Mixed-Mode Calculations within the Nuclear Shell Model.

- **The idea:** to combine the best shell model methods available:
  - m-scheme spherical shell-model
  - SU(3) symmetry based shell-model
- **Developers …**
  - Vesselin Gueorguiev
  - Jerry Draayer
  - Erich Ormand
  - Calvin Johnson

Oblique SMC

H□ = E□

m-scheme states

+

SU(3) states

(H - E*g*)□ =0

Problems that we understand well

exactly solvable (symmetry at play)

$H = H_0$

perturbative regime

$H = H_0 + V$

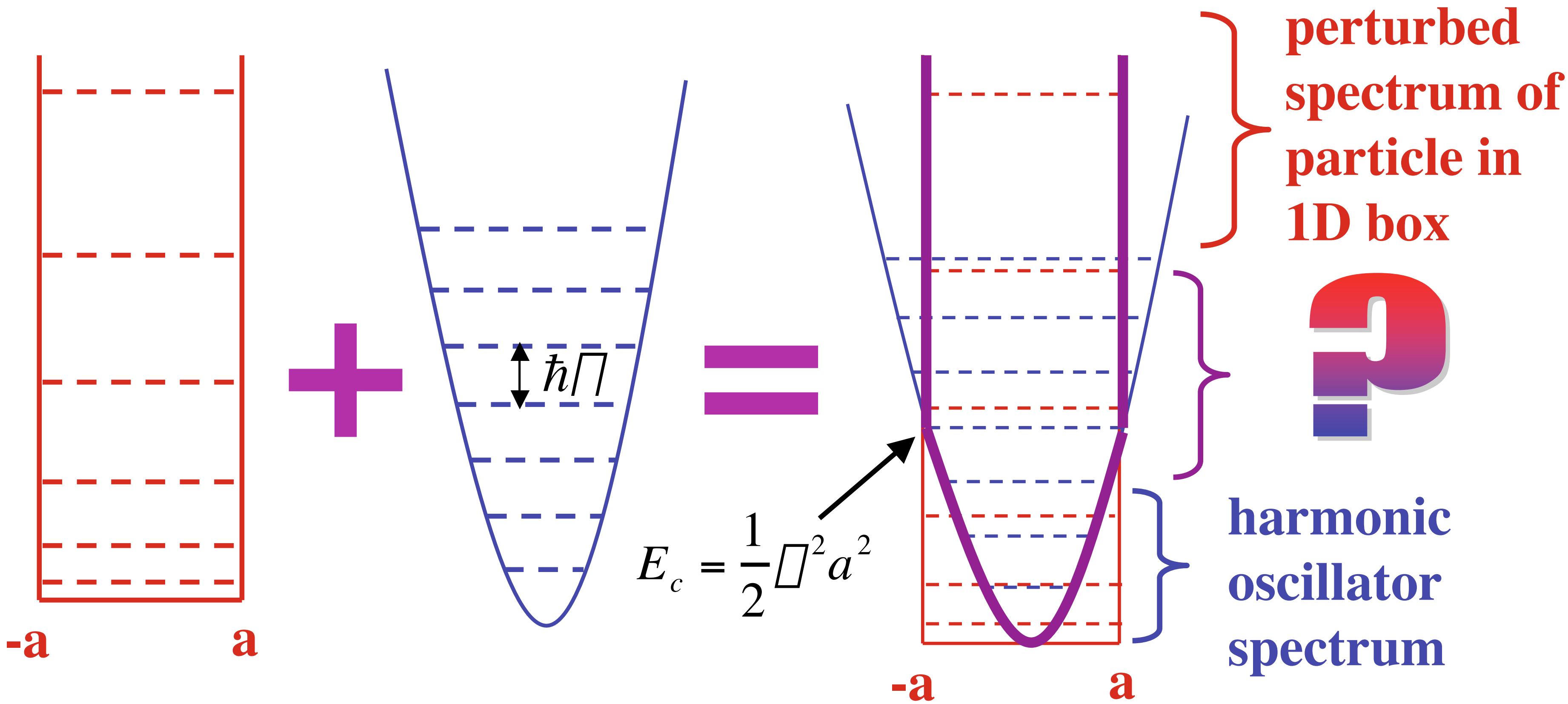
$H = H_1(\square) + H_2(\square) + \ldots$

What about **more than one** *exactly solvable part* beyond **the perturbative regime?**

Transition from **phase one** to **phase two** should occur.

Maybe two or more different sets of basis states could be employed to understand such problems …

Two-Mode Toy System



Particle in 1D box

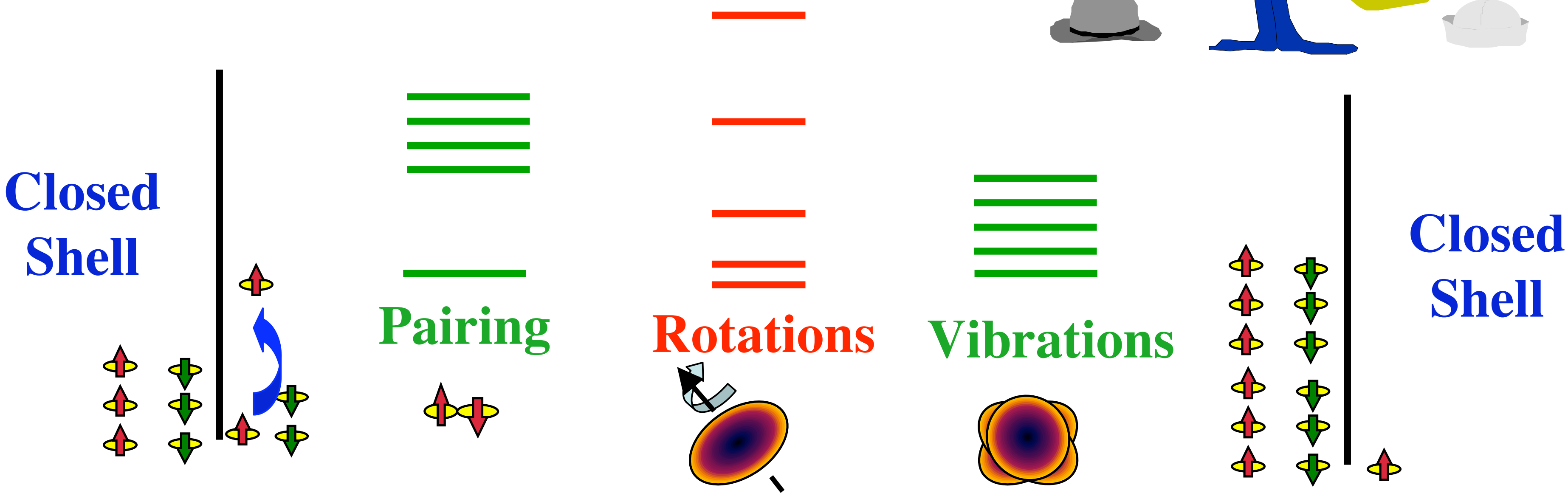
Harmonic Oscillator

Particle in a 1D box subject to harmonic oscillator potential.

The Challenge in Nuclei...

Nuclei display unique characteristics:

- Single-particle Features
- Pairing Correlations
- Deformation/Rotations



Nuclear Shell-Model Hamiltonian

$$H = \sum_i \varepsilon_i a_i^+ a_i + \sum_{i,j,k,l} V_{ijkl} a_i^+ a_j^+ a_k a_l = \sum_i \varepsilon_i N_i + \chi Q \cdot Q + U_{\text{residual}}$$

where a_i^+ and a_i are fermion creation and annihilation operators,

ε_i and V_{ijkl} are real and $V_{ijkl} = V_{klij} = -V_{jikl} = -V_{ijlk}$

- Spherical shell-model basis states are eigenstates of the one-body part of the Hamiltonian - **single-particle states**.

- The two-body part of the Hamiltonian H is dominated by the **quadrupole-quadrupole interaction** Q·Q ~ C_2 of SU(3).

- SU(3) basis states - **collective states** - are eigenstates of H for degenerate single particle energies ε and a pure Q·Q interaction.

SU(3) Basics

The SU(3)   SO(3) Reduction

- **SU(3)** generators as **SO(3)** tensors:

$$\left[L_m, L_{m'}\right] = \Box\sqrt{2}(1m1m' \mid 1m+m')L_{m+m'}$$

$$\left[Q_m, L_{m'}\right] = \Box\sqrt{6}(2m1m' \mid 2m+m')Q_{m+m'}$$

$$\left[Q_m, Q_{m'}\right] = 3\sqrt{10}(2m2m' \mid 1m+m')L_{m+m'}$$

Algebraic quadruple operator:

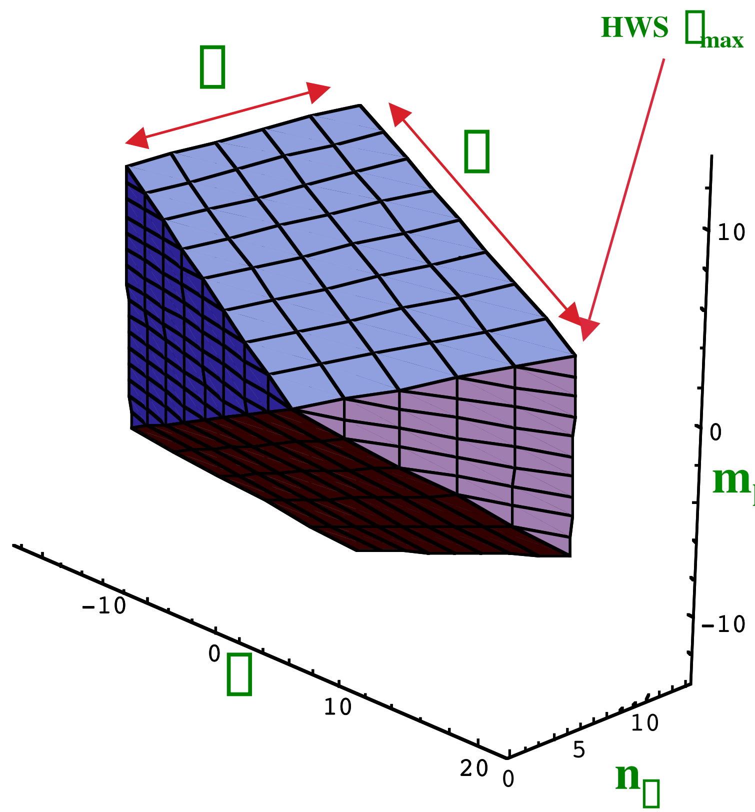
$$Q_m^{(a)} = (4\Box/5)^{1/2}\left(r^2 Y_{2,m}(\Box_r,\Box_r) + b^4 p^2 Y_{2,m}(\Box_p,\Box_p)\right)$$

$$H_0 = r^2 + b^4 p^2, \qquad \vec{L} = \vec{r}\,\Box\,\vec{p}$$

- State labels: **|(□,□)□lm_l>**
  - **(□,□) - SU(3)** irrep labels
  - **l** - total orbital angular momentum
  - **m_l** - angular momentum projection (laboratory axis)
  - **□** - angular momentum projection (body-fixed axis)

The reduction SU(3)   SU(2)   U(1)

- **SU(3)** generators as **SU(2)** tensors:
  - **{Q_0; L_0,Q_{+2}, Q_{-2}} □ U(1) ; SU(2)**
  - **{L_{+1} ,Q_{+1}, L_{-1}, Q_{-1}} □ 2 conjugate [1/2] irreps of SU(2) with □= ± 3**
- State labels: **|(□,□) □$n_{□}$ m_l>**
  - **(□,□)** - **SU(3)** irrep labels
  - **□** - quadruple moment
  - **m_l** - third projection of the angular momentum
  - **$n_{□}$** - number of oscillator quanta in (x,y) plane for **(□,0)** irreps
- Label's values:
  - **□ = □□□2□, □□□2□ +3, ..., 2□+□**
  - **$n_{□}$ = 0, 1, ..., □+□**
  - **m_l = □$n_{□}$,□$n_{□}$+2, ..., $n_{□}$**



Basis States

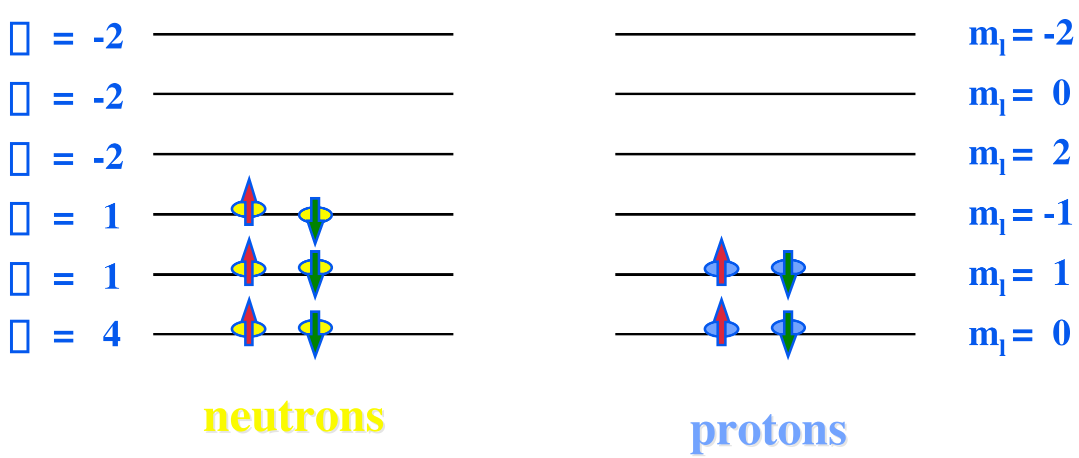
Strong **SU(3)** coupling:

$| N\, S\, (□,□)\, □,\ n_{□},\ m_l,\ m_S >=$

$□\ <SU_p(3)\ SU_n(3)\ |\ SU(3)>\ <SU_{Sp}(2)\ SU_{Sn}(2)\ |\ SU_S(2)>$

$| N_p\, S\, (□,□)\, □,\ n_{□},\ m_l,\ m_S >_p \quad | N_n\, S\, (□,□)\, □,\ n_{□},\ m_l,\ m_S >_n$

✦ **SU(3) SU(4) SU(3) $SU_S(2)$ $SU_T(2)$ leading irreps**



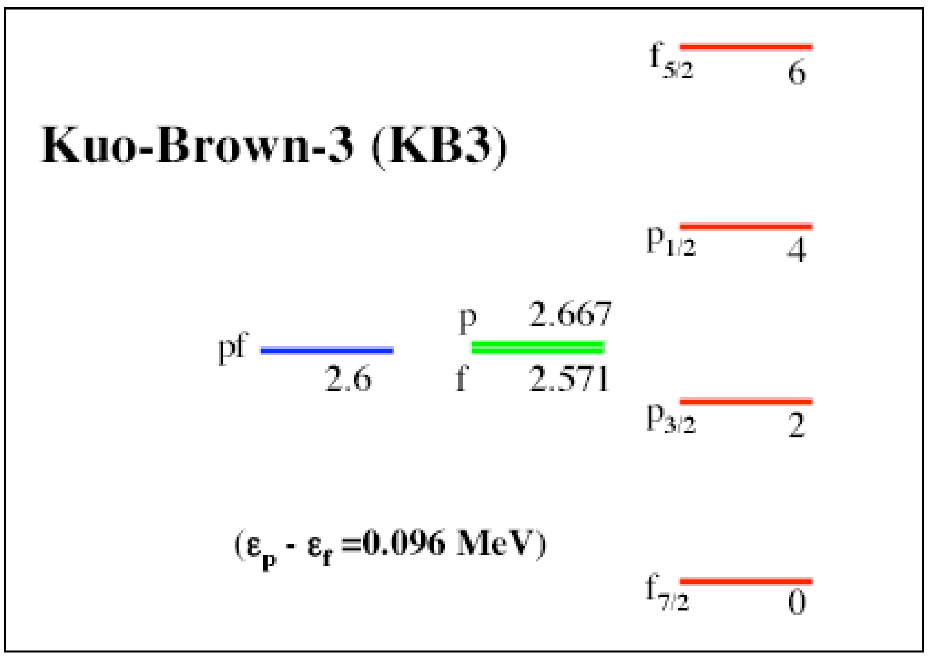
Similar, but much simpler construction of **m-scheme** basis states:

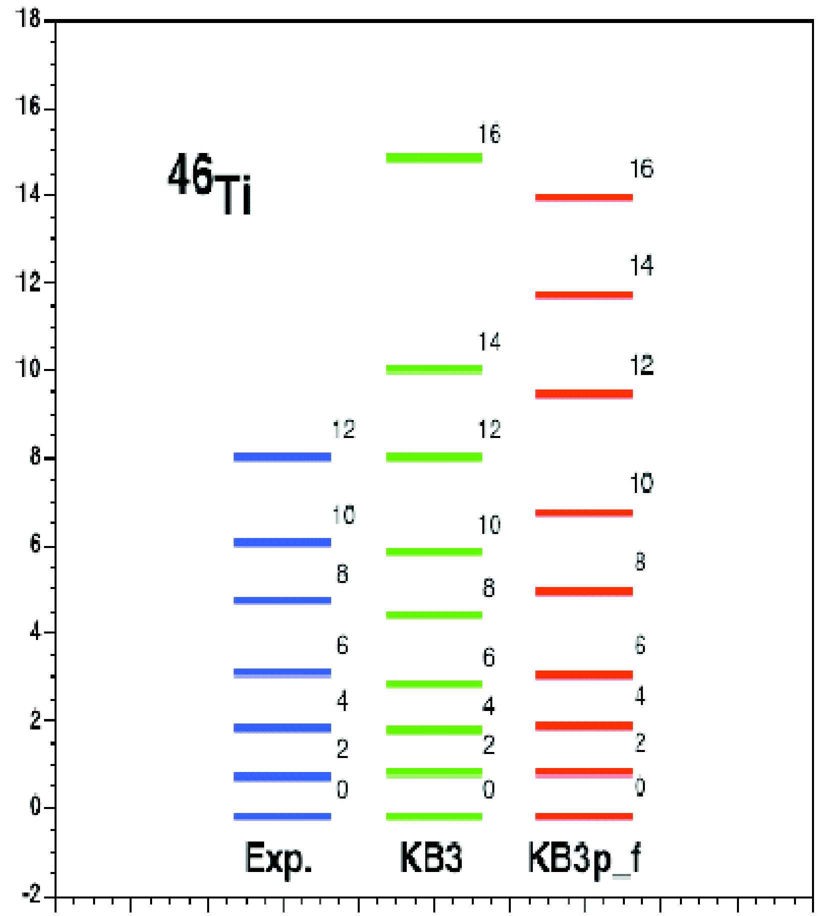
just configurations with same total M_J.

The Shell-Model Hamiltonian

$$H = \sum_i \varepsilon_i a_i^+ a_i + \sum_{i,j,k,l} V_{ijkl} a_i^+ a_j^+ a_k a_l, \qquad \sum_i \varepsilon_i a_i^+ a_i \approx \hbar\omega\left(\sum n_i + \alpha_i l_i \cdot s_i + \beta_i l_i^2\right)$$

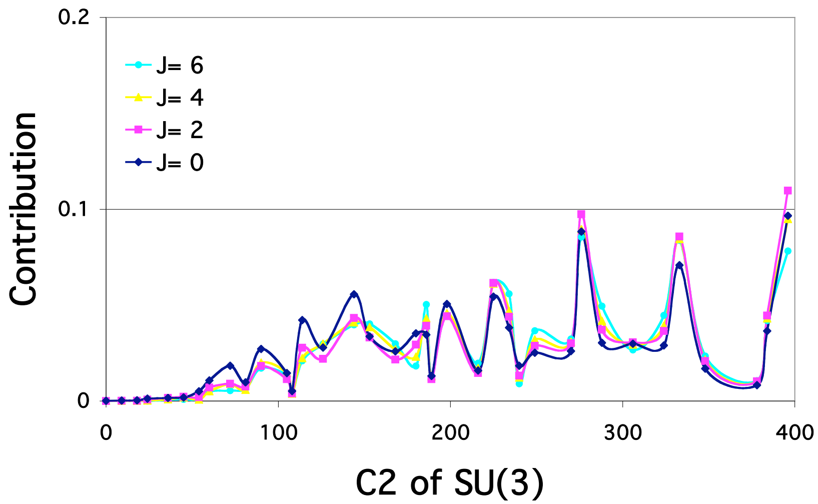
Single-particle energies





SU(3) Symmetry Breaking in the pf-shell nuclei

Low Energy States SU(3) Structure

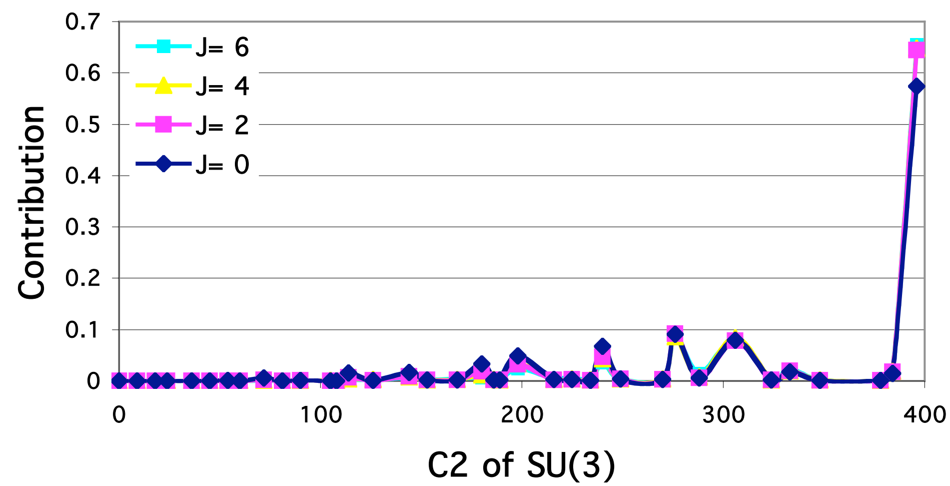


Realistic spin-orbit (*l·s*) single particle energy splitting!

~~(*l·s*)~~

Turn off the s.p.e. spin-orbit splitting!

Low Energy States SU(3) Structure, No Spin-Orbit Interaction



Coherent state structure in ^{48}Cr using Kuo-Brown-3 interaction.

Eigenvalue Problem in an Oblique Basis

Spherical basis states **e_i** SU(3) basis states **E_α**

Overlap matrix **g**

$$\hat{g} = \begin{pmatrix} \langle e_i | e_j \rangle & \langle e_i | E_\alpha \rangle \\ \langle E_\alpha | e_j \rangle & \langle E_\alpha | E_\beta \rangle \end{pmatrix} = \begin{pmatrix} 1 & \Theta \\ \Theta^{+} & 1 \end{pmatrix}$$

The eigenvalue problem

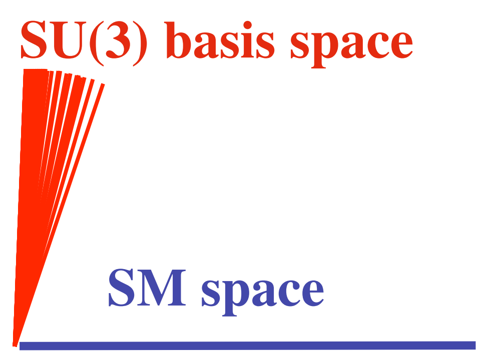
$$H\Psi = E\Psi \quad \Rightarrow \quad \hat{H} \cdot \hat{\Psi} = E\, \hat{g} \cdot \hat{\Psi}$$

Example of an Oblique Basis Calculation: ^{24}Mg

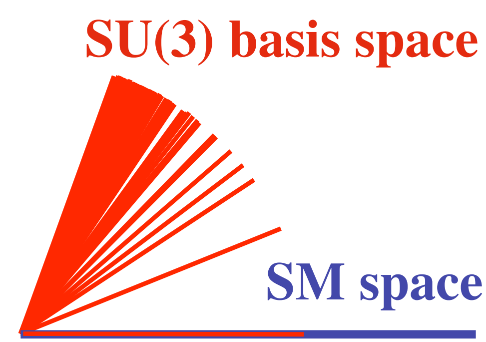
We use the **Wildenthal USD interaction** and denote the **spherical basis** by SM(#) where # is the number of nucleons outside the $d_{5/2}$ shell, the **SU(3) basis** consists of the leading irrep (8,4) and the next to the leading irrep, (9,2).

Model Space	**SU3 (8,4)**	**SU3+ (8,4) & (9,2)**	**GT100**	**SM(0)**	**SM(1)**	**SM(2)**	**SM(4)**	**Full**
Dimension (m-scheme)	**23**	**128**	**500**	**29**	**449**	**2829**	**18290**	**28503**
%	**0.08**	**0.45**	**1.75**	**0.10**	**1.57**	**9.92**	**64.17**	**100**

Visualizing the SU(3) space with respect to the SM space using the naturally induced basis in the SU(3) space.



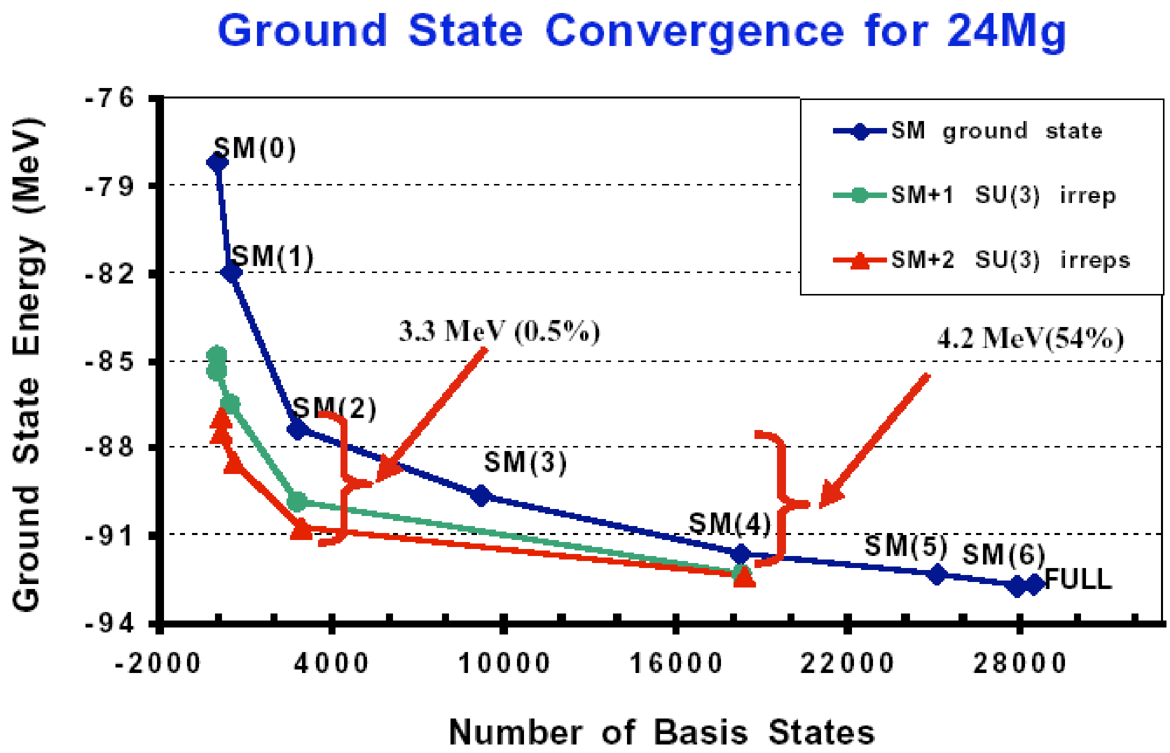
SM(2) & SU3+

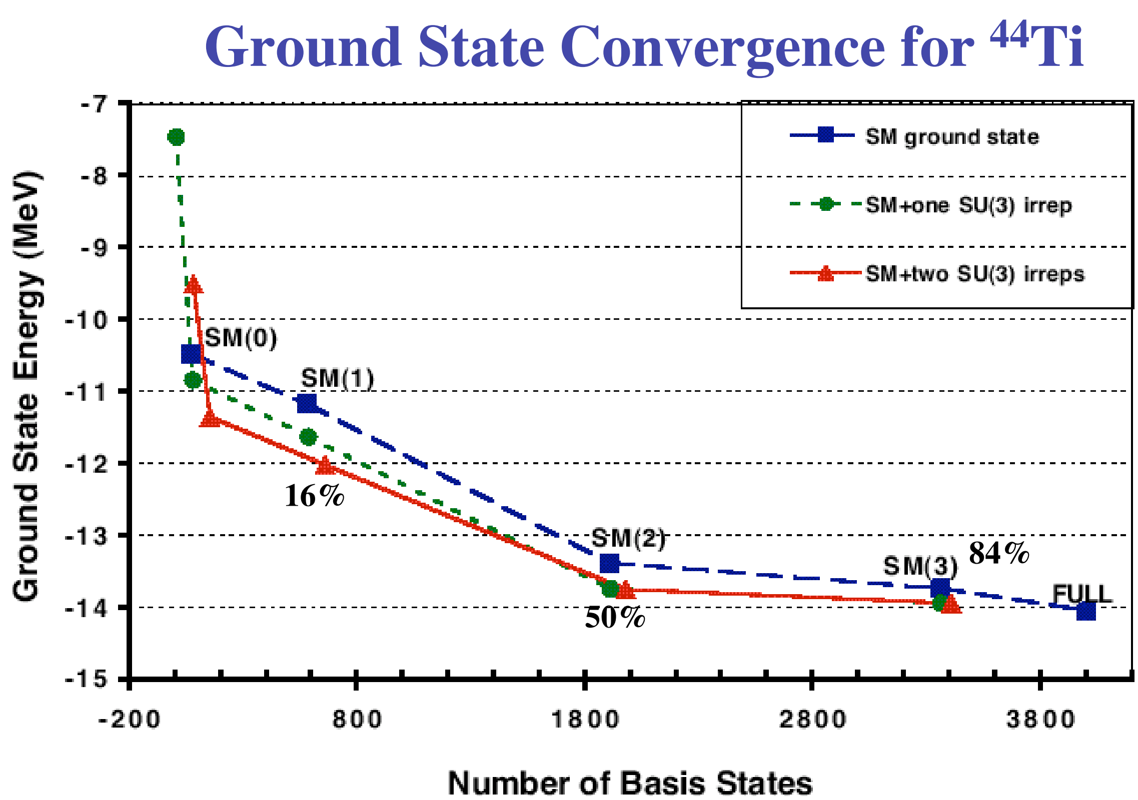


SM(4) & SU3+

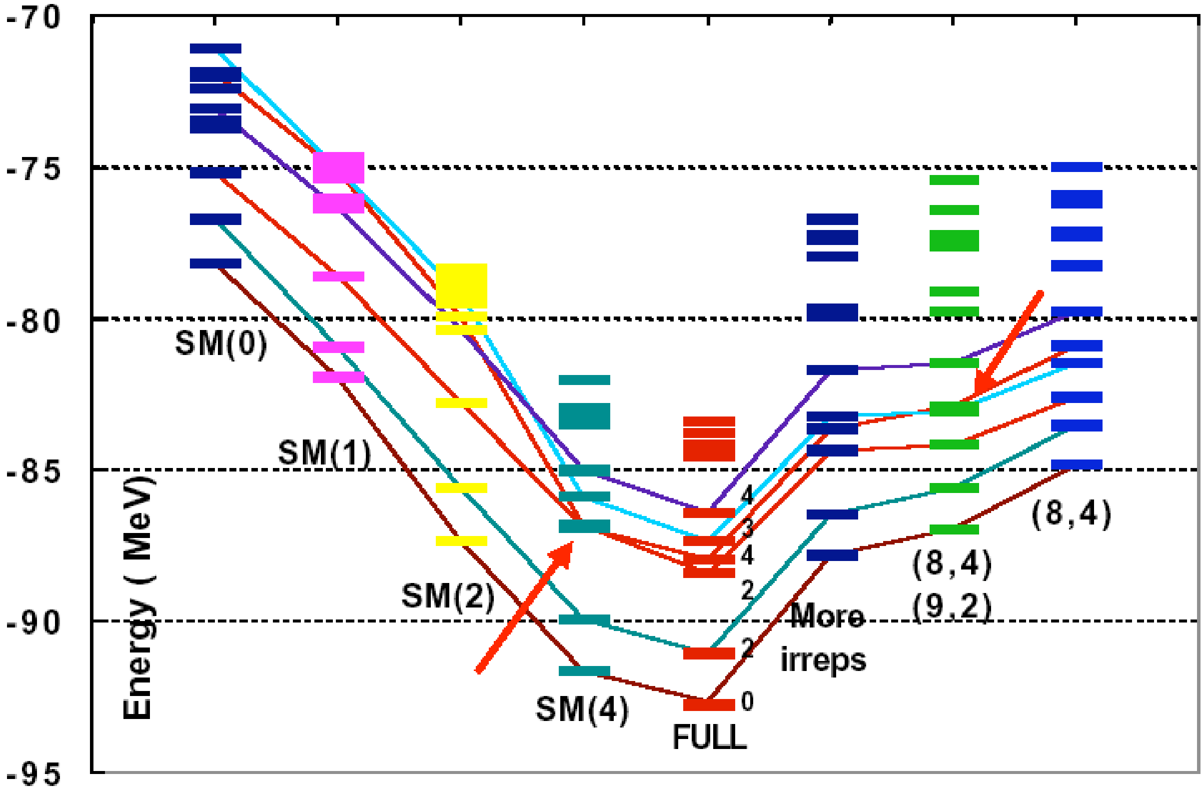
Insert portrait slides

Better Dimensional Convergence!

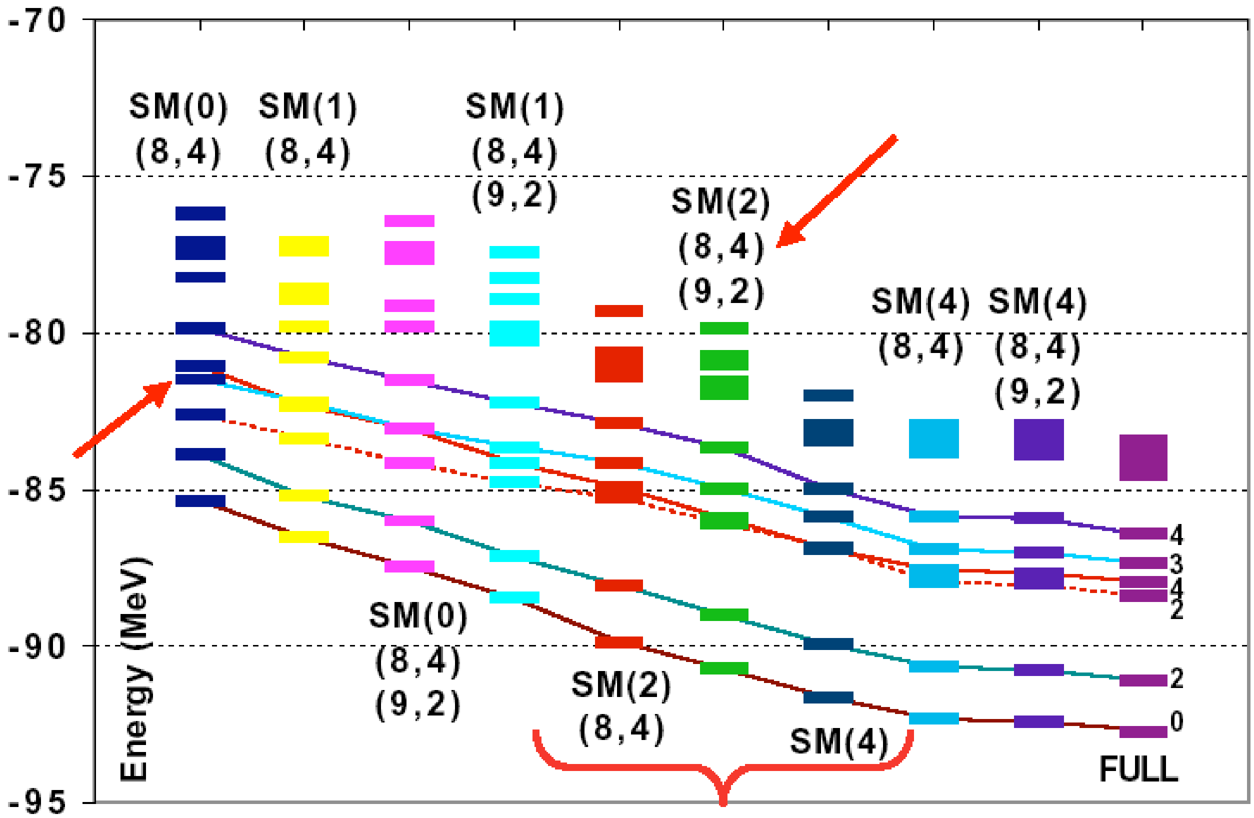




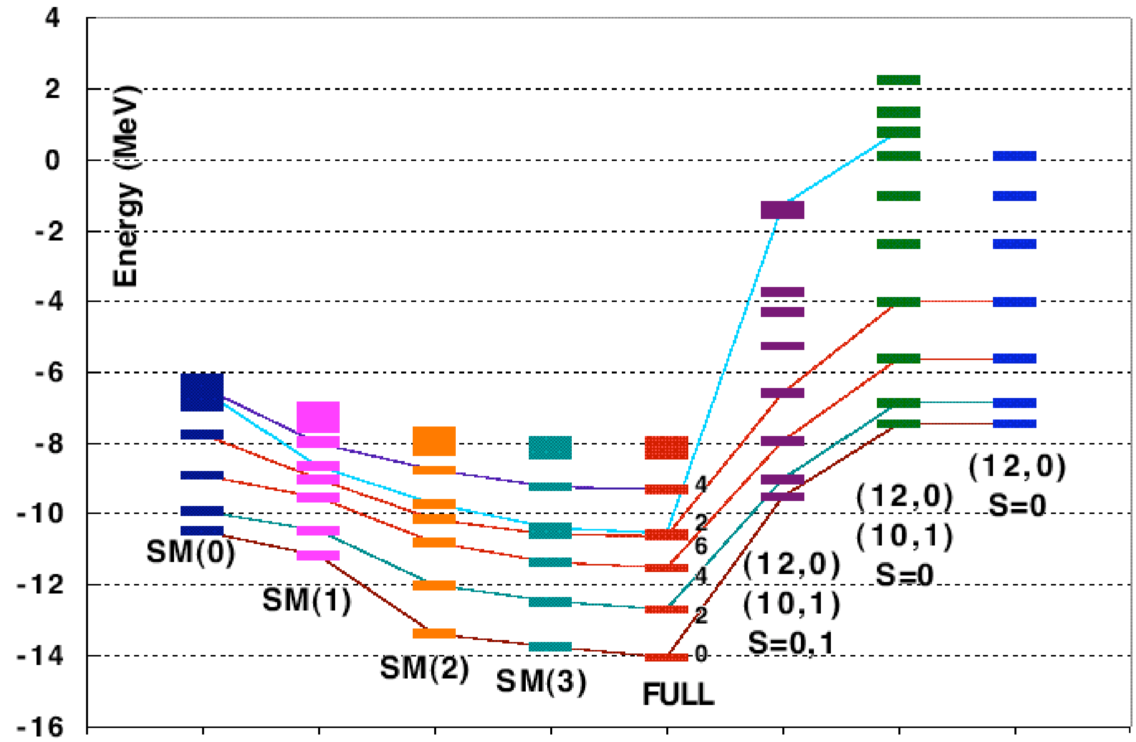
Level Structure ^{24}Mg



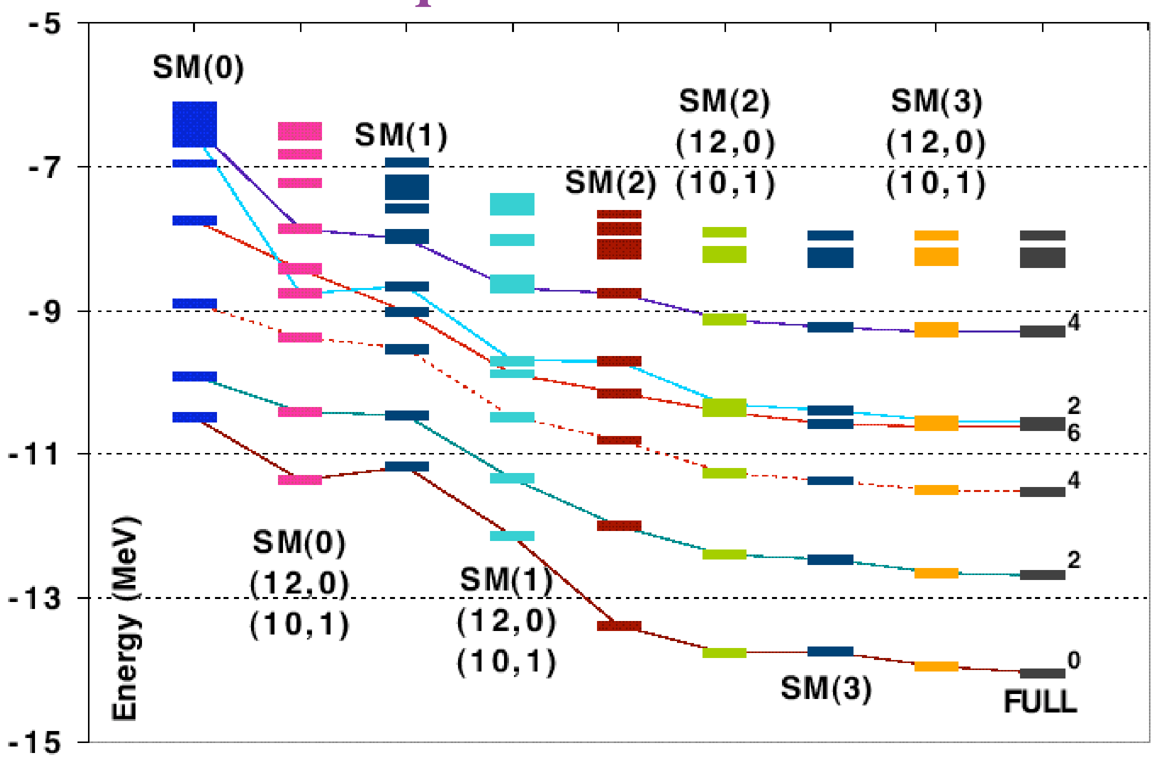
Oblique Basis Spectral Results



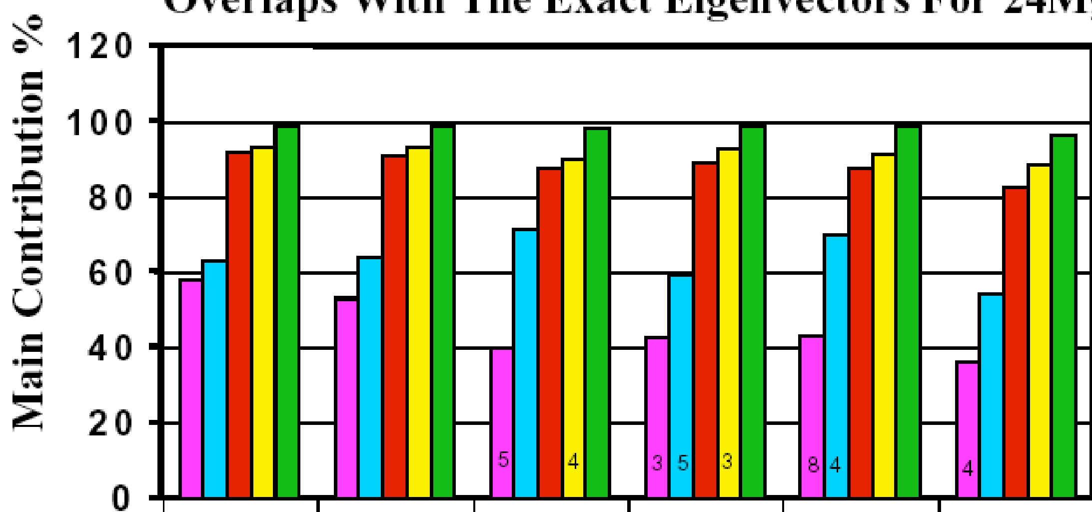
Level Structure for ^{44}Ti



Oblique Basis Results



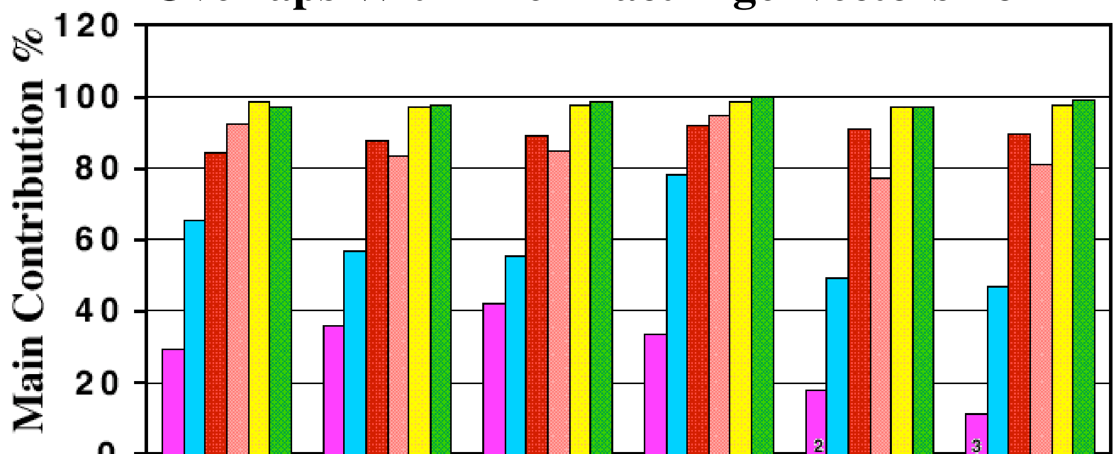
Overlaps With The Exact Eigenvectors For 24Mg



		1	2	3	4	5	6
	SM(2)	57.77	53.02	39.78	42.50	42.99	35.92
	SU(3)+	63.02	63.77	71.49	59.46	70.15	54.14
10%	SM(2)+SU(3)+	91.58	90.95	87.72	89.06	87.35	82.23
64%	SM(4)	93.25	92.81	89.98	92.47	91.10	88.33
	SM(4)+SU(3)+	98.57	98.73	97.92	98.41	98.55	96.59

Eigenvectors

Overlaps With The Exact Eigenvectors For 44Ti



		1	2	3	4	5	6
	SU(3)+	29.11	35.97	41.95	33.78	17.88	11.41
	SM(1)	65.47	56.71	55.55	78.08	49.22	46.96
	SM(1)+SU(3)+	84.11	87.78	89.14	91.82	90.80	89.34
	SM(2)	92.16	83.46	84.95	94.56	77.20	81.10
50%	SM(2)+SU(3)+	98.31	97.32	97.81	98.48	97.06	97.69
84%	SM(3)	97.36	97.39	98.67	99.79	97.22	98.93

Eigenvectors

Summary

- The **spin-orbit interaction** drives the **breaking of the SU(3)** symmetry in the lower pf-shell.
- The nuclear interaction has a **clear two-mode structure**:

  s.p.e. and **SU(3) invariant two-body** part…

- ***Use of two different sets of states*** *can enhance our understanding of complex systems*.
  - There is better dimensional convergence.
  - Correct level order of the low-lying states.
  - Significant overlap with the exact states.
    - 10% versus 64% for ^{24}Mg (good SU(3) limit)
    - 50% versus 84% for ^{44}Ti (poor SU(3) limit)