US Army Corps
of Engineers
Water Resources Support Center
Institute for Water Resources

AD-A197 011

Multiobjective Risk Partitioning: An Application to Dam Safety Risk Analysis

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Risk Analysis Research Program

April 1988

IWR Report 88-R--4

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved
OMB No 0704-0188
Exp Date Jun 30, 1986

1a REPORT SECURITY CLASSIFICATION: UNCLASSIFIED	1b RESTRICTIVE MARKINGS
2a SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT: UNLIMITED
2b DECLASSIFICATION/DOWNGRADING SCHEDULE	
4 PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S): IWR Report 88-R-4
6a NAME OF PERFORMING ORGANIZATION: Environmental Systems Management Inc. / 6b OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION: Water REsources Support Center Institute for Water Resources
6c. ADDRESS (City, State, and ZIP Code): 3160 Waverly Drive Charlottesville, VA 22901	7b. ADDRESS (City, State, and ZIP Code): Casey Building, #2594 Fort Belvoir, VA 22060-5586
8a. NAME OF FUNDING/SPONSORING ORGANIZATION: USACE, Institute for Water Resources / 8b. OFFICE SYMBOL (If applicable): CEWRC-IWR	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: WRC-1A-86-5
8c. ADDRESS (City, State, and ZIP Code): Casey Buidling Ft. Belvoir, VA 22060-5586	10. SOURCE OF FUNDING NUMBERS: PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO. / WORK UNIT ACCESSION NO

11 TITLE (Include Security Classification)

Multiobjective Risk-Partitioning: An Application to Dam Safety Risk Analysis

12 PERSONAL AUTHOR(S)

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13a TYPE OF REPORT	13b TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
FINAL	FROM ____ TO ____	April 1988	

16. SUPPLEMENTARY NOTATION

17 COSATI CODES		
FIELD	GROUP	SUB-GROUP

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)

Dam Safety, Risk Analysis, Risk-Cost Analysis, Risk-Partitioning, Expected Annual Damages, Low Probability-High Consequence Events.

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

Risk -- a measure of the probability and severity of adverse events -- has commonly been measured by the traditional Bayesian expected value approach. While a reasonable measure for some cases, the expected value approach is inadequate and may lead to fallacious conclusions when applied to risks associated with extreme and catastrophic events and where public policy issues are involved. Furthermore, risk analysis is often divided into two components: risk assessment of hazards, both natural and technological, and risk management options designed to solve or ameliorate a hazardous situation. While conventional, statistically based risk assessment methods are appropriate in characterizing hazards, they are not always appropriate for the evaluation and management of those hazards.

In particular, the use of the traditional expected value in the assessment of low-probability/high-consequences (LP/HC) risk is inadequate because this approach does not distinguish between events with high probability of exceedance and low-damage consequence

20 DISTRIBUTION/AVAILABILITY OF ABSTRACT: ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	21 ABSTRACT SECURITY CLASSIFICATION
22a NAME OF RESPONSIBLE INDIVIDUAL: Eugene Stakhiv	22b TELEPHONE (Include Area Code): 202-355-2468 / 22c OFFICE SYMBOL: CEWRC-IWR-R

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

UNCLASSIFIED

and events with low probability of exceedance and catastrophic consequence. To study the risks associated with dam failure, the traditional unconditional expectation will be augmented with the conditional expectation generated by the partitioned multiobjective risk method (PMRM).

This report documents an application of the PMRM to a real, albeit somewhat idealized, dam safety case study. During the course of the analysis, useful relationships are derived that greatly facilitate the applications of the PMRM not only to dam safety problems, but also to a variety of other risk-related problems. Apart from the theoretical investigations, the practical usefulness of the PMRM is examined in detail by using it as an aid in the evaluation of alternative dam safety remedial actions. It is shown that the use of the PMRM allows decisionmakers to enhance their understanding of the problem's characteristics, especially those characteristics which are of an extreme and catastrophic nature.

Three main objectives for the study can be identified:

(a) evaluate the applicability of the PMRM to a realistic dam safety problem,
(b) examine the sensitivity of the results generated by the PMRM to variations in the value of the return period of the PMF, and
(c) determine the sensitivity of the PMRM to changes in the probability distribution used to describe extreme flood flows.

Results obtained for the first of the above objectives showed that the PMRM was indeed superior to the use of the unconditional expected value.

To address objective (b) and (c) -- evaluating the sensitivity of the PMRM both to the choice of the distribution describing an extreme flood and to the choice of the return period of the PMF -- the PMRM calculations were performed for the dam modification problems in question assuming four different distributions (the Log-normal, Paretor, Weibull, and Gumbel), and four different values of the return period of the PMF (namely, $T_4 = 10^4$, $T_5 = 10^5$, $T_6 = 10^6$ and $T_7 = 10^7$). The results showed conclusively that, in general, the absolute magnitude of the conditional expected risk of LP/HC events is sensitive to the value of the return period of the PMF -- it increased with the increasing value of the return period of the PMF. At the same time, the conventional (unconditional) expected value of damage showed an insignificant sensitivity to changes in the value of the return period of the PMF.

The correct citation for this report is:

Haimes, Y.Y., Petrakian, R., P.-O. Karlsson and J. Mitsiopoulos, 1988. Multiobjective Risk-Partitioning: An Application to Dam Safety Risk Analysis. Prepared for U.S. Army Institute for Water Resources by Environmental Systems Management, Inc. IWR Report 88-R-4.

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MULTIOBJECTIVE RISK-PARTITIONING:

AN APPLICATION TO DAM SAFETY RISK ANALYSIS

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A Report Submitted

to the

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by

Environmental Systems Management, Inc.
3160 Waverly Drive
Charlottesville, VA 22901

April 1988

IWR Report 88-R-4

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Approved for public release;
Distribution Unlimited

88 6 9 011

PREFACE

This report is one of the products of a number of related research efforts that fall under the Corps of Engineers "Risk Analysis Research Program," managed by the Institute for Water Resources (CEWRC-IWR) in conjunction with the Hydrologic Engineering (CEWRC-HEC) of the U.S. Army Corps of Engineers Water Resources Support Center (CEWRC) as part of the initiatives and directives of the Office of the Chief of Engineers. Specifically, the examination of "Multiobjective Risk Partitioning: An Application to Dam Safety Risk Analysis" is one of the products of the research plan for the dam safety risk analysis research element. However, the evaluation principles developed in this report supports a facet of risk considerations that underlies, and is common to, most applications of risk and uncertainty analysis in water resources planning, especially those in the low probability - high consequence category of natural and man-made hazards.

The genesis of the Corps of Engineers "Risk Analysis Research Program" evolved out of a request by the Office of the Assistant Secretary of the Army for Civil Works to develop a uniform approach to evaluating dam safety by way of ".. a substantial program of research which addresses the issue of dam safety assurance for existing structures as it relates to the criteria used for spillway design...." (letter of 28 Sept 1983, by Assistant Secretary of the Army William R. Gianelli). The risk analysis research effort was geared initially to focus on hydrologic and spillway-related dam safety issues.

Subsequently, the notion of extending risk and uncertainty analysis to a larger set of planning and design-oriented issues emerged, culminating in a memorandum from the Assistant Secretary of the Army for Civil Works, Mr. Robert Dawson (8 Feb 1985) asking the Chief of Engineers to "...develop a plan of action to provide guidance to FOAs on the use of risk evaluation procedures appropriate to Corps programs." This request was followed by a plan of action for incorporating risk assessment methods into Corps planning and a training and technology transfer program. The plan consisted of a broad research program that expanded on the technical bases developed for dam safety and included a series of regional workshops on applying risk analysis to dam safety problems and in planning for flood control and navigation purposes and associated environmental consequences. A formal course in risk analysis techniques applied to planning is part of the training program.

The expanded risk analysis research program is conducted at the Institute for Water Resources (CEWRC-IWR). The risk research program manager is Eugene Z. Stakhiv, assisted by Dr. David Moser, both of the CEWRC-IWR. The research program consists of discrete work units for dam safety risk analysis; navigation planning; risk perception and communication; environmental risk analysis; and hydrologic risk analysis (conducted at the Hydrologic Engineering Center). The hydrologic and hydraulic aspects of risk analysis are conducted under the management of Arlen Feldman at CEWRC-HEC. The work is part of the broader Water

Resources Planning Studies research program conducted through the Research Division, Institute for Water Resources, which is headed by Michael R. Krouse. J.R. Hanchey is the Director of the Institute for Water Resources. The technical monitors for this research programs are Robert Daniel (Planning Division), Donald Duncan (Office of Policy), and Roy Huffman (Hydrologic and Hydraulics Division) of the Office of the Chief of Engineers.

ACKNOWLEDGEMENTS

It is difficult to envision the conception and completion of our study on dam safety that is documented in this report without the vision, inspiration, and continuous guidance of Eugene Stakhiv, Program Manager for risk analysis research, and the tireless advise of David A. Moser, Economist, U.S. Army Institute for Water Resources. The successful collaboration throughout the study between the two groups is indeed a model of an effective partnership among government, universities, and consulting firms. For their tutorial and other didactic assistance in the art and science of dams--their structure, economics, and safety--we are most grateful. We also acknowledge with appreciation the review and comments provided by Russell Lee, Program Manager, Oak Ridge National Laboratory. Finally, we thank Virginia Benade and Susan Hitchcock for their valuable editorial work, and Angie Amato and Melanie Farrish for their typing and retyping the manuscript.

Executive Summary

Risk -- a measure of the probability and severity of adverse events -- has commonly been measured by the traditional Bayesian expected value approach. While a reasonable measure for some cases, the expected value approach is inadequate and may lead to fallacious conclusions when applied to risks associated with extreme and catastrophic events and where public policy issues are involved. Furthermore, risk analysis is often divided into two components: risk assessment of hazards, both natural and technological, and risk management options designed to solve or ameliorate a hazardous situation. While conventional, statistically based risk assessment methods are appropriate in characterizing hazards, they are not always appropriate for the evaluation and management of those hazards. In particular, the use of the traditional expected value in the assessment of low-probability/high-consequences (LP/HC) risk is inadequate because this approach does not distinguish between events with high probability of exceedance and low-damage consequence and events with low probability of exceedance and catastrophic consequence. To study the risks associated with dam failure, the traditional unconditional expectation will be augmented with the conditional expectation generated by the partitioned multiobjective risk method (PMRM).

This report documents an application of the PMRM to a real, albeit somewhat idealized, dam safety case study. During the course of the analysis, useful relationships are derived that greatly facilitate the applications of the PMRM not only to dam safety problems, but also to a variety of other risk-related problems. Apart from the theoretical investigations, the practical usefulness of the PMRM is examined in detail by using it as an aid in the evaluation of alternative dam safety remedial actions. It is shown that the use of the PMRM allows decisionmakers to enhance their understanding of the problem's characteristics, especially those characteristics which are of an extreme and catastrophic nature.

The PMRM was developed in order to avoid the theoretical and philosophical problems associated with traditional expectational analysis. The PMRM supplements and complements the traditional benefit-cost analysis and ensures that the approach comprises a valid evaluation tool for low-probability/high-consequence events. Namely, risk-cost tradeoffs constitute a valid approach for selecting a preferred and acceptable policy, whether the costs are expressed in terms of dollars or lives or both. In contrast to the use of the unconditional expected value, the PMRM collapses the risk curve into a set of points, each of which represents a conditional expected value of damage falling within a particular probability range. These points are obtained by partitioning the exceedance probability axis into different ranges and then calculating the conditional expected value of damages corresponding to the exceedance probabilities that fall within a particular range. Typically, the three ranges considered are the high-probability/low-consequence (HP/LC) range, the intermediate-probability/intermediate-consequence (IP/IC) range, and the low-probability/high-consequence (LP/HC) range. The generation of these conditional expected values allows the decisionmakers to evaluate risk-cost tradeoffs in the particular probability domain that interests them. Ultimately, the risk curves generated by the conditional expected values are compared with the curve generated by the conventional expected value. By providing information on the various domains encountered in choosing an appropriate policy (especially in the LP/HC domain), the PMRM allows the decisionmakers to appreciate the impact of alternative actions corresponding to the risk-cost tradeoff curve.

Engineers have been successful in designing water systems that meet some a priori standards for a system's performance. In the study documented here, the objective was to design a dam that would withstand a probable maximum flood (PMF) for which the probability of occurrence is not known. Risk analysis tools, such as the method of moments or the two-point boundary value curve fitting, can be used to define a probability distribution that incorporates the return period of the PMF and the return period of the 100-year flood, as was done in this study. More important is the realization that there are alternative ways to meet the design standard, where the benefits, costs, and risks associated with each alternative are kept in the analysis in their original noncommensurate units.

In the instance of dam safety, two simple alternatives exist: (a) raising the dam height to hold a greater volume of the PMF runoff, and/or (b) widening the spillway so that its larger capacity can discharge more of the volume without causing dam failure. Both alternatives meet the objective of improving dam safety.

In this work, the PMRM was used to evaluate dam modification options for an idealized dam/reservoir system. To aid in this evaluation, a computer-based decision support system (DSS) was developed and implemented. This DSS provides an interactive framework in which the decisionmaker can evaluate the options with respect to the risk-cost tradeoffs in the different probability regimes and, subsequently, make an informed decision.

Three main objectives for the study can be identified:

(a) evaluate the applicability of the PMRM to a realistic dam safety problem,
(b) examine the sensitivity of the results generated by the PMRM to variations in the value of the return period of the PMF, and
(c) determine the sensitivity of the PMRM to changes in the probability distribution used to describe extreme flood flows.

Results obtained for the first of the above objectives showed that the PMRM was indeed superior to the use of the unconditional expected value.

To address objectives (b) and (c) -- evaluating the sensitivity of the PMRM both to the choice of the distribution describing an extreme flood and to the choice of the return period of the PMF -- the PMRM calculations were performed for the dam modification problem in question assuming four different distributions (the Log-normal, Pareto, Weibull, and Gumbel), and four different values of the return period of the PMF (namely, $T_4 = 10^4$, $T_5 = 10^5$, $T_6 = 10^6$ and $T_7 = 10^7$). The results showed conclusively that, in general, the absolute magnitude of the conditional expected risk of LP/HC events is sensitive to the value of the return period of the PMF -- it increased with the increasing value of the return period of the PMF. At the same time, the conventional (unconditional) expected value of damage showed an insignificant sensitivity to changes in the value of the return period of the PMF.

Major Findings

(1) The commensuration of events of low-probability/high-consequences (LP/HC) with events of high-probability/low-consequences (HP/LC) through the traditional unconditional expectation distorts, and almost eliminates, the distinctive features of many viable alternative policy options that could lead to the reduction of the risk of dam failure. On the other hand, the conditional expectation generated by the PMRM clearly delineates the attributes of each policy option, and thus markedly improves the management of risk by maintaining a wider range of options for the decisionmakers. In particular, sixteen alternative policy options were generated in the study -- each with a specific increase in the dam's height and/or in its spillway capacity. When the cost associated with each alternative design configuration was plotted versus the corresponding unconditional risk of dam failure, the resulting almost vertical line of the curve leads to the inescapable conclusion that the most cost-effective policy is the doing-nothing alternative. On the other hand, when the same pairs of costs and their corresponding conditional risks of dam failure were plotted against each other, a clear distinction among the various options became evident. Furthermore, the tradeoffs generated by the surrogate worth tradeoff (SWT) method, which is used as part of the PMRM, provide an invaluable quantitative knowledge-base to the decisionmakers as they discriminate among the various available options. The following graph depicts two risk-cost tradeoff curves, one generated by using the unconditional expected value of damage and the other by using the conditional expected damage over LP/HC events (i.e., the extreme-event risk):

Note that increasing the spending level (for widening the spillway capacity and/or increasing the height of the dam) from 20 to 30 million dollars would contribute to a negligible reduction of 0.1 units of conventional (unconditional) expected social and economic damage, and thus would likely make such an investment economically unjustifiable. On the other hand, the same investment would significantly reduce the conditional expected value of damage (due to extreme flooding) by one unit of social and economic cost, and thus would likely make such an investment economically and socially acceptable. In other words, while the conventional expected value of damage shows a tradeoff of $10 million/0.1 units of social and economic cost, the conditional expected value of extreme risk events shows a tradeoff of $10 million/1 unit of social and economic cost. One can see that the curve representing the extreme-event damages is, in a sense, more representative of a catastrophic scenario than the curve representing expected damages. The implications are that the partitioning does indeed induce a separation of events that consequently alters both the absolute magnitude of the risk involved and the values of the risk-cost tradeoffs. The changes in these quantities ultimately influence the decisionmaker.

(2) One of the most uncertain factors in the quantification of risk of dam failures is ascertaining the proper and representative value of the return period of the PMF for that specific dam. In this study, a wide range (10^4 - 10^7) of return periods of the PMF was used and the risks corresponding to the LP/HC conditional expectation and to the unconditional expectation were calculated. The sensitivity of the unconditional expected risk to variations in the return period of the PMF (from 10^4 to 10^7) was minor and insignificant -- a variation within a few percentages. On the other hand, the LP/HC conditional risk exhibited a major sensitivity to this variation in the return period of the PMF -- a variation on the order of 100%.

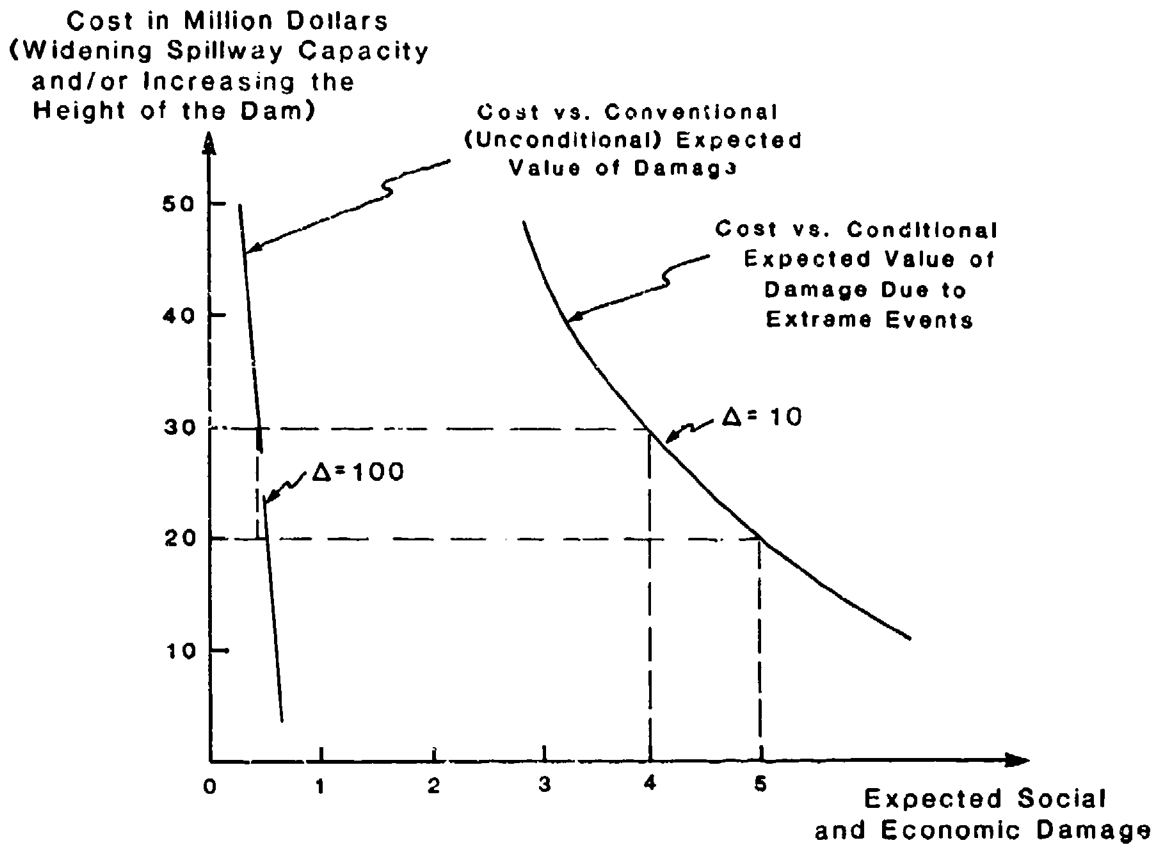
This finding has two major implications:

(a) Contrary to the conclusions advanced by the traditional unconditional expectation of risk, the proper and representative value of the return period of the PMF used in the analysis of dam safety has major significance.

(b) Corollary implications are that (i) there is an efficacy in improving the data base, especially with respect to the data associated with extreme events -- the return period of the PMF (the tail of the distribution function), and (ii) the study of the statistics of extremes is of particular importance to managing the risk of dam failure.

(3) When the type of probability distribution that best represents the hydrology of the dam and its region cannot be determined with an acceptable certainty -- a prevailing situation in most dam-safety studies -- then the selection of the specific distribution function for that hydrology should not be taken lightly. Although the traditional unconditional expected risk demonstrates no great sensitivity to the type of the selected distribution (and to its tail), the conditional risk function associated with LP/HC exhibits much more sensitivity and thus calls for more care and prudence in the

selection of the probability distribution function. This result has led to further studies which developed distribution-free results that would assist in the quantification of dam failure risk independent of the specific distribution's parameters -- by developing an upper bound on the LP/HC conditional risk function.



Trade-offs between cost and conditional and unconditional expected value of risk

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CHAPTER 1
INTRODUCTION

This study is aimed primarly at illustrating how the partitioned multiobjective risk method (PMRM) -- a new risk analysis approach that is designed to aid the evaluation of low-probability/high-consequence events -- can be applied to the problem of dam safety. In particular, the study addresses the choice of appropriate modification measures under conditions of highly uncertain knowledge about extreme floods that are considered primary loading events for possible dam failure.

1.1 Dam Safety

Dams are designed to control the extreme variability in natural hazards (floods and droughts), but they simultaneously impose even larger, though much less frequent technological hazard - potential dam failure.

First, let us define the word "safe". By saying that a structure is safe we mean that risks associated with this structure are "acceptable" to society. Risk is defined as a measure of the probability of occurence of a potentially hazardous event and of the event's consequence to society. It is important to understand that risks can never be reduced to zero, and therefore it becomes necessary to determine a risk level that can be considered to be acceptable. Thus, one of the main tasks of construction and regulatory agencies is to determine acceptable safety criteria.

At present, a large number of dams in the U.S. are more than thirty years old, well into their planned lifetime and in need of restoration. Many of these older dams are becoming increasingly more dangerous because of structural deterioration or inadequate spillway capacity, which increases the likelihood of failure, and also because of downstream development, which increases the hazard potential (NRC [1983a]).

Strong earthquakes or exceptionally large floods are major causes of dam failure. While few dams are under the threat of earthquakes, most, if not all, dams are exposed to floods. This is one of the reasons why this study will focus on the important issue of dam failures caused by extreme floods.

In a joint study on dam failures and accidents, the American Society of Civil Engineers (ASCE) [1975] and the United States Committee on Large Dams (USCOLD) showed that overtopping accounts for 26% of all reported dam failures and that the principal reason for overtopping is inadequate spillway capacity. Thus, the evaluation of adequate spillway capacity is a vital issue in dam safety analysis, and it comprises the focus of this analysis. Two types of corrective or remedial actions will be considered: widening the spillway and increasing the dam's height.

The main function of a spillway is to protect the dam itself during extreme floods. Spillways help to avoid dam failure by passing excess water -- that is, the volume of water above the design flood for which the storage capacity was optimized along with other project purposes -- which otherwise might have caused the dam to be overtopped or breached. The hazards posed by any of these situations might approach or even exceed damages that would have occurred under natural flood conditions, that is, without the dam.

Many dams in the U.S. were designed before or during the time when the concept of, and knowledge about the probable maximum floods (PMF) as a design criterion was being formulated and subsequently refined as a specific computational procedure. As a result, a number of existing dams have inadequate spillway capacities by today's standards. It was estimated in the National Research Council (NRC) report (NRC [1983a]) that about 25% of the 9000 non-federal dams inspected in 1979 had insufficient spillway capacity and were therefore designated unsafe.

1.2 Frequency Distributions of Extreme Floods

Gumbel [1958] introduces the flood problem by defining the annual flood:

> For the statistical treatment, consider the mean daily discharge of a river through a given profile at a specific station, measured in cubic meters (or cubic feet) per second. Among the 365 (or 366) daily discharges during a year, there is one measure which is the largest. This discharge is called the annual flood.

A large number of studies have been done to determine the statistical distribution that provides the best fit for the existing records of annual floods and that can be extrapolated beyond the period of records with maximum confidence. But, since most flood records in the U.S. do not exceed 50 to 100 years of data, it becomes very difficult to determine which distribution is most appropriate for extreme floods larger than the 100-year flood.

Bulletin 17B of the Interagency Advisory Committee on Water Data [1982], which was written as a guide for determining flood flow frequency, recommended the use of the log-Pearson type-III distribution for the description of the "normal" range of floods used in flood control design. But the choice of this distribution was not without controversy (Wood and Hebsan [1985], Wood and Rodrique-Iturbe [1975], Wallis and Wood [1985], Hebson and Wood [1982]) and did not attempt to solve the issue of the description of floods larger than the 100-year flood.

A more recent NRC report [1985] on dam safety recommends that extrapolations not be made to floods much larger than the 100-year flood by flood-frequency distributions derived from recorded data of less than 100 year old floods. The report presents two approaches to help overcome this problem.

The first approach is to use information from regional sites and combine them to increase the accuracy of the estimates of the frequency of extreme floods. There are two classes of procedures for this approach. The first one is the index flood procedure, which was recently revived by Wallis [1980]. A dimensionless flood-frequency curve for a region is defined; when it is used for a specific site, it has to be scaled by an estimate of the mean flow at that site. Stedinger [1983] pointed out problems associated with this procedure. In particular he argued that annual floods at different sites in the same region are not independent random events.

The second class of procedures is based on Bayesian analysis. In this approach, different techniques are employed to make use of available information on historical floods. This information can be obtained either from written records or from the physical evidence that paleohydrology techniques can provide. The NRC report adds, however, that it does not result in significant improvement in estimates of flow frequency of extreme floods.

The NRC report makes the point that neither of these approaches provides sufficient accuracy in estimates of extreme floods with return periods on the order of 10^4 to 10^6 years. The report also discusses whether the return period of the PMF can be estimated with reasonable accuracy. It was observed that if antecedent conditions had little impact on the PMF values, then the required calculations could be simplified and could provide credible solutions, but otherwise estimates of the return period of the PMF could be quite unreliable.

In Appendix E of the NRC report, a procedure for estimating flood frequency for extreme floods was presented. It was based on extrapolating the flood-frequency curve that was obtained for floods with small return periods. As discussed earlier, this approach does not, in general, yield credible results. However, if the return period of the PMF can be estimated, then such an extension of the flood-frequency curve to the PMF can be reliable. Since the return period of the PMF is often difficult to estimate, the report affirms that the use of either 10^6 or 10^4 years can give reasonable results. According to Newton [1983], however, the PMF is estimated on an antecedent flood of fairly large magnitude, and the joint probability of both events is about 10^{-12} per year(i.e., the return period of the joint events is about 10^{12} years). The report also recommends that research be done on the impact and advisability of using different flood-frequency distributions.

Stedinger and Grygier [1985] have studied the sensitivity of the results of a dam safety risk analysis to the value of the assigned return period of the PMF and to the choice of the flood-frequency distribution used to extend the frequency curve to the PMF. They found that the results could be easily influenced by either a change in the return period of the PMF or by the choice of the distribution. Therefore, it is concluded that any risk analysis on dam safety should include a sensitivity study with respect to these factors. It is hoped that through such sensitivity analyses the range of uncertainty could be somewhat bounded.

Consequently, in this study one of the key issues is to perform sensitivity analysis on the decision situations by varying both the frequency distribution of the inflow events and the return period of the PMF. This analysis includes an adequate representation of the wide variety of flood-frequency distributions used in different studies: the log-normal, Gumbel, log-Gumbel, and Weibull distributions. Of particular interest are the log-normal distribution, which is a special case of the log-Pearson type-III distribution and which was recommended (as we have seen) by Bulletin 17B, and the Gumbel distribution (type-I extreme value distribution), which is characterized by its thin tail. The values of 10^4 and 10^6 years have been assigned to the return period of the PMF, thus following the recommendation of Appendix D of the 1985 NRC report on dam safety.

1.3 Risk Analysis and Dam Safety

Risk analysis can be a useful tool to assist the decision maker (DM) in evaluating the impact of the various policies and remedial actions on dam safety. Risk analysis can also help the DM determine the amount of protection that should be added to a dam given the construction and maintenance costs needed to modify the dam's characteristics to the desired level. Any decision the DM will take will involve the consideration of a trade-off between somewhat more certain expenditures and relatively uncertain benefits and economic losses.

Different levels of complexity in risk analysis emerge for different types of problems. The amount of information needed is a function of these levels. For example, in prioritizing dams for safety evaluation, approximate methods are adequate, but a more detailed risk analysis of the safety of a given dam requires that engineering, economic, social, and environmental aspects be factored into the evaluation.

Risk analysis as a public decision-aiding tool is a controversial and evolving field: there are scientists charging that risk assessment is much more of an art than a science. This might be true to a certain extent, but the main merit of risk analysis is that it provides a framework for organizing and summarizing information about costs and risks such that communication among the different interests and groups that are involved can be enhanced.

Dams belong to the category of structures that have the potential to produce catastrophic events through their failure which, though infrequent, can nonetheless cause damage that is greater than that caused by naturally occuring floods. Therefore, a low-probability/high-consequence (LP/HC) risk analysis of dams is the most appropriate approach to tackle the issue of dam safety. But, risk analysis of LP/HC events is still an evolving field, and it must cope with complex, incommensurable issues. This requires analysts who posses a diversity of expertise. In general, information related to LP/HC events is scarce, and the different statistical tools that are used try to make full use of sparse information. Such events as nuclear power-plant accidents, dam failures, and toxic chemical spills constitute major LP/HC events, and many risk assessment studies of these events have been done. These studies have, in general, relied on the application of traditional risk analysis tools to characterize the hazards associated with LP/HC events, which may be inadequate for the solution of the problem.

1.4 The Partitioned Multiobjective Risk Method and Dam Safety

Kaplan and Garrick [1981] stated that expressing risk as probability times consequence would implicitly reduce risk to expected damage. This in turn would lead to equating low-probability/high-damage alternatives with high-probability/low-damage ones, which are clearly not equivalent events unless the decision maker is risk neutral. Kaplan and Garrick insisted that risk is not the mean of the risk curve but is rather the curve itself. They stated that "a single number is not a big enough concept to communicate the idea of risk. It takes a whole curve." Moreover, if the uncertainties due to our incomplete knowledge must be considered, then a whole family of curves would be needed to express the idea of risk.

Vohra [1984] even presented a quantitative definition of risk that accounts for the higher impact of extreme events on society. In many other reports and papers, scientists have expressed reluctance or discomfort when they had to use the traditional expected value method to evaluate risks associated with extreme events. For example, the 1985 NRC report on dam safety borrows this quotation from Raiffa [1968]:

The issue is how much members of society are willing to pay to avoid such unlikely events. It is highly plausible that they are ready to pay more than the expected cost.

There is obviously a strong need for a risk analysis method that would allow us to consider explicitly the low-frequency/high-damage domain. The partitioned multiobjective risk method, or PMRM (Asbeck [1982], Asbeck and Haimes [1984], and Leach [1984]), provides the capability to quantify risks for extreme events.

The PMRM attempts to avoid the problems associated with the concept of traditional expected value by collapsing the risk curve into a set of points that represent the conditional expected values for the different damage domains. These points are obtained by partitioning the exceedence probabilty axis into different ranges, and then taking for each range the expected value for damages that have their exceedence frequencies lying within that range. This method allows us to represent a distribution by a number of points instead of just one point, as in the traditional expected value method, and therefore more information about the risk curve is preserved. Ideally, we would like to keep the whole risk curve, but the PMRM is still an improvement on the method of traditional expected value. Through an appropriate partitioning of the probability axis, we can calculate the conditional expected value for damages that correspond to the LP/HC events, thus quantifying the risk of extreme events.

The PMRM also has another advantage: it avoids the explicit use of utility functions to represent the decision maker's preferences. Utility theory has often been criticized because it is based on assumptions of the behavior of the individual that are sometimes inconsistent with reality. For example, Slovic and Tversky [1974] showed that Savage's independence principle, which is at the heart of expected-utility theory axioms, is not always satisfied. MacCrimmon and Larsson [1975] and Shoemaker [1980] argued along similar lines.

Moreover, the PMRM does not replace the judgment of the decision maker (DM); it is merely a tool to help the DM express individual preferences through the consideration of trade-off information among the different objectives. The surrogate worth trade-off (SWT) method and its extensions (Haimes and Hall [1974], Chankong [1977], Haimes and Chankong [1979], and Haimes [1980]) are used to develop trade-offs and, through interaction with the DM, to obtain a preferred solution. This means that the SWT allows implicit expression of the decision maker's utility function through the use of trade-off information among the several objectives. In the case of dam safety, these objectives could be, for example, the desire to reduce risks associated with moderate or extreme floods, and to simultaneously to minimize the cost of remedial actions or to minimize loss-of-life.

It is a distinctive characteristic of the PMRM that it allows an analysis of the dam safety problem in a multiobjective framework. Haimes [1984] has illustrated the advantages of performing risk assessment using a multiobjective approach. First, more than one objective can be taken into consideration and therefore a better approximation of the real decision-making process is obtained. Also, the analyst can limit the scope of the work to such areas as system modeling, the quantification of risks and objectives, and the calculation of trade-offs. The actual decision-making process is left to the DM, who uses subjective preferences and judgment, interprets the results and determines appropriate policies.

In summary, the partitioning of the probability axis in the PMRM can be done in such a way that risks associated with extreme floods can be explicitly quantified and can therefore be compar ' to the costs of the different corrective or remedial actions. The decision maker performs this comparison by examining the calculated trade-offs between the conditional expectations for each domain of the damage axis and the annual modification costs.

The purpose of this work is to apply the PMRM to a case study in the context of dam safety and to investigate the usefulness of the PMRM in dam safety analysis. After a review of the literature in Chapter 2, a real-world case study problem is introduced in Chapter 3. In Chapter 4, approximations are used to derive some new relationships for the PMRM in order to facilitate the application of the method to the case study. In Chapter 5, an analysis of the results of this application is presented and the way in which the PMRM adds insight to the dam safety problem is shown. Chapter 5 also includes a sensitivity analysis of the trade-offs with respect to the choice of the probability distribution used to extrapolate the flood-frequency curve to the PMF, with respect to the partitioning of the probability axis, and with respect to the return period of the PMF. Chapter 6 deals with the importance of the distribution and Chapter 7 discusses extensions of the PMRM. Finally, Chapter 8 provides a summary and an evaluation of the study, as well as recommendations for future applications of the PMRM and for future research in the field. An appendix is also included, which contains an application of the PMRM to a model presented by Stedinger and Grygier [1985].

CHAPTER 2
LITERATURE REVIEW

This chapter, presents a review of the literature relevant to this study. Different reports and papers that investigate the issue of dam safety are discussed. Next, the different approaches used to evaluate flood flow frequency are examined. Finally, risk analysis and low-probability/high consequence risk analysis are briefly reviewed.

2.1 The NRC Reports on Dam Safety

Two excellent reports that cover a wide range of issues and methods in dam safety analysis are the National Research Council reports of 1983 and 1985 on the safety of dams.

The objective of the NRC's Safety of Existing Dams--Evaluation and Improvement report [1983a] was to provide a comprehensive overview of the status of dam safety and technical issues related to monitoring and evaluation. It reviewed and evaluated risk analysis techniques, possible modifications to remedy the deficiencies in existing dams, and methodologies to assess the impact of a catastrophic event such as dam failure. The report pointed out numerous areas where more research is needed, and it suggested the directions this research should take in certain cases.

This report was innovative in that it placed some emphasis on risk-based decision analysis which, according to the report, is a discipline with which practicing engineers feel uncomfortable. Overall, they believe that the uncertainties that characterize risk analysis techniques render its results unreliable.

The NRC report, however, proved to be a very useful starting point for this study. First, it was shown that overtopping was the main cause of failure (26% of all dam failures) and that the principal reason for overtopping was inadequate spillway capacity. It was also observed that the modes and causes of dam failure were numerous, different, and sometimes interrelated.

Next, the report focused on risk analysis and made it clear that it is not meant to replace engineering judgment and intuition but to complement them. Also, risk analysis was said to be able to assist decision makers by summarizing available information and quantifying any uncertainties associated with this information. The report pointed out that if risk analysis is used in prioritizing dams for safety evaluation, no further extensive probabilistic studies are needed. However, a subsequent NRC report

[1985] on dam safety criteria specified a set of conditions under which risk-cost analysis is to be performed (see p.244 of the NRC report).

The authors summarized the formal risk assessment procedure by the following steps:

- Identification of the events or sequences of events that can lead to dam failure and evaluation of their (relative) likelihood of occurrence.
- Identification of the potential modes of failure that might result from the adverse initiating events.
- Evaluation of the likelihood that a particular mode of dam failure will occur given a particular level of loading.
- Determination of the consequences of failure for each potential failure mode.
- Calculation of the risk costs, i.e., the summation of expected losses (economic and social), from potential dam failure.

The report also described briefly the Stanford university risk-based screening procedure, the methodologies developed at MIT, the Corps' (Hagen) method of index-based risk assessment, and the safety evaluation of existing dams (SEED) procedure of the U.S. Bureau of Reclamation.

In the discussion of flood risk assessment, it was suggested that a good approach for determining the exceedence probability for rare floods would be to extend a smooth curve from the limit of the relation obtained by historical records of flood events until the curve becomes asymptotic to the PMF value.

It was recognized that the choice of a remedial action involves a fundamental trade-off between expenditures and future gains and losses. The possible alternative actions are: maintain the status quo; modify the dam; modify the spillway; construct upstream facilities; perform corrective maintenance; survey intensively; regulate reservoirs; install emergency action plans; and consider flood plain management. The report also presented examples of the application of risk analysis to dam safety.

Next, an excellent overview of hydrologic and hydraulic analysis was provided. The procedures and criteria most utilized currently were described in some detail. Some of the interesting topics discussed were the generalized estimates of the probable maximum flood peak discharge, the bases for assessing spillway capacity, spillway capacity criteria, and design floods.

NRC's Safety of Dams--Flood and Earthquake Criteria report [1985] might be considered to be the risk analyst's handbook for dam safety analysis.

Appendices C, D, and E are of particular interest, since the authors took a practical point of view in discussing issues on dam safety.

Appendix C investigates the different methods used to estimate the probable maximum precipitation (PMP), in particular the progressive increase in the magnitude of the PMP in successive National Weather Service hydrometeorological reports.

Within Appendix D is presented a summary of some statistical relations and a review of Bulletin 17B's procedures for flood-frequency analysis. Next, a study of the problems associated with the determination of the return period of the PMF and more generally with the specification of the frequencies of rare floods is presented. Appendix D contains the reminder that the log-Pearson type-III probability distribution that Bulletin 17B of the Interagency Advisory Committee [1982] assumes to be the best estimate of flood-frequency distribution can, in general, only hold for floods with return periods smaller than 100 years. The report also acknowledges the uncertainties associated with the estimation of the return period of the PMF. But, the report conludes that it is generally accepted that the return period of the PMF lies between 10^4 and 10^6 years. The suggestion is then made in the report to use a linear extrapolation on log-normal paper of the flood-frequency curve through the 100-year flood and the PMF in order to obtain the cumulative probability function of flood occurrence for floods between the PMF and the 100-year flood. By taking the derivative of the cumulative probability function, the probability density function f(q) is obtained, which yields:

$$f(q)=[(2\pi)^{(1/2)}vq]^{-1} \exp[-0.5(\ln q - m)^2/v^2]$$

where q is flood magnitude in cfs and m and v are two parameters that can be determined from boundary conditions.

The estimation of a damage function D(q) should be based on the results of appropriate flood routing exercises. In general, D(q) is continuous except at the critical flow above which the dam fails. Finally, an example is provided to facilitate the comprehension of these concepts.

Appendix E focuses on risk analysis and in particular on the extension of the frequency curve to the PMF, the evaluation of the damages caused by floods, the matrix decision approach, and the calculation and use of expected cost.

In this last section, the report describes the problems that the use of the concept of expected cost raises. First, in the calculation of expected costs, multiplying estimates of large costs by rather poor estimates of small probabilities will yield poor results. Also, the concept of expected value might not be the right approach to describe extreme events with a small chance of occurrence, because it is likely that the public is willing to pay more than the expected cost to avoid potentially catastrophic events (i.e., the public is risk averse). The report also addresses the issue of estimating the worth to society of avoiding fatalities; however, it does not give any definitive answer to this problem.

2.2 Other Reports on Dam Safety

Stedinger and Grygier [1985] conducted a sensitivity analysis based on the risk analysis example presented in the 1985 NRC report on dam safety. They varied the values of different parameters, such as the return period of the PMF, the magnitude of the PMF, the flood-frequency distribution, and the shape of the damage function. Changes in the value of the return period of the PMF and in the choice of the frequency distribution used to interpolate between the 100-year flood and the PMF proved to have a significant impact on the relative attractiveness of the alternative designs. Variations in the magnitude of the PMF and the damage function shape were less critical in influencing the final results. The true difficulty lay in the fact that the return period of the PMF is highly uncertain: different reports recommend values ranging from 10^4 to 10^{12} years. Most of these recommendations are actually based on subjective judgment and experience than on statistical evidence. Similar problems cripple the estimation of the flood-frequency distribution for floods ranging from the 100-year event to the PMF. The authors concluded that for risk analysis to be considered a reliable tool, careful and accurate estimation of the return period of the PMF and of the flood-frequency distribution of extreme flood events is needed.

An important report on dam safety is the review by the NRC's Committee on the Safety of Dams [1977] of the program of the U.S. Bureau of Reclamation. The task of the committee was to investigate and evaluate the criteria, procedures, and practices used by the U.S. Bureau of Reclamation to manage dams under its jurisdiction. The committee recommended some improvements that would enhance the effectiveness of the Bureau's dam safety program. One of these recommendations was that the Bureau should use risk analysis to rank existing Bureau dams in accordance with the probability of failure and hazard potentials of the dam. But, the report did not mention the problems associated with the use of risk analysis in dam safety.

McCann et al. [1984], from Stanford University, prepared for the Federal Emergency Management Agency (FEMA) a report composed of two volumes: the first one presents a screening process and the concepts behind it; the second is a user's manual for this methodology. By measuring the relative risk associated with different dams, the screening process is supposed to help the dam safety manager allocate funds for the improvement of the safety of the different dams owned by his firm or agency.

This report does not actually introduce any new concept or approach; on the other hand, it structures traditional risk analysis in a framework appropriate for helping the DM allocate resources by prioritizing dams. Since only limited funds are normally allocated for the preliminary stage of prioritizing dams, simplified techniques not necessarily supported by extensive studies are often used. Therefore, this report might prove to be very useful to the analyst who just needs to take an approximate approach: it presents a wide range of approximate techniques to such problems as the evaluation of flood frequency, overtopping and failure criteria, and the estimation of life loss and property damage due to flooding. The authors of this report stress that the screening process can under no circumstances be an alternative to an in-depth risk analysis of dam safety, and that the approach is only valid for the relative ranking of dams.

The Bohnenblust and Vanmarcke [1982] and Vanmarcke and Bohnenblust [1982] reports presented a similar methodology that was also supposed to provide a framework for organizing available information on dam safety. The method was illustrated by an application to a case study involving sixteen dams owned by the state of Vermont. Baye's theorem was used to update subjective prior assessments of risks, with information particular to each dam. The authors have emphasized the flexibility of the model, arguing that their method allows the user to perform a sensitivity analysis on the decision criteria and the input data.

It should be noted that neither the Stanford/FEMA risk-based screening procedure nor the Bohnenblust and Vanmarcke approach have explicitly taken into consideration the low-probability/high-consequence (LP/HC) nature of dam safety problems. They have resorted to the use of the expected value which, the NRC committee on Dam Safety (1983, 1985) has shown to be an incomplete and perhaps inadequate way to express risk for LP/HC problems.

Karaa and Krzysztofowicz [1984] describe a Bayesian methodology that can assist dam owners in developing dam maintenance programs. It allows the decision maker to explicitly quantify uncertainty about the actual state of the dam, and the owner to specify his risk preference. Moreover, the engineer should be able to use this method to combine his engineering judgment with facts from geotechnical tests. The authors draw the conclusion that extensive inspection programs are likely to be economically justifiable. They also conclude that the expected-disutility criterion might be more appropriate than the expected-loss criterion, since it can take into account the fact that the decision maker might be willing to pay more than the expected loss to reduce the chance of failure and of catastrophic losses.

2.3 The Assessment of Flood Flow Frequency

The Interagency Advisory Committee on Water Data [1982] published Bulletin 17B, a guide for flood flow frequency analysis that includes and integrates accepted recent technical methods. Bulletin 17B established a procedure that is based on the use of the log-Pearson type-III probability distribution as the flood-frequency distribution. This guide assumes that a systematic record of annual floods is available and can be used for determining the flood flow frequency. In fact, this procedure can make use of three types of data: systematic flow records, historical records of unusual floods, and regional information. Historical records and regional information, if available, can be used to enhance the reliability of the flood frequency estimates obtained from Bulletin 17B's procedure. Federal agencies are supposed to follow the prescribed guidelines, and nonfederal groups are urged to do the same. These guidelines allow the user to deviate from the procedure for situations in which there is strong evidence that a more site-specific appropriate approach could be taken.

Wallis et al. [1974], in a series of papers on flood flow frequency, obtained (by Monte Carlo simulation) three statistics: the mean, the standard deviation, and the coefficient of skewness for small samples with sizes varying from 10 to 90. Different distributions such as the normal, the Gumbel, the log-normal, the Pearson type-III, the Weibull, and the Pareto, were considered. It was observed that the sampling properties of the statistics showed strong skews, biases, and constraints.

In another paper, Slack et al. [1975] used Monte Carlo simulations to study the value of information to flood-frequency analysis. They observed that in the absence of information on the flood-frequency distribution or the damage function, if the underlying distribution is indeed Gumbel, the best "fitting" distribution was found to be Gumbel; otherwise, however, the use of the normal distribution was better than the Gumbel, log-normal, or Weibull distributions. They also studied the impact of the level of information available on the expected opportunity design losses.

Matalas et al. [1975] also compared the mean and standard deviation of regional estimates of skewness derived from historical flood sequences, with the statistics for seven hypothetical flood distributions. They were found to be inconsistent with each other. The hypothetical flood distributions were the normal, uniform, Gumbel, log-normal, Pearson type-III, Weibull , and Pareto distributions.

Wallis et al. [1976] found, on the basis of Monte Carlo experiments, that the length of flood sequences did not have significant influence on the relative attractiveness of the distributions and that the differences in the analytical form between the chosen distributions and the underlying distributions cannot be accounted for by the bias in the estimate of the design flood.

Haimes et al. [1979] presented the results of an analysis of the worth of streamflow data in water resources planning. They showed that the expected objective values depended both on the length of data records and on the planning model. This study, which was done in a multiobjective framework, yielded a set of noninferior data collection policies.

Hosking et al. [1985a] examined the properties of the probability-weighted moments (PWM) estimation method when it is used to evaluate the parameters and quantiles of the generalized extreme-value (GEV) distribution. They defined the probability-weighted moments of a random variable X with a cumulative distribution F(X) to be the quantities

$$M_{p,r,s} = E\ [X^p\ \{F(X)\}^r\ \{1-F(X)\}^s]$$

where p, r, and s are real numbers. But, they only used the moments

$$\beta_r = M_{1,r,0} = E[X\ \{F(x)\}^r]$$

The GEV distribution, which was first introduced by Jenkinson [1955], combines into a single form the three possible extreme-value distributions: Fisher-Tippett types I, II, and III. When compared to the maximum likelihood estimator (MLE) or the sextiles method for estimating the parameters and quantiles of the GEV distribution, the PWM is shown to have many advantages over the other methods. First, the PWM method is fast, requires easy computations, and always gives feasible values for the estimated parameters. Also the variance of the PWM estimators is similar to that of the MLE for moderate sample sizes, but is better for small sample sizes (15 to 25 years of annual flood data). The PWM has no severe biases except for quantiles in the extreme tail of the GEV distribution when the sample is small. Moreover, the PWM can be used to test the hypothesis that the extreme-value distribution is of type I. As we will see, this method was later tested in different studies against other methods under various conditions.

Hosking, Wallis, and Wood [1985b] compared the procedure for estimating the regional flood-frequency that was prescribed in the National Environmental Research Council [1975] Flood Studies Report (FSR) with the regional generalized extreme value (GEV) distribution and the regional Wakeby distribution (WAK) algorithms. Probability weighted moments (PWM) were used to estimate the parameters of the GEV and WAK distributions. The authors found that the GEV/PWM and WAK/PWM algorithms are superior to the FSR procedure with respect to the variances and biases of the estimates. They also suggested that the use of the FSR algorithm should be discontinued in the FSR.

Using Monte Carlo experiments, Wallis and Wood [1985] investigated the properties of the log-Pearson type-III distribution when fitted by the method of moments. They compare the flood quantiles estimates obtained by different procedures, such as that of the U.S. Water Resources Council (WRC), GEV/PWM, and WAK/PWM , and came to the conclusion that the performance of the Water Resources Council procedures is relatively poor. Wallis and Wood therefore warn against the use of Bulletin 17B's procedures given the importance of the facts they had presented. Moreover, they call for a reassessment of current flood-frequency procedures and guidelines in the United States.

Hosking and Wallis [1985a] examined with computer simulations whether the incorporation of the thousands-of-years-old flood events that can be obtained by recent palaeohydrology techniques would significantly improve the accuracy of flood frequency estimates of extreme events. They found that palaeological information is useful only if a three-parameter flood-frequency distribution is used to fit the data for a single site and only if the gauged record is short.

Hosking and Wallis [1985b] also investigated whether, in general, estimates of historical large floods, when included in the flood-frequency analysis procedure, improved the accuracy of the results. The conclusions are very similar to the ones obtained in their previous paper.

Lettenmaier et al. [1986] explored the robustness of flood frequency estimates obtained by recent index flood estimators such as the regional GEV/PWM and regional WAK/PWM, mainly with respect to the underlying flood distributions, the regional heterogeneity in the moments of these distributions, and the record length over the region. They show that the three-parameter GEV/PWM method yields estimates that are stable relative to modest regional heterogeneity in the coefficient of variation and relative to the regional variation in the skew coefficient, only if the regional mean coefficient of variation is not too high.

Hebson and Wood [1982] found that the flood distribution derived from the geomorphologic unit hydrograph (GUH) model of catchment response shows good agreement with historical data. Wood and Hebson [1985] also developed a dimensionless geomorphologic unit hydrograph (DGUH) that was used to derive a flood-frequency distribution. They did not show whether this distribution improved the quantile estimates; it was also obvious that more research needed to be done.

Bayesian methods have often been sought to improve the estimates of the flood-frequency distibution parameters. For example, Vicens et al. [1975] tried to incorporate the uncertainties associated with the estimation of streamflow parametersinto the generation of synthetic streamflows. They concluded that, overall, the use of Bayesian methods leads to better designs under uncertainty conditions. A similar approach was adopted by Wood and Rodriguez-Iturbe [1975], who used a composite Bayesian distribution to take into account both parameter uncertainty and model uncertainty.

2.4 Risk Analysis

A detailed taxonomy of the numerous studies done in risk analysis will not be provided here. Rather, the focus will be on papers and books that have contributed to the development of risk analysis as a discipline. A section of this survey will be on low-probability/high-consequence risk analysis. Various papers that are at the basis of this present work will also be discussed.

Lowrance's [1976] defines risk as "a compound measure of the probability and magnitude of adverse effect." Lowrance also discussed in detail classes of hazard and their different characteristics. He described a four-step procedure to assess risk. Moreover, he set several guidelines for the judgment of the acceptability of risk.

In a similar fashion, the Committee on Institutional Means for Assessment of Risks to Public Health [NRC, 1983b] recommended a four-step procedure for measuring risk, but used different terminology.

Fischhoff et al. [1981] reached the heart of the problem of judging safety by asking "How safe is safe enough?" To define acceptable risk, he outlined five steps: (1) specify objectives, (2) define the options including the no-action one, (3) identify all consequences, (4) specify desirable consequences and their likelihoods, and (5) analyze the alternatives and select the best one. A detailed discussion of the problems associated with finding acceptable risk was also provided.

Starr [1969] presented a methodology based on historical data to quantify the measure of benefit relative to cost. He made an interesting distinction between voluntary and involuntary risks. In the first case, the individual participates voluntarily in a risky activity, while in the second case the individual's participation is decided by a society which is controlled by a regulating body. He concluded by suggesting that the risk of death from disease can be used as a yardstick for establishing risk acceptability.

More recently, Starr [1985] strongly recommended that emphasis should be shifted from the quantitative assessment of the probability and consequence of rare events to a management program of these extreme events that would give more importance to a positive human intervention. He believes that such an approach will "create the public confidence needed for public acceptance of new technologies with their accompanying uncertainties." Starr also affirms that the risk management program will not be able to gain the trust of the public unless there is a reasonable chance that the program will be successful in avoiding extreme situations with catastrophic implications. He did not, however, examine the means needed to determine whether a risk management program actually has a reasonable chance for success. For example, we might need to measure with some accuracy the risks corresponding to different options in management policies. But, Starr seemed more concerned about generating public confidence in risky technologies than in trying to address the issue of effective management of risk.

Kaplan and Garrick [1981] introduced a quantitative definition of risk, where risk is defined as the set of triples:

$$Risk=\{<s_i,p_i,x_i>\} \quad i= 1,2,\ldots,N$$

where s_i = scenario identification
p_i = probability of that scenario
x_i = the measure of damage for that scenario

They also defined the risk curve as the exceedence frequency of damage; it can be obtained from the set of triples. They next described as "misleading" the traditional definition of risk as probability times consequence and prefer, instead, in keeping with the set of triples idea, to say that "risk is probability and consequence." The definition was also extended to include uncertainty and completeness and to permit the use of Bayes's theorem. Finally, notions of relative risk, relativity of risk, and acceptability of risk were discussed in some detail.

Moser and Stakhiv [1987] discuss the various risk evaluation frameworks used for public decision making, particularly in the area of dam safety. They also show how the use of design standards can result in non-uniform protection from hazards and can lead to ignorance of the magnitude of potential damage and human loss. Next, they presented (as depicted in Fig. 2-1) a categorization of the various methods used in dam risk analysis. Three main categories were examined: (1) the cost-effectiveness approach, which limits the role of the analyst to a search for the least-cost design for given fixed standards and criteria, (2) benefit-cost analysis, which allows choice of the solution that satisfies the constraints and generates the greatest net benefits, and (3) the multiobjective approach, where no predetermined decision rules are used and where benefits, costs, and reduction of risk (e.g. loss of life) are often considered as distinct objectives.

Moser and Stakhiv also identified five sources of uncertainty in dam safety risk analysis:

- hydrologic uncertainty (probable maximum precipitation, probable maximum flood, antecedent conditions)
- dam structural reliability (static, dynamic loading; auxiliary spillway failure, breaching characteristics; overtopping duration; extent of freeboard use)
- resevoir and downstream routing uncertainty (hydraulic characteristics; floodwave travel time, inundation depth; flow velocity)
- flood damage uncertainty (forecasting of economic development; population forecasts; time-dependent damages for recreation and agriculture; loss of communication networks)
- uncertainty about the effectiveness of alternative fixes (evacuation and warning systems; widening the spillway; use of freeboard, etc.)

The use of sensitivity analysis could explicitly help evaluate these uncertainties. Moser and Stakhiv have, in addition, divided the risk analysis procedures into three parts: risk assessment, risk evaluation, and risk management. They also discuss risk-benefit analysis (or risk-cost analysis)

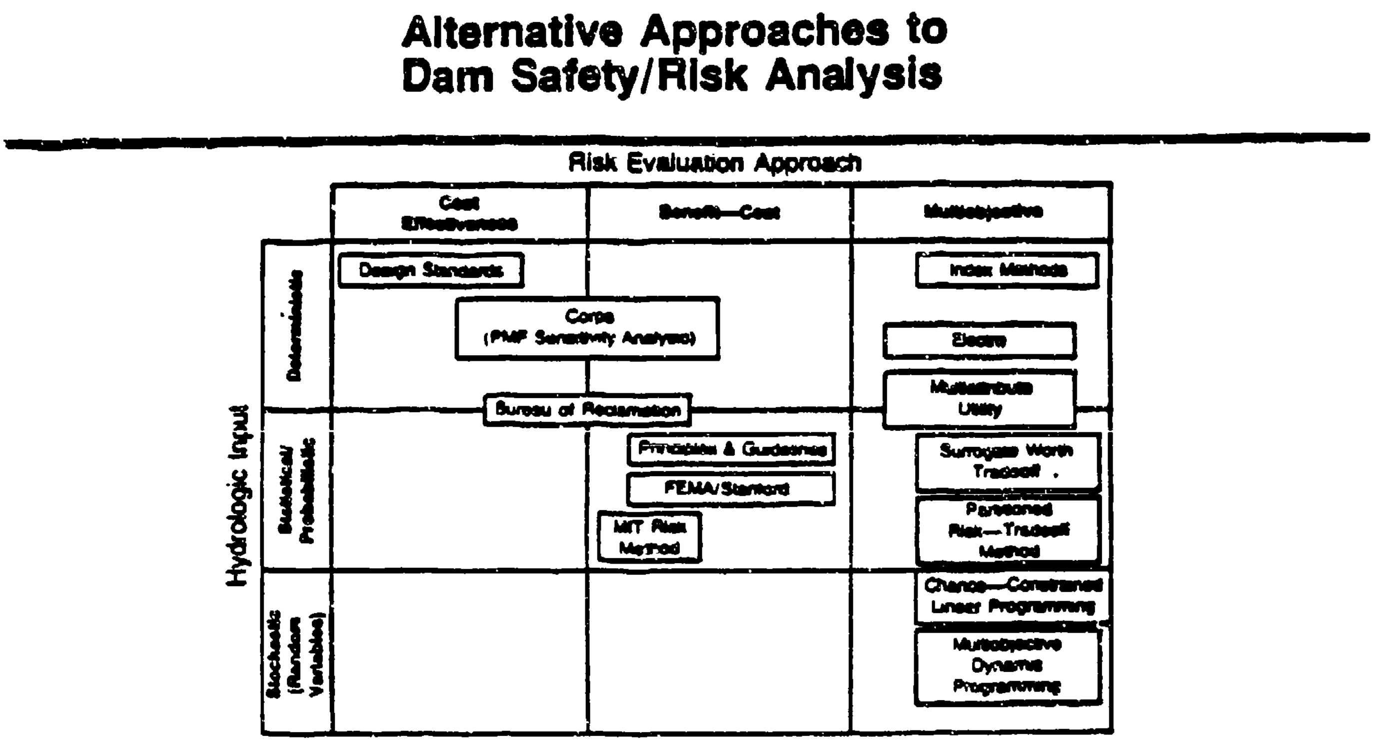


Figure 2-1 Alternative approaches to dam safety/risk analysis (source: Moser and Stakhiv, Risk Evaluation Frameworks for Public Decisionmaking, 1985)

and state that it is the approach that takes into consideration the economic costs and risks in the most complete fashion within the traditional water resource decision framework, as defined by the WRC's "principles and Guidelines" (1983).

They show that there is considerable uncertainty in the estimation of the magnitude of the PMF and the probability of the PMF, and that risk-cost analysis is extremely sensitive to the value of the return period of the PMF. Moser [1985a] even said that

> as long as there are significant disagreements about the probability distribution of rare floods, risk analysis will not provide a definitive answer on the appropriate level of hydrologic safety in dam design.

Sage and White's paper [1980] included a survey of the various methodologies in risk and hazard assessment with strong illustrations of the many dimensions and concerns in risk analysis. The authors also presented a methodological framework for risk analysis based on systems engineering methods. Future methodological needs in the field were also discussed.

The uncertainty/sensitivity index method (USIM) (see Haimes and Hall [1977], Haimes [1982], or Chankong and Haimes [1983]) can be used to assess and minimize the effect of uncertainties and errors on the decision-making process. These uncertainties are associated with six major parts of risk analysis: the model's topology, parameters, scope, data, the optimization techniques used for solution, and human subjectivity. Sensitivity can be considered as an objective function and be used with the original cost function in a multiobjective optimization analysis. For example, if $f_1(x,\alpha)$ is the cost function, where x denotes the model's uncertain decision variable and α denotes the model's parameter, then $f_2(x,\alpha)$, the objective function which represents the sensitivity index, is defined as

$$f_2(x,\alpha) = \left[\frac{\delta f_1(x,\alpha)}{\delta\alpha} \right]^2$$

It is obvious that the above two objectives, considered together, form a multiobjective optimization problem that can, perhaps, be solved by using the surrogate worth trade-off method (SWT).

Leach and Haimes [1985] combined the PMRM with the multiobjective, multistage impact analysis method (MMIAM), forming a risk analysis methodology that explicitly includes time as a dimension. This methodology, the multiobjective risk-impact analysis method (MRIAM), was applied to a hypothetical model representing impact in terms of resource damage of pollutant emissions on an environmental system over a number of years. The PMRM, which has already been described in Chapter 1, generates the risk functions. These functions are then used by the MMIAM to develop trade-offs between different objectives, including the risk objectives, at different stages and in a multiobjective framework. The MMIAM, which was introduced by Gomide [1983] and Gomide and Haimes [1984], defined impact analysis as the study of the effect of decisions upon the decision making problem. In addition, Leach [1984] has developed theoretical extensions to the PMRM such that multidimensional decision variables and different types of damage can be included in the use of the PMRM.

2.5 Low-Probability/High-Consequence Risk Analysis

The Society for Risk Analysis organized an international workshop in 1982 on, "Low-Probability/High-Consequence Risk Analysis: Issues, Methods, and Cases Studies," that was held in Washington, D.C. A number of papers were presented, the most relevant (for our purposes) of which will be reviewed here.

Martz and Bryson [1984] proposed a Bayes/empirical Bayes data-pooling procedure which, by combining five types of data, could improve the accuracy of the quantitative assessment of risk. But it is not obvious whether this procedure actually significantly enhances the precision of the risk estimates. Moreover, the validity of the choice of the prior distribution is questionable. The authors illustrated the procedure by using it in assessing the probability of failure of a hypothetical dam.

Vohra [1984] reviewed the use of the dose-effect model, the regression model, and the event-tree and fault-tree model for assessing risks of low-probability/high-consequence (LP/HC) events. He found that all these methods possess uncertainties. Vohra also presented a generic quantitative definition of risk that avoids the drawback associated with the use of expected value, that is, equating low-probability/high-consequence events with high-probability/low-consequence events. He favors the following definition of risk:

$$\text{Risk} = \sum_{i=1}^{n} p(i) \times C(i) \times W(i)$$

where p(i) = probability per unit cause of an event i
C(i) = consequence of an event i
W(i) = weight factor for event i

Vohra argued that the weight factor W(i) could be used to account for the higher effect of catastrophic events on society. Unfortunately, Vohra did not provide any quantitative application of this definition of risk; he also did not provide any guideline or procedure for choosing the values of the weight factors. On the other hand, he followed Kaplan and Garrick's recommendations by describing the risk situation by a family of risk curves to account for uncertainties.

Ballestero and Simons [1984] presented a causal analysis approach to estimate low-probability flood events. First, available extreme-event data are grouped into subsets corresponding to the different physical processes that cause the extreme floods. Next, flood-frequency distributions are fitted to each subset, and are then are aggregated to give a joint probability distribution. The case study provided did not actually show how the knowledge of the joint-probability distribution will be used to predict low-probability flood events.

Wagner et al. [1984] examined a methodology to be used in the investigation of the effects of floods on nuclear power plant safety systems. They provided an accident sequence occurrence frequency equation that could be used to combine the probability of floods with their impact in a probabilistic risk assessment.

Barlow et al. [1984] used compounded Kalman filtering for modeling stochastic processes such as block and trickle special nuclear material losses. The Bayesian approach that they used, although quite involved, seemed to provide a significant addition of information about this LP/HC event.

Most of these authors presented an LP/HC risk analysis that focused mainly on the estimation of the probability of extreme events. They often discussed methods to quantify losses caused by LP/HC events, and most used the "expected value" approach to express risk the economic consequences of LP/HC events. However, they failed (Vohra excluded) to discuss other methods that would allow analysis to combine the estimates of the probability of LP/HC events with estimates of the economic impact of these LP/HC events so that the most appropriate way to represent risk of events could be determined.

CHAPTER 3

BACKGROUND OF THE CASE STUDY

In this chapter, the approach to risk analysis of dam safety used by the U.S. Army Corps of Engineers will be discussed. A model that was developed at the U.S. Army Institute for Water Resources will be presented. It is this model that we will be using to illustrate an application of the PMRM.

3.1 The Corps Approach to Risk Analysis of Dam Safety

Recently, the U.S. Army Corps of Engineers has been trying to encourage the use of risk analysis in dam safety studies. In particular, it has suggested that risk analysis could be a valuable tool in evaluating alternative modification options of existing dams to prevent hydrologic deficiences. In a letter dated April 8, 1985, the U.S. Army Corps of Engineers [1985a] stated its central criterion:

> The base safety standard will be met when a dam failure related to hydrologic capacity will result in no significant increase in downstream hazard (loss of life and economic damages) over the hazard which would have existed if the dam had not failed. Recommendations for modifications that would accommodate floods larger than the flood identified by the base safety standard must be supported by an analysis that presents the incremental costs and benefits of the enhanced design in a manner that demonstrates the merits of the recommendation.

The Corps required that, for each alternative remedial action, the relationship between flood flows and both economic damages and loss of life should be evaluated under two conditions -- with and without dam failure. The results obtained are to be used in an incremental cost analysis framework to allow the decision maker to evaluate the different scale combinations of modifications needed to improve the existing dam structure to a safe level (the base safety standard). The comparison of the total average annual benefits with the annualized modification costs would be the final step in the evaluation process, should the decision be made to justify a management measure beyond the base safety standard based solely on incremental cost analysis and comparison of with and without dam failure.

The Corps also recommended that the analyst should allow an appropriate freeboard necessary to accommodate any winds or waves that might occur in the reservoir. Moreover, the calculation of the size of the population at risk should be based on more than just the population living downstream. The calculation should take into consideration:

prefailure flooding, warning time available, evacuation opportunities and other factors that might affect the occupancy of the incrementally inundated area at the time the failure occurs.

The Corps believes that the amount and quality of information generated by risk analysis will be valuable in the decision-making process; therefore, this information should be presented to the decision maker in a form that gives him a better understanding of the trade-offs involved. Finally, the Corps has recommended a format of display to be used in showing information on downstream hazard and on modification costs, and to be used also in the application of benefit-cost analysis (Stakhiv and Moser [1986]).

3.2 <u>The U.S. Army Institute of Water Resources Model</u>

Moser and Stakhiv of the U.S. Army Institute of Water Resources (IWR) developed a simulation model of dam failure on LOTUS 123 spreadsheets to complement both phases of risk analysis (hazard assessment and risk evaluation). Four sources of economic benefits and costs were considered: (1) prevention of downstream property damages due to failure, (2) preservation of benefits from the reservoir outputs, (3) construction costs for the modification of the spillway size, and (4) downstream property damages when no failure occurs. Also measured was the population at risk (PAR), the threatened population (TP) and the loss-of-life (LOL). The hazard assessment phase <u>could</u> use either economic damages or LOL as the decision criterion to justify setting the new Base Safety Standard.

Moser [1985b] provided a detailed description of the IWR dam safety risk-cost analysis model, which is used extensively in the rest of this chapter. He followed the assumptions in the Corp's guidelines, that overtopping in excess of the assumed safe amount would cause the dam to fail with certainty. Other circumstances that might cause the dam to fail were ignored for the sake of simplification.

Two preventive remedial actions are of interest: widening the spillway and raising the dam's height. Inherent to each one of these actions is a trade-off between two situations. For example, the widening of the spillway reduces the chances of a failure caused by rare floods with high magnitudes by overtopping of the dam; but, on the other hand, greater damage is incurred downstream by medium-sized floods that pass through the spillway. Similarly, augmenting the dam's height reduces the likelihood of a dam failure but <u>increases</u> the <u>severity</u> of downstream damages in the event of failure. This reflects an incommensurable trade-off in risk reduction. Each alternative can meet a stated design objective, but the damages occur in different parts of the frequency spectrum. The expected-value approach cannot capture this risk-reduction.

Moser adopted the common engineering approach of simulating the routing of alternative flooding events through the reservoir to obtain estimates of the failure and non-failure downstream flows and inundation levels. The simulation model also included a stage damage relationship for the inundated areas downstream. Results which were obtained from the previous steps were then used to perform a net benefit analysis.

The simulation model was developed on LOTUS 123 spreadsheets because they allow great flexibility and add useful graphic capabilities. However, because of memory constraints, some sections of the computer model must be linked manually. The model has two main subdivisions: hydrologic and economic.

Of particular interest is the hydrologic subdivision, which contains a dam and reservoir model. To construct the model of a specific dam first, various categories of information such as the dam's dimensions, the spillway's dimensions, the outlet's dimensions, and the storage volume must be specified. Moser's setting of the above parameter values corresponds to an approximate model of the Tomahawk Dam and Reservoir (see Fig. 3-1) located in Ohio.

Different design options are available to allow the user to change the spillway's width, the dam's height, and a few other characteristics of the model, as depicted in Fig. 3-2. This feature will be used in the analysis of the different structural modifications. The different combination of scales of the remedial actions, which combine changes of the spillway width and the dam height were reduced to sixteen discrete alternatives.

The peak rate of inflow as a percentage of the probable maximum flood (PMF) must be specified. The PMF was assumed here to be 432,000 cfs instead of the actual value, which for the Tomahawk Dam is 380,000 cfs. This arbitrary change will have the effect of reducing the level of safety provided by the dam, the fact being that the Tomahawk Dam is not as unreliable as would be desired for the purposes of this study.

When the computerized model is executed, the hydrologic model routes the specified peak inflow event through the dam and calculates the corresponding peak outflows for both cases of dam failure and nonfailure. A normalized hydrograph is first used to generate the rate of inflow in 2-hour increments for each peak rate of inflow. Then for each increment, the total volume of inflow, the head at the spillway, the total head at the dam, and the rate and volume of outflow are calculated iteratively. The basic equations used are:

$$V_1 = \frac{(Q_0^I + Q_1^I)}{2} \times 7{,}200$$

$$H_i = f(v_i)$$

$$Q_i^0 = 3.33 \times L \times H_i^{1.5}$$

$$V_{i+1} = V_i + \frac{(Q_{i+1}^I + Q_i^I)}{2} x7{,}200 - \frac{(Q_{i-1}^0 + Q_i^0)}{2} x7{,}200$$

where V_1 = addition in cubic feet to volume of storage during first 2-hour period

Q_0^I = initial rate of inflow in cfs, assumed equal to zero

Q_i^I = rate of inflow in cfs at end of i<u>th</u> 2-hour period

H_i = head in feet at spillway after i<u>th</u> period inflow but prior to outflow

A = reservoir surface area in square feet, assumed constant

Q_i^0 = rate of outflow in cfs through spillway

L = width in feet of spillway

V = net volume of storage in cubic feet after inflow and outflow outflow of i<u>th</u> period plus inflow in i+1<u>th</u> period but prior to i+1th period outflow volume

If the dam is overtopped, then the nonfailure outflow rate is assumed to be equal to the sum of the rate through the spillway and the rate over the top of the dam. It is also assumed that a breach is initiated if the dam is overtopped for more than two hours. Once this occurs the breach's dimensions increase with time. The model will calculate peak outflows at the breach but it will also continue to calculate nonfailure outflows as if the dam was not breached.

	A	B	C	D	E	F
5		DAM				
6		----------				
7		Elevation of Dam Crest			910.00	f.t.,m.s.l.
8		Elevation of Stream Bed			799.20	f.t.,m.s.l.
9		Dam Heighth			110.8	ft.
10		Dam Length			2330	ft.
11		Freeboard			3	ft.
12		Initial Reservoir Surface Area			7950	acres
13						
14		OUTLET				
15		----------				
16		Outlet, Invert Ele.			799.20	f.t.,m.s.l.
17		Outlet Conduit Diameter			20	feet
18		Number of Conduits			2	
19		Power Outlet, Invert Ela			NONE	f.t.,m.s.l.
20		Power Conduit Diameter			NONE	ft.
21		Number of Concuits			NONE	
22						
23		SPILLWAY				
24		----------				
	A	B	C	D	E	F
25		Elevation of Spillway Cr.			890.00	f.t.,m.s.l.
26		Spillway Design Depth			17.00	ft.
27		Spillway Width			620	ft.
28		Approach Depth			10	ft.
29		Maximum Spillway Discharge Coeff.			4.186	
30		Overtopping Discharge Coeff.			2.500	
31						
32		AVAILABLE STORAGE				
33		-----------------				
34		Storage at Spillway Crest			285000	acre-ft
35		Initial Reservoir Storage			285000	acre-ft
36		Flood Control Storage			0	acre-ft
37		Free Storage			0	acre-ft
38		Initial Water Surface Ela.			890.0	f.t.,m.s.l.
39		INFLOW EVENT				
40		------------				
41		Peak PMF Inflow			432000	cfs
42		Peak Inflow			432000	cfs
43		Hydrograph Time Increment			7200	sec
44		Total Flood Event Volume			1207777	acre-feet

Figure 3-1. Dam, Outlet, Spillway, Storage and Inflow Characteristics

	A B C D	E	F
45			
46	DESIGN PARAMETERS		
47	------------------		
48	Increment in Dam Height	0.0	ft.
49	Inflow Event as % PMF	1.000	PMF
50	Spillway Capacity	1.00	Ex. Cap
51	Freeboard standard	3	ft.
52	Overtopping Allowance	0.0	ft.
53	Outlet Gate Open (proportion)	0.75	
54	Power Outlet Open (1=YES,0=NO)	NONE	
55	Spillway Gates Open (1=YES,0=NO)	NONE	
56			
57	SIMULATION OUTPUT		
58	------------------		
59	Peak Outflow (spillway+outlet)	332058	cfs
60	no failure (total)	428924	cfs
61	failure	680371	cfs
62	Water Surface Ela. at Peak	913.52	f.t.,m.s.l.
63	Freeboard at Peak Water Ela.	-3.52	ft.
64	Error in Breach Side Slope	0	

Figure 3-2. Design options and output of simulation

Next, Moser used a shortcut method, presented by McCann et al. [1983], to calculate the peak flows and the inundation depths at different points downstream from the dam. The following relationships were used for this purpose:

$$Q(x) = 10^{(\log Q_D - kx)}$$

where

$Q(x)$ = rate flow in cfs at distance x downstream from the dam

Q_D = $\max_i Q_i^O$, peak rate of outflow in cfs

k = a constant representing the contour of the flood plain

x = distance downstream from the dam in miles

and

$$d_s = \{.7533\ Q(x)\ (80+mx)^{-5/3}[1+(80+mx)^2]^{1/3}\}^{3/8}$$

where

d_s = depth in feet of flow (stage at reach number s) at x miles below dam

m = parameter describing the rate at which the flood plain broadens out, assumed equal to 0.005

To estimate damages at all inundation levels, a single-stage damage function was used. It was assumed that the stage-damage relationship is free of uncertainties and is described by:

$$D_s = [0.7\ (\hat{d}_s)^2 - 0.0085\ (\hat{d}_s)^3] \cdot R_s$$

where

R_s = length of reach number s in miles

$\hat{d}_s = (d_s - d_{s+1})/2$ = average depth in feet of inundation along reach number s

d_s = depth in feet of inundation at start of reach number s below dam

D_s = damages in millions of US$ along reach number s

To obtain the total damages predicted to occur for the specified peak inflow, the damages across all downstream reaches are summed.

As has been stated earlier, this model contains a section that performs an economic analysis. This section will not be discussed here, because a multiobjective perspective will be used instead of the traditional benefit-cost analysis approach that was employed in the model.

It is felt that the Corp's model (Moser, 1985a) is an adequate abstraction of a real-world dam problem that is suitable to the purposes of this study. It is flexible relative to the general dam and reservoir characteristics. The user can also specify a number of parameters such as the spillway's width, the dam's height, the peak inflow, the freeboard standard, the overtopping allowance, and the opened proportion of the outlet gate. Moreover, the model's equations are not oversimplified and are sometimes quite sophisticated.

CHAPTER 4

APPLICATION OF THE PARTITIONED MULTIOBJECTIVE RISK METHOD TO THE CASE STUDY

This chapter offers a discussion of the application of the partitioned multiobjective risk method (PMRM) to the dam safety problem that was introduced in Chapter 3. First, a brief overview of the PMRM, a method, that is described in detail by Asbeck [1982] and by Asbeck and Haimes [1984] is presented. Then, the assumptions related to the issue of antecedent floods are examined. Next, it is shown how to obtain the flood-frequency distribution for both ordinary and rare floods; different procedures will be used to obtain this distribution, depending on the type of floods studied. In the following two sections, some relationships are derived through approximations, that allow us to find the probability density function of damages. This function is needed for the application of the PMRM, a topic that is discussed in the last section after the scenarios are defined and the cost estimates are derived. A procedure is described that will facilitate the application of the PMRM to the case study; this procedure can even be used, as will be seen in Chapter 6, for problems with the same general structure as this case study.

4.1 An Overview of the PMRM

The PMRM is based on the use of conditional expectation, which is defined as follows. Given $p_X(x)$, the marginal probability distribution of the random variable X, and assuming that

$$p_X(x) \begin{cases} \geq 0 & \text{for } 0 \leq x \leq \infty \\ = 0 & \text{for } -\infty \leq x < 0 \end{cases}$$

the conditional expectation of an event $D=\{x/\ x\epsilon[a,b]\}$ is given by

$$E[X \mid D] = \frac{\int_b^a x\, p_x(x)\, dx}{\int_b^a p_x(x)\, dx}$$

4.1.1 Finding Marginal Probability Density Functions

The use of the PMRM requires knowledge of the probability density function of losses and this funciton is often dependent on the policy option. Let this function be denoted by $h_x(x,s_j)$, where x is the magnitude of losses and s_j (j=1,2,...,n) is the policy option or scenario.

Given $h_x(x,s_j)$, it is possible to calculate the exceedence probability function, defined as

$$1 - H_x(x,s_j)=1 - \int_0^x h_x(y,s_j)\,dy \qquad j=1,2,...,n$$

4.1.2 Partitioning the Probability Axis

The next step is to partition the exceedence probability axis into a set of ranges. These ranges should be compatible with the nature of the problem of interest. The partitioning should be done in such a way that the decision maker's understanding of the problem will be enhanced. In other words, the analyst should try to capture the subtleties of the problem by adequately determining the number of ranges m into which he will partition the probability axis and the positions of the partitioning points α_i (i=1,2,...,m+1). The partitioning points on the risk curves should be exactly the same for all the various policy options. Note that partitioning the cumulative probability axis would produce the same final results as partitioning the exceedence axis.

4.1.3 Mapping Partitions to the Damage Axis

Before the conditional expectations are used, the values of the partitioning points of the probability axis should be mapped onto the damage axis. Therefore, for each partitioning point α_i (i=1,2,...,m+1) and each policy option s_j (j=1,2,...,n), it is necessary to find an $a_{ij} \geq 0$ such that

$$1 - H(a_{ij}\ ,\ s_j) = \alpha_i$$

Notice that a_{ij} is actually the projection of the partitioning point α_i on the damage axis.

These values of a_{ij} (i=1,2,...,m+1 ; j=1,2,...,n) will be used to calculate the conditional expectations for the m domains of the damage axis that correspond to the m ranges of the cumulative probability axis. These domains have been defined in the following way:

$D_{1j} = [a_{1j}, a_{2j}]$ $\quad j=1,2,\ldots,n$

$D_{ij} = (a_{ij}, a_{i+1,j}]$ $\quad j=1,2,\ldots,n$

$i=2,3,\ldots,m$

(Here $c \varepsilon (a,b]$ means that $a < c \leq b$)

4.1.4 Finding Conditional Expectations

The conditional expectations can be computed using

$$E[X \setminus D_{ij}; s_j] = \frac{\int_{a_{ij}}^{a_{i+1,j}} x \, h_X(x,s_j) \, dx}{\int_{a_{ij}}^{a_{i+1,j}} h_X(x,s_j) \, dx} \qquad \begin{array}{l} i=1,2,\ldots,m \\ j=1,2,\ldots,n \end{array}$$

The denominator can be reduced to

$$\int_{a_{ij}}^{a_{i+1,j}} h_X(x,s_j) \, dx = \alpha_i - \alpha_{i+1} \quad i=1,2,\ldots,m$$

Since the partitioning points are invariant for all policy options, the value of the denominator, which can be considered as a weighting factor, will be unaffected by the policy option. On the other hand, the damage regions $[a_{ij}, a_{i+1,j}]$ $(i=1,2,\ldots,m; j=1,2,\ldots,n)$ vary with the policy options.

If the damage axis had been partitioned, a reversal of the above would occur. The weighting factors would be variant while the damage regions would be invariant. The task of Chapter 5 is to determine which of these partitioning approaches would be more appropriate for the problem.

4.1.5 Generating Functional Relationships

A set of risk functions $f_i(s_j)$ can be generated from the conditional expectations by setting

$$f_{i+1}(s_j) = E[X \setminus D_{ij}; s_j] \qquad \begin{array}{l} i=1,2,\ldots,m \\ j=1,2,\ldots,n \end{array}$$

For a given policy s_j, each $f_i(s_j)$ represents the particular risk associated with the corresponding partitioning range $[\alpha_i, \alpha_{i+1}]$.

In addition to the m risk functions defined above, the unconditional expectation (or expected value of damages), which still has some use, should be computed for all policy options. It will be utilized to form the m + 1st risk function that will be denoted by $f_{m+2}(s_j)(j-1,2,\ldots,n)$.

Moreover, the cost function is a vital part of the analysis in many problems. It represents the costs associated with the different policies. The notation $f_1(s_j)$ $(j-1,2,\ldots,n)$ will be used to represent the cost function.

If it can be assumed that the decision policy is continuous between s_j and s_{j+1} $(j-1,2,\ldots,n-1)$ and if, in addition, it can be assumed that the m + 2 risk functions defined above are continuous in a simple way, then by the use of regression, it is possible to fit (for $i-1,2,\ldots,m+2$) a smooth curve $f_i(s)$ to the points $\{s_j, f_i(s_j)\}$ $(j-1,2,\ldots,n)$.

4.1.6 Employing the Surrogate Worth Trade-off Method

At this stage, the proposed decision-making problem involves m + 1 risk objective functions and one cost objective function. Since as little information as possible should be lost in the analysis, we need to try to make use of all the objective functions. Only a multiple-objective decision-making methodology would be appropriate in this case. The surrogate worth trade-off (SWT) method is one such method. Its advantage over other methodologies is that it allows the decision maker (DM) to express his preferences during the decision-making process. Basically, the SWT method provides the DM with the Pareto optimal policies and the associated trade-offs among the various objectives.

Of particular interest are the trade-offs between f_1, the cost function, and the m + 1 risk objectives.

The multiobjective problem can be formulated as

$$\min_{s_j \in S} \begin{cases} f_1(s_j) \\ f_2(s_j) \\ \vdots \\ f_{m+2}(s_j) \end{cases}$$

where S is the set of feasible decisions.

Assuming that s is a continuous variable and that the objective functions are continuously differentiable over s, we can reformulate the problem as

$$\min_{s \in S} \; f_1(s)$$

$$\text{s.t.} \quad f_i(s) \le \varepsilon_i \qquad i=2,3,\ldots,m+2$$

$$\varepsilon_i = \bar{f}_i + \bar{\varepsilon}_i$$

where $\bar{f}_i$ is the minimum value of f_i, and $\bar{\varepsilon}_i$ represents the deviations from the minimum value. The above problem can be solved to find a set of noninferior solutions of a multiobjective optimization problem (MOP) from which the final preferred solution can be chosen by the DM.

Haimes et al. [1975] define the noninferior solution as being a solution in which no decrease can be obtained in any of the objectives without causing a simultaneous increase in at least one of the objectives. It can be shown that any solution of Problem P is a noninferior solution of MOP, and that if $\lambda_{12},\ldots, \lambda_{1,m+2}$ are corresponding (optimum) multipliers associated with a noninferior solution (which is a solution to P), then $\lambda_{1i} > 0$ for at least one $i = 2,\ldots,m+2$ (see, for example, Chankong and Haimes, 1983).

Moreover, the multipliers are required to satisfy the Kuhn-Tucker conditions for the above Problem P. It can be shown that for any $\lambda_{1i} > 0$, (see Chankong and Haimes [1983] and Haimes and Chankong [1979]) we have:

$$\frac{\delta f_1}{\delta f_i} = -\lambda_{1i}(s) \qquad i=2,3,\ldots,m+2$$

From this, the negative of a multiplier λ_{1i} is equivalent to the trade-off rate function $T_{1i}(s)$, which is defined to be the rate of change of $f_1(s)$ with respect to $f_i(s)$.

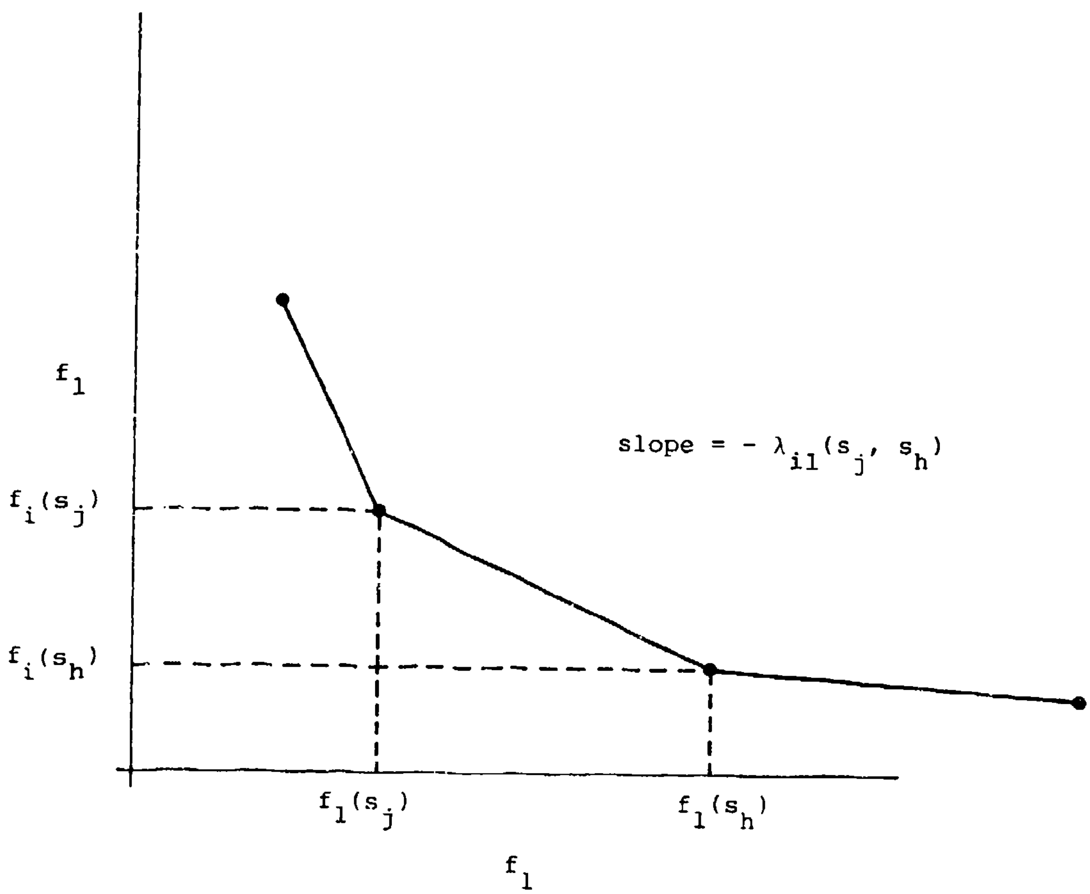
Haimes et al. [1975] and Chankong and Haimes [1983] discussed the various approaches that can be used to generate the required noninferior solutions and to construct the trade-off rate functions. For example, in one approach, we can vary ε_i parametrically.

The trade-off rate function between $f_1(s)$ and $f_i(s)$ can also be defined as

$$\lambda_{i1}(s) = -\frac{\delta f_i(s)}{\delta f_1(s)} = \frac{1}{\lambda_{1i}(s)} \qquad i=2,3,\ldots,m+2$$

Notice that the trade-off rate functions $\lambda_{i1}(s)$ are defined as the negative of the partial derivatives of f_i relative to f_1. Thus, if the decision variable is discrete, partial derivatives do not exist and consequently the trade-off rate functions cannot be used. In this case, the concept of total trade-off is introduced, and it will be used in this work for discrete problems. When it is stated that the total trade-off between f_1 and f_i from s_j to s_h is $\lambda_{i1}(s_j,s_h)$, this means that by using policy s_h instead of policy s_j, a change of magnitude $\lambda_{i1}(s_j,s_h)$ in f_i will correspond to a change of 1 unit in f_1. The total trade-off is defined by

$$\lambda_{i1}(s_j,s_h) = -\frac{f_i(s_j)-f_i(s_h)}{f_1(s_j)-f_1(s_h)} \qquad \text{for } i=2,\ldots,m+2$$



For convenience, before calculating the total trade-offs the alternatives should be reordered to obtain an increasing function f_1. Notice that s_h has to be the alternative immediately next to s_j, according to this new order. Notice, also, that this trade-off is not partial, in the sense that all other objectives are unchanged. When going from one alternative to another, all objectives change, and the total trade-off must be evaluated by the decision maker considering all trade-offs simultaneously.

In this work, a heuristic procedure based on computer simulations was used to reorder the alternatives and to determine all noninferior solutions.

Other concepts that are central to this work must now be defined. The indifference band is defined as the subset of the set of noninferior solutions in which the worth of an improvement in one objective is equivalent (in the mind of the decision maker) to the correponding negative change in another objective. The preferred solution is any noninferior solution that belongs to the indifference band.

The aim of any multiobjective optimization procedure is to determine the preferred solution. To achieve this purpose, if the continuity assumptions hold, the SWT uses the surrogate worth function (SWF) associated with the risk objective function f_i and the cost objective function f_1. This surrogate worth function is defined as $W_{i1}(f_1)$. It is a monotonic function defined on the interval [-10, 10]; in our case it will be constructed as a function of f_1. In addition, it should satisfy [Haimes et al., 1975]:

$$W_{i1}(f_1) \begin{cases} > 0 & \text{when } \lambda_{i1} \text{ marginal units of } f_i(s_j) \text{ are preferred over one marginal unit of } f_1(s_j) \text{, given the level of achievement the objectives.} \\ = 0 & \text{when } \lambda_{i1} \text{ marginal units of } f_i(s_j) \text{ are equivalent to one marginal unit of } f_1(s_j) \text{, given the level of achievement of all the objectives.} \\ < 0 & \text{when } \lambda_{i1} \text{ marginal units of } f_i(s_j) \text{ are not preferred over one marginal unit of } f_1(s_j) \text{, given the level of achievement of all the objectives.} \end{cases}$$

$W_{i1}(\lambda_{i1})$, thus, reflects the DM's preference for the prescribed trade-off.

To determine the surrogate worth function, the analyst should ask the decision maker to assess the value of W_{i1} at a certain number of points that correspond to noninferior solutions. The functions $W_{i1}(\lambda_{21},\ldots,\lambda_{m+2,1})$ are then constructed, possibly by regression, foreach i=2,...,m+2. The analyst can then estimate the solution to the equation $W_{i1}(\lambda_{21},\ldots,\lambda_{m+2,1}) = 0$, i=2,...,m+2. The solution obtained, λ_{i1}, represents the preferred value of λ_{i1}. Finally, we can find s_j^* (the preferred policy option), $\lambda_{i1}^* = \lambda_{i1}(s_j^*)$, and $f_i^* = f_i(s_j^*)$ (for i=2,3,...,m+2) by solving the corresponding Lagrangian of Pr. lem P with multipliers fixed as found above.

If the continuity assumptions do not hold, the above discussion must be modified. Essentially, the total trade-off concept must be used rather than the partial trade-off concept. It is still possible to employ a function similar to the surrogate worth function; this will be called SWF2. This, however, can be written in the form of a table with each row entry corresponding to each discrete alternative. This function can thus be written as a function of f_1 (or any f_i) alone. Since the preferred solution must be chosen from the given discrete set, it is assumed that $W(f_1) = 0$ for at least one of these solution points. Such a point, therefore, is a candidate for the preferred solution. This SWF2 is denoted by $w(f_1)$. It is also a monotonic function of f_1 defined on the interval [-10, 10], and it should satisfy the same general properties of a surrogate worth function.

4.2 Antecedent Floods

In Chapter 3, it was explained how the IWR model was used to simulate the routing of various inflow events through the example dam and to quantify downstream damage caused by outflows from the dam. Simulations were performed for sixteen (16) alternatives that combined changes in the spillway's width and in the dam's height. Fifteen (15) inflow events varying in size from 0 cfs to 432,000 cfs (the assumed PMF)were routed for each alternative.

In the simulations, the following assumptions have been made concerning flood conditions antecedent to the routed flood inflow event:

- Since the Tomahawk Dam is operated mainly for flood control and therefore has no conservation or recreation pool, the initial reservoir storage is assumed to be only 300 acre-feet.

- No antecedent floods could either cause the dam to be overtopped or produce any downstream damages. This assumption is contradicted by the results of some simulations that have been performed, and it tends to artificially reduce risk estimates.

- If at the arrival of the routed inflow event, antecedent conditions have caused the pool to be filled to the spillway's crest, then it is assumed that the outlet will be opened to 75% of its capacity. If antecedent floods have not filled the reservoir, it is assumed that the routed inflow event will not result in any downstream damages. This assumption will have the same negative effect on risk estimates as the previous one.

- The antecedent flood and the following flood of interest represent two independent events.

- The probability of having two successive floods within a short period of time (in terms of days) equals the probability of having the same sequence of floods within a year.

Given these assumptions, it can be said that only inflow flood events following antecedent flood conditions that have filled the pool to the spillway's crest will produce downstream damages. Therefore it is possible to write:

Since

$$h_x(x) = PFP \times Pr(x\backslash \text{full pool}) + (1-PFP) \times Pr(x\backslash \text{no full pool})$$

and
$$Pr(x\backslash \text{no full pool}) = \begin{cases} 0 & \text{for } x \neq 0 \\ 1 & \text{for } x = 0 \end{cases}$$

then

$$h_x(x) = \begin{cases} PFP \times Pr(x\backslash \text{full pool}) & \text{for } x \neq 0 \\ PFP \times Pr(x\backslash \text{full pool}) + (1-PFP) & \text{for } x = 0 \end{cases}$$

where

x :downstream damages in millions of US$

$h_x(x)$:probability density function of damage

PFP :probability of having antecedent flood conditions filling the pool to the spillway's crest

Because a full pool has been assumed when inflow events have been routed in the simulations, it will be possible to estimate Pr(x\full pool), as it will be shown later in this chapter.

S ılations showed that, given an initial pool of 300 acre-feet (empty pool), antecedent floods of 101,952 cfs magnitude or larger will fill the reservoir to the top of the spillway's crest. The probability of having, in any year, inflows larger than 101,952 cfs or, equivalently, of having a full pool was found to vary from 4.53×10^{-3} to 6.54×10^{-5}, depending on the assumed flood-frequency distribution.

It was stated earlier that some of the assumptions made above were at odds with simulations that were performed, and that they would actually give risk estimates that are smaller than their actual values. These errors can be compensated for in an ad hoc manner by specifying an artificially large value to the probability of having a full pool (PFP).

Although, in the analysis, that follows, the PFP will be treated as a value between 0 and 1 to be determined by the DM, the recommendation made here is to assign the value 1 to the PFP. A priori, this number might seem to be too high, particularly in light of the fact that the actual PFP actually lies somewhere between 4.53×10^{-3} and 6.54×10^{-5}. However, an example can illustrate the point. An extremely severe antecedent flood, such as the probable maximum flood (PMF), will overtop the dam and cause it to fail in its present state, regardless of the reservoir's initial storage and of the outlet's status. For an empty pool, it will also cause high damages of magnitude x', and we can see that $Pr(x > x')$ should be between 10^{-4} and 10^{-7}, depending on the assumed value of the return period of the PMF. If the assumptions had been followed and the PFP had been given its actual value, then the of $Pr(x > x')$ would have been between 10^{-6} and 10^{-10}; thus, it would be smaller by two to three orders of magnitude than what it should be. When our assumptions are followed and PFP is set at 1, risk estimates will generally be obtained that are within the proper ranges; for the above example, it will be found that $Pr(x > x')$ has a value that belongs to the interval $[3 \times 10^{-4}, 8 \times 10^{-7}]$. These estimates might in fact be somewhat inflated and conservative, but not as much as might have been expected originally. It is obvious that this approach will still create errors in the risk estimates, but, since the same errors will be consistently repeated in the analysis for all the alternatives, they should cancel out each other when the scenarios' benefits and risks are compared. Therefore, these errors should have little impact on the DM's decision. Essentially, by setting PFP = 1, it will be possible to provide the DM with risk estimates that are within the proper ranges and that are sufficiently accurate. Therefore, it is anticipated that the decision-making process will not be seriously affected by the assumptions.

4.3 Flood-Frequency Distribution

In this section the flood-frequency distribution of the inflows to the Tomahawk Reservoir will be derived. Different procedures will be used, depending on whether the floods of interest are smaller or larger than the 100-year flood. The National Research Council [1985] report on dam safety showed that if the flood frequency distribution derived from 20 to 80 years of systematic records is extrapolated to floods larger than the 100-year flood, inaccurate estimates for floods with high return periods will be obtained. To simplify the analysis, no regional or historical information was taken into consideration in our calculations.

4.3.1 Flood-Frequency Distribution for Ordinary Floods

For floods smaller than the 100-year flood, systematic data records on maximum yearly inflows into the Tomahawk Reservoir were used. It has been assumed that peak flood inflows smaller than the 100-year flood follow a log-normal distribution. The log-normal distribution is in fact a special case of the log-Pearson type-III distribution that was recommended in Bulletin 17B [Interagency Advisory Committee on Water Data, 1982]. The method of moments was used to fit the log-normal distribution to the available data; the statistics for the available 24 years of record were computed. Let us denote the mean and the standard deviation of ln Q_i (Q_i is the maximum inflow for data year i) by m_1 and s_1, respectively, where

$$m_1 = \sum_{i=1}^{24} (\ln Q_i)/24$$

$$s1 = \sqrt{\frac{\sum_{i=1}^{24} \left(\ln Q_i\right)^2}{23} - \frac{\left(\sum_{i=1}^{24} \ln Q_i\right)^2}{23(24)}}$$

It was found that $m_1 = 9.5954$ and $s_1 = 0.3511$.

It has been assumed that the peak yearly inflows smaller than the 100-year flood follow the log-normal distribution function, which has the form

$$f_Q(q) = \frac{1}{q\ a\ (2\pi)^{1/2}} \exp\left\{-\frac{1}{2}\left(\frac{\ln q - m}{a}\right)^2\right\}, \quad q \geq 0$$

In this case m_1 and s_1 can be used respectively as the maximum likelihood estimators of m and a.

$$\Longrightarrow \begin{cases} m = 9.5954 \\ a = 0.3511 \end{cases}$$

Using the standard normal tables, the 100-year flood q_{100} can be found

$$F_Q(q_{100}) = 0.99 \Longrightarrow \frac{\ln q_{100} - m}{a} = 2.326$$

$$\Longrightarrow q_{100} = 33{,}258 \text{ cfs.}$$

4.3.2 Flood-Frequency Distribution for Rare Floods

For floods larger than the 100-year flood, the recommendations of the National Research Council [1985] on dam safety will be followed concerning the estimation of the flood-frequency distribution. Since there is much uncertainty about the form of the real flood-frequency distribution, the analysis here has considered four probability distributions that have often been used in the literature: log-normal, Pareto, Weibull, and Gumbel. In Chapter 5, the impact of the various assumed distributions on the decision-making process will be studied.

The log-normal distribution has been widely used as a flood-frequency distribution, in particular for floods with moderate return periods. The Pareto distribution (Pearson type-IV), which has a tail similar to that of the log-Gumbel, is often used by the Bureau of Reclamation as a flood-frequency distribution. The Weibull distribution is widely employed in reliability models; it takes on shapes similar to the gamma distribution. The Weibull distribution is also known as the extreme value type-III distribution of the smallest value. The Gumbel (or extreme value type-I) distribution is still very popular among European scientists and particularly among British scientists, who use it extensively as a flood-frequency distribution. The Gumbel distribution is also the limiting form to which the probability distributions of extreme values (largest values) from initial distributions with exponential tails converge. It seems, therefore, that the Gumbel distribution might be proper for representing maximum yearly floods, which can be considered as the extreme values of daily floods. Moreover, the Gumbel distribution has a thinner tail than the other distributions considered in this analysis.

The cumulative distribution derived from the assumed flood-frequency distribution between the probable maximum flood (PMF) and the 100-year flood will be interpolated, but first, it will be necessary to estimate T, the return period of the PMF. This is a task which involves many uncertainties and which yields, in general, inaccurate estimates. The return period of the PMF is sometimes estimated to be as low as 10^4, but the American Nuclear Society [1981], for example, has estimated it to be larger than 10^7. Therefore, it has been decided to perform a sensitivity analysis on the value of the return period of the PMF; the values 10^4, 10^5, 10^6, and 10^7 were examined. The following notation, $T_4 = 10^4$, $T_5 = 10^5$, $T_6 = 10^6$, $T_7 = 10^7$ will be used.

Next, the distribution parameters for the four assumed flood-frequency distributions and the four assumed return periods of the PMF will be derived.

Log-Normal Distribution

$$f(q) = \frac{1}{a\, q\, (2\pi)^{(1/2)}} \exp\left\{-\frac{1}{2}\left(\frac{\ln q - m}{a}\right)^2\right\}, \quad q \geq 0$$

If q is lognormally distributed, then y = (ln q - m)/a has a standard normal distribution, and therefore

$$F_Q(q) = F_Y\left[\frac{\ln q - m}{a}\right]$$

Using this relationship yields

$$\begin{cases} F_Q(q_{100}) = 0.99 \\ F_Q(\mathrm{PMF}) = 1-(1/T) \end{cases} \qquad \begin{cases} F_Q(33{,}258) = 0.99 \\ F_Q(432{,}000) = 1-(1/T) \end{cases}$$

$$\Longrightarrow \begin{cases} (\ln 33{,}258 - m)/a = 2.326 \\ (\ln 432{,}000 - m)/a = \begin{cases} 3.72 & \text{for } T=T_4 \\ 4.27 & \text{for } T=T_5 \\ 4.75 & \text{for } T=T_6 \\ 5.20 & \text{for } T=T_7 \end{cases} \end{cases}$$

The following results are derived by solving the above equations:

	$T=T_4$	$T=T_5$	$T=T_6$	$T=T_7$
a=	1.83938	1.31898	1.0578	.892171
m=	6.1337	7.34415	7.95166	8.33691

Pareto distribution

$f_Q(q) = (b\, a^b)/(q^{b+1}) \qquad a>0,\ b>0,\ q\geq a$

and $F_Q(q) = 1 - (a/q)^b$

Therefore, it is possible to write:

$$\begin{cases} F_Q(q_{100})=0.99 \\ F_Q(PMF)=1-(1/T) \end{cases} \Longrightarrow \begin{cases} F_Q(33{,}258)=0.99 \\ F_Q(432{,}000)=1-(1/T) \end{cases}$$

$$\Longrightarrow \begin{cases} (a/33{,}258)^b = 0.01 \\ (a/432{,}000)^b = 1/T \end{cases}$$

$$\Longrightarrow \begin{cases} b = \dfrac{\ln 0.01 - \ln (1/T)}{\ln 432{,}000 - \ln 33{,}258} \\ \\ a = 33{,}258\ (0.01)^{(1/b)} \end{cases}$$

And the following is obtained:

	$T=T_4$	$T=T_5$	$T=T_6$	$T=T_7$
b=	1.79600	2.69400	3.59199	4.4900
a=	2560.404	6018.72	9227.888	11925.07

Weibull Distribution

$$f_Q(q) = \left[\frac{c}{a}\right]\left[\frac{q}{a}\right]^{c-1} \exp\left[-\left(\frac{q}{a}\right)^{c}\right] \qquad q \geq 0$$

and $F_Q(q) = 1 - \exp\left[-\left(\frac{q}{a}\right)^c\right]$

It can then be written:

$$\begin{cases} F_Q(q_{100}) = 0.99 \\ F_Q(PMF) = 1 - (1/T) \end{cases} \Longrightarrow \begin{cases} F_Q(33,258) = 0.99 \\ F_Q(432,000) = 1 - (1/T) \end{cases}$$

$$\Longrightarrow \begin{cases} -(33,258/a)^c = \ln 0.01 \\ -(432,000/a)^c = \ln (1/T) \end{cases}$$

$$\Longrightarrow \begin{cases} c = \dfrac{\ln [-\ln (1/T)] - \ln [-\ln 0.01]}{\ln 432,000 - \ln 33,258} \\ a = \exp [\ln 33,258 - (1/c) \ln(-\ln 0.01)] \end{cases}$$

And the following is obtained:

	$T=T_4$	$T=T_5$	$T=T_6$	$T=T_7$
c=	.270325	.357350	.428454	.488572
a=	117.0521	463.3213	941.6713	1460.085

Gumbel Distribution (Extreme-Value Type-I)

$$f_Q(q) = \left[\frac{1}{a}\right] \exp\left[-\frac{q-m}{a}\right] \exp\left\{-\exp\left[-\frac{q-m}{a}\right]\right\}$$

and $F_Q(q) = \exp\left\{-\exp\left[-\dfrac{q-m}{a}\right]\right\}$

It is possible then to write

$$\begin{cases} F_Q(q_{100}) = 0.99 \\ F_Q(PMF) = 1 - (1/T) \end{cases} \Longrightarrow \begin{cases} F_Q(33{,}258) = 0.99 \\ F_Q(432{,}000) = 1 - (1/T) \end{cases}$$

$$\Longrightarrow \begin{cases} m - 33{,}258 = a \ln(- \ln 0.99) \\ m - 432{,}000 = a \ln \{- \ln [1-(1/T)]\} \end{cases}$$

When these equations are solved, the following is obtained:

	$T=T_4$	$T=T_5$	$T=T_6$	$T=T_7$
m=	-364620	-232087	-165787	-125995
a=	86492.4	57681.9	43269.3	34619.2

4.4 The Damage Array

For each inflow event routed through the Tomahawk Dam, the IWR model is used to compute the downstream damages. If the dam is overtopped, the model will then determine damages for both cases -- nonfailure and failure. Here, the dam is said to fail if it is breached by floods. In our analysis, it is assumed that the dam is breached as soon as the dam is overtopped for more than two hours.

4.5 The Probability Function of Damages

For each alternative s_j $(j=1,2,\ldots,n)$, we have calculated the fifteen damage values y_{kj} have been calculated that correspond to the fifteen routed inflows q_k $(k=1,2,\ldots,K;\ K=15)$, assuming that the reservoir is filled to the spillway's crest prior to design flood. There are therefore 15 data points (q_k, y_{kj}) for each of the n scenarios. The following notation will be used in the rest of this work:

s_j : alternative j $j=1,2,\ldots,m$; here $m=16$
q_k : inflow event k $k=1,2,\ldots,l$; here $l=15$

$y(q_k, s_j) = y_{kj}$:downstream damages in millions of US$ for inflow q_k under alternative s_j (assuming full pool and the outlet is open at 75%)

$x(q_k, s_j) = x_{kj}$:downstream damages in millions of US$ for inflow q_k under alternative s_j; $x_{kj}=y_{kj}$

$f(q)$: probability density function of inflow q

$F(q)$: cumulative probability function of inflow q

$\phi(y)$: conditional probability density function of damages given that antecedent floods have filled the reservoir

$\Phi(y)$: cumulative probability function of damages given that antecedent floods have filled the reservoir

$h(x)$: probability density function of damages

$H(x)$: cumulative probability function of damages

It has been found that

$$h(x) = \begin{cases} PFP\ \phi(y)+0 \times (1-PFP)= PFP\ \phi(y) & \text{for } x\neq 0 \\ PFP\ \phi(0)+1 \times (1-PFP) & \text{for } x=0 \end{cases}$$

Next, it will be shown how, for a given alternative s_j, $h(x)$ can be derived from $F(q)$, given the K points (q_k, y_{kj}). It is assumed that q_k and y_{kj} are related by the function g_j in the following way:

$$y_{kj} = g_j(q_k) \text{ or } q_k = g_j^{-1}(y_{kj})$$

Note that $q_k < q_{k+1} \Longrightarrow y_{kj} < y_{k+1,j}$ for all values of k; therefore, it will be assumed that g_j is a continuous strictly monotone increasing function of the inflow variable q. Such properties guarantee that the mapping of $g_j^{-1}(y)$ on the set of images of $g(q)$ to the domain of $g(q)$ is a one-to-one function.

For $y_{kj} \leq y \leq y_{k+1,j}$, g_j can also be approximated by G_j, using piecewise linearization:

if $y_{kj} = g_j(q_k)$ and $y_{k+1,j} = g_j(q_{k+1})$

then $$y = y_{kj} + \left[\frac{y_{k+1,j} - y_{kj}}{q_{k+1} - q_k}\right](q - q_k) = G_j(q)$$

for $q_k \leq q \leq q_{k+1}$

Let a constant K be defined as

$$K = \left[\frac{q_{k+1} - q_k}{y_{k+1,j} - y_{kj}}\right]$$

Note that $q = G_j^{-1}(y) = q_k + K(y - y_{kj})$ for $y_{kj} \leq y \leq y_{k+1,j}$

Given that

$$\Phi(y) = \Pr(Y \leq y) = \Pr(G_j(Q) \leq y)$$

$$= \Pr(Q \leq G_j^{-1}(y)) = F(G_j^{-1}(y))$$

By differentiating $\Phi(y)$ with respect to y, the following is obtained by using the chain rule:

$$\phi(y) = \frac{d\Phi(y)}{dy} = \frac{dF(G_j^{-1}(y))}{dG_j^{-1}(y)} \times \frac{dG_j^{-1}(y)}{dy}$$

$$\Longrightarrow \phi(y) = \frac{dF(G_j^{-1}(y))}{dG_j^{-1}(y)} \times K$$

$$\Longrightarrow \phi(y) = K\, f(G_j^{-1}(y))$$

Since $G_j^{-1}(y)$ does not have one closed form relationship, it will be easier to use the following approximation for the derivations:

$$\phi(y) \simeq \begin{cases} \dfrac{\Phi(y_{k+1,j}) - \Phi(y_{kj})}{y_{k+1,j} - y_{kj}} & \text{for } y_{kj} \le y \le y_{k+1,j} \\ & \text{where } k=1,\dots,K-1 \\ 0 & \text{otherwise} \end{cases}$$

But, since

$$\Phi(y_{kj})=F(G_j^{-1}(y_{kj}))=F(q_k) \qquad \text{for } k=1,\dots,K$$

then the above equality becomes

$$\phi(y) \simeq \begin{cases} \dfrac{F(q_{k+1})-F(q_k)}{y_{k+1,j}-y_{kj}} & \text{for } y_{kj} \le y \le y_{k+1,j} \\ & \text{where } k=1,\dots,K-1 \\ 0 & \text{otherwise} \end{cases}$$

Moreover, the unconditional probability of damages is then

$$h(x) \simeq \begin{cases} PFP \left[\dfrac{F(q_{k+1}) - F(q_k)}{y_{k+1,j} - y_{kj}}\right] & \text{for } y_{kj} \le x \le y_{k+1,j} \\ & \text{where } 1=1,\dots,K-1 \\ 1-PFP & \text{for } x=0 \\ 0 & \text{otherwise} \end{cases}$$

4.6 Alternatives and Cost Estimates

The sixteen alternatives we are studying combine such remedial actions as raising the dam's height and increasing the spillway's width. They are described in detail in Table 4.1.

INCREASE IN DAM HEIGHT	SPILLWAY WIDTH (1 UNIT = 620 FT.)			
	1	1.5	2	2.4
0 FT.	s_1	s_5	s_9	s_{13}
3 FT.	s_2	s_6	s_{10}	s_{14}
6 FT.	s_3	s_7	s_{11}	s_{15}
10 FT.	s_4	s_8	s_{12}	s_{16}

Table 4-1 Description of the Atlernatives s_j (j=1,2,...,16)

If the dam's height is raised by 10 feet to an elevation of 920 ft above sea level and if the present spillway width is maintained, the dam will safely pass the PMF. Similarly, if the present dam's height is kept and if the spillway is widened to 2.4 times its current size, the dam will also safely pass the PMF. Alternatives such as increasing the spillway's width by more than 2.4 times or raising the dam's height by more than 10 feet were disregarded for the reason that the only effect of the corresponding added construction costs would be that the dam would safely pass floods larger than the PMF. However, floods of such large magnitude are considered to be very unlikely and have generally been ignored by analysts in the field of dam safety.

For the Tomahawk Dam,generic cost estimated for remedial action were based on studies by the U.S. Army Corps of Engineers [1983] and [1985b]. These values have been used to obtain the cost estimates for the 16 alternative actions (see Table 4-2). It has been assumed that if the dam's height is raised by less than 3 feet, then a concrete parapet wall will be used. On the other hand, if the dam's height is to be increased by more than three feet, then, to consolidate the dam, earthfill will be used; in addition, a three-foot

INCREASE IN DAM HEIGHT	SPILLWAY WIDTH (1 UNIT = 620 FT.)			
	1	1.5	2.0	2.4
0 FT.	0	19.32	25.88	31.12
3 FT.	0.8	20.12	26.68	31.92
6 FT.	5.15	22.02	27.93	32.64
10 FT.	20.83	36.14	42.04	46.76

Table 4-2 Construction costs for the remedial actions (in millions of US$)

concrete parapet wall will be used. If the spillway is also to be widened then, to reduce costs, the material from the spillway's excavations would be utilized as the stabilizer earthfill. In fact, while constructing the concrete parapet wall is relatively cheap, using earthfill can be very costly, particularly if it is not available at a location close to the dam. On the other hand, widening the spillway requires extremely costly modifications.

4.7 Application of the Partitioned Multiobjective Risk Method

In the first section of this chapter the partitioned multiobjective risk method (PMRM) was described, and, in the later sections, all the information was assembled that needed to apply the PMRM to the case study. In this section, the PMRM is applied to our problem, following the procedure for discrete decision variables outlined at the beginning of the chapter.

4.7.1 Finding the Probability Distribution of Damages

In section 4.3, the probability distribution function of the inflows, $f(q)$, was derived along with the corresponding cumulative probability function, $F(q)$, for the four assumed distributions. Section 4.4 contained the derivation of the relationships between the probability distribution of damages $h(x)$ and $F(q)$. It was found that

$$h(x) \simeq \begin{cases} PFP \left[\dfrac{F(q_{k+1})-F(q_k)}{y_{k+1,j}-y_{kj}} \right] & \text{for } y_{kj} \leq x \leq y_{k+1,j} \\ & \text{where } k = 1,\ldots,K-1 \\ 1-PFP & \text{for } x=0 \\ 0 & \text{otherwise} \end{cases}$$

where y_{kj} represent the damages resulting from inflow q_k under alternative s_j and assuming that antecedent floods have filled the pool to the spillway's crest. These values of y_{kj} were computed through computer simulations based on the IWR model. It was also found in section 4.4 that

$$\Phi(y) = F(G_j^{-1}(y))$$

and, in particular, for the data points obtained by simulation,

$$\Phi(y_{kj}) = F(q_k)$$

4.7.2 Partitioning the Probability Axis

Next, it is necessary to partition the exceedence probability axis (or, alternatively, the cumulative probability axis) into various ranges that enhance the understanding of the different risk-related aspects of the problem. The analyst should perform the partitioning only after he studies carefully the risk curves (exceedence probability curves) for the various alternatives. For example, Figures 4.1, 4.2, and 4.3 represent, respectively, the risk curves that correspond to alternatives s_1 (the status quo), s_2, and s_5. Note that only the relevant part of the risk curve is shown on these figures. After examining the available information, we decided to partition the probability axis into four ranges representing: (1) no hazards, (2) high-probability/low-consequence (HP/LC) risk, (3) intermediate risk, (4) and low-probabilty/high consequence (LP/HC) risk. We will be using the following notation for the partitioning points:

$[\alpha_1, \alpha_2]$: no damages range

$[\alpha_2, \alpha_3]$: HP/LC risk range

$[\alpha_3, \alpha_4]$: intermediate risk range

$[\alpha_4, \alpha_5]$: LP/HC risk range

Since the full exceedence probability axis is being partitioned, α_1 will be set equal to 1 and α_5 will be set to zero. Moreover, the range $[\alpha_1, \alpha_2]$ corresponds to the no-damage domain, which in turn corresponds to the case where antecedent floods do not fill all of the empty reservoir; therefore, α_2 will be set equal to the PFF (where the PFP is the probability of having antecedent floods filling the pool to the spillway's crest). In the following analysis it will be assumed, as reccmmended in section 4.2, that PFP = 1; thus yielding α_2=1. Therefore, the range $[\alpha_1, \alpha_2]$ becomes the range [1,1] and the number of ranges is then reduced to three. For the sake of simplicity, the values of α_3 and α_4 will be chosen from among the values of $[1-\Phi(y_{kj})]$ (k=1,2,...,K) or, equivalently, from among the values of $[1-F(q_k)]$ (k=1,2,...,K).A sensitivity analysis will also be performed on each of α_3 and α_4, but, for convenience, they will be allowed to take values only among $[1-F(q_k)]$ (k=1,2,...,K).

4.7.3 Mapping Partitions to the Damage Axis

Because of the above simplification, the mapping of the partitions on the probability axis onto the damage axis is simplified.

If α_i (where i=2,3,4) is set equal to $[1-F(q_{k'})]$ (where k'=2,...,K), then the following relations hold:

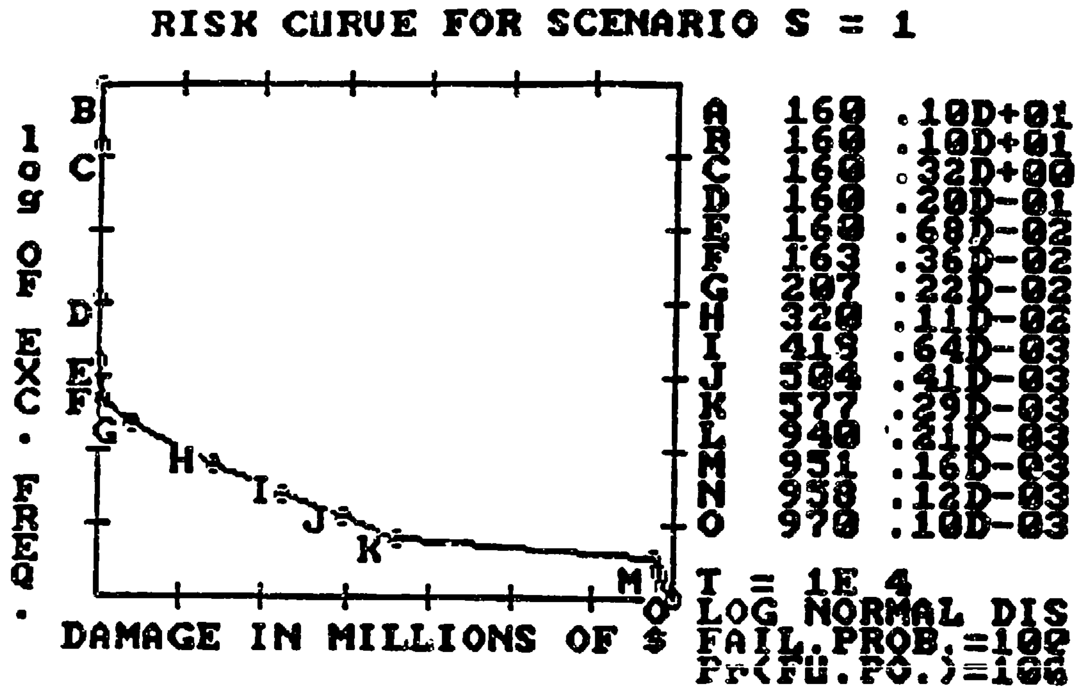


Figure 4.1 Risk Curve for scenarios s_1

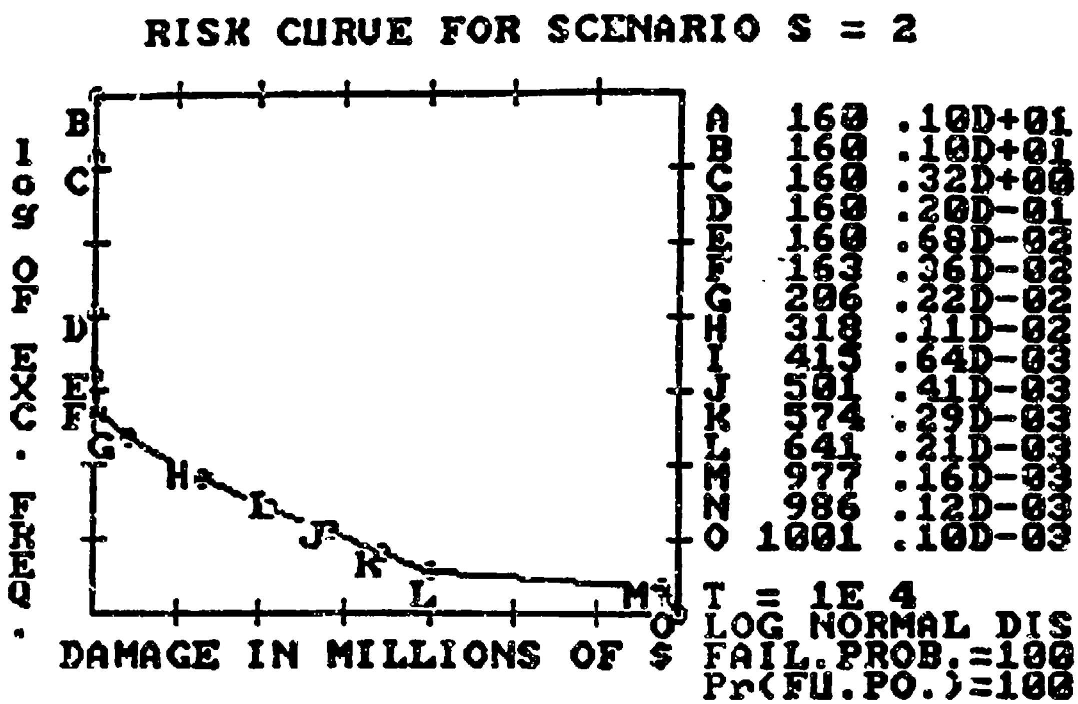


Figure 4.2 Risk curve for scenario s_2

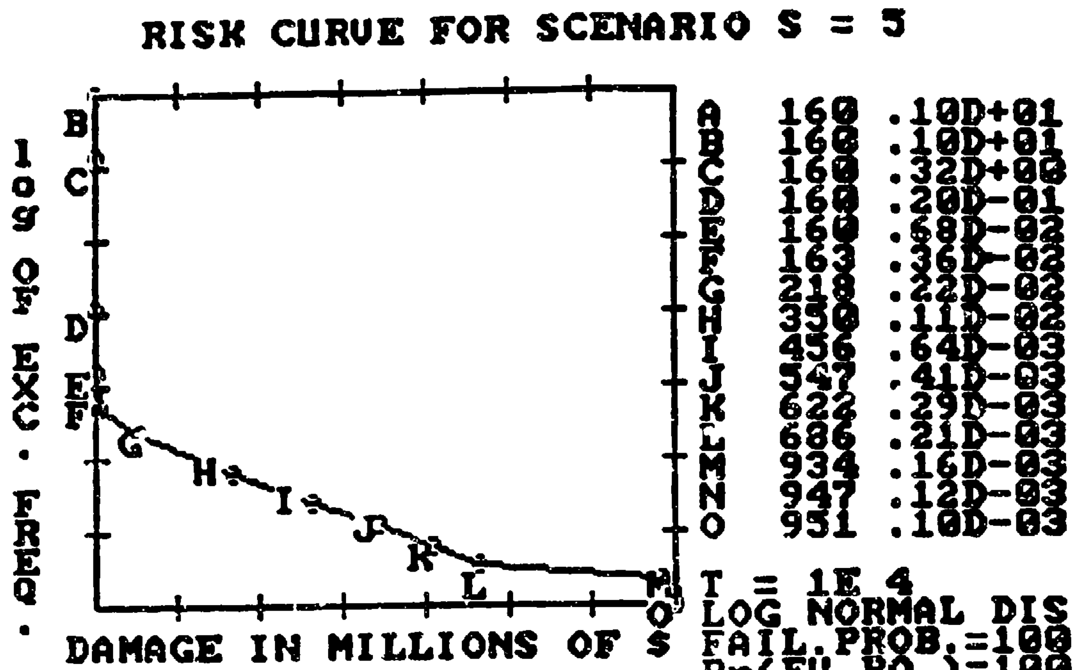


Figure 4.3 Risk curve for scenario s_5

$$F(q_{k'}) = \Phi(y_{k',j}) = [H(x_{k',j})-(1-PFP)]/PFP$$

$$= H(x_{k',j}) ,$$

it follows that

$$1-\alpha_i = H(x_{k',j})$$

$$H^{-1}(1-\alpha_i) = x_{k',j}$$

$$a_{ij} = x_{k',j}$$

$$a_{ij} = y_{k',j}$$

This value of k′ will be denoted by γ_i.

Since $q_1=0$, then $F(q_1) = 0$, and since $\alpha_2 = 1$

$$\Longrightarrow \alpha_2 = 1 - F(q_1) \text{ and } \gamma_2 = 1$$

From the above results, it follows that

$$a_{2j} = y_{1j}$$

For $\alpha_5 = 0$, a_{5j} can be approximated by $x_{15',j}$ and $\gamma_5=15$.

The three domains corresponding to the three ranges of the probability axis are

domain I : $[a_{2j}, a_{3j}]$ HP/LC risk

domain II : $[a_{3j}, a_{4j}]$ intermediate risk

domain III : $[a_{4j}, a_{5j}]$ LP/HC risk

4.7.4 Finding Conditional Expectations

For alternative s_j and for the damage domain D_{ij} defined by the interval $[a_{ij}, a_{i+1,j}]$, the conditional expectation is

$$E[X \setminus D_{ij}; s_j] = \frac{\int_{a_{ij}}^{a_{i+1,j}} x\, h_X(x)\, dx}{\int_{a_{ij}}^{a_{i+1,j}} h_X(x)\, dx}$$

$$= \frac{\sum_{z=\gamma_i}^{(\gamma_{i+1}-1)} \int_{y_{zi}}^{y_{z+1,j}} x\, h_X(x)\, dx}{\sum_{z=\gamma_i}^{(\gamma_{i+1}-1)} \int_{y_{zi}}^{y_{z+1,j}} h_X(x)\, dx}$$

$$= \frac{\sum_{z=\gamma_i}^{(\gamma_{i+1}-1)} \int_{y_{z+1}}^{y_{z+1,j}} x \text{ PFP} \left[\frac{F(q_{z+1})-F(q_z)}{y_{z+1,j}-y_{zj}}\right] dx}{\sum_{z=\gamma_i}^{(\gamma_{i+1}-1)} \int_{y_{zj}}^{y_{z+1,j}} \text{PFP} \left[\frac{F(q_{z+1})-F(q_z)}{y_{z+1,j}-y_{zj}}\right] dx}$$

$$= \frac{PFP \sum_{z=\gamma_i}^{(\gamma_{i+1}-1)} \left[\frac{F(q_{z+1})-F(q_z)}{y_{z+1,j}-y_{zj}}\right] \int_{y_{zj}}^{y_{z+1,j}} x\, dx}{PFP \sum_{z=\gamma_i}^{(\gamma_{i+1}-1)} \left[\frac{F(q_{z+1}-F(q_z)}{y_{z+1,j}-y_{zj}}\right] \int_{y_{zj}}^{y_{z+1,j}} dx}$$

$$= \frac{\sum_{z=\gamma_i}^{(\gamma_{i+1}-1)} [F(q_{z+1}-F(q_z)] \left[\frac{y_{z+1,j}+y_{zj}}{2}\right]}{\sum_{z=\gamma_i}^{(\gamma_{i+1}-1)} [F(q_{z+1})-F(q_z)]}$$

$$= \left[\frac{1}{\alpha_i-\alpha_{i+1}}\right] \sum_{z=\gamma_i}^{(\gamma_{i+1}-1)} [F(q_{z+1})-F(q_z)] \left[\frac{y_{z+1,j}+y_{zj}}{2}\right]$$

This derivation is valid for the three domains, I, II, and III, that correspond, respectively, in the above equations to $i = 2, 3,$ and 4. We computed the conditional expectations of domains I, II, and III for all alternatives.

The probability of having damage that falls in the interval $[a_{ij}, a_{i+1,j}]$ is

$$Pr(x \epsilon D_{ij}) = \alpha_i - \alpha_{i+1}$$

This probability is the weight coefficient for the conditional expectation, and it represents the relative importance of this conditional expected value.

4.7.5 Generating Functional Relationships

The expected conditional values $E[x \backslash D_{2j};s_j]$, $E[x \backslash D_{3j};s_j]$, and $E[x \backslash D_{4j};s_j]$ (where $j=1,2,\ldots,16$) will be used to define the set of risk objective functions f_2, f_3, and f_4, where

$$f_i(s_j) = E[x \backslash D_{ij};s_j] \qquad (i=2,3,4)$$

Thus, $f_2(s_j)$ will correspond to domain I or HP/LC risk, $f_3(s_j)$ will correspond to domain II or intermediate risk, and $f_4(s_j)$ will correspond to domain III or LP/HC risk.

Another risk objective function is the unconditional expected value of damages, which will be denoted by $f_5(s_j)$; it can be approximated by

$$f_5(s_j) \simeq \sum_{i=2}^{4} [\alpha_i - \alpha_{i+1}]\, f_i(s_j)$$

Figure 4.4 contains an example of the graph of the risk objective functions f_3, f_4, and f_5.

The cost function $f_1(s_j)$ was also constructed from Tables 4.1 and 4.2. The graph of the cost function is shown in Figure 4.5.

4.7.6 Employing the Surrogate Worth Trade-off Method

So far, five objective functions have been determined. To find the decision maker's preferred solution, a multiobjective optimization problem will have to be solved. Since the decision variable is discrete, the modified version of the SWT method that makes use of total trade-offs will be used, as discussed in Section 4.1.

Although the main interest here is in the trade-offs between the cost function f_1 and the LP/HC risk objective function f_4, the trade-offs between f_1 and the risk objective function f_5 which represents the unconditional expected value will also be analyzed.

All noninterior solutions were determined through an exhaustive search. The modified version of the surrogate worth function (SWF2; see Section 4.7) can then be used to determine the preferred solution. A decision support system based on this function was developed to assist the DM in this crucial task.

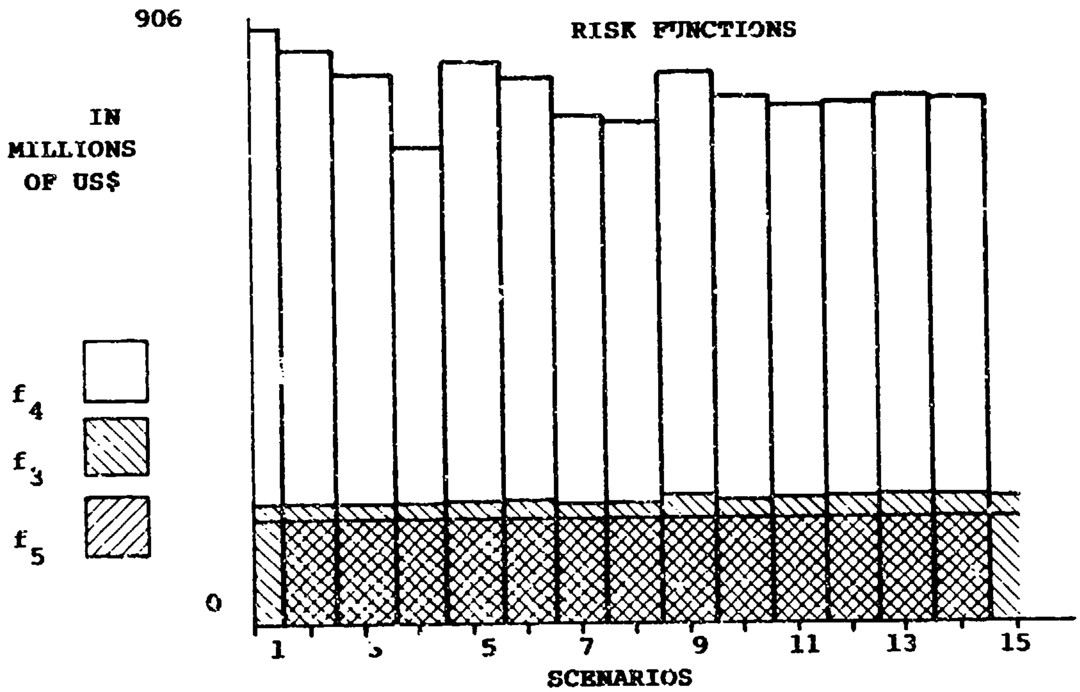


Figure 4.4 Risk objective functions $f_3(s)$, $f_4(s)$, and $f_5(s)$

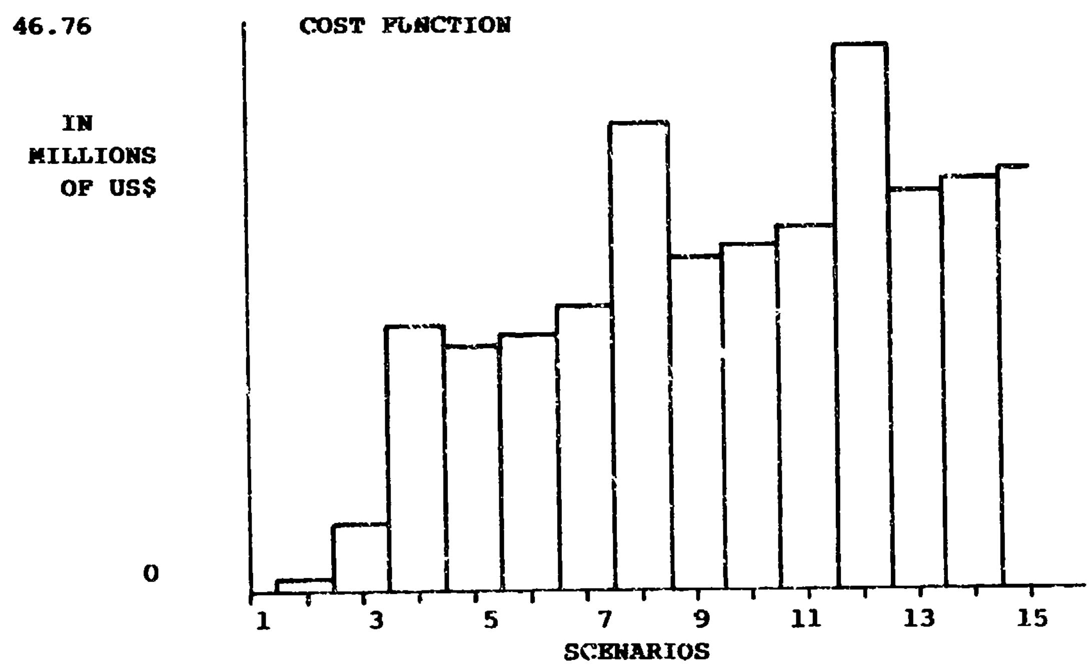


Figure 4.5 Cost objective function

CHAPTER 5

ANALYSIS OF RESULTS

This chapter contains a discussion of the results that were obtained by applying the partitioned multiobjective risk method (PMRM) to the dam safety problem described in Chapters 3 and 4. In particular, a sensitivity analysis will be performed on the distribution used to extrapolate the frequency curve to the PMF, the return period of the PMF, and on the partitioning points. But first, the decision support system that contains the computer model used to calculate the results will be described.

5.1 A Decision Support System

A decision support system (DSS) based on the PMRM was developed to help the decision maker (DM) determine the preferred solution that would enhance dam safety to the desirable level with acceptable costs.

The DSS is divided into three main modules. The first one is a simplified database management system that allows the analyst to construct and display the database needed to apply the PMRM. For each alternative s_j ($j = 1,2,\ldots,n$), the database should contain information relative to damages caused by inflow q_k ($k = 1,2,\ldots,K$) for both dam failure and nonfailure. Information such as the nonfailure outflow, the failure outflow, and the peak freeboard can be provided, but they will not be used by this version of the DSS.

In the second module, the cost-objective function f_1 and the risk-objective functions f_2, f_3, f_4, and f_5 are generated. First, the decision maker (or analyst) who is using the DSS will be asked to specify the values of the following parameters: (1) the conditional probability of dam failure given that the dam was overtopped for more than two hours, (2) the PFP, which is the probability of having antecedent floods filling the reservoir to the spillway's crest, (3) the flood-frequency distribution assumed for floods larger than the 100-year flood event (a choice can be made from among the log-Normal, Pareto, Weibull, and Gumbel probability distributions), (4) the return period of the PMF (either 10^4, 10^5, 10^6, or 10^7).

Before he is asked to partition the probability axis, the DM (or analyst) will be able to review at will the various risk curves that correspond to the alternatives. In fact, the DM who will have to assign the values of α_3 and α_4, the partitioning points, can only pick values among the K values, $[1-F(q_k)]$.

After computing the conditional expectations for each domain and each scenario, the DSS will display a tableau containing all the generated objective functions. In addition, the tableau will contain the values of all the parameters, and it will contain, for each risk domain the probability of having an event cause a damage that is within the range of the domain.

The last module of the DSS allows the DM to use the surrogate worth trade-off (SWT) method or its modified version for discrete decision situations (see section 4.1.6). The DM will be given many options as to what scenarios he would like to consider. For example, he can fix either one of the two decision variables--the spillway's width or the dam's height--and vary the other. He can also vary both decision variables simultaneously, obtaining discrete decision options. The DM will also be able to display the cost function, the risk functions, and the Pareto frontiers (noninferior solutions). When using the SWT, the DM will be asked to evaluate the surrogate worth function (or its modified version for the discrete decision situations--see section 4.1.6) at various points of the Pareto optimum frontier between f_1 and f_4. The DM will be given the values of f_1, f_4, f_5, λ_{41}, and λ_{51} for each point. In making the assessment, the weight coefficients $(\alpha_i - \alpha_{i+1})$ of each risk objective, f_i, should be kept in mind as they reflect, to a certain degree, the relative importance of the risk objectives. Based on this information, the DM should be able to assign, for each point, a value to $W_{41}(f_1)$ [or $W(f_1)$, for the discrete decision variables] that reflects his preferences. The DSS will then find the DM's preferred solution by using the surrogate worth function. If the decision situations are discrete, then the DM will have to select a preferred solution from the set of alternatives considered.

In the next sections a modified version of the second module of the DSS will be used to obtain all the results needed for the sensitivity analyses.

5.2 Explanation of the Chapter's Figures

Consider Fig. 5.1, which is a typical sample of the kind of figures that will be used in this chapter. Notice that it contains two tableaus. In the first one, the objective functions $f_i(s_j)$ $(i=1,\ldots,5)$ are shown for all the alternatives, while in the second one, the total trade-off functions $\lambda_{i1}(s_j,s_h)$ $(i=2,\ldots,5)$ are listed for the noninferior solutions. In the figure, the value of -1 was assigned to the total trade-off functions whenever the value of these functions was negative. When the total trade-off functions are assigned the symbol "**" for some alternative action, it means that the alternative corresponds to a noninferior solution but that the total trade-off function cannot be computed because there are no noninferior solution with higher cost exists.

Notice that the alternatives are ordered according to a continuously increasing cost function. The total trade-off functions were calculated using this order. Also, all the values in the first tableau are in units of have 10^6 US$. Moreover, the figure contains a display of the values that were assigned to the various parameters, and it contains the yearly probabilities of having an event belonging to various risk domains. This explanation of Fig. 5.1 holds for the rest of the figures in this chapter (excluding Figs. 5.2 and 5.3).

5.3 Trade-offs in the Dam Safety Problem

The main advantage that the PMRM has over other risk analysis methodologies is that it does not collapse the risk curve into one point, the yearly expected value. Instead, it represents this curve by a number of points that correspond to the yearly conditional expected values. It will be demonstrated that this advantage improves the decision-making process for our dam safety problem. For this demonstration, the numbers in Fig. 5.1 will be examined and Fig. 5.2 will be used to illustrate the trade-offs involved.

Traditionally, risk analysis has relied heavily on the concept of the yearly expected value, which corresponds in the figure to $f_5(s_j)$. First, note that f_5, the yearly expected damage, takes unusually high values (on the order of 161×10^6 US$ to 162×10^6 US$). This is due to the assumptions concerning antecedent floods; in particular, the assumptions that the reservoir is filled to the spillway's crest and that the outlet is open to 75% of its capacity. Therefore, any small inflow into the reservoir will cause large damages on the order of 160×10^6 US$. Notice that these two assumptions were made to comply with the guidelines and recommendations established by the U.S Army Corps of Engineers.

It is also apparent that when the dam's height is increased ($s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow s_4$, $s_5 \rightarrow s_6 \rightarrow s_7 \rightarrow s_8$, ...), f_5 decreases, but by less than 0.3%. Next, when the spillway's width is increased ($s_1 \rightarrow s_5 \rightarrow s_9 \rightarrow s_{13}$, $s_2 \rightarrow s_6 \rightarrow s_{10} \rightarrow s_{14}$, ...), f_5 increases in general, and when it decreases, it does so by less than 0.02%. These observations could lead the DM to conclude that increasing the spillway's width is not an attractive solution because any investment in such an action will mainly increase the risks. By looking at λ_{51}, the DM could also find some incentives not to invest money to raise the dam since under alternative s_2, an investment of one million US$ will not reduce the expected yearly damages by more than $25,386.

S SCENARIO	F1(S) COST FUNC.	F2(S) HP/LC RISK	F3(S) INTER.RISK	F4(S) LP/HC RISK	F5(S) EXPE. VALUE
1	0	159.9796	209.5453	1260.525	161.7427
2	0.8	159.9796	209.0715	1038.298	161.5697
3	5.15	159.9796	208.652	899.4269	161.4593
5	19.32	159.9796	217.3968	1028.719	161.7225
6	20.12	159.9796	216.9271	834.9428	161.5707
4	20.83	159.9796	208.2824	678.3266	161.2892
7	22.02	159.9796	216.5687	721.5613	161.4802
9	25.88	159.9796	223.0323	908.4608	161.7421
10	26.68	159.9796	222.6253	746.3894	161.6148
11	27.93	159.9796	222.3008	744.2904	161.6071
13	31.12	159.9796	226.3684	758.2729	161.6955
14	31.92	159.9796	225.946	756.9135	161.6863
15	32.64	159.9796	225.6023	755.37	161.6786
8	36.14	159.9796	216.1759	718.7776	161.4706
12	42.04	159.9796	221.9517	741.6782	161.5984
16	46.76	159.9796	225.2417	753.775	161.6705

RETURN PERIOD OF PMF = 1E 4 THERE IS 0.00000% CHANCE THAT F0 OCCURS
GUMBEL DISTRIB. FOR RARE FLOODS THERE IS 98.00605% CHANCE THAT F2 OCCURS
THE PARTITIONING POINTS ARE D AND K THERE IS 1.92024% CHANCE THAT F3 OCCURS
CONDITION. PROBABI. OF FAILURE=100% THERE IS 0.07371% CHANCE THAT F4 OCCURS
DM'S ESTIMATE OF PR(FULL POOL)=100% PRESS <SPACE> BAR TO CONTINUE

S SCENARIO	F1(S) COST FUNC.	LAM(2,1) HP/LC RISK	LAM(3,1) INTER RISK	LAM(4,1) LP/HC RISK	LAM(5,1) EXPE. VALUE
1	0	**	0.5921555	277.784900	0.2161406
2	0.8	-1.0000000	0.0964531	31.9242800	0.0253857
3	5.15	-1.0000000	0.0235674	4.3075570	0.0108466
5	19.32	-1.0000000	-1.0000000	-1.0000000	-1.0000000
6	20.12	-1.0000000	-1.0000000	220.586500	-1.0000000
4	20.83	-1.0000000	**	**	**
7	22.02	-1.0000000	-1.0000000	-1.0000000	-1.0000000
9	25.88	-1.0000000	-1.0000000	-1.0000000	-1.0000000
10	26.68	-1.0000000	-1.0000000	-1.0000000	-1.0000000
11	27.93	-1.0000000	-1.0000000	-1.0000000	-1.0000000
13	31.12	-1.0000000	-1.0000000	-1.0000000	-1.0000000
14	31.92	-1.0000000	-1.0000000	-1.0000000	-1.0000000
15	32.64	-1.0000000	-1.0000000	-1.0000000	-1.0000000
8	36.14	-1.0000000	-1.0000000	-1.0000000	-1.0000000
12	42.04	-1.0000000	-1.0000000	-1.0000000	-1.0000000
16	46.76	-1.0000000	-1.0000000	-1.0000000	-1.0000000

Figure 5.1 Risk objective functions ($x10^6$) and total trade-off functions

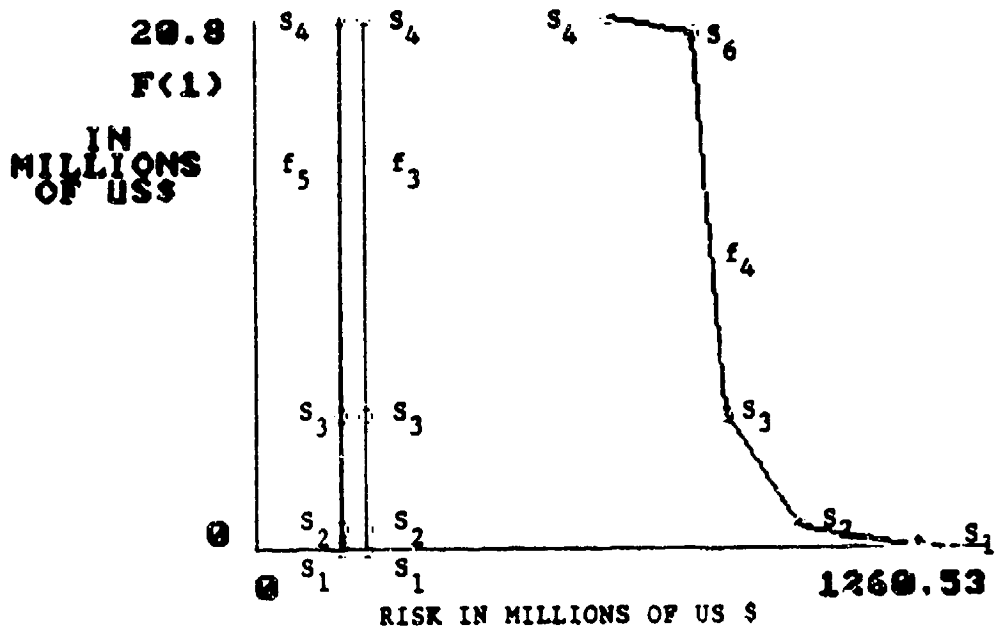


Figure 5.2 Pareto Optimal frontiers

But, if the DM takes into consideration the rest of the risk objective functions, in particular $f_4(s_j)$ and $f_3(s_j)$, then his picture of the problem might radically change. First, he will notice that f_4 decreases greatly when the spillway's width is increased, but that f_3 increases. In other words, the DM will be able to see that by increasing the spillway's width, he is reducing risks in the low-probability/high-consequence (LP/HC) domain because spillway widening reduces both the probability of dam failure and the damages in case of failure. On the other hand, the DM will also see that the risks associated with less extreme events are increasing, because floods which are relatively frequent will cause more downstream damages. Moreover, even when compared to increasing the dam's height, spillway widening could still be an attractive solution. For example, s_6, which would have been disregarded if traditional risk analysis methods were used, becomes a noninferior solution if the risk objective f_4 is considered. Thus, by using the PMRM, the DM can better understand the trade-offs among risks that correspond to the various risk domains.

Moreover, regarding the alternative of increasing the dam's height, the use of f_4 allows explicit quantification of risks in the LP/HC risk domain, and this might induce the DM to invest money in some situations where he might have been reluctant to do so if he had just used f_5. Using the same example as above, investing one million US$ under alternative s_2, only reduces the expected yearly damages by $25,386. It is apparent that if f_4 is included, then, in the case of an extreme event, up to $31,924,280 in yearly damages might be saved with a probability of 7.371×10^{-4}.

Notice that for this problem, because smaller inflows caused the same amount of damages for all alternatives, $f_2(s_j)$ is constant for all alternatives and therefore is of no interest to the decision maker. This can be interpreted to mean that the high-probability/low-consequence (HP/LC) risk domain provides no information for this problem in the decision-making process, and for this reason it will be disregarded in the rest of this chapter.

It is obvious that by using the PMRM, the DM is able to grasp certain aspects of the problem which would have been completely ignored had he simply used the yearly expected value of damages. These aspects were mainly associated with LP/HC risks in this case, but this is not a general restriction.

5.4 Sensitivity to the Flood Frequency Function

Chapters 1 and 2 called attention to the fact that there is very little knowledge concerning the type of probability distribution function that should be used to extrapolate the flood-frequency curve beyond the 100-year flood to the PMF. Moreover, Stedinger and Grygier [1985] showed that the results of their risk analysis could be influenced by the choice of this probability distribution function. Thus, it was decided to perform a sensitivity analysis to try to determine the impact that this choice would have on the decision-making process for this case study.

The approach that has been used here to facilitate the application of the PMRM (see Chapter 4) does not allow partitioning of the probabilty axis at the same points for all the distributions. Therefore, to be able to compare the results for the various distributions, linear interpolation has been used to approximate the risk objective functions. These results are partially listed in Tables 5-1, 5-2, 5-3, and 5-4.

By studying the results of this sensitivity analysis, one notices that f_5 increases by less than 1% when the flood-frequency distribution function of rare floods is changed using, alternatively, the Pareto, log-Normal, Weibull, and Gumbel distributions. But it can also be observed that the total trade-off function λ_{51} increases dramatically from 200% to more than 300% when the Gumbel distribution is used instead of the Pareto distribution.

Therefore, if only traditional risk analysis methods which focus only on f_5 are used, then the use of the Gumbel distribution is likely to give results differ from the ones obtained using the other distributions.

From our numerical results it is apparent that when the distributions are changed in the same order as above, f_4 increases up to 37%, but λ_{41} does not vary much until the Gumbel distribution is used. Because of its thin tail, the Gumbel distribution puts more weight on the extreme range and therefore causes a change in some alternatives: these might have seemed attractive to the DM if f_5 alone had been considered, but they are less attractive in the LP/HC domain.

It can also be shown that f_3 and λ_{31} increase dramatically if the distributions are changed in the same order as described above. Here too, the use of the Gumbel distribution has great impact on the results.

It is clear, therefore, that the decision-making process in the PMRM is also sensitive to a change of distributions. In particular, the use of the Gumbel distribution tends to give high estimates of risk which might induce the DM to choose conservative and expensive remedial actions. Therefore, it is recommended that all studies on dam safety include a sensitivity analysis that examines the effects that changes in the selection of a flood-frequency distribution function have on the results. In this sensitivity analysis, the Gumbel distribution should be used in addition to any other distribution that does not have a thin tail such as the Pareto distribution, the log-Normal distribution, or the log-Gumbel distribution.

Notice that by using the PMRM, it was possible to see how the choice of the probability distribution for rare floods effects the risk estimates in the various risk domains, and therefore a better understanding of the problem was achieved.

5.5 Sensitivity of Risk-Cost Analysis to the Return Period of the PMF

The problems associated with estimating the return period of the PMF and the uncertainties that characterize this parameter have already been discussed. In fact, Stedinger and Grygier [1985] also found that their results were sensitive to changes in the return period of the PMF. Thus, in this section, there will be an attempt to determine how changes in T, the estimate of the return period of the PMF, influence the choices of the decision maker. The PMRM has been used, assuming, alternatively, 10^4, 10^5, 10^6, or 10^7 to be the value of the return period of the PMF.

SCE. s	f1(s) COST FUN	f3(s) INTE RIS	f4(s) LP/HC RI	f5(s) EXP VALU
s1	0	166.676	1147.482	160.162
s2	0.8	166.619	998.343	160.154
s3	5.15	166.572	905.176	160.148
s5	19.32	167.851	991.702	160.178
s6	20.12	167.793	859.103	160.170
s4	20.83	166.527	665.780	160.136

Table 5-1 Objective functions for the Pareto distribution

SCE. s	f1(s) COST FUN	f3(s) INTE RIS	f4(s) LP/HC RI	f5(s) EXP VALU
s1	0	170.103	1264.684	160.238
s2	0.8	170.015	1055.099	160.226
s3	5.15	169.941	932.060	160.218
s5	19.32	171.877	1042.946	160.262
s6	20.12	171.786	871.349	160.251
s4	20.83	169.871	684.473	160.204

Table 5-2 Objective functions for the log-normal distribution

SCE. s	$f_1(s)$ COST FUN	$f_3(s)$ INTE RIS	$f_4(s)$ LP/HC RI	$f_5(s)$ EXP VALU
s_1	0	174.112	1356.605	160.321
s_2	0.8	173.670	1193.864	160.303
s_3	5.15	173.566	1011.948	160.292
s_5	19.32	176.175	1164.029	160.352
s_6	20.12	176.050	913.897	160.337
s_4	20.83	173.468	707.526	160.276

Table 5-3 Objective functions for the Weibull distribution

SCE. s	$f_1(s)$ COST FUN	$f_3(s)$ INTE RIS	$f_4(s)$ LP/HC RI	$f_5(s)$ EXP VALU
s_1	0	202.479	1438.588	160.891
s_2	0.8	199.313	1451.709	160.827
s_3	5.15	198.179	1259.544	160.794
s_5	19.32	205.791	1389.484	160.953
s_6	20.12	204.729	991.343	160 912
s_4	20.83	197.888	751.035	160.764

Table 5-4 Objective functions for the Gumbel distribution

Here, too, the risk objective functions were approximated by linear interpolation because the structure of the problem does not allow partitioning of the probability axis at the same points for all the values of the return period of the PMF. The results obtained are shown, partially, in Tables 5-5, 5-6, 5-7, and 5-8.

When T is increased from $T_4 = 10^4$ to $T_7 = 10^7$, the following occurs: (1) f_5 decreases by less than 0.4%, but λ_{51} decreases by more than 190%--thus, a DM using traditional risk analysis will tend to take more conservative actions if he assumes a higher return period of the PMF; (2) f_4 decreases by more than 100% and λ_{41} decreases by more than 150%--thus it is obvious that changes in T impact most on LP/HC risks; (3) f_3 also decreases but not as drastically as f_4, and λ_{31} decreases as well.

It can be seen that here, too, although the use of the PMRM did not improve the robustness of the results, it added more insight to the problem.

This short exposition concludes with the recommendation that sensitivity analysis be performed on the return period of the PMF for all risk analyses on dam safety.

5.6 Where and How to Partition

In this section, the emphasis is on determinatin of the partitioning points on the probability axis. For this case study, the probability axis has been partitioned using the DSS described earlier in the chapter. In section 4.7.2 of Chapter 4, it was shown that the partitioning points α_3 and α_4 could be chosen from a specified set of points $[1-F(q_k)]$ (where $k=1,2,\ldots,K$). In the DSS, this set of points corresponds to the set of points A, B, C,..., O. These points can be seen on the graph of Fig. 5.3, which represents the risk curve for s_1.

Ideally, the objective is to partition the probability axis in a way that would allow isolation of the extreme risks and of ordinary risks. More specifically, an attempt is being made to construct an LP/HC risk domain that corresponds to dam failure, and an HP/LC risk domain that corresponds to damages caused by floods smaller than the 100-year flood.

SCE. s	$f_1(s)$ COST FUN	$f_3(s)$ INTE RIS	$f_4(s)$ LP/HC RI	$f_5(s)$ EXP VALU
s_1	0	178.709	1320.635	160.682
s_2	0.8	178.537	1177.850	160.638
s_3	5.15	178.389	1076.94	160.606
s_5	19.32	181.853	1148.869	160.695
s_6	20.12	181.680	998.460	160.648
s_4	20.83	178.254	714.677	160.499

Table 5-5 Objective functions for $T = 10^4$

SCE. s	$f_1(s)$ COST FUN	$f_3(s)$ INTE RIS	$f_4(s)$ LP/HC RI	$f_5(s)$ EXP VALU
s_1	0	166.968	616.034	160.238
s_2	0.8	166.911	569.755	160.226
s_3	5.15	166.867	541.796	160.218
s_5	19.32	168.304	600.075	160.262
s_6	20.12	168.243	561.865	160.251
s_4	20.83	166.820	488.039	160.203

Table 5-6 Objective functions for $T = 10^5$

SCE. s	$f_1(s)$ COST FUN	$f_3(s)$ INTE RIS	$f_4(s)$ LP/HC RI	$f_5(s)$ EXP VALU
s_1	0	163.359	413.191	160.105
s_2	0.8	163.337	397.940	160.102
s_3	5.15	163.320	389.547	160.100
s_5	19.32	163.993	431.163	160.123
s_6	20.12	163.968	420.515	160.121
s_4	20.83	163.301	379.498	160.097

Table 5-7 Objective functions for $T = 10^6$

SCE. s	$f_1(s)$ COST FUN	$f_3(s)$ INTE RIS	$f_4(s)$ LP/HC RI	$f_5(s)$ EXP VALU
s_1	0	161.615	314.332	160.051
s_2	0.8	161.608	310.009	160.050
s_3	5.15	161.603	307.570	160.049
s_5	19.32	161.885	337.099	160.063
s_6	20.12	161.877	334.029	160.062
s_4	20.83	161.596	305.175	160.049

Table 5-8 Objective functions for $T = 10^7$

Notice that the probability of dam failure is largest for the status-quo alternative (alternative s_1). If, through a certain partitioning, we include all failure damages that are included in the LP/HC risk domain, this domain will then contain all failure damages for all of the remaining alternatives. Therefore, if the analyst partitions the probability axis, the risk curve for alternative s_1 would be most useful.

A study of the graph in Fig. 5.3, reveals that by adopting the following partitioning of the probability axis, $\alpha_3 = [1-F(q_4)] = .02$ (point D in the graph) and $\alpha_4 = [1-F(q_{11})]$ (point K in the graph), which for $T = 10^4$ gives $\alpha_4 = 0.29 \times 10^{-3}$, most of the risks associated with floods smaller than the 100-year flood can be isolated in domain I, and all the risk associated with dam failure plus also some risks that are not very extreme can be isolated in Domain III. This partitioning seems to be the best approach to achieve, although partially, the objective stated above.

5.7 Sensitivity to the Partitioning

Since the choice of the partitioning points on the probability axis is a somewhat arbitrary process, it is necessary to examine the sensitivity of the results to changes in the partitioning points. The PMRM has therefore been applied using various partitioning points in the neighborhood of D and K.

Figures 5.4, 5.5, 5.6, 5.7, and 5.8 contain the results of the PMRM when the partitioning points were varied from (D,I) to (D,J), (D,K), (D,L), and (D,M). A study of the tableaus in these figures reveals, that the magnitudes of the risk objective functions f_3 and f_4 in rease, especially f_4, whose magnitude increases, for some scenarios, by as much as 38%. But it is notable that these increases are smaller when the partitioning point around K (less than 32% increase for f_4) is moved. Moreover, since the DM uses the probababilities of the risk domains to implicitly weigh the importance of the corresponding risk objective functions, these increases in f_4 are partially compensated for by decreases in the probability of having LP/HC events. A study of the total trade-off functions makes is clear that the set of non-inferior solutions does not change when the partitioning point is varied in the neighborhood of K. But, if a partitioning point is chosen that is further away from K, then this set can vary greatly (compare Figs. 5-4 and 5-8; s_5 is inferior in one but not the other).

Figures 5.9, 5.6, and 5.10 correspond, respectively, to the partitioning points (C,K), (D,K), and (E,K). It can be seen that f_2 increases by less than 3% while f_3 increases by as much as 34% for some alternatives. But here, too, the increases in f_3 are matched by a very important decrease in the probability of intermediate risks (domain II).

Therefore, it can be said that, for this problem, the results of the decision-making process would be relatively stable in the neighborhood of the partitioning points. But, some more sensitivity analyses showed that the results of the decision-making process become sensitive to the partitioning if the magnitude of failure damages is not much larger than the magnitude of nonfailure damages.

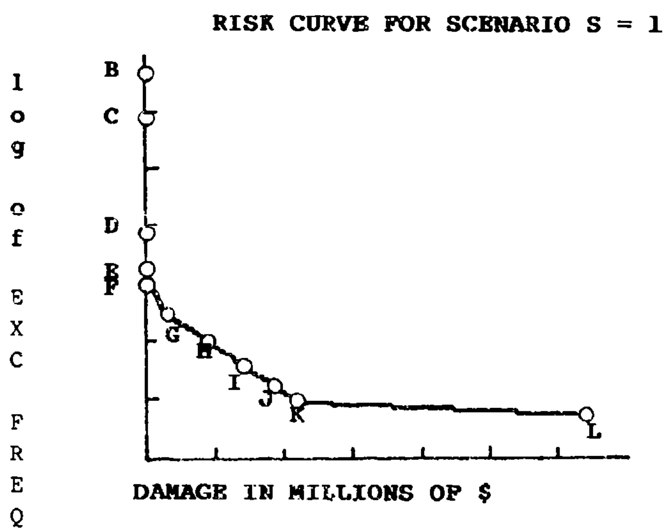
Since these partitioning points are arbitrary points, it might be more appropriate to obtain more robust results. The partitioning of the damage axis might be an adequate solution to this problem. In particular, it would allow very stable results to be obtained for the LP/HC risk domain. But, on the other hand, the risk objectives would be somewhat invariant with the different alternatives. In fact, more theoretical research needs to be done to investigate the partitioning of the damage axis approach.

5.8 Why Are the LP/HC Risk Estimates So Sensitive?

It has been apparent throughout this chapter that f_4 is quite sensitive to changes in the parameters and in the partitioning points. This issue should be elaborated on in an attempt to understand the mechanism behind the behavior of f_4. Figure 5.11 represents an approximate sketch of the exceedence probability function of damages for the three alternatives s_1, s_2 (increase the dam's height by three feet), and s_6 (widen the spillway to 1.5 times its present size and increase the dam's height by three feet).

Observe how, by a gradual decrease in α_4, s_6 becomes noninferior after which s_2 will become inferior in the LP/HC risk domain. Notice that this problem arises only if there is no first-degree stochastic dominance among the alternatives (for a discussion on stochastic dominance see Zeleny [1982]). Imagine that someone is moving downward a horizontal line from the actual position of α_4. For each alternative, the value of f_4 can be visualized as the product of $(1/\alpha_4)$ and the area bounded by the X-axis, the Y-axis, the horizontal line passing through α_4, and the risk curve. First, it can be seen that since α_4 is invariant for all scenarios, only the magnitude of the area defined above will determine which decision situations (or alternatives) are inferior in the LP/HC risk domain. These areas will be called A_1, A_2, and A_6 for alternatives s_1, s_2, and s_6, respectively.

Figures 5.12 and 5.13 help make clear that when α_4 is gradually decreased, first $f_4(s_6)$ is larger than $f_4(s_1)$, but then it becomes smaller, and therefore s_6 becomes a noninferior solution in the LP/HC risk domain. But on the other hand, $f_4(s_2)$, which at first is smaller than $f_4(s_1)$, will become larger eventually, and thus s_2 will become an inferior solution in the LP/HC risk domain.



A 160.10D+01
B 160.10D+01
C 160.32D+00
D 160.20D-01
E 160.71D-02
F 163.40D-02
G 207.26D-02
H 320.13D-02
I 418.75D-03
J 504.48D-03
K 577.33D-03
L 1410.23D-03
M 1427.17D-03
N 1437.13D-03
O 1455.10D-03

T = 1E4
WEIBULL DISTRI
FAIL. PROB. = 100
Pr(FU. PO.) = 100

Figure 5.3 Risk curve for scenario s_1

S SCENARIO	F1(S) COST FUNC.	F2(S) HP/LC RISK	F3(S) INTER.RISK	F4(S) LP/HC RISK	F5(S) EXPE. VALUE
1	0	159.9796	170.6886	897.2678	160.5698
2	0.8	159.9796	170.5917	835.427	160.5359
3	5.15	159.9796	170.5158	790.0118	160.5109
5	19.32	159.9796	172.6828	842.4341	160.5801
6	20.12	159.9796	172.5833	774.8003	160.5431
4	20.83	159.9796	170.4393	598.0398	160.4099
7	22.02	159.9796	172.5109	640.6646	160.4722
9	25.88	159.9796	174.2641	808.079	160.5931
10	26.68	159.9796	174.1715	666.5029	160.5178
11	27.93	159.9796	174.1021	664.5985	160.5155
13	31.12	159.9796	175.2195	680.1681	160.5453
14	31.92	159.9796	175.1216	678.4303	160.5425
15	32.64	159.9796	175.0441	676.7976	160.5401
8	36.14	159.9796	172.4314	637.93[illegible]5	160.4692
12	42.04	159.9796	174.0296	662.3223	160.5129
16	46.76	159.9796	174.9654	674.9736	160.5376

RETURN PERIOD OF PMF = 1E 4
PARETO DISTRIB. FOR RARE FLOODS
THE PARTITIONING POINTS ARE D AND I
CONDITION. PROBABI. OF FAILURE=100%
DM'S ESTIMATE OF PR(FULL POOL)=100%

THERE IS 0.00000% CHANCE THAT F0 OCCURS
THERE IS 98.00605% CHANCE THAT F2 OCCURS
THERE IS 1.94211% CHANCE THAT F3 OCCURS
THERE IS 0.05184% CHANCE THAT F4 OCCURS
PRESS <SPACE> BAR TO CONTINUE

S SCENARIO	F1(S) COST FUNC.	LAM(2,1) HP/LC RISK	LAM(3,1) INTER RISK	LAM(4,1) LP/HC RISK	LAM(5,1) EXPE. VALUE
1	0	**	0.1210785	77.3010300	0.0424385
2	0.8	-1.0000000	0.0174476	10.4402800	0.0057492
3	5.15	-1.0000000	0.0048793	1.0161350	0.0064431
5	19.32	-1.0000000	-1.0000000	-1.0000000	-1.0000000
6	20.12	-1.0000000	-1.0000000	248.958700	-1.0000000
4	20.83	-1.0000000	**	**	**
7	22.02	-1.0000000	-1.0000000	-1.0000000	-1.0000000
9	25.88	-1.0000000	-1.0000000	-1.0000000	-1.0000000
10	26.68	-1.0000000	-1.0000000	-1.0000000	-1.0000000
11	27.93	-1.0000000	-1.0000000	-1.0000000	-1.0000000
13	31.12	-1.0000000	-1.0000000	-1.0000000	-1.0000000
14	31.92	-1.0000000	-1.0000000	-1.0000000	-1.0000000
15	32.64	-1.0000000	-1.0000000	-1.0000000	-1.0000000
8	36.14	-1.0000000	-1.0000000	-1.0000000	-1.0000000
12	42.04	-1.0000000	-1.0000000	-1.0000000	-1.0000000
16	46.76	-1.0000000	-1.0000000	-1.0000000	-1.0000000

Figure 5.4 PMRM results for partitioning points D and I

S SCENARIO	F1(S) COST FUNC.	F2(S) HP/LC RISK	F3(S) INTER.RISK	F4(S) LP/HC RISK	F5(S) EXPE. VALUE
1	0	159.9796	173.2247	1112.358	160.5698
2	0.8	159.9796	173.102	1021.535	160.5359
3	5.15	159.9796	173.0004	955.2206	160.5109
5	19.32	159.9796	175.5553	1010.526	160.5801
6	20.12	159.9796	175.4311	910.9949	160.5431
4	20.83	159.9796	172.9056	669.6812	160.4099
7	22.02	159.9796	175.3399	711.8328	160.4722
9	25.88	159.9796	177.3512	946.3501	160.593
10	26.68	159.9796	177.24	736.0798	160.5179
11	27.93	159.9796	177.1549	734.1538	160.5155
13	31.12	159.9796	178.427	748.1218	160.5453
14	31.92	159.9796	178.3097	746.6701	160.5425
15	32.64	159.9796	178.2161	745.1796	160.5401
8	36.14	159.9796	175.2398	708.9613	160.4692
12	42.04	159.9796	177.0652	731.7629	160.5129
16	46.76	159.9796	178.1197	743.494	160.5376

RETURN PERIOD OF PMF = 1E 4 THERE IS 0.00000% CHANCE THAT F0 OCCURS
PARETO DISTRIB. FOR RARE FLOODS THERE IS 98.00605% CHANCE THAT F2 OCCURS
THE PARTITIONING POINTS ARE D AND J THERE IS 1.95922% CHANCE THAT F3 OCCURS
CONDITION. PROBABI. OF FAILURE=100% THERE IS 0.03473% CHANCE THAT F4 OCCURS
DM'S ESTIMATE OF PR(FULL POOL)=100% PRESS <SPACE> BAR TO CONTINUE

S SCENARIO	F1(S) COST FUNC.	LAM(2,1) HP/LC RISK	LAM(3,1) INTER RISK	LAM(4,1) LP/HC RISK	LAM(5,1) EXPE. VALUE
1	0	**	0.1533318	113.528200	0.0424385
2	0.8	-1.0000000	0.0233582	15.2447600	0.0057492
3	5.15	-1.0000000	0.0060442	2.9542930	0.0064431
5	19.32	-1.0000000	-1.0000000	-1.0000000	-1.0000000
6	20.12	-1.0000000	-1.0000000	339.878900	-1.0000000
4	20.83	-1.0000000	**	**	**
7	22.02	-1.0000000	-1.0000000	-1.0000000	-1.0000000
9	25.88	-1.0000000	-1.0000000	-1.0000000	-1.0000000
10	26.68	-1.0000000	-1.0000000	-1.0000000	-1.0000000
11	27.93	-1.0000000	-1.0000000	-1.0000000	-1.0000000
13	31.12	-1.0000000	-1.0000000	-1.0000000	-1.0000000
14	31.92	-1.0000000	-1.0000000	-1.0000000	-1.0000000
15	32.64	-1.0000000	-1.0000000	-1.0000000	-1.0000000
8	36.14	-1.0000000	-1.0000000	-1.0000000	-1.0000000
12	42.04	-1.0000000	-1.0000000	-1.0000000	-1.0000000
16	46.76	-1.0000000	-1.0000000	-1.0000000	-1.0000000

Figure 5.5 PMRM results for partitioning points D and J

S SCENARIO	F1(S) COST FUNC.	F2(S) HP/LC RISK	F3(S) INTER.RISK	F4(S) LP/HC RISK	F5(S) EXPE. VALUE
1	0	159.9796	175.0322	1333.776	160.5698
2	0.8	159.9796	174.8957	1206.926	160.5359
3	5.15	159.9796	174.779	1118.166	160.5109
5	19.32	159.9796	177.5669	1175.565	160.5801
6	20.12	159.9796	177.4289	1038.634	160.5431
4	20.83	159.9796	174.6712	723.1444	160.4099
7	22.02	159.9796	177.3253	763.3835	160.4722
9	25.88	159.9796	179.4694	1077.463	160.593
10	26.68	159.9796	179.3463	786.7682	160.5178
11	27.93	159.9796	179.2506	784.9703	160.5155
13	31.12	159.9796	180.6136	796.6943	160.5453
14	31.92	159.9796	180.4845	795.6606	160.5425
15	32.64	159.9796	180.3805	794.4435	160.5401
8	36.14	159.9796	177.2111	760.5455	160.4692
12	42.04	159.9796	179.1487	782.6455	160.5129
16	46.76	159.9796	180.2727	793.038	160.5377

RETURN PERIOD OF PMF = 1E 4 THERE IS 0.00000% CHANCE THAT F0 OCCURS
PARETO DISTRIB. FOR RARE FLOODS THERE IS 98.00605% CHANCE THAT F2 OCCURS
THE PARTITIONING POINTS ARE D AND K THERE IS 1.96892% CHANCE THAT F3 OCCURS
CONDITION. PROBABI. OF FAILURE=100% THERE IS 0.02503% CHANCE THAT F4 OCCURS
DM'S ESTIMATE OF PR(FULL POOL)=100% PRESS <SPACE> BAR TO CONTINUE

S SCENARIO	F1(S) COST FUNC.	LAM(2,1) HP/LC RISK	LAM(3,1) INTER RISK	LAM(4,1) LP/HC RISK	LAM(5,1) EXPE. VALUE
1	0	**	0.1706123	156.062900	0.0424385
2	0.8	-1.0000000	0.0268309	20.8643200	0.0057492
3	5.15	-1.0000000	0.0068762	5.3127410	0.0064422
5	19.32	-1.0000000	-1.0000000	-1.0000000	-1.0000000
6	20.12	-1.0000000	-1.0000000	444.352300	-1.0000000
4	20.83	-1.0000000	**	**	**
7	22.02	-1.0000000	-1.0000000	-1.0000000	-1.0000000
9	25.88	-1.0000000	-1.0000000	-1.0000000	-1.0000000
10	26.68	-1.0000000	-1.0000000	-1.0000000	-1.0000000
11	27.93	-1.0000000	-1.0000000	-1.0000000	-1.0000000
13	31.12	-1.0000000	-1.0000000	-1.0000000	-1.0000000
14	31.92	-1.0000000	-1.0000000	-1.0000000	-1.0000000
15	32.64	-1.0000000	-1.0000000	-1.0000000	-1.0000000
8	36.14	-1.0000000	-1.0000000	-1.0000000	-1.0000000
12	42.04	-1.0000000	-1.0000000	-1.0000000	-1.0000000
16	46.76	-1.0000000	-1.0000000	-1.0000000	-1.0000000

Figure 5.6 PMRM results for partitioning points D and K

S SCENARIO	F1(S) COST FUNC.	F2(S) HP/LC RISK	F3(S) INTER.RISK	F4(S) LP/HC RISK	F5(S) EXPE. VALUE
1	0	159.9796	177.5416	1442.379	160.5698
2	0.8	159.9796	176.2233	1400.721	160.5359
3	5.15	159.9796	176.0977	1281.964	160.5109
5	19.32	159.9796	179.0272	1342.058	160.5801
6	20.12	159.9796	178.8823	1162.193	160.5432
4	20.83	159.9796	175.9788	762.0803	160.4099
7	22.02	159.9796	178.7717	799.8699	160.4722
9	25.88	159.9796	180.9972	1205.022	160.593
10	26.68	159.9796	180.8658	822.4636	160.5178
11	27.93	159.9796	180.7625	820.9095	160.5155
13	31.12	159.9796	182.1768	830.613	160.5453
14	31.92	159.9796	182.0421	829.8723	160.5425
15	32.64	159.9796	181.9319	828.9383	160.5401
8	36.14	159 9796	178.6493	797.0218	160.4692
12	42.04	159.9796	180.6517	818.8064	160.5129
16	46.76	159.9796	181.8186	827.6878	160.5376

RETURN PERIOD OF PMF = 1E 4 THERE IS 0.00000% CHANCE THAT F0 OCCURS
PARETO DISTRIB. FOR RARE FLOODS THERE IS 98.00605% CHANCE THAT F2 OCCURS
THE PARTITIONING POINTS ARE D AND L THERE IS 1.97497% CHANCE THAT F3 OCCURS
CONDITION. PROBABI. OF FAILURE=100% THERE IS 0.01898% CHANCE THAT F4 OCCURS
DM'S ESTIMATE OF PR(FULL POOL)=100% PRESS <SPACE> BAR TO CONTINUE

S SCENARIO	F1(S) COST FUNC.	LAM(2,1) HP/LC RISK	LAM(3,1) INTER RISK	LAM(4,1) LP/HC RISK	LAM(5,1) EXPE. VALUE
1	0	**	1.6478920	52.0715400	0.0424385
2	0.8	-1.0000000	0.0288795	27.3005900	0.0057527
3	5.15	-1.0000000	0.0075827	8.0007510	0.0064422
5	19.32	-1.0000000	-1.0000000	-1.0000000	-1.0000000
6	20.12	-1.0000000	-1.0000000	563.539200	-1.0000000
4	20.83	-1.0000000	**	**	**
7	22.02	-1.0000000	-1.0000000	-1.0000000	-1.0000000
9	25.88	-1.0000000	-1.0000000	-1.0000000	-1.0000000
10	26.68	-1.0000000	-1.0000000	-1.0000000	-1.0000000
11	27.93	-1.0000000	-1.0000000	-1.0000000	-1.0000000
13	31.12	-1.0000000	-1.0000000	-1.0000000	-1.0000000
14	31.92	-1.0000000	-1.0000000	-1.0000000	-1.0000000
15	32.64	-1.0000000	-1.0000000	-1.0000000	-1.0000000
8	36.14	-1.0000000	-1.0000000	-1.0000000	-1.0000000
12	42.04	-1.0000000	-1.0000000	-1.0000000	-1.0000000
16	46.76	-1.0000000	-1.0000000	-1.0000000	-1.0000000

Figure 5.7 PMRM results for partitioning points D and L

S SCENARIO	F1(S) COST FUNC.	F2(S) HP/LC RISK	F3(S) INTER.RISK	F4(S) LP/HC RISK	F5(S) EXPE. VALUE
1	0	159.9796	180.0789	1448.919	160.5698
2	0.8	159.9796	178.0176	1494.803	160.5359
3	5.15	159.9796	177.1065	1448	160.5109
5	19.32	159.9796	180.7943	1423.082	160.5801
6	20.12	159.9796	179.9734	1284.116	160.5431
4	20.83	159.9796	176.9796	788.2978	160.4099
7	22.02	159.9796	179.859	824.1115	160.4722
9	25.88	159.9796	182.138	1331.379	160.5931
10	26.68	159.9796	182.0024	845.7278	160.5179
11	27.[illegible]3	159.9796	181.8948	844.3559	160.5155
13	31.[illegible]2	159.9796	183.3357	852.7628	160.5453
14	31.92	159.9796	183.1984	852.[illegible]99	160.5425
15	32.64	159.9796	183.085	851.4856	160.5401
8	36.14	159.9796	179.7318	821.161	160.4692
12	42.04	159.9796	181.7785	842.4351	160.5129
16	46.76	159.9796	182.9686	850.3344	160.5377

RETURN PERIOD OF PMF = 1E 4 THERE IS 0.00000% CHANCE THAT F0 OCCURS
PARETO DISTRIB. FOR RARE FLOODS THERE IS 98.00[illegible]5% CHANCE THAT F2 OCCURS
THE PARTITIONING POINTS ARE D AND M THERE IS 1.97902% CHANCE THAT F3 OCCURS
CONDITION. PROBABI. OF FAILURE=100% THERE IS 0.01493% CHANCE THAT F4 OCCURS
DM'S ESTIMATE OF PR(FULL POOL)=100% PRESS <SPACE> BAR TO CONTINUE

S SCENARIO	F1(S) COST FUNC.	LAM(2,1) HP/LC RISK	LAM(3,1) INTER RISK	LAM(4,1) LP/HC RISK	LAM(5,1) EXPE. VALUE
1	0	**	2.5765800	0.1783886	0.0424194
2	0.8	-1.0000000	0.2094488	-1.0000000	0.0057527
3	5.15	-1.0000000	0.0080965	1.7585360	0.0064422
5	19.32	-1.0000000	-1.0000000	173.707500	-1.0000000
6	20.12	-1.0000000	-1.0000000	698.335800	-1.0000000
4	20.83	-1.0000000	**	**	**
7	22.02	-1.0000000	-1.0000000	-1.0000000	-1.0000000
9	25.88	-1.0000000	-1.0000000	-1.0000000	-1.0000000
10	26.68	-1.0000000	-1.0000000	-1.0000000	-1.0000000
11	27.93	-1.0000000	-1.0000000	-1.0000000	-1.0000000
13	31.12	-1.0000000	-1.0000000	-1.0000000	-1.0000000
14	31.92	-1.0000000	-1.0000000	-1.0000000	-1.0000000
15	32.64	-1.0000000	-1.0000000	-1.0000000	-1.0000000
8	36.14	-1.0000000	-1.0000000	-1.0000000	-1.0000000
12	42.04	-1.0000000	-1.0000000	-1.0000000	-1.0000000
16	46.76	-1.0000000	-1.0000000	-1.0000000	-1.0000000

Figure 5.8 PMRM results for partitioning points D and M

S SCENARIO	F1(S) COST FUNC.	F2(S) HP/LC RISK	F3(S) INTER.RISK	F4(S) LP/HC RISK	F5(S) EXPE. VALUE
1	0	159.942	160.9789	1333.776	160.5698
2	0.8	159.942	160.9706	1208.926	160.5359
3	5.15	159.942	160.9634	1118.166	160.5109
5	19.32	159.942	161.1339	1175.565	160.5802
6	20.12	159.942	161.1254	1038.634	160.5432
4	20.83	159.942	160.9569	723.1444	160.4099
7	22.02	159.942	161.1191	763.3835	160.4722
9	25.88	159.942	161.2502	1077.463	160.593
10	26.68	159.942	161.2426	786.7682	160.5179
11	27.93	159.942	161.2368	784.9703	160.5155
13	31.12	159.942	161.3201	796.6943	160.5455
14	31.92	159.942	161.3122	795.6606	160.5425
15	32.64	159.942	161.3058	794.4435	160.5401
8	36.14	159.942	161.1121	760.5455	160.4692
12	42.04	159.942	161.2306	782.6455	160.5129
16	46.76	159.942	161.2992	793.038	160.5377

RETURN PERIOD OF PMF = 1E 4 THERE IS 0.00000% CHANCE THAT F0 OCCURS
PARETO DISTRIB. FOR RARE FLOODS THERE IS 67.76366% CHANCE THAT F2 OCCURS
THE PARTITIONING POINTS ARE C AND K THERE IS 32.21131% CHANCE THAT F3 OCCURS
CONDITION. PROBABI. OF FAILURE=100% THERE IS 0.02503% CHANCE THAT F4 OCCURS
DM'S ESTIMATE OF PR(FULL POOL)=100% PRESS <SPACE> BAR TO CONTINUE

S SCENARIO	F1(S) COST FUNC.	LAM(2,1) HP/LC RISK	LAM(3,1) INTER RISK	LAM(4,1) LP/HC RISK	LAM(5,1) EXPE. VALUE
1	0	**	0.0104332	156.062900	0.0424385
2	0.8	-1.0000000	0.0016381	20.8643200	0.0057492
3	5.15	-1.0000000	0.0004204	5.3127410	0.0064422
5	19.32	-1.0000000	-1.0000000	-1.0000000	-1.0000000
6	20.12	-1.0000000	-1.0000000	444.352300	-1.0000000
4	20.83	-1.0000000	**	**	**
7	22.02	-1.0000000	-1.0000000	-1.0000000	-1.0000000
9	25.88	-1.0000000	-1.0000000	-1.0000000	-1.0000000
10	26.68	-1.0000000	-1.0000000	-1.0000000	-1.0000000
11	27.93	-1.0000000	-1.0000000	-1.0000000	-1.0000000
13	31.12	-1.0000000	-1.0000000	-1.0000000	-1.0000000
14	31.92	-1.0000000	-1.0000000	-1.0000000	-1.0000000
15	32.64	-1.0000000	-1.0000000	-1.0000000	-1.0000000
8	36.14	-1.0000000	-1.0000000	-1.0000000	-1.0000000
12	42.04	-1.0000000	-1.0000000	-1.0000000	-1.0000000
16	46.76	-1.0000000	-1.0000000	-1.0000000	-1.0000000

Figure 5.9 PMRM results for partitioning points C and K

S SCENARIO	F1(S) COST FUNC.	F2(S) HP/LC RISK	F3(S) INTER.RISK	F4(S) LP/HC RISK	F5(S) EXPE. VALUE
1	0	159.9825	208.8936	1333.776	160.5698
2	0.8	159.9825	208.4459	1208.926	160.5359
3	5.15	159.9825	208.0629	1118.166	160.5109
5	19.32	159.9825	217.2098	1173.565	160.5802
6	20.12	159.9825	216.757	1038.634	160.5432
4	20.83	159.9825	207.7092	723.1444	160.4099
7	22.02	159.9825	216.4168	763.3835	160.4722
9	25.88	159.9825	223.4515	1077.463	160.5931
10	26.68	159.9825	223.0474	786.7682	160.5179
11	27.93	159.9825	222.7334	784.9703	160.5155
13	31.12	159.9825	227.2054	796.6943	160.5453
14	31.92	159.9825	226.7818	795.6606	160.5425
15	32.64	159.9825	226.4404	794.4435	160.5401
8	36.14	159.9825	216.0424	760.5455	160.4693
12	42.04	159.9825	222.3991	782.6455	160.5129
16	46.76	159.9825	226.0868	793.038	160.5377

RETURN PERIOD OF PMF = 1E 4 THERE IS 0.00000% CHANCE THAT F0 OCCURS
PARETO DISTRIB. FOR RARE FLOODS THERE IS 99.37484% CHANCE THAT F2 OCCURS
THE PARTITIONING POINTS ARE E AND K THERE IS 0.60013% CHANCE THAT F3 OCCURS
CONDITION. PROBABI. OF FAILURE=100% THERE IS 0.02503% CHANCE THAT F4 OCCURS
DM'S ESTIMATE OF PR(FULL POOL)=100% PRESS <SPACE> BAR TO CONTINUE

S SCENARIO	F1(S) COST FUNC.	LAM(2,1) HP/LC RISK	LAM(3,1) INTER RISK	LAM(4,1) LP/HC RISK	LAM(5,1) EXPE. VALUE
1	0	**	0.5596352	156.062900	0.0424194
2	0.8	-1.0000000	0.0880344	20.8643200	0.0057527
3	5.15	-1.0000000	0.0225573	5.3127410	0.0064422
5	19.32	-1.0000000	-1.0000000	-1.0000000	-1.0000000
6	20.12	-1.0000000	-1.0000000	444.352300	-1.0000000
4	20.83	-1.0000000	**	**	**
7	22.02	-1.0000000	-1.0000000	-1.0000000	-1.0000000
9	25.88	-1.0000000	-1.0000000	-1.0000000	-1.0000000
10	26.68	-1.0000000	-1.0000000	-1.0000000	-1.0000000
11	27.93	-1.0000000	-1.0000000	-1.0000000	-1.0000000
13	31.12	-1.0000000	-1.0000000	-1.0000000	-1.0000000
14	31.92	-1.0000000	-1.0000000	-1.0000000	-1.0000000
15	32.64	-1.0000000	-1.0000000	-1.0000000	-1.0000000
8	36.14	-1.0000000	-1.0000000	-1.0000000	-1.0000000
12	42.04	-1.0000000	-1.0000000	-1.0000000	-1.0000000
16	46.76	-1.0000000	-1.0000000	-1.0000000	-1.0000000

Figure 5.10 PMRM results for partitioning points E and K

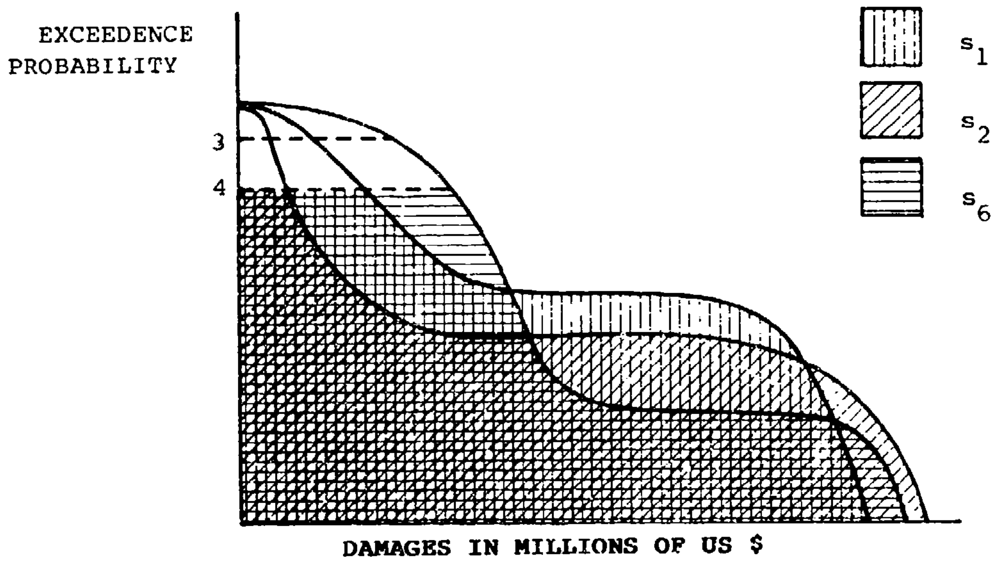


Figure 5.11 Risk curves for s_1, s_2 s_6. Partitioning point is H (not on scale)

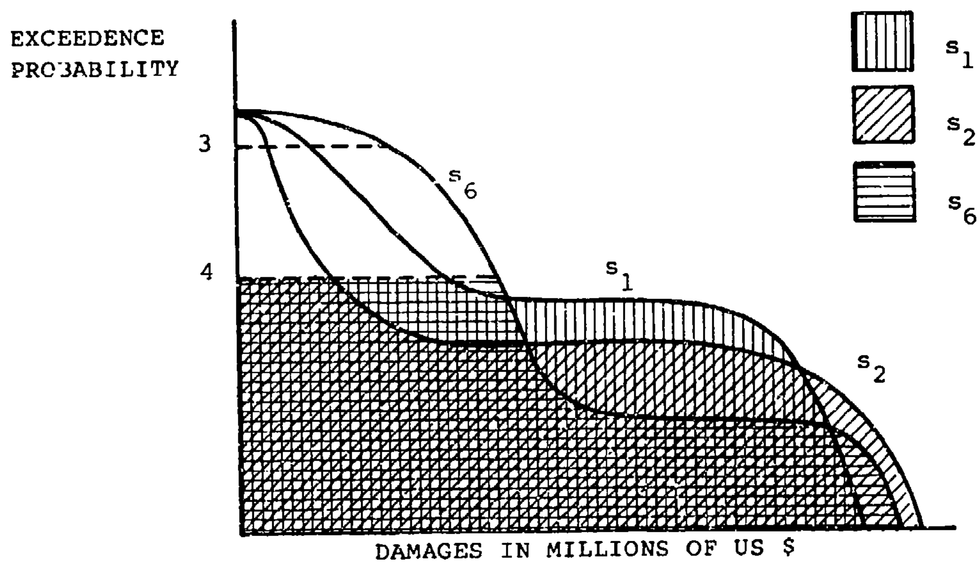


Figure 5.12 Risk curves for s_1, s_2, and s_6. Partitioning point is k (not on scale)

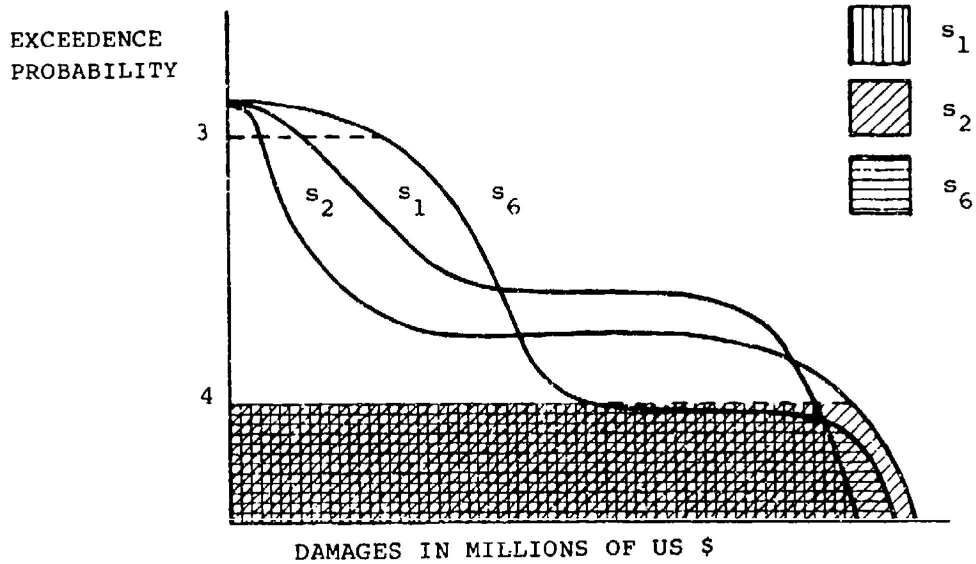


Figure 5.13 Risk curves for s_1, s_2, and s_6. Partitioning is $\tilde{N}$ (not on scale)

CHAPTER 6

THE IMPORTANCE OF THE CHOICE OF PROBABILITY DISTRIBUTION

When the PMRM is used, it is obvious that the choice of partition points will effect the result. Just as important, however, is the choice of the probability density function (pdf) to represent the flood flows. Very seldom, if ever, is it possible to state that a specific distribution should be used. In some cases one can exclude some pdfs or guess that some are more likely than others. Quite often, one is given a very limited set of data that does not contain any information about extreme events. In flood control, for example, records have only been kept for the last 50 or 100 years and it is virtually impossible to draw any conclusions about floods with return periods of more than 100 years. In particular, nothing can be said with certainty about the probable maximum flood (PMF), that is, the flood with a return period greater than 10^3 years. Events of a more extreme character are, however, very important and are what determine the low-probability expectation f_4.

Since it is a very difficult task to decide which distribution best represents the damage, one would like to know what effect the choice of distribution has on the conditional expectations and consequently the results. In many cases, none of the well-known distributions fit the data perfectly. The random damage may be the result of a stochastic process involving several random variables and the joint density function of these random variables is not necessarily one of the well-known few. Therefore, the problem is not always one of finding the correct pdf but in selecting one that is sufficiently accurate. The best situation would be to have some guidelines as how to choose the density function.

6.1 The Range of Low- and Intermediate-Damage Expectation (f_2, f_3)

As when studying the effect of the partition scheme on the conditional expectations f_2 and f_3, very little can be said about the importance of the distribution using only analytical means. Instead, one is forced to use empirical evidence from simulations.

The low-damage expectation f_2 is basically a result from the shape of the distribution, which represents the bulk of the damage events. Since the majority of all well-known pdfs are asymmetrical (having, for example, lognormal, gamma or Weibull distributions) and change their shapes with the quotient s/m, it is impossible to generalize and say which distribution gives the largest value for the expectation f_2. In fact, even for a given value of s/m, different partition points may give different relationships between the values of f_2 corresponding to the different distributions (Fig. 6.1).

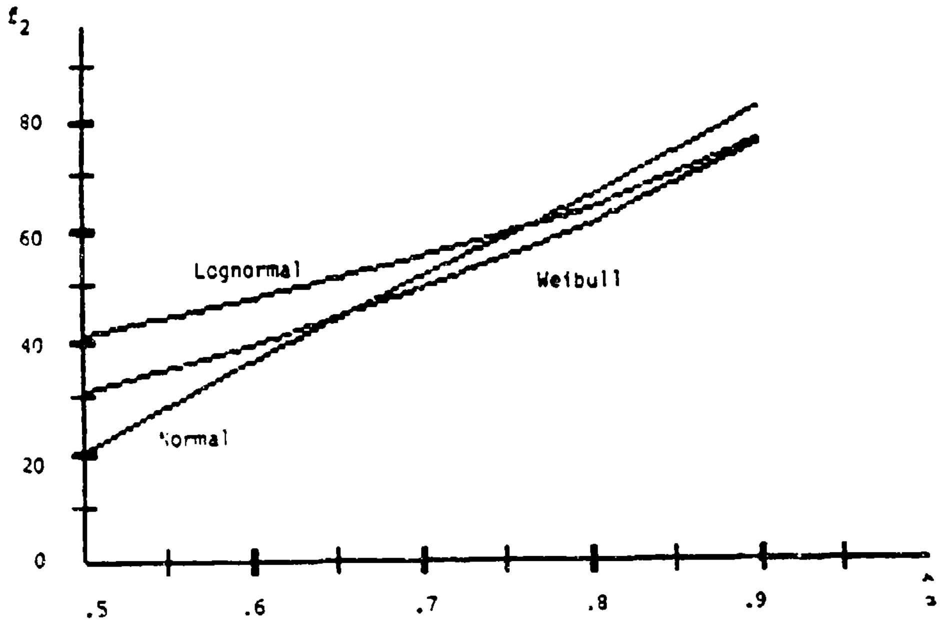


Figure 6.1. The low-damage expectation f_2 versus $\hat{\alpha}$ for initial normal, lognormal, and Weibull distributions (m = 100 and s = 100)

It seems that the values of the expectations are rather similar for all three distributions, at least for large $\hat{\alpha}$. In some cases, the values coincide even better and the choice of distribution seems rather irrelevant (see Fig. 5.8). We have already observed that for quotients of s/m between 0.05 and 0.5, the values of f_2 are almost identical for our three distributions. This is, however, not a result which may be generalized to all types of distributions without more simulations. There is not much to be said about the intermediate f_3 either (Fig. 6.2). Once again, the asymmetrical pdfs make it impossible to relate the different values of f_3 with one another.

For some partition points ($\hat{\alpha} < 0.87$), the normal distribution gives the greatest values of f_3, while for $\hat{\alpha} > 0.87$ the Weibull oduces the largest. Notice, in Fig. 5.10, that the lognormal distribution may yield large values of f_3 as well.

In conclusion, it is impossible to make any general statements about how the values of f_2 and f_3, corresponding to different initial distributions, relate to one another.

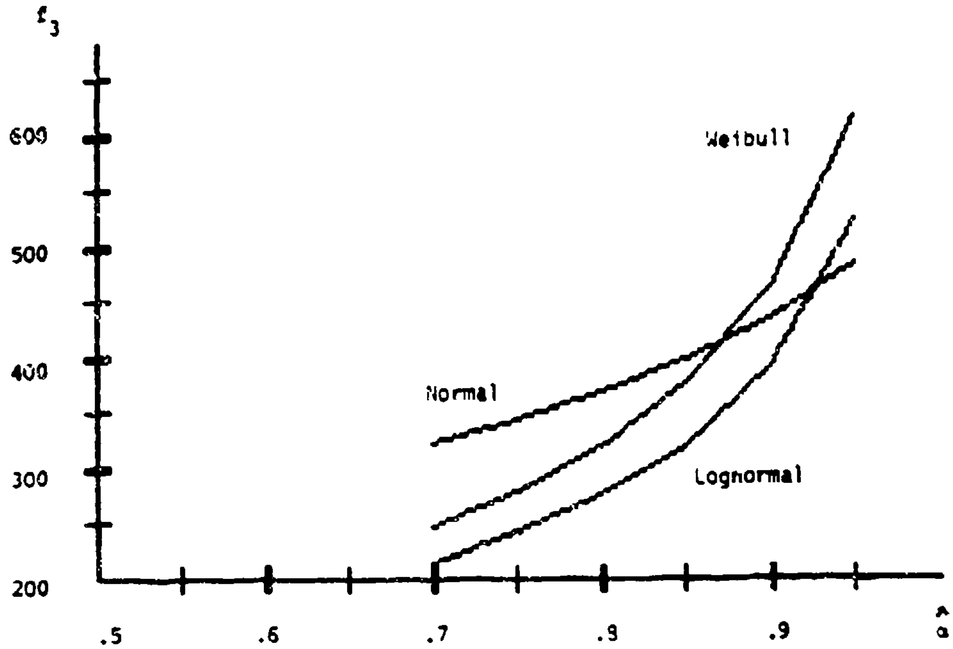


Figure. 6.2 The intermediate expectation of $f_3(\hat{\alpha})$ versus $\hat{\alpha}$ for initial normal, lognormal and Weibull distributions with m = 100, s = 200 and higher partition point $\alpha' = 0.999$

6.2 The Low-Probability Expectation (f_4)

As pointed out in the previous section, the values of f_2 and f_3 are largely determined by the shape of the initial variate. This is however not the case for the low-probability expectation f_4. There is a closer relationship between f_4 and the statistics of extremes, and the extremes are determined by the behavior of the distribution's tail and not its shape (Ang and Tang [19084]). Consequently, it should be possible to say more about f_4's dependence on the chosen pdf than was possible for f_2 and f_3.

The initial variate's asymptotic form (type I or Type II) plays an important role here; that is, it matters whether the pdf's tail decays exponentially or polynomially. A polynomially decaying tail produces greater probabilities of occurrence for extreme events than an exponentially decaying tail. This implies that trully extreme events are given more weight during the integration of the conditional expectations for the Type II asymptotic form than for the Type I. Consequently, the values of f_4 are generally greater for a Type II distribution than for a Type I, although the partitioning of both is the same. Of the three distributions of interest, the lognormal belongs to the Type II form while both the normal and Weibull are of Type I. Therefore, the values corresponding to the lognormal distribution are expected to be greater than those for both the others.

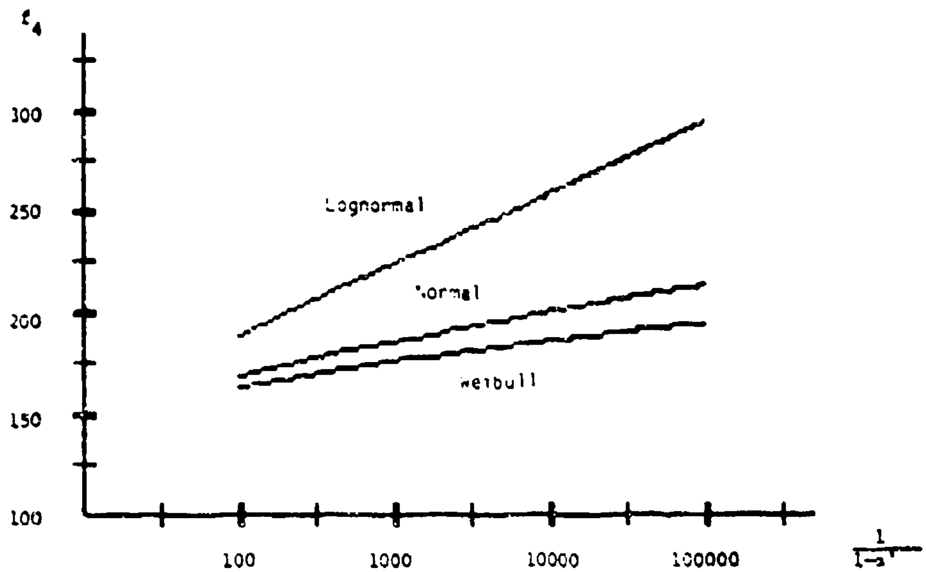


Figure 6.3 The expectation f_4 $\frac{1}{1-\alpha'}$ for initial normal, lognormal, and Weibull distributions (m = 100 and s = 25)

However, it is very difficult to state anything about how different Type I or II distributions relate to one another in terms of the values of f_4 they produce. In Fig. 6.3, the normal distribution gives greater values of f_4 than the Weibull, but this will no longer be true as the value of s/m is altered (increased or decreased).

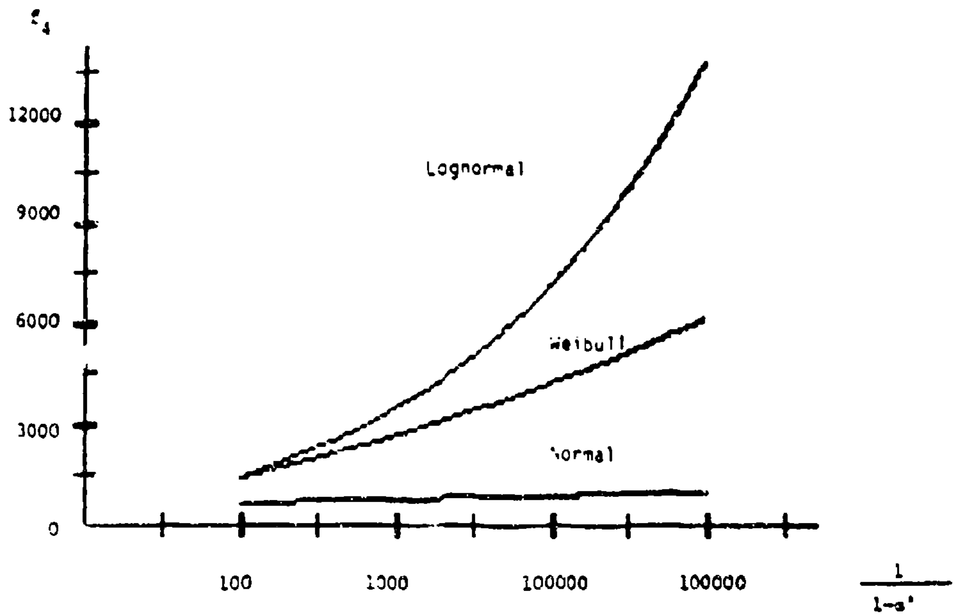


Figure 6.4 The expectation f_4 versus $\frac{1}{1-\alpha'}$ for initial normal, lognormal, and Weibull distributions (m = 100 and s = 200)

Clearly, one cannot generalize and say that one distribution will always give greater values for f_4 than another when they have the same asymptotic form. This may be explained by looking at the behavior of the distributions' respective tails. Take, for example, the normal and Weibull distributions. Their corresponding pdfs are

$$P_X^N(x) = \frac{1}{\alpha\sqrt{2\pi}} \cdot \exp[-1/2(\frac{x-\mu}{\sigma})^2]$$

and

$$P_X^W(x) = \frac{c}{a} \cdot (\frac{x}{a})^{c-1} \cdot \exp[-(\frac{x}{a})^c]$$

The normal distribution's tail will always decay as e^{-x^2}, while that of the Weibull decays differently for different values of its parameter c. For large values of c, the decay of $x^{c-1}\exp[-x^c]$ will be faster than that of the normal and consequently, the values of f_4^W will be less than those of f_4^N. The opposite is true for small values of c. Moreover, small values of s/m correspond to large values of c and, conversely, large values of s/m correspond to small values c. In Fig. 6.3, s/m = 0.25, a relatively small value which corresponds to a c-value of about 4.444 (see Appendix 1). One would expect the tail of the Weibull to decay faster than that of the normal; that is, $x^{c-1}\exp[-x^c]$ decays faster than e^{-x} for large x. Consequently, the values of f_4^N are greater than those of f_4^W.

In Fig. 6.4, however, s/m = 2, which corresponds to a c-value of 0.5423. Clearly, the tail of the normal decays much faster than that of the Weibull, and we expect f_4^N to be less than f_4^W, just as suggested by the simulations.

It should be mentioned that this is a somewhat simplified picture. Actually, the values of both the parameters for the respective distributions should be taken into consideration, thus making the whole issue rather complicated.

From the simulations that have been performed, it seems that for very small quotients of s/m, the normal and the lognormal give similar values of f_4 while the Weibull's is considerably lower. For large quotients, however, the normal will always given the lowest values of the three and the Weibull will take somewhat of an intermediate position (see Fig. 6.4).

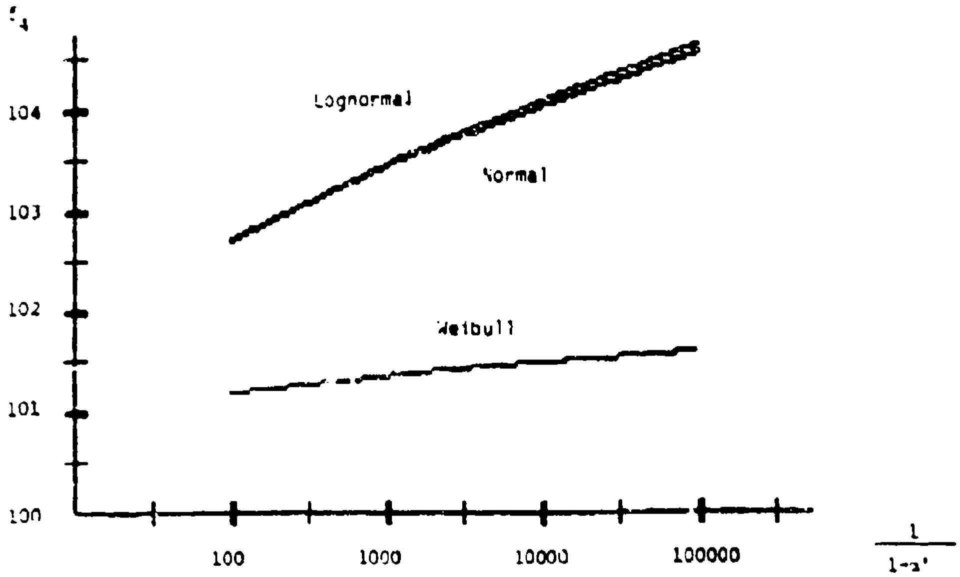


Figure 6.5 The expectation f_4 versus $\frac{1}{1-\alpha'}$ for initial normal, lognormal, and Weibull distributions (m = 100 and s = 1)

In all the simulations performed on here, the lognormal distribution gave, without exception, the largest values of f_4 (see Table 6.1). This implies that if the decision maker is a pessimist and wishes to emphasize catastrophic events, he should choose an initial lognormal distribution over a normal or a Weibull.

Partition	Normal	Lognormal	Weibull
	s/m = 0.01		
0.99	102.67	102.67	101.15
0.999	103.37	103.50	101.33
0.9999	103.96	104.02	101.46
0.99999	103.48	104.60	101.56
	s/m = 0.25		
0.99	166.62	187.60	161.25
0.999	184.24	222.94	174.43
0.9999	198.97	257.50	184.85
0.99999	211.95	292.60	193.55
	s/m = 1.00		
0.99	366.49	676.09	560.52
0.999	436.95	1198.90	790.98
0.9999	495.87	1948.00	1021.03
0.99999	547.80	2992.10	1251.29
	s/m = 5.00		
0.99	1432.45	3005.48	2281.08
0.999	1784.75	9941.85	6485.85
0.9999	2079.36	27815.30	14388.97
0.99999	2339.00	69487.60	27329.33

Table 6.1 Comparison between the values of f_4 for initial normal, lognormal, and Weibull distributions. Different ratios of s to m are considered as well.

It cannot be overemphasized that these results are only true when only these three distributions are considered. There exist many more distributions that correspond to both the Type I and Type II asymptotic forms--for example, the gamma and the Pareto distributions. Nothing can be concluded about them without doing more simulations where these new distributions are included. By studying the decay of their tails, one may guess the relative order between the values of f_4. However, an initial variate belonging to the Type II asymptotic form generally gives greater values of f_4 than one of the Type I form.

6.3 A Comparison of Two Case Studies

There are many ways of fitting a distribution function to a sample of events or a set of data. It is impossible to make a general statement that one method is superior to the others. Every single problem has its own individual characteristics that make one method more suitable for it than another.

Not only is it difficult to choose which method to use but also which type of distribution function makes the best fit. However, a number of statistical tests exist that are helpful in this latter issue. Among these, the chi^2 and the Kolmogorov-Smirnov are widely used.

There are basically three different methods for fitting a distribution function: the method of moments, the method of maximum likelihood, and the use of two boundary conditions. Clearly, these three methods will generally give different values for the pdfs' parameters. This latter issue has great impact on the PMRM, especially on the low-probability conditional expectation f_4.

Karlsson [1986] has shown that when the method of moments is used, thin-tailed distributions such as the normal tend to give much lower values of f_4 than thick-tailed ones. His results cannot be generalized to the use of the other two methods without further investigation. Karlsson does, however, use boundary conditions in his example problem on dam safety [see Karlsson, 1986, Chapter 8]. This problem is a slight modification of one constructed by Stedinger and Grygier [1985]. Just as when using the method of moments, he found that thin-tailed distributions are associated with smaller values of f_4. The differences are, however, much less prominent than when using the method of moments.

Petrakian [1986] has also addressed a very similar problem. He simply took the problem posed by Stedinger and Grygier [1985] and studied the applicability of the PMRM to it. Surprisingly, he found that thin-tailed distributions give greater values for f_4 than thick-tailed ones. His conclusions are therefore opposite to the ones made by Karlsson. Fortunately, there is an explanation for this discrepancy.

When two boundary conditions are used to determine the flood frequency distribution, one at the 100-yr flood and one at the PMF, the pdfs are forced to attain the same exceedence probabilities at these two particular floods. The boundary conditions are

$$1 - P_Q(q_{100}) = 0.01$$

and

$$1 - P_Q(PMF) = 1/T$$

where T is the return period of the PMF. However, nothing is said about floods of any other magnitude.

It is known that the tails of different distributions decay with different velocities. Thin-tailed distributions, such as the normal, decay much faster than thick-tailed, such as the Pareto (Fig. 6.6). In mathematical terms, the exceedance probability of a very large flood q^* is less for a normal distribution than for a Pareto. In fact, this is true for all floods exceeding (in this problem) the PMF.

It has already been mentioned that the PMF, being one of boundary points, is one of the two floods that have the same exceedance probability for any distribution. Consequently, when graphing the corresponding cumulative distribution functions (cdfs), all the curves will intersect at

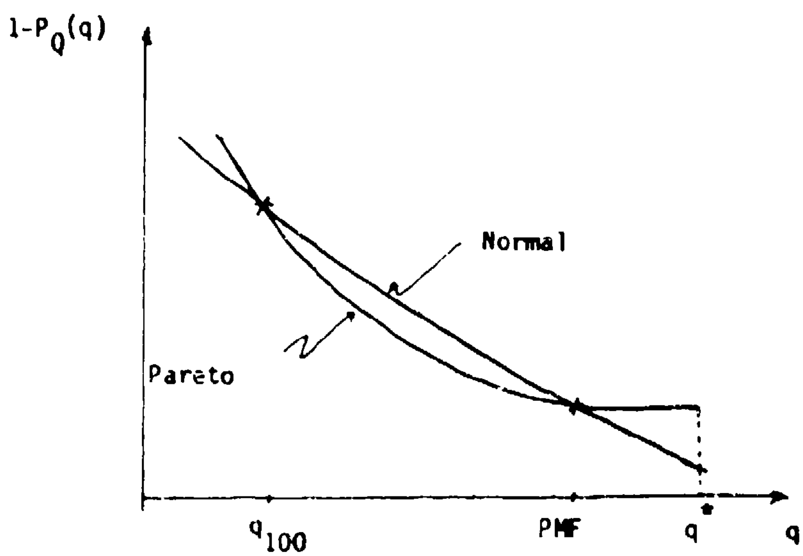


Figure 6.6 The cumulative distribution functions for the normal and Pareto distributions fitted by two boundary conditions.

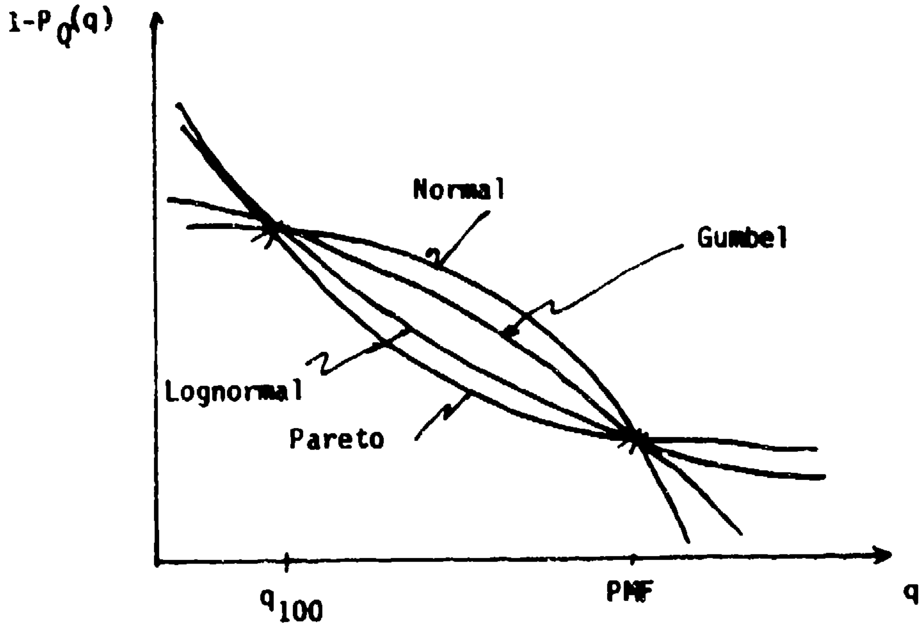


Figure 6.7 The cdfs of the normal, Gumbel, lognormal and Pareto when using boundary conditions.

the 100-yr flood as well as at the PMF. For floods exceeding the PMF, thin-tailed distributions will decay faster than thick-tailed. By fitting four different distribution functions (the normal, Gumbel, lognormal, and Pareto), the Fig. 6.7 is obtained. The ordinate is made logarithmic for the purpose of making the trend easier to appreciate.

Clearly, distributions that have small exceedence probabilities for floods exceeding the PMF also have a large exceedance probabilities for floods below the PMF. Therefore, it may seem that thin-tailed distributions decay slower than thick-tailed ones for these latter floods.

When using the PMRM, one can either partition the probability axis or the damage axis. In this explanation of the discrepancies between Karlsson's and Petrakian's results, only the partitioning of the probability axis will be discussed. The reason for this is that it is not so intuitive and much more difficult to explain what happens when using the other partitioning.

In Petrakian's study, all floods exceeding the PMF are ignored. There are basically two arguments behind this assumption. First of all, Stedinger and Grygier [1985] suggest in their example that these floods may be ignored, and since Petrakian does not want to alter their problem, he makes the same assumption. The second reason is that the low-probability expectation f_4 has to be computed numerically, and some technical difficulties arise when floods of infinite magnitude are permitted. The low-probability expectations computed by Petrakian are, therefore, solely based on floods below the PMF.

On the other hand, Karlsson does not ignore the floods exceeding the PMF. In fact, his approach with the statistics of extremes requires that no floods be ignored. Moreover, he partitions the probability axis at 1-1/T, where T is the return period of the PMF, and is consequently only concerned with floods exceeding the PMF.

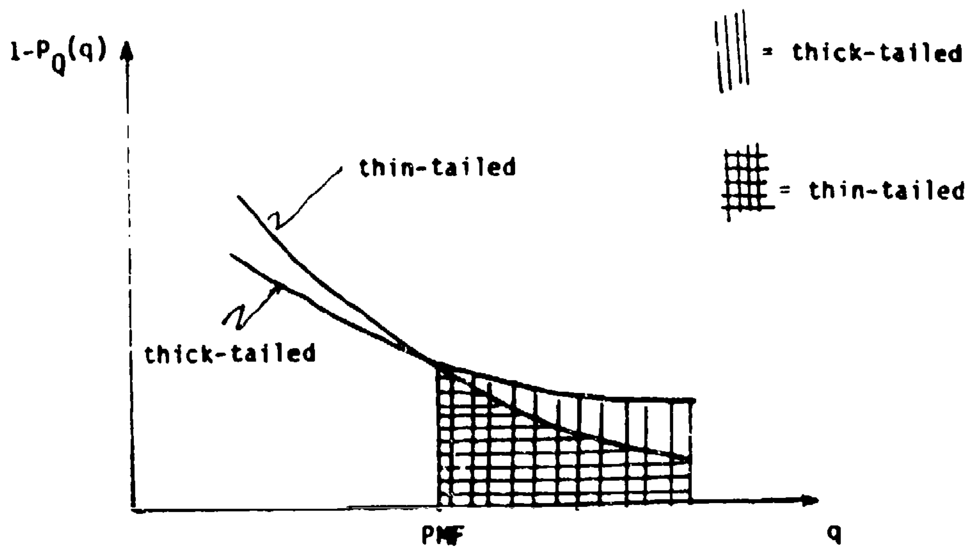


Figure 6.8 The areas under the curves are in a sense proportional to f_4. Karlsson's approach is used.

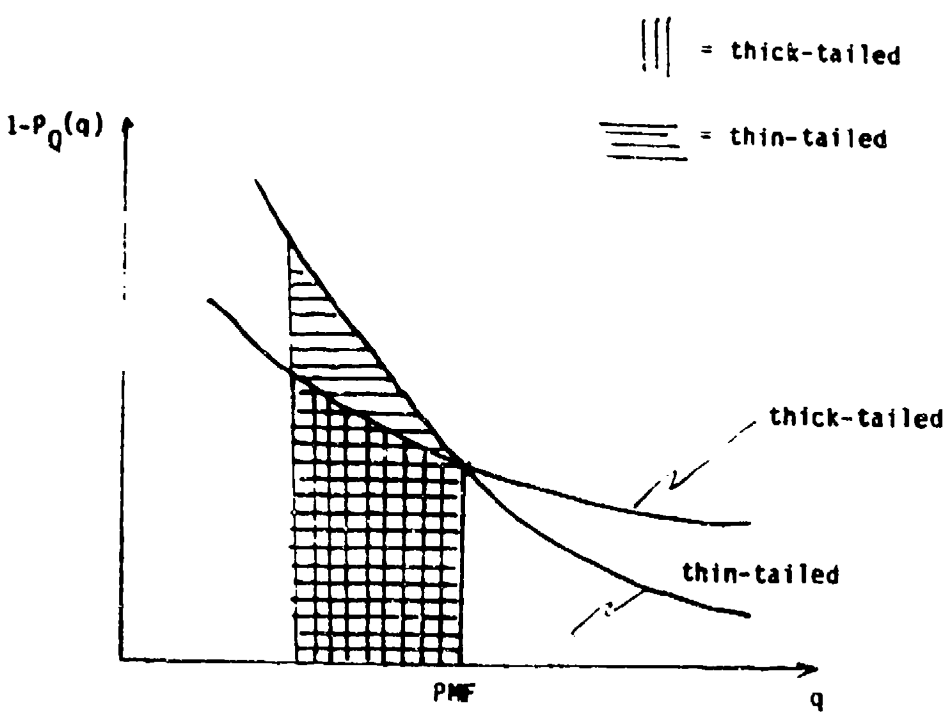


Figure 6.9 The areas are in a sense proportional to f_4. Petrakian's approach is used.

The low-probability expectation f_4 is in a sense proportional to part of the area under the curves in Fig. 6.8. In Karlsson's example, the probability axis was partitioned at the PMF clearly, the area corresponding to a thick-tailed distribution is greater than that of a thin-tailed.

Clearly, thin-tailed distributions are associated with smaller values of the expectation f_4 than are thick-tailed ones. On the other hand, the situation is reversed for Petrakian's study, which only considers floods below the PMF (Fig. 6.9).

Obviously, thin-tailed, distribution functions are now associated with the greater values of f_4. Had the floods exceeding the PMF not been ignored, his results would have been altered, but not necessarily drastically. The major reason behind Petrakian's results is that he simultaneously partitions at a flood below the PMF and uses the PMF as one of his boundary conditions.

It should be mentioned that the discussion above is rather simplified and should only be considered as an intuitive explanation. Actually, the damage function should have been included in the discussion, and instead of discussing the flood frequency distribution the distribution function of the random change should have been discussed. This incorporation of a damage function makes things much more difficult to understand intuitively, but the end results would have been similar.

The discrepancies that exist between Karlsson's and Petrakian's results are a consequence of the use of two boundary conditions instead of, for instance, the method of moments. The use of boundary conditions is well suited for some applications (for instance, this study on dam safety), and the method as such is not deficient. However, to be able to draw the best conclusions from the results, the analyst should always understand and fully appreciate the implications that follow his choice of approach.

CHAPTER 7

CONCLUSIONS: EXTENSIONS OF THE PARTIONED MULTIOBJECTIVE RISK METHOD

When the partitioned multiobjective risk method (PMRM) is extended to a multiobjective risk problem, it is desirable for the risk functions (the conditional and unconditional expected value functions) to be expressed as functions of the decision policy s. This is generally not possible when the traditional method, relying on brute-force integrations is used. Quite often, one is mainly interested in the low-probability expectation $f_4(\underline{s})$ and might totally ignore $f_2(\underline{s})$ and $f_3(\underline{s})$. There exists one class of problems where the relationship between f_4 in the PMRM and the statistics of extremes permits us to derive a closed-form expression for f_4 as a function of the partition point and the decision policy $\underline{s}$.

7.1 Damage Functions of One Random Variable.

In many multiobjective optimization problems, one is given some sort of cost function $f_1(s)$ and a damage function $g(\underline{y};\underline{s})$, where $\underline{s}$ denotes the decision variables. The damage function $g(\underline{y};\underline{s})$ may depend on one or more random variables y; thus, the damage $g(\underline{y};\underline{s})$ is itself a random variable with its own probability density function. As will be seen, the special case with only one random variable is especially interesting. It is very seldom possible to find analytical functional relationships between the conditional expectations and the partition points and decision options. However, when the damage function is dependent on only one random variable Y, it is possible to find a closed-form expression for the low-probability risk function $f_4(\alpha';\underline{s})$. That is, what is needed is an analytic solution of

$$f_4(\alpha';\underline{s}) = \frac{\int_{P_X^{-1}(\alpha')}^{\infty} x p_X(x;\underline{s})dx}{\int_{P_X^{-1}(\alpha')}^{\infty} p_X(x;\underline{s})dx} \tag{7.1}$$

where $p_X(x;\underline{s})$ is the probability density function of the variate $X = g(Y;\underline{s})$. One of the difficulties lies in finding the damage's pdf $p_X(x;\underline{s})$ of the damage given the damage function $g(\underline{y};\underline{s})$ and the variate Y's pdf $p_Y(y)$. A way of finding this function exists.

Let Y be a continuous random variable, nonzero for all values within the range of Y, with probability density function $P_y(Y)$. If $X = g(y;\underline{s})$ is a strictly monotone increasing or decreasing function for each decision policy $\underline{s}$ (its inverse exists for all values of Y and $\underline{s}$) and if it is differentiable for all y, then the probability density function of the random variable $X = g(Y;\underline{s})$ as given by Tsokos [1972] is

$$p_X(x;\underline{s}) = p_Y[g^{-1}(x;\underline{s})\cdot|\frac{dg^{-1}(x;\underline{s})}{dx}| \qquad (7.2)$$

or

$$p_X(x;\underline{s} = p_Y[g^{-1}(x;\underline{s})] \qquad (7.3)$$

Clearly, it is possible to find an analytic expression for the density function of the random x. This expression is generally very complex and leads to an unsolvable integral in Eq. (7.1). The relationship between the expectation f_4 and the statistics of extremes will now come in handy.

Let the superscripts X and Y indicate that the variable or parameter corresponds to the random variable X or Y, respectively.

From previous chapters, we know that Eq. (7.1) may be written

$$f_4(n';\underline{s}) = n' \int_{u^X_{n'}(\underline{s})}^{\infty} x\, p_X(x;\underline{s})dx \qquad (7.4)$$

where $u^x_{n'}$ denotes the characteristic largest value associated with the variate X, the damage (see Fig. 7.1). It has already been proven that if the probability density function of the largest value Y (corresponding to the random damage X) converges in distribution to one of the three asymptotic forms s defined by Gumbel, then f_4 may be evaluated from

$$f_4(n'\underline{s}) = u^X_{n'}(\underline{s}) + \sum_{j=1}^{\infty} \frac{d^j}{d(\ln n')^j}\, u^X_{n'}(\underline{s}) \qquad (7.5)$$

However, it has not been shown that this expression is correct for all distributions. Since there is no guarantee that the random damage $X = g(Y;\underline{s})$ will belong to one of these three forms (even though the initial variate Y does), it is necessary to be convinced that the derivation of the relationship between f_4 and the statistics of extremes [and therefore also Eq. (7.5)] is valid for all continuous distributions.

The case with the unlimited variate is much more complicated. There are, however, two facts that suggest that, for these distributions as well, μ_n does not increase more slowly than u_n. Karlsson [1986] shows that among the population of possible largest values from samples of size n, about 37% are less than u_n, or 63% greater than u_n, for any initial distribution.

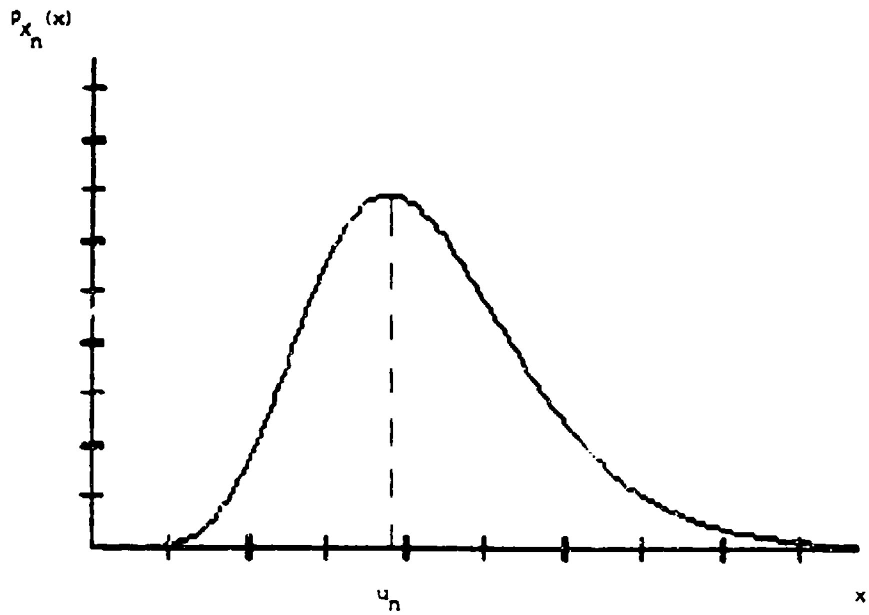


Figure 7.1 Definition of the characteristic largest value u_n

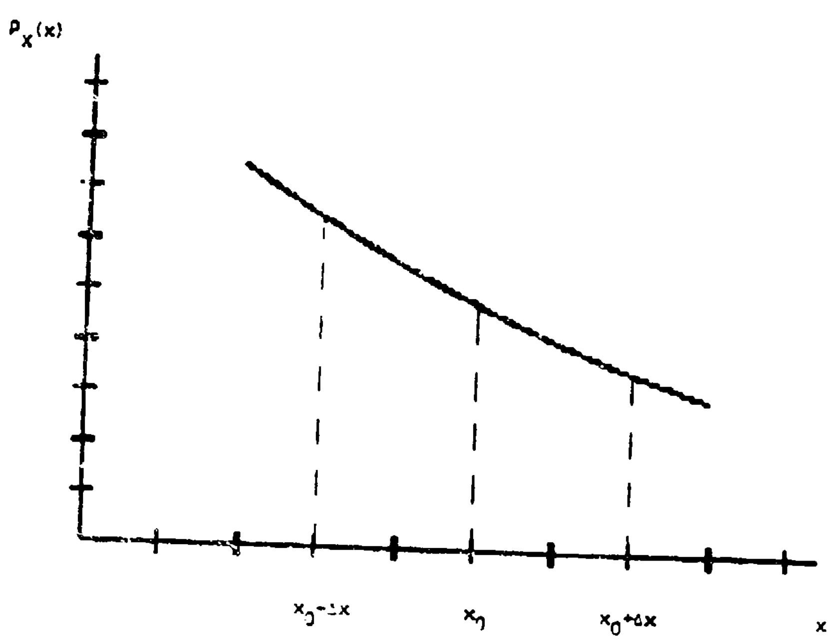


Figure 7.2 $Pr(x_0 - \Delta x < x < x_0) > Pr(x_0 < x < x_0 + \Delta x)$

Furthermore, each unlimited distribution decays to zero and in the limit (as x_0 approaches infinity) the probability of an event falling within the range $x \varepsilon [x_0 - \Delta x, x_0]$ is greater than it falling within $x \varepsilon [x_0, x_0 + \Delta x]$ resulting from the monotonic decreasing of the tail.

If x_0 is thought of as the characteristic largest value of u_n, it seems likely that the largest values drawn from samples of n′ independently and identically distributed random variables will be more concentrated for values below u_n than for values above. This, together with the knowledge that $P_X(u_n) = 0.37$, implies that the probability density function of the largest value will always have the same shape as in Fig. 7.1. Whenever this occurs, μ_n will always be greater than u_n.

In conclusion, it seems very likely that μ_n will never increase more slowly than u_n for any initial distribution and that the relationship between f_4 in the PMRM and the statistics of extremes derived in chapter 4 is indeed true for any initial variate, limited as well as unlimited. It will henceforth be assumed that Eq. (7.5) is valid for any initial variate.

7.2 The Characteristic Largest Values

It has now been shown that it is possible, at least theoretically, to find a closed-form expression for the low-probability expected-value function $f_4(n';\underline{s})$. However, even where it is possible, it is generally very difficult to use Eqs. (7.2) through (7.5) for obtaining this expression. Equation (7.2) requires the inverse of the damage function, $g^{-1}(x;\underline{s})$, which might not be analytically attainable. Furthermore, in order to obtain the characteristic largest value u_n, the inverse $P_X^{-1}(x;\underline{s})$ must be found, which is generally a very difficult task. Fortunately, there exists a way of circumventing all these difficulties.

By definition

$$P_X[u_n^X(\underline{s});\underline{s}] = 1 - \frac{1}{n} \tag{7.6}$$

or

$$\int_{-\infty}^{u_n^X(\underline{s})} p_X(x;\underline{s})dx = 1 - \frac{1}{n} \tag{7.7}$$

This equation may be further developed:

$$\int_{-\infty}^{u_n^X(\underline{s})} p_X(x;\underline{s})dx = \int_{-\infty}^{u_n^X(\underline{s})} p_y[g^{-1}(x;\underline{s}0] \mid \frac{dg^{-1}(x;\underline{s})}{dx} \mid dx \tag{7.8}$$

We will only study the case where $g(y;\underline{s})$ is strictly monotone increasing (see Fig. 7.3). Thus, we may omit the magnitude operation. (Similar results can easily be derived for strictly monotone decreasing damage functions using the same procedures as those that follow.

$$\int_{-\infty}^{u_n^X(\underline{s})} p_y(g^{-1}(x;\underline{s}))\ dg^{-1}(x;\underline{s}) = 1 - \frac{1}{n} \tag{7.9}$$

Let

$$x = g(y;\underline{s})$$

or

$$y = g^{-1}(x;\underline{s})$$

We also have that

$$dy = dg^{-1}(x;\underline{s})$$

Equation (7.9) becomes

$$\int_{-\infty}^{g^{-1}[u_n^X(\underline{s});\underline{s}]} p_y(y)\ dy = 1 - \frac{1}{n} \tag{7.10}$$

But, by the definition of u_n^y,

$$\int_{-\infty}^{u_n^y} p_y(y)\ dy = 1 - \frac{1}{n} \tag{7.11}$$

Clearly,

$$u_n^y = g^{-1}[u_n^X(\underline{s});\underline{s}] \tag{7.12}$$

or

$$u_n^X(\underline{s}) = g(u_n^y;\underline{s}) \tag{7.13}$$

In the special, although not very unusual, situation where the damage function $g(y;\underline{s})$ depends on only one random variable, the characteristic largest value $u_n^x(\underline{s})$ corresponding to the variate $X = g(Y;\underline{s})$ is easily obtained given the variable Y's characteristic largest value, , u_n^Y. It is simply the damage function evaluated at u_n^Y.

Equation (7.5) may be rewritten as

$$f_4(n';\underline{s}) = g(u_{n'}^Y;\underline{s}) + \sum_{j=1}^{\infty} \frac{d^j}{d(\ln n')^j} g(u_{n'}^Y;\underline{s}) \tag{7.14}$$

Although this equation involves an infinite sum, we are now very close to the desired closed-form expression for $f_4(n';\underline{s})$. If only the first four terms in the sum are used, Eq. (7.14) will never give errors of more than about one percent. The sum converges very quickly to its limit.

All the difficulties described in the onset of this section have now been overcome and an easy-to-use method has been derived for finding a closed-form expression for the low-probability expected value function $f_4(n';\underline{s})$.

7.3 Interpretation

The low-probability risk function $f_4(\underline{s})$ is a measure for the expected damage, given that the damage exceeds a level chosen, subjectively, by the analyst. This preset level is actually determined by the way the analyst partitions the probability axis. Unfortunately, it may be difficult to appreciate the true meaning of this partition.

For a continuous variate, there is no probability for a certain value, but only a density of probability. However, there is a probability $1-P_X(x)$ of exceeding x. The reciprocal of this exceedence probability,

$$T(x) = \frac{1}{1-P_X(x)} \tag{7.15}$$

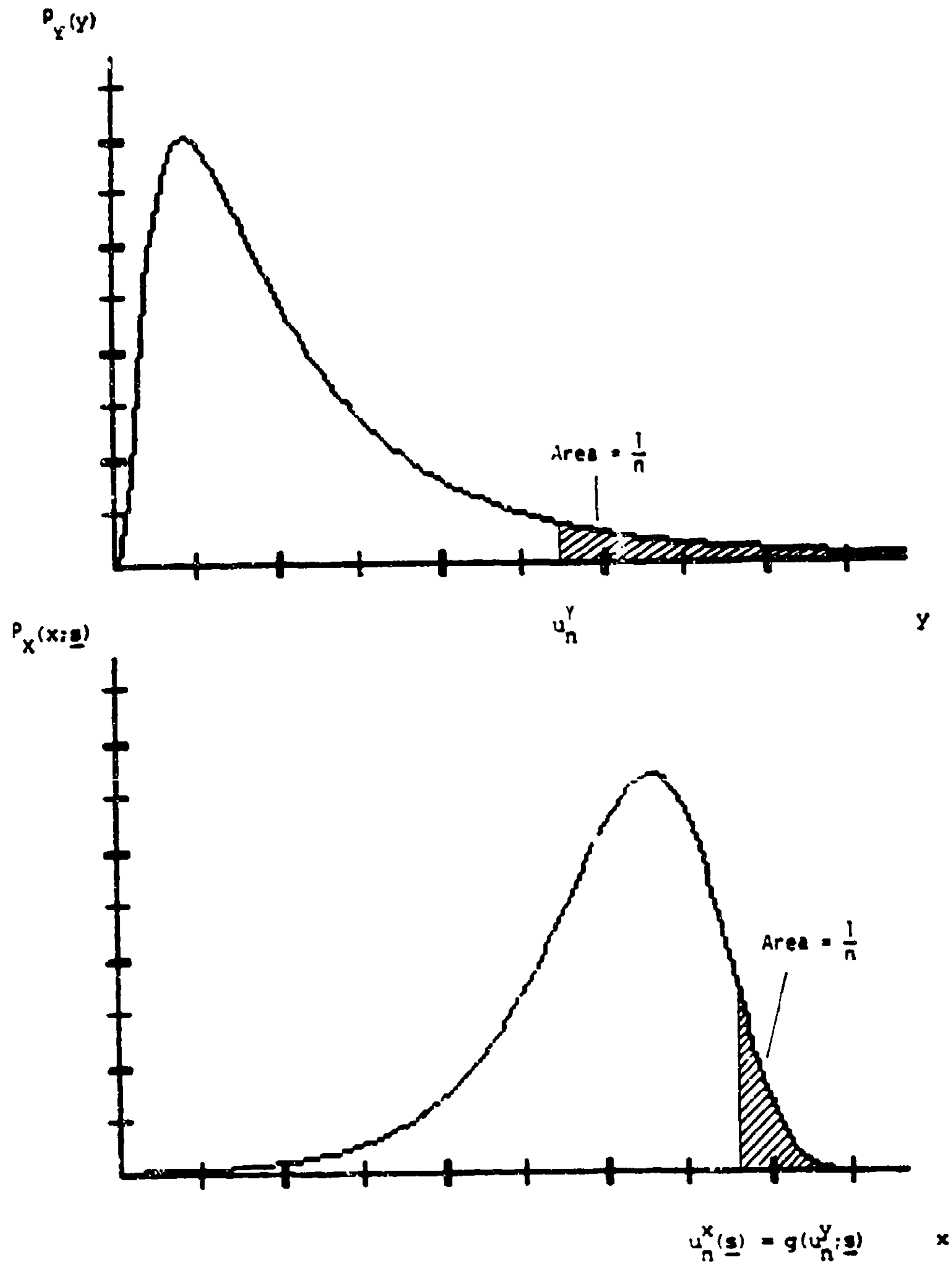


Fig. 7.3 Relationship Between u_n^X and u_n^Y

is called the return period, which is the number of observations such that, on average, there is one observation equalling or exceeding x. In the special case where $x = u_{n'}$,

$$T(u_{n'}) = n' \tag{7.16}$$

This n′ relates to the partition point through

$$n' = \frac{1}{1 - \alpha'}$$

The partition α′ may consequently be viewed as a parameter corresponding to a return period of n′. It is natural to think of $f_4(n';\underline{s})$ as a measure of the expected damage given that an event with a return period equalling or exceeding n′ occurs. Equation (7.14) is simply a closed-form expression for this expected damage with the decision options as well as the return period as variables. This equation's practical importance cannot be over emphasized.

7.4 Summary

The effort in combining f_4 in the PMRM with the statistics of extremes has proven very successful. Not only has the sensitivity to the partition points been evaluated but, maybe more importantly, closed-form expressions for $f_4(\alpha';\underline{s})$ have been found, and these have a damage function $g(y;\underline{s})$ that is only dependent on one random variable Y. The expression

$$f_4(n';\underline{s}) = g(u^Y_{n'};\underline{s}) + \sum_{j=1}^{\infty} \frac{d^j}{d(\ln n')^j}\, g(u^Y_{n'};\underline{s})$$

involves an infinite sum, but fortunately, this sum converges very fast. It seem that only four terms need be included to achieve accurate values within about one percent. Although the terms involved in this equation may be rather tedious to develop analytically, they lead to a closed-form expression from which the corresponding values of f_4 for any combination of return period n and decision option s may be obtained. The equation $f_4(n';\underline{s})$ may take some time to develop but once it is obtained all the values for $f_4(n';\underline{s})$ fall out automatically. For problems with a large number of interesting combination of n′ and $\underline{s}$, it is much more time-consuming to do individual numerical integrations for each combination and then also try find a functional relationship typing these discrete points together.

The statistics-of-extremes approach has a number of advantages and can be quite useful. There are plans to apply this new approach to a problem on flooding and dam safety.

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APPENDIX I

THE PARTITIONED MULTIOBJECTIVE RISK METHOD APPLIED TO STEDINGER AND GRYGIER MODEL

In this appendix, the partitioned multiobjective risk method (PMRM) is applied to the model presented by Stedinger and Grygier*.

A1.1 Introduction

The aim of this study is to illustrate the impact of the selected flood frequency distribution on the ranking of design alternatives. The dam being considered here can safely pass 40,000 cfs. It was estimated that the probable maximum flood (PMF) is 150,000 cfs and that the 100-year flood is 20,000 cfs. Table A1-1 describes the four alternative remedial actions that are examined here. The decision variable is denoted by s (where s=1,2,3, or 4 depending on the chosen option).

A1.2 Damage Function

It was assumed that overtopping of the dam would result in a dam failure. Damages in US$ to downstream

OPTION	DESIGN FLOW IN CFS $q_c(s)$	AMORTIZED CONSTRUCTION COST $/YEAR
DO NOTHING (S=1)	40,000	0
MODIFY SPILLWAY (S=2)	60,000	50,000
REBUILD SPILLWAY & RAISE DAM (S=3)	120,000	120,000
REBUILD SPILLWAY WITH LOWER CREST (S=4)	150,000	80,000

Table A1-1 Design options and cost

*J. Stedinger and J. Grygier, Risk-Cost Analysis and Spillway Design, in Computer Applications in Water Resources, H. Torn. (ed.), 1985.

and upstream properties were approximated by the following damage function:

$$X(q,s)=\begin{cases} 0 & 0 \leq q < 10,000 \text{ cfs.} \\ \left(\dfrac{M}{1+\left[\dfrac{q}{v}\right]^{-r}}\right) & 10,000 \leq q < q_c(s) \\ M+L & q_c(s) \leq q \end{cases}$$

Here, $q_c(s)$ denotes the critical design flow which was defined in table A1-1. It is also assumed that the parameters of the above function take the following values: $M=100\text{x}10^6$ \$, $L=30\text{x}10^6$\$, r=3; also for s=1, 2, 3 $v=60\text{x}10^3$cfs, and for s=4, $v=55\text{x}10^3$ cfs.

A1.3 Flood-Frequency Distributions

The next step is to estimate the flood-frequency distribution.

A1.3.1 The Flood-Frequency Distribution for Frequent Floods

It was assumed that for floods smaller than the one-hundred year flood, the flood frequency distribution is log-normal. To determine the parameters of the probability function, an interpolation was made of the corresponding cumulative probability distribution between the 10-year flood (10,000 cfs) and the 100-year flood (20,000 cfs).

For $0 \leq q < 20,000$ cfs

$$f_Q(q)=\frac{1}{q\ a\ \sqrt{2\pi}} \exp\left[-0.5\left(\frac{\ln q - m}{a}\right)^2\right]$$

m=8.3592
a=0.6639

proof If q ~ LN and if u ~ N(0,1) then $u=(\ln q - m)/a$

and $F_Q(q)=F_U(u)=F_U((\ln q - m)/a)$

We have

$$\begin{cases} F_Q(10{,}000)=0.9 \\ F_Q(20{,}000)=0.99 \end{cases} \Rightarrow \begin{cases} (\ln 10{,}000-m/a=1.282 \\ (\ln 20{,}000\ -m/a=2.326 \end{cases}$$

$$\Rightarrow \begin{cases} m = 8.3592 \\ a = 0.6639 \end{cases}$$

A1.3.2 Flood-Frequency Distribution for Rare Floods

For floods between the 100 year flood and the PMF, it has been assumed that the flood flow frequency distribution could possibly be normal, Gumbel, Pareto, log-normal, log-Gumbel, or Weibull. The analytical expression of these distributions can be determined by interpolating the corresponding cumulative probability distribution between the 100 year flood and the PMF. (It has been assumed that the return period of the PMF is $1\text{x}10^6$ years.)

A1.3.2.1 Normal Distribution

$$f_Q(q)=\frac{1}{a\sqrt{2\pi}}\exp\left[-0.5\left(\frac{q-m}{a}\right)^2\right]$$

m=-98117.1875
a=50781.25

proof

$$\begin{cases} F_Q(20{,}000)=0.99 \\ F_Q(150{,}000)=0.999999 \end{cases} \Rightarrow \begin{cases} (20{,}000-m)/a=2.326 \\ (150{,}000-m)/a=4.886 \end{cases}$$

$$\Rightarrow \begin{cases} a = 50781.25 \\ m = -98117.1875 \end{cases}$$

A1.3.2.2 Gumbel Distribution

$$f_Q(q)=(1/a)\ \exp[-(q-m)/a]\ \exp\{-\exp[-(q-m)/a]\}$$

m=-44893.75841
a= 14106.88115

proof $F_Q(q) = \exp\{ -\exp[-(q-m)/a] \}$

$$\begin{cases} F_Q(20{,}000)=0.99 \\ F_Q(150{,}000)=0.999999 \end{cases}$$

$$\Rightarrow \begin{cases} \exp\{-\exp[-(20{,}000-m)/a]\}=.99 \\ \exp\{-\exp[-(150{,}000-m)/a]\}=.999999 \end{cases}$$

$$\Rightarrow \begin{cases} a = 14106.88115 \\ m = -44893.75841 \end{cases}$$

A1.3.2.3 Pareto Distribution

$f_Q(q) = b\, a^b / q^{b+1}$ $\qquad q > a$

b=4.571109
a=7302.968206

proof $F_Q(q) = 1-(a/q)^b$

$$\begin{cases} F_Q(q) = 0.99 \\ F_Q(q) = 0.999999 \end{cases} \Rightarrow \begin{cases} 1-(a/20{,}000)^b = 0.99 \\ 1-(a/150{,}000)^b = 0.999999 \end{cases}$$

$$\Rightarrow \begin{cases} b = (\ln 0.01 - \ln 1\times10^{-6})/(\ln 150000 - \ln 20000) \\ a = \exp\{[(\ln 0.01)/b] + \ln 20{,}000\} \end{cases}$$

$$\Rightarrow \begin{cases} b = 4.571109 \\ a = 7302.968206 \end{cases}$$

A1.3.2.4 Log-Normal Distribution

$$f_Q(q) = \frac{1}{a\, q\, \sqrt{2\pi}} \exp[-.5\,(\ln q - m)^2/ a^2]$$

a=0.787072
m=8.072759

proof If q is log-Normally distributed and $U \sim N(0,1)$ then $U = (\ln Q - M/a$ and $F_Q(q) = F_U[(\ln q - m)/a]$

$$\Rightarrow \begin{cases} F_Q(20000) = .99 \\ F_Q(150000) = .999999 \end{cases} \Rightarrow \begin{cases} (\ln 20000-m)/a=2.326 \\ (\ln 150000-m)/a=4.886 \end{cases}$$

$$\Rightarrow \begin{cases} a = 0.78072 \\ m = 8.072759 \end{cases}$$

A1.3.2.5 Log-Gumbel Distribution

$$f_Q(q) = \left[\frac{1}{a\ q}\right] \exp\left[-\left(\frac{\ln q-m}{a}\right)\right] \exp\left\{-\exp\left[-\left(\frac{\ln q-m}{a}\right)\right]\right\}$$

This expression for the Log-Gumbel distribution from the Gumbel distribution was obtained from the Gumbel distribution using the following transformation: if Q ~ log-Gumbel then X = ln Q ~ log-Gumbel. Next, it is possible we can get

$$f_Q(q)=\left[\frac{1}{a\ q}\right] \left[\frac{q}{e^m}\right]^{(-1/a)} \exp\left[-\left(\frac{q}{e^m}\right)^{(-1/a)}\right]$$

$$\Longrightarrow f_Q(q)=-\left[\frac{-1}{a\ e^m}\right] \left[\frac{q}{e^m}\right]^{[(-1/a)-1]} \exp\left[-\left(\frac{q}{e^m}\right)^{(-1/a)}\right]$$

Let $c'=-1/a$ and $a'=e^m$

$$\Longrightarrow f_Q(q)=-(c'/a')\ (q/a')^{(c'-1)} \exp[-(q/a')^{c'}] \quad q>0$$

It is also possible to get $F_Q(q)= \exp\ [-(q/a')^{c'}]$

$$\begin{cases} F_Q(20{,}000) = 0.99 \\ F_Q(150{,}000) = 0.999999 \end{cases}$$

$$\Longrightarrow \begin{cases} \exp[-(20{,}000/a')^{c'}] = 0.99 \\ \exp[-(150{,}000/a')^{c'}] = 0.999999 \end{cases}$$

$$\Longrightarrow \begin{cases} c'=-4.573600187 \\ a'= 7315.002883 \end{cases} \Longrightarrow \begin{cases} a = 0.2186461341 \\ m = 8.897682708 \end{cases}$$

A1.3.2.6 Weibull Distribution

$$f_Q(q) = (c/a) \quad (q/a)^{c-1} \quad \exp[-(q/a)^c]$$

$a = 1215.0899822$
$c = 0.545243$

proof $F_Q(q)=1-\exp[-(q/a)^c]$

$$\begin{cases} F_Q(20,000)=0.99 \\ F_Q(150,000)=0.999999 \end{cases}$$

$$\Longrightarrow \begin{cases} 1 - \exp[-(20,000/a)^c] = 0.99 \\ 1 - \exp[-(150,000/a)^c] = 0.999999 \end{cases}$$

$$\Longrightarrow \begin{cases} a = 1215.0899822 \\ c = 0.545243 \end{cases}$$

A1.4 Application of the Partitioned Multiobjective Risk Method

In the partitioned multiobjective risk method, the step that follows the estimation of the marginal probability is the partitioning of the cumulative probability axis. This was done in the following fashion:

RANGES OF THE PARTITIONING OF PROBABILITY AXIS	CORRESPONDING DOMAIN OF THE DAMAGE AXIS	Pr(XεDOM.)
0-0.99	I	99%
0.99-0.9999	II	0.99%
0.9999-0.999999	III	0.0099%
0.999999-1	IV	0.0001%

Notice that domain I covers the range of recorded floods up to the 100 year event flood. Domain II covers the range of rare floods with small likelihood of occurrence. Domain III covers the range spanning from the 10^4 year flood to the 10^6 year flood, which is the range where the actual return period of the PMF is estimated to lie. Domain IV includes floods that are larger than the PMF; and since any flows of such magnitude will cause the dam to fail independently of the design option, the resulting damages would not depend on the design selected. It was therefore decided to neglect domain IV in this design comparison.

The next step is to calculate the conditional expected damage for the different domains defined above. $f_2(s)$, $f_3(s)$, $f_4(s)$ will represent the conditional expected damage for domain I, domain II, domain III respectively. $f_5(s)$ will denote the unconditional expected annual cost. The following definition of conditional expectation will be used:

$$f_2(s) = \frac{\int_0^{20,000} X(q,s)\, f_Q(q)\, dq}{\int_0^{20,000} f_Q(q)\, dq}$$

It can be seen that

$$f_2(s) = \frac{0 + \int_{10,000}^{20,000} \left[\frac{100\text{x}10^6}{[1+(q/v)^{-3}]}\right] f_Q(q)\, dq}{0.99}$$

where

$$v = \begin{cases} 60\text{x}10^6 & \text{for } s = 1, 2, 3 \\ 55\text{x}10^6 & \text{for } s = 4 \end{cases}$$

Since the integral in the denominator has value equal to 0.99, it is [on]ly necessary to calculate the integral in the numerator. This integral was [co]mputed by numerical integration using the Romberg method. Similarly,

$$f_3(s) = \begin{cases} \dfrac{\int_{20,000}^{q_F} \left[\frac{100\text{x}10^6}{[1+(q/v)^{-3}]}\right] f_Q(q)\, dq}{0.0099} & \text{if } q_F \leq q_c(s) \\[2ex] \dfrac{\int_{20,000}^{q_c(s)} \left[\frac{100\text{x}10^6}{[1+(q/v)^{-3}]}\right] f_Q(q)\, dq}{0.0099} + \\[2ex] \dfrac{\int_{q_c(s)}^{q_F} [130\text{x}10^6]\, f_Q(q)\, dq}{0.0099} & \text{if } q_F > q_c(s) \end{cases}$$

where

$$\begin{cases} v = 60\text{x}10^3 \text{ for } s = 1, 2, 3 \\ v = 55\text{x}10^3 \text{ for } s = 4 \end{cases}$$

Note that q_F is defined as the flood q such that $F_Q(q_F)=0.9999$. It is clear that q_F is different for each flow frequency distribution; it can be calculated easily. Also,

$$f_4(s)=\begin{cases} \dfrac{\int_{q_F}^{150,000} \left[\dfrac{100\text{x}10^6}{1+(q/v)^{-3}}\right] f_Q(q)\, dq}{0.000099} & \text{if } q_F \geq q_c(s) \\[2ex] \dfrac{\int_{q_F}^{q_c(d)} \left[\dfrac{100\text{x}10^6}{1+(q/v)^{-3}}\right] f_Q(q)\, dq}{0.000099} + \dfrac{\int_{q_c(s)}^{150,000} [130\text{x}10^6]\, f_Q(q)\, dq}{0.000099} & \text{if } q_F < q_c(s) \end{cases}$$

where

$$\begin{cases} v = 60\text{x}10^3 & \text{for } s = 1, 2, 3 \\ v = 55\text{x}10^3 & \text{for } s = 4 \end{cases}$$

Here also, numerical integration with the Romberg method will be used to calculate the values of the integrals.

$f_5(s)$ has been using

$$f_5(s)=0.99\ f_2(s)+0.0099\ f_3(s)+0.000099\ f_4(s)$$

$f_2(s)$, $f_3(s)$, $f_4(s)$ and $f_5(s)$ were calculated for each of the six flow frequency distributions assumed to describe floods between the 100-year flood event and the PMF. The final results are shown in table A1-2.

A1.5 Interpretation of Results

First, note that when the results reported here are checked against the ones obtained by Stedinger and Grygier, Table 4 of their paper can be derived, obtaining the same numbers for all distributions except for the log-normal distribution. In this later case, there were some differences in the results but they were not larger than 6%. These errors can be accounted to the computational method used to evaluate the integrals and to the accuracy of the cumulative normal distribution tables that were used.

Consider table A1-2 in which the Pareto and log-Gumbel distributions show very similar results which are also somewhat close to the results obtained when the log-normal distribution was assumed. These three distributions seem to favor policies that require small investments or no investments at all. For example, assuming log-Gumbel, the decision maker who invests (DM) $ 50,000 (option s=2) has a 0.99% chance per year of saving $3,248,000, less than 0.01 % chance per year of saving $28,659,000, and 99 % chance per year that of saving nothing at all. On the other hand, assuming Gumbel, by investing $ 50,000 (option s=2) there is 0.99 % chance per year that the DM will save $17,968,000 and 99.01 % chance per year that he will save nothing at all. The DM who assumed a Gumbel distribution will certainly have more incentive to invest his money since he can expect much larger savings for the same probability (0.99 % per year) than if he had assumed a log-Gumbel distribution. By performing comparisons similar to the one done above one can see that the Gumbel distribution favors conservative policies.

One can also notice that, in general, the choice of the assumed distribution does not have on $f_4(s)$ an impact large enough for it to be considered seriously by the DM. The reason is that the tradeoffs between $f_4(s)$ and $f_1(s)$ for the different design alternatives vary around 30 % from one distribution to the other. But this number is too low to have an impact on the D.M.'s choice because Pr(x ε domain III)=0.0099 % per year.

It can also be seen that the tradeoffs between $f_3(s)$ and $f_1(s)$ for different alternatives are magnified by distributions such as Gumbel, normal and Weibull. Moreover, since Pr(x ε domain II)=0.99% per year, which is a much larger probability than for domain III, it seems that any difference in the tradeoffs between $f_3(s)$ and $f_1(s)$ caused by the choice of the assumed distribution will have a significant impact on the DM's choice of the design option. Therefore, it can be concluded that the use of the Gumbel, normal or Weibull probability distributions as the flood frequency distribution will be more likely to induce the decision maker to adopt a conservative policy. If the distributions were ranked by how much increasingly they favor conservative policies, their order would be the Pareto, log-Gumbel, log-normal, Weibull, Gumbel, and normal distributions.

In conclusion, it is clear that the choice of the flow frequency distribution for floods between the 100-year flood and the PMF has a considerable influence on the attractiveness of the different design options. This influence is mainly present in domain II. For future dam safety studies involving flow frequency distributions it might be wise to perform a sensitivity analysis for at least two distributions such as the log-Gumbel and the Gumbel distributions. These two distributions seem to be a good representation of the two categories of distributions: the ones favoring little preventive actions and the one favoring conservative policies involving significant investments.

APPENDIX II
THE PARTITIONED MULTIOBJECTIVE RISK METHOD

A2.1 Introduction

This appendix contains a description of the partitioned multiobjective risk method (P{MRM). First, the concepts of the conditional expectation and the surrogate worth trade-off (SWT) method are introduced. Next, the six-step procedure that forms the PMRM is described. The next section contains comments and observations that should allow the reader to enhance his understanding of the method. Finally, an attempt is made to evaluate the PMRM, and possible extensions to the method are presented.

A2.2 Mathematical Foundation

The partitioned multiobjective risk method (PMRM) employs several concepts. The theory of random variables may be used to find an unknown marginal probability density function (marginal pdf). A conditional expectation may be defined using this marginal pdf, and both may be approximated through Monte Carlo techniques. Finally, the surrogate worth trade-off (SWT) method, a multiobjective decision-making technique, is valuable in risk-related decisions.

A2.2.1 Conditional Expectation

A conditional expectation based on a marginal pdf may be defined as follows. Given the marginal pdf $P_x(x) = Pr\{X=x\}$ governed by the axioms

$$P_X(x) \geq 0 , \quad - \infty < x < \infty$$

$$\int_{-\infty}^{\infty} P_X(x)dx = 1$$

$$P_X(x) = \int_{-\infty}^{x} P_X(y)dy \quad \text{is nondecreasing}$$

$$Pr\{a < X < b\} = \int_{a}^{b} P_X(x)dx$$

and assuming $p(x) \geq 0$ for $0 < x < \infty$ and $p(x) = 0$ for $-\infty < x \leq 0$, the conditional expectation of an event $D = \{x | x \varepsilon [a,b]\}$ where the notation $c \varepsilon (a,b]$ means that $a < c \leq b$, is given by

$$E[X|D] = \frac{\int_a^b x p_X(x)dx}{\int_a^b p_X(x)dx} \quad (A2.2)$$

A2.2.2. The Surrogate Worth Trade-off (SWT) Method

The multiple-objective optimization problem is also known as a vector optimization problem. Two approaches, the parametric, and the ε-constraint (which is employed in the SWT method) are outlined here.

A2.2.2.2.1 Vector Optimization Problems and Noninferior Solutions

The vector optimization problem defines a decision vector $\underline{s} = (s_1, s_2, \ldots, s_n)$, an objective vector $\underline{f} = (f_1, f_2, \ldots, f_m)$ with $f_i: R^n \to R^1$ and $\underline{f}: R^n \to R^m$, and a set of feasible solutions $S = \{\underline{s} | g(\underline{s}) \leq 0\}$ with $\underline{g} = (g_1, g_2, \ldots, g_p)$, where $g_i: R^n \to R^1$ and $\underline{g}: R^n \to R^p$. The notation $f: R^n \to R^m$ means that the function $\underline{f}(\underline{s})$ maps values from the space of real numbers with dimension n into the space of real numbers with dimension m. Assuming the f_i have noncoincident minima, they should be minimized. A point $\underline{s} \in S \subseteq R^n$ is a noninferior (Pareto) point for a mapping $\underline{f}$ if and only if no change $\Delta\underline{s} \in R^n$ exists such that, for all $i = 1, 2, \ldots, m$,

$$f_i(\underline{s} + \Delta\underline{s}) \leq f_i(\underline{s}) \quad (A2.3)$$

with strict inequality for at least one function when $\underline{s} + \Delta\underline{s} \in S \subseteq R^n$. Consider a scalar decision variable s and two conflicting quadratic objective functions f_1 and f_2 in the decision space (see Fig. A2.1), where the region N represents the noninferior solutions. Those noninferior solutions are shown in the functional (or objective) space in Fig. A2.2. To decrease the value of one objective, the value of (at least) one other objective must increase. This is the essential quality of noninferiority.

A2.2.2.2 The Parametric Approach

The parametric approach further defines the vector optimization problem as

$$\min_{\underline{s} \in S} \underline{w}^T \underline{f}$$

$$\text{subject to } \underline{w}^T \underline{e} = 1$$

$$\underline{w}_i \geq 0 \qquad i = 1, 2, \ldots, m \quad (A2.4)$$

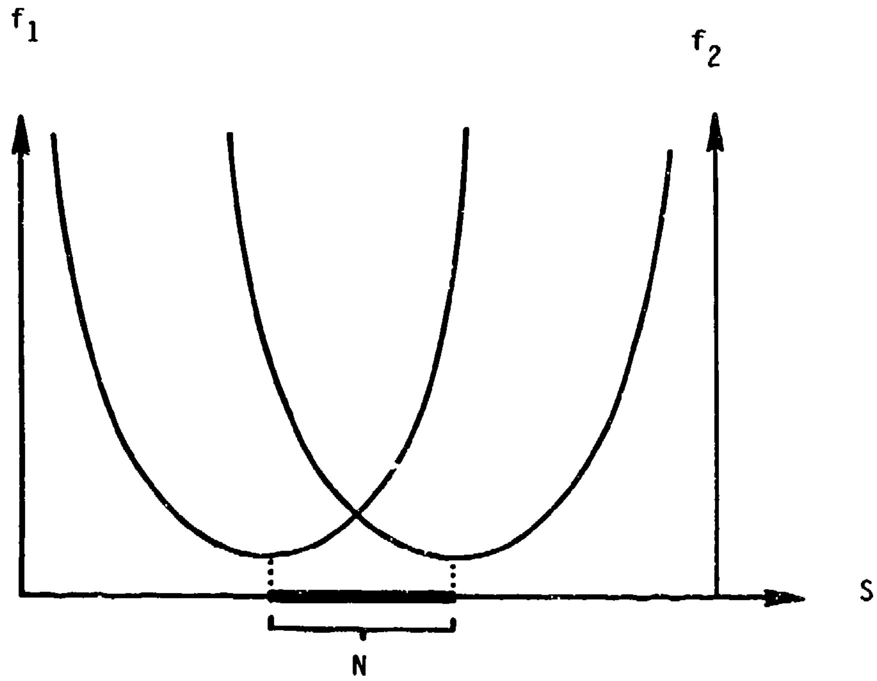


Figure A2.1 Two conflicting Quadradic Objectives, graphed in the Decision Space

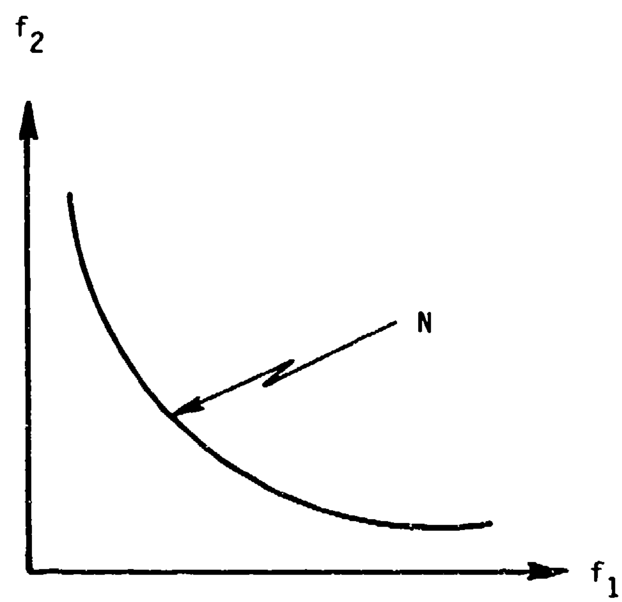


Figure A2.2 The Noninferior Solutions from Figure A2.1, Graphed in the Functional Space where $\underline{w} = (w_1, w_2, \ldots, w_m)$ and $\underline{e} = (1, \ldots, 1) \varepsilon R^m$. This is a well-defined minimization problem. The solution is unique when all f_i are convex; otherwise, some noninferior solutions are unobtainable. For example, Fig. A2.3 shows a two-objective minimization problem with a compact, nonconvex, feasible decision set in the function space. The noninferior solutions are in bold lines. Solutions shown in the heaviest bold line (a pocket of nonconvexity) are not obtainable by the parametric approach, which finds the minimal-valued hyperplane tangent to the convex hull of the feasible set when given a weight $\underline{w} = (w_1, w_2)$. The hyperplanes H_1 and H_2 in Fig. A2.3 both have slopes corresponding to the same weights $\underline{w}$ and contain noninferior solutions; however, H_1 is the minimal hyperplane for this $\underline{w}$, so H_2 and its associated noninferior solutions are never obtained by the parametric approach.

A2.2.2.3 The ε-Constraint Approach: The Basis of the SWT Method

The ε-constraint approach clarifies the vector optimization problem differently. Choose an $i \varepsilon \{1, 2, \ldots, m\}$ and define the problem

$$\min_{\underline{s} \varepsilon S} f_i(\underline{s}) \tag{A2.5}$$

subject to $f_j(\underline{s}) \leq \varepsilon_j, \quad j \neq i, \quad j = 1, 2, \ldots, m$

where each component of - is once continuously differentiable on S. From the Lagrangian $L(\underline{s}, \underline{\lambda}) = f_i(\underline{s}) + \sum_{j \neq i} \lambda_{ij} (f_j) \underline{s}) - \varepsilon_j)$ and for each $\underline{s} \varepsilon R^n$ with $\lambda_{ij} > 0$ satisfying the Kuhn-Tucker conditions, it follows that $L(\underline{s}) = f_i(\underline{s})$ and $\varepsilon_j = f_j(\underline{s})$; thus,

$$\frac{\partial L}{\partial \varepsilon_j} = \frac{\partial f_i}{\partial f_j} \triangleq - \lambda_{ij}(\underline{s}) \tag{A2.6}$$

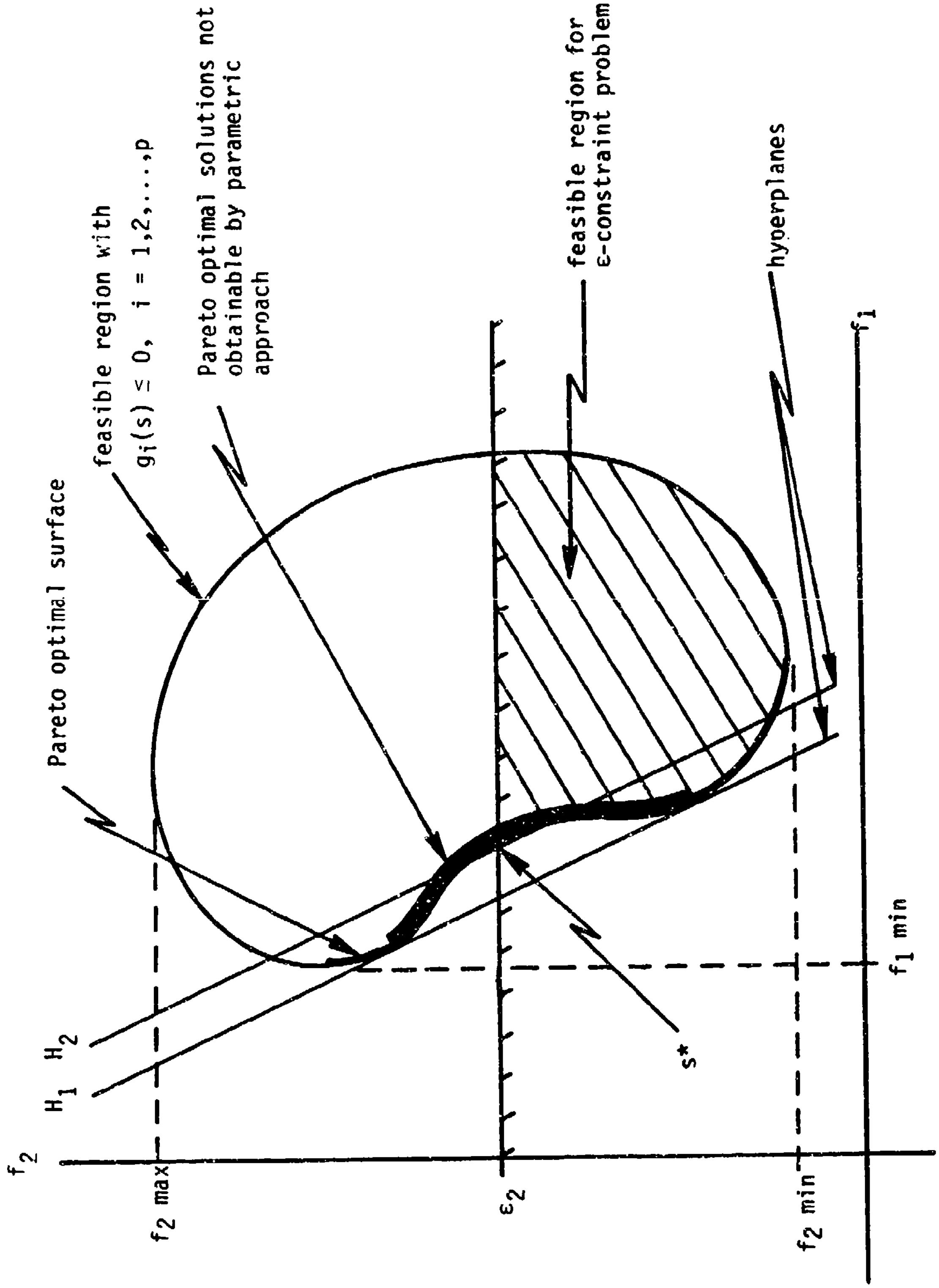
The ε-constraint approach varies the ε_j parametrically to generate all needed noninferior solutions as well as their associated trade-off values, λ_{ij}.

As shown in Fig. A2.3, ε_2 defines an artificial upper bound on the feasible set. The associated ε-constraint problem is

$$\min_{s \varepsilon S} f_1(s) \tag{A2.7}$$

subject to $f_2(s) \leq \varepsilon_2$

Figure A2.3 The ε-Constraint Approach



and its solution is the noninferior point s^*. The associated strictly positive trade-off at s^* between f_1 and f_2 is given by the Lagrange multiplier λ_{12} related to the Lagrangian $L(s) = f_1(s) + \lambda_{12}(f_2(s) - \varepsilon_2)$. The PMRM employs the static n-objective ε-constraint (SNE) algorithm

A2.2.2.4 The Surrogate Worth Function

The surrogate worth function is defined as W_{ij} ε[-10,+10], i =/ j and i,j = 1,2,...,m. For a given λ_{ij}, there are $f_i(\underline{s})$ and $f_j(\underline{s})$, $i \neq j$ and i,j = 1,2,...,m, associated with a particular $\underline{s} \in S$ _ R^n. The decision makers choose $W_{ij}(>,=<)0$ when they prefer λ_{ij} units of $f_i(\underline{s})$ (more, equally, less) than 1 unit of $f_j(\underline{s})$. Using the W_{ij} and the values of $f_i(\underline{s})$ and $\lambda_{ij}(\underline{s})$, the analyst helps the decision maker(s) search the noninferior surface defined by $\lambda_{ij} > 0$ until all $W_{ij} = 0$. Associated with these $W_{ij} = 0$ are a set of λ_{ij}^* and f_i^*. Solving the problem

$$\min_{\underline{s} \in S} f_i(\underline{s}) \qquad \text{(A2.8)}$$

$$\text{subject to } f_j(\underline{s}) < f_j^*, \quad j \neq i \quad \text{and } j = 1,2,\ldots,m$$

yields the preferred decision $\underline{s}^* \in S$ _ R^n.

A2.2.2.5 Strengths of the SWT Method

The vector optimization problem may be solved parametrically if the weights w_j are known; however, they usually are not know. The SWT method allows the decision maker indirectly to discover the preferred weights by searching the noninferior surface for a preferred solution. The responsibilities in the risk assessment process are thus distributed more equitably. The analyst obtains, structures, and presents the data. The decision makers determine the importance of the various decision factors in view of objective function values and trade-offs expressed in familiar measures.

Risk-related decisions are often made by groups. The surrogate worth trade-off method with multiple decision makers allows for using compromise, negotiation, and any quantifiable decision rule in the decision process.

A2.3 The Method

The PMRM involves a six-step procedure:

1) Find marginal probability density functions.
2) Partition the probability axis to provide a fuller risk description.
3) Map the probability partitions onto the damage axis.
4) Find conditional expectations.
5) Generate functional relationships between conditional expectations and policy choices.
6) Employ the SWT method to generate Pareto optimal solutions and their associated trade-offs and to choose a preferred policy.

An overview of the process is presented in Fig. A2.4, while Fig. A2.5 describes more detailed branch points in flowchart fashion.

A2.3.1 Find the Marginal Probability Density Functions

The PMRM requires the marginal probability density functions (pdf), $P_x(s;s_i)$, relating the probability of loss to the magnitude of loss for each of the policy options s_i, $i = 1,2,\ldots,q$. The s_i are considered scalar in this discussion, although extension to the vector case should not present significant theoretical difficulties. These probability density functions may be explicitly known, obtained through random variable techniques, useful and inexpensive in simple problems, is exact but computationally cumbersome; the Monte Carlo approach is approximate but more broadly applicable.

From these $p_x(x;s_i)$, a set of probability distribution functions (cdf) may be defined as

$$P_X(x;s_i) = \int_0^x p_X(y;s_i)dy, \quad i = 1,2,\ldots,q \quad , \tag{A2.9}$$

where $p_x(x;s_i) = 0$ for $x \leq 0$. Each of these cdfs is a description of the distribution of "risk" [Kaplan and Garrick, 1981] for the policy choice s_i; that is, the cdfs relate the loss x and its probability of occurrence $p_x(x;s_i)$. One way to extract essential information is through mathematical expectation:

$$E[X] = \int_{-\infty}^{\infty} x p_X(x;s_i)dx \tag{A2.10}$$

This condensation loses information about losses at the extreme tails of the loss-distribution.

A2.3.2 Partition the Probability Axis

The PMRM partitions the probability axis into a set of ranges. The ultimate intention of this partitioning is to provide the decision maker with a more complete view of the distribution of risk. One application concerns events that represent extremely large losses with a low probability of occurrence, while another is concerned with describing optimistic, middle-of-the-road, and pessimistic viewpoints. Some guidelines based on the standard normal distribution N(0,1) for choosing the partitioning values α_i, $i = 1,2,\ldots,n+1$, on the probability axis are presented using Fig. A2.6. In general literature, catastrophic events have 10^{-5} or less probability of occurrence; this relates to events exceeding $+4\sigma$ on N(0,1). If the N(0,12) exceedence probability function $1 - P_x(x;s_i)$ is employed as a heuristic Fig. A2.7), it can be seen, or example, that if three ranges were needed to represent the bulk of the low-damage events, an intermediate-damage range, and a range representing "catastrophic" low-probability events, the $+1\sigma$ and $+4\sigma$ partitioning values would provide an effective rule of thumb in the normal distribution case; the low range contains 84% of the loss events, the intermediate range contains just under 16% of the loss events, and the higher range contains about 0.0032% (or 3.2×10^{-5} probability) of the loss events. Alternatively, using $+2\sigma$ and $+4\sigma$ as the partitioning values results in 97.7% ~ 2.3%, and 0.0032% for the respective ranges.

As another example, again using the heuristic of Fig. A2.7 the probability axis could be partitioned into optimistic/middle-of-the-road/pessimistic ranges. This could be done by choosing the partitioning values associated with $\pm 1\sigma$ for the sample. This results in the lower 15.9% of the damage observations, the middle 68.2%, and the higher 15.9%.

A2.3.3. Map the Partitions to the Damage Axis

Once the partition values on the probability axis have been deluded, these values are mapped onto the damage axis (as in Fig. A2.7). Solutions must be found to the following problem:

For each partition value α_i, $i = 1,2,\ldots,n+1$, and each policy option s_j, $j = 1,2,\ldots,q$, find an $a_{ij} > 0$ such that $P(a_{ij};s_j) = \alpha_i$.

These a_{ij} are used in defining conditional expectations for the next step of the PMRM. If $P(x;s_j)$ has a closed-form expression for the inverse (that is,

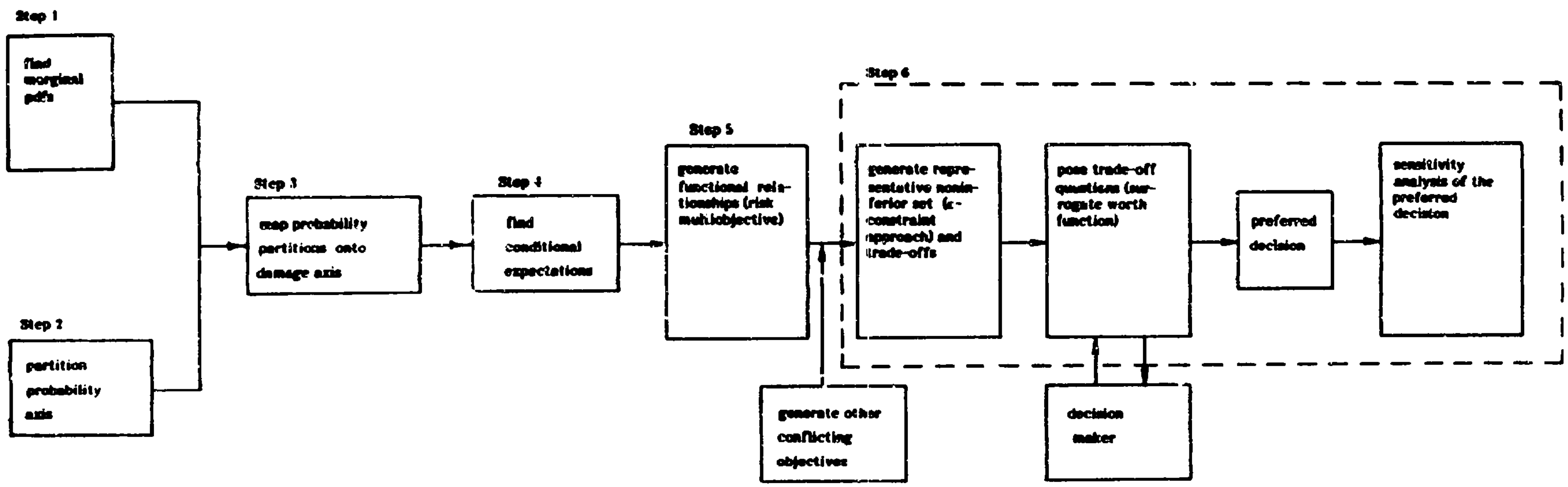
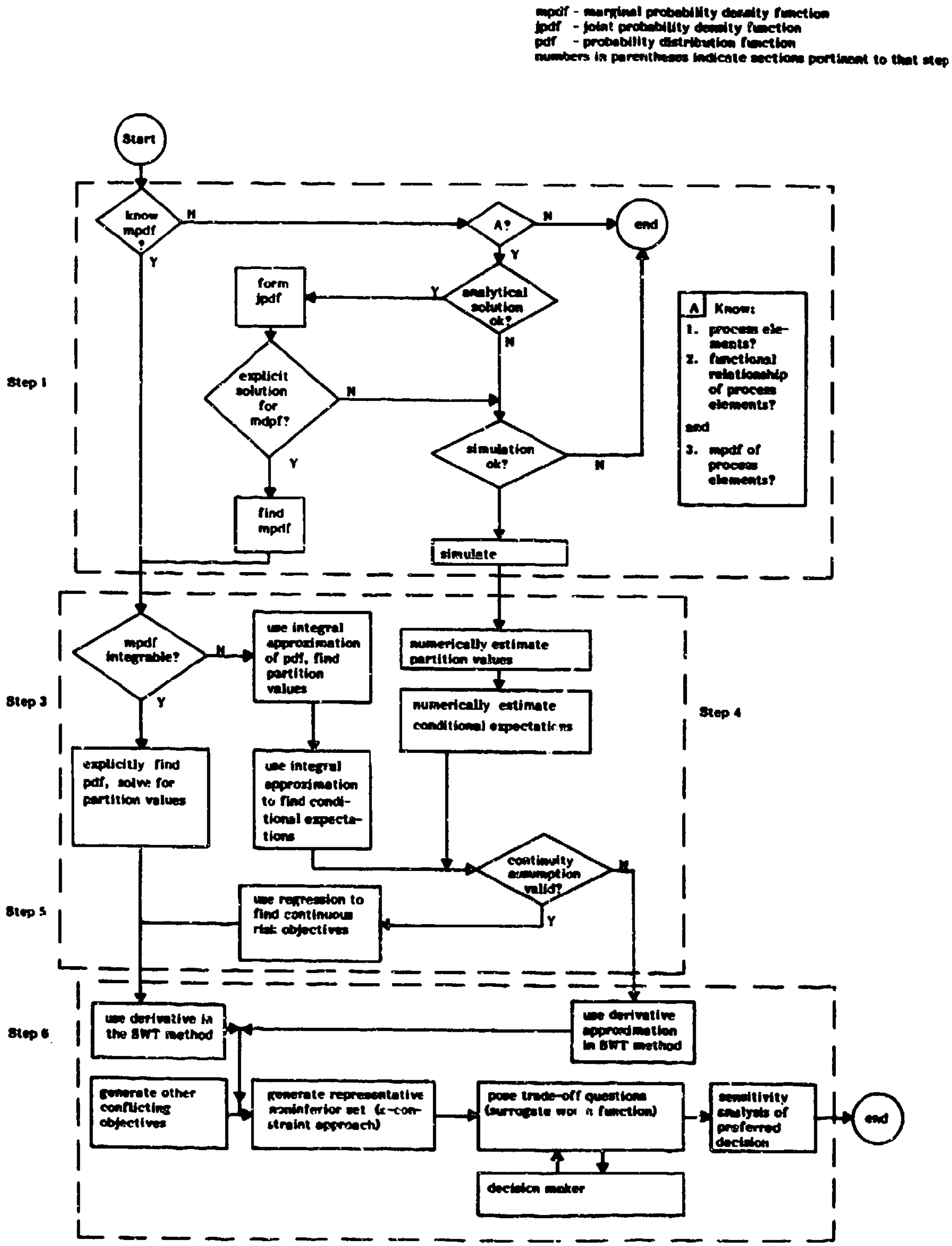


Figure A2.4 Major Steps in the PMRM

Figure A2.5 Detailed Flowchart of the PMRM Procedure



unknown a_{ij} may be found explicitly; otherwise those a_{ij} may be found by approximation through bisection, false position, or other line-search techniques.

A2.3.4 Find the Conditional Expectations

Conditional expectations must be found for each $P(x;s_j)$, $j = 1,2,\ldots,q$, with domains on the damage axis defined by the a_{ij}, $i = 1,2,\ldots,n+1$ and $j = 1,2,\ldots,q$. Let

$$D_{1j} = [a_{1j},a_{2j}] \quad , \quad j\ j = 1,2,\ldots,q$$

$$D_{ij} = (a_{ij},a_{i+1,j}] \quad , \quad i = 1,2,\ldots,n; \quad j = 1,2,\ldots,q \tag{A2.11}$$

The expectations are computed [see Eq. (A2.7)] to be

$$E[X|D_{ij}] = \frac{\int_{a_{i+1,j}}^{a_{i+1,j}} xp_X(x;s_j)dx}{\int_{a_{ij}}^{a_{i+1,j}} p_X(x;s_j)dx} \tag{A2.12}$$

$$i = 1,2,\ldots,n, \quad j = 1,2,\ldots,q$$

Note that the denominator of Eq. (A2.12) is actually

$$\int_{a_{ij}}^{a_{i+1,j}} p_X(x;s_j)dx \simeq \alpha_{i+1} - \alpha_i \tag{A2.13}$$

$$i = 1,2,\ldots,n$$

but the use of :e integral denominator reduces the computational error arising from the use of approximate values for the a_i and α_i in Eq. (A2.12).

A2.3.5 Generate Functional Relationships

Given the $E[X|D_{ij}]$, a set of risk functions $f_i(s)$, $i = 1,2,\ldots,n$, may be found as follows. If it can be assumed that the conditional expectations for values of s between the known data points act in a continuous and simple way, then for any region or the probability axis D_{ij} with $i \in \{i,2,\ldots,n\}$, regression may be used to fit a smooth curve $f_i(s)$ to the point pairs $\{s_j,E[X|D_{ij}]\}$, $j = 1,2,\ldots,q$. If the continuity assumptions cannot be made or if a smooth curve cannot be found to fit the data points to the analyst's

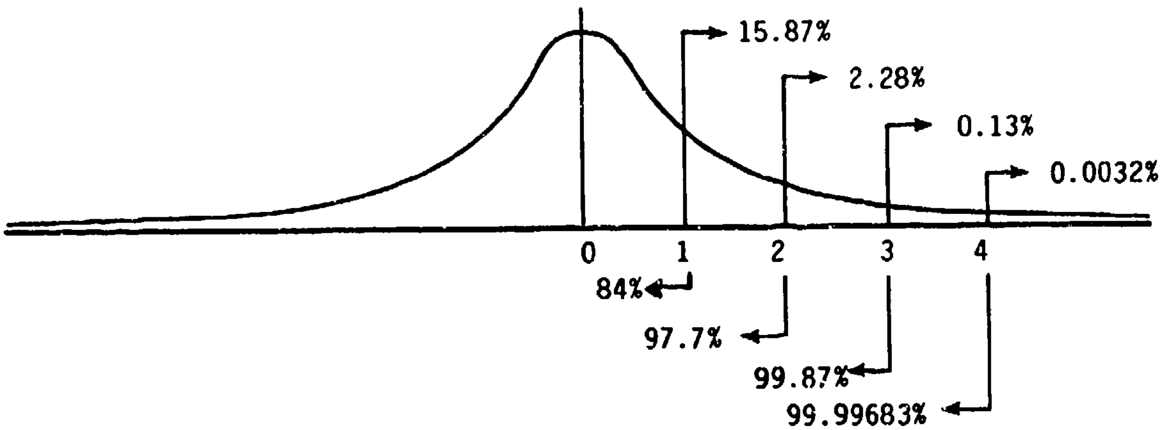


Figure A2.6 Percent of Observations Falling above and below several σ-limits

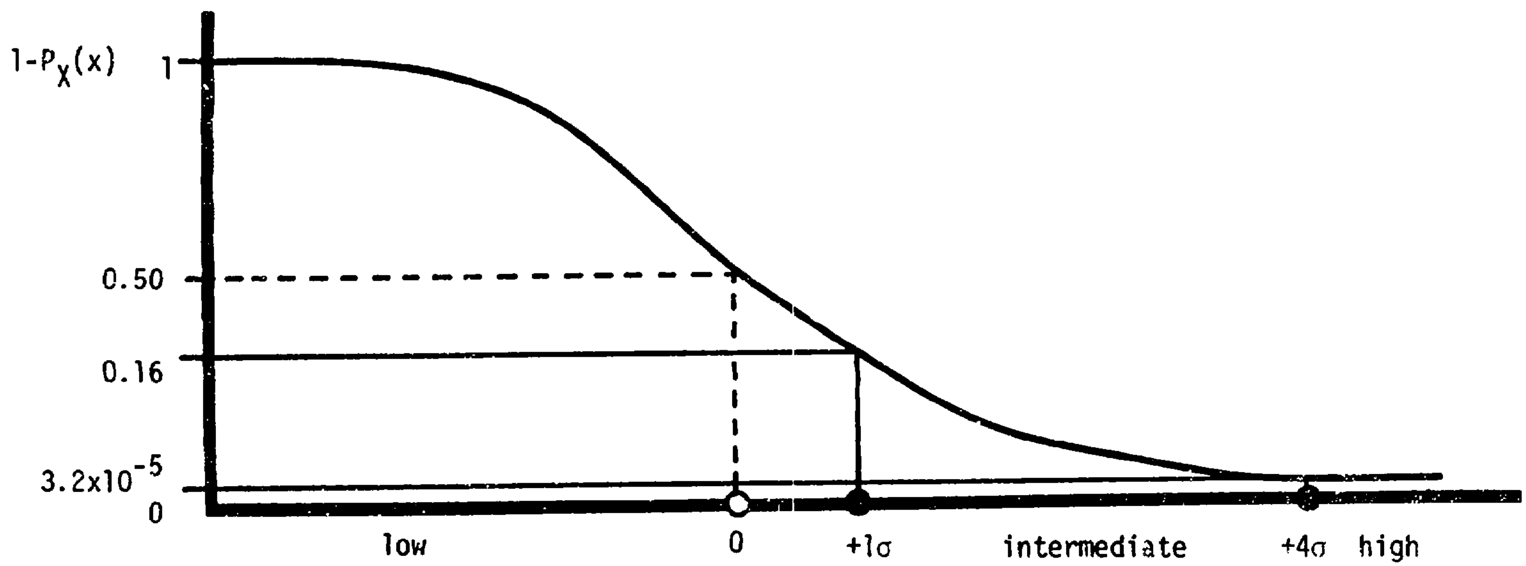


Figure A2.7 Partitioning the Probability Axis satisfaction, the data-point pairs may be used in lieu of the $f_i(s)$ to obtain a less general result in the next step. Each $f_i(s)$ relates the damage domains associated with the partitioned regions on the probability axis to the policy variable s.

A2.3.6 Employ the Surrogate Worth Trade-Off Method

In section A2.3.5 a set of risk objectives was created that, in combination, can provide some insight into how risk is distributed over the range of losses for each decision choice. A structure technique is required for effectively employing this information and valuing each decision choice. Trade-off information for the decision maker(s) is required; furthermore, risk is only one component of the broader context of the decision-making process. These criteria suggests the necessity of a multiple-objective decision-making methodology that allows decision makers to express their implicit values and/or those of their constituents during the decision-making process; the surrogate worth trade-off (SWT) method (section A2.2.2) satisfies these needs by providing trade-offs among the several objectives.

Through the SWT method, the $f_i(s)$, $i = 1,2,\ldots,n$, may be used in conjunction with a set of conflicting nonrisk objective functions $f_j(s)$, $j = 1,2,\ldots,m$, and a feasible decision set $S = \{s \mid g_i(s) \leq 0,\ i = 1,2,\ldots,p\}$, as follows. Arbitrarily choosing the first nonrisk objective as the primary objective (although experience has shown that the objective measured in monetary units to be the best selection as the primary objective) and for any one risk function $f_h(s)$ with $h \in \{1,2,\ldots,\}$, solve the problem P_h (see below) to obtain trade-offs between the risk function $f_h(s)$ and the nonrisk objectives.

$$P_h:\ \min_{s \in S} f_1(s)$$

$$\text{subject to } f_j(s) \leq \varepsilon_j, \qquad j = 2,3,\ldots,m$$

$$f_h(s) \leq \varepsilon_h, \qquad h \varepsilon \{1,2,\ldots,n\}$$

In practice, the trade-off between the m conflicting nonrisk objectives need be obtained only once, while the trade-offs related to each of the n risk objectives can be obtained by swapping one risk objective for another in p_h. This process of swapping the risk objectives one at a time is necessary because these risk functions are dependent on each other by construction (see section A2.4.3). The trade-offs provide extremely useful information in the decision-making process.

If the continuity assumption in section A2.3.5 cannot be justified, the trade-offs in the SWT method may be obtained by approximation of the partial derivative: that is,

$$\lambda_{1h} = \frac{\partial f_1(s_j)}{\partial f_h(s_j)} \simeq \frac{f_1(s_j) - f_1(s_k)}{E[X|D_{hj}, s_j] - E[X|D_{hk}, s_k]} \qquad \text{(A2.14)}$$

where $k = j + 1$ for an increase of s_j to s_{j+1} and $k = j - 1$ for a decrease to s_{j-1}. Although heuristically appealing, technical details of this approximation have not been confirmed.

These trade-offs allow decision makers to see the marginal cost of a small change in an objective, given a particular level of risk assurance for each of the partitioned risk regions. A knowledge of marginal costs gives the decision maker insights that are useful for determining acceptable risk levels. In general, trade-offs between the risk functions associated with any one loss dimension cannot be found; however, if more than one risk axis is used--say mortality, morbidity, dollars lost, etc.--trade-offs between these risks should be obtainable if the objectives are in conflict.

A2.4 Comments and Observations

A2.4.1 On Creating the Risk Functions

In the spirit of regarding risk as a distribution of probability and damage, the decision maker should ideally be presented with the entire distribution of risk for each policy option. This approach quickly becomes confusing and cannot provide the marginal worth of one decision over another, nor can it show the relations between various nonrisk objectives and the risk aspects of a decision. The PMRM includes risk distribution information through the functions $f_i(s)$, $i = 1,2,\ldots,n$, that relate the conditional expectations associated with the probability axis partitions to the policy variable s (section A2.3.5); this provides information across the entire domain of the damage x.

A2.4.2 A Temporal Interpretation of the Risk Functions

Consider three risk functions generated by partitioning the probability axis. When viewed temporarily, the lower, intermediate, and catastrophic damage levels could be interpreted as short-, intermediate-, and long-term effects of that decision choice. The lower-damage events could be interpreted as every-day-type occurrences representing the short-term effects of the system reflected in the operational costs or the long-term cumulative effects from radiation, carcinogens, or other etiological (disease-causing) agents. The intermediate effects could be interpreted as representing near-future effects reflected in the system's evolution to its next phases of development. The catastrophic losses could be viewed as the long-term effects of a decision choice that requires finding the most reduction affordable. The catastrophic events tend to be perceived as having broader societal impact, which makes them more politically charged issues and more subject to influence by the public's risk perceptions.

Defining additional risk functions allows for a finer grid of damage levels, and thus a finer division of choices over the time horizon. The probabilistic nature of the risk distributions gives no guarantee that a catastrophic event would not occur tomorrow; thus, the temporal interpretation should be employed only as a guideline in the analytical process. Creating more risk functions also yields an increased number of trade-offs and objective function values for each decision choice. As a rule of thumb, an individual is normally able to keep in mind only about 7 ± 2 pieces of information at one time. This should be given consideration at the time the number of risk functions is chosen.

A2.4.3 Relating Conditional and Unconditional Expectations

A relation between the conditional [Eq. (A2.2)] and unconditional [Eq. (A2.10)] expectations may be found. Define the following functions:

$f_1(s)$ = a nonrisk function which serves as the primary objective function in the ε-constraint format

$f_i(s)$ = the N-1 conditional expectation risk functions, $i = 2,3,4,\ldots,N$ (A2.15)

F_{N+1} = the unconditional expectation risk functions; that is, the expected-value function

Furthermore, let $0 = P(x_2) < P(x_3) < \ldots < P(x_{N+1}) = 1$ be partition values used to define the N-1 conditional expectation risk functions. Note $P(x_i) = \alpha_i$ for the α_i in sections A2.3.2 and A2.3.3 with $i = 2,3,\ldots,n+1$ and that $i \neq 1$ because the risk functions in this example begin with $f_2(\cdot)$.

Assuming that P(x) is a monotonically increasing (vs. nondecreasing) function of x, it may be observed that $0 = x_2 < x_3 < \ldots < x_N < x_{N+1} = +\infty$.

Define the following constant weights:

$$\theta_i(s) = \int_{x_i}^{x_{i+1}} p(x,s)dx \ , \qquad i = 2,3,\ldots,N \qquad \text{(A2.16)}$$

Because $\theta_i(s)$ is constant with changing s, then

$$f_{N+1}(s) = \sum_{i=2}^{N} \theta_i f_i(s) \qquad \text{(A2.17)}$$

Further effort should be made to find some similarly simple relation between the conditional and unconditional trade-offs, such that $\lambda_{N+1,1} = \psi[\lambda_{21},\ldots,\lambda_{N1}]$. A2.4.4 Catastrophic Losses and Decision Making

Using the notation from section A2.4.3 consider $f_{N+1}(s)$ (the unconditional expectation), $f_N(s)$ (the conditional expectation of the catastrophic damage events), and $f_1(s)$ (the cost function). Figure A2.8 plots $f_1(s)$ against $f_N(s)$ and $f_{N+1}(s)$. Note that $f_{N+1}(s)$ characteristically takes values less than $f_N(s)$. When decision makers are presented with a value for $f_N(s)$ as well as $f_{N+1}(s)$, they are being reminded that besides the lesser value for $f_{N+1}(s)$ there is a nonzero probability of a major loss of $f_N(S)$; therefore, catastrophic events are considered as a component of the decision process.

For example, policy alternative s = A gives the resulting values of $f_1(A)$, $f_N(A)$, and $f_{N+1}(A)$. If the business-as-usual approach is followed, $f_{N+1}(A)$ alone would be available as the risk-representing function. The nonzero probability of the significantly larger loss $f_N(A)$ would have been ignored from the decision maker's point of view; thus, this valuable information would have been lost.

A2.5 Evaluation of the Method and Extensions

A2.51 Evaluation of the PMRM

Fischhoff et al.* suggested seven criteria against which a risk-related decision-making methodology might be measured; comprehensiveness; logical soundness; practicality in relation to real problems, people, and resource constraints; openness to evaluation; political acceptability; compatibility vithe existing institutions; and conduciveness to learning for future risk decisions.

A2.5.1.1 Attributes

In view of these seven criteria, the following observations can be made. An increase in the comprehensiveness and practicality of an analysis can be brought about when information is presented about the distribution of risk in a multiple-objective format with other objective functions, allowing risk to be viewed in its perspective with other important criteria. The PMRM proceeds in a structured and logical manner. Explicit steps to create the risk functions and the trade-offs provide an openness to later evaluation. Transforming the risk problem into a multiple-objective problem allows the decision maker to consider the political acceptability and institutional compatibility of various alternatives through multiobjective decision-making techniques such as the surrogate worth trade-off (SWT) method. Finally, the structured format allows each well-documented study to be a learning experience for improving the method for future analyses.

Beyond the seven-evaluation criteria, the PMRM provides some additional strong points. Proper development of the risk objectives in their multiple aspects separates information about catastrophic (primarily low-probability) events and low-damage (primarily high-probability) events, thereby circumventing a point of contention with the traditional expected value; thus, more information is available to the decision maker about the distribution of risk. Employing the SWT method further strengthens the PMRM by avoiding the need to explicitly assess each decision maker's utility function(s).

A2.5.1.2 Shortcomings

This section describes some of the shortcomings of mathematical decision-making systems in general and of the PMRM in particular.

A2.5.1.2.1 Shortcomings indigenous to mathematical decision-making systems

Several difficulties are shared by all mathematical decision-making systems. The comprehensiveness of the analysis is formulation dependent; that is, broad and clear formulation, care in the actual decision making, and a diligent thorough sensitivity analysis provide for the soundest results. Practicality in terms of real people, problems, and resource constraints

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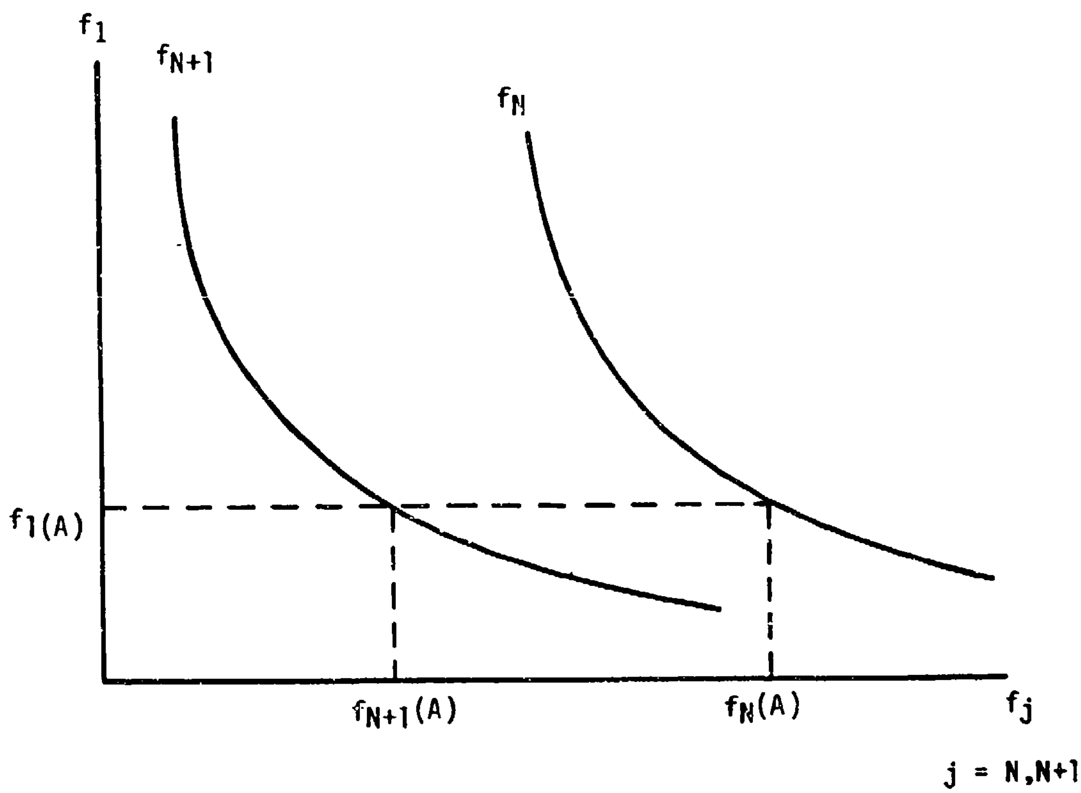
*B. Fischoff, S. Lichtenstein, P. Slovic, R. Keeney, and S. Derby, Approaches to Acceptable Risk: A Critical Guide, Oak Ridge National Laboratory, Oak Ri'ge, TN, NUREG/OR-1614 and ORNL/Sub-7656/1, 1980.

Figure A2.8 f_{N+1}, The Unconditional Expectation, and f_N, the Catastrophic Conditional Expectation vs. f_1, the Cost Objective, in the Functional Space



requires the use of robust, proven mathematical models of the risks, other objectives, and constraints. For a study to provide the opportunity of learning for future decisions, documentation of that study must be as complete and thorough as possible. Finally, the alternatives included in any analysis must be structured in such a way that compatability with existing institutions is kept in mind.

A2.5.1.1.2 PMRM Shortcomings

There are some specific shortcomings indigenous to the PMRM itself. The decision maker's utility function(s) are not made explicit during the procedure, so the basis for the decision retains some subjectivity; however, explicit utility functions are subject to some question in any analysis. The basis by which the probability range is partitioned could be strengthened. The interpretation of the risk functions could be more complete. Although there is no reason to doubt their solvability, the PMRM has yet to be applied to problems involving multidimensional decision and/or risk vectors. Partitioning on the damage axis rather than the probability axis has been suggested, but the efficacy and practical application of this option has yet to be demonstrated.

The simulator approach requires a large amount of (computer) calculation and, therefore, requires either easily solvable models for the risky-loss variables and/or computer packages for solution approximations of multiple integrals. On the brighter side, a micro- or minicomputer with hard disk storage capacity should provide adequate computer capacity for many problems.

A2.5.2 Extensions

The PMRM at present offers exciting and widespread potential use in risk-related decision-making problems, and several suggestions came to mind. More studies employing the PMRM should be done, particularly examples involving multidimensional decision and/or damage vectors and partitioning on the damage (rather than the probability) axis. These studies could also be used in refining the interpretation of the risk functions. Development of a more theoretical basis for assigning the partitioning ranges could provide for a better communication of the distribution of risk for a given alternative. Other theoretical investigations intended to find an explicit relationship between risk function trade-offs similar to the relation among risk functions found in section A2.4.3 could be quite useful.

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