{GeV}$ SM Dirac mass

$M \sim V_{GUT} \sim \Lambda_{GUT}$ SM singlet Majorana mass

- diagonalize mass matrix

$$m_1 \sim \frac{m^2}{M} \sim 10^{-3}\,\mathrm{eV} \qquad m_2 \sim M \sim 10^{16}\,\mathrm{GeV}$$

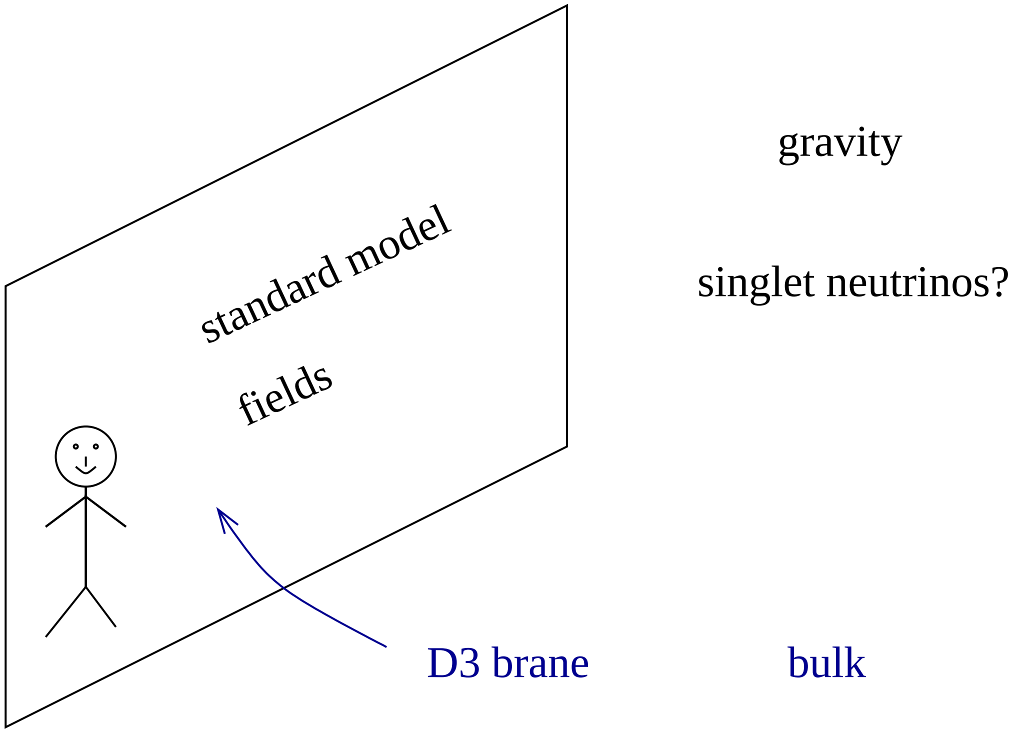
get the right mass scale for light neutrino



Large Extra Dimensions

- Study of extra dimensions is motivated by string theory which requires 10 dimensions for consistency
- old idea: all but 4 dimensions are compactified with sizes on the order of the Planck length
- new idea: extra dimensions could be large as long as standard model fields are confined to 3+1 dimensional brane

Arkani-Hamed, Dimopolous, Dvali,…



- important constraints

size of extra dimensions $R < 1\,\mathrm{mm}$

(1/r law of gravity)

string scale $M_* > 1\,\mathrm{TeV}$

(no evidence for new physics at the Tevatron)

Kaluza Klein Modes

- particles in $4 + n$ dim looks like a tower of states in 4 dim

  Example: 1 extra dim which looks like a circle of radius R

$$\Phi(x^\mu, y) = \sum_{k=-\infty}^{\infty} \Phi_k(x^\mu) e^{iyk/R}$$

- consider scalar field

$$\left(\partial_\mu \partial^\mu - \partial_y^2 + m^2\right) \Phi(x, y) = 0$$

$$\sum_k \left(\partial_\mu \partial^\mu + \frac{k^2}{R^2} + m^2\right) \Phi_k(x) = 0$$

tower of states with $m_k^2 = \dfrac{k^2}{R^2} + m^2$

Neutrino Mass from Extra Dimensions

- introduce SM singlet neutrino field in $4 + n$ dim $\Rightarrow$ tower of states

$$m_{\nu_k} = \left(\frac{k_1^2}{R_1^2} + \frac{k_2^2}{R_2^2} + \ldots + \frac{k_n^2}{R_n^2} \right)^{1/2}$$

- couple to SM doublet neutrinos $\nu_{e\mu,\tau}$ on the brane

$$\frac{yv}{(M_* R)^{n/2}} \bar{\nu}_R \nu_e = m_D \bar{\nu}_R \nu_e$$

  pre-factor comes from coupling $(4 + n)$ dim field ν_R to 4 dim field ν_e

- mass matrix

$$\begin{array}{cc} & \begin{array}{cccccc} \nu_0^R & \nu_1^R & \nu_2^R & \nu_3^R & \ldots & \nu_k^R \end{array} \\ \begin{array}{c} \nu_e^L \\ \nu_1^L \\ \nu_2^L \\ \nu_3^L \\ \\ \nu_k^L \end{array} & \begin{pmatrix} m_D & \sqrt{2} m_D & \sqrt{2} m_D & \sqrt{2} m_D & \ldots & \sqrt{2} m_D \\ & 1/R & & & & \\ & & 2/R & & & \\ & & & 3/R & & \\ & & & & \ldots & \\ & & & & & k/R \end{pmatrix} \end{array}$$

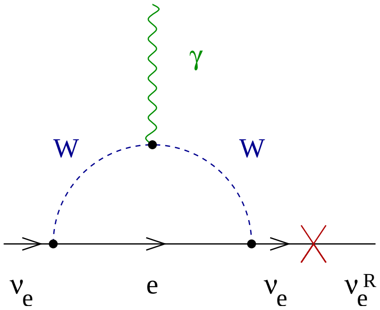
$$\Rightarrow \quad |\nu_e\rangle = \frac{1}{N} \left[|\nu_0^M\rangle + (m_D R)|\nu_1^M\rangle + \ldots + \frac{(m_D R)}{k} |\nu_k^M\rangle \right]$$

mass eigenstates, $m_0 \simeq m_D$, $m_k \simeq k/R$

- small neutrino mass from small overlap between 4-dim and higher dimensional neutrino

Signature: Energy dependent ν magnetic moment

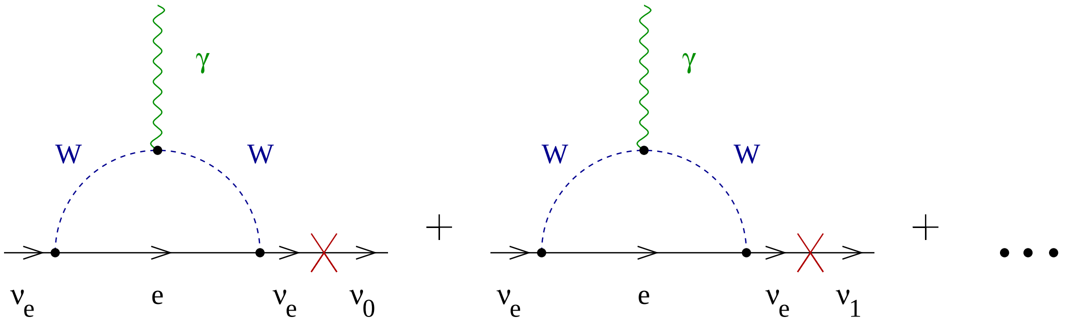
- $m_\nu \neq 0 \Rightarrow \nu$ has a magnetic moment. SM Dirac ν



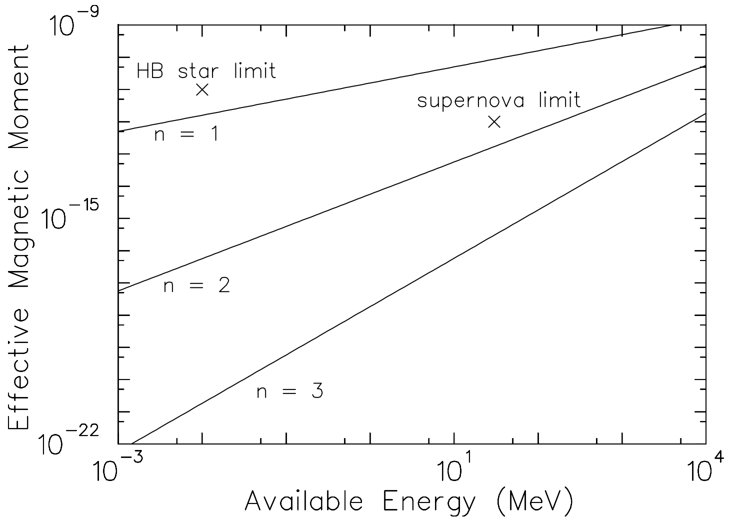
$$\mu_\nu = 3 \cdot 10^{-18} \left(\frac{m_\nu}{1\,\mathrm{eV}}\right) \mu_B$$

current limits: $$\mu_\nu < \begin{cases} 1.8 \cdot 10^{-10} \mu_B & \nu_e e \text{ scattering} \\ 1.0 \cdot 10^{-12} \mu_B & \text{supernova} \end{cases}$$

- extra dimensional scenario

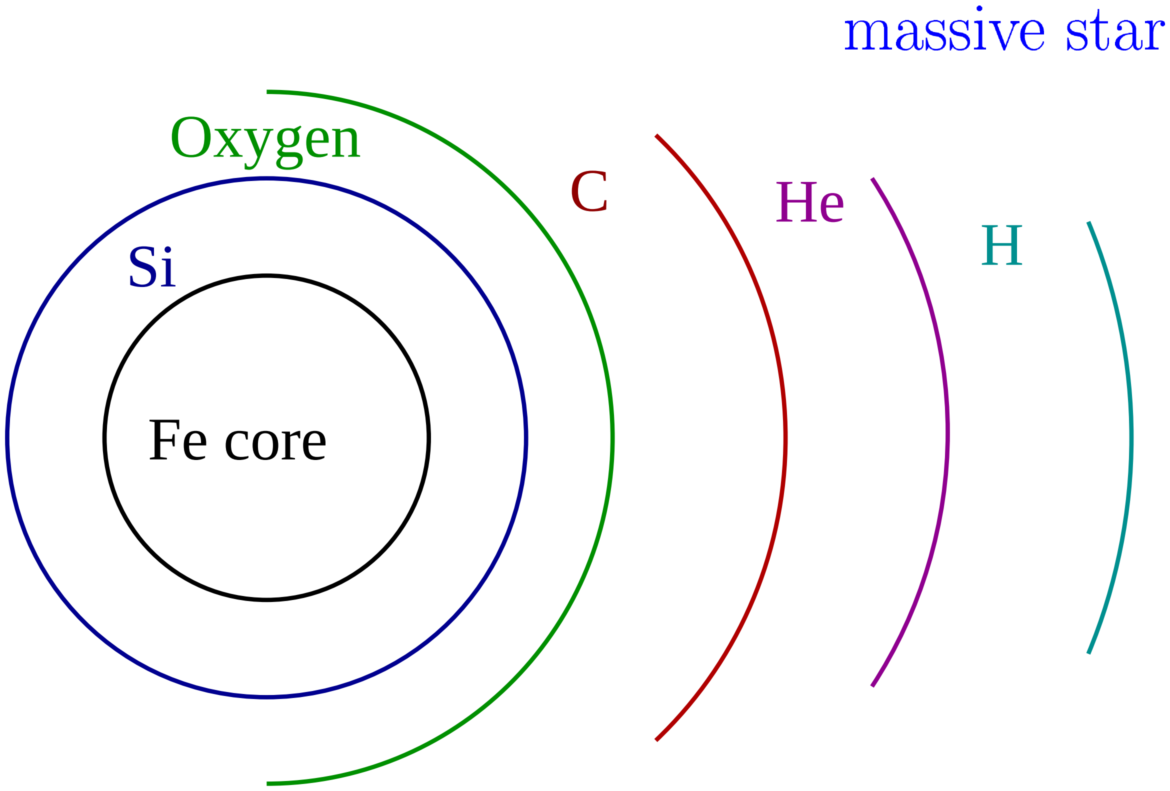


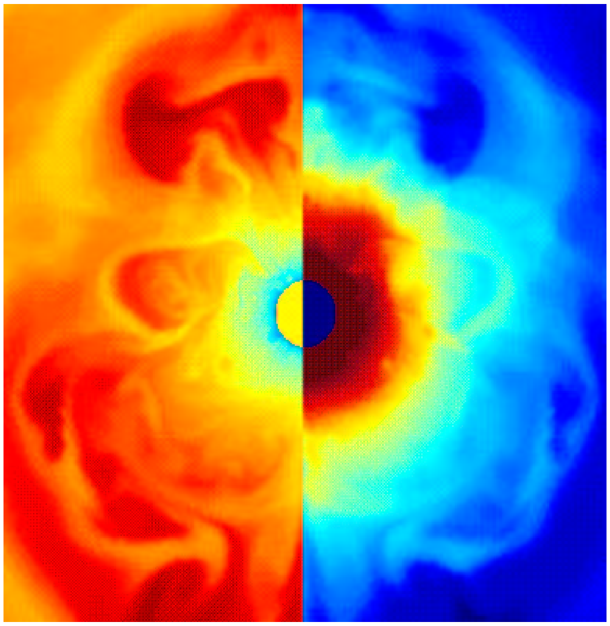
number of νs that contribute depends on energy



McLaughlin, Ng (1999)

Type II Supernova



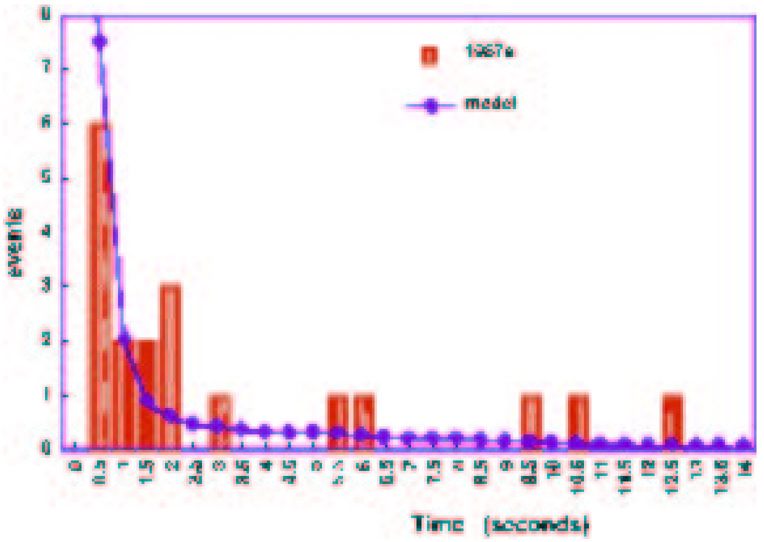


John Blondin, NC State

- core unstable
  $M_{core} \sim 1.5 M_{sun}$
- collapse to nuclear density
- core bounce
- shock produced
- shock stalls
- neutrinos diffuse out of core, energize shock

Supernova Neutrinos

- Most neutrinos emitted during the first $\sim$ 10 sec.
- Galactic supernovae estimated to occur $\sim$ 1 every 30 years.
- Supernova neutrinos detected from SN1987a: $\sim$ 20 events observed in Kamiokande and IMB.



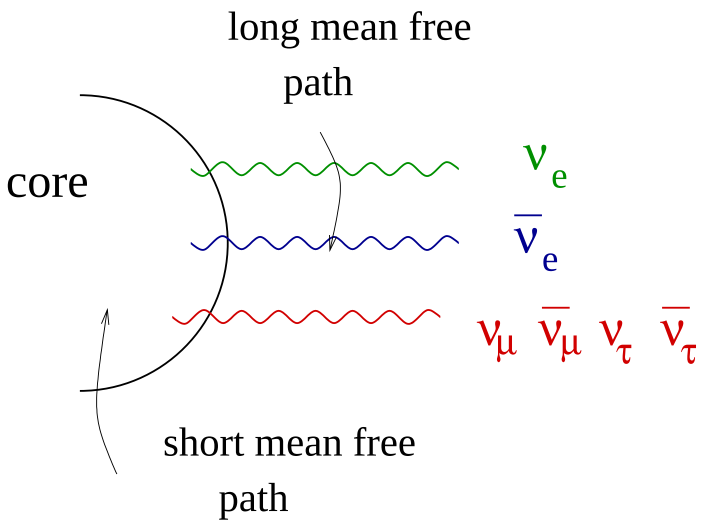
- Current detectors will record 1000's of events.

SuperK: $\sim$ 8000 $\bar{\nu}_e + p \to e^+ + n$ events
SNO: $\sim$ 500 $\nu + d \to \nu + p + n$
SNO: $\sim$ 80 $\nu_e + d \to e^- + p + n$
KamLAND: $\sim$ 300 $\bar{\nu}_e + p \to n + e^+$
KamLAND: $\sim$ 100s $\nu + p \to \nu + p$

Beacom and Vogel

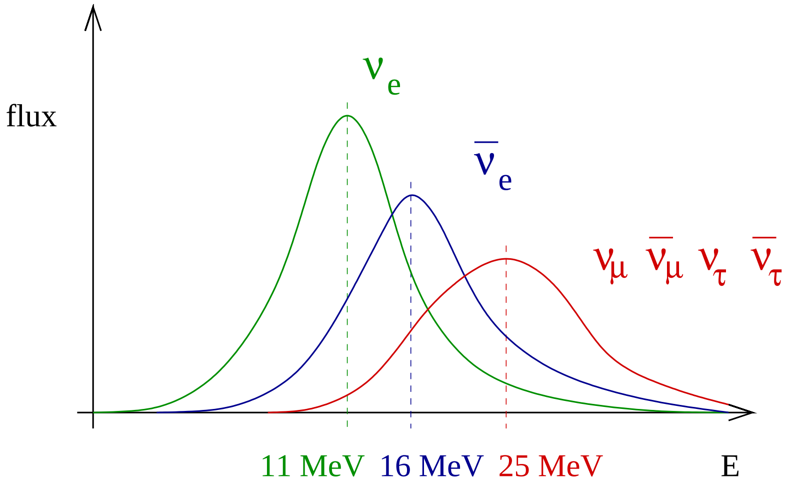
Supernova Neutrinos

- All types of neutrinos emanate from the core. They travel through the outer layers of the SN, then to earth.



SN neutrinos provide unique opportunity to “see” the center of a SN.

- neutrino spectra determined by surface of last scattering in SN core



$$f_\nu(E_\nu) \sim \frac{1}{\exp(\frac{E_\nu}{T_\nu} + \eta_\nu) + 1}$$

Fermi – Dirac distribution

(approximate)

- large uncertainty in spectra

$$\begin{aligned} E_{\nu_\mu} &= E_{\bar{\nu}_\mu} = E_{\nu_\tau} = E_{\bar{\nu}_\tau} = 20 - 30\,\mathrm{MeV} \\ E_{\bar{\nu}_e} &= 13 - 19\,\mathrm{MeV} \\ E_{\nu_e} &= 8 - 13\,\mathrm{MeV} \\ \eta_\nu &= 0 - 3 \end{aligned}$$

Supernova Neutrino Oscillations I

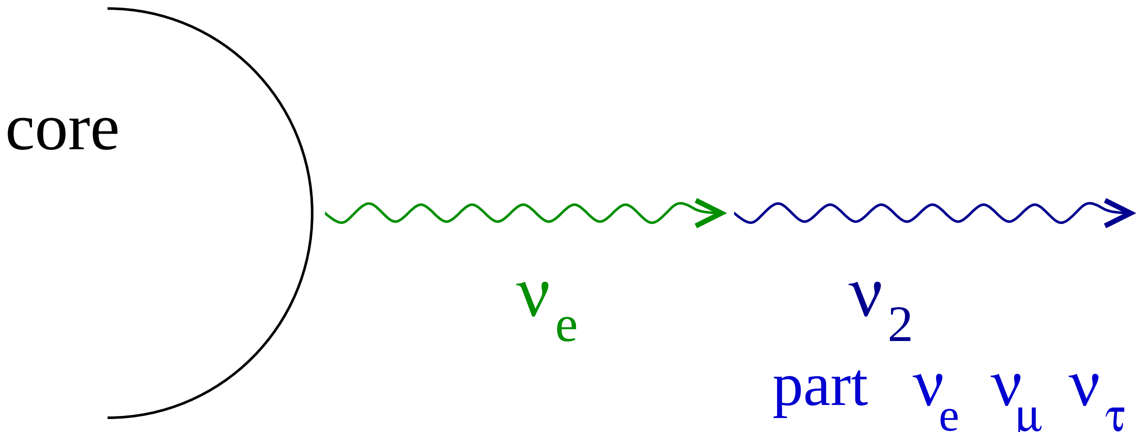
- The MSW resonance that affects solar ν's also occurs in the supernova

Calculation? No problem!

- We have everything we need

$$\begin{array}{rcl} \delta m_{12}^2 & \leftarrow & \text{measured} \\ \text{density in SN} & \leftarrow & \text{calculated} \\ \sin^2 2\theta_{12} & \leftarrow & \text{measured} \end{array}$$

- Conclusion: $\nu_e \to \nu_2$ resonance



Supernova Neutrino Oscillations II

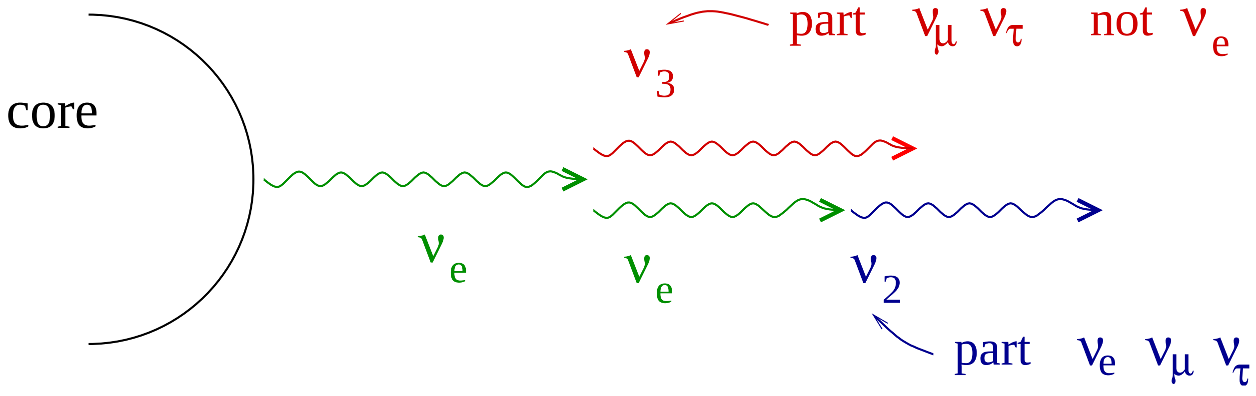
- Any more resonances? Yes! 13-resonance (does not occur in the sun)

Calculation? Big problem!

- We do not have everything we need

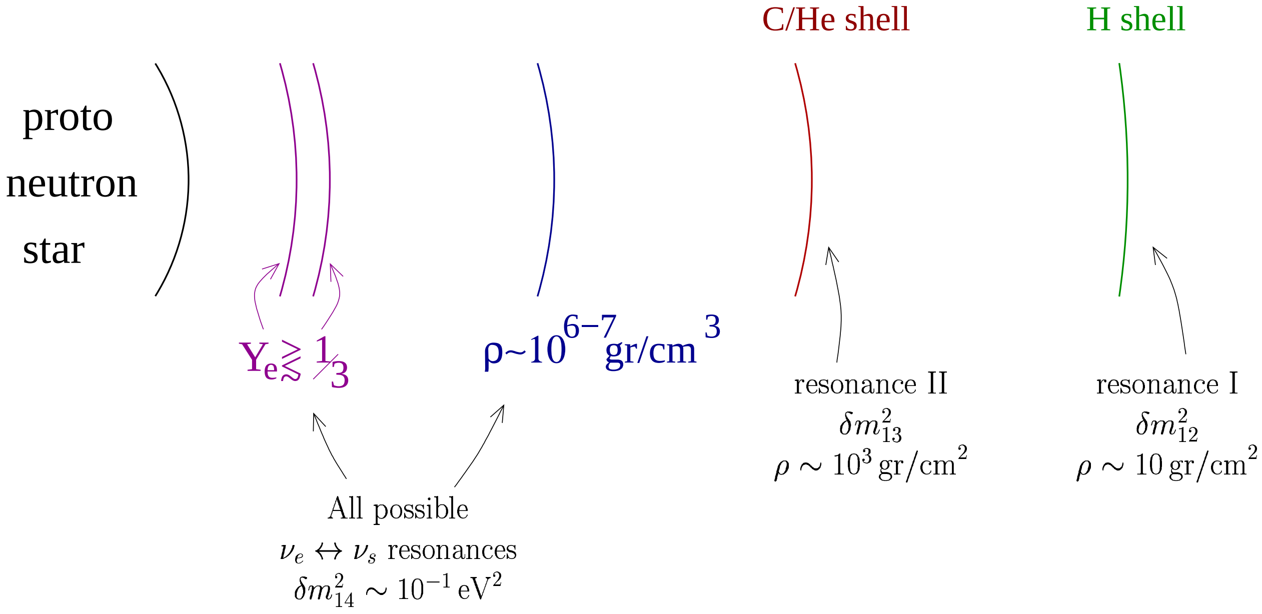
δm^2_{13} ← measured (inferred)
density in SN ← calculated
$\sin^2 2\theta_{13}$ ← unknown

- oscillation depends on θ_{13}, hierarchy



Supernova Neutrino Oscillations III

include sterile neutrino



C/He shell

H shell

proto
neutron
star

$Y_e \gtrless 1/3$

$\rho \sim 10^{6-7}$ gr/cm ^{3}

resonance II
δm^2_{13}
$\rho \sim 10^3\,\mathrm{gr/cm^2}$

resonance I
δm^2_{12}
$\rho \sim 10\,\mathrm{gr/cm^2}$

All possible
$\nu_e \leftrightarrow \nu_s$ resonances
$\delta m^2_{14} \sim 10^{-1}\,\mathrm{eV}^2$

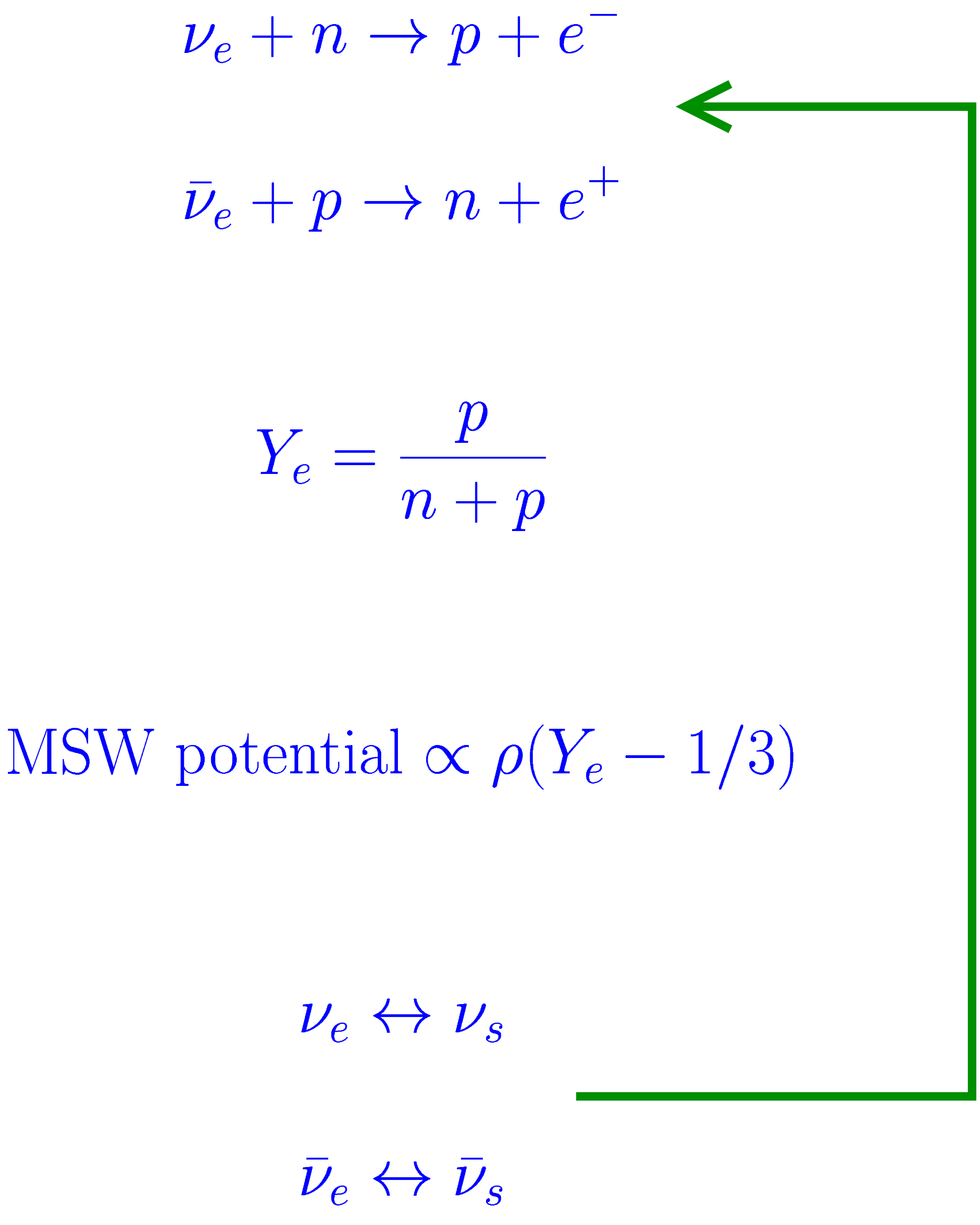
Adiabaticity for $\nu_e \leftrightarrow \nu_s$?

- Depends strongly on Θ_{14}
- Density profile changes with time

Feedback!

- Unique to supernova
- no feedback in the sum

Feedback in Flavor Transformations



$$\nu_e + n \rightarrow p + e^-$$

$$\bar{\nu}_e + p \rightarrow n + e^+$$

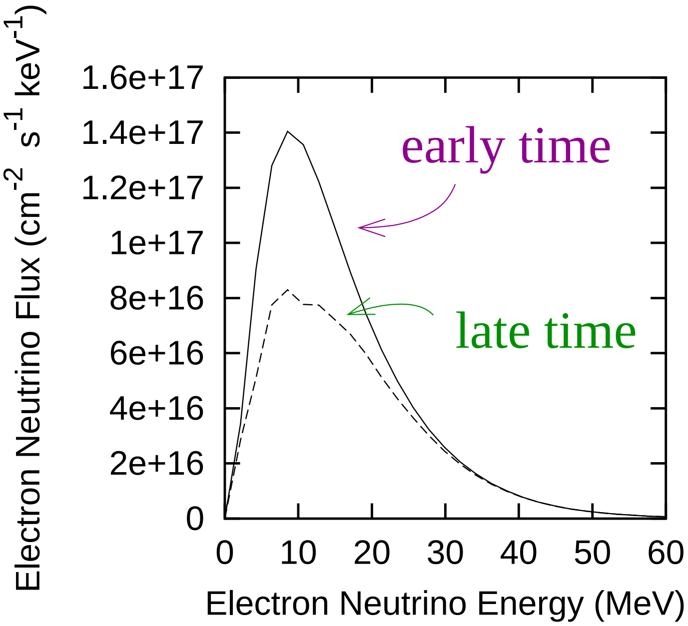
$$Y_e = \frac{p}{n+p}$$

MSW potential $\propto \rho(Y_e - 1/3)$

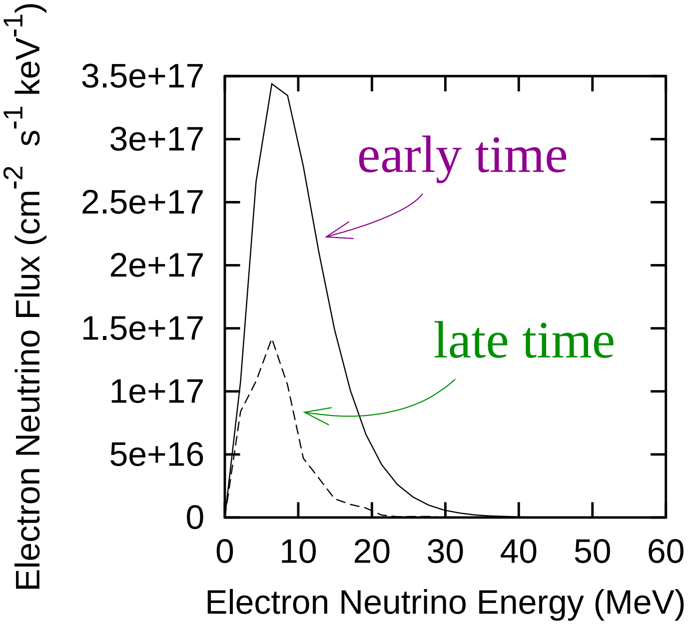
$$\nu_e \leftrightarrow \nu_s$$

$$\bar{\nu}_e \leftrightarrow \bar{\nu}_s$$

Electron neutrino spectrum for $\nu_e \leftrightarrow \nu_s$ solution to r-process



with LMA



w/o LMA, no osc

McLaughlin, Fetter, Fuller, Balantekin 2002

Summary

Compelling evidence for neutrino oscillations

- SuperK atmospheric neutrinos
- SNO CC and NC measurements
- standard solar model vindicated: sun produces energy via the *pp* chain

Many questions remain

- what precisely are the masses and mixing angles? (what are the elements of the MNS matrix?)
- is CP (or even CPT) violated? are neutrinos Majorana or Dirac? do they decay? do they have a magnetic moment?
- what about LSND? (⇒ BooNe). do sterile neutrinos exist?
- where does the neutrino mass come from? why are neutrinos so much lighter than quarks and charged leptons?

Neutrino astrophysics

- supernova produce neutrino bursts. detailed observation will provide constraints on supernova and neutrino physics
- neutrinos figure crucially in the r process. are sterile neutrinos the solution to the r process problem?