Cosmic Calibration - Statistical Modeling for Dark Energy

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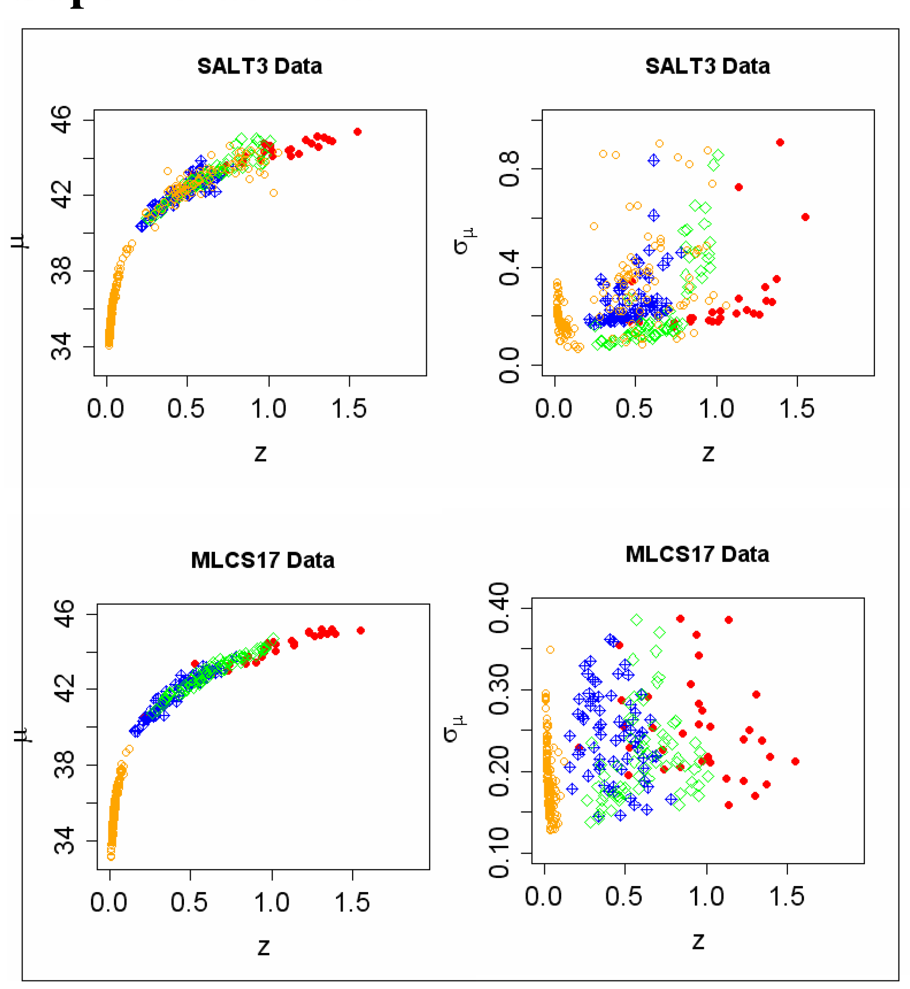
Abstract

The fact that the Universe is expanding has been known since the 1920's. If the Universe was filled with ordinary matter, the expansion should be decelerating. Beginning in 1998, however, observational evidence has been accumulating in favor of an accelerating expansion of the Universe. The unknown driver of the acceleration has been termed dark energy. The nature of dark energy can be investigated by studying its equation of state, that is the relationship of its pressure to its density. The equation of state can be measured via a study of the luminosity distance-redshift relation for supernovae. In this study, we employ supernovae data, including measurement errors, to determine whether the equation of state is constant or not. Our method is based on Bayesian analysis of a differential equation and modeling w(z) directly, where w(z) is the equation of state parameter. This work stems from collaboration between UCSC and Los Alamos National Laboratory (LANL) in the context of the Institute for Scalable Scientific Data Management (ISSDM) project.

Supernovae Data



SALT3 data set is the third supernovae dataset released using the SALT method of light curve fitting and contains 397 observations. MLCS17 is a data set of 372 observations using the MLCS light curve fitting method. The data has a redshift (z) value for each supernova and a value for μ (observed distance modulus.) The first plot is of z vs μ. The plots are colored by the telescope that cataloged the data.

We will look specifically at two models to fit these data sets. The first will be a parametric model that has been studied in depth and the other is a Gaussian Process model. Both of these models will be implemented using Bayesian methods and MCMC algorithms.

Equations and Parameters of Interest

The main parameter of interest is w(u). There are also two other known parameters: H_0=72.0 and Ω_{m0} = 0.27. The main equation of interest is a transformation:

$$T(z,H_0,\Omega_m,w(u)) = 25 + 5\log_{10}\left(\frac{c(1+z)}{H_0}\int_0^z\left(\Omega_m(1+s)^3 + (1-\Omega_m)(1+s)^3 e^{3\int_0^s \frac{w(u)}{1+u}du}\right)^{-0.5} ds\right)$$

To be able to use this equation we will need to specify a form for w(u). This also leads to a likelihood as follows:

$$L(\sigma,H_0,\Omega_m,w(u)) \propto \left(\frac{1}{\tau_i\sigma}\right)^n e^{-\frac{1}{2}\sum_{i=1}^n\left(\frac{\mu_i - T(z_i,H_0,\Omega_m,w(u))}{\tau_i\sigma}\right)^2}$$

To be able to use this likelihood we will need priors for σ and whatever parameters we used to specify w(u). As a note the τ's are the standard deviations for μ and part of the observed data set.

Parametric Model Theory

This non-linear model is advocated by Linder, a cosmologist, as a good alternative to w(u) set equal to a constant or just a simple line.

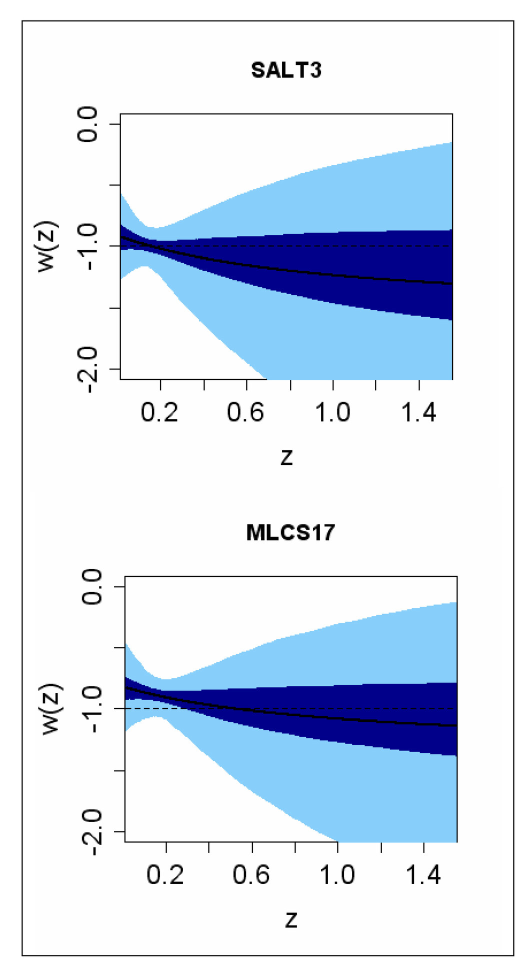
$$w(u) = a + b\left(\frac{1}{1+u} - 1\right)$$

This leads to a simplified version of our equation, namely we were able to do one of the integrations analytically.

$$T(z,H_0,\Omega_m) = 25 + 5\log_{10}\left(\frac{c(1+z)}{H_0}\int_0^z\left(\Omega_m(1+s)^3 + (1-\Omega_m)(1+s)^{3(a-b+1)} e^{\frac{3bs}{1+s}}\right)^{-0.5} ds\right)$$

To be able to use this likelihood we will need priors: π(a)~U(-25,1), π(b)~U(-25,25), and π (σ^2)~IG(10,9). We will use this model to compare against our Gaussian process model.

Parametric Model Results



The dashed line is at negative one, which is a value of interest. The black line is the mean fit of the MCMC simulation and the dark blue is 68% probability interval and the light blue are the 95% probability intervals. The fits here do not preclude the possibility that w(z) is negative one.

We also estimated the unknown parameters H_0 and Ω_m in this model. It should be noted that in the MCMC, the unknown parameters are correlated and drawn jointly.

Table 1 –Parameter Estimates

95% Probability interval	SALT3	MLCS17
a	(-1.286, -0.525)	(-1.184, -0.442)
b	(-1.805, 3.699)	(-1.707, 3.110)
Ω_m	(0.232, 0.336)	(0.232, 0.332)
H_0	(64.16, 66.19)	(64.16, 65.82)
σ^2	(1.04, 1.32)	(0.96, 1.21)

Gaussian Process Model Theory

We will consider a model where w(u) is a Gaussian process (GP): w(u) ~ GP(-1, κ^2 K(u,u')) where K(u,u')=$\rho^{|u-u'|^\alpha}$. The correlation function is of great importance in this method because we are going to use it to do integration and cannot use a nugget term with our method, this precludes a Gaussian or Matern correlation function. So, we will use α=1.999 as an approximation to a Gaussian correlation.

w(u) is a GP therefore its integral is also a GP: $y(s) = \int_0^s \frac{w(u)}{1+u}du$. The integral of a GP can be found by integrating the correlation function.

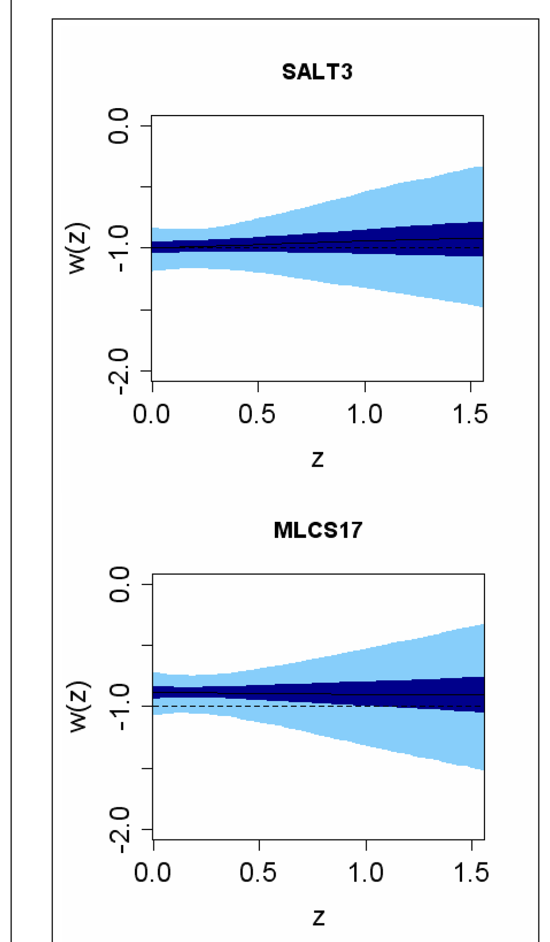
$$w(u) \sim GP\left(-1, \Sigma_{22} = \kappa^2\rho^{|u-u'|^\alpha}\right) \qquad y(s) \sim GP\left(-\ln(1+s), \Sigma_{11} = \kappa^2\int_0^s\int_0^{s'}\frac{\rho^{|u-u'|^\alpha}}{(1+u)(1+u')}du\,du'\right)$$

The mean of y(s) given w(u) can be found through the following relation:

$$y(s)\,|\,w(u) = -1\ln(1+s) + \Sigma_{12}\Sigma_{22}^{-1}\left(w(u) - (-1)\right) \qquad \Sigma_{12} = \kappa^2\int_0^s\frac{\rho^{|u-u'|^\alpha}}{(1+u)}du$$

To be able to use this likelihood we will need priors: π(κ^2)~ IG(25,9), π(ρ)~Be(6,1) and π (σ^2)~IG(10,9).

Gaussian Process Model Results



The dashed line is at negative one, which is a value of interest. The black line is the mean fit of the MCMC simulation and the dark blue is 68% probability interval and the light blue are the 95% probability intervals. The fits here do not preclude the possibility that w(z) is negative one.

The Gaussian process is a non-parametric model that allows for a flexible fit for w(z). The mixing is slow for the GP and it must be run many iterations. It should be noted that in the MCMC that the parameters are correlated and drawn jointly.

Table 1 – Parameter Estimates

95% Probability interval	SALT3	MLCS17
Ω_m	(0.231 ,0.324)	(0.226, 0.323)
H_0	(64.47, 66.16)	(64.35, 65.77)
σ^2	(1.04, 1.31)	(0.96, 1.21)

Conclusions

- We have shown that a typical parametric model, as well as, a non-parametric Gaussian process model can be fitted to the equation of state.
- The benefit of using a GP model is that it allows us to fit the equation of state without specifying a parametric form, which at this time is unknown.
- The GP produces smaller probability bands than the parametric model
- H_0 and Ω_m parameters are estimated in both models

Future Work

- Use an orthogonal basis of damped Hermite polynomials to approximate w(u)
- Add other probes to this analysis and reduce the uncertainty in the fit of w(u)
- Set up an experimental design to find where more data is need (on the z axis). In the experimental design also test how shrinking uncertainty for μ, Ω_{m0}, and H_0 would help in drawing more conclusive statements about w(z).
- Look into which type of measurement error could be reduced to help make conclusive statements about the parameters of interest; especially the standard deviations associated with μ

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