



PUBLIC NOTICE

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DA 02-260
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AUCTION OF LICENSES IN THE 747-762 AND 777-792 MHz BANDS SCHEDULED FOR JUNE 19, 2002

COMMENT SOUGHT ON ADDING TO AUCTION INVENTORY AND FURTHER MODIFYING PACKAGE BIDDING PROCEDURES

Report No. AUC-02-31-A (Auction No. 31)

By this Public Notice, the Wireless Telecommunications Bureau (“Bureau”) seeks comment on the potential inclusion of 24 additional licenses in Auction No. 31 and potential modifications of previously-announced procedures for Auction No. 31. In the *Auction No. 31 Procedures Public Notice*,¹ the Bureau established procedures for the auction of the twelve licenses in the 747-762 and 777-792 MHz bands (the “Upper 700 MHz” bands). The Bureau later revised its procedures to include package bidding in *Auction No. 31 Package Bidding Procedures Public Notice*,² and subsequently modified those procedures in the *Auction No. 31 Package Bidding Procedures Modification Public Notice*.³ In light of subsequent developments and further analysis, we have determined that it may be appropriate to make further refinements. Accordingly, this Public Notice seeks comment on whether to:

1. include 24 licenses from the 698-746 MHz band (the “Lower 700 MHz” band) in Auction No. 31;
2. increase the number of bidder-defined packages if the Lower 700 MHz band licenses are included in Auction No. 31;

¹ See Auction of Licenses in the 747-762 and 777-792 MHz Bands; Auction Notice and Filing Requirements for 12 Licenses in the 700 MHz Bands Auction Scheduled for May 10, 2000; Minimum Opening Bids and Other Auction Procedural Issues, DA 00-292, *Public Notice*, 15 FCC Rcd 2921 (2000) (“*Auction No. 31 Procedures Public Notice*”).

² See Auction of Licenses in the 747-762 and 777-792 MHz Bands Scheduled for September 6, 2000, Procedures Implementing Package Bidding for Auction No. 31; Bidder Seminar Scheduled for July 24, 2000, DA 00-1486, *Public Notice*, 15 FCC Rcd 11,526 (2000) (“*Auction No. 31 Package Bidding Procedures Public Notice*”).

³ See Auction of Licenses in the 747-762 and 777-792 MHz Bands Scheduled for March 6, 2001; Modifications to the Calculation for Determining Minimum Acceptable Bids and the Provisions Concerning “Last and Best Bids” and Other Procedural Issues; Bidder Seminar Scheduled for January 23, 2001, and Test of Package Bidding System Set for January 24, 25, and 26, 2001, DA 01-12, *Public Notice*, 16 FCC Rcd 217 (2001) (“*Auction No. 31 Package Bidding Procedures Modification Public Notice*”).

3. change part (iii) of the minimum acceptable bid calculation by either substituting a new method of calculating minimum acceptable bids (Option 1) or modifying the existing formula for calculating minimum acceptable bids for new packages (Option 2) and, if Option 1 is adopted, consider all bids submitted when determining provisionally winning bid sets;⁴ and
4. establish an aggregate reserve price or increase the previously-established minimum opening bids.

I. WHETHER TO INCLUDE 24 LICENSES FROM THE LOWER 700 MHZ BAND IN AUCTION NO. 31

The Bureau seeks comment on whether we should include 24 licenses from the Lower 700 MHz band in Auction No. 31. These 24 licenses consist of licenses for four blocks of spectrum in each of the six regions known as the 700 MHz band economic area groupings (“700 MHz band EAGs”). In each 700 MHz band EAG, four Lower 700 MHz band licenses are for the use of two 12 megahertz blocks (each block consists of paired 6 megahertz segments) and two 6 megahertz blocks that are unpaired and contiguous.⁵

Including these 24 Lower 700 MHz band licenses in Auction No. 31 would enable bidders to create and bid on packages containing 700 MHz band EAG licenses from both the Upper and Lower 700 MHz bands. Commenters should address any potential advantages or disadvantages of permitting package bidding on such combinations and facilitating the substitution of spectrum in the Upper and Lower 700 MHz bands. Interested parties are advised that the Bureau also seeks comment regarding this issue in a separate Public Notice addressing procedures for the auction of licenses in the Lower 700 MHz bands (Auction No. 44).⁶

If these 24 additional licenses are included in Auction No. 31, operational considerations would require the Bureau also to adopt the more comprehensive revision to the minimum acceptable bid calculation described as Option 1 in Section III(B) below.

II. WHETHER TO INCREASE THE NUMBER OF BIDDER-DEFINED PACKAGES

The Bureau seeks comment on whether to increase the number of bidder-defined packages if the Bureau includes 24 licenses from the Lower 700 MHz band in Auction No. 31. Taking into account various factors, including the available licenses, the Bureau previously announced that Auction No. 31 bidders would be permitted to create and bid on up to twelve different packages of their own choosing during Auction No. 31.⁷ Commenters are asked to address whether the inclusion of additional licenses in Auction No. 31 would create any need to increase the number of bidder-defined packages, considering

⁴ The *provisionally winning bid set* at the end of each round consists of those non-mutually exclusive considered bids that maximize revenue while assigning each license only once.

⁵ See Auction of Licenses in the 698-746 MHz Band Scheduled for June 19, 2002, Comment Sought on Reserve Prices or Minimum Opening Bids and Other Auction Procedural Issues, *Public Notice*, DA 02-200 (rel. Jan. 24, 2002) (“*Auction No. 44 Procedures Public Notice*”) at 1-2.

⁶ See *Auction No. 44 Procedures Public Notice* at 2.

⁷ *Auction No. 31 Package Bidding Procedures Public Notice*, 15 FCC Rcd at 11,532.

that bidder-defined packages may contain any number of licenses, up to all the licenses available in the auction.

III. WHETHER TO CHANGE THE MINIMUM ACCEPTABLE BID CALCULATION

A. Summary of Existing Procedures

The Bureau established the current three-part formula for calculating minimum acceptable bids in the *Auction No. 31 Package Bidding Procedures Modification Public Notice*.⁸ Pursuant to the existing procedure, the minimum acceptable bid for any particular license or package would be the greatest of: (i) the applicable minimum opening bid; (ii) the bidder's previous high bid on that license/package plus x%, where the Bureau would specify the value of x in each round; and (iii) the bidder's previous high bid on that particular license/package plus an amount based on the increase in the bidder's previous high bid needed to create a tie with the provisional winners.⁹ If the bidder has not bid on a license or an already constructed package, the bidder's previous high bid for purposes of calculating part (iii) would be the applicable minimum opening bid.

With respect to part (iii), the Bureau established that it would determine the amount to add to a given previous high bid in three steps. First, the Bureau would calculate a shortfall for the given bid, with the shortfall being the difference between the revenue of the provisionally winning bid set and the maximum total revenue associated with a set of bids that includes the given bid and satisfies all other constraints for determining a provisionally winning set. Second, the Bureau would determine a deficit for the given bid, or its share of the shortfall produced by the set, in two stages. Initially, the Bureau would determine the ratio of bidding units in the given bid's particular license/package to bidding units for all the non-provisionally winning bids in the set of bids that produced the shortfall. (If more than one set of bids yields the same shortfall for a given bid, the Bureau would use the set that includes the most provisionally winning bidding units.) The Bureau then would multiply the given bid's shortfall by this ratio to arrive at the deficit for the given bid. Third, and finally, the Bureau would determine the amount to be added to the given previous high bid by multiplying the given bid's deficit by y%. At the beginning of the auction, the Bureau would set y at 100 but would retain the discretion to adjust the amount during the course of the auction.

However, part (iii) cannot be used to calculate the minimum acceptable bid for a package in the same round that the particular package is created because no shortfall or deficit was calculated with respect to that particular package at the end of the prior round. For such a new package, the Bureau established the following procedures. Generally, the third part of the initial minimum acceptable bid formula would be calculated for new packages by multiplying the number of bidding units in the package by the lowest \$/bidding unit of any provisionally winning bid in the last five rounds. However, in the event the new package is the global package of all licenses available in the auction, the minimum acceptable bid would be the revenue generated by the provisionally winning bid set in the previous round plus w%.¹⁰ This assures that no bid for the global package would be accepted that does not exceed the maximum revenue in the preceding round.

⁸ See *Auction No. 31 Package Bidding Procedures Modification Public Notice*, 16 FCC Rcd at 219-221.

⁹ *Id.*

¹⁰ *Id.*, at 211.

Pursuant to current Auction No. 31 procedures, provisionally winning bids would be determined by considering: (1) for each bidder, bids from the most recent two rounds in which the bidder submitted bids or renewed bids; and (2) all provisionally winning bids from the prior round; and (3) a set of FCC bids on each license at some small amount less than the applicable minimum opening bid.¹¹ Furthermore, a bidder's bids submitted in a given round would be treated as mutually exclusive of the bidder's bids submitted in other rounds.¹²

B. Proposed Procedures -- Option 1

Option 1 involves two significant changes to current procedures, the first being a comprehensive revision in calculating part (iii) of the minimum acceptable bid formula and the second regarding the bids considered when determining provisionally winning bids. These changes would be made jointly or not at all. Adopting the first proposed change would enable the Bureau to include additional licenses in Auction No. 31, as discussed in Section I, above; could improve the pace of the auction; and would make it feasible to consider all bids submitted during the auction when determining provisionally winning bids. Considering all bids submitted during the auction when determining provisionally winning bids would make bidders responsible for all bids throughout the auction and may discourage strategic bidding. On the other hand, considering all bids submitted might make it more complex for participants and other observers to track the auction.

We seek comment on both aspects of the proposal, as described below.

1. Minimum Acceptable Bids

As the first part of Option 1, the Bureau proposes to replace part (iii) of the minimum acceptable bid formula with a current price estimate of the license or package plus $z\%$. A current price estimate for each license would be calculated at the close of each round using the Smoothed Anchoring Method (see Attachment A). The current price estimate for a package would be the sum of the current price estimates of the licenses that comprise the package. Pursuant to this proposal, the Bureau would set z to be zero at the beginning of the auction but would retain the discretion to adjust the amount, including setting it at less than zero, during the course of the auction in order to provide control over the pace of the auction.

Implementing current price estimates for licenses and packages would preclude the need for a special version of part (iii) of the minimum acceptable bid formula for newly-created packages. However, the Bureau would retain the exception for calculating the minimum acceptable bid for a newly-created global package. After the first round of the auction, the minimum acceptable bid for the global package would always be the revenue generated by the provisionally winning bid set in the previous round plus $w\%$. The Bureau would make this distinction in order to retain the ability to ensure that bids for the global package would continue to increase even if we were to employ a percentage z that would not guarantee that outcome. That is, we might want to set w higher than z .

2. Bids Considered

As the second part of Option 1, the Bureau would consider all bids submitted during the auction when determining the provisionally winning set of bids. It is impractical to consider every bid submitted

¹¹ See *Auction No. 31 Package Bidding Procedures Modification Public Notice*, 16 FCC Rcd at 224, 231-32.

¹² See *id.* at 231.

during the auction when calculating shortfalls and deficits in each round pursuant to the existing minimum acceptable bid formula. In the event that the Bureau calculates minimum acceptable bids using current price estimates, as described above, it would become feasible to consider every bid submitted when calculating minimum acceptable bids. As in the previously-established procedures, bids placed and/or renewed in different rounds would be considered as mutually exclusive – a bidder could be a provisional winner only on bids placed in the same round.

C. Proposed Procedures -- Option 2

Option 2 involves a more modest change to current procedures, limited to revising the third part of the minimum acceptable bid formula applicable to newly-created packages. Pursuant to this alternative proposal, for newly-created packages, the third part of the minimum acceptable bid formula would be calculated by multiplying the number of bidding units in the package by the *average* dollar per bidding unit of all provisionally winning bids in the past five rounds. The average dollar per bidding unit would be calculated by dividing the sum of gross revenues from the provisionally winning bids sets¹³ over the last five rounds by five times the sum of all of bidding units in the auction. This exception would not apply to bids for the global package, however, for which the minimum acceptable bid would be the maximum revenue from the previous round plus w%.

This alternative proposed revision to the calculation of minimum acceptable bids could improve the pace of the auction. We seek comment on this proposal.

IV. WHETHER TO ESTABLISH AN AGGREGATE RESERVE PRICE OR INCREASE PREVIOUSLY-ESTABLISHED MINIMUM OPENING BIDS

The Communications Act calls upon the Commission to prescribe methods by which a reasonable reserve price will be required, or a minimum opening bid will be established, when the Commission auctions licenses, unless the Commission determines that a reserve price or a minimum opening bid is not in the public interest.¹⁴ Consistent with this mandate, the Commission has directed the Bureau to seek comment on the use of a minimum opening bid and/or reserve price and on the methodology to be employed in establishing each of these mechanisms prior to the start of each auction.¹⁵ After seeking comment on relevant issues, the Bureau established minimum opening bids for Auction No. 31.¹⁶ The

¹³ The provisionally winning bid set would include any license for which no bid has been submitted, with any such license treated as though a bid had been submitted in the amount of the minimum opening bid. *See Auction No. 31 Package Bidding Procedures Public Notice*, 15 FCC Rcd at 11,550-51. Accordingly, the divisor for determining the average dollar per bidding unit includes all the bidding units in the auction, as described above.

¹⁴ *See* 47 U.S.C. §309(j)(4)(F). The Commission's authority to establish a reserve price or minimum opening bid is set forth in 47 C.F.R. § 1.2104(c) and (d).

¹⁵ *See* Amendment of Part 1 of the Commission's Rules – Competitive Bidding Procedures; Allocation of Spectrum Below 5 GHz Transferred from Federal Government Use, 4660–4685 MHz, WT Docket No. 97-82, ET Docket No. 94-32, *Third Report and Order and Second Further Notice of Proposed Rule Making*, 13 FCC Rcd 374, 455-456 ¶141 (1997) (“*Part 1 Third Report and Order*”).

¹⁶ A minimum opening bid is the minimum bid price set at the beginning of the auction below which no bids are accepted. It is generally used to accelerate the competitive bidding process. Also, in a minimum opening bid scenario, the auctioneer generally has the discretion to lower the minimum opening bid later in the auction. *See Auction No. 31 Package Bidding Procedures Public Notice*, 15 FCC Rcd at 11,543 and 11,561.

Bureau further established that the minimum opening bids for packages would be the sum of the minimum opening bids for the licenses comprising the package.¹⁷

For Auction No. 31, to supplement the established minimum opening bids, the Bureau seeks comment on establishing an aggregate reserve price that exceeds the sum of the minimum opening bids. A reserve price is an absolute minimum price below which specified goods will not be sold. Bidders, however, may place bids below reserve prices. Furthermore, reserve prices can be either public or undisclosed. The spectrum in Auction No. 31 is subject to a unique statutory clearance process, which the Commission has created incentives to accelerate.¹⁸ In light of these complexities, using an aggregate reserve price in addition to minimum opening bids may be appropriate to ensure “a recovery for the public of a portion of the value of the public spectrum resource”¹⁹ commensurate with Congressional expectations.²⁰

With respect to the appropriate level of an aggregate reserve price, Congress has estimated that the Upper 700 MHz band licenses available for auction will generate \$2.6 billion in revenue.²¹ The Commission’s previous 700 MHz guard bands auctions raised nearly \$541 million.²² Therefore, we seek comment on the possibility of an aggregate reserve in Auction No. 31 equal to \$2.6 billion in gross bids. This gross bid figure should result in net revenues approximating the Congressional revenue estimate minus net revenues from the 700 MHz guard bands auctions.²³ Under this approach, the Commission only would accept the provisionally winning set of bids at the close of Auction No. 31 if the gross amount of the provisionally winning set of bids equals or exceeds the \$2.6 billion aggregate reserve price.²⁴

Commenters are asked to address how the public interest would or would not be served by adopting an aggregate reserve price. In particular, given all the circumstances of this auction, we seek comment on whether the public interest would be served by establishing an aggregate reserve price to ensure that the Commission recovers for the public a portion of the value of the public spectrum resource

¹⁷ See *id.*, at 11,543.

¹⁸ See Service Rules for the 746-764 and 776-794 MHz Bands, and Revisions to Part 27 of the Commission’s Rules; Carriage of the Transmissions of Digital Television Broadcast Stations; Review of the Commission’s Rules and Policies Affecting the Conversion to Digital Television, WT Docket No. 99-168, CS Docket No. 98-120, MM Docket No. 00-39, *Order on Reconsideration of the Third Report and Order*, 16 FCC Rcd 21,633 (2001).

¹⁹ 47 U.S.C. §309(j)(3)(C)(one Commission statutory objective when proceeding to auction is “recovery for the public of a portion of the value of the public spectrum resource”).

²⁰ See, generally, *Part 1 Third Report and Order*, 13 FCC Rcd at 455 (regarding the use of reserve prices in Commission auctions).

²¹ See 145 Cong. Rec. H9907-04, H9919 (daily ed. October 13, 1999).

²² See 700 MHz Guard Bands Auction Closes, DA 00-2154, *Public Notice*, 15 FCC Rcd 18,026 (2000); 700 MHz Guard Bands Auction Closes, DA 01-478, *Public Notice*, 16 FCC Rcd 4590 (2001).

²³ This gross bid figure should approximate the amount necessary to result in \$2.6 billion in net bids when added to the net results from the 700 MHz guard bands auctions. Unavoidably, there are uncertainties in estimating the gross bids required to meet Congressional expectations regarding net revenues.

²⁴ Pursuant to previously established procedures, the winning set of bids would value any FCC-held licenses at the close of the auction at the minimum opening bid for those licenses. See *Auction No. 31 Package Bidding Procedures Public Notice*, 15 FCC Rcd at 11,550-51.

commensurate with Congressional expectations. Additionally, we seek comment on whether an aggregate reserve price would introduce an added threshold problem²⁵ with which bidders would have to contend.

Commenters also should address whether any aggregate reserve price should be made public or remain undisclosed.²⁶ Commenters supporting disclosure should address when and how any aggregate reserve price would be made public. For example, should it be disclosed when the Bureau announces the qualified bidders for the auction? Or when the auction begins? In addition, commenters should address whether the Bureau should make periodic announcements during the auction regarding whether bids meet the aggregate reserve price or postpone any announcements until after the auction closes. Commenters supporting an undisclosed aggregate reserve price should comment on whether the Bureau should announce whether bids meet any aggregate reserve at any time before the auction closes. For example, should the Bureau announce whether bids meet any aggregate reserve price if the Bureau keeps the auction open by exercising its discretion under the stopping rule?²⁷

Commenters addressing the level of the proposed aggregate reserve price should support their claims with valuation analyses and suggested reserve prices or formulas.²⁸ Any commenters suggesting license-by-license reserve prices, in place of the proposed aggregate reserve price, should explain with specificity how reserve prices for individual licenses would be established. We particularly seek comment on such factors as, among other things, the amount of spectrum being auctioned, levels and circumstances of incumbency, the availability of technology to provide service, the size of the geographic service areas, issues of interference with other spectrum bands and any other relevant factors that reasonably could have an impact on valuation of the 747–762 and 777–792 MHz bands.

As an alternative, the Bureau also seeks comment on increasing the previously-established minimum opening bids in Auction No. 31. The previously-established minimum opening bids total \$720 million.²⁹ If commenters believe that the public interest would be served by increasing the current minimum opening bids, they should support their claims with valuation analyses and suggested minimum opening bid levels or formulas. With respect to the amounts of the minimum opening bids, we particularly seek comment on such factors as, among other things, the amount of spectrum being auctioned, levels of incumbency, the availability of technology to provide service, the size of the geographic service areas, issues of interference with other spectrum bands and any other relevant factors that reasonably could have an impact on valuation of the 747–762 and 777–792 MHz bands.

²⁵ See *Auction No. 31 Package Bidding Procedures Public Notice*, 15 FCC Rcd at 11,529-30 (defining and discussing the threshold problem for purposes of Auction No. 31).

²⁶ Any commenters suggesting license-by-license reserve prices, in place of the proposed aggregate reserve price, should address the issue of disclosure as well.

²⁷ See *Auction No. 31 Procedures Public Notice*, 15 FCC Rcd at 2946-47 (discussing stopping rule for Auction No. 31); 47 C.F.R. §1.2104(e).

²⁸ In addressing these issues, commenters may also wish to address the possibility that 24 Lower 700 MHz bands licenses may be added to the inventory of Auction No. 31.

²⁹ See *Auction No. 31 Procedures Public Notice* at 11,561.

CONCLUSION

Comments are due on or before February 19, 2002, and reply comments are due on or before February 26, 2002. Because of the disruption of regular mail and other deliveries in Washington, DC, the Bureau requires that all comments and reply comment be filed electronically. Comments and reply comments must be sent by electronic mail to the following address: auction31@fcc.gov.³⁰ The electronic mail containing the comments or reply comments must include a subject or caption referring to Auction No. 31 Comments. The Bureau requests that parties format any attachments to electronic mail as Adobe[®] Acrobat[®] (pdf) or Microsoft[®] Word documents. Copies of comments and reply comments will be available for public inspection during regular business hours in the FCC Public Reference Room, Room CY-A257, 445 12th Street, SW, Washington, DC 20554.

In addition, the Bureau requests that commenters fax a courtesy copy of their comments and reply comments to the attention of Kathryn Garland at (717) 338-2850.

This proceeding has been designated as a “permit-but-disclose” proceeding in accordance with the Commission’s *ex parte* rules.³¹ Persons making oral *ex parte* presentations are reminded that memoranda summarizing the presentations must contain summaries of the substance of the presentations and not merely a listing of the subjects discussed. More than a one- or two-sentence description of the views and arguments presented is generally required.³² Other rules pertaining to oral and written *ex parte* presentations in permit-but-disclose proceedings are set forth in Section 1.1206(b) of the Commission’s rules.³³

For further information concerning this proceeding, contact the following employees of Auctions and Industry Analysis Division, Wireless Telecommunications Bureau, at (202) 418-0660:

For legal questions: Howard Davenport

For general auction questions: Craig Bomberger or Martha Stancill

For software questions: Karen Wrege

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³⁰ Comments and reply comments in response to this Public Notice may not be filed using the Commission’s Electronic Comment Filing System, which is used for the Commission’s docketed rule making proceedings.

³¹ See 47 C.F.R. §§ 1.1200(a), 1.1206.

³² See 47 C.F.R. § 1.1206(b).

³³ See 47 C.F.R. § 1.1206(b).

ATTACHMENT A

Using the Smoothed Anchoring Method to Obtain Current Price Estimates

This appendix describes the proposed method for estimating a current price associated with each license at the close of every round. If the Bureau decides to implement the comprehensive revision of the procedures described as Option 1 in “Whether To Change The Minimum Acceptable Bid Calculation,” these “current price estimates,” as they will be called, will be used in the next round when calculating part *iii* of the minimum acceptable bid formula. Specifically, for a license, this value will be the current price estimate of the license plus $z\%$. For a package, the value will be the sum of the current price estimates of the licenses that make up the package plus $z\%$ of the sum.

The current price estimates of the licenses are based on the concept that every linear optimization problem has a dual problem that provides pricing information. We begin by presenting the FCC winner determination problem as an integer program, (P1), and then discuss its linear programming representation before displaying the dual problem of interest.

$$(P1): \quad \begin{aligned} & \max \sum_{j \in B^t} b_j x_j \\ & s.t. \quad \sum_{j \in B^t} a_{ij} x_j = 1, \quad \text{for all } i \in L \quad (1) \\ & \quad \quad x_j \in \{0,1\}, \quad \text{for all } j \in B^t \end{aligned}$$

where B^t is the set of considered bids in round t ,

b_j is the bid amount of bid j ,

L is the set of licenses being auctioned,

$$a_{ij} = \begin{cases} 1, & \text{if license } i \text{ is in bid } j \\ 0, & \text{otherwise} \end{cases} \text{ and,}$$

$$x_j = \begin{cases} 1, & \text{if bid } j \text{ is in the winning set} \\ 0, & \text{otherwise} \end{cases}$$

In this formulation, x_j is an indicator variable that equals one if bid j is in the provisionally winning set and zero otherwise. Thus, the sum of the bid amounts of all provisionally winning bids produces the maximum obtainable revenue for round t . Constraints (1) ensure that each license is awarded exactly once. The constraints that ensure that a bidder’s bids between rounds are mutually exclusive are not represented in (P1) since they will be ignored in the linear representation of the problem.¹

¹ These constraints will be ignored in the linear program representation since they are rarely binding in the relaxation of the integer-programming problem and because adding such constraints to the dual problem creates “degeneracy” in the solution thereby causing multiple alternative solutions.

The linear program of (P1) relaxes the restriction on the variables x_j , for all $j \in B^t$, allowing these variables to take on any value between zero and one. The linear programming representation of (P1) is shown in (P2):

$$(P2): \quad \begin{aligned} & \max \sum_{j \in B^t} b_j x_j \\ & \text{s.t.} \quad \sum_{j \in B^t} a_{ij} x_j = 1, \quad \text{for all } i \in L \\ & \quad \quad x_j \geq 0, \quad \text{for all } j \in B^t \end{aligned}$$

The dual formulation of (P2) can be used to identify a price, π_i , for each license i , and is shown in the following linear program (P3):

$$(P3): \quad \begin{aligned} & \min \sum_{i \in L} \pi_i \\ & \text{s.t.} \quad \sum_{i \in L} a_{ji} \pi_i \geq b_j, \quad \text{for all } j \in B^t \setminus F \\ & \quad \quad \pi_i \geq b_j, \quad \text{for all } j \in F \end{aligned} \quad (2)$$

and i is the license index associated with bid j (3)

where $F \subset B^t$ is the set of FCC bids on each license² and,

$$a_{ji} = \begin{cases} 1, & \text{if bid } j \text{ contains license } i \\ 0, & \text{otherwise} \end{cases}.$$

The optimal value of each variable, π_i , in (P3) corresponds to a dual price³ – often called a “shadow price” – for each constraint, i.e., each license, in (P2). The dual price of each license measures the monetary cost of not awarding the license to whom it has been provisionally assigned under the solution to (P2). Thus, this monetary cost has a clear and natural use in estimating the current price of a license given the bids considered in the current round.

Constraints (2) in (P3) ensure that the dual price of a license must be at least as large as the greatest bid made on that license. For a package, these constraints ensure that the sum of the dual prices of the licenses that make up a particular package must be at least as large as the greatest bid made on that package. Constraints (3) in (P3) ensure that

² The bid amount for a FCC bid is some small amount less than the minimum-opening bid for that license. See Auction Of Licenses in the 747-762 and 777-792 MHz Bands Scheduled for March 6, 2001; Modifications to the Calculation for Determining Minimum Acceptable Bids and the Provisions Concerning “Last and Best Bids” and Other Procedural Issues, DA 01-12, *Public Notice*, 16 FCC Rcd 217 (2001)(Section III discusses the reasons for this approach).

³ We note that for non-linear problems, these dual prices are also known as *Lagrange multipliers*.

if a license has not been bid on, the dual price of that license is at least as large as the FCC bid amount.

Ideally, the solution to (P2) is identical to the solution of (P1). When this occurs, the sum of the dual prices of the licenses comprising any provisionally winning bid equals the winning bid amount. However, (P2) is only an approximation to the integer problem⁴ and often *overestimates* the maximum revenue of (P1). When this occurs, the sum of the dual prices of the licenses in at least one provisionally winning bid will be greater than the respective bid amount. Thus, using the dual prices of (P3) can result in minimum acceptable bid amounts that are too high.

We propose to resolve this issue by using *pseudo-dual prices*,⁵ rather than the dual prices of (P3). These pseudo-dual prices are obtained by forcing the sum of the dual prices of the licenses comprising a provisionally winning bid to equal its respective bid amount. For example, suppose there are two bids in the provisionally winning set in round t : a bid on license A for \$10 and a bid on package BC for \$25. The pseudo-dual price of A would exactly equal \$10 and the sum of the pseudo-dual prices of B and C would exactly equal \$25. These restrictions ensure that the sum of the pseudo-dual prices equals the maximum revenue for the round (e.g. \$35) and that minimum acceptable bid amounts reflect the bid amounts of bids in the provisionally winning set.

Pseudo-dual prices for each license i , denoted π_i , satisfy the following constraints:

$$\sum_{i \in L} a_{ji} \pi_i + \delta_j \geq b_j, \text{ for all } j \in B^t \setminus (W^t \cup F) \quad (4)$$

$$\sum_{i \in L} a_{ji} \pi_i = b_j, \quad \text{for all } j \in W^t \quad (5)$$

$$\pi_i \geq b_j, \quad \text{for all } j \in F \setminus (W^t \cap F) \quad (6)$$

and i is the license index associated with bid j

$$\delta_j \geq 0, \quad \text{for all } j \in B^t \setminus (W^t \cup F) \quad (7)$$

where $W^t \subset B^t$ is the provisionally winning bid set in round t and,

δ_j is a slack variable that represents the difference between the bid amount of non-winning bid j and the sum of pseudo-dual prices of the licenses contained in non-winning bid j

⁴ When the problem is a convex optimization problem, the primal and dual problems yield the same objective function values. This is called strong-duality. These conditions do not hold for integer programming problems, often resulting in a gap between the linear programming and integer programming solution values.

⁵ In our research we found this term first applied to auction pricing in the paper by Rassenti, Smith and Bulfin (1982), "A combinatorial auction mechanism for airport slot allocation," *Bell Journal of Economics*, vol. 13, pp. 402-417.

Constraints (5) ensure that for each provisionally winning bid, the sum of the dual prices of the licenses comprising that bid equal its respective bid amount. This new restriction requires that we ease restriction (2) in (P3) for non-winning bids in order to ensure that a feasible solution exists. Constraints (4) provide this needed slack. Constraints (6) are equivalent to constraints (3) in (P3) and constraints (7) force the slack variables to be non-negative.

Satisfying constraints (5) implies that the sum of the pseudo-dual prices always yields the maximum revenue for the round. There are likely to be many sets of pseudo-dual prices that satisfy this constraint set. For instance, in the example provided earlier, the pseudo-dual prices of B and C might be any two numbers that together sum to \$25.

By keeping constraints (4)-(7), we have the flexibility to choose an objective function that will help in selecting among multiple solutions while still ensuring that the sum of the pseudo-dual prices yields the maximum revenue of the round. We would like an objective function that minimizes the values of the slack variables δ_j , for all $j \in B^t \setminus (W^t \cup F)$ in order to obtain pseudo-dual prices that are close to the dual prices of (P3). We have tested a number of alternative objective functions:

1. Minimization of the maximum δ_j for all $j \in B^t \setminus (W^t \cup F)$ followed by maximization of the minimum π_i for all i in license set L , in an iterative manner. (DeMartini, Kwasnica, Ledyard and Porter, 1999)
2. Minimization of the sum of the squares of δ_j for all $j \in B^t \setminus (W^t \cup F)$. (also DeMartini, Kwasnica, Ledyard and Porter, 1999)
3. Minimization of the sum of the δ_j for all $j \in B^t \setminus (W^t \cup F)$ using a “centering” algorithm⁶ to solve, essentially finding an average among all sets of optimal pseudo-dual prices.

In testing the above alternatives, we frequently observed instances where the pseudo-dual price of a license significantly changed from round to round. We acknowledge that prices of licenses should be allowed to reflect real changes, both increases and decreases, in the way bidders value the licenses over time. However, we believe that large oscillations in minimum acceptable bid amounts for the same bid that are due to irrelevant factors such as multiple optimal solutions, can be confusing to bidders. We have therefore chosen a method that attempts to balance minimizing the slack variables and reducing the fluctuations in pseudo-dual prices from round to round. This method requires solving two optimization problems, the first of which is alternative 3 above, which we present as (P4):

⁶ The centering algorithm used in this testing was the barrier method available in CPLEX, a commercial optimization package.

$$\begin{aligned}
\Omega^* &= \min \sum_{j \in B^t \setminus (W^t \cup F)} \delta_j \\
s.t. \quad & \sum_{i \in L} a_{ji} \pi_i + \delta_j \geq b_j, \text{ for all } j \in B^t \setminus (W^t \cup F) \\
(P4): \quad & \sum_{i \in L} a_{ji} \pi_i = b_j, \quad \text{for all } j \in W^t \\
& \pi_i \geq b_j, \quad \text{for all } j \in F \setminus (W^t \cap F) \\
& \quad \quad \quad \text{and } i \text{ is the license index associated with bid } j \\
& \delta_j \geq 0, \quad \text{for all } j \in B^t \setminus (W^t \cup F)
\end{aligned}$$

Since multiple optimal solutions can exist to (P4) we solve a second optimization problem that chooses a solution in a way that reduces the magnitude of price fluctuations between rounds. Specifically, we use an objective function that applies the concepts of exponential smoothing⁷ to choose among alternative pseudo-dual prices with the additional constraint on the problem that the sum of the slack variables equals Ω^* (the optimal value of (P4)). This objective function minimizes the sum of the squared distances of the resulting pseudo-dual prices in round t from their respective smoothed prices in round $t-1$.⁸ At the start of the auction, we use the minimum opening bid prices as the prior smoothed prices. Since these opening prices are based on bandwidth and population, the pricing algorithm begins with *a priori* information about the differences among licenses.

Let π_i^t be the pseudo-dual price of license i in round t . The smoothed price for license i in round t is calculated using the following exponential smoothing formula:

$$p_i^t = \alpha \pi_i^t + (1 - \alpha) p_i^{t-1}$$

where p_i^{t-1} is the smoothed price in round $t-1$,

$$0 \leq \alpha \leq 1, \text{ and}$$

$$p_i^0 = \text{the minimum opening bid amount for license } i.$$

Consistent with prior practice of the Commission, a weighting factor of $\alpha = 0.5$ has been chosen but can change, as the Commission requires.

⁷ Exponential smoothing often is used in determining minimum acceptable bids in FCC auctions. See, e.g., Auction of Licenses in the 747-762 and 777-792 MHz Bands; Auction Notice and Filing Requirements for 12 Licenses in the 700 MHz Bands Auction Scheduled for May 10, 2000; Minimum Opening Bids and Other Procedural Issues, DA 00-292, *Public Notice*, 15 FCC Rcd 2921, Attachment G (2000).

⁸ This objective function is a convex, quadratic function. This quadratic optimization problem is solved using the barrier method.

The following quadratic program (QP) will find the pseudo-dual price, π_i^t , for each license i in round t that minimizes the sum of the squared distances from the respective smoothed price in round $t-1$ while ensuring that the pseudo-dual prices sum up to the provisionally winning bid amounts and that the sum of the slack variables is minimized.

$$\begin{aligned}
 \text{(QP):} \quad & \min \sum_{i \in L} (\pi_i^t - p_i^{t-1})^2 \\
 & \text{s.t.} \quad \sum_{i \in L} a_{ji} \pi_i^t + \delta_j \geq b_j, \quad \text{for all } j \in B^t \setminus (W^t \cup F) \\
 & \quad \quad \sum_{i \in L} a_{ji} \pi_i^t = b_j, \quad \text{for all } j \in W^t \\
 & \quad \quad \sum_{j \in B^t \setminus (W^t \cup F)} \delta_j = \Omega^* \\
 & \quad \quad \pi_i^t \geq b_j, \quad \text{for all } j \in F \setminus (W^t \cap F) \\
 & \quad \quad \quad \quad \quad \quad \quad \quad \text{and } i \text{ is the license index associated with bid } j \\
 & \quad \quad \delta_j \geq 0, \quad \text{for all } j \in B^t \setminus (W^t \cup F)
 \end{aligned}$$

The objective function of (QP) minimizes the difference between the current round's pseudo-dual price and the previous round's smoothed price, where p_i^{t-1} is known and treated as a constant within the optimization.⁹

Among alternative prices that satisfy all constraints, the objective function of this optimization problem chooses one that forces the pseudo-dual prices to be as close as possible to the previous round's smoothed price. Thus, we call this the *Smoothed Anchoring Method* since we “anchor” on the smoothed prices when solving for the pseudo-dual prices. We define the “current price estimate” for license i in round t as the pseudo-dual price, π_i^t , obtained by solving (QP).

If the Bureau decides to implement the comprehensive revision of the procedures described as Option 1 in “Whether To Change The Minimum Acceptable Bid Calculation,” the minimum acceptable bid amount for a license in round $t+1$ under part *iii* will be the current price estimate of the license plus $z\%$. For a package, the value will be the sum of the current price estimates of the licenses that make up the package plus $z\%$ of the sum.

⁹ Once the pseudo-dual prices, π_i^t , have been determined, the smoothed prices, p_i^t , can be calculated and used for solving (QP) in round $t+1$.