Final National Models for the United States: Development of GEOID09

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Introduction

The National Geodetic Survey (NGS) has been developing gravimetric geoid models for nearly two decades starting with GEOID90 (Milbert, 1991; Smith and Milbert, 1999; Smith and Roman, 2001; Roman et al 2004). Since 1996, these models have been combined with GPS/leveling information to create hybrid geoid height models. These models use the control data available in the NGS database at the time of their creation. The control data consist of bench marks where both the GPS-derived NAD 83 ellipsoidal height and leveled NAVD 88 orthometric height are known, and are called "GPSBM" data. The difference between these two heights provides an estimate of the separation between NAD 83 and NAVD 88 at that location.

Since these points are insufficient in coverage to develop a model of the NAD 83-NAVD 88 separation across the country, the gravimetric geoid model is used as a base and the control data used to warp the gravimetric geoid to fit between the two datums. This takes place by removing an interpolated gravimetric geoid height (N) from the GPSBM-derived geoid height (ellipsoidal height (h) - orthometric height (H)):

residual = (h – H) - N

If these values were without error and in the same reference frame, then the residual value would be zero. Naturally, it is not. The GPS-derived ellipsoidal height usually has 2-4 cm of random error associated with it. There is a known meter level trend, centered on an approximately half-meter bias in the NAVD 88 datum across the country with additional multi-decimeter features around the country (Figure 1). There are errors in the leveling, GPS, and gravimetric geoid development that create systematic effects.

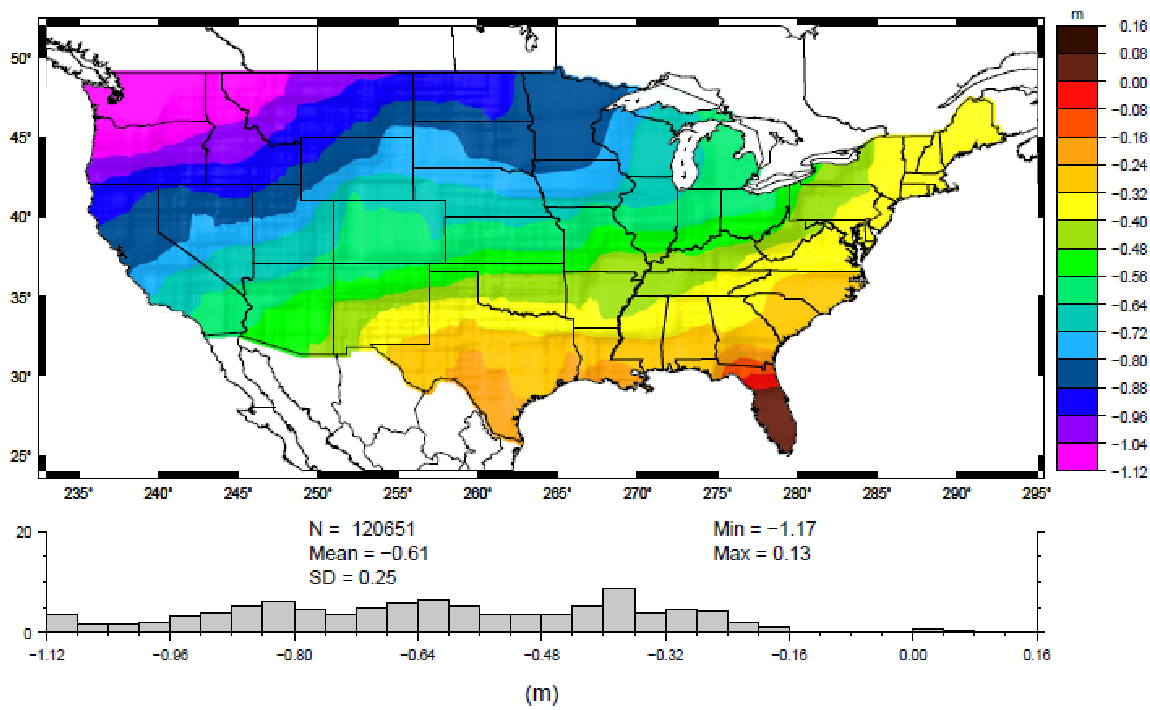

Figure 1: The long wavelength errors of NAVD 88 as determined from comparison to GRACE gravity field data.

In the end, the source of these errors that create the residual values doesn't matter. What matters from a pragmatic point of view is whether or not they can be modeled. Least Squares Collocation (LSC) is used to extract the correlated signal between these residuals. Using multi-matrix LSC (MMLSC) (Roman et al. 2004), systematic effects at many scales can be resolved in one adjustment. These features can be modeled and incorporated into a conversion surface that effectively translates between the gravimetric geoid surface (e.g., the geoid surface implied by USGG2009) and the zero elevation surface of the NAVD 88 datum.

The development of the GEOID09 model largely follows previous procedures but uses the most recent data from the NGS database. Hence, this model provides the most consistency with values given on NGS datasheets and held in the NGS database right now. Note the changes in values in the database contribute some of the biggest changes seen between GEOID03 and GEOID09.

GPSBM2009

The GPSBM data were drawn from the database in the summer of 2009. Further details can be found at http://www.ngs.noaa.gov/GEOID/GPSonBM09/. The database pull was delayed for a number of reasons. Waiting until this past summer permitted resolution of the appropriate data to extract and a more finalized set of data from which to perform the extraction. Many changes occurred between 2003 and 2009 for both orthometric and ellipsoidal heights. It should be noted that after the National Readjustment of 2007, many qualities of the ellipsoidal heights in the database changed. Details can be found here (http://www.ngs.noaa.gov/NationalReadjustment/).

Since the hybrid model is fit to these points, changes in these heights have a direct impact on the resulting model. Hence, it is necessary to use GEOID09 to develop values consistent with the heights given in the database as of 2009. Differences in the database are given below and are developed from GPSBM's that were common to database pulls made in 2003, 2007 (just prior to the National Readjustment of 2007) and 2009. The respective differences graphically show the changes over time and permit a better understanding of when the changes occurred.

Changes in the ellipsoidal heights from 2003 to 2009 are shown in Figure 2. This represents the net change in the knowledge of ellipsoid heights (not necessarily actual movement of the mark itself) over that period of time. While the overall mean (-1.2 cm) and standard deviation, or SD (one sigma = 2.7 cm), are fairly low, note that this is a national number. Local effects in California highlight about a -10 cm bias with a 10 cm SD. Other states with little variability bring the overall national number down. Hence, it is worth it to break down the changes over time and space.

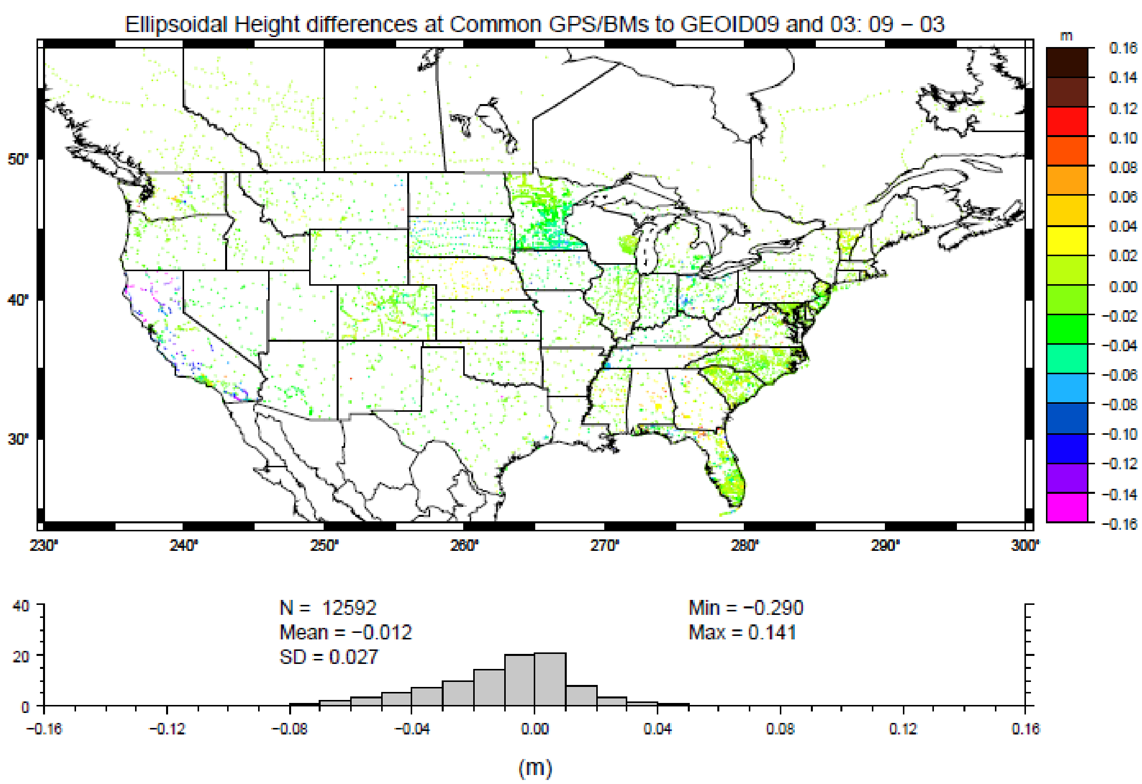

Figure 2: Differences in the ellipsoidal heights at GPSBMs common to GEOID03 and GEOID09.

Figures 3 and 4, respectively, show the ellipsoidal height changes from 2003 to 2007 and from 2007 to 2009. Note that the shifts seen in Figure 4 represent a subset of those seen in the National Readjustment of 2007 (Figure 5).

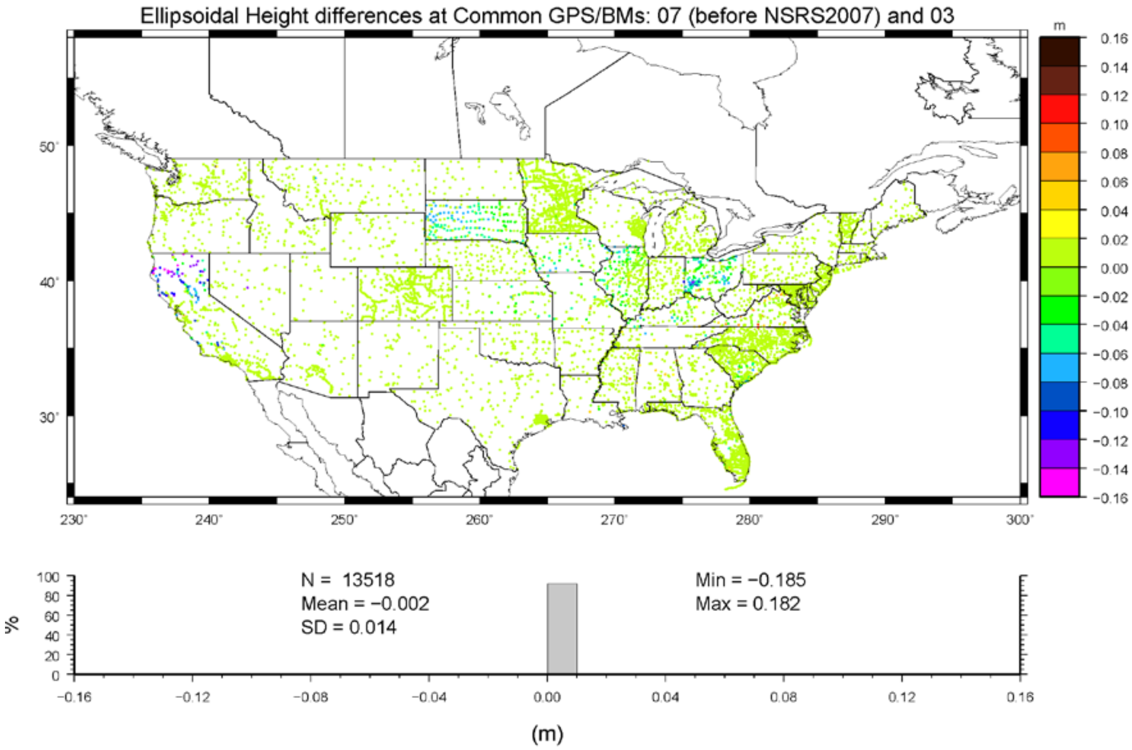

Figure 3: Differences in ellipsoidal heights in common GPSBM's from 2003 to 2007. These *were not* caused by the National Readjustment of 2007.

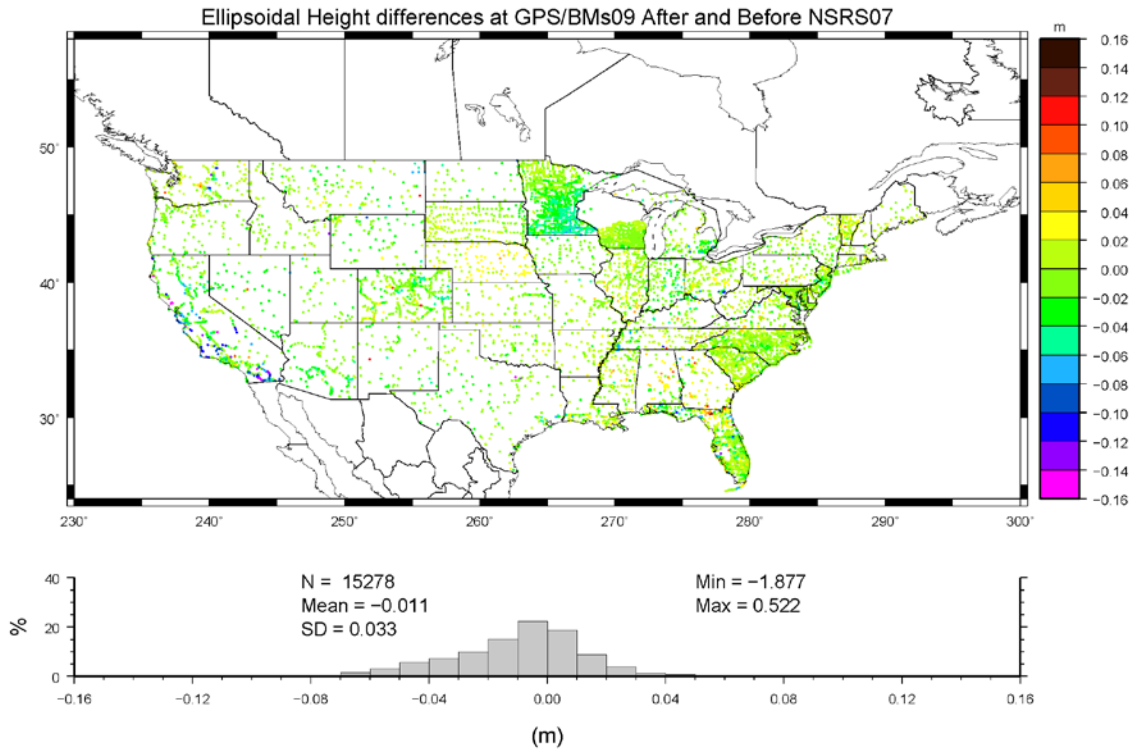

Figure 4: Differences in ellipsoidal heights in common GPSBM's from 2007 to 2009. These *<u>were</u>* caused by the National Readjustment of 2007.

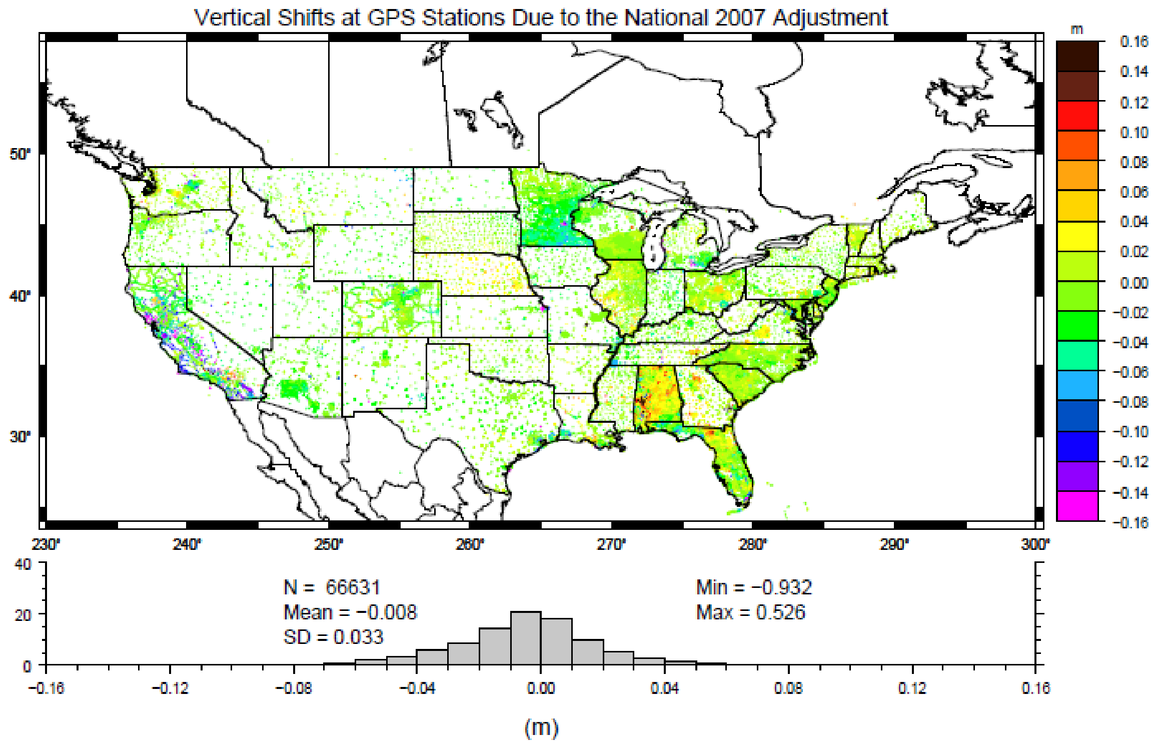

Figure 5: Vertical shifts at all GPS stations due to the NSRS2007. Those points given in Figure 4 represent a subset of this group.

Interestingly, there were also changes in orthometric heights from 2003 to 2009 (Figure 6). These are also broken down into the same two periods: 2007 minus 2003 (Figure 7) and 2009 minus 2007 (Figure 8). Changes occurred between 2003 and 2007 in Minnesota and Wisconsin. Between 2007 and 2009, the changes resulted from the changes to GPS-derived ellipsoidal heights in the subsidence region. Since the PID's differed from 2003 and 2009, the direct comparison between 2003 and 2009 seen in Figure 6 doesn't highlight that change. Clearly, the major changes were in ellipsoidal heights.

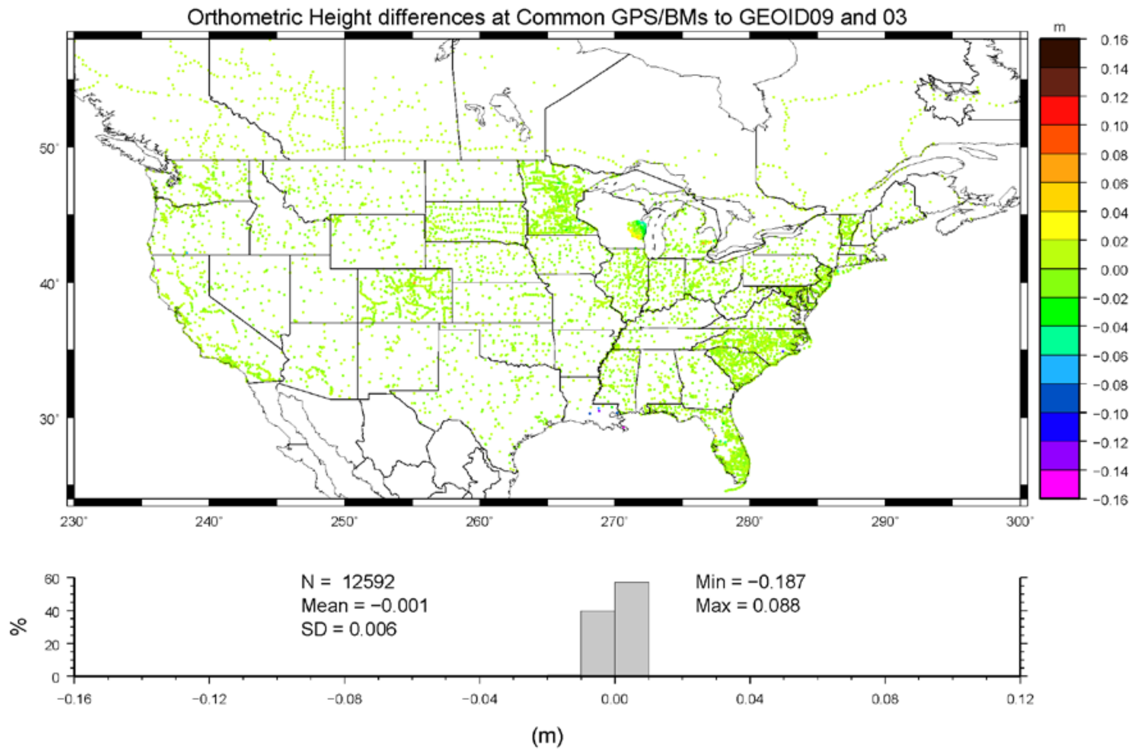

Figure 6: Orthometric height differences at GPS/BMs common to GEOID03 and GEOID09. Most changes were limited to Wisconsin and Minnesota, which are very subtle because we limited the color scale to that used in the other plots.. Note that Louisiana doesn't show here because the PID's changed between 2003 and 2009 in the subsidence region.

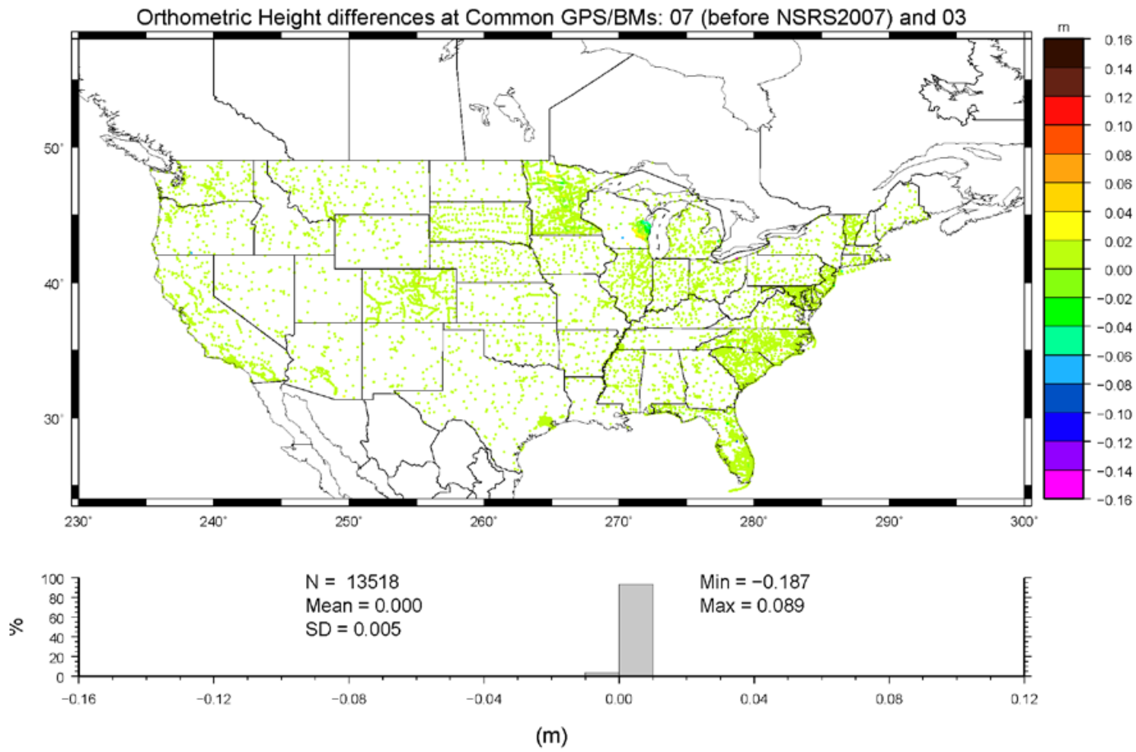

Figure 7: Differences in orthometric heights in common GPSBM's from 2003 to 2007. Note the similarity with Figure 6. The relatively few changes in orthometric heights occurred in this period.

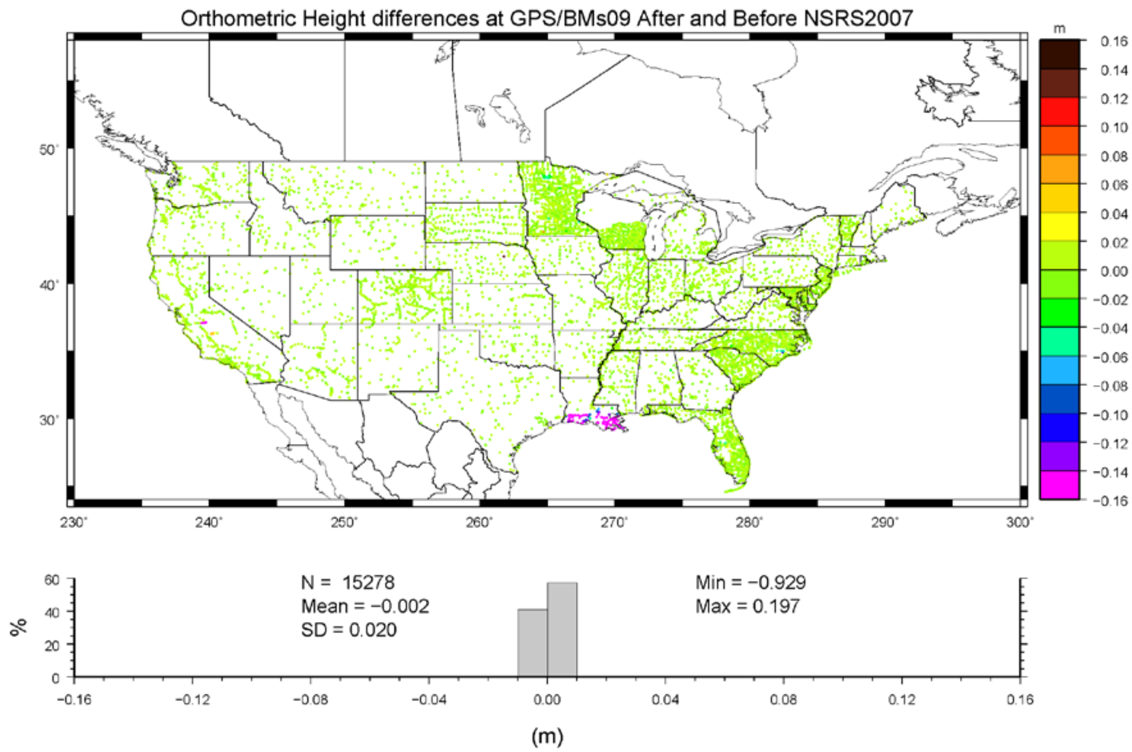

Figure 8: Differences in orthometric heights in common GPSBM's from 2007 to 2009. Essentially zero except for Louisiana where updated GPS-derived orthometric heights were used.

The cumulative effect of all of these changes is then seen in the raw differences between 2003 and 2009 for both ellipsoidal and orthometric heights. This is seen below in Figure 9.

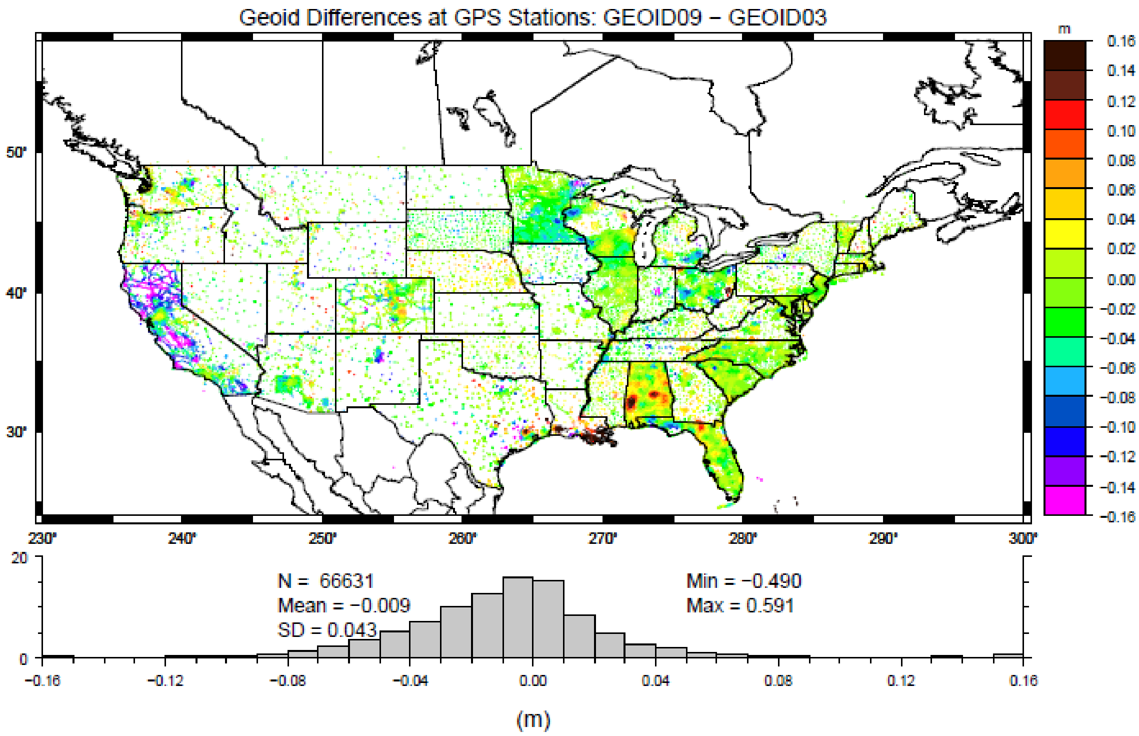

Figure 9: Geoid differences between GEOID09 and GEOID03 at GPSBM lcoations

In general, east CONUS is less mountainous than the west and has non-active geologic boundaries (no faulting, subduction, etc.). Hence, the gravimetric geoid is usually better defined due to this improved data and the reduced effect of assumptions (density, etc.). Additionally, it has a greater density of GPSBM's, so there are more control data to fit the conversion through. This is why the bulk of the changes seen in the GPSBM's greatly explain the shifts between GEOID03 and GEOID09 in the east.

For the west, the GPSBM's are sparser. Examining the differences between GEOID03 and GEOID09 in the west shows that many of the differences between USGG2003 and USGG2009 can be seen. See the section on USGG2009 (link) for more details.

MMLSC for CONUS

The MMLSC method was used combining six different positive definite matrices to account for systematic effects at multiple scales. Keep in mind that density of GPSBM's varies quite significantly across the country. The Gaussian models with shorter correlation lengths are only valid in regions where the GPSBM locations are sufficiently close. In some western states, GPSBM's are more than 100 km apart. Hence, the shortest models have no validity there since there is no signal that can be modeled in the first place. The use of multiple functions was pursued to mitigate some of the problems that arose after GEOID03 was developed.

In GEOID99, only a single Gaussian function at 400 km correlation length was utilized. For GEODI03, two functions were used with correlation lengths of 650 km and 60 km. For the western states such as Arizona, the function with the 60 km correlation length did not work because of the spacing of the GPSBM's. This left only the 650 km correlation length to model the remaining signal. This caused signal between 400 km and 650 km to be neglected, resulting in a slight degradation for those regions from GEOID99 to GEOID03.

To mitigate this in GEOID09, multiple models were used. The GPSBM spacing will determine which of the functions will actually apply for each region. Incrementing from 30 km to 60km to 90 km to 120 km creates thresholds that can better capture the correlated signal present in the GPSBM residuals. For example, the GPSBM station spacing may be insufficient for the function with a 60 km correlation length but it is sufficient for the function with the 90 km correlation length. This prevents a jump to longer wavelength function and neglecting correlated signal.

See Table 1 for a listing of the correlation lengths (scale) and standard deviation (square root of the power). Figure 6 shows the fit of this model to the empirical data (GPSBM residuals). Note that the model doesn't fit perfectly. Also, a random error of up to (1.3 cm)^2 was allowed to account for uncertainties in the ellipsoidal heights determined from GPS. Hence, there should be no expectation that the GEOID09 determined height (N) could be removed at GPSBM2009 control point and produce a residual of zero. Except in rare cases, there will always be a residual to the equation given above. However, this residual value should conform to the value for any given state in Table 2 below.

Table 1: The correlation lengths and powers of the different Gaussians combined to model the final covariance function.

#	Correlation length (km)	Standard deviation (cm)
1	600	2.9
2	260	3.5
3	120	0.1
4	90	1.4
5	60	1.6
6	30	3.2

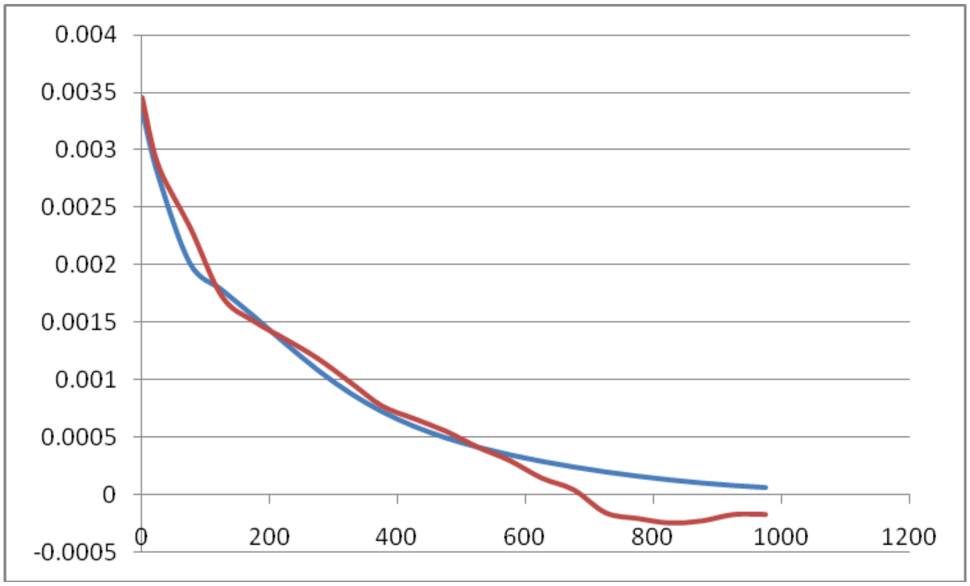

Figure 10: The covariance model (blue curve) fit to the GPS/Leveling residuals (red curve).

The six covariance functions are then stacked and used to predict a conversion surface based on the empirical data given at the GPSBM's. The national bias of 0.523 meter is restored as is a first order trend surface describing the tilt seen in Figure 1. Figure 11 then shows the conversion surface between the geoid surface of USGG2009 and the zero surface of the NAVD 88 datum realized by the GPSBM2009 dataset. Note that the major features and statistics are similar to those seen in Figure 1, highlighting the fact that the major differences derive from the systematic errors in the NAVD 88 datum. However, Figure 1 represents a smoothed version of the difference: NAVD88 – USGG09 while Figure 11 represents USGG2009 – NAVD88, hence the reversed sign. The higher frequency differences correspond with a number of the features seen above in the differences between orthometric and ellipsoidal heights from 2003 to 2009.

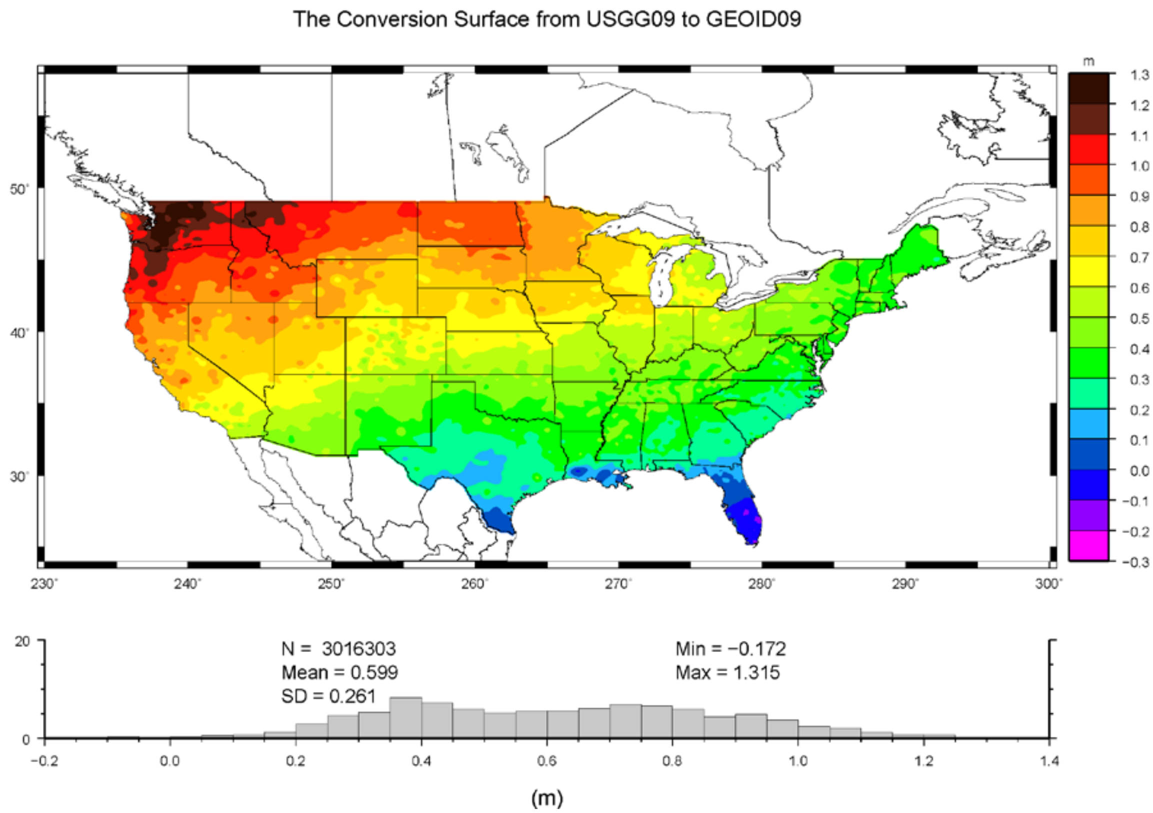

Figure 11: GEOID09 conversion surface from geoid to NAVD 88. Note that most of the difference (with a negative sign) is accounted for by the NAVD 88 trend seen in Figure 1.

Finally, a comparison is made between GEOID09 and GEOID03 in Figure 12. Areas outside of CONUS have been blocked out so that the statistics relate to the differences in CONUS. Many of the differences can be related to the changes in ellipsoidal and orthometric heights noted above. This is particularly true for the eastern states where the nature of the gravity field is better resolved and density of GPSBM's is greater. For the western states, the differences are based more on changes to the gravimetric geoids (USGG2003 to USGG2009) due to the sparser GPSBM coverage there. Major changes are Louisiana (changes to "orthometric" heights), California (changes to ellipsoidal heights), Texas (better accounting for orthometric heights), and the mountainous western states (changes in the gravimetric geoid).

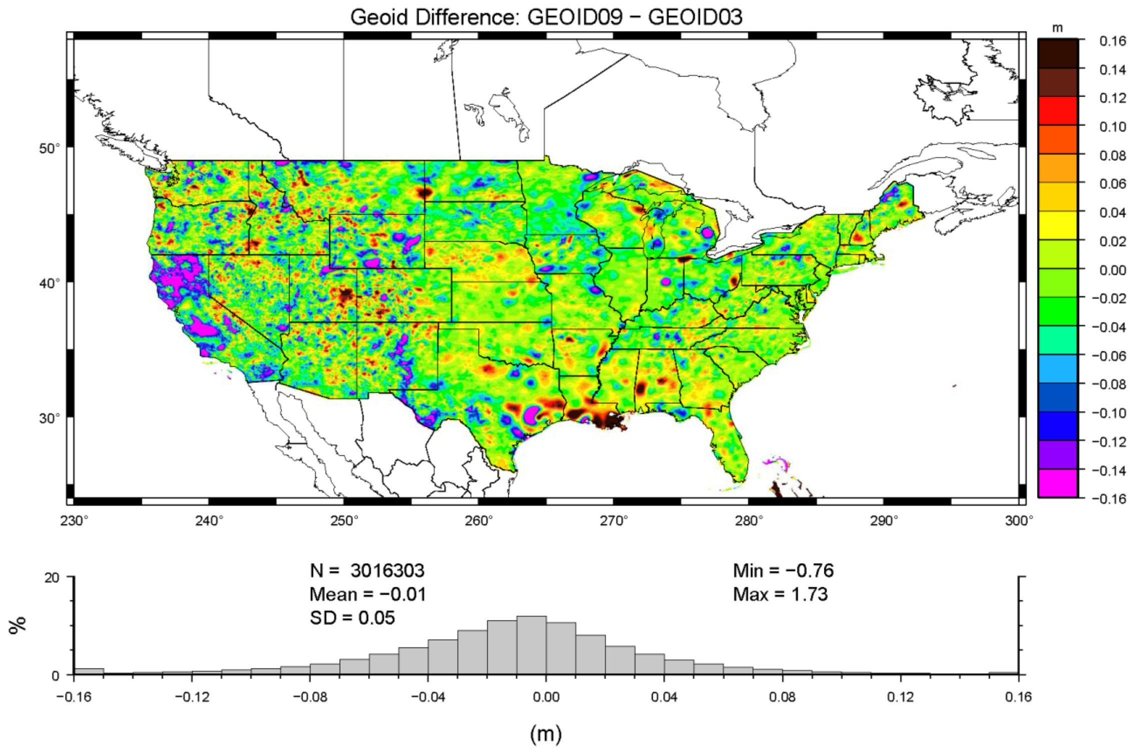

Figure 12: Differences between GEOID03 and GEOID09.

In Table 2, the GPSBM2009 data are broken out by state and compared to both USGG2009 and GEOID09. The state bias and standard deviation are given for both models. Note that you should multiply the standard deviation by 1.96 to get the 95% confidence value. For example, the national value for GEOID09 is 1.4 cm. Hence, 95% of the points will be within +/- 2.8 cm. Note also that there are significant residual values for USGG2009, because it represents the raw fit to the GPSBM's and demonstrates the magnitude of the systematic residuals (between a gravimetric geoid, NAD 83 ellipsoid heights and NAVD 88 orthometric heights) seen by state. GEOID09 fits better because it has already been fit to the GPSBM2009. Hence, use of GEOID09 is the correct geoid model to use when trying to match the NAD 83 and NAVD 88 heights given in the NGS database.

Table 2: Given are the statistics of the residual differences between GPSBM2009 and USGG2009 and GEOID09 by state. Listed for each state: the number of GPSBM's, average (bias), and standard deviation (SD) compared to USGG2009 and GEOID09. This table is for CONUS: all states plus DC but not HI or AK. CONUS value lists the national number.

ST ID	No. Pts.	USGG2009		GEOID09			ST ID	No. Pts.	USGG2009		GEOID09	
		Ave (m)	SD (m)	Ave (m)	SD (m)				Ave (m)	SD (m)	Ave (m)	SD (m)
AL	283	-0.206	0.050	0.000	0.011		NE	145	0.177	0.047	0.000	0.007
AZ	227	0.015	0.087	0.000	0.016		NV	70	0.247	0.089	0.001	0.012
AR	133	-0.116	0.034	0.001	0.018		NH	14	-0.141	0.018	-0.003	0.009

CA	738	0.234	0.132	0.000	0.022	NJ	326	-0.144	0.028	0.000	0.011
CO	562	0.106	0.083	0.000	0.025	NM	107	-0.103	0.091	0.000	0.015
CT	20	-0.142	0.035	0.000	0.015	NY	185	-0.104	0.064	0.000	0.011
DE	35	-0.179	0.046	0.001	0.012	NC	1676	-0.226	0.046	0.000	0.015
DC	16	-0.118	0.021	0.004	0.020	ND	47	0.412	0.033	0.001	0.007
FL	2181	-0.541	0.083	0.000	0.014	OH	297	0.022	0.047	0.000	0.022
GA	137	-0.265	0.064	0.000	0.014	OK	79	-0.089	0.057	0.000	0.008
ID	97	0.469	0.079	0.001	0.011	OR	202	0.523	0.081	0.000	0.015
IL	334	0.106	0.091	0.001	0.011	PA	96	-0.080	0.045	-0.001	0.013
IN	119	0.026	0.057	0.000	0.013	RI	29	-0.147	0.023	0.000	0.018
IA	100	0.189	0.060	-0.001	0.009	SC	1315	-0.221	0.057	0.000	0.012
KS	105	0.070	0.058	0.000	0.009	SD	242	0.285	0.062	0.000	0.008
KY	123	-0.086	0.038	-0.001	0.013	TN	302	-0.106	0.031	0.000	0.018
LA	217	-0.355	0.106	-0.001	0.012	TX	218	-0.257	0.085	0.000	0.012
ME	65	-0.144	0.043	0.000	0.011	UT	55	0.223	0.090	0.000	0.016
MD	511	-0.126	0.037	0.000	0.016	VT	317	-0.141	0.030	0.000	0.013
MA	35	-0.163	0.041	0.000	0.012	VA	434	-0.141	0.040	0.000	0.021
MI	410	0.087	0.043	0.000	0.015	WA	259	0.610	0.083	0.000	0.017
MN	4089	0.309	0.038	0.000	0.009	WV	55	-0.059	0.045	0.001	0.013
MS	243	-0.151	0.048	0.000	0.019	WI	758	0.172	0.036	0.000	0.007
MO	138	0.008	0.074	0.000	0.010	WY	101	0.270	0.089	-0.001	0.017
MT	151	0.469	0.091	0.000	0.009	CONUS	18398	-0.010	0.063	0.000	0.014

MMLSC for Outlying Regions

Similar approaches were followed for Guam, the Commonwealth of the Northern Marianas Islands (CNMI), American Samoa, and Alaska. Modeling for Puerto Rico and the U.S. Virgin Islands is on hold pending the formal adoption of the Virgin Islands Vertical Datum of 2009 (VIVD09). When that is ratified, a final GEOID09 model will be created for that region.

The GEOID09 model for Hawaii is in fact the same as the gravimetric geoid model except for being referenced to NAD 83 (PAC00) instead of ITRF 00. Hawaii has not adopted a vertical datum.

MMLSC for the outlying regions was much simpler due to the sparser data and irregular coverage. Empirical data were clustered close together (points on the same island) or at intermediate distances (points on different islands). There was no in between. Hence, the structure was much simpler. Typically, only two covariance matrices were stacked to develop the final models. Table 3 below lists the same comparisons given in Table 2 but instead for Guam/CNMI, American Samoa, Alaska, and (eventually) Puerto Rico and the U.S. Virgin Islands.

Table 3: Comparison of GPSBM2009 data to USGG2009 and GEOID09 for outlying U.S. regions. Same values as Table 2.

Territory /State	No. Pts.	USGG2009		GEOID09	
		Ave (m)	SD (m)	Ave (m)	SD (m)

Guam/CNMI	70	-1.046	0.071	0.000	0.006
-Guam	16	-1.061	0.046	0.000	0.004
-Saipan	10	-0.995	0.027	0.000	0.001
-Tinian	35	-1.091	0.020	0.000	0.008
-Rota	9	-0.900	0.025	0.001	0.002
American Samoa (Tutuila only)	22	0.538	0.053	-0.001	0.020
Alaska	176	1.270	0.243	0.000	0.006
PR/USVI	*****	*****	*****	*****	*****
-Puerto Rico	29	-0.330	0.021	*****	*****
-St. Thomas	*****	*****	*****	*****	*****
-St. John	*****	*****	*****	*****	*****
-St. Croix	*****	*****	*****	*****	*****

One final note, GEOID09 is intended solely for converting heights between NAD 83 and the official vertical datums of the United States (e.g., NAVD 88 in CONUS and Alaska, ASVD02 in American Samoa, etc) inside the respective region. Vertical datums have been established in many outlying regions, and the GEOID09 model for those regions will fit to those datum points. GEOID09 is not valid in Canada or Mexico or offshore to any great extent. While control data were available in Canada to make GEOID09, NAVD 88 is not the official datum of Canada and should not be used there. Do not use it for any other transformations. This is because MMLSC provides a valid interpolation inside the GPSBM2009 control data. Outside of these points, there is no control and the transformation has no real validity. Hence, it is only designed to be used to transform between NAD 83 ellipsoid heights and the official vertical datums in the land areas of CONUS, Alaska, Guam, Saipan, Tinian, Rota and Tutuila (and eventually Puerto Rico, St. Thomas, St. John and St. Croix). Other uses of GEOID09 are not endorsed by NGS.