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Fairfax,  
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*DMA TECHNICAL MANUAL*

# **THE UNIVERSAL GRIDS: Universal Transverse Mercator (UTM) and Universal Polar Stereographic (UPS)**

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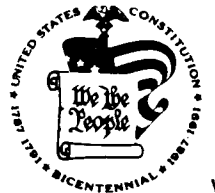
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THE UNIVERSAL GRIDS :  
UNIVERSAL TRANSVERSE MERCATOR (UTM)  
AND  
UNIVERSAL POLAR STEREOGRAPHIC (UPS)

### FOREWORD

1. This manual describes geographic to grid and grid to geographic conversions for the Universal Transverse Mercator (UTM) and the Universal Polar Stereographic (UPS) grids. It also discusses computations for convergence from geographic positions and from grid coordinates for the UTM grid, and convergence in the Polar Stereographic projection. Both mathematical and tabular methods are illustrated for the above items. A discussion of scale corrections is included. Transformations between the two grids, the Military Grid Reference System and the World Geographic Reference System are discussed. Finally, datum transformation methods, formulas and definitions are illustrated. Diagrams, textual information and software are provided. All software for grid calculations, including those from manual DMA TM 8358.1, are included as ANNEXES A and B.
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## CHAPTER 1

### GENERAL

#### 1-1 AUTHORITY.

This document is issued under the authority delegated by DoD Directive 5105.40, subject: Defense Mapping Agency (DMA).

#### 1-2 CANCELLATION.

DMA TM 8358.2 replaces Department of the Army Technical Manuals 5-241-8 and 5-241-9.

#### 1-3 PURPOSE.

1-3.1 This manual provides DoD Mapping, Charting and Geodetic (MC&G) production elements, product users and system developers with information necessary for the use of the UTM and UPS grids in surveying and mapping operations.

1-3.2 It also updates the methods and equations used to perform coordinate conversions, convergence and scale adjustments to efficiently program high-speed computers used in these operations.

#### 1-4 SCOPE.

1-4.1 This manual contains the recommended procedures and tables necessary to convert between grid and geographic coordinates within the Universal Transverse Mercator and Universal Polar Stereographic grid systems. Computations to determine convergence in the two systems and corrections for scale variations are also discussed and illustrated. One chapter is dedicated to the methods used to perform datum transformations and grid/geographic coordinate transformations.

1-4.2 Annex A refers to software and documentation available for the two universal grids. Annex B refers to software and documentation available for selected non-universal grids.

#### 1-5 UTILIZATION.

1-5.1 DMA TM 8358.2 is to be used by DoD MC&G production elements, product users and DoD system developers in the application of datums, ellipsoids, grids, grid/geographic coordinate conversions and grid reference systems.

1-5.2 Users are cautioned that the information contained herein applies to current and future MC&G production and does not necessarily apply to products that are currently available through the DoD supply system.

#### 1-6 DEFINITIONS.

1-6.1 Datum. As used in this manual, datum refers to the geodetic or horizontal datum. The classical datum is defined by five elements giving the position of the origin (two elements), the orientation of the network (one element) and the parameters of a reference ellipsoid (two elements). More recent definitions express the position and orientation as a function of the deviations in the meridian and in the prime vertical, the geoid-ellipsoid separations, and the parameters of a reference ellipsoid. The World Geodetic System datum gives positions on a specified ellipsoid with respect to the center of mass of the earth.

1-6.2 Easting. Eastward (that is left to right) reading of grid values on a map.

1-6.3 Ellipsoid. A three-dimensional figure generated by the revolution of an ellipse about one of its axes. The ellipsoid that approximates the geoid is an ellipsoid rotated about its minor axis, or an oblate spheroid.

1-6.4 Geoid. The equipotential surface in the Earth's gravity field approximates the undisturbed mean sea level extended continuously through the continents. The direction of gravity is perpendicular to the geoid at every point. The geoid is the surface of reference for astronomic observations and for geodetic leveling.

1-6.5 Graticule. A network of lines representing parallels of latitude and meridians of longitude forming a map projection.

1-6.6 Grid. Two sets of parallel lines intersecting at right angles and forming squares; a rectangular Cartesian coordinate system that is superimposed on maps, charts and other similar representations of the Earth's surface in an accurate and consistent manner to permit identification of ground locations with respect to other locations and the computation of direction and distance to other points.

1-6.7 Map Projection. An orderly system of lines on a plane representing a corresponding system of imaginary lines on an adopted terrestrial datum surface. A map projection may be derived by geometrical construction or by mathematical analysis.

1-6.8 Military Grid Reference System (MGRS). The alphanumeric position reporting system used by the U.S. military. A full description can be found in DMA TM 8358.1, chapter 3.

1-6.9 Non-Universal Grids. Grids other than UTM and UPS grids, such as British National Grid, Irish Transverse Mercator Grid, Madagascar Grid, and New Zealand Grid. Also referred to as non-standard grids.

1-6.10 Nothing. Northward (bottom to top) reading of grid values on a map.

1-6.11 Spheroid. A mathematical figure closely approaching the geoid in form and size, used as a surface of reference for geodetic surveys (See also ellipsoid.).

1-6.12 Universal Grids. The Universal Transverse Mercator (UTM) grid and the Universal Polar Stereographic (UPS) grid. Also referred to as standard grids.

1-6.13 World Geographic Reference System (GEOREF). A worldwide position reference system that may be applied to any map or chart graduated in latitude and longitude (with Greenwich as the prime meridian) regardless of projection. It provides a method of expressing positions in a form suitable for reporting and plotting. The primary use is for interservice and interallied reporting of aircraft and air target positions.

## 1-7 CROSS REFERENCE TO OTHER VOLUMES.

1-7.1 DMA TM 8358.1, Datums, Ellipsoids, Grids and Grid Reference Systems.

1-7.2 The DMA TM 8358 series replaces the Department of the Army TM 5-241 series manuals.

## 1-8 TECHNICAL PUBLICATION INFORMATION.

1-8.1 Any questions/comments concerning this manual should be directed to HQ DMA, ATTN: PR.

1-8.2 Additional copies of this manual are available through DMA Combat Support Center (DMACSC), ATTN: DDCP.

1-8.3 For information regarding other Department of the Army or Defense Mapping Agency manuals, tables, etc., direct requests to: Defense Technical Information Center, Cameron Station, Alexandria, VA, 22314-6145.

1-8.4 The Annexes to this manual will be distributed to DoD Components only. For these Annexes in software form, direct requests to DMA Hydrographic/Topographic Center (DMAHTC), ATTN: PRT, Washington D.C. 20315-0030.

## CHAPTER 2

UNIVERSAL TRANSVERSE MERCATOR GRID (UTM)2-1 DESCRIPTION OF UTM.

For a narrative description of the Universal Transverse Mercator Grid, see DMA TM 8358.1, chapter 2.

2-2 SUMMARY OF NOTATION.

For the following definitions of terms, note that all lengths are in meters, all geographic coordinates are in radians unless specified otherwise. The sign notation is negative for the Southern and Western hemispheres.

2-2.1 Ellipsoid Parameters.

By convention, ellipsoid parameters are defined as follows:

a = semi-major axis of the ellipsoid

b = semi-minor axis of the ellipsoid

f = flattening or ellipticity =  $\frac{a - b}{a}$

$e^2$  = (first eccentricity)<sup>2</sup> =  $\frac{a^2 - b^2}{a^2}$  = f(2 - f)

$e'^2$  = (second eccentricity)<sup>2</sup> =  $\frac{a^2 - b^2}{b^2}$  =  $\frac{e^2}{1 - e^2}$  =  $\frac{f(2 - f)}{(1 - f)^2}$

n =  $\frac{a - b}{a + b}$  =  $\frac{f}{2 - f}$

$\rho$  = radius of curvature in the meridian =  $\frac{a(1 - e^2)}{(1 - e^2 \sin^2 \phi)^{3/2}}$

$\nu$  = radius of curvature in the prime vertical; also defined as the normal to the ellipsoid terminating at the minor axis

=  $\frac{a}{(1 - e^2 \sin^2 \phi)^{1/2}}$  =  $\rho(1 + e'^2 \cos^2 \phi)$

S = meridional arc, the true meridional distance on the ellipsoid from the equator

=  $A'\phi - B'\sin 2\phi + C'\sin 4\phi - D'\sin 6\phi + E'\sin 8\phi$

where:

$$A' = a \left[ 1 - n + \frac{5}{4} (n^2 - n^3) + \frac{81}{64} (n^4 - n^5) + \dots \right]$$

$$B' = \frac{3}{2} a \left[ n - n^2 + \frac{7}{8} (n^3 - n^4) + \frac{55}{64} n^5 + \dots \right]$$

$$C' = \frac{15}{16} a \left[ n^2 - n^3 + \frac{3}{4} (n^4 - n^5) + \dots \right]$$

$$D' = \frac{35}{48} a \left[ n^3 - n^4 + \frac{11}{16} n^5 + \dots \right]$$

$$E' = \frac{315}{512} a \left[ n^4 - n^5 + \dots \right] \quad (E' \approx 0.03mm)$$

### 2-2.2 Universal Transverse Mercator Projection Parameters.

$\phi$  = latitude

$\lambda$  = longitude

$\phi'$  = latitude of the foot of the perpendicular from the point to the central meridian

$\lambda_0$  = longitude of the origin (the central meridian) of the projection

$\Delta\lambda$  =  $\lambda - \lambda_0$  = difference of longitude from the central meridian (for general formula use, value is sign dependent; for use in tables, value always considered positive)

$k_0$  = central scale factor, an arbitrary reduction applied to all geodetic lengths to reduce the maximum scale distortion of the projection.  
For the U.T.M.,  $k_0 = 0.9996$

$k$  = scale factor at the working point on the projection.

**FN** = False Northing (0 for the Northern Hemisphere;  
10,000,000 for the Southern Hemisphere)

**FE** = False Easting (500,000)

$\Delta E$  =  $E - FE$  (for general formula use, value is sign dependent; for use in tables, value always considered positive)

**E** = grid easting

**N** = grid northing

**C** = convergence of the meridians (i.e. the angle between true north and grid north)

### 2-2.3 Terms Used to Calculate General Equations

The following terms are used to calculate the general equations which follow in this chapter. With some modification, they are also used to produce the tables in chapter four. These terms are derived from the Functions found in the U.S. Department of Commerce, Special Publication No. 251. They differ slightly from the Functions used in DA TM 5-241-8 in that they are the fully expanded terms.

$$\begin{aligned}
 T1 &= Sk_0 \\
 T2 &= \frac{v \sin \phi \cos \phi k_0}{2} \\
 T3 &= \frac{v \sin \phi \cos^3 \phi k_0}{24} (5 - \tan^2 \phi + 9e'^2 \cos^2 \phi + 4e'^4 \cos^4 \phi) \\
 T4 &= \frac{v \sin \phi \cos^5 \phi k_0}{720} (61 - 58 \tan^2 \phi + \tan^4 \phi + 270e'^2 \cos^2 \phi - 330 \tan^2 \phi e'^2 \cos^2 \phi \\
 &\quad + 445e'^4 \cos^4 \phi + 324e'^6 \cos^6 \phi - 680 \tan^2 \phi e'^4 \cos^4 \phi \\
 &\quad + 88e'^8 \cos^8 \phi - 600 \tan^2 \phi e'^6 \cos^6 \phi - 192 \tan^2 \phi e'^8 \cos^8 \phi) \\
 T5 &= \frac{v \sin \phi \cos^7 \phi k_0}{40320} (1385 - 3111 \tan^2 \phi + 543 \tan^4 \phi - \tan^6 \phi) \\
 T6 &= v \cos \phi k_0 \\
 T7 &= \frac{v \cos^3 \phi k_0}{6} (1 - \tan^2 \phi + e'^2 \cos^2 \phi) \\
 T8 &= \frac{v \cos^5 \phi k_0}{120} (5 - 18 \tan^2 \phi + \tan^4 \phi + 14e'^2 \cos^2 \phi - 58 \tan^2 \phi e'^2 \cos^2 \phi + 13e'^4 \cos^4 \phi \\
 &\quad + 4e'^6 \cos^6 \phi - 64 \tan^2 \phi e'^4 \cos^4 \phi - 24 \tan^2 \phi e'^6 \cos^6 \phi) \\
 T9 &= \frac{v \cos^7 \phi k_0}{5040} (61 - 479 \tan^2 \phi + 179 \tan^4 \phi - \tan^6 \phi) \\
 T10 &= \frac{\tan \phi'}{2\rho v k_0^2} \\
 T11 &= \frac{\tan \phi'}{24\rho v^3 k_0^4} (5 + 3 \tan^2 \phi' + e'^2 \cos^2 \phi' - 4e'^4 \cos^4 \phi' - 9 \tan^2 \phi' e'^2 \cos^2 \phi')
 \end{aligned}$$

$$\begin{aligned}
T12 &= \frac{\tan\phi'}{720\rho\nu^5k_o^6} (61 + 90\tan^2\phi' + 46e'^2\cos^2\phi' + 45\tan^4\phi' - 252\tan^2\phi'e'^2\cos^2\phi' \\
&\quad - 3e'^4\cos^4\phi' + 100e'^6\cos^6\phi' - 66\tan^2\phi'e'^4\cos^4\phi' \\
&\quad - 90\tan^4\phi'e'^2\cos^2\phi' + 88e'^8\cos^8\phi' + 225\tan^4\phi'e'^4\cos^4\phi' \\
&\quad + 84\tan^2\phi'e'^6\cos^6\phi' - 192\tan^2\phi'e'^8\cos^8\phi') \\
T13 &= \frac{\tan\phi'}{40320\rho\nu^7k_o^8} (1385 + 3633\tan^2\phi' + 4095\tan^4\phi' + 1575\tan^6\phi') \\
T14 &= \frac{1}{\nu\cos\phi k_o} \\
T15 &= \frac{1}{6\nu^3\cos\phi k_o^3} (1 + 2\tan^2\phi' + e'^2\cos^2\phi') \\
T16 &= \frac{1}{120\nu^5\cos\phi k_o^5} (5 + 6e'^2\cos^2\phi' + 28\tan^2\phi' - 3e'^4\cos^4\phi' + 8\tan^2\phi'e'^2\cos^2\phi' \\
&\quad + 24\tan^4\phi' - 4e'^6\cos^6\phi' + 4\tan^2\phi'e'^4\cos^4\phi' + 24\tan^2\phi'e'^6\cos^6\phi') \\
T17 &= \frac{1}{5040\nu^7\cos\phi k_o^7} (61 + 662\tan^2\phi' + 1320\tan^4\phi' + 720\tan^6\phi') \\
T18 &= \sin\phi \\
T19 &= \frac{\sin\phi\cos^2\phi}{3} (1 + 3e'^2\cos^2\phi + 2e'^4\cos^4\phi) \\
T20 &= \frac{\sin\phi\cos^4\phi}{15} (2 - \tan^2\phi + 15e'^2\cos^2\phi + 35e'^4\cos^4\phi - 15\tan^2\phi e'^2\cos^2\phi + 33e'^6\cos^6\phi \\
&\quad - 50\tan^2\phi e'^4\cos^4\phi + 11e'^8\cos^8\phi - 60\tan^2\phi e'^6\cos^6\phi \\
&\quad - 24\tan^2\phi e'^8\cos^8\phi) \\
T21 &= \frac{\sin\phi\cos^6\phi}{315} (17 - 26\tan^2\phi + 2\tan^4\phi) \\
T22 &= \frac{\tan\phi'}{\nu k_o}
\end{aligned}$$



$$T23 = \frac{\tan\phi'}{3\nu^3k_0^3} (1 + \tan^2\phi' - e'^2\cos^2\phi' - 2e'^4\cos^4\phi')$$

$$T24 = \frac{\tan\phi'}{15\nu^5k_0^5} (2 + 5\tan^2\phi' + 2e'^2\cos^2\phi' + 3\tan^4\phi' + \tan^2\phi'e'^2\cos^2\phi' + 9e'^4\cos^4\phi' + 20e'^6\cos^6\phi' - 7\tan^2\phi'e'^4\cos^4\phi' - 27\tan^2\phi'e'^6\cos^6\phi' + 11e'^8\cos^8\phi' - 24\tan^2\phi'e'^8\cos^8\phi')$$

$$T25 = \frac{\tan\phi'}{315\nu^7k_0^7} (17 + 77\tan^2\phi' + 105\tan^4\phi' + 45\tan^6\phi')$$

$$T26 = \frac{\cos^2\phi}{2} (1 + e'^2\cos^2\phi)$$

$$T27 = \frac{\cos^4\phi}{24} (5 - 4\tan^2\phi + 14e'^2\cos^2\phi + 13e'^4\cos^4\phi - 28\tan^2\phi e'^2\cos^2\phi + 4e'^6\cos^6\phi - 48\tan^2\phi e'^4\cos^4\phi - 24\tan^2\phi e'^6\cos^6\phi)$$

$$T28 = \frac{\cos^6\phi}{720} (61 - 148\tan^2\phi + 16\tan^4\phi)$$

$$T29 = \frac{1}{2\nu^2k_0^2} (1 + e'^2\cos^2\phi)$$

$$T30 = \frac{1}{24\nu^4k_0^4} (1 + 6e'^2\cos^2\phi' + 9e'^4\cos^4\phi' + 4e'^6\cos^6\phi' - 24\tan^2\phi'e'^4\cos^4\phi' - 24\tan^2\phi'e'^6\cos^6\phi')$$

$$T31 = \frac{1}{720\nu^6k_0^6}$$

### 2-3 SPECIFICATIONS OF THE UTM.

Projection: Transverse Mercator (Gauss-Kruger type), in zones 6° wide.  
Unit of measurement: Meter

Zone numbering: Starting with 1 for the zone from 180°W to 174°W, and increasing eastward to 60 for the zone from 174°E to 180°E.

Latitude limits: North: 84°N  
South: 80°S

Zone limits and overlap: The zones are bounded by meridians whose longitudes are multiples of 6° west or east of Greenwich. On large-scale maps, an overlap of approximately 40km on either side of the junction is provided for engineer surveyors and for artillery survey and firing. This overlap is never used in giving a grid reference.

Polar region overlap: The U.T.M. overlaps 30' onto the Universal Polar Stereographic Grid, which extends from the poles to 83°30'N or 79°30'S respectively.

### 2-4 ACCURACY OF THE EQUATIONS.

The computations in this chapter, using geodetic latitude, are accurate to the nearest .001 arc second for geographic coordinates and to the nearest .01 meter for grid coordinates. More accurate formulas are available. These formulas contain more terms and utilize isometric latitude. If desired, FORTRAN subroutines and programs utilizing these formulas may be obtained from HQ DMA, ATTN: PR.

### 2-5 CONVERSION OF GEOGRAPHIC COORDINATES TO GRID COORDINATES.

The general formulas for the computation of N and E are:

$$N = FN + (T1 + \Delta\lambda^2T2 + \Delta\lambda^4T3 + \Delta\lambda^6T4 + \Delta\lambda^8T5)$$

$$E = FE + (\Delta\lambda T6 + \Delta\lambda^3T7 + \Delta\lambda^5T8 + \Delta\lambda^7T9)$$

### 2-6 CONVERSION OF GRID COORDINATES TO GEOGRAPHIC COORDINATES.

The general formulas for the computation of  $\phi$  and  $\lambda$  are:

$$\phi = \phi' - \Delta E^2T10 + \Delta E^4T11 - \Delta E^6T12 + \Delta E^8T13$$

$$\lambda = \lambda_0 + \Delta E T14 - \Delta E^3T15 + \Delta E^5T16 - \Delta E^7T17$$

Programmer's Note: The footpoint latitude,  $\phi'$ , is normally derived by iteration (successive approximation), based on term T1.

### 2-7 MERIDIAN CONVERGENCE FROM GEOGRAPHIC COORDINATES.

The general formula for meridian convergence is:

$$C = \Delta\lambda T18 + \Delta\lambda^3T19 + \Delta\lambda^5T20 + \Delta\lambda^7T21$$

2-8 MERIDIAN CONVERGENCE FROM GRID COORDINATES.

The general formula for meridian convergence is:

$$C = \Delta E T^2 - \Delta E^3 T^3 + \Delta E^5 T^4 - \Delta E^7 T^5$$

2-9 SCALE FACTOR FROM GEOGRAPHIC COORDINATES.

The general formula for the scale factor is:

$$k = k_0(1 + \Delta \lambda^2 T^6 + \Delta \lambda^4 T^7 + \Delta \lambda^6 T^8)$$

2-10 SCALE FACTOR FROM GRID COORDINATES.

The general formula for the scale factor is:

$$k = k_0(1 + \Delta E^2 T^9 + \Delta E^4 T^30 + \Delta E^6 T^31)$$

NOTE: The computation of both convergence and scale factor has generally been replaced by geodetic inverse and forward computer programs to determine geodetic azimuths and distances. Formulas such as those by Vincenty and Sodano are available from DMAHTC/PR.

2-11 SAMPLE OUTPUT FOR THE PRECEEDING COMPUTATIONS.

## COORDINATE CONVERSIONS ON THE UNIVERSAL TRANSVERSE MERCATOR (UTM) GRID

## ELLIPSOID DATA

A	1/F	NAME	B
6378388.000	297.000000000	INTERNATIONAL	6356911.94613
E**2	EB**2		
.006722670022	.006768170197		

## LATITUDE AND LONGITUDE TO ZONE, NORTHING AND EASTING

ID	LATITUDE	LONGITUDE	ZONE	NORTHING	EASTING	CONVERGENCE	SCALE FACTOR
1	73 0 .000N	45 0 .000E	38	8100702.90	500000.00	0 0 .00 E	.99960000
2	30 0 .000N	102 0 .000E	47	3322624.35	789422.07	1 30 3.76 E	1.00063354
			48	3322624.35	210577.93	1 30 3.76 W	1.00063354
3	72 4 32.110N	113 54 43.321W	12	8000000.01	400000.00	2 46 15.31 W	.99972228
			11	8000301.04	606036.97	2 56 18.08 E	.99973749

## ZONE, NORTHING AND EASTING TO LATITUDE AND LONGITUDE

ID	LATITUDE	LONGITUDE	ZONE	NORTHING	EASTING	CONVERGENCE	SCALE FACTOR
4	30 0 6.489N	101 59 59.805E	48	3322824.35	210577.93	1 30 4.15 W	1.00063354
			47	3322824.08	789411.59	1 30 3.96 E	1.00063346
5	9 2 10.706N	0 16 17.099E	31	1000000.00	200000.00	0 25 43.95 W	1.00071386
			30	1000491.75	859739.88	0 30 51.72 E	1.00120178
6	81 3 30.487N	75 0 .000E	43	9000000.00	500000.00	0 00 .00 E	.99960000
7	54 6 28.992S	0 3 33.695E	30	4000000.00	700000.00	2 28 45.39 W	1.00009080
			31	4000329.42	307758.89	2 22 58.83 E	1.00005345

## CHAPTER 3

UNIVERSAL POLAR STEREOGRAPHIC GRID (UPS)3-1 DESCRIPTION OF UPS.

For a narrative description of the Universal Polar Stereographic Grid, see DMA TM 8358.1, chapter 2.

3-2 SUMMARY OF NOTATION.

For the following definitions of terms, note that all lengths are in meters and all geographic coordinates are in radians unless specified otherwise. All sign notation is indicated where appropriate.

3-2.1 Ellipsoid Parameters.

3-2.1.1 The Universal Polar Stereographic Grid is defined on the WGS-84 Ellipsoid. The formulas which follow have been generalized, however, to allow construction of the polar stereographic grid on any ellipsoid. The WGS-84 Ellipsoid has the following parameters:

$$a = 6,378,137 \text{ meters}$$

$$1/f = 298.257223563$$

3-2.1.2 The adopted notation for the remainder of the ellipsoid parameters used in subsequent calculations in this chapter is as follows:

$$b = a(1 - f)$$

$$e^2 = (\text{first eccentricity})^2 = \frac{a^2 - b^2}{a^2} = f(2 - f)$$

$$e'^2 = (\text{second eccentricity})^2 = \frac{a^2 - b^2}{b^2} = \frac{e^2}{1 - e^2} = \frac{f(2 - f)}{(1 - f)^2}$$

$$v = \text{radius of curvature in the prime vertical; also defined as the normal to the ellipsoid terminating at the minor axis}$$

3-2.2 Universal Polar Stereographic Projection Parameters.

$$\phi = \text{geodetic latitude}$$

$$\chi = \text{isometric latitude} = \text{an auxiliary latitude used in the conformal mapping of the spheroid on a sphere. By transforming geographic latitudes on the spheroid into isometric latitudes on a sphere, a conformal map projection (the Mercator) may be calculated, using spherical formulas, for the plotting of geographic data.}$$

$$\lambda = \text{longitude}$$

$$z = \text{isometric colatitude} = 90^\circ \text{ minus isometric latitude.}$$

$k_0$  = scale factor at the pole, an arbitrary reduction applied to all geodetic lengths to reduce the maximum scale distortion of the projection.

$k$  = scale factor at the working point on the projection.

$R$  = radius of the parallel of latitude from the pole.

$N$  = Northing

$E$  = Easting

$FN$  = False Northing

$FE$  = False Easting

$\Delta N$  =  $N - FN$

$\Delta E$  =  $E - FE$

### 3-2.3 Formulas.

$$C_0 = \text{constant} = \frac{2a}{(1 - e^2)^{1/2}} \cdot \left[ \frac{1 - e}{1 + e} \right]^{e/2}$$

$$\tan \frac{z}{2} = \left[ \frac{1 + e \sin \phi}{1 - e \sin \phi} \right]^{e/2} \cdot \tan \left[ \frac{\pi - \phi}{4} - \frac{\phi}{2} \right]$$

$$R = \text{radius} = k_0 C_0 \tan \frac{z}{2}$$

Constants for computing isometric to geodetic latitude:

$$\bar{A} = \frac{e^2}{2} + \frac{5e^4}{24} + \frac{e^6}{12} + \frac{13e^8}{360}$$

$$\bar{B} = \frac{7e^4}{48} + \frac{29e^6}{240} + \frac{811e^8}{11520}$$

$$\bar{C} = \frac{7e^6}{120} + \frac{81e^8}{1120}$$

$$\bar{D} = \frac{4279e^8}{161280}$$

The preceding terms are used to calculate the general equations which follow in this chapter and the tables in chapter four.

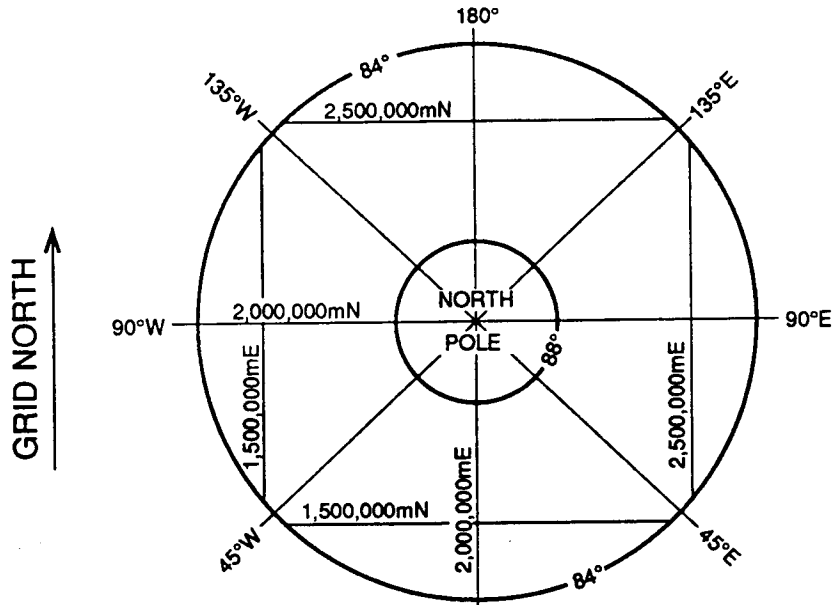


FIGURE 1(a). North Zone

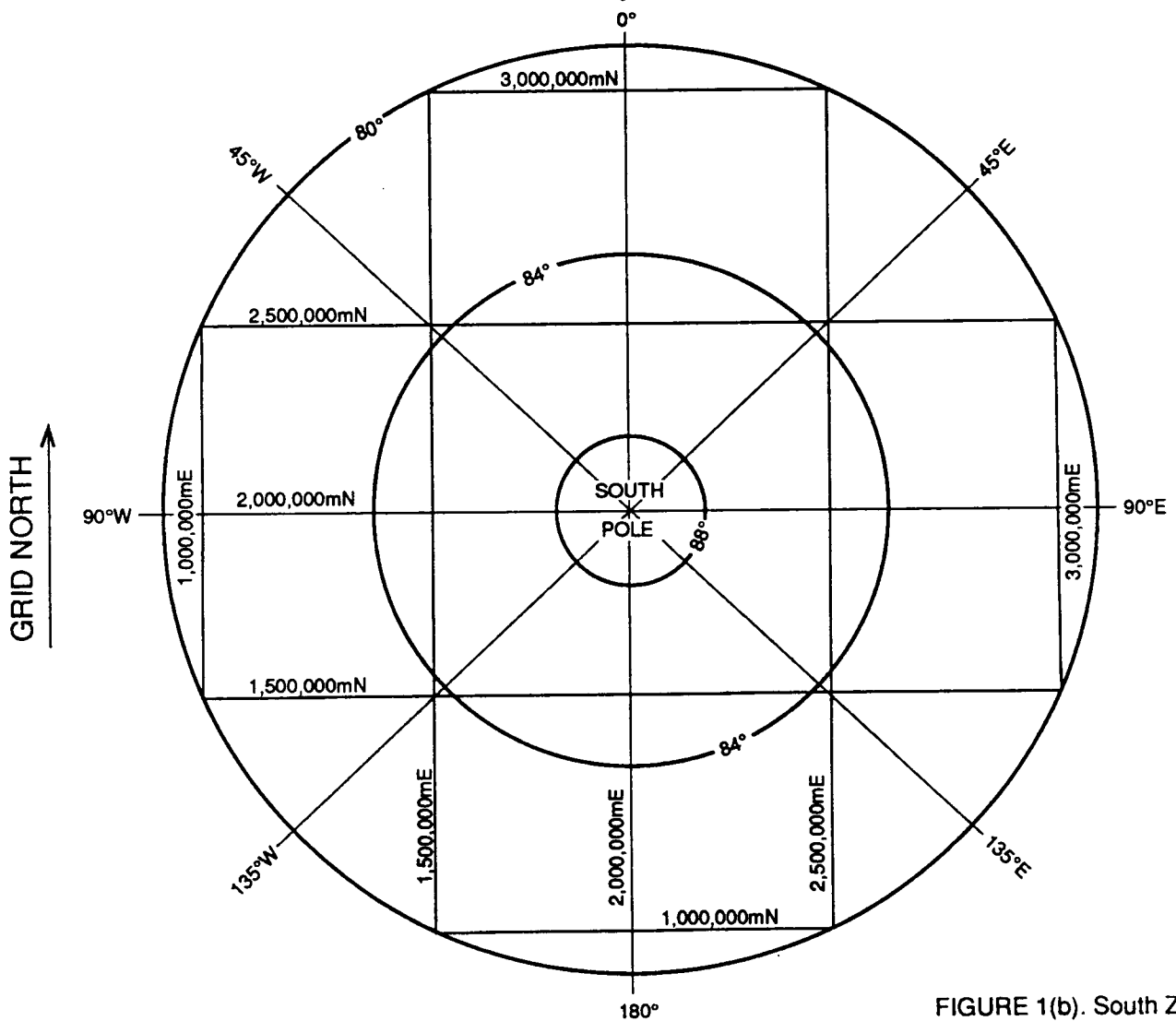


FIGURE 1(b). South Zone

3-2.4 Specifications of the UPS.

Unit of Measurement:	Meter
Ellipsoid:	WGS 84
False Northing:	2,000,000 meters
False Easting:	2,000,000 meters
Scale Factor at the Origin:	0.994

Orientation of the grid systems: In both zones, the 2,000,000 meter easting line coincides with the 0° and the 180° meridians and grid north is equal to true north on the 0° meridian (See figure 1).

Limits of system:

North zone:	The north polar area 84° – 90°
South zone:	The south polar area 80° – 90°

Overlap with the UTM: The U.P.S. grid will be extended to 83°30'N and 79°30'S to provide a 30' overlap with the Universal Transverse Mercator Grid System.

3-3 CONVERSION OF GEOGRAPHIC COORDINATES TO GRID COORDINATES.

The general formulas for the computation of N and E are:

$$N = FN - R \cos \lambda \quad \text{for the North Zone}$$

$$N = FN + R \cos \lambda \quad \text{for the South Zone}$$

$$E = FE + R \sin \lambda \quad \text{for Both Zones}$$

3-4 CONVERSION OF GRID COORDINATES TO GEOGRAPHIC COORDINATES.

3-4.1 From the notation in Section 3-2.3, it is given:

$$\Delta N = N - FN$$

$$\Delta E = E - FE$$

To compute the longitude, let:

$$L = \arctan \frac{\Delta E}{-\Delta N} \quad \text{for the North Zone where } N \neq 0$$

$$L = \arctan \frac{\Delta E}{\Delta N} \quad \text{for the South Zone where } N \neq 0$$

A single argument arctan routine returns an angle 'L' in the range of  $\pi/2$  (90°) to  $-\pi/2$  (-90°). The longitude of the point is equal to 'L' if the denominator of the above equations is positive. If the denominator is negative, then the longitude is obtained as follows:

$$\lambda = \pi + L \quad \text{when } \Delta E \text{ is positive and angle 'L' is negative}$$

$$\lambda = -\pi + L \quad \text{when } \Delta E \text{ is negative and angle 'L' is positive}$$

A two argument arctan routine returns the value of  $\lambda$  directly (range of  $\pi$  to  $-\pi$ ) except in the case where both arguments are equal to zero.

If  $\Delta N = 0$ , then  $\lambda = 90^\circ$  E or W, depending on the sign of  $\Delta E$  (see figure 1).  
If  $\Delta N = \Delta E = 0$ , then the point is at the pole and  $\lambda$  is undefined.

When computing longitude by hand, use the above arctan functions when  $\Delta E$  is numerically less than  $\Delta N$ . When  $\Delta E$  is numerically greater than  $\Delta N$ , use the following arccot functions for greater accuracy:

$$\lambda = \operatorname{arccot} \frac{-\Delta N}{\Delta E} \quad \text{for the North Zone}$$

$$\lambda = \operatorname{arccot} \frac{\Delta N}{\Delta E} \quad \text{for the South Zone}$$

NOTE: The programmed equations utilize only the arctan function since this routine is standard in most systems. Normally, using only the arctan function will not result in any loss of accuracy on the computer as the machine defaults to the arccot routine where appropriate (i.e. above  $45^\circ$ ).

3-4.2 The computation of latitude begins by defining R:

$$R = \left| \frac{\Delta E}{\sin \lambda} \right| \quad \text{where } \Delta E \neq 0$$

$$\text{if } \Delta E = \Delta N = 0, \quad \text{then } \phi = \frac{\pi}{2} = 90 = \text{pole}$$

$$\text{if } \Delta N = 0, \quad \text{then } R = |\Delta E|$$

$$\text{if } \Delta E = 0, \quad \text{then } R = |\Delta N|$$

When computing R by hand, use the previous equation when  $\Delta N$  is numerically less than  $\Delta E$ . When  $\Delta N$  is numerically greater than  $\Delta E$ , use the following equation for greater accuracy:

$$R = \left| \frac{\Delta N}{\cos \lambda} \right| \quad \text{where } \Delta N \neq 0$$

From R and  $\lambda$ , determine the isometric colatitude (z) from the following:

$$\tan \frac{z}{2} = \frac{R}{(k_0 C_0)}$$



Using the isometric latitude, determine  $\phi$  from the following:

$$\phi = \chi + \bar{A}\sin 2\chi + \bar{B}\sin 4\chi + \bar{C}\sin 6\chi + \bar{D}\sin 8\chi$$

where:

$$\chi = \frac{\pi}{2} - z$$

NOTE: Latitude, in the Polar Stereographic System, is always treated as positive, regardless of hemisphere.

### 3-5 CONVERGENCE

Convergence in the Polar Stereographic Projection is equal to the longitude,  $\lambda$ , in numeric value. In the north polar area, it has the same sign as  $\lambda$  and in the south polar area it has the opposite sign. Unlike most grid systems, where convergence is a small angle, the polar system may have convergence angles up to  $180^\circ$  east or west.

### 3-6 SCALE FACTOR FOR GEOGRAPHIC COORDINATES

The scale factor is the factor by which true distances over short lines must be multiplied to obtain grid distances. The scale factor is constant for any given latitude.

The general formula for the scale factor is:

$$k = \frac{R}{\nu \cos \phi}$$

NOTE: The computation of both convergence and scale factor has generally been replaced by geodetic inverse and forward computer programs to determine geodetic azimuths and distances. Formulas such as those by Vincenty and Sodano are available from HQ DMA, ATTN: PR.

**3-7 SAMPLE OUTPUT FOR THE PRECEDING COMPUTATIONS.****COORDINATE CONVERSIONS ON THE UNIVERSAL POLAR STEREOGRAPHIC (UPS) GRID****ELLIPSOID DATA**

<b>A</b>	<b>1/F</b>	<b>NAME</b>	<b>UNITS</b>
6378137.000	298.257223563	WGS-84	METERS
<b>B</b>	<b>E**2</b>	<b>EB**2</b>	
6356911.94613	.006694379990	.006739496742	

**LATITUDE AND LONGITUDE -TO- NORTHING AND EASTING**

<b>ID</b>	<b>LATITUDE</b>	<b>LONGITUDE</b>	<b>NORTHING</b>	<b>EASTING</b>	<b>CONVERGENCE</b>	<b>SCALE FACTOR</b>
1	84 17 14.042N	132 14 52.761W	2426773.60	1530125.78	132 14 52.76 W	.99647445
2	73 0 .000N	44 0 .000E	632668.43	3320416.75	44 0 .00 E	1.01619505
3	87 17 14.400S	132 14 52.303E	1797474.90	2222979.47	132 14 52.30 W	.99455723

**NORTHING AND EASTING -TO- LATITUDE AND LONGITUDE**

<b>ID</b>	<b>LATITUDE</b>	<b>LONGITUDE</b>	<b>NORTHING</b>	<b>EASTING</b>	<b>CONVERGENCE</b>	<b>SCALE FACTOR</b>
4	84 17 14.042N	132 14 52.762W	2426773.60	1530125.78	132 14 52.76 W	.99647445
5	73 0 .000N	44 0 .000E	632668.43	3320416.75	44 0 .00 E	1.01619505
6	83 38 14.343S	135 0 .000E	1500000.00	2500000.00	135 0 .00 E	.99707070

## CHAPTER 4

SAMPLE GRID TABLES FOR THE  
UNIVERSAL TRANSVERSE MERCATOR (UTM)  
AND  
UNIVERSAL POLAR STEREOGRAPHIC (UPS)  
GRIDS

4-1 GENERAL

Tables were traditionally used to solve complex, repetitive calculations. With the advent and ready access to computers and hand-held programmable calculators, tables are now rarely used. Their discussion has been included in this manual, however, to maintain continuity with manuals DA TM 5-241-8 and DA TM 5-241-9, which this manual replaces. The equations have been adapted from those in the previous manuals to produce the tables in this manual. A representative example of the function tables for the International Ellipsoid is included at the end of this chapter.

4-2 DESIGN AND PREPARATION OF UNIVERSAL TRANSVERSE MERCATOR GRID TABLES

4-2.1 The tables in this chapter were computed using the International Ellipsoid where:

$$\begin{aligned} a &= 6,378,388 \text{ meters} \\ 1/f &= 297 \\ b &= 6,356,911.946 \text{ meters} \\ e^2 &= 0.006\ 722\ 670\ 022 \end{aligned}$$

Tables for other ellipsoids should be constructed using the constants provided in DMA TM 8358.1.

4-2.2 Other Notation:

$$\begin{aligned} k_0 &= .9996 \\ p &= .0001\Delta\lambda'' \\ q &= .000001\Delta E \quad \text{where } \Delta E = |E - FE| \\ &\quad \text{(FE = false easting)} \end{aligned}$$

$$S = A'\phi - B'\sin 2\phi + C'\sin 4\phi - D'\sin 6\phi + E'\sin 8\phi$$

where (for the International Ellipsoid):

$$\begin{aligned} A' &= 6,367,654.500\ 058 \text{ meters} \\ B' &= 16,107.034\ 678 \text{ meters} \\ C' &= 16.976\ 211 \text{ meters} \\ D' &= 0.022\ 266 \text{ meters} \\ E' &= 0.000\ 032 \text{ meters} \end{aligned}$$

NOTE: The values A' through E' were derived from the formulas listed in Section 2-2.1.

4-2.3 The functions represented by Roman Numerals (I) to (XIX) in DA TM 5-241-8, and functions (XX) and (XXI) are calculated from the "T" terms of Section 2-2.3 herein as follows:

(I)	=	$T1 = Sk_0$
(II)	=	$(T2\sin^2 1^\circ) \times 10^8$
(III)	=	$(T3\sin^4 1^\circ) \times 10^{16}$
(IV)	=	$(T6\sin 1^\circ) \times 10^4$
(V)	=	$(T7\sin^3 1^\circ) \times 10^{12}$
(VII)	=	$(T10/\sin 1^\circ) \times 10^{12}$
(VIII)	=	$(T11/\sin 1^\circ) \times 10^{24}$
(IX)	=	$(T14/\sin 1^\circ) \times 10^6$
(X)	=	$(T15/\sin 1^\circ) \times 10^{18}$
(XII)	=	$T18 \times 10^4$
(XIII)	=	$(T19\sin^2 1^\circ) \times 10^{12}$
(XV)	=	$(T22/\sin 1^\circ) \times 10^6$
(XVI)	=	$(T23/\sin 1^\circ) \times 10^{18}$
(XVIII)	=	$T29 \times 10^{12}$
(XIX)	=	$T30 \times 10^{24}$
(XX)	=	$(T26\sin^2 1^\circ) \times 10^8$
(XXI)	=	$(T27\sin^4 1^\circ) \times 10^{16}$

4-2.4 The terms represented graphically in DA TM 5-241-8 are represented as values in the new tables. They are calculated from the following:

$A_6$	=	$(T4\sin^6 1^\circ) \times 10^{24}$
$B_5$	=	$(T8\sin^5 1^\circ) \times 10^{20}$
$C_5$	=	$(T20\sin^4 1^\circ) \times 10^{20}$
$D_6$	=	$(T12/\sin 1^\circ) \times 10^{36}$
$E_5$	=	$(T16/\sin 1^\circ) \times 10^{30}$
$F_5$	=	$(T24/\sin 1^\circ) \times 10^{30}$

NOTE: Unlike DA TM 5-241-8, the terms p and q are not incorporated into the above terms, but rather, are included in the equations used to perform grid/geographic conversions and convergence computations when using the tables.

4-2.5 Two correction terms,  $\Delta^2(\text{IV})$  and  $\Delta^2(\text{IX})$ , which were shown graphically in the old tables were eliminated in the construction of the new tables. They are difficult to interpolate with precision and therefore, for accuracy requirements greater than that provided by the table, it is recommended that calculations be done using the computer programs listed in the appropriate Annexes.

NOTE: The tables presented in this chapter are constructed at 1' intervals, however, they can be requested from HQ DMA, ATTN: PR at any desired interval.

#### 4-3 COMPUTATION OF UTM GRID COORDINATES FROM GEOGRAPHIC COORDINATES.

The formulas for the computation of E' and N are:

$$\begin{aligned} N &= (I) + (II)p^2 + (III)p^4 + A_6p^6 \\ E' &= (IV)p + (V)p^3 + B_5p^5 \end{aligned}$$

NOTE: E' is added to or subtracted from 500,000, depending on whether the point is east or west of the central meridian. This E' should not be confused with the E' in Section 4-2.2.

South of the equator:

$$N = 10,000,000 - [(I) + (II)p^2 + (III)p^4 + A_6p^6]$$

#### 4-4 COMPUTATION OF GEOGRAPHIC COORDINATES FROM UTM GRID COORDINATES.

4-4.1 The formulas for computing geographic coordinates from UTM grid coordinates are:

$$\begin{aligned} \phi &= \phi' - (VII)q^2 + (VIII)q^4 - D_6q^6 \\ \Delta\lambda &= (IX)q - (X)q^3 + E_5q^5 \\ \lambda &= \lambda_0 \pm \Delta\lambda \end{aligned} \quad \text{where } \lambda_0 \text{ is the longitude of origin of the projection (the central meridian).}$$

4-4.2 The footpoint latitude ( $\phi'$ ) is obtained by entering the table through Function I with N as the argument in the northern hemisphere or 10,000,000 - N as the argument in the southern hemisphere, and making an inverse interpolation.

#### 4-5 COMPUTATION OF THE CONVERGENCE FOR THE UTM.

The formula for the computation of convergence from geographic coordinates is:

$$C = (XII)p + (XIII)p^3 + C_5p^5$$

and from grid coordinates is:

$$C = (XV)q - (XVI)q^3 + F_5q^5$$

#### 4-6 SCALE CORRECTION FOR THE UTM.

The formula for scale correction for geographic coordinates is:

$$k = k_0[1 + (XX)p^2 + (XXI)p^4]$$

and for grid coordinates is:

$$k = k_0[1 + (\text{XVIII})q^2 + (\text{XIX})q^4]$$

NOTE: Functions (XVIII) and (XIX) can be determined from the tables by either N or footpoint latitude.

#### 4-7 SAMPLE COMPUTATION FOR THE UTM.

A sample computation is provided for computing UTM grid coordinates from geographic coordinates as follows:

Given:

Latitude	=	34° 15' 34".742 N
Longitude	=	96° 02' 43".158 E
Central Meridian	=	99° 00' 00" E

From the preceding, it follows:

$$\begin{aligned}\Delta\lambda &= 2^\circ 57'16".842 = 10,636.842'' \\ p &= 1.0636842\end{aligned}$$

To determine the Northing, derive the following functions using the tables:

Function I:

Even minutes of $\phi$	=	3789935.119
Interpolation for seconds of $\phi$	=	<u>1070.106</u>
I	=	3791005.225

Function II:

Even minutes of $\phi$	=	3489.536
Interpolation for seconds of $\phi$	=	<u>0.465</u>
II	=	3490.001

Function III:

$$\text{III} = 2.138$$

Function A<sub>6</sub>:

$$A_6 = 0.0009$$

The above functions are then multiplied by the appropriate powers of p as indicated in the formula and then summed to give:

$$\begin{aligned}I &= 3791005.225 \\ IIp^2 &= 3948.671 \\ IIIp^4 &= 2.737 \\ A_6p^6 &= \frac{0.001}{3794956.630} = \text{NORTHING}\end{aligned}$$

To determine the Easting, derive the following functions using the tables:

Function IV:

$$\begin{array}{rcl} \text{Even minutes of } \phi & = & 255779.038 \\ \text{Interpolation for seconds of } \phi & = & \underline{-29.205} \\ \text{IV} & = & 255749.833 \end{array}$$

Function V:

$$\begin{array}{rcl} \text{Even minutes of } \phi & = & 37.040 \\ \text{Interpolation for seconds of } \phi & = & \underline{-0.036} \\ \text{V} & = & 37.004 \end{array}$$

Function B<sub>5</sub>:

$$B_5 = -0.0175$$

The preceding functions are then multiplied by the appropriate powers of p as indicated in the formula and then summed to give:

$$\begin{array}{rcl} IVp & = & 272037.057 \\ Vp^3 & = & 44.534 \\ B_5p^5 & = & -0.024 \\ \text{plus False Easting}^* & \underline{500000.000} & \\ & 772081.570 & = \text{EASTING} \end{array}$$

\*NOTE: False Easting is either added or subtracted dependent on the location of the point relative to the central meridian.

All other computations are performed in a similar fashion and therefore need not be discussed in this manual.

#### 4-8 DESIGN AND PREPARATION OF THE UNIVERSAL POLAR STEREOGRAPHIC GRID.

4-8.1 The table of radii is produced from the basic formula for the Polar Stereographic Projection as follows:

$$\begin{array}{rcl} R & = & k_0 \left[ \frac{2a}{(1 - e^2)^{1/2}} \right] \left[ \frac{(1 - e)}{(1 + e)} \right]^{e/2} \tan(z/2) \\ z/2 & = & \text{the isometric semi-colatitude} \end{array}$$

4-8.2 The tables of scale factors were computed from the following formula:

$$\begin{array}{rcl} k & = & R/(v \cos \phi) \\ v & = & \text{the radius of curvature in the prime vertical.} \end{array}$$

#### 4-9 FORMULAS NEEDED TO USE THE UPS TABLES.

The formulas to utilize the tables for the Universal Polar Stereographic Grid System are identical to those presented in chapter 3 of this manual.

4-10 SAMPLE CALCULATION

A sample computation is provided for computing UPS grid coordinates from geographic coordinates as follows:

$$\begin{aligned} \text{Given:} \quad \phi &= 84^{\circ} 17' 14''.042 \text{ N} \\ \lambda &= 132^{\circ} 14' 52''.761 \text{ W} \end{aligned}$$

From the table:

$$\begin{array}{rcl} \text{R (even minutes of } \phi \text{)} & & 635191.905 \\ \text{Interpolation for seconds of } \phi & & \underline{-434.088} \\ \text{R} & = & 634757.817 \end{array}$$

Therefore:

$$\begin{aligned} \cos\lambda &= \cos(-132^{\circ} 14' 52''.761) = -\cos(47^{\circ} 45' 07''.239) \\ &= -0.67234083 \\ \sin\lambda &= \sin(-132^{\circ} 14' 52''.761) = -\sin(47^{\circ} 45' 07''.239) \\ &= -0.74024172 \end{aligned}$$

And finally:

$$\begin{array}{rcl} \text{Northing} & = & 2,000,000 - R\cos\lambda & = & 2426773.60 \\ \text{Easting} & = & 2,000,000 + R\sin\lambda & = & 1530125.78 \end{array}$$



4-11 SAMPLE TABLES FOR THE UTM AND UPS GRIDS.

For the UTM grid:

ELLIPSOID DATA				
A	1/F	NAME	UNITS	
6378388.000	297.000000000	INTERNATIONAL	METERS	
B	E**2	EB**2		
6356911.94613	.006722670022	.006768170197		

LATITUDE	N	DIFF. 1"	R	DIFF. 1"
34 0 0	6385102.774	.09669	6355538.173	.28873
34 1 0	6385108.576	.09671	6355555.496	.28880
34 2 0	6385114.378	.09674	6355572.824	.28887
34 3 0	6385120.183	.09676	6355590.157	.28894
34 4 0	6385125.988	.09678	6355607.493	.28901
34 5 0	6385131.795	.09681	6355624.833	.28907
34 6 0	6385137.604	.09683	6355642.178	.28914
34 7 0	6385143.413	.09685	6355659.526	.28921
34 8 0	6385149.224	.09687	6355676.879	.28928
34 9 0	6385155.037	.09690	6355694.236	.28935
34 10 0	6385160.851	.09692	6355711.597	.28942
34 11 0	6385166.666	.09694	6355728.962	.28948
34 12 0	6385172.482	.09696	6355746.331	.28955
34 13 0	6385178.300	.09699	6355763.704	.28962
34 14 0	6385184.119	.09701	6355781.081	.28969
34 15 0	6385189.940	.09703	6355798.462	.28976
34 16 0	6385195.762	.09705	6355815.848	.28982
34 17 0	6385201.585	.09708	6355833.237	.28989
34 18 0	6385207.410	.09710	6355850.631	.28996
34 19 0	6385213.236	.09712	6355868.028	.29003
34 20 0	6385219.063	.09714	6355885.430	.29009
34 21 0	6385224.891	.09717	6355902.835	.29016
34 22 0	6385230.721	.09719	6355920.245	.29023
34 23 0	6385236.553	.09721	6355937.658	.29029
34 24 0	6385242.385	.09723	6355955.076	.29036
34 25 0	6385248.219	.09726	6355972.498	.29043
34 26 0	6385254.055	.09728	6355989.923	.29049
34 27 0	6385259.891	.09730	6356007.353	.29056
34 28 0	6385265.729	.09732	6356024.787	.29063
34 29 0	6385271.569	.09734	6356042.224	.29069
34 30 0	6385277.409	.09737	6356059.666	.29076

LATITUDE	MERIDIONAL ARC	DIFF. 1"	METERS PER SECOND	
			R·SIN1"	N·SIN1"·COS(LAT)
			LATITUDE	LONGITUDE
34 0 0	3763719.865	30.81256	30.81252	25.66356
34 1 0	3765568.619	30.81264	30.81260	25.65855
34 2 0	3767417.378	30.81273	30.81269	25.65354
34 3 0	3769256.141	30.81281	30.81277	25.64852
34 4 0	3771114.910	30.81290	30.81285	25.64350
34 5 0	3772963.684	30.81298	30.81294	25.63848
34 6 0	3774812.463	30.81306	30.81302	25.63345
34 7 0	3776661.247	30.81315	30.81311	25.62843
34 8 0	3778510.036	30.81323	30.81319	25.62340
34 9 0	3780358.830	30.81332	30.81328	25.61837
34 10 0	3782207.629	30.81340	30.81336	25.61334
34 11 0	3784056.433	30.81349	30.81344	25.60830
34 12 0	3785905.242	30.81357	30.81353	25.60326
34 13 0	3787754.056	30.81365	30.81361	25.59822
34 14 0	3789602.875	30.81374	30.81370	25.59318
34 15 0	3791451.700	30.81382	30.81378	25.58814
34 16 0	3793300.529	30.81391	30.81386	25.58309
34 17 0	3795149.363	30.81399	30.81395	25.57805
34 18 0	3796998.203	30.81408	30.81403	25.57300
34 19 0	3798847.048	30.81416	30.81412	25.56794
34 20 0	3800695.897	30.81424	30.81420	25.56289
34 21 0	3802544.752	30.81433	30.81429	25.55783
34 22 0	3804393.611	30.81441	30.81437	25.55277
34 23 0	3806242.476	30.81450	30.81446	25.54771
34 24 0	3808091.346	30.81458	30.81454	25.54265
34 25 0	3809940.221	30.81467	30.81462	25.53758
34 26 0	3811789.101	30.81475	30.81471	25.53252
34 27 0	3813637.986	30.81484	30.81479	25.52745
34 28 0	3815486.876	30.81492	30.81488	25.52238
34 29 0	3817335.771	30.81500	30.81496	25.51730
34 30 0	3819184.672	30.81509	30.81505	25.51223

LATITUDE	(I)	DIFF. 1"	(II)	DIFF. 1"	(III)	(A6)
34 0 0	3762214.378	30.80024	3477.361	.01367	2.147	.0009
34 1 0	3764062.392	30.80032	3478.181	.01365	2.147	.0009
34 2 0	3765910.411	30.80040	3478.999	.01363	2.146	.0009
34 3 0	3767758.435	30.80049	3479.817	.01361	2.145	.0009
34 4 0	3769606.464	30.80057	3480.633	.01359	2.145	.0009
34 5 0	3771454.499	30.80066	3481.449	.01357	2.144	.0009
34 6 0	3773302.538	30.80074	3482.263	.01355	2.144	.0009
34 7 0	3775150.582	30.80082	3483.076	.01353	2.143	.0009
34 8 0	3776998.632	30.80091	3483.887	.01351	2.142	.0009
34 9 0	3778846.686	30.80099	3484.698	.01349	2.142	.0009
34 10 0	3780694.746	30.80108	3485.507	.01347	2.141	.0009
34 11 0	3782542.810	30.30116	3486.315	.01345	2.140	.0009
34 12 0	3784390.880	30.30124	3487.122	.01343	2.140	.0009
34 13 0	3786238.955	30.80133	3487.928	.01341	2.139	.0009
34 14 0	3788087.034	30.80141	3488.733	.01339	2.139	.0009
34 15 0	3789935.119	30.80150	3489.536	.01337	2.138	.0009
34 16 0	3791783.209	30.80158	3490.339	.01335	2.137	.0009
34 17 0	3793631.304	30.80167	3491.140	.01333	2.137	.0009
34 18 0	3795479.404	30.80175	3491.940	.01331	2.136	.0009
34 19 0	3797327.509	30.80183	3492.738	.01329	2.135	.0009
34 20 0	3799175.619	30.80192	3493.536	.01327	2.135	.0009
34 21 0	3801023.734	30.80200	3494.332	.01325	2.134	.0009
34 22 0	3802871.854	30.80209	3495.127	.01323	2.134	.0009
34 23 0	3804719.979	30.80217	3495.921	.01321	2.133	.0009
34 24 0	3806568.110	30.80226	3496.714	.01319	2.132	.0009
34 25 0	3808416.245	30.80234	3497.506	.01317	2.132	.0009
34 26 0	3810264.335	30.80243	3498.296	.01315	2.131	.0009
34 27 0	3812112.531	30.80251	3499.036	.01314	2.130	.0009
34 28 0	3813960.681	30.80259	3499.874	.01312	2.130	.0009
34 29 0	3815808.837	30.80268	3500.661	.01310	2.129	.0009
34 30 0	3817656.998	30.30276	3501.447	.01308	2.128	.0009

NORTHING = (I) + (II)P\*\*2 + (III)P\*\*4 + A(6)P\*\*6 NORTHERN HEMISPHERE  
SUBTRACT FROM 10 MILLIONS FOR SOUTHERN HEMISPHERE

P = .0001DELTA(LONGITUDE) IN SECONDS FROM CENTRAL MERIDIAN

DMA TM 8358.2

LATITUDE	(IV)	DIFF. 1"	(V)	DIFF. 1"	(B5)
34 0 0	256532.988	-.83519	37.976	-.00103	-.0170
34 1 0	256482.877	-.83555	37.905	-.00103	-.0170
34 2 0	256432.744	-.83591	37.843	-.00103	-.0170
34 3 0	256382.589	-.83627	37.782	-.00103	-.0171
34 4 0	256332.413	-.83664	37.720	-.00103	-.0171
34 5 0	256282.214	-.83700	37.658	-.00103	-.0172
34 6 0	256231.994	-.83736	37.596	-.00103	-.0172
34 7 0	256181.753	-.83772	37.534	-.00103	-.0172
34 8 0	256131.489	-.83808	37.472	-.00103	-.0173
34 9 0	256081.204	-.83845	37.410	-.00103	-.0173
34 10 0	256030.898	-.83881	37.349	-.00103	-.0173
34 11 0	255980.569	-.83917	37.287	-.00103	-.0174
34 12 0	255930.219	-.83953	37.225	-.00103	-.0174
34 13 0	255879.847	-.83989	37.163	-.00103	-.0175
34 14 0	255829.454	-.84025	37.101	-.00103	-.0175
34 15 0	255779.038	-.84062	37.040	-.00103	-.0175
34 16 0	255728.601	-.84098	36.978	-.00103	-.0176
34 17 0	255678.143	-.84134	36.916	-.00103	-.0176
34 18 0	255627.662	-.84170	36.854	-.00103	-.0176
34 19 0	255577.161	-.84206	36.793	-.00103	-.0177
34 20 0	255526.637	-.84242	36.731	-.00103	-.0177
34 21 0	255476.092	-.84278	36.669	-.00103	-.0178
34 22 0	255425.525	-.84314	36.607	-.00103	-.0178
34 23 0	255374.936	-.84353	36.546	-.00103	-.0178
34 24 0	255324.326	-.84386	36.484	-.00103	-.0179
34 25 0	255273.694	-.84423	36.442	-.00103	-.0179
34 26 0	255223.041	-.84459	36.360	-.00103	-.0179
34 27 0	255172.365	-.84495	36.299	-.00103	-.0180
34 28 0	255121.669	-.84531	36.237	-.00103	-.0180
34 29 0	255070.950	-.84567	36.175	-.00103	-.0181
34 30 0	251020.210	-.84603	36.113	-.00103	-.0181

DELTA(EASTING) = (IV)P + (V)P\*\*3 + (B5)P\*\*5  
 ADD OR SUBTRACT FROM FALSE EASTING (500,000)

P = .0001DELTA(LONGITUDE) IN SECONDS FROM CENTRAL MERIDIAN

LATITUDE	(I)	DIFF. 1"	(VII)	DIFF. 1"	(VIII)	(D6)
34 0 0	3762214.378	30.80024	1715.576	.01784	22.286	.32
34 1 0	3764062.392	30.80032	1716.646	.01785	22.306	.32
34 2 0	3765910.411	30.80040	1717.717	.01785	22.326	.32
34 3 0	3767758.435	30.80049	1718.788	.01786	22.346	.32
34 4 0	3769606.464	30.80057	1719.860	.01787	22.366	.32
34 5 0	3771454.499	30.80066	1720.932	.01788	22.386	.32
34 6 0	3773302.538	30.80074	1722.004	.01788	22.406	.32
34 7 0	3775150.582	30.80082	1723.077	.01789	22.426	.32
34 8 0	3776998.632	30.80091	1724.151	.01790	22.446	.32
34 9 0	3778846.686	30.80099	1725.224	.01790	22.466	.32
34 10 0	3780694.746	30.80108	1726.299	.01791	22.486	.32
34 11 0	3782542.810	30.80116	1727.373	.01792	22.506	.32
34 12 0	3784390.830	30.80124	1728.448	.01792	22.526	.32
34 13 0	3786238.955	30.80133	1729.524	.01793	22.546	.32
34 14 0	3788087.034	30.80141	1730.600	.01794	22.566	.32
34 15 0	3789935.119	30.80153	1731.676	.01794	22.586	.32
34 16 0	3791783.209	30.80158	1732.752	.01795	22.606	.32
34 17 0	3793631.304	30.80167	1733.830	.01796	22.626	.33
34 18 0	3795479.404	30.80175	1734.907	.01797	22.646	.33
34 19 0	3797327.509	30.80183	1735.935	.01797	22.666	.33
34 20 0	3799175.619	30.80192	1737.063	.01798	22.686	.33
34 21 0	3801023.734	30.80200	1738.142	.01799	22.707	.33
34 22 0	3802871.854	30.80209	1739.221	.01799	22.727	.33
34 23 0	3804719.979	30.80217	1740.301	.01800	22.747	.33
34 24 0	3806568.110	30.80226	1741.381	.01801	22.767	.33
34 25 0	3808416.245	30.80234	1742.462	.01802	22.788	.33
34 26 0	3810264.335	30.80243	1743.542	.01802	22.808	.33
34 27 0	3812112.531	30.80251	1744.624	.01803	22.828	.33
34 28 0	3813960.681	30.80259	1745.706	.01804	22.849	.33
34 29 0	3815808.837	30.80268	1746.738	.01804	22.869	.33
34 30 0	3817656.998	30.80276	1747.870	.01805	22.889	.33

DELTA(LATITUDE) = (VII)Q\*\*2 - (VIII)Q\*\*4 + (D6)Q\*\*6  
SUBTRACT FROM FOOTPOINT LATITUDE IN SECONDS

Q = .000001DELTA(EASTING) = DIFFERENCE IN EASTING FROM FALSE EASTING (500,000)

TABLE FOR USE WITH FOOTPOINT LATITUDE

DMA TM 8358.2

LATITUDE	(IX)	DIFF. 1"	(X)	DIFF. 1"	(E5)
34 0 0	38981.341	.12694	305.344	.00402	4.454
34 1 0	38988.958	.12704	305.585	.00402	4.460
34 2 0	38996.580	.12714	305.827	.00403	4.466
34 3 0	39004.209	.12725	306.063	.00403	4.473
34 4 0	39011.844	.12735	306.310	.00404	4.479
34 5 0	39019.485	.12746	306.553	.00404	4.486
34 6 0	39027.133	.12756	306.795	.00405	4.492
34 7 0	39034.786	.12767	307.038	.00405	4.499
34 8 0	39042.447	.12778	307.281	.00406	4.505
34 9 0	39050.113	.12788	307.525	.00406	4.512
34 10 0	39057.786	.12799	307.747	.00407	4.518
34 11 0	39065.465	.12809	308.013	.00407	4.525
34 12 0	39073.151	.12820	308.257	.00408	4.532
34 13 0	39080.842	.12830	308.502	.00408	4.538
34 14 0	39088.541	.12841	308.747	.00409	4.545
34 15 0	39096.245	.12851	308.992	.00409	4.551
34 16 0	39103.956	.12862	309.233	.00410	4.558
34 17 0	39111.673	.12873	309.484	.00411	4.564
34 18 0	39119.397	.12883	309.730	.00411	4.571
34 19 0	39127.127	.12894	309.977	.00412	4.578
34 20 0	39134.863	.12905	310.224	.00412	4.584
34 21 0	39142.606	.12915	310.471	.00413	4.591
34 22 0	39150.355	.12926	310.719	.00413	4.598
34 23 0	39158.111	.12936	310.966	.00414	4.604
34 24 0	39165.873	.12947	311.215	.00414	4.611
34 25 0	39173.641	.12958	311.463	.00415	4.618
34 26 0	39181.416	.12969	311.712	.00415	4.625
34 27 0	39189.197	.12979	311.961	.00416	4.631
34 28 0	39196.984	.12990	312.211	.00416	4.638
34 29 0	39204.778	.13001	312.460	.00417	4.645
34 30 0	39212.578	.13011	312.710	.00417	4.652

DELTA(LONGITUDE) = (IX)Q - (X)Q\*\*3 + (E5)Q\*\*5  
 ADD OR SUBTRACT FROM CENTRAL MERIDIAN IN SECONDS

Q = .000001DELTA(EASTING) = DIFFERENCE IN EASTING FROM FALSE EASTING (500,000)

TABLE FOR USE WITH FOOTPOINT LATITUDE

LATITUDE	(XII)	DIFF. 1"	(XIII)	(C5)
34 0 0	5591.929	.04019	3.053	.00154
34 1 0	5594.340	.04018	3.053	.00154
34 2 0	5596.751	.04017	3.054	.00154
34 3 0	5599.162	.04017	3.054	.00154
34 4 0	5601.572	.04016	3.054	.00154
34 5 0	5603.981	.04015	3.054	.00153
34 6 0	5606.390	.04014	3.054	.00153
34 7 0	5608.798	.04013	3.054	.00153
34 8 0	5611.206	.04013	3.054	.00153
34 9 0	5613.614	.04012	3.054	.00153
34 10 0	5616.021	.04011	3.054	.00153
34 11 0	5618.428	.04010	3.054	.00153
34 12 0	5620.834	.04009	3.054	.00153
34 13 0	5623.239	.04009	3.055	.00153
34 14 0	5625.645	.04008	3.055	.00152
34 15 0	5628.049	.04007	3.055	.00152
34 16 0	5230.453	.04006	3.055	.00152
34 17 0	5632.857	.04005	3.055	.00152
34 18 0	5635.260	.04005	3.055	.00152
34 19 0	5637.663	.04004	3.055	.00152
34 20 0	5640.066	.04003	3.055	.00152
34 21 0	5642.467	.04002	3.055	.00152
34 22 0	5644.869	.04001	3.055	.00151
34 23 0	5647.270	.04001	3.055	.00151
34 24 0	5649.670	.04000	3.055	.00151
34 25 0	5652.070	.03999	3.055	.00151
34 26 0	5654.469	.03998	3.055	.00151
34 27 0	5656.868	.03997	3.056	.00151
34 28 0	5659.267	.03997	3.056	.00151
34 29 0	5661.665	.03996	3.056	.00151
34 30 0	5664.062	.03995	3.056	.00151

CONVERGENCE IN SECONDS = (XII)P + (XIII)P\*\*3 + (C5)P\*\*5

P = .0001DELTA(LONGITUDE) IN SECONDS FROM CENTRAL MERIDIAN

DMA TM 8358.2

LATITUDE	(XV)	DIFF. 1"	(XVI)	(F5)
34 0 0	21798.090	.2277	258.7	4.297
34 1 0	21811.750	.2278	258.9	4.304
34 2 0	21825.416	.2279	259.2	4.310
34 3 0	21839.087	.2279	259.5	4.317
34 4 0	21852.763	.2280	259.7	4.324
34 5 0	21866.445	.2281	260.0	4.330
34 6 0	21880.132	.2282	260.3	4.337
34 7 0	21893.825	.2283	260.5	4.343
34 8 0	21907.523	.2284	260.8	4.350
34 9 0	21921.226	.2285	261.1	4.357
34 10 0	21934.935	.2286	261.3	4.363
34 11 0	21948.649	.2287	261.6	4.370
34 12 0	21962.368	.2287	261.9	4.377
34 13 0	21976.093	.2288	262.1	4.383
34 14 0	21989.824	.2289	262.4	4.390
34 15 0	22003.559	.2290	262.7	4.397
34 16 0	22017.301	.2291	262.9	4.403
34 17 0	22031.047	.2292	263.2	4.410
34 18 0	22044.799	.2293	263.5	4.417
34 19 0	22058.557	.2294	263.7	4.424
34 20 0	22072.320	.2295	264.0	4.430
34 21 0	22086.088	.2296	264.3	4.437
34 22 0	22099.862	.2297	264.6	4.444
34 23 0	22113.641	.2297	264.8	4.451
34 24 0	22127.426	.2298	265.1	4.458
34 25 0	22141.216	.2299	265.4	4.464
34 26 0	22155.011	.2300	265.6	4.471
34 27 0	22168.813	.2301	265.9	4.478
34 28 0	22182.619	.2302	266.2	4.485
34 29 0	22196.431	.2303	266.5	4.492
34 30 0	22210.249	.2304	266.7	4.499

CONVERGENCE IN SECONDS = (XV)Q + (XVI)Q\*\*3 + (F5)Q\*\*5

Q = .000001DELTA(EASTING) = DIFFERENCE IN EASTING FROM FALSE EASTING (500,000)

TABLE FOR USE WITH FOOTPOINT LATITUDE



LATITUDE	(XX)	(XXI)
34 0 0	.00081149	.00000035
34 1 0	.00081117	.00000035
34 2 0	.00081085	.00000035
34 3 0	.00081053	.00000034
34 4 0	.00081021	.00000034
34 5 0	.00080989	.00000034
34 6 0	.00080957	.00000034
34 7 0	.00080925	.00000034
34 8 0	.00080893	.00000034
34 9 0	.00080861	.00000034
34 10 0	.00080829	.00000034
34 11 0	.00080797	.00000034
34 12 0	.00080765	.00000034
34 13 0	.00080733	.00000034
34 14 0	.00080701	.00000034
34 15 0	.00080668	.00000034
34 16 0	.00080636	.00000034
34 17 0	.00080604	.00000034
34 18 0	.00080572	.00000034
34 19 0	.00380540	.00000034
34 20 0	.00080508	.00000034
34 21 0	.00080476	.00000034
34 22 0	.00080444	.00000033
34 23 0	.00080411	.00000033
34 24 0	.00080379	.00000033
34 25 0	.00080347	.00000033
34 26 0	.00080315	.00000033
34 27 0	.00080283	.00000033
34 28 0	.00080250	.00000033
34 29 0	.00380218	.00000033
34 30 0	.00380186	.00000033

SCALE FACTOR =  $.9996(1.0 + (XX)P^{**2} + (XXI)P^{**4})$

P =  $.0001\text{DELTA}(\text{LONGITUDE})$  IN SECONDS FROM CENTRAL MERIDIAN

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LATITUDE	(I)	(XVIII)	(XIX)
34 0 0	3762214.378	.0123310	.000026
34 1 0	3764062.392	.0123309	.000026
34 2 0	3765910.411	.0123309	.000026
34 3 0	3767758.435	.0123308	.000026
34 4 0	3769606.464	.0123308	.000026
34 5 0	3771454.499	.0123307	.000026
34 6 0	3773302.538	.0123307	.000026
34 7 0	3775150.582	.0123307	.000026
34 8 0	3776998.632	.0123306	.000026
34 9 0	3778846.686	.0123306	.000026
34 10 0	3780694.746	.0123305	.000026
34 11 0	3782542.810	.0123305	.000026
34 12 0	3784390.880	.0123304	.000026
34 13 0	3786238.955	.0123304	.000026
34 14 0	3788087.034	.0123303	.000026
34 15 0	3789935.119	.0123303	.000026
34 16 0	3781783.209	.0123303	.000026
34 17 0	3793631.304	.0123302	.000026
34 18 0	3795479.404	.0123302	.000026
34 19 0	3797327.509	.0123301	.000026
34 20 0	3799175.619	.0123301	.000026
34 21 0	3801023.734	.0123300	.000026
34 22 0	3802871.854	.0123300	.000026
34 23 0	3804719.979	.0123299	.000026
34 24 0	3806568.110	.0123299	.000026
34 25 0	3808416.245	.0123298	.000026
34 26 0	3810264.385	.0123298	.000026
34 27 0	3812112.531	.0123298	.000026
34 28 0	3813960.681	.0123297	.000026
34 29 0	3815808.837	.0123297	.000026
34 30 0	3817656.998	.0123296	.000026

SCALE FACTOR = .9996( 1.0 + (XVIII)Q\*\*2 + (XIV)Q\*\*4 )

Q = .000001DELTA(EASTING) = DIFFERENCE IN EASTING FROM FALSE EASTING (500,000)

TABLE FOR USE WITH FOOTPOINT LATITUDE

For the UPS Grid:

ELLIPSOID					
		A	1/F	NAME	UNITS
		6378137.00000	298.257223563	WGS-84	METERS
LATITUDE	R	DIFF. 1"	SCALE FACTOR	DIFF.1" X 10**7	
84 0 0	666727.704	-30.92101	.9967300	-2.53	
84 1 0	664872.443	-30.92056	.9967148	-2.52	
84 2 0	663017.210	-30.92011	.9966997	-2.51	
84 3 0	661162.003	-30.91966	.9966846	-2.51	
84 4 0	659306.824	-30.91921	.9966696	-2.50	
84 5 0	657451.671	-30.91877	.9966546	-2.49	
84 6 0	655596.545	-30.91832	.9966396	-2.49	
84 7 0	653741.446	-30.91788	.9966247	-2.48	
84 8 0	651886.373	-30.91743	.9966098	-2.47	
84 9 0	650031.327	-30.91699	.9965950	-2.47	
84 10 0	648176.308	-30.91655	.9965802	-2.46	
84 11 0	646321.315	-30.91612	.9965654	-2.45	
84 12 0	644466.348	-30.91568	.9965507	-2.44	
84 13 0	642611.407	-30.91524	.9965361	-2.44	
84 14 0	640756.492	-30.91481	.9965214	-2.43	
84 15 0	638901.604	-30.91437	.9965069	-2.42	
84 16 0	637046.742	-30.91394	.9964923	-2.42	
84 17 0	635191.905	-30.91351	.9964778	-2.41	
84 18 0	633337.094	-30.91308	.9964634	-2.40	
84 19 0	631482.310	-30.91265	.9964490	-2.39	
84 20 0	629627.550	-30.91223	.9964346	-2.39	
84 21 0	627772.817	-30.91180	.9964203	-2.38	
84 22 0	625918.109	-30.91137	.9964060	-2.37	
84 23 0	624063.427	-30.91095	.9963918	-2.37	
84 24 0	622208.769	-30.91053	.9963776	-2.36	
84 25 0	620354.138	-30.91011	.9963634	-2.35	
84 26 0	618499.531	-30.90969	.9963493	-2.34	
84 27 0	616644.950	-30.90927	.9963352	-2.34	
84 28 0	614790.394	-30.90885	.9963212	-2.33	
84 29 0	612935.862	-30.90844	.9963072	-2.32	
84 30 0	611081.356	-30.90802	.9962933	-2.32	
84 31 0	609226.875	-30.90761	.9962794	-2.31	
84 32 0	607372.418	-30.90720	.9962655	-2.30	
84 33 0	605517.986	-30.90679	.9962517	-2.30	
84 34 0	603663.579	-30.90638	.9962380	-2.29	
84 35 0	601809.196	-30.90597	.9962242	-2.28	
84 36 0	599954.838	-30.90556	.9962105	-2.27	
84 37 0	598100.504	-30.90516	.9961969	-2.27	
84 38 0	596246.195	-30.90475	.9961833	-2.26	
84 39 0	594391.910	-30.90435	.9961697	-2.25	

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84 40 0	592537.649	-30.90395	.9961562	-2.25
84 41 0	590683.412	-30.90355	.9961427	-2.24
84 42 0	588829.199	-30.90315	.9961293	-2.23
84 43 0	586975.010	-30.90275	.9961159	-2.22
84 44 0	585120.845	-30.90235	.9961026	-2.22
84 45 0	583266.704	-30.90196	.9960893	-2.21
84 46 0	581412.586	-30.90156	.9960760	-2.20
84 47 0	579558.493	-30.90117	.9960628	-2.20
84 48 0	577704.422	-30.90078	.9960496	-2.19
84 49 0	575850.376	-30.90039	.9960365	-2.18
84 50 0	573996.352	-30.90000	.9960234	-2.18
84 51 0	572142.352	-30.89961	.9960103	-2.17
84 52 0	570288.376	-30.89922	.9959973	-2.16
84 53 0	568434.422	-30.89884	.9959844	-2.15
84 54 0	566580.492	-30.89845	.9959714	-2.15
84 55 0	564726.585	-30.89807	.9959586	-2.14
84 56 0	562872.700	-30.89769	.9959457	-2.13
84 57 0	561018.839	-30.89731	.9959329	-2.13
84 58 0	559165.000	-30.89693	.9959202	-2.12
84 59 0	557311.185	-30.89655	.9959075	-2.11
85 0 0	555457.391	-30.89618	.9958948	-2.10

NORTHING = 2,000,000 - (R)COS(LONGITUDE) NORTH ZONE  
 NORTHING = 2,000,000 + (R)COS(LONGITUDE) SOUTH ZONE  
 EASTING = 2,000,000 + (R)SIN(LONGITUDE) BOTH ZONES

## CHAPTER 5

### OTHER COORDINATE CONVERSIONS AND TRANSFORMATIONS

#### 5-1 GRID CONVERSION BETWEEN ZONES WITHIN THE UTM SYSTEM AND GRID CONVERSION BETWEEN THE UTM AND UPS SYSTEMS.

5-1.1 To accurately convert grid coordinates between zones or systems, the grid coordinates are first converted to geographic coordinates in the known zone or system. Once the grid coordinates have been converted to geographic coordinates, they are then converted to grid coordinates in the new zone or system. These conversions utilize the formulas found in Chapters 2 and 3 of this manual.

5-1.2 Although direct grid to grid conversions exist, which can be performed with the use of tables, this method is not as accurate as the conversion method discussed above. Other apparent direct grid to grid conversions exist, however they involve the intermediate conversion to geographic coordinates.

#### 5-2 MILITARY GRID REFERENCE SYSTEM.

5-2.1 The U.S. Military Grid Reference System (MGRS) is designed for use with the UTM and UPS grids. An MGRS position location is an alpha-numeric version of a numerical UTM or UPS grid coordinate.

5-2.2 Chapter 3 and Appendix B of DMA TM 8358.1 describes and shows the method for finding the 100,000-meter square identifications. Software to convert between UTM or UPS coordinates and MGRS positions is listed as Annex A of this manual and can be obtained from DMA HTC/PRT.

#### 5-3 WORLD GEOGRAPHIC REFERENCE SYSTEM.

5-3.1 The World Geographic Reference System (GEOREF) is a system used for position reporting. It is not a military grid, but rather an area-designation method. Positions are expressed in a form suitable for reporting and plotting on any map or chart graduated in latitude and longitude (with Greenwich as prime meridian) regardless of map projection.

5-3.2 Section 5-4 of DMA TM 8358.1 describes and illustrates the World Geographic Reference System. Software to convert between UTM or UPS coordinates and GEOREF positions is provided as Annex A-4 of this manual.

#### 5-4 DATUM TO DATUM COORDINATE TRANSFORMATIONS.

##### 5-4.1 Molodenskiy Coordinate Transformation Formulas.

###### 5-4.1.1 Definition Of Terms.

Terms previously defined in Chapters 2 and 3 are not redefined below.

$\phi, \lambda$  = geodetic coordinates (input ellipsoid)

$H$  =  $N + h$  = the distance of a point from the ellipsoid center measured along the ellipsoidal normal through the point

- N = geoid-ellipsoid separation = the distance of the geoid above (+ N) or below (- N) the ellipsoid
- h = distance of a point from the geoid = the elevation above or below mean sea level
- $\Delta\phi, \Delta\lambda, \Delta H$  = corrections to transform the geodetic coordinates from the input datum to the output datum (output minus input)
- $\Delta X, \Delta Y, \Delta Z$  = shifts between ellipsoid centers of the input datum and output datum (output minus input)
- $\Delta a, \Delta f$  = differences between the parameters of the input ellipsoid and the output ellipsoid (output minus input)
- $\rho$  = radius of curvature in the meridian
- $\nu$  = radius of curvature in the prime meridian

#### 5-4.1.2 Standard Molodenskiy Formulas.

The Standard Molodenskiy formulas are as follows:

$$\Delta\phi'' = \{-\Delta X \sin\phi \cos\lambda - \Delta Y \sin\phi \sin\lambda + \Delta Z \cos\phi + \Delta a (R_N e^2 \sin\phi \cos\phi) / a + \Delta f [R_M (a/b) + R_N (b/a)] \sin\phi \cos\phi\} / [(R_M + H) \sin 1'']$$

$$\Delta\lambda'' = [-\Delta X \sin\lambda + \Delta Y \cos\lambda] / [(R_N + H) \cos\phi \sin 1'']$$

$$\Delta H = \Delta X \cos\phi \cos\lambda + \Delta Y \cos\phi \sin\lambda + \Delta Z \sin\phi - \Delta a (a/R_N) + \Delta f (b/a) R_N \sin^2\phi$$

#### 5-4.1.3 Abridged Molodenskiy Formulas.

The Abridged Molodenskiy formulas are as follows:

$$\Delta\phi'' = [-\Delta X \sin\phi \cos\lambda - \Delta Y \sin\phi \sin\lambda + \Delta Z \cos\phi + (a\Delta f + f\Delta a) \sin 2\phi] / [R_M \sin 1'']$$

$$\Delta\lambda'' = [-\Delta X \sin\lambda + \Delta Y \cos\lambda] / [R_N \cos\phi \sin 1'']$$

$$\Delta H = \Delta X \cos\phi \cos\lambda + \Delta Y \cos\phi \sin\lambda + \Delta Z \sin\phi + (a\Delta f + f\Delta a) \sin^2\phi - \Delta a$$

#### 5-4.2 Multiple Regression Equations.

Multiple regression equations are developed from polynomial equations to the nth degree to fit predetermined accuracy requirements. The terms that are included in the multiple regression equations are a function of the desired accuracy of the solution. In developing the solution, the terms of first degree to nth degree are tested and in the process of deriving the solution to fit the accuracy requirements, certain terms drop out. Because of this, there is no generalized equation to present as a guideline. However, as an example, a datum shift from European Datum to WGS 1972 is

presented to illustrate multiple regressions. This shift is not to be construed as the most up-to-date transformation for the area under consideration. For a more in-depth discussion of The Multiple Regression Equations, see DMA TR 8350.2, Department of Defense World Geodetic System 1984, Its Definition and Relationships with Local Geodetic Systems.

$$\begin{aligned}
 \Delta\phi'' &= -2.5830 + 2.0782X + 0.6631Y + 0.6144X^2 + 1.0456XY \\
 &\quad - 0.7752Y^2 + 0.8414X^3 + 0.1058XY^2 - 14.4049X^3Y \\
 &\quad + 4.4291XY^3 + 0.0166Y^4 + 59.9408X^5Y - 4.2792X^3Y^3 \\
 &\quad - 3.9642XY^5 + 0.7818Y^6 - 93.5475X^7Y - 4.5053X^7Y^2 \\
 &\quad + 48.8445X^9Y + 11.0197X^5Y^7 + 4.5980X^9Y^4 - 5.4256X^5Y^9 \\
 \Delta\lambda'' &= -4.8255 - 1.8094X + 1.8479Y - 2.0174X^2 + 1.1912XY \\
 &\quad - 0.3288X^3 + 3.3287X^4 - 4.1036X^3Y + 1.6161XY^3 \\
 &\quad + 0.6259Y^4 + 1.5379XY^4 - 2.5285XY^5 - 1.1917X^8 \\
 &\quad + 4.8445X^7Y - 10.0979X^6Y^2 - 0.7021Y^9 + 10.6185X^6Y^4 \\
 &\quad + 6.2369X^4Y^6 - 9.1252X^6Y^8 \\
 \Delta H_m &= 36.604 - 28.206X - 18.351Y + 9.525X^2 + 2.107Y^2 \\
 &\quad + 11.094X^3 - 0.684X^2Y + 25.268Y^3 + 86.856X^2Y^2 \\
 &\quad + 6.040XY^3 - 34.469Y^5 - 22.520X^5Y - 77.583X^4Y^2 \\
 &\quad - 125.318X^2Y^4 - 18.485XY^6 + 57.003X^5Y^3 + 72.140X^2Y^6 \\
 &\quad - 24.950X^8Y + 16.896Y^9 + 37.821X^8Y^2 + 41.265X^4Y^9
 \end{aligned}$$

$$\text{Where: } X = 3\phi - 2.7969898$$

$$Y = 3\lambda - 0.5248325$$

$$\phi, \lambda = \text{ED 50 latitude and longitude (in radians)}$$

$$\Delta H_m = \text{ED 50 height (in meters)}$$

### 5-4.3 ACCURACIES

5-4.3.1 The Multiple Regression Equations will provide a more accurate fit than either the Standard or Abridged Molodenskiy Transformations within its specific area of application. Outside of the specific area of application, the accuracies deteriorate rapidly.

5-4.3.2 Within a small area, such as a degree square, locally determined Molodenskiy Transformation constants will generally provide accuracies commensurate with the multiple regression. The simplicity and versatility of the Molodenskiy Transformation are also advantageous.

5-4.3.3 As long as the accuracy of the positions determined by satellite point positioning is in the 2-meter range, the Standard Molodenskiy Transformation will not give significantly more accurate results than the Abridged Molodenskiy Transformation.

5-4.3.4 When available, the Multiple Regression Equations will be the preferred method of datum transformation. This is followed by the Standard Molodenskiy Transformation and then the Abridged Molodenskiy Transformation.



**ANNEXES A and B**

These annexes list the software and documentation available for the transformation of geographic coordinates to/from grid coordinates. These products can be obtained by contacting DMA HTC(PRT).

**ANNEX A SOFTWARE SUPPORT FOR UNIVERSAL GRIDS****Section A-1 Software and Documentation for Geographic Coordinate Transformations to/from Grid Coordinates**

- A-1.1 UTM Grid
- A-1.2 UPS Grid
- A-2 Software and Documentation for Datum to Datum Coordinate Transformations
  - A-2.1 Molodenskiy
  - A-2.2 Abridged Molodenskiy
- A-3 Software and Documentation for the UTM and the UPS Grid Coordinate Transformation to/from the MGRS
- A-4 Software and Documentation for Geographic Coordinate Transformation to/from GEOREF Coordinates
- A-5 Software and Documentation for Grid Coefficients and Latitude Functions

**ANNEX B SOFTWARE SUPPORT FOR NON-UNIVERSAL GRIDS****Section B-1 Software and Documentation for the Transverse Mercator (TM) Grid**

- B-2 Software and Documentation for the Mercator Grid
- B-3 Software and Documentation for the Lambert Conical Orthomorphic Grid
- B-4 Software and Documentation for the Madagascar Gauss Laborde Grid
- B-5 Software and Documentation for the Rectified Skewed Orthomorphic Grid
- B-6 Software and Documentation for the New Zealand Map Grid
- B-7 Software and Documentation for the Guam Azimuthal Equidistant Grid