The Navigation Economic Technologies Program

November 1, 2004

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SPATIAL MODELING IN TRANSPORTATION

Congestion and Mode Choice

US Army Corps of Engineers®

IWR Report 04-NETS-P-06

Navigation Economic Technologies

The purpose of the Navigation Economic Technologies (NETS) research program is to develop a standardized and defensible suite of economic tools for navigation improvement evaluation. NETS addresses specific navigation economic evaluation and modeling issues that have been raised inside and outside the Corps and is responsive to our commitment to develop and use peer-reviewed tools, techniques and procedures as expressed in the Civil Works strategic plan. The new tools and techniques developed by the NETS research program are to be based on 1) reviews of economic theory, 2) current practices across the Corps (and elsewhere), 3) data needs and availability, and 4) peer recommendations.

The NETS research program has two focus points: expansion of the body of knowledge about the economics underlying uses of the waterways; and creation of a toolbox of practical planning models, methods and techniques that can be applied to a variety of situations.

Expanding the Body of Knowledge

NETS will strive to expand the available body of knowledge about core concepts underlying navigation economic models through the development of scientific papers and reports. For example, NETS will explore how the economic benefits of building new navigation projects are affected by market conditions and/or changes in shipper behaviors, particularly decisions to switch to non-water modes of transportation. The results of such studies will help Corps planners determine whether their economic models are based on realistic premises.

Creating a Planning Toolbox

The NETS research program will develop a series of practical tools and techniques that can be used by Corps navigation planners. The centerpiece of these efforts will be a suite of simulation models. The suite will include models for forecasting international and domestic traffic flows and how they may change with project improvements. It will also include a regional traffic routing model that identifies the annual quantities from each origin and the routes used to satisfy the forecasted demand at each destination. Finally, the suite will include a microscopic event model that generates and routes individual shipments through a system from commodity origin to destination to evaluate non-structural and reliability based measures.

This suite of economic models will enable Corps planners across the country to develop consistent, accurate, useful and comparable analyses regarding the likely impact of changes to navigation infrastructure or systems.

NETS research has been accomplished by a team of academicians, contractors and Corps employees in consultation with other Federal agencies, including the US DOT and USDA; and the Corps Planning Centers of Expertise for Inland and Deep Draft Navigation.

For further information on the NETS research program, please contact:

Mr. Keith Hofseth
NETS Technical Director
703-428-6468

Dr. John Singley
NETS Program Manager
703-428-6219

U.S. Department of the Army
Corps of Engineers
Institute for Water Resources
Casey Building, 7701 Telegraph Road
Alexandria, VA 22315-3868

The NETS program was overseen by Mr. Robert Pietrowsky, Director of the Institute for Water Resources.

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Prepared by:

Simon P Anderson
Department of Economics
University of Virginia

Wesley W Wilson
Department of Economics
University of Oregon

For the:

Institute for Water Resources
U.S. Army Corps of Engineers
Alexandria, Virginia

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*Spatial Modeling in Transportation: Congestion and Mode Choice**

Simon P. Anderson†and Wesley W. Wilson‡

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Abstract

We construct an equilibrium model of sequential bottlenecks in a transportation network. The model is applied to transport mode usage in the context of river (barge) traffic. The main alternative is rail, and barge must be supplemented by trucking (which also enters as a stand-alone option). We allow for congestion on the waterway, as well as congestion avoidance by trucking past congested areas (such as locks). Finally, we endogenize the price of river transport and show when a unique equilibrium exists. The equilibrium specifies the mode choice, congestion times, and barge rates for the system.

KEYWORDS: Spatial equilibrium, congestion, transportation networks, mode choice, equilibrium mode price, sequential bottlenecks.

1 Introduction

Many transportation networks entail a sequence of bottlenecks. This is true for waterways where the locks are frequently a source of congestion. It is also true for highways where certain interchanges (and toll collection areas) are prone to block up, and for city streets that tend to congest at certain intersections. In this paper, we construct an equilibrium model of sequential bottlenecks in a transportation network. We build up the model from a simple starting point and extend it to analyze first one and then several bottlenecks. We also allow for the possibility of bottleneck avoidance (or "bypass"), meaning that a commuter or shipper may choose to join the transport facility "downstream" of a congestion bottleneck.

The analysis is phrased in the context of river transportation, and the paper develops an equilibrium model of the demand for river transportation, but it may be applied to other markets too. The micro-economic foundations for the

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†Department of Economics, University of Virginia.

‡Department of Economics, University of Oregon, and Institute for Water Resources.

model are rooted in a spatial representation of the production of output that is to be shipped. Output may be shipped by several alternative modes, and the equilibrium specifies what is shipped and how. Thus, the demand for shipping is seen as the aggregation of individual producers' choices of shipping modes. That is, instead of assuming a demand for river services, we explicitly derive this from the underlying demand for shipping that generates it. In doing so, we allow for shippers to choose the preferred mode of transport, taking into account the congestion costs. We allow for the choice of transport mode to also include the routing choice of whether to by-pass a lock or locks. If there is a bottleneck point near where a shipper would otherwise begin barge shipping, the shipper may instead choose to truck past the bottleneck point and begin using the river below the bottleneck. In our model, it is also possible for the shipper to bypass several locks.

The framework allows for an explicit model of a transportation network. We introduce various geographic features, such as locks along the waterway, and we also allow for the congestion costs that locks may cause. This allows us to introduce an equilibrium that accounts for feedback effects in the full equilibrium system. In particular, the flow demand of traffic down-river along with the lock capacity jointly determine the lock cost which in turn determines the demand for traffic. Traffic demand though depends on the costs through ALL locks. Thus, the benefits of an improvement must take into account the induced changes at all of the others. This is indeed one of the benefits of an explicit spatial model of traffic.

We further deal with equilibrium pricing of the barges used to haul freight. This determination of an endogenous equilibrium price is done within the full spatial equilibrium structure with endogenous congestion levels. There are, of course, many aspects missing from this model, and these form important areas of further inquiry. Some of these are described more in the concluding section. In particular, though we assume throughout that shippers care only about the price they will pay for shipping - reliability in terms of when it arrives, or indeed the time taken to arrive are assumed not to be important determinants of choice. In extensions of this work, we envisage taking account explicitly of time on the river and the variance of the trip length. For the moment though, these features are not explicitly included. We note that such extensions are unlikely to change the overall structure of the model. First, we may include a separate time cost to barge travel within the existing structure: this is readily done given that the model already uses time spent on trips as part of the construction of the demand for barge time. Secondly, a reliability factor may be added both to the fixed costs of the affected modes, and/or to their marginal costs. While these are shorthand expedients to describing more complex economic phenomena, the treatment just outlined may be seen as a first step to including these additional factors.

The following sections develop the model from a simple and sparse description up to a more complex equilibrium model. The first sketch lays out the basic spatial framework (that can be compared to other models in which the demand is exogenously specified) and then adds various important considerations germane to the situation. We first allow for fixed costs (with respect to distance) in transportation, then we allow for lock costs and address the possibility of lock by-pass. Next, congestion is introduced, as an endogenous feature that depends on the amount of traffic coming downstream. We show that there is a unique equilibrium to the level of congestion at each lock and the catchment area of the river shippers. We then endogenize the price of barge services (barge rate) by showing that there is a unique price that clears the market. This means that the equilibrium model can be fully solved from the primitives. In the conclusion we describe some further possible extensions.

2 Base Model

We begin with a very simple model that forms a basic template from which the assumptions are dropped in the ensuing subsections. Shippers are located across a region. They ship to a single terminal market which is located at the mouth of a river. Shippers may choose to ship to the river in trucks. Shipments then proceed down the river, traversing a set of locks en route. Shippers may instead choose to use an alternative mode (rail).[1]

There is a river that runs from North to South, with length l. Space is two dimensional, and $y \in [0, l]$ represents the latitude (vertical, or N-S coordinate) of a map reference point, and x is its longitude (horizontal, or E-W coordinate), taking the river as longitude zero. The terminal market point is the river source, with coordinates $(0, 0)$.[2] Transport follows the "Manhattan metric," meaning that distances are traversed horizontally and vertically only.[3] The basic geography is given in Figure 1.

INSERT FIGURE 1

Trucks carry output to the river, and trucking costs t per unit distance per unit shipped.[4] As will be clear from what follows, trucks are only used in the

[1] We use truck-barge versus rail as the two options of the shipper for simplicity of presentation. The model can readily be adapted to allow for rail-to-river traffic, and more complex patterns of shipping that reflect the transportation network more finely. The first extension below does capture the idea of using one shipping method (trucking) to bypass bottlenecks on another (locks).

[2] We let the terminal market be at the river mouth for expediency. We assume here too that there is a single terminal market. The analysis can be readily extended to multiple terminal markets once the principles from the single market case are determined.

[3] This is also known as the "block" metric reflecting the idea of city blocks. The "crow-flies" or Euclidean metric is more cumbersome to work with analytically, and does not appear to give different insights. Arguably the block metric applies better to river, rail, and road since traffic needs to follow specific routes in transportation networks.

[4] We thus assume constant returns to volume and to distance in transportation. Empirically, shipping costs are often felt to be concave in distance. This finding, though, likely represents the possibility of substituting from high marginal cost options (with low fixed cost) to lower marginal cost options when the distance shipped is greater. That is, any particular option is perhaps satisfactorily represented as entailing a fixed cost for loading/unloading, and then a constant unit cost per mile. Trucks likely have the lowest fixed costs, and so are cheapest over short hauls. For longer hauls, barge traffic (if feasible) may be more economical, with its

horizontal dimension (this is modified later below). River traffic carries output down the river, along which shipping passes at cost b per unit (b is for barge) shipped per unit distance, in the vertical dimension. There is an alternative transport technology, rail, which can carry output. Rail transport is assumed to carry output at rate r per unit distance per unit output: recall that it must first be shipped from North to South, and then from East to West. Assume that $t > r > b$ so trucking costs per mile are higher than rail costs, which are in turn higher than barge costs. For now, we treat these costs per mile as constants. This is a simple way to get a "funnel" relation to describe the catchment area for river transport.[5] Since shipping by barge means a combination of the lowest cost mode with the highest one (the truck needed to bring the output to the river), the assumption that $t > r > b$ implies that river transport is preferred for locations close by the river banks, and rail transport is preferred otherwise. It will also be seen below that river transportation will tend to be preferred further upstream (for any given distance from the river) because then the low per-mile cost advantage of barge traffic will more easily defray trucking costs.

Then road-river transport will be used from coordinate (y, x) as long as

$$rx + ry > tx + by, \tag{1}$$

or

$$y \geq \hat{y}(x) = \frac{t-r}{r-b}x \tag{2}$$

where the numerator and denominator of the fraction term are both positive.[6] Note that $\hat{y}(0) = 0$, so that the river catchment area forms a triangle, symmetric about the river, with a point (vertex) at the destination [7] The catchment area broadens up towards the source of the river.[8] The idea here is that the advantage

higher fixed cost offset by lower marginal cost. Taking the lower envelope of the costs across alternatives yields a concave overall cost as a function of distance. For the present, we ignore the presence of fixed costs in the different transport modes. However, they are treated in the first extension sub-section below.

[5] With the alternative assumption of a crow-flies distance for the rail net, the cost of bringing output to the destination is

$$r\left(y^2 + x^2\right)^{1/2}.$$

[6] These transport rates will differ across different items shipped, and so the margins will be different for different commodities. Shippers may also place different weights on other features of shipping modes, such as time, reliability, etc. Systematic differences may be readily introduced into an extended model, and idiosyncratic choice effects can be effectively modeled with a discrete choice approach.

[7] Below we introduce fixed costs in transport, and these imply that all shippers close enough to the terminal market use trucks. However, it may also be that having the commodity already loaded onto barges is useful for further trans-shipment! That is, it may also be important to consider what happens beyond the terminal point.

[8] As in Figure 2 below, the source of the river is also synonymous with the end of the territory (or the Canadian border, assuming that Canadian shippers use routes in their own territory). For now we have not allowed for trucks to move in the vertical dimension, but we shall allow for that below. Then, if the river stops short of the end of the territory, the extra catchment area north of the river's source is simply an extra triangle atop the existing cone illustrated in Figure 2. This extra triangle is like the one at the bottom of Figure 5 around the terminal market, except the other way up.

of river transport rises with "vertical" distance (i.e., N-S) but decreases with "horizontal" distance. Hence the "indifferent" location, which is captured as $\hat{y}(x)$, is a diagonal line in (y, x) space. It is illustrated in Figure 2 (in this Figure, as with those that follow, m denotes the slope of a line.)

INSERT FIGURE 2

The rate at which the boundary line rises with distance from the river in (2) is quite subtle. It is the ratio of the extra cost of trucks over rail to the extra cost of rail over barge. For example, suppose that $t = .0025$, $r = .0005$, and $b = .00035$ (these rates are an approximation of the costs of shipping corn, and their genesis is detailed in the Appendix). With these numbers, the critical ratio is $\frac{t-r}{r-b} = \frac{.002}{.00015} = \frac{200}{15} = 13.33$. For each mile further from the river, the critical latitude rises over 13 miles. This gradient steepness reflects the high differentials of the cost truck-barge relative to rail. Put another way, one would have to go 1300 miles upriver to find shippers choosing to truck as far as 100 miles in order to use the river. Other commodities entail different costs per mile and so will have different catchment areas.

The relations above are the key to determining demand. Indeed, once we know what is $\hat{y}(x)$, we can then readily determine the total demand for barge traffic by integrating up over the space within the catchment area. Below we elaborate by introducing various other elements that influence costs and hence the choice of shipping mode.

2.1 Fixed costs

The above analysis has assumed that transportation (conditional upon a mode) takes place at constant cost per unit distance traveled and constant cost per unit shipped, so that we have assumed constant returns in both volume and distance. At the level of the truck-load, the volume assumption seems not too egregious. However, we have disregarded fixed costs associated with making a shipment and the relationship between rates and distance seems crucial to observed patterns.[9] In practice, there are fixed cost components with respect to distance because goods need to be loaded for shipment onto and off the truck, railcar, and barge. We now allow for these fixed costs, while retaining the assumption that marginal shipping costs per mile are constant (at rates t, r, and b respectively). That is, as a function of distance, each mode will entail a fixed cost as well as a constant marginal cost per unit distance. The existence of such fixed costs implies that the (average) cost per mile decreases with distance shipped (just like average total costs are always decreasing in output when fixed costs with respect to output are positive and average variable costs are constant in Principles classes) In the context of Transportation, this phenomenon is known as "tapering" (see Locklin, 1972). These concepts are developed in Figures 3 and 4. Our cost functions take the form $F_m + md$, where

[9] Without fixed costs, the model allows for barge and rail movements even for very short distances.

d is distance and $m = t, r, b$ denotes truck, rail and barge, respectively. However, since we assume it is always necessary to use trucks to arrive at the waterway (effectively, the number of farms or other input sources close enough to load directly onto barges is so negligible as to be disregarded) the relevant rate for barge transport, even right next to the riverbank, is $F_b + F_t + bd$. In Figure 3, we map rates against distance. The cost function to the shipper is the lower envelope of the modal cost lines. There are two cases illustrated in Figure 3. In the upper panel, each mode is used from some distance. In the lower panel, rail is dominated in the sense that it is not the cheapest mode from any distance.

INSERT FIGURE 3

In Figure 4, we have indicated both total costs (upper panel) and average costs (lower panel) as functions of distance, for the case in which rail movements are not dominated. The tapering principle (diminishing rates per unit distance) is clearly shown in the lower panel. In both panels, given the cost-minimizing mode choice, the actual costs are the lower envelope of those associated to the modes.

INSERT FIGURE 4

We assume that fixed costs are lowest for trucks and highest for barges, so $F_t < F_r < F_b$, as in the Figures. Intuitively, this implies that only truck is the least expensive option from locations close by the terminal point. This intuition is borne out in the description that follows.

The per unit costs associated with using only trucks from location (y, x) are then $F_t + tx + ty$. Using only rail costs $F_r + rx + ry$ per unit. Finally, the river option entails a barge cost of $F_b + by$ plus a truck cost of $F_t + tx$, for a total cost of $F_b + F_t + tx + by$. The lowest cost option is found by comparing these magnitudes.

First, truck-only is preferred to rail for $F_t + tx + ty < F_r + rx + ry$, or $y < \bar{y} = \frac{(F_r - F_t)}{(t-r)} - x$. If it were a matter of just truck and rail, this would generate a tent-shaped catchment area for truck around the destination point (see Figure 5).

INSERT FIGURE 5

If the fixed cost for barge transport is relatively high, we have the case illustrated in the Figure, with a rail segment dividing the truck-only catchment area from the truck-and barge one. In that case, the catchment are for truck and barge is given by comparing the corresponding costs. Indeed, truck-and-barge is then preferred as long as $F_b + F_t + tx + by < F_r + rx + ry$. This we can rewrite as $y > \hat{y}(x) = \frac{\Delta F + (t-r)x}{r-b}$, where we have defined $\Delta F = F_b + F_t - F_r$, which is positive because barge shipments have higher fixed costs than rail. Indeed, $\frac{\Delta F}{r-b}$

is the latitude at which rail start to give way to truck and-barge. Note that the critical vertical distance $\hat{y}(x)$ is then *increasing* in the horizontal distance.

On the other hand, if the barge fixed costs are quite low, then, following the river north, there is no intervening rail usage between the truck and truck-and-barge segments. The comparison of mode usage is quite clean because a pair-wise comparison of truck versus barge indicates that there is a critical latitude South of which only trucks are preferred, and North of which both barges and trucks are employed. Indeed, the truck-only option beats truck and barge if and only if $F_t + tx + ty < F_b + F_t + tx + by$ which boils down to $ty < F_b + by$. The economics of this condition are that only the vertical component matters since trucks must necessarily be used for the horizontal one: moreover, the truck fixed cost is already expensed for the horizontal distance. Rephrasing the condition, truck-only is preferred to truck-barge for

$$y < \tilde{y} = \frac{F_b}{t-b}, \tag{3}$$

meaning that the barge fixed cost cannot be paid for from its per-mile cost advantage over trucks for such locations.

Given this simple condition, the full equilibrium is now simply determined by comparing truck-only to rail for $y < \tilde{y}$, and truck-and-barge to rail for $y > \tilde{y}$. As above, truck-only is preferred to rail for $F_t + tx + ty < F_r + rx + ry$, or $y < \bar{y} = \frac{(F_r - F_t)}{(t-r)} - x$. This means that the further North (below $\tilde{y}$), the smaller the truck-only catchment area. Above $\tilde{y}$ though, the larger the catchment area for truck-and-barge shipments. Indeed, truck-and-barge is then preferred as long as $F_b + F_t + tx + by < F_r + rx + ry$, which we can rewrite as $y > \hat{y}(x) = \frac{\Delta F + (t-r)x}{r-b}$, where we recall that $\Delta F = F_b + F_t - F_r > 0$. The equilibrium catchment area configuration is indicated in Figure 6.

INSERT FIGURE 6

In what follows, we shall treat primarily the case of relatively large barge fixed costs, the one illustrated in Figure 5, and corresponding to Figure 4. This does seem loosely to correspond to the case of the Mississippi river system, where it is relatively unusual for trucks alone to be used from the same point that truck-and-barge are used (i.e., a location such as $\tilde{y}$ does not arise). We shall mention the other case, the one illustrated in Figure 5, in footnotes where pertinent in what follows.

2.2 Locks

Suppose that there are n locks on the river, at locations y_i, $i = 1, ..., n$. In the absence of congestion (the system is below its free-flow capacity level), passing lock j costs C_j.[10] Assume too that the first lock (at y_1) is far enough upstream

[10]These costs include both time costs and usage fees. The time cost is a design factor that differs across locks. Below, when we consider congestion, we separate out the time cost from the access fee charged to those using the lock.

that barge transport is used down-river from that lock (i.e., $y_1 > \tilde{y}$). Then, road-river transport will be used from coordinate (y, x), with $y \in (y_j, y_{j+1})$ as long as

$$F_r + rx + ry > F_t + F_b + tx + by + \sum_{i=1}^{j} C_i, \quad (4)$$

where the last term represents the costs of traversing the j locks down to the market terminal.

Rearranging the above equation yields

$$\hat{y}_j(x) = \frac{(t-r)}{(r-b)}x + \frac{\Delta F + \sum_{i=1}^{j} C_i}{(r-b)}, \text{ for } \hat{y}_j \in (y_j, y_{j+1}), \quad (5)$$

with $y(x) \in [\max\{y_j, \hat{y}_j(x)\}, y_{j+1}]$ as the set of locations for which river transport is preferred to rail. As above, truck-only is preferred for $y < \tilde{y}$, where $\tilde{y}$ is given by (3), and under the assumption that $y_1 > \tilde{y}$, so that the first lock is sufficiently far up-river.

Notice that the RHS of the equation (5) falls every time the y value passes through a lock location. Visually, think of a set of stacked funnels (the funnels may have different sizes at their bases); each time a lock location is passed, the river gets less attractive as a transport option. As the distance North increases, though, since the advantage of river transport rises with "vertical" distance, the river gets more attractive again as long as no further locks are passed. Thus the pattern is like a pile of "Z's," as illustrated in Figure 7.

INSERT FIGURE 7

2.3 Lock by-pass (single lock by-pass)

Given that it costs a fixed (i.e., distance independent) charge to pass a lock, it is cheaper to by-pass close-by locks with truck transport. That is, we now extend the model just above to allow for the by-pass option, using road transportation to get just below a lock and enter the river at that point. Note that it is always an option for a shipper to *not* by-pass (though not by-passing is not always the least costly option) and so the set of river-using locations identified in (5) is contained in the new set once we allow this option. However, other locations nearby the lock and just above it, previously precluded, will now also use the river.

The vertical cost component of using road transport down to the nearest lock, assuming the Manhattan metric, is $t(y - y_j)$ for a location $y \in (y_j, y_{j+1})$.[11] The net cost (net of the vertical cost of transporting the same shipment down the

[11] The shipment enters the river just below the lock at x_j. This corresponds to a river port terminal. We assumed above that shipments can enter at any point on the river. The framework may be readily adapted to a finite number of entry points (river port terminals) using similar methods.

river by barge) is $(t-b)(y-y_j)$, and so by-passing the lock is preferred as long as this net cost is smaller than the lock-crossing cost, C_j (notice that loading fixed costs do not enter these incremental calculations). This condition can be written as $(t-b)(y-y_j) < C_j$. Rewriting this as $(y-y_j) < \frac{C_j}{(t-b)}$, shows that this option is preferred for all distances above the lock below the critical value on the RHS. This means that the pattern given above is the same as in (5) for $y \in \left(y_j + \frac{C_j}{(t-b)}, y_{j+1}\right)$ and must be modified otherwise.[12] In the interval $y \in \left(y_j, y_j + \frac{C_j}{(t-b)}\right)$, the costs of river shipping are $F_t + F_b + tx + t(y-y_j) + by_j + \sum_{i=1}^{j-1} C_i$, since the j-th lock is by-passed. Comparing to the rail shipping cost of $F_r + rx + ry$, river is preferred as long as

$$tx + t(y-y_j) + by_j + \Delta F + \sum_{i=1}^{j-1} C_i > rx + ry,$$

which enables us to define the critical y-value where this holds with equality as

$$\hat{y}(x) = \frac{1}{(t-r)}\left\{(t-b)y_j - \Delta F - \sum_{i=1}^{j-1} C_i\right\} - x.$$

Clearly the critical y is now decreasing with x in this region. Putting all this together, the set of locations using the river now zigzags in and out, as illustrated in Figure 8.

INSERT FIGURE 8

2.4 Lock by-pass (multiple lock by-pass)

It is straightforward, though somewhat cumbersome, to write down the full problem with multiple by-pass. It may be that the cost-minimizing shipment solution involves by-passing several locks with trucks. We next provide a sufficient condition for this to be so. Indeed, suppose that a shipper at location y_{j+1} prefers to by-pass the lock at y_j in the type of simple comparison undertaken in the previous sub-section. Suppose too that a shipper at location $y \in (y_{j+1}, y_{j+2})$ prefers to by-pass the lock at y_{j+1} in the same type of simple comparison. This implies that a shipper at location y prefers to by-pass both locks. Indeed, the first condition is

$$(t-b)(y_{j+1} - y_j) < C_j$$

[12] For simplicity, we are assuming here that the interval over which some shippers do NOT bypass the lock is not empty. The analysis is readily adapted to cover the converse case. Notice that if the interval is not empty, then necessarily at least two locks would be by-passed by some shippers. This type of behavior does correspond, in the context of the Mississippi, to those shippers from the northern reaches who ship down below the locks (around St. Louis) before using barges. Some sufficient conditions are derived below.

while the second one is

$$(t-b)(y-y_{j+1}) < C_{j+1}.$$

Summing these inequalities yields

$$(t-b)(y-y_j) < C_j + C_{j+1},$$

which is the condition for a shipper at y to prefer to truck past the next two downstream locks rather than ship by barge from y. This type of reasoning easily extends to multiple by-pass. However, the complete shipping pattern may be quite complex. For example, if there is a series of locks in quite close proximity, all shippers around these locks may by-pass them directly. However, shippers further upstream may use barge transport rather than incurring high trucking costs to reach the series of locks. The situation on the Mississippi would appear to be that a substantial amount of output is only put onto barges below the lowest lock (Lock 27), which is in St. Louis. By doing so, shippers avoid several locks upstream.

3 Congestion

To illustrate the equilibrium construction, we first consider the case of a single lock. We then consider multiple locks. Suppose too, for further simplicity and to clarify the insights, that lock by-pass is not an option, so that truck transport is constrained to be in the E-W plane. The objective here is to describe how congestion can be treated within the current framework. Let the cost of passing the lock be a non-decreasing function of the amount of traffic flowing through the lock from up-river. This could be a constant up to the "free-flow capacity" of the lock, and thenceforth an increasing function. It is easier though to first assume that the time cost of traversing the lock is simply (linearly) proportional to the amount of traffic it handles. This assumption aids in envisaging the equilibrium conditions that must be satisfied.

3.1 Single lock case

The objective here is to the endogenous equilibrium value of C, the lock cost, is determined. For a given value of C, the amount of shipping traffic through the lock is

$$D = 2\int_{y_1}^{l} \hat{x}(y) f(y)\, dy.$$

Here $f(y)$ is the density of shipments emanating from each square mile of land at distance y (assuming here that the density is independent of x and other parameters, for now), the "2" comes from symmetry, and recall l is the length of the river (which coincides with the whole territory). Furthermore, $\hat{x}(y)$ is the inverse of $\hat{y}(x)$, as above, and is given as

$$\hat{x}(y) = \frac{(r-b)y - C}{(t-r)}.$$

For illustration, let $f(y) = 1$.[13] The amount of output traversing the lock is then equal to the demand from points between y_1 and l:

$$D(C) = \left[\frac{(r-b)\,y^2 - 2Cy}{(t-r)}\right]_{y_1}^{l}.$$

Let the cost of traversing the lock be a constant, c_0, plus γ times the amount of shipping, so the equilibrium value of the traversing cost (including congestion) is $C^* = c_0 + \gamma D(C^*)$.[14] Thus, the equilibrium cost solves

$$C^*(t-r) = c_0(t-r) + \gamma\left[(r-b)\left[l^2 - y_1^2\right] - 2C^*\left[l - y_1\right]\right].$$

This reduces to

$$C^* = \frac{c_0(t-r) + \gamma\left[(r-b)\left[l^2 - y_1^2\right]\right]}{(t - r + 2\gamma\left[l - y_1\right])}.$$

Of course, shipment patterns below y_1 are unaffected by those upstream when there is a single lock. The value of C^* just determined is to be reinserted into the expression for $\hat{y}(x)$ in order to generate the catchment area for barge shipping.

Analogous techniques apply for more complex congestion functions (and also for more locks). First, find the amount of traffic passing through a lock as a function of the cost of traversing it. Then find the cost as a function of the amount of traffic. The equilibrium solution is the "fixed point" to this process. The first relation is a decreasing one: there is less traffic the higher the cost of passing a lock. The second one is an increasing one: costs increase with the amount of traffic. Therefore, there is a unique solution to the equilibrium congestion cost level. We next show how this process works when there are several locks.

3.2 Multiple locks and congestion

The problem has an interesting structure when there are multiple locks prone to congestion. The traffic from above the highest lock faces a congestion cost from all the locks downstream. Traffic from between the two locks furthest upstream faces congestion costs from just the $n-1$ locks downstream; and traffic from just above the first lock simply faces congestion from the first lock. However, the level of congestion at the first lock depends on the total traffic from all points further upriver, while the level at the last lock (lock n) depends just on

[13] The unit density of agricultural production is purely for reducing notational clutter. Any constant density makes no real difference at this point: with non-constant density the formula would need to be amended, but the crucial property (for equilibrium existence) that demand be continuous and decreasing in cost will be preserved.

[14] Under current practices, tows do not pay directly for lock services. Most, however, pay a fuel tax that goes into a trust fund from which some lock expenses are paid. Not all locks are financed through the fund, and tow vessels that do not lock through locks that receive trust fund financing are exempted. Nonetheless, most locks are financed in part through the fund, and most tow companies do pay the tax. There are currently no use (access) fees, and there is no congestion pricing.

the traffic from above it (which depends on all lock congestion costs, etc.). We can write these equations out as follows. On the demand side,

$$D_n = D_n\left(\sum_{i=1}^{n} C_i\right)$$

is the traffic from North of the nth lock, which depends (in a decreasing manner) on the total costs from passing through all locks. Define then D_{n-1} as the demand emanating from between locks $n-1$ and n, so

$$D_n = D_{n-1}\left(\sum_{i=1}^{n-1} C_i\right),$$

right down to the traffic between the first and second locks, which we write as

$$D_1 = D_1\left(C_1\right).$$

The endogenous lock costs, C_1 through C_n, are determined in an analogous fashion by all the traffic passing through them. Hence, we write

$$\begin{aligned}
C_1 &= C_1\left(\sum_{i=1}^{n} D_i\right),\\
C_2 &= C_2\left(\sum_{i=2}^{n} D_i\right)\\
&\cdots\\
C_{n-1} &= C_{n-1}\left(D_{n-1}+D_n\right)\\
C_n &= C_n\left(D_n\right)
\end{aligned}$$

which are increasing functions. The solution to these equations jointly determines lock costs and demands at each level.

3.2.1 Existence of an equilibrium solution

The existence of a solution follows from Brouwer's Fixed Point Theorem. The theorem says that any continuous mapping from a convex and compact set into itself has a fixed point. We assume that the D's and C's are continuous functions, that the D's are finite even when costs are zero, and that the C's are finite (and so bounded above) even when the D's are at their maximum possible values (the values associated to zero lock costs). We now show that the set of D's determined by the system of equations has a fixed point. First, any set of demands (D's) determines a set of costs (C's). These in turn determine a new set of D's, and the mapping from original D's to new D's is continuous by the continuity of the D's and the C's. Furthermore, the D's can never exceed the D's that are elicited from zero values for the D's. Thus the equation system has a fixed point, meaning that there is a solution for these quantities: at these quantities demanded, the cost functions determine equilibrium costs which in turn generate the quantities demanded. Since the quantities are determined, then the costs are also determined. Thus, we can be sure there exists a solution to these equations.

3.2.2 Uniqueness of the equilibrium solution

The uniqueness argument may be expressed as follows. The proof is by contradiction. Suppose there existed another solution, and suppose that the associated demand for lock n, D_n, were (weakly) lower than under the first solution. Then the congestion cost at the furthest upstream lock, lock n with cost C_n, would be (weakly) lower because fewer shippers pass through lock n. Thus, it must be that congestion costs are (weakly) higher at subsequent locks (than in the first solution) in order to render D_n lower. However, with these purported (weakly) higher costs downstream, then D_{n-1} must also be (weakly) lower. Consequently, C_{n-1} must be (weakly) higher, meaning that it must be the (weakly) higher congestion at the locks downstream of $n-1$ that are responsible for the (weak) demand reduction. But then D_{n-2} must be (weakly) lower, etc., until we reach the conclusion that D_1 must be (weakly) lower, and so C_1 must be (weakly) lower too. But this means that all congestion costs are (weakly) lower than at the first equilibrium, and so that the demand for lock n, D_n, must be (weakly) higher than under the first solution.

The only way these statements can be reconciled is if all quantities are exactly the same, otherwise we arrive at a contradiction. That is, no quantity may be strictly lower than under the first solution. Clearly, the analogous argument applies to show that no quantity may be strictly higher than under the first solution, and so the solution is necessarily unique.

Existence and uniqueness of equilibria are important properties. They mean that when a solution is sought, the problem guarantees one and only one is present.

4 Barge shipping rates

We assumed in the template model that per unit shipping costs themselves are independent of quantities shipped. However, at least in the short-run, when the number of barges is fixed, prices of shipping by barge reflects this capacity constraint. The shipping price is endogenously determined, as is the catchment area for barge shipping. The techniques for finding the equilibrium rates are rather similar to those for dealing with congestion at locks, and are given in detail below. There are though key economic differences between congestion and barge rates although higher levels of both are a reaction to higher market demand for river transport. It should be recognized that congestion at a lock only raises costs (with no corresponding direct benefits) and so is a pure deadweight loss, while capacity constraints in the barge industry raise shipping prices and so accrue as rents to barge owners. The higher prices do, though, alter the composition of barge traffic.

Barge prices (shipping prices) may be determined in the above conceptual framework by looking at equilibrium in the barge market. Suppose the supply of barges is fixed (short-run), and so we need to determine the demand.[15] We proceed as follows. In the preceding model, interpret now the transport rate b as the time spent times the barge rate per day: so let $b = \tau\tilde{b}$ where τ is the time spent and $\tilde{b}$ is the barge rate.[16] Similarly, decompose the lock cost into time components and direct monetary access charges: so we may write

$$C_j = c_j + \tilde{b}T_j, \quad j = 1, ..., n,$$

where c_j is the money charge for passing the lock, and T_j is the time taken.[17] Then the equilibrium (rental) rate for barge time, $\tilde{b}$, is determined as the price that equates demand and supply for barge services.

In particular, we now show that the equilibrium exists and is unique.[18] Any barge rate, $\tilde{b}$, determines a corresponding unique set of demands in each inter-lock interval, and so determines aggregate demand for barge services. Consider now the effects of decreasing $\tilde{b}$. Suppose the demand D_n were to fall. Then C_n would also fall. But this would mean that the sum of costs, $\sum_{i=1}^{n-1} C_i$ would have to be higher for D_n to fall. Hence D_{n-1} must fall. But this, in conjunction with the lower D_n, implies C_{n-1} must fall, and so this could only happen if $\sum_{i=1}^{n-2} C_i$ rose. The logical chain continues until we arrive at the conclusion that all costs must have fallen, which is inconsistent with the initial premise of a lower D_n. Thus, what must happen is that D_n must rise. A priori, C_n could rise or fall: there is more traffic through the lock but at a lower cost per unit time. However, if C_n rose, then $\sum_{i=1}^{n-1} C_i$ would have to have fallen for D_n to rise.

In summary (and briefly sketching the broader concept here), the equilibrium to this model specifies modal choice by origin-destination-commodity, congestion costs (both monetary and in time) at locks, and barge rates. The equilibrium may be viewed as a fixed point: at the equilibrium barge rate, shippers choose modes taking into account the full costs (in terms of time and money congestion), and the demand for barges equals the supply.We now argue that the

volume of shipping through every single lock must rise when the price of barge transportation falls. We just showed that the volume of shipping through the uppermost lock must rise. Suppose then that the volume of shipping through

[15] Since the argument that follows determines that the demand for barge services is a decreasing function of the price per unit time of barge services, then the argument applies equally well when the supply of barge services is a general non-decreasing function of the price of barge services.

[16] This assumes that there is no peak-load problem, and shipping is priced at the same level over the whole shipping season.

[17] We may interpret all times as including the trip up the river to pick up the cargo and then the trip down. This ignores the possibility of back-hauling different commodities.

[18] We again suppose that there is no lock by-pass. Otherwise, the demand equations no longer depend in a simple manner on the sum of costs at all locks lower down the river.

lock $n-1$ fell. Then, since D_n necessarily rose, it must be that D_{n-1} fell so much that the total fell. Then C_{n-1} would fall too: so it would have to be the case that $\sum_{i=1}^{n-2} C_i$ would have to have risen to explain the purported fall in D_{n-1}. This then implies that D_{n-2} must fall. Hence the total volume of shipping through lock $n-2$ must fall (less coming down from lock $n-2$ and then less added). But this implies a lower C_{n-2} and so it would have to be the case that $\sum_{i=1}^{n-3} C_i$ had risen to explain the purportedly lower D_{n-2}. This argument proceeds to the logical conclusion that all costs must have fallen; but then it cannot be true that the initial volume through lock $n-1$ (i.e., $D_n + D_{n-1}$) can have fallen. Hence that volume must have risen. This argument then applies for all subsequent locks to show that the volume at each lock must rise. Hence total demand for barges must necessarily rise. The aggregate demand curve slopes down, and there can be only one intersection with the supply curve. This in turn induces a unique allocation of demand and congestion costs.

5 Some comparative static properties

There are a number of comparative static results that are of interest. These relate to demand drivers and to supply-side differences across the modes. Congestion is influenced by each of these (any change in an exogenous variable may have an affect on volumes in the model and, hence, congestion levels). We focus here on yields (a demand driver), rates of rail and truck, and lock performance.

In the model, total transportation demand is given, and the model determines modal splits. Transportation demand, however, can change through increased yields (and, ignored here, an increase in the region of analysis). If yields increase, only barge rates and quantities are affected. Naturally, barge rates will rise as will quantities from a rightward shift in demand. However, the quantities shipped by barge will increase by a factor less than that of the total quantity produced. That is, as yields increase, the catchment areas for barge will be no larger than before the increase. The reason is that both rail and truck rates are fixed. This means that relative to rail and truck, barge rates will increase due to increased traffic on the waterway which increases congestion and, hence, barge rates. Apart from increases in yields there are no other forms through which the demand-side of the model can affect either rates or volumes.

The supply-side has a number of different sources of comparative statics. Changes in either truck or rail rates can affect the catchment areas as well as the demand for barge. For example, suppose that the marginal rail rate rose. Ceteris paribus, this increases the catchment areas for river transport and the demand for barge. However, the catchment areas will adjust to the new congestion costs at the various locks, and new barge prices will result. Suppose, for illustration, that the supply of barges is fixed. This means that the total amount of time spent for all shipments is fixed. The adjustment in the pattern of lock use, and the change in the equilibrium areas, is quite interesting.

In particular, there will be a relative rise in the price of using barge time as compared to other costs of locks.

If truck rates change the effects include a further source of ambiguity. In particular, there is both a substitution effect from truck to truck-barge (with the truck component of the latter being small), but there is also a substitution from truck-barge to rail. The first effect raises demand for barge transportation, but the latter one diminishes it

Of tantamount interest to the present study, a slow down in lock performance can be viewed as a rise in the congestion cost function at that point. This will affect all shipments higher upriver (that pass through the lock and are subject to higher costs). It will also end up affecting costs at locks downstream because of induced traffic changes due to lower congestion from reduced traffic upstream. Finally, if a lock breaks, it must be by passed. This will lead to changes in the whole system through equilibrium adjustment.

6 Conclusions

The model laid out in this paper provides an equilibrium model for the waterway market. Congestion bottlenecks and barge prices are endogenous to the model. We start with explicit micro-economic foundations for the demand for barge transportation. The model can readily be calibrated and used to provide values for costs and benefits accruing from changes to the fundamental parameters of the model. This foundation also allows for various possible extensions of merit in a more detailed picture of equilibrium modeling of river shipping. The current model can also be used to indicate the likely effects of different scenarios. For example, a change in yields per acre (a bumper harvest in corn, for example) will raise demand (given existing congestion and barge costs). This will then put pressure on both the locks through additional congestion, and on the price of barge traffic. The model of this paper, includes through its explicit construction such general equilibrium effects, and therefore, can address the consequences for other river transport, congestion, etc., of such changes in the parameter values. It can also be directly used to calibrate the costs and benefits from improvements to the efficiency of the lock system. For example, suppose that the congestion costs were reduced at one (or several) locks in the system. Then more traffic would use the system, and this would cause more congestion at other locks. However, some of this pressure would also build up as increased demand for barge services, which would raise the equilibrium price of barges. This would raise the rents accruing to barge owners, at least in the short term (before more barges could get allocated to the river). Shippers would benefit from overall lower costs in shipping. The model can provide estimates of the net benefits of such improvements in infrastructure. The model can also be used to look at transportation improvements in concert with other changes, such as congestion pricing of certain locks in the system.

Several extensions are likely to be important. First both the rail rate and the truck rate are exogenous. An important extension is to look at endogenous rail pricing given the oligopolistic nature of the industry. Second, there is frequently market power at the level of the shippers. Indeed, much grain comes through grain elevators that buy up grain form farmers and then ship it. These need to be modeled explicitly (at least where the grain industry is an important shipper). More attention needs to be paid to the market structure in the barge industry and its vertical integration with shippers. The current model is also missing back-haul traffic opportunities - the possibility of hauling a different commodity back-upriver on an otherwise empty journey to pick up a subsequent cargo. It is readily straightforward to extend the model to multiple destination points, and to endogenize the price of the final output.

Other factors that are important to shippers' choice of mode include the reliability of the mode, and the time it takes to ship.[19] These were mentioned briefly in the introduction, and simple expedient methods were described for thinking about how to address these factors within the current framework. However, a complete treatment would require a fuller description of these issues in the heart of the model

7 Appendix: Calculation of illustrative transport rates and farm yields

It is useful to have reasonable "ball-park" estimates of transport rates to help guide the intuition as to the size of various effects in the model.

7.1 Transportation Rates

The Grain Transportation Report (GTR) provides several figures that can be converted to per bushel per mile rates for alternative modes. The statistics for 9/16/2004 can be found at
http://www.ams.usda.gov/tmdtsb/grain/2004/09_16_04.pdf).

7.1.1 Truck rate per bushel per mile: =.00278

GTR provides rates per mile (based on an 80lb gross weight) for different regions of the country and shipment distances. We used a shipment distance of 25 miles. The cost per mile (North Central region) for corn is: 2.68 per mile (this is a truckload per mile: we took a truckload to be 27 tons). We then converted to a per bushel basis (corn weighs about 56 per bushel). Total paid is 2.68*25=67. Total bushels=27*2000/56=964. Total bushel-miles=964*25=24107. Total cost per bushel mile=.00278.

7.1.2 Rail Rate per bushel per mile: = .0005633

GTR provides rates per bushel. We used a rate from Council Bluffs, IA, to Baton Rouge, LA. The rate per bushel is .61. To convert to the rate per bushel

[19] For example, there are several reasons why diamonds are not sent by barges!

per mile, we used the Mapquest distance (1083 miles). The result is: .0005633 per bushel mile. Notice that the truck rate is almost 5 times higher. However, the truck rate given is for a small shipment (27 tons) taken a short distance (25 miles). This is big on its own account. Rail is a unit train taken a long distance (and it should therefore be expected to be small). A commonly used number is the rate per ton-mile. The rate per ton is 21.75. The rate per ton-mile is 21.75/1083=.0201 or about 2 cents per ton-mile. This is in the right neighborhood for this type of movement. The conversion from rate/ton-mile is 56/2000*rate/ton-mile.

7.1.3 Barge Rate per bushel per mile= .00035

GTR provides a rate index for the "Mid-mississippi" which covers IA. This index is 222 which means the rate is 2.22 times the base tariff rate, which is $5.32 per ton for shipments to the New Orleans area. There are 2000/56 bushels per ton (about 35.7 bushels per ton). This translates into 5.32*56/2000=.33069 per bushel. The distance traveled is about 945 miles.[20] Thus, the cost per bushel mile is about .00035 cents per bushel mile. The corresponding rail number for a similar movement is about .0005633 which is larger by a factor of 1.609424.[21]

7.2 Yields Per Square Mile

We aggregated output over space in the paper. It is useful to have reasonable values for yields per acre. We have data on a set of elevators that ship by barge from a data set taken from USDA sources. Corresponding yields are given per acre in the county. The mean values indicate yields of about 138 bushels per acre. There are 640 acres per square mile. Thus, if bushels were uniformly distributed across acres, there would be about 88320 bushels per square mile.

7.3 Price Spreads

Another indication of transportation costs can be found from the spot prices at various locations.

The bid prices available for corn in the GTR areas follows.

Origins:

[20] We used mapquest distances here. The convention is to use river miles for barge transport, and short-line rail distances for railroads. Of course, the shipper does not care, and our model does not have "routes" in them. We used mapquest distances to have something comparable across the modes. The distance difference can be marked though. For example, the meandering of the Mississippi gives a river-mile distance of about 1425 miles or so.

[21] Long haul truck moves may also be less expensive than is indicated above. The data source only gives rates for distances up to 200 miles. However, at 200 miles the rate is 1.75 per mile (for 27 tons). This means that the rate for truck is about .065 per ton-mile, or about .00182 per bushel mile. Rates per unit-distance fall at a non-linear rate with distance. The figures given here are intended to be reasonable approximations to real world data, and bear out our assumption that rates for trucks are higher than for rail, and that rail rates are higher than for barges. The treatment in the text allows for fixed costs to loading onto the various modes.

IA 2.02 per bushel
IL 2.04
NE 2.27

Terminal Markets:
New Orleans 2.57
Toledo 2.21

We can compare these spreads with the costs of shipping indicated earlier in this Appendix. The New Orleans to IA spread is about .55 cents per bushel (this is the maximum willingness to pay for transportation). Consider a shipper located about 25 miles off the river, shipping to Davenport, IA. The truck-barge rate is .0695 per bushel by truck, plus .3307 per bushel by barge, for a total of .4002 per bushel The corresponding rail rate is .61. This shipper would therefore choose truck-barge. It would make a profit of .55-.4002 per bushel shipped. They would not ship by rail, and if truck-barge was not an option they would not ship at all.

References

[1] Locklin, D. Philip (1972) *The Economics of Transportation.* Irwin (publisher).

Figure 1.—The Transportation Network with a Single Representative Source

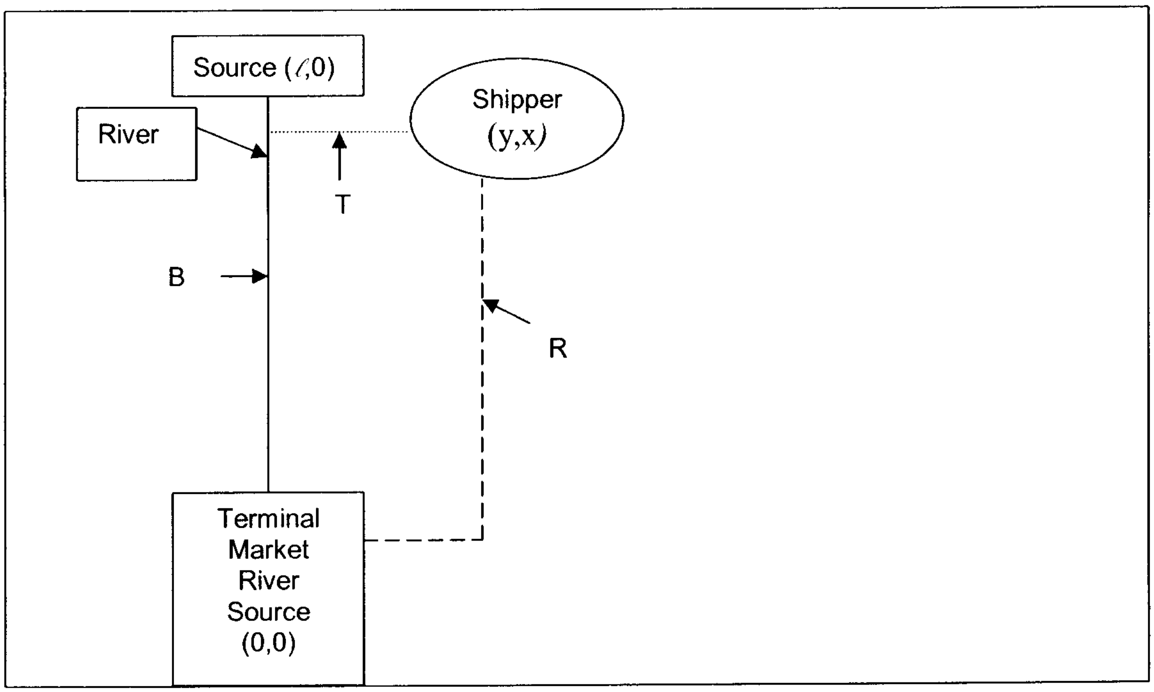


Figure 2.—Modal Catchment Areas.

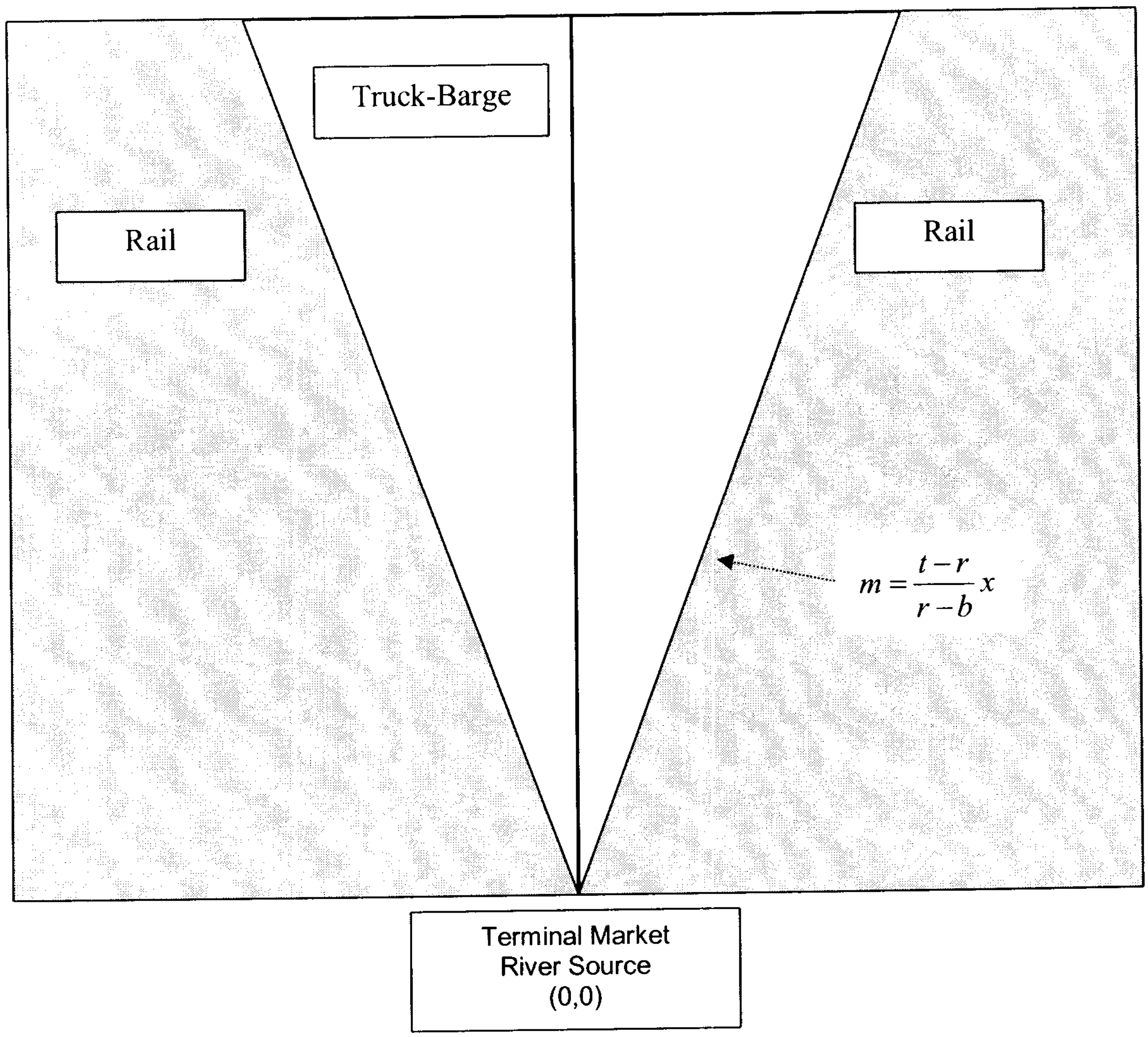
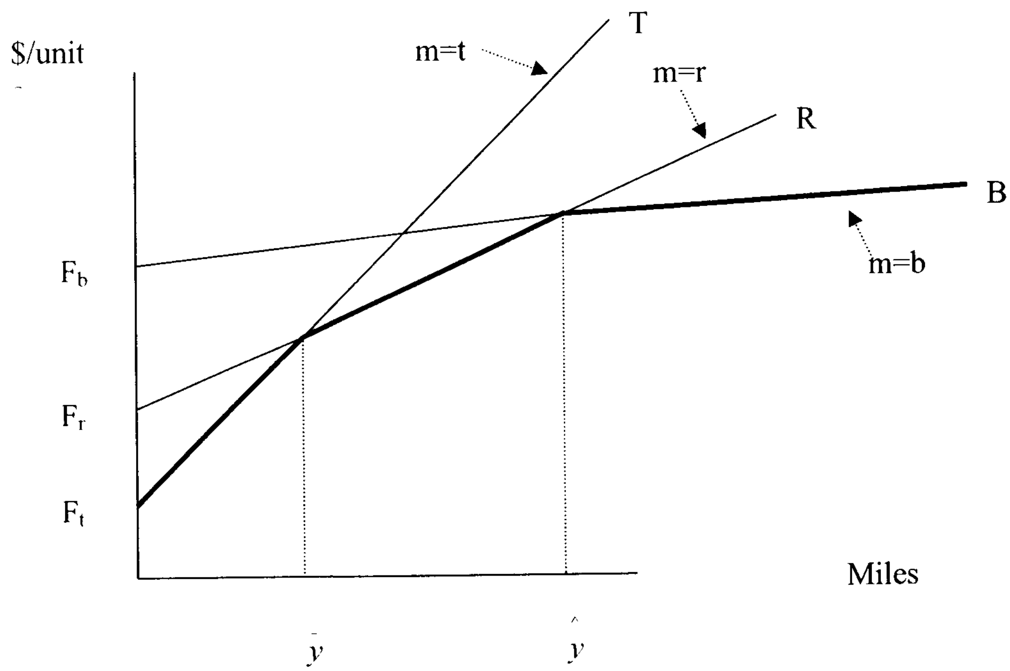


Figure 3. Fixed Costs and Shipment Costs

Panel a: Rail movements are not dominated.



Panel b: Rail movements are dominated along the riverside.

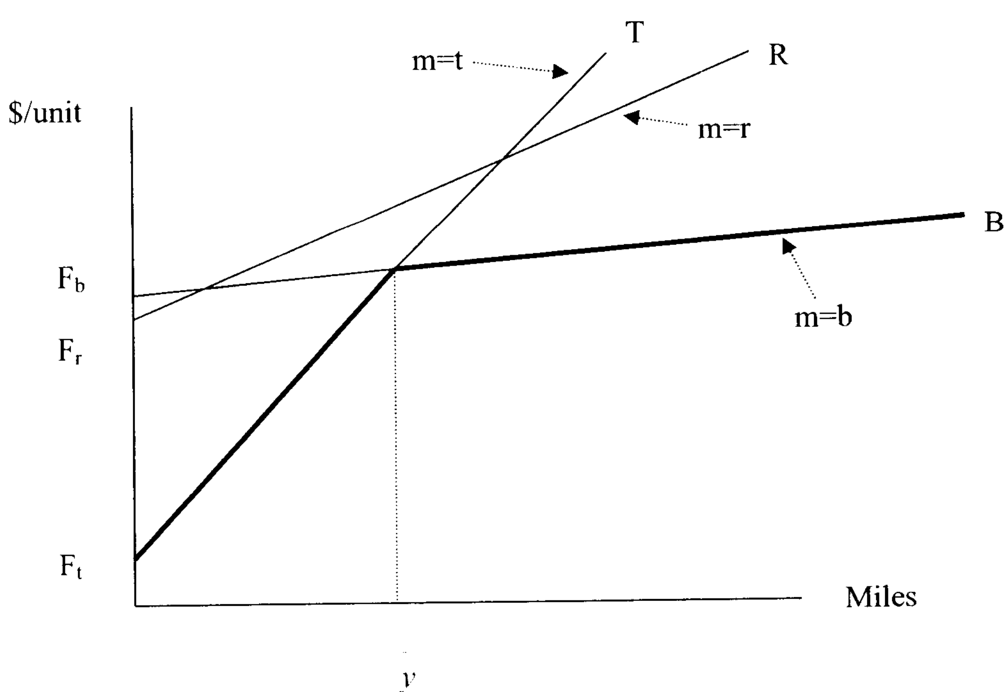


Figure 4.—Per Unit Costs and Per Unit Distance Tapers (Rail Movements not Dominated)

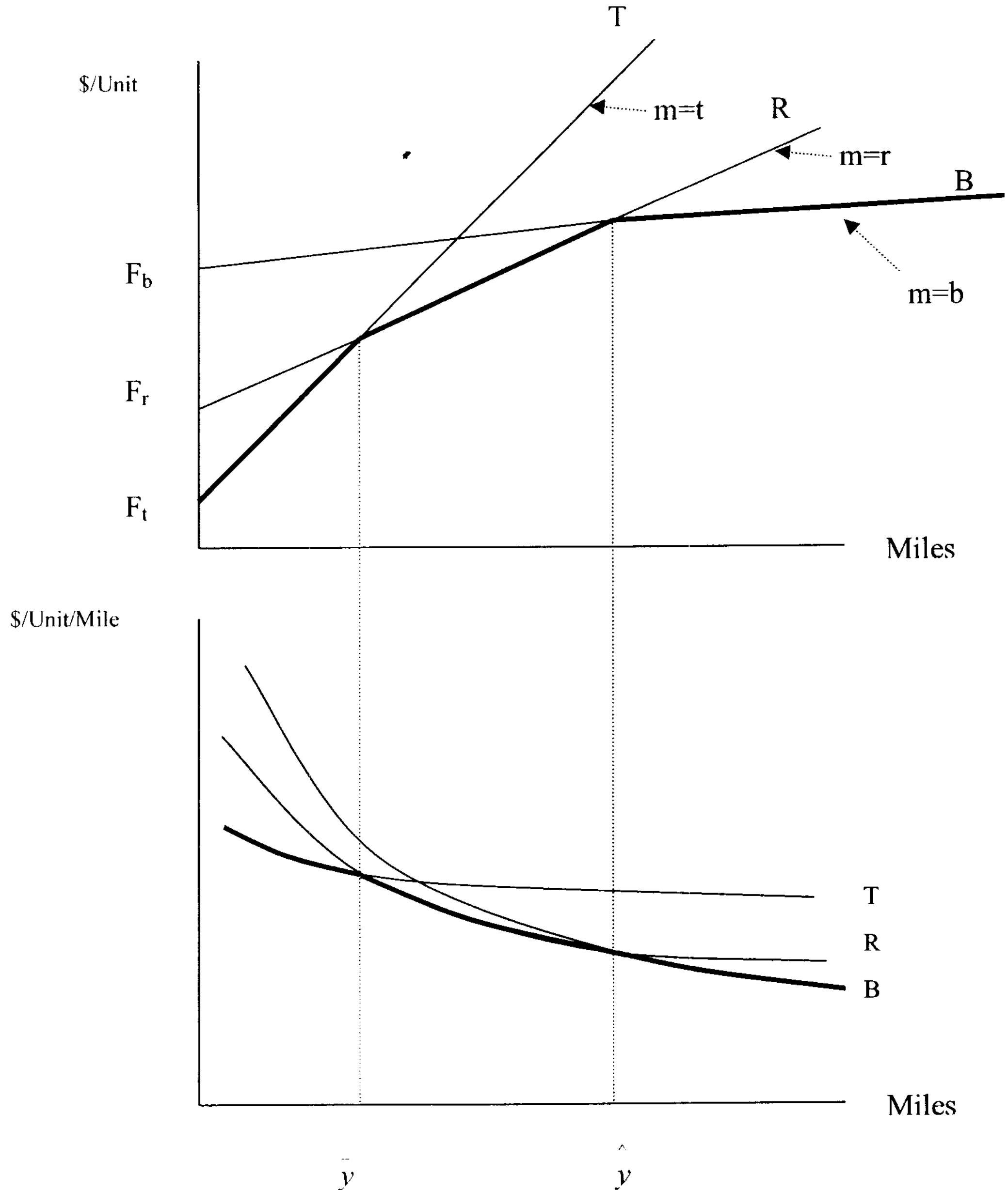


Figure 5.—Fixed Costs and Modal Catchment Areas (Rail not dominated)

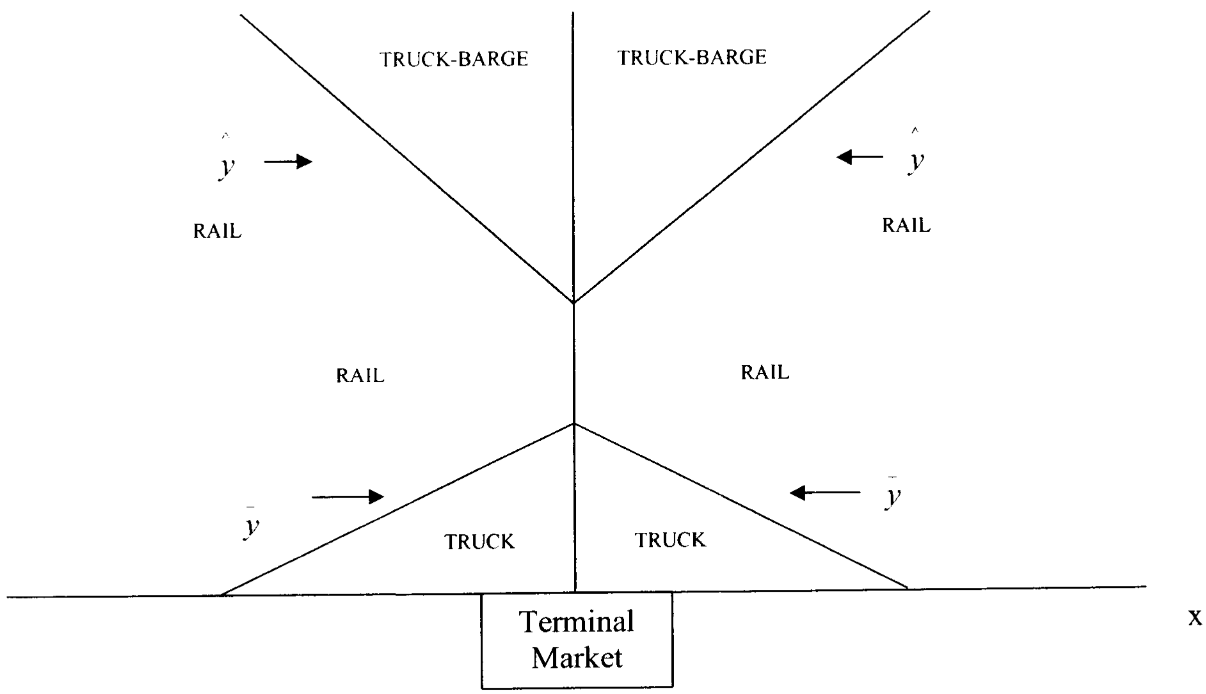


Figure 6. Fixed costs and Mode Catchment Areas (Rail Dominated)

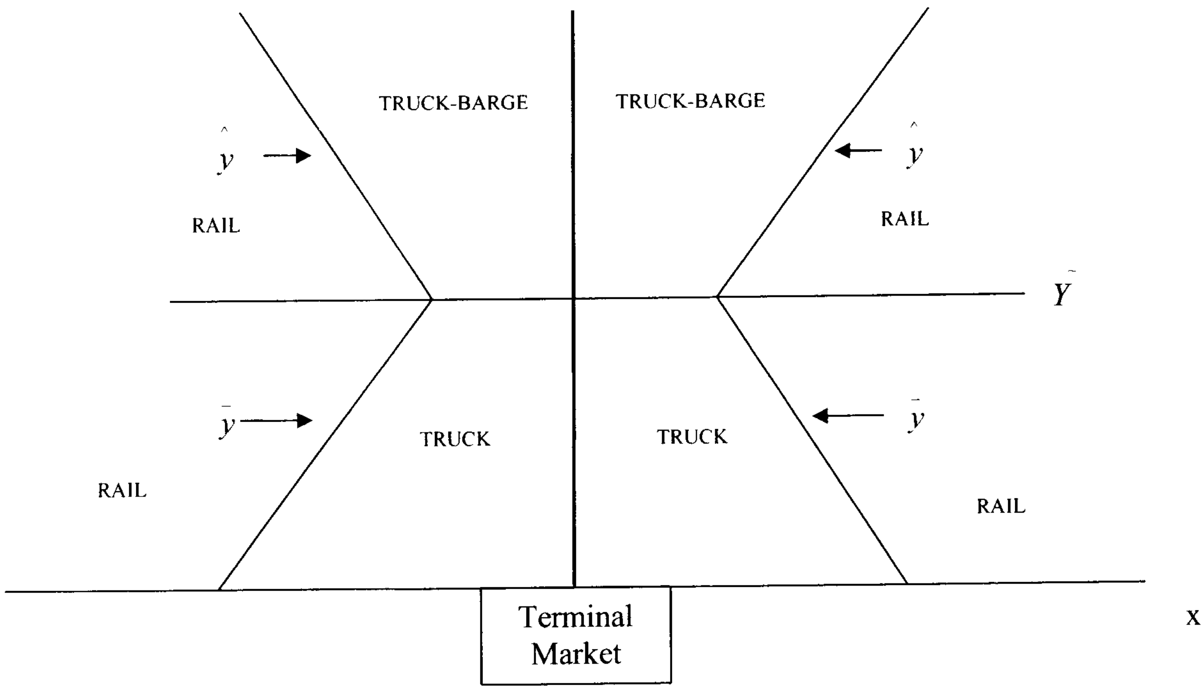


Figure 7. Fixed Costs and Locks.

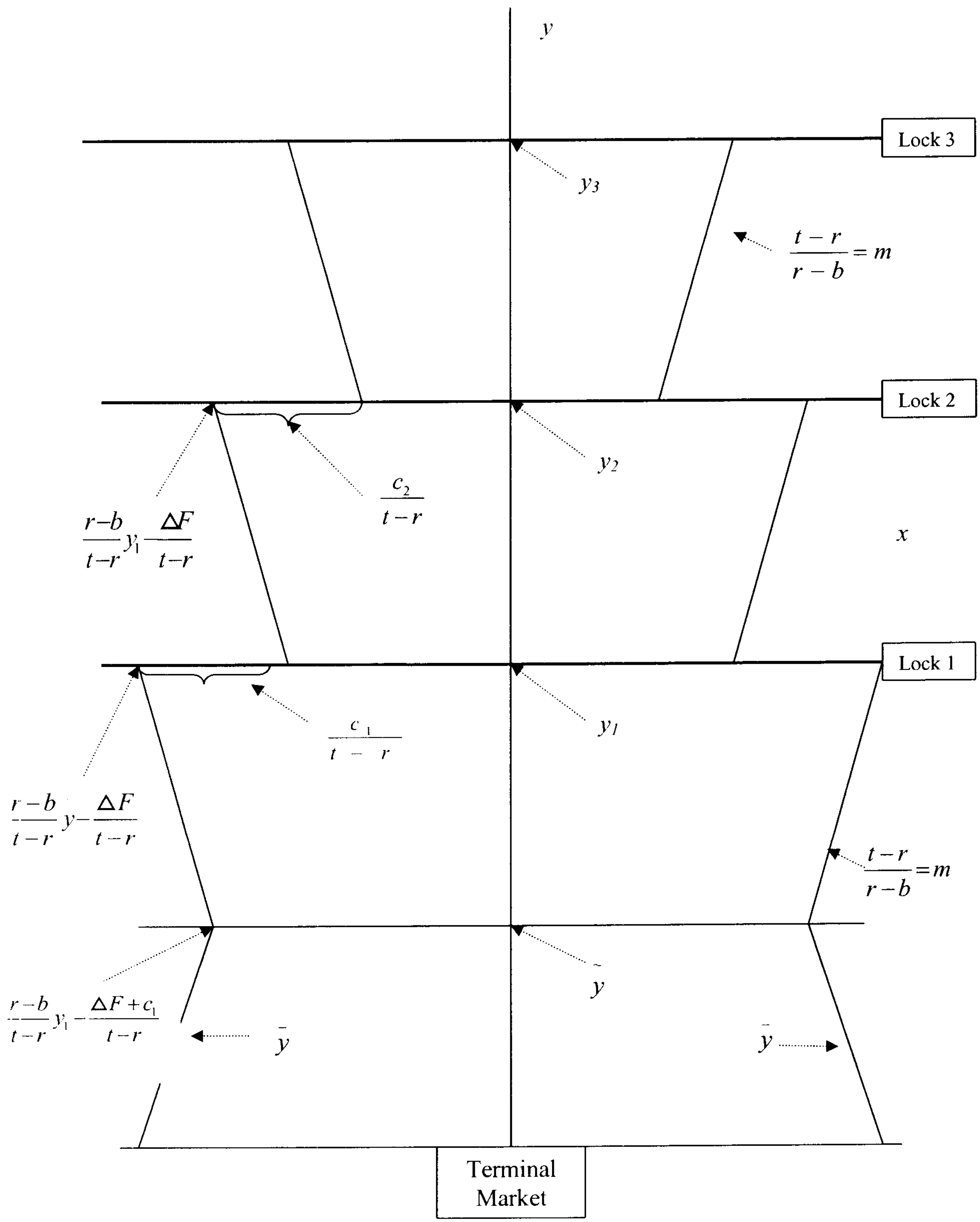
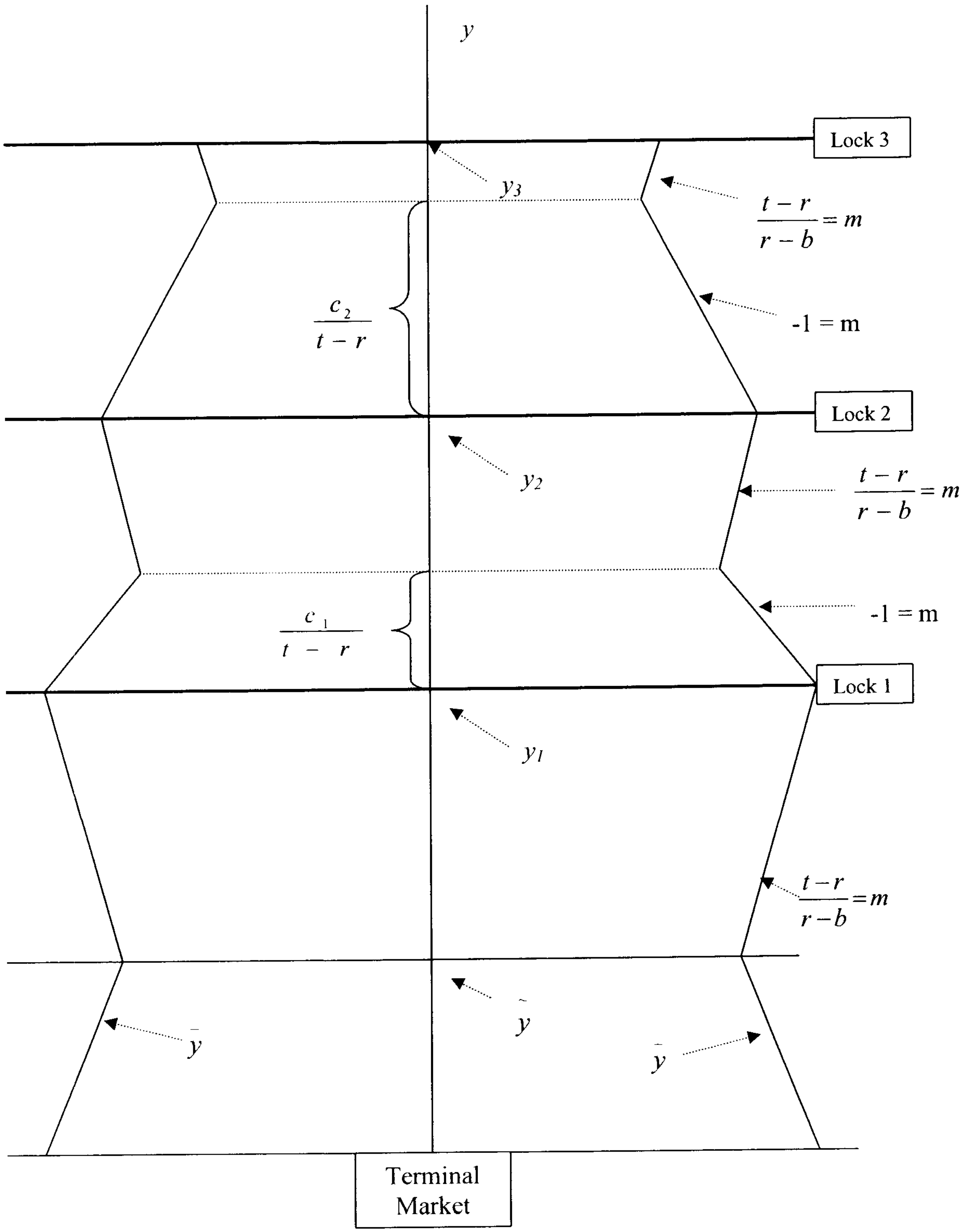


Figure 8. Fixed Costs and Simple Lock By-Pass



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