GREATER LOWER BOUNDS FOR ODD PERFECT NUMBERS

By

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To Mary Abney
With love
Beauregard Stubblefield
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INTRODUCTION. A number N is perfect if the sum of its factors is twice that number. The problem of finding perfect numbers dates back to ancient times. The foremost pioneer in this area of study was Euclid [1] who proved in his ninth book the following theorem:

If P and 2**P - 1 are prime numbers, then the number

(2**(P-1))(2**P - 1)

is an even perfect number. This condition is both necessary and sufficient.

Euler has discovered eight values of P satisfying the above conditions. They are: P = 2, 3, 5, 7, 13, 17, 19, and 31.

As of today there are twenty-four known perfect numbers. Each of these is even. Neither has anyone found an odd perfect number nor has anyone determined whether or not one exists.

Kanold [2] has proved that if an odd perfect number exists then that number is greater than 10**20. Tuckerman [5] showed that there is no odd perfect number less than 10**36. More recently the writer [4] provided a newer method by which, when a natural number M is given, one either (a) can find an odd perfect number less than M or (b) can determine that an odd perfect number less than M does not exist. Using this method the writer showed that any odd perfect number, if one exists, is necessarily greater than 10**50. Using this same method, now it is shown that there is no odd perfect number less than 10**100. In this case, we let M be 10**100 and prove both Theorem A and Theorem B below.

Theorem A: If an odd perfect number has a prime factor less than 8, then that number is greater than 10**100.

Theorem B: If an odd perfect number has no prime factor less than 8, then that number is greater than 10**100.

Proof of Theorem A: This theorem is proved by exhausting all of the possible cases for the primes 7, 3, and 5 in that order. Several propositions are used to eliminate these cases. Most of these propositions have proofs which are based on the following considerations:

(1) If N = P1**E1 x P2**E2 x ... x Pn**En where the Pi's are distinct primes and the sum of the factors of N is S, then the sum of the factors of 1 is given by

$$S \,/\, \prod_{i=1}^{n} \sigma(Pi^{**}Ei) \qquad (1)$$

(2) If N is a perfect number, (i.e., the sum of its factors is 2 x N) then

$$1/2 = \prod_{i=1}^{n} Pi^{**}Ei \,/\, \sigma(Pi^{**}Ei) \qquad (2)$$

This latter condition is sufficient as well as necessary.

The following propositions are used repeatedly in the detailed proofs of lemmas and theorems which are included in this paper. In the propositions, let P**E||N stand for "E is the largest power of P in N." In the proofs of the lemmas and theorems, we let P**E stand for P**E||N.

PROPOSITION 1: Suppose

(1) that N is an odd perfect number,

(2) for some prime P and natural number E, P**E||N,

(3) σ(P**E) has a factor Q that is not a perfect square, and

(4) there is no prime factor F of Q and natural number X, where X(Mod 4) = 1, such that F**X||N.

Then, for some prime factor R of Q it is true that R x Q divides N. In particular, if in addition Q has no prime factor less than or equal to its cube root, then Q**2 divides N.

PROPOSITION 2: If N is an odd perfect number, exactly one of the applicable numbers σ(Pi**Ei) is even and that one contains 2 as a factor exactly once. (Note: This can happen only for primes P(Mod 4) = 1.)

The proof follows from (2) in that each Pi is odd and the expression on the right reduces to 1/2.

PROPOSITION 3: No prime P can appear as a factor of an odd perfect number exactly E times where E(Mod 4) = 3. The proof follows from the fact that if P is a prime and E(Mod 4) = 3, then the integer σ(P**E) is a multiple of 4.

PROPOSITION 4: If N is an odd perfect number written as a product of powers of distinct primes, then no prime P(Mod 4) = 3 can appear to an odd power.

PROPOSITION 5: For an arbitrary integer M, let O be an odd perfect number less than M, P be any prime number and let E be a positive integer such that P**(2xE) is greater than 2 x M. Then P**E cannot be a factor of O.

PROPOSITION 6: If

$$\prod_{i=1}^{n} (Pi^{**}Ei)/\ \sigma(Pi^{**}Ei) < 1/2$$

for n distinct primes Pi, then

$$\prod_{i=1}^{n} (Pi^{**}Ei)$$

cannot be a factor of an odd perfect number.

PROPOSITION 7: The number 3x5x7 cannot be a factor of an odd perfect number.

For the indicated primes P the numbers listed in the next proposition cannot be factors of any perfect number.

PROPOSITION 8: (A) The number 3x5x11xP where 12 < P < 72.

(B) The number 3x7x11xP where 16 < P < 24.

(C) The number 3x7x11x29xP where 30 < P < 138.

(D) The number 3x7x11x31xP where 36 < P < 104.

(E) The number 3x5**2x11xP where for any P.

(F) The number 3x5**2x13xP where 16 < P < 44.

(G) The number 3x5**2x17xP where 18 < P < 128.

(H) The number 3x5**2x19xP where 22 < P < 90.

(I) The number 3x5**2x23xP where 28 < P < 48.

PROPOSITION 9: Let N be an odd perfect number.

If:

(A) P(Mod 10) = 1 and P**Z||N or

(B) P(Mod 10) = -1 and P**X||N,

where Z(Mod 5) = 4 and X(Mod 4) = 1, then

3 x 7 cannot divide N.

PROPOSITION 10: No odd perfect number N which is relatively prime to 105 can be less than 10**100. (This is actually Theorem B.)

LEMMA O : If O is an odd perfect number, P is a prime, E is a natural number such that P**E||O, then there exist m distinct primes P1, P2, ..., Pm and natural numbers E1, E2, ...,Em such that for each i,

$$Pi^{**}Ei \| O, \quad P \mid \sigma(Pi^{**}Ei)$$

and

$$P^{**}E \mid \prod_{i=1}^{m} \sigma(Pi^{**}Ei). \qquad \text{Note: } E > m.$$

Proof: The proof follows immediately from (1).

THEOREM O: Let the Pi's and the Ei's be given as provided in Lemma O for the given P**E, then

$$\prod_{i=1}^{m} (Pi^{**}Ei) \quad > \quad P^{**}E/2^{**}E.$$

Proof: For each i, from

$$Pi^{**}Ei \quad > \quad \sigma Pi^{**}(Ei-1)$$

it follows that

$$Pi^{**}Ei \quad > \quad 1/2 \ \sigma(Pi^{**}Ei).$$

Hence:

$$\prod_{i=1}^{m}(Pi^{**}Ei) > \prod_{i=1}^{m}(1/2\ \sigma(Pi^{**}Ei) \geq P^{**}E/2^{**}E.$$

DEFINITION 1: Let N be an odd perfect number and M be a natural number. Two primes P and Q are said to be minimal with respect to N and M provided

(A) P**T||N and Q| σ(P**T), for some natural number T.

(B) Other than the prime R = P or possibly for primes R(Mod Q) = Q-1, and except possibly for W(Mod 4) = 3, there is no ordered pair (R,W) where R is a prime and W is a natural number for which σ(R**W) is divisible by Q and at the same time, R**W < M.

PROPOSITION 11: Let N be an odd perfect number and M be a natural number. If two primes P and Q are minimal with respect to N and M, and if at the same time there exists a prime R(other than R=P), and a natural number W such that R**W||N and Q| σ(R**W), then either R**W > M or W(Mod 4) = 1.

THE MAIN THEOREM

The main result of this paper follows immediately from the following two theorems.

THEOREM A: IF AN ODD PERFECT NUMBER EXISTS AND CONTAINS A PRIME FACTOR LESS THAN 8, THEN THAT NUMBER IS GREATER THAN 10**100.

THEOREM B: IF AN ODD PERFECT NUMBER EXISTS AND CONTAINS NO PRIME FACTOR LESS THAN 8, THEN THAT NUMBER IS GREATER THAN 10**100.

The proof of Theorem A may be obtained on microfiche at the Environmental Research Laboratories, National Oceanic and Atmospheric Administration, Boulder, Colorado, 80302. The proof of Theorem B is appended.

Now, we can state our main Theorem.

The Main Theorem. There is no odd perfect number less than 10**100.

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PROOF FOR THEOREM A

Except where otherwise indicated, for each case or sub-case of each block or sub-block, lemma or theorem, used in the proof of Theorem A, we let

X(Mod 4) = 1, Y(Mod 3) = 2, Z(Mod 5) = 4, U(Mod 7) = 6,

V(Mod 11) = 10 and W(Mod 13) = 12.

In the proof of a theorem, in many cases, we represent a large factor of S(P**E) by the letter Q. Although in most cases this factor is determined to be composite, usually it is stated that Q has no prime factor less than a certain number, say 10,000,000. Sometimes this is indicated by

QHNPFLT 10,000,000.

The following sub-block is used in Block 151.

(A)	3041**X 2x3x3x13x13	Prop 1	(D)	3041**6 29x337x9871x811651xP	N Exceeds M
(B)	3041**Y P	N Exceeds M	(E)	3041**A	N Exceeds M
(C)	3041**Z 5x11x6481xP	N Exceeds M			

Block 151 This block, labeled Block 151, is used in the proof of several of the following related theorems. Except where indicated to the contrary, each subcase is eliminated by N exceeding M. It is used only when it is assumed that the prime 5 divides N or whenever sufficiently small primes divide N.

Note S(411448913311**4) = 5x101x191x251x251x80651x415861xQ

(1)	151**Y 3x7x1093				Proposition 6
(2)	151**Z 5xP	104670301**X 2x11x109xP	43649**Y 7x37x277xP	26557**Y 3x283xP	Proposition 7
(3)	151**Z 5xP	104670301**X 2x11x109xP	43649**Y 7x37x277xP	26557**Z 11x101x541xP	N Exceeds M
(4)	151**Z 5xP	104670301**X 2x11x109xP	43649**Y 7x37x277xP	26557**A	N Exceeds M
(5)	151**Z 5xP	104670301**X 2x11x109xP	43649**4 61x61x2371xP	411448913311**2 3x7x109x439x2311x22039xP	Proposition 7
(6)	151**Z 5xP	104670301**X 2x11x109xP	43649**4 61x61x2371xP	411448913311**4 (See above)	N Exceeds M
(7)	151**Z 5xP	104670301**X 2x11x109xP	43649**4 61x61x2371xP	411448913311**B	Proposition 5
(8)	151**Z 5xP	104670301**X 2x11x109xP	43649**6 43xQ	Q is composite and QHNPFLT 10,000,000	N Exceeds M

```
( 9)     151**Z    104670301**X      43649**10                           BK     18742307
          5xP       2x11x109xP       11x23x89x18742307xQ QHNPFLT 23,199,947
(10)     151**Z    104670301**X      43649**C                            BK        43649
          5xP       2x11x109xP
(11)     151**Z    104670301**Y                    2357622555649**X     BK    339226267
          5xP        3x1549xP                    2x5x5x139x339226267
(12)     151**Z    104670301**Y                    2357622555649**2     Proposition  7
          5xP        3x1549xP                       3x7x31x20959xQ
(13)     151**Z    104670301**Y                    2357622555649**4     Proposition  6
          5xP        3x1549xP QHNPFLT 3 million          11xQ
(14)     151**Z    104670301**Y                    2357622555649**D     Proposition  5
          5xP        3x1549xP
(15)     151**Z    104670301**4     174405421565054777963837 41**1      Proposition  1
          5xP      5x1376461xP               2x53x17417xP
(16)     151**Z    104670301**4     17440542156505477796383741**2       Proposition  7
          5xP      5x1376461xP         3x7x109x763x124819xQ
(17)     151**Z    104670301**4     17440542156505477796383741**DD      Proposition  5
          5xP      5x1376461xP
(18)     151**Z    104670301**6                                         BK          631
          5xP         7x631xQ       Q  is composite and QHNPFLT 10,000,000
(19)     151**Z    104670301**E                                         Proposition  5
          5xP
(20)     151**6   7960598843**Y     63371133947133537493**1             Proposition 11
         1499xP          P              2x113x1493x1736347xP
(21)     151**6   7960598843**Y     63371133947133537493**2             BK       694951
         1499xP          P              3x103x1009x694951xQ  QHNPFLT 8,799,997
(22)     151**6   7960598843**Y     63371133947133537493**EE            Proposition  5
         1499xP          P
(23)     151**6   7960598843**4                                         N  Exceeds   M
         1499xP 31x41x11897351xQ  QHNPFLT   11,897,351
(24)     151**6   7960598843**6                                         N  Exceeds   M
         1499xP  7x197x337x743x3389x117209xQ  QHNPFLT 4,200,001
(25)     151**6   7960598843**F                                         Proposition  5
         1499xP
(26)     151**10                                                        N  Exceeds   M
23x14864609x18145704541823
(27)     151**12 141451988007859836896332l713**1                        Proposition  1
            P             2x7x29x31xQ    Q  is composite and QHNPFLT 48,499,999
(28)     151**12 1414519880078598368963321713**2                        BK           43
            P             3x43x2647xQ  QHNPFLT  600001
(29)     151**12 1414519880078598368963321713**G                        Proposition  5
            P
(30)     151**16                                                        N  Exceeds   M
        148768021xQ    Q has no prime factor less than 148,000,000
(31)     151**18                                                        Block     3041
         3041xP
(32)     151**22                                                        BK         9109
       599x9109xQ Q  is composite and has no prime factor less than 213 million
(33)     151**H                                                         N  Exceeds   M
```

Block 79 This block is used as a sub-block in Block 23. The details are given elsewhere in this paper.

79**Z	39449441**X	PR11	79**6 1289**Y 13093**Y 57146581**Y	PR 6
79**Z	39449441**Y	PR 6	79**6 1289**Y 13093**Y 57146581**Z	N>M
79**Z	39449441**4	PR 7	79**6 1289**Y 13093**Y 57146581**C	N>M
79**Z	39449441**6	N>M	79**6 1289**Y 13093**4	N>M
79**Z	39449441**A	PR 5	79**6 1289**Y 13093**6	N>M
79**6	1289**X	PR 7	79**6 1289**Y 13093**D	N>M
79**6	1289**Y 10393**X 6547**Y	PR 6	79**10 1750258119644519**E	N>M
79**6	1289**Y 10393**X 6547**6	PR 6	79**12	PR 1
79**6	1289**Y 10393**X 6547**10	N>M	79**16	PR 1
79**6	1289**Y 10393**X 6547**12	N>M	79**18	N>M
79**6	1289**Y 10393**X 6547**B	PR 5	79**22	N>M
79**6	1289**Y 10393**Y 57146581**X	N>M	79**F	N>M

Block 23 This block of sub-cases is used in several lemmas and theorems. Except where indicated otherwise, each sub-case leads to the contradiction, N Exceeds M. Whenever this block is used, the assumption is that 3**4 divides N and at least one of the primes 11 and 13 divides N. An alternative is to have a collection of sufficiently small primes each to divide N.

Note S(75013**4) = 11 x 491 x 2851 x 1040881 x 1975511
S(480393499**2) = 3 x 19 x 43 x 181 x 463 x 1123547317
S(889453**4) = 11 x 1051 x 1301 x 4691 x P

23**Y	79**Y		PR 6	23**6	5336717**X	889453**Y	N>M
7x79	3x7x7x43			29xP	2x3x889453	3x37xP	
23**Y	79**R		Bk79	23**6	5336717**X	889453**Z	N>M
7x79				29xP	2x3x889453	(See above)	
23**Z	292561**X	7699**Y 24919**Y		23**6	5336717**X	889453**F	N>M
P	2x19x7699	3x13x61xP 3xP		29xP	2x3x889453		
23**Z	292561**X	7699**Y 24919**4		23**6	5336717**Y	70671349069**X	PR7
P	2x19x7699	3x13x61xP 11xP		29xP	13x31xP	2x5x7x7xP	
23**Z	292561**X	7699**Y 24919**A		23**6	5336717**Y	70671349069**L	N>M
P	2x19x7699	3x13x61xP		29xP	13x31xP		
23**Z	292561**X	7699**4	N> M	23**6	5336717**4		N>M
P	2x19x7699	1474675lxP		29xP	P		
23**Z	292561**X	7699**6	N> M	23**6	5336717**6	7**G	N>M
P	2x19x7699	379xP		29xP	7xQ	Q is composite	10m
23**Z	292561**X	7699**B	N> M	23**6	5336717**H		PR5
P	2x19x7699			29xP			
23**Z	292561**Y	168820969**X	PR 6	23**10	393723040460З**2		N>M
P	3x13x13xP	2x5x16882097		11xP	463x3511xP		
23**Z	292561**Y	168820969**Y	PR 6	23**10	3937230404603**I		PR5
P	3x13x13xP	3x7x79x97xP		11xP			
23**Z	292561**Y	168820969**4	N> M	23**12	480393499**2		N>M
P	3x13x13xP	11x661x153071xQ(com)10m		47691619xP	(See above)		
23**Z	292561**Y	168820969**C	N> M	23**12	480393499**J		N>M
P	3x13x13xP			47691619xP			

23**Z	292561**4	7151**Y	PR 7	23**16	62246266355102810647**2	N>M
7x79	5x11x7151xP	7x19x19x37x547		103xP	3x7x21751x526657xQ	9.6M
23**Z	292561**4	7151**4	PR8E	23**16	62246266355102810647**K	PR5
P	5x11x7151xP	5x211x751xP		103xP		
23**Z	292561**4	7151**6	N> M	23**18	2129**1	N>M
7x79	5x11x7151xP	1051xP		2129xQ	2x3x5x71	
23**Z	292561**4	7151**D	N> M	23**18	2129**L	N>M
P	5x11x7151xP			2129xQ	Q is composite	216m
23**Z	292561**6		N> M	23**R		N>M
P	911x75013x180307x17551493x2899469274619					
23**Z	292561**E		PR 5			

In Block 23 it is given that S(7151**6) = 1051 x 1 2725026295 6073990307. To show that Q = 1 2725026295 6073990307 is a prime number, for each prime factor P of Q - 1 = 2 x 3 x 7 x 47 x 53 x 49843 x 24402341261, we find a prime Px which is relatively prime to Q such that both of the following are true.

Px**(Q-1)[Mod Q] is 1 and Px**[(Q-1)/P] [Mod Q] is not 1

(See Table I below)

P	Px	Px**(Q-1)[Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	3	1	1 2725026295 6073990306
3	5	1	7057563201 8757572883
7	3	1	1 0854797986 7438308051
47	3	1	6483710127 1726662474
53	3	1	9164338850 1012710895
49843	3	1	8258383564 5609679796
24402341261	3	1	7370846911 6711502041

TABLE I

Lemma 1.1 If 5 x 11 x 13 x 17351**2 x 21787**2 is a factor of an odd perfect number N which is less than M, then

(1) there is no Y such that Y(Mod 3) = 2 for which 1063**Y||N
(2) there is no Z such that Z(Mod 5) = 4 for which 1063**Z||N

In the proof of this Lemma, we consider the following fractions in which each denominator is the sum of the factors of the corresponding numerator.

(A) 1063**2 / 3x377011

(B) 1063**4 / 1291x989955251 989955251**2 / 43x823x877x31576445701 31576445701**1 / 2x103x503x304739

(C) 1063**4 / 1291x989955251 989955251**2 / 43x823x877x31576445701 31576445701**2 / 3xQ(composite)

Proof for (1)

First, we assume that the hypothesis of Lemma 1.1 is true and that for some Y such that Y(Mod 3) = 2, 1063**Y||N. S(1063**2) which is equal to 3x377011 necessarily divides N contradicting Prop 8A. Therefore, the first assumption is false and (1) is proved.

Proof for (2)

Next, we assume that our hypothesis is true and that for some Z such that Z(Mod 5) = 4, 1063**Z||N. S(1063**4) = 1291x989955251 divides N. By Prop. 4, if 989955251**W||N for positive W, W is even. By Prop. 5, there is no integer W greater than 5 for which 989955251**W||N. Hence, either (A) 989955251**2||N or (B) 989955251**4||N.

If (A), then S(989955251**2) = 43x823x877x31576445701 divides N. If for some positive integer X such that X(Mod 4) = 1, it follows that 31576445701**X||N, then 103x503x304739 divides N making N greater than M, contradicting our hypothesis. Prop. 8A makes 3156445701**2||N false and Prop. 3 shows that the expression 31576445701**3||N is false. If 31576445701**4||N, then N is greater than M, again contradicting our hypothesis. All of the possibilities for 3176445701 have been exhausted.

Finally, if 989955251**4||N, then it is true that the product of (A) 5x11x11x13x17351x17351x21787x21787x1063**4x989955251**4 and (B) the sum of the factors of 989955251**4 also divides N. It follows that N is greater than M under these conditions. Thus, we have a contradiction of our hypothesis. Hence, our second assumption is false and Lemma 1.1 is true.

Theorem 1 Suppose both that N is an odd perfect number less than M and that for some Y such that Y(Mod 3) = 2, 17351**Y||N. Then for no Z such that Z(Mod 5) = 4 does 31**Z||N.

Note S(31**4) = 5 x 11 x 17351 S(17351**2) = 13 x 1063 x 21787

Block 1409

1409**X 2x3x5xP		PR8E	1409**6 P	7830225998024225671**Y 3x1543xQ	PR8A
1409**Y 7xP	283813**X 2xP	PR11	1409**6 P	7830225998024225671**B	PR 5
1409**Y 7xP	283813**Y 3x433xP	PR 7	1409**10 11x23x331x419x617x34871xP		PR 1
1409**Y 7xP	283813**A	N> M	1409**C		N> M
1409**4 431x10151xP		N> M			

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

1063**Y 3xP	where Y is congruent to 2 (Mod 3)		Lemma 1.1
1063**Z 1291xP	where Z is congruent to 4 (Mod 5)		Lemma 1.1
1063**6 337x2423xP	1768581264143**2 19x19x139291xP	62204210394254443**2 3xQ	Proposition 8A
1063**6 337x2423xP	1768581264143**2 19x19x139291xP	62204210394254443**A	Proposition 5
1063**6 337x2423xP	1768581264143**B		N Exceeds M
1063**10 1409xQ	1409**E Q is composite and has no prime factor less than 100 million		Block 1409
1063**12 1951xP	1951**Y 3x79xP		Proposition 8A
1063**12 1951xP	1951**4 5x11xP		N Exceeds M
1063**12 1951xP	1951**6 43x113x197x12097xP	4765137847**Y 3xQ	Proposition 8A
1063**12 1951xP	1951**6 43x113x197x12097xP	4765137847**4 Q(Comp) QHNPFLT 22 million	N Exceeds M
1063**12 1951xP	1951**6 43x113x197x12097xP	4765137847**F	Proposition 5
1063**12 1951xP	1951**G		N Exceeds M
1063**H			N Exceeds M

Lemma 2.1 If N is an odd perfect number less than M, and if for some Z, Z(Mod 5) = 4, 31**Z||N and also 17351**6||N, then neither of the following can happen.

(A) 11**Y||N (B) 11**Z||N

Note S(31**4)= 5 x 11 x 17351 and S(17351**6) is composite and has no prime factor less than 1,000,000,000.

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

11**Y	19**Y			Proposition 7
7x19	3x127			
11**Y	19**Z			Block 151
7x19	151x911			
11**Y	19**6	70841**X		Proposition 8E
7x19	701xP	2x3xP		
11**Y	19**6	70841**Y	39103**Y	Proposition 7
7x19	701xP	128341xP	3x7561xP	
11**Y	19**6	70841**Y	39103**Z	N Exceeds M
7x19	701xP	128341xP	11x57344741xP	
11**Y	19**6	70841**Y	39103**6	N Exceeds M
7x19	701xP	128341xP	7x43x7547x13004671xP	
11**Y	19**6	70841**Y	39103**A	N Exceeds M
7x19	701xP	128341xP		
11**Y	19**6	70841**4	1163018639068051**2	Proposition 7
7x19	701xP	5x61x71xP	3xQ	
11**Y	19**6	70841**4	1163018639068051**B	Proposition 5
7x19	701xP	5x61x71xP		
11**Y	19**6	70841**6		N Exceeds M
7x19	701xP	7x29x6301xP		
11**Y	19**6	70841**C		N Exceeds M
7x19	701xP			
11**Y	19**10	62060021**X		Proposition 8E
7x19	104281xP	2x3x3x47x109x673		
11**Y	19**10	62060021**Y	17748600316039**2	Proposition 7
7x19	104281xP	7x31xP	3xQ	
11**Y	19**10	62060021**Y	17748600316039**D	Proposition 5
7x19	104281xP	7x31xP		
11**Y	19**10	62060021**4		N Exceeds M
7x19	104281xP	5x71x251xQ	Q is composite and QHNPFLT 10,000,000	
11**Y	19**10	62060021**E		N Exceeds M
7x19	104281xP			
11**Y	19**12	133338869**X		Proposition 7
7x19	599x29251xP	2x3x3x5x7x113x1873		

```
11**Y     19**12          133338869**2              29251**Y    Proposition  7
7x19    599x29251xP        56094673xP               3x193xP
11**Y     19**12          133338869**2              29251**4    N  Exceeds   M
7x19    599x29251xP        56094673xP                5x41xP
11**Y     19**12          133338869**2              29251**F    N  Exceeds   M
7x19    599x29251xP        56094673xP
11**Y     19**12          133338869**4                          N  Exceeds   M
7x19    599x29251xP          11xP
11**Y     19**12          133338869**G                          N  Exceeds   M
7x19    599x29251xP
11**Y     19**16     99995282631947**2                          N  Exceeds   M
7x19     3044803xP   67x87403x1311127x10050613x129574807
11**Y     19**16     99995282631947**H                          Proposition  5
7x19     3044803xP
11**Y     19**18     109912203092239643840221**1                Proposition  1
7x19        P                   2xQ(composite)   QHNPFLT 40,000,000
11**Y     19**18     109912203092239643840221**I                N  Exceeds   M
7x19        P
11**Y     19**22                                                N  Exceeds   M
7x19    277x2347xQ  Q  is composite          QHNPFLT 141 million
11**Y     19**28                                                N  Exceeds   M
7x19     59x233xQ  Q  is composite            QHNPFLT 100 million
11**Y     19**30  24327031889148383810359338159515l809701**1   Proposition  1
7x19        P       2x24229x32579x327689x886799x1085785lxP
11**Y     19**30  243270318891483838103593381595151809701**J   Proposition  5
7x19        P
11**Y     19**K                                                 N  Exceeds   M
7x19
11**Z   3221**L                                                 Block     3221
 5xP
```

Note:

S(99995282631947**2) = 67 x 87403 x Q where Q = 17 0748887314 7914650757.

To imply that Q is composite, it is sufficient to state the fact that

3**(Q-1) [Mod Q] = 8 5463011244 5861196967

which is not congruent to 1 modulo Q.

Lemma 2.2 If N is an odd perfect number less than M, and if for some Z, Z(Mod 5) = 4, 31**Z||N, then neither of the following can happen.

(A) 17351**6||N (B) 17531**10||N

Note S(31**4) = 5 x 11 x 17351

Block 15797

15797**X 2x3x2633	Proposition 8E	15797**6 (Comp)43x337xQ	N Exceeds M QHNPFLT 200 million
15797**Y 249561007	N Exceeds M	15797**10 (Comp)11x463xQ	N Exceeds M QHNPFLT 10,000,000
15797**Z 17311991xP	N Exceeds M	15797**A	Proposition 5

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(17351**6) has no prime factor less than 1,000,000,000

Possibilities And Reasons By Which They May Be Excluded

17351**6	11**Y	where Y(Mod 3) = 2		Lemma 2.1
17351**6	11**Z	where Z(Mod 5) = 4		Lemma 2.1
17351**6	11**6 43xP	45319**Y 3x127xP		Proposition 8A
17351**6	11**6 43xP	45319**4 151xP	27935336611728311**A	N Exceeds M
17351**6	11**6 43xP	45319**6 7x953x4327x7841x108866969xP		N Exceeds M
17351**6	11**6 43xP	45319**B		N Exceeds M
17351**6	11**10 15797xP	1806113**X 2x3x17xP	where X(Mod 4) = 1	Proposition 8E
17351**6	11**10 15797xP	1806113**Y 19xP	171686630257**X 2x11x13x7727xP	Proposition 1
17351**6	11**10 15797xP	1806113**Y 19xP	171686630257**C	N Exceeds M
17351**6	11**10 15797xP	1806113**4 4051xQ	4051**Y 3xP	Block 15797
17351**6	11**10 15797xP	1806113**4 4051xQ	4051**D Q is composite and QHNPFLT 11,100,000	Block 15797
17351**6	11**10 15797xP	1806113**E		N Exceeds M
17351**6	11**12 1093xP	3158528101**X 2xP	1579264051**2 3x19xP	Proposition 8E

17351**6	11**12	3158528101**X	1579264051**F	Proposition 1
	1093xP	2xP		
17351**6	11**12	3158528101**2		Proposition 6
	1093xP	3xP		
17351**6	11**12	3158528101**G		N Exceeds M
	1093xP			
17351**6	11**16	50544702849929377**1		Proposition 1
	P	2x23x4591xP		
17351**6	11**16	50544702849929377**H		N Exceeds M
	P			
17351**6	11**18	6115909044841454629**1		Proposition 1
	P	2x5x31x70911041xP		
17351**6	11**18	6115909044841454629**I		N Exceeds M
	P			
17351**6	11**22	829**X		Proposition 1
	829x28878847xP	2x5x83		
17351**6	11**22	829**Y	1087**2	Proposition 7
	829x28878847xP	3x211xP	3x7x199xP	
17351**6	11**22	829**Y	1087**J	N Exceeds M
	829x28878847xP	3x211xP		
17351**6	11**22	829**4		N Exceeds M
	829x28878847xP	11x461xP		
17351**6	11**22	829**6		N Exceeds M
	829x28878847xP	71x31149623xP		
17351**6	11**22	829**K		N Exceeds M
	829x28878847xP			
17351**6	11**28	523**Y		Proposition 8A
	523xP	3x13xP		
17351**6	11**28	523**4		N Exceeds M
	523xP	3491xP		
17351**6	11**28	523**L		N Exceeds M
	523xP			
17351**6	11**30	2428541**1		Proposition 8E
	2428541xQ	2x3x3x3x3x3x3x19x263		
17351**6	11**30	2428541**R		N Exceeds M
	2428541xQ	Q is composite and has no prime factor		less than 1Bil
17351**6	11**36			N Exceeds M
	2591x36855109x136151713xP			
17351**6	11**S			N Exceeds M
17351**10	11**T			Block 11
23xQ	Q is composite and	QHNPFLT 100 million		

S(11**30) = 2428541 x Q where Q = 79036 5182048606 4699099041. The condition "Q is composite" is implied by the fact that

5**(Q-1) [Mod Q] = 8153 2140900143 4360868085.

Lemma 2.3 If N is an odd perfect number less than M, then not both of the following can happen simultaneously.

(A) 31**6||N (B) 917087137**4||N

Note Q = S(917087137**4) is composite, is not a perfect square and has no prime factor less than 1,000,000,000. Hence, if Q divides N and if Q has no factor which appears to an odd power in the prime factorization of N, then there is some factor P(>1,000,000,000) of Q such that P x Q divides N also.

Proof It is shown easily that if N is an odd perfect number less than M and both 31**6||N and 917087137**4||N, then none of the following primes can be a factor of N.

19, 7, 3.

With this fact N must contain at least 14 factors and consequently, it must be greater than M. We have a contradiction.

Case (1) The prime 19 divides N.

Possibilities And Reasons By Which They May Be Excluded

19**Y	127**Y	5419**2	N Exceeds M
3x127	3x5419	3x31x313x1009	
19**Y	127**Y	5419**A	N Exceeds M
3x127	3x5419		
19**Y	127**4	262209281**1	Proposition 1
3x127	P	2x3x3137xP	
19**Y	127**4	262209281**B	N Exceeds M
3x127	P		
19**Y	127**6		N Exceeds M
3x127	7x43x86353xP		
19**Y	127**C		N Exceeds M
3x127			
19**Z	151**2		N Exceeds M
151x911	3x7x1093		
19**Z	151**4		N Exceeds M
151x911	5xP		
19**Z	151**6		N Exceeds M
151x911	1499xP		
19**Z	151**D		N Exceeds M
151x911			
19**6	70841**1		Proposition 1
701xP	2x3xP		
19**6	70841**2		N Exceeds M
701xP	128341xP		
19**6	70841**E		N Exceeds M
701xP			

19**10			N Exceeds M
104281xP			
19**12			N Exceeds M
599x29251xP			
19**16			N Exceeds M
3044803xP			
19**F			N Exceeds M

Case (2) The prime 7 divides N.

Possibilities And Reasons By Which They May Be Excluded

7**Y			Case 1
3x19			
7**4	2801**X		Proposition 11
2801	2x3xP		
7**Z	2801**2	4933**1	Proposition 1
2801	37x43xP	2xP	
7**Z	2801**2	4933**A	N Exceeds M
2801	37x43xP		
7**Z	2801**B		N Exceeds M
2801			
7**6	4733**1		Proposition 1
29xP	2x3x3x263		
7**6	4733**2		N Exceeds M
29xP	P		
7**6	4733**C		N Exceeds M
29xP			
7**10			N Exceeds M
1123xP			
7**12			N Exceeds M
P			
7**16			N Exceeds M
14009xP			
7**18			N Exceeds M
419xP			
7**22			N Exceeds M
47x3083xP			
7**D			N Exceeds M

Case (3) The prime 3 divides N.

Details for the proof in Case (3) are found elsewhere herein.

Lemma 2.4 Suppose N is an odd perfect number less than M, then neither 31**Z||N nor 31**6||N where Z(Mod 5) = 4.

```
   1451**Y                          PR8B 1451**6                  PR1
 7x7x19x31x73                            2381x52584967x74590391
   1451**4                          PR8E 1451**A                  N>M
    5x41xP
```

Block 93169

```
   93169**Y       163411**Y  2045761**Y  PR6   93169**Y    163411**E      PR5
  3x17707xP      3x19x229xP 3x7x13x43x277xP   3x17707xP
   93169**Y       163411**Y  2045761**4  PR6   93169**Z                Bk1451
  3x17707xP      3x19x229xP    5x11xQ        181x4662101xP
   93169**Y       163411**Y  2045761**D  N>M   93169**6                 N>M
  3x17707xP      3x19x229xP                     12377xP
   93169**Y       163411**Z              PR8E  93169**10                N>M
  3x17707xP      5x3011xQ                       1155lxQ(composite) QHNPFLT 10m
   93169**Y       163411**6              N>M   93169**F                 PR5
  3x17707xP        29x43xQ  Q is composite and  QHNPFLT 13,599,979
```

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

```
  31**Z      17351**Y                                         Theorem      1
5x11x17351  13x1063x21787
  31**Z      17351**4        1648012040336791**2              Proposition 8E
5x11x17351     5x11xP               3x163xQ
  31**Z      17351**4        1648012040336791**B              Proposition  5
5x11x17351     5x11xP
  31**Z      17351**6                                         Lemma      2.2
5x11x17351      Q   Q is composite and  QHNPFLT 1,000,000,000
  31**Z      17351**10                                        Lemma      2.2
5x11x17351     23xQ   QHNPFLT  100,000,000
  31**Z      17351**C                                         N  Exceeds   M
5x11x17351
  31**6     917087137**X     28729**Y                         Block    93169
    P       2x11x1451xP    3x2953x93169
  31**6     917087137**X     28729**4        7670501**2       N  Exceeds   M
    P       2x11x1451xP  2179391x7670501xP 19x47161xP
  31**6     917087137**X     28729**4        7670501**D       N  Exceeds   M
    P       2x11x1451xP  2179391x7670501xP
  31**6     917087137**X     28729**6                         N  Exceeds   M
    P       2x11x1451xP    7x71x197xP
  31**6     917087137**X     28729**10                        N  Exceeds   M
    P       2x11x1451xP      23xP
```

31**6	917087137**X	28729**E		Proposition 5
P	2x11x1451xP			
31**6	917087137**Y	38533987**Y	38047**Y	Proposition 6
P	3x43x4447x38047xP	3x7x19x241xP	3x7x13x5302609	
31**6	917087137**Y	38533987**Y	38047**4	Proposition 8B
P	3x43x4447x38047xP	3x7x19x241xP	11xQ	
31**6	917087137**Y	38533987**Y	38047**6	N Exceeds M
P	3x43x4447x38047xP	3x7x19x241xP	43x491x547xP	
31**6	917087137**Y	38533987**Y	38047**F	N Exceeds M
P	3x43x4447x38047xP	3x7x19x241xP		
31**6	917087137**Y	38533987**4		N Exceeds M
P	3x43x4447x38047xP	93151x347071xP		
31**6	917087137**Y	38533987**6		N Exceeds M
P	3x43x4447x38047xP	11243x2402107xQ(comp) QHNPFLT 15,000,000		
31**6	917087137**Y	38533987**G		Proposition 5
P	3x43x4447x38047xP			
31**6	917087137**4			Lemma 2.3
	Q	Q is composite and QHNPFLT 1,000,000,000		
31**6	917087137**H			Proposition 5

For the tenth case of Lemma 2.4 it is assumed that 28729**6||N. This implies that S(28729**6) = 7 x 71 x 197 x 57 4269244945 2968574979 divides N. Showing that Q = 57 4269244945 2968574979 is prime, for each prime factor P of

Q - 1 = 2 x 3**3 x 7 x 47 x 241 x 7109 x 7177 x 26287991

we find a prime Px which is relatively prime to Q such that both of the following are true.

Px**(Q-1) [Mod Q] is 1 and Px**[(Q-1)/P] [Mod Q] is not 1

(See Table II below)

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	3	1	57 4269244945 2968574979
3	3	1	5 3905888536 5124446960
7	3	1	68121160 3988304481
47	3	1	18 4772962671 6346081454
241	3	1	2031848477 3439737581
7109	3	1	16 2546610740 8802747656
7177	3	1	1 1512839171 0459804237
26287991	3	1	54 1893495241 1447549849

TABLE II

Theorem 2 If 31 divides an odd perfect number N that is less than M, then for some Y, Y(Mod 3) = 2, 31**Y||N.

Block 42407. The block labeled, Block 42407, is used in Theorem 2. Except where indicated otherwise, each sub-case is eliminated because N exceeds M.

42407**Y	33331**Y	1230331**Y	N>M	42407**Y	33331**F	PR5
42407**Y	33331**Y	1230331**4	PR9	42407**4	3234152111453204401**1	N>M
42407**Y	33331**Y	1230331**6	N>M	42407**4	3234152111453204401**2	N>M
42407**Y	33331**Y	1230331**A	PR5	42407**4	3234152111453204401**C	PR5
42407**Y	33331**4		N>M	42407**6	617**D	N>M
42407**Y	33331**6		N>M	42407**10		N>M
42407**Y	33331**10		N>M	42407**E		PR5

Block 642646908601

642646908601**X	321323454301**2	N>M	642646908601**2	N>M
2xP	3x433x317071xP		3x13xP	
642646908601**X	321323454301**4	N>M	642646908601**4	N>M
2xP	5x11x11x11x265711x969011xQ		5x11x37021x717091xQ	
642646908601**X	321323454301**A	PR5	642646908601**C	PR5
2xP				

Block 1509997

1509997**X	107857**Y			PR1	1509997**X	107857**C	PR5
2x7xP	3x13x463x631x1021				2x7xP		
1509997**X	107857**4	61**Y	97**2	N>M	1509997**Y	389559619**2	N>M
2x7xP	31x61xP	3x13x97	3x3169		3x1951xP	3x373x33967xP	
1509997**X	107857**4	61**Y	97**A	N>M	1509997**Y	389559619**D	N>M
2x7xP	31x61xP	3x13x97			3x1951xP		
1509997**X	107857**4	61**4		N>M	1509997**4		N>M
2x7xP	31x61xP	5x131x21491			11x17049871xP		
1509997**X	107857**4	61**6		N>M	1509997**6		N>M
2x7xP	31x61xP	P			29xQ	QHNPFLT 16,000,000	
1509997**X	107857**4	61**B		N>M	1509997**E		N>M
2x7xP	31x61xP						
1509997**X	107857**6			N>M			
2x7xP	7xQ Q is composite and			QHNPFLT 10,000,000			

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(31**10) = 23 x 397 x 617 x 150332843 S(31**12) = 42407 x 2426789 x P

Possibilities And Reasons By Which They May Be Excluded

31**Z			Lemma 2.4
31**6			Lemma 2.4
31**10	150332843**Y		See Block
23x397x617xP	139x253x642646908601		
31**10	150332843**4	617**1	Block 23
23x397x617xP	Q	2x3x103	
31**10	150332843**4	617**C	See Block 617
23x397x617xP	Q	Q is composite and QHNPFLT 10,000,000	
31**10	150332843**6		N Exceeds M
23x397x617xP	113xQ	Q is composite and QHNPFLT 10,000,000	
31**10	150332843**A		Proposition 5
23x397x617xP			
31**12	7908811**2		Block 42407
	3x43x73x349x571x33331		
31**12	7908811**4		N Exceeds M
	5x11x11xP		
31**12	7908811**6		N Exceeds M
	7x463xQ	Q is composite and QHNPFLT 10,000,000	
31**12	7908811**B		Proposition 5
31**16	751670559138758105956097**1		N Exceeds M
P	2x3x11xP		
31**16	751670559138758105956097**D		N Exceeds M
P			
31**18	887706663326107621 69**1	807006057569188747**2	Proposition 7
571x14251xP	2x5x11xP	3x7xQ	
31**18	88770666332610762169**1	807006057569188747**E	Proposition 5
571x14251xP	2x5x11xP		
31**18	88770666332610762169**F		N Exceeds M
571x14251xP			
31**22			Block 1509997
1509997x61562537x929592461824389			
31**28	10789**1	349**2	Proposition 8F
349x10789xQ	2x5x13x83	3x19x2143	
31**28	10789**1	349**H	N Exceeds M
349x10789xQ	2x5x13x83		
31**28	10789**2		N Exceeds M
349x10789xQ	3x7x5543491		
31**28	10789**G		N Exceeds M
349x10789xQ	Q is composite and QHNPFLT 263,124,541		
31**30	56897247102410786528702143430197715853482448 1**1		N Exceeds M
P	2x71713xQ		
31**30	568972471024107865287021434301977158534824481**I		Proposition 5
P			
31**L			Proposition 5

Lemma 3.1 If N is an odd perfect number less than M and if 3 divides N, then it is not true that 331**Z||N.

Note S(331**4) = 5 x 37861 x 63601

Block 37861

37861**Y	10753**Y	Bk151	37861**Z	99244068137581**2	P8F
3x37x1201xP	3x151x397x643		5x41x101xP	3x13xQ	
37861**Y	10753**Z	N>M	37861**Z	99244068137581**B	PR5
3x37x1201xP	P		5x41x101xP		
37861**Y	10753**6	PR7	37861**6		PR6
3x37x1201xP	7x29x71xQ		29x43xP		
37861**Y	10753**10	PR6	37861**10		PR6
3x37x1201xP	23xQ		23x376729xP		
37861**Y	10753**A	N>M	37861**C		PR6
3x37x1201xP					

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(63601**4) = 5 x 41 x 271 x 1381 x 4231 x 50408381

Possibilities And Reasons By Which They May Be Excluded

63601**X				Proposition 7
2x7x7x11x59				
63601**Y	612067**Y	26010319**Y	527135461**X	Proposition 7
3x2203xP	3x4801xP	3x43x9949xP	2x7x37652533	
63601**Y	612067**Y	26010319**Y	527135461**2	N Exceeds M
3x2203xP	3x4801xP	3x43x9949xP	3x41233xP	
63601**Y	612067**Y	26010319**Y	527135461**4	N Exceeds M
3x2203xP	3x4801xP	3x43x9949xP	5x491x2537551x3563501xP	
63601**Y	612067**Y	26010319**Y	527135461**A	Proposition 5
3x2203xP	3x4801xP	3x43x9949xP		
63601**Y	612067**Y	26010319**4		N Exceeds M
3x2203xP	3x4801xP	11x271x5806121xP		
63601**Y	612067**Y	26010319**6		N Exceeds M
3x2203xP	3x4801xP	29x71xP		
63601**Y	612067**Y	26010319**B		Proposition 5
3x2203xP	3x4801xP			
63601**Y	612067**4	37861**X		Proposition 8E
3x2203xP	11x151x421x4861x6701xP	2x11x1721		
63601**Y	612067**4	37861**C		N Exceeds M
3x2203xP	11x151x421x4861x6701xP			
63601**Y	612067**6			Proposition 7
3x2203xP	7xP			
63601**Y	612067**D			Proposition 5
3x2203xP				

```
63601**4        50408381**X      37861**E                    Block     37861
                  2x3xP
63601**4        50408381**Y                                  Proposition  7
                  7x31xQ
63601**4        50408381**4                                  N  Exceeds   M
                 5x241xP
63601**4        50408381**6                                  N  Exceeds   M
                     Q  Q is composite and  QHNPFLT 16,000,000
63601**4        50408381**P                                  Proposition  5

63601**6          37861**X                                   N  Exceeds   M
  43xP            2x11x1721
63601**6          37861**G                                   N  Exceeds   M
  43xP
63601**10                                                    N  Exceeds   M
23x2927xP
63601**R                                                     Proposition  5
```

In the following lemma S(331**10) = 11 x 23 x 89 x 7 0320745112 1161180833. To show that Q = 7 0320745112 1161180833 is a prime number, for each prime factor P of Q-1 we find a prime Px which is relatively prime to Q such that both of the following are true.

Px**(Q-1) [Mod Q] is 1 and Px[(Q-1)/P] [Mod Q] is not 1

(See Table III below)

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	5	1	7 0320745112 1161180832
3	7	1	5541221865 0601489820
11	3	1	5 7780455063 6451775560
71	3	1	2 2741809490 4723824845
97	3	1	4 8591758280 4934717341
1373	3	1	4 4083584758 9682025719
21881	3	1	8138717322 6267932939
1072829	3	1	3 0949054808 0933938960

TABLE III

Lemma 3.2 If N is an odd perfect number less than M and both 3 and 331 divide N, then neither 331**6||N nor 331**10||N.

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(331**6) = 2180921 x 604842179 S(331**10) = 11 x 23 x 89 x P

Possibilities And Reasons By Which They May Be Eliminated

331**6	604842197**X	100807033**2	3387352667690041**2	N Exceeds M
2180921xP	2x3xP	3xP	3x67x283xP	
331**6	604842197**X	100807033**2	3387352667690041**A	Proposition 5
2180921xP	2x3xP	3xP		
331**6	604842197**X	100807033**4		N Exceeds M
2180921xP	2x3xP	229181xQ Q is composite and	QHNPFLT 10,000,000	
331**6	604842197**X	100807033**B		N Exceeds M
2180921xP	2x3xP			
331**6	604842197**2	365834083876629007**2		N Exceeds M
2180921xP	P	3x13x19x23563x1510819xP		
331**6	604842197**2	365834083876629007**E		Proposition 5
2180921xP	P			
331**6	604842197**4	2180921**1		N Exceeds M
2180921xP	11x2291041xQ	2x3x103xP		
331**6	604842197**4	2180921**2		N Exceeds M
2180921xP	11x2291041xQ	3217xP		
331**6	604842197**4	2180921**C		N Exceeds M
2180921xP	11x2291041xQ	Q is composite and QHNPFLT 10,000,000		
331**6	604842197**D			Proposition 5
2180921xP				
331**10	703207451121161180833**1		595938517899289136З**2	Proposition 8B
11x23x89xP	2x59xP		7xQ	
331**10	703207451121161180833**1		5959385178992891363**F	Proposition 5
11x23x89xP	2x59xP			
331**10	703207451121161180833**2			Proposition 8B
11x23x89xP	3x7xQ			
331**10	703207451121161180833**G			Proposition 5
11x23x89xP				

Theorem 3 If N is an odd perfect number less than M and both 3 and 331 divide N, then for some Y where Y(Mod 3) = 2, 331**Y||N.

Block 307

307**Y	733**1	367**2	N>M	307**Y	733**B	N>M
3x43xP	3xP	3x13xP		3x43xP		
307**Y	733**1	367**4	N>M	307**4		N>M
3x43xP	3xP	11x281xP		1051x5231x1621		
307**Y	733**1	367**A	N>M	307**6		N>M
3x43xP	3xP			659xP		
307**Y	733**2		N>M	307**10		N>M
3x43xP	3x19xP			23x26731xP		
307**Y	733**4		N>M	307**C		N>M
3x43xP	5641xP					

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

331**Z			Lemma 3.1
331**6			Lemma 3.2
331**10			Lemma 3.2
331**12	10999171**Y		N Exceeds M
53x37181x10999171xP	3x1929637xP		
331**12	10999171**4		N Exceeds M
53x37181x10999171xP	5x250031x537091xP		
331**12	10999171**6		N Exceeds M
53x37181x10999171xP	7x449xQ	QHNPFLT 10,000,000	
331**12	10999171**A		Proposition 5
53x37181x10999171xP			
331**16			Block 307
307xQ	Q is composite and QHNPFLT 100,000,000		
331**E			N Exceeds M

Corollary 3.1 If N is an odd perfect number less than M then it is not true that 5 x 31 divides N.

Corollary 3.2 If N is an odd perfect number less than M then it is not true that 3 x 5 x 331 divides N.

Lemma 4.1 If N is an odd perfect number less than M, then not both of the following can happen simultaneously.

```
409**X||N               and          41**Y||N

where  X(Mod 4) = 1     and     Y(Mod 3) = 2.

S(409**1)  =  2 x 5 x 41        S(41**2)  =  1723
```

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(1723**6) = 71 x 113 x 5503 x 6014337547541

Block 990151

```
990151**Y       2212009**Y            PR7    990151**4    131**2    17293**2    PR6
3x147739xP   3x7x232999334671              5x131x1021xQ  17293      3x13xP
990151**Y       2212009**4         Cor3.1    990151**4    131**2    17293**B    PR1
3x147739xP        31xQ                     5x131x1021xQ  17293
990151**Y       2212009**6  147739**2 PR7    990151**4    131**C                PR1
3x147739xP       421xP  3x7x9721x106921    5x131x1021x493450561x2912698091
990151**Y       2212009**6  147739**E N>M    990151**6                          N>M
3x147739xP       421xP                        7x747xQ(composite)  QHNPFLT 34.4m
990151**Y       2212009**A            PR5    990151**D                          PR5
3x147739xP
```

Block 1445413861

```
 1445413861**2      43853794861**2   PR7   1445413861**4   457091**2           N>M
3x43x73x5059xP         3x7x19xQ           5x72161x457091xP  22093xP
 1445413861**2      43853794861**4   N>M   1445413861**4   457091**B           N>M
3x43x73x5059xP           5x211xQ(comp)12m  5x72161x457091xP
 1445413861**2      43853794861**A   PR5   1445413861**C                       PR5
3x43x73x5059xP
```

Block 2377 (For more details, see Lemma 13.4)

```
  2377**Y                                   PR7   2377**6                      N>M
 3x7x629167                                    2213x2927149xP
  2377**4        467531**2   9800731**2    N>M   2377**10                      PR1
401x170351xP      22303xP  3x241x256561xP 11x23x199x10781x6963023xP
  2377**4        467531**2   9800731**A    N>M   2377**12                      N>M
401x170351xP      22303xP                         4993xP
  2377**4        467531**4                 N>M   2377**C                       PR5
401x170351xP      5x11xP
  2377**4        467531**B                 N>M
401x170351xP
```

Note: S(467531**4) = 5 x 11 x 8 6872013774 0337952351. To show that Q = 8 6872013774 0337952351 is a prime number, the entries in Table IV are given.

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	3	1	8 6872013774 0337952350
3	7	1	1 8255192103 9125825032
5	3	1	8 6861794214 7798166567
6421	3	1	5 9118800537 6861825527
100217473653041	3	1	2 8033735183 9966439692

TABLE IV

Possibilities And Reasons By Which They May Be Excluded

```
  1723**Y                                                          Block    990151
    3xP
  1723**4                                                          Bk  1445413861
  6101xP
  1723**6        6014337547541**2        Note  S(P)> 10**25        Proposition  5
7x113x5503xP              P
  1723**6        6014337547541**B                                  Proposition  5
7x113x5503xP
  1723**10                                                         Block      2377
89x617x2377xQ  QHNPFLT 1,210,000,000
  1723**12        79**Y                                            Proposition  7
  79x157xQ       3x7x7xP
  1723**12        79**Z                  39449441**2               N  Exceeds   M
  79x157xQ         P                 19x271x349x6163xP
  1723**12        79**Z                  39449441**C               N  Exceeds   M
  79x157xQ         P
  1723**12        79**6                                            N  Exceeds   M
  79x157xQ  281x337x1289x2017
  1723**12        79**10                                           N  Exceeds   M
  79x157xQ        5479xP
  1723**12        79**12                 157**2                    Proposition  6
  79x157xQ         13xQ                   3xP
  1723**12        79**12                 157**4                    Corollary  3.1
  79x157xQ         13xQ                 11x31xP
  1723**12        79**12                 157**D                    N  Exceeds   M
  79x157xQ         13xQ
  1723**12        79**E                                            N  Exceeds   M
  79x157xQ  Q is composite and  QHNPFLT 30,000,000
  1723**F                                                          Proposition  5
```

Lemma 4.2 If N is an odd perfect number less than M, then not both of the following can happen simultaneously.

(A) 409**X||N (B) 41**Z||N

where X(Mod 4) = 1 and Z(Mod 5) = 4 .

Note S(409**1) = 2 x 5 x 41 S(41**4) = 5 x 579281

```
  1051**Y     368551**Y    14347**Y PR7    1051**6        165161219987**2  N>M
   3xP     3x367x14347xP 3x7x31x181xP    7x29x40237xP    (Comp)Q QHNPFLT 30m
  1051**Y     368551**Y    14347**Z P8E    1051**6        165161219987**4  N>M
   3xP     3x367x14347xP    11x71xP      7x29x40237xP     11x11x101xQ(com)4m
  1051**Y     368551**Y    14347**6 N>M    1051**6        165161219987**D  PR5
   3xP     3x367x14347xP   113x421x3697x12769xP
  1051**Y     368551**Y    14347**A N>M    1051**10                        N>M
   3xP     3x367x14347xP                      23xQ Q is composite QHNPFLT 59m
  1051**Y     368551**4             N>M    1051**12      4447**Y           N>M
   3xP     5x2741x448631xP                53x4447xQ     3x7x79xP
  1051**Y     368551**6             PR7    1051**12      4447**4           N>M
   3xP         7xQ                        53x4447xQ      11x281xP
  1051**Y     368551**B             PR5    1051**12      4447**6           PR1
   3xP                                    53x4447xQ      71x127xP
  1051**Z   14275091**Y             N>M    1051**12      4447**E           N>M
5x71x241xP  19x19x31x37x811xP             53x4447x263953xP
  1051**Z   14275091**4             N>M    1051**16                        N>M
5x71x241xP  5x251x4051xP                    2687xQ
  1051**Z   14275091**C             N>M    1051**F                         PR5
5x71x241xP
```

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

```
   579281**Y      1783**Y                                           Corollary  3.2
331x541x1051xP  3x829x1279
   579281**Y      1783**4                                           Corollary  3.1
331x541x1051xP     31xP
   579281**Y      1783**6     32147832919817717593**2               Corollary  3.2
331x541x1051xP      P                   3xQ
   579281**Y      1783**6     32147832919817717593**A               Proposition  5
331x541x1051xP      P
   579281**Y      1783**10                                          Block      1051
331x541x1051xP  11x23x727xQ    Q  is composite  QHNPFLT 22,374,901
   579281**Y      1783**12                                          Block      1051
331x541x1051xP  131x9049xP
```

```
  579281**Y     1783**B                                              Proposition  5
331x541x1051xP
  579281**4     2131**Y                    4933**Y                   Corollary  3.2
  5x2131xQ      3x307xP                 3x127x193x331
  579281**4     2131**Y                    4933**4                   Proposition 8E
  5x2131xQ      3x307xP                 11x31x7541xP
  579281**4     2131**Y                    4933**6                   N  Exceeds   M
  5x2131xQ      3x307xP                 3221x360851xP
  579281**4     2131**Y                    4933**10                  Proposition  1
  5x2131xQ      3x307xP                       Q  Q is composite  59,499,901
  579281**4     2131**Y                    4933**12                  N  Exceeds   M
  5x2131xQ      3x307xP                       Q  Q is composite  24,624,991
  579281**4     2131**Y                    4933**C                   Proposition  5
  5x2131xQ      3x307xP
  579281**4     2131**Z           4126364997061**2                   N  Exceeds   M
  5x2131xQ        5xP       3x43x163x4774387x6859933xP
  579281**4     2131**Z           4126364997061**D                   Proposition  5
  5x2131xQ        5xP
  579281**4     2131**6     93692438982092641237**2                  N  Exceeds   M
  5x2131xQ         P                 3xQ  Q is composite and  QHNPFLT 8,274,001
  579281**4     2131**6     93692438982092641237**Z                  Proposition  5
  5x2131xQ         P
  579281**4     2131**10                                             N  Exceeds   M
  5x2131xQ      1231xQ  Q is composite and  QHNPFLT 20,000,000
  579281**4     2131**12                                             N  Exceeds   M
  5x2131xQ       7541xQ   Q is composite and QHNPFLT 39,249,991
  579281**4     2131**F                                              Proposition  5
  5x2131xQ   Q is composite and QHNPFLT 147,999,811 (Apply Prop 1)
  579281**6    28351**Y                    9241**Y        3559**Y Corollary  3.1
 9241x28351xP  3x61x631xP                3x19x421xP    3x31x136237
  579281**6    28351**Y                    9241**Y        3559**G N  Exceeds   M
 9241x28351xP  3x61x631xP                3x19x421xP
  579281**6    28351**Y                    9241**Z                   N  Exceeds   M
 9241x28351xP  3x61x631xP                  5x41xP
  579281**6    28351**Y                    9241**6                   Proposition  7
 9241x28351xP  3x61x631xP                    7xQ
  579281**6    28351**Y                    9241**10                  Proposition 8E
 9241x28351xP  3x61x631xP                    11xQ
  579281**6    28351**Y                    9241**H                   N  Exceeds   M
 9241x28351xP  3x61x631xP
  579281**6    28351**4               4101224731**I                  N  Exceeds   M
 9241x28351xP 5x11x2864261xP
  579281**6    28351**6                                              Proposition  1
 9241x28351xP  7x1544957xQ  QHNPFLT 22,599,991 Q is composite
  579281**6    28351**10                                             N  Exceeds   M
 9241x28351xP  5347x6733xQ   Q  is composite  QHNPFLT 59,499,901
  579281**6    28351**J                                              Proposition  5
 9241x28351xP
  579281**K                                                          Proposition  5
```

Lemma 4.3 If N is an odd perfect number less than M, then there is no X such that X(Mod 4) = 1 for which 409**X||N and one of the following is true.

(A) 41**6||N (B) 41**10||N (C) 41**12||N (D) 41**16||N
(E) 41**18||N

Note S(409**1) = 2 x 5 x 41

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

41**6	113229229**2		Proposition 8F
43xP	3x13x249811xP		
41**6	113229229**4	83791**2	Proposition 8E
43xP	11x83791xQ	3x409xP	
41**6	113229229**4	83791**4	N Exceeds M
43xP	11x83791xQ	5x11x6271x140611xP	
41**6	113229229**4	83791**6	N Exceeds M
43xP	11x83791xQ	7x164011x631751xP	
41**6	113229229**4	83791**A	N Exceeds M
43xP	11x83791xQ	Q is composite and QHNPFLT 31,000,001	
41**6	113229229**6		N Exceeds M
43xP	7x3767xQ	Q is composite and QHNPFLT 10,000,000	
41**6	113229229**B		Proposition 5
43xP			
41**10	4499415031**2	6748245208562715331**BB	Block 23
23x132947xP	3xP		
41**10	4499415031**4		N Exceeds M
23x132947xP	5xP		
41**10	4499415031**C		Proposition 5
23x132947xP			
41**12	17615988547**2	1764479442181**2	N Exceeds M
	3x379x154681xP	3x61x193x1618807xP	
41**12	17615988547**2	1764479442181**4	Corollary 3.1
	3x379x154681xP	5x11x31x101x1511xQ	
41**12	17615988547**2	1764479442181**D	Proposition 5
	3x379x154681xP		
41**12	17615988547**4		Corollary 3.1
11831x110969xP	11x31xQ		
41**12	17615988547**F		Proposition 5
11831x110969xP			
41**16	201815909**2	31330508713311707**2	N Exceeds M
201815909xP	13xP	7x7x13x37x139xP	
41**16	201815909**2	31330508713311707**G	N Exceeds M
201815909xP	13xP		

41**16	201815909**4		Proposition 1
201815909xP	11x11x41x402551xQ	Q is composite and QHNPFLT	100,000,000
41**16	201815909**H		N Exceeds M
201815909xP			
41**18	12541**Y		Proposition 7
12541xQ	3x7x13xP		
41**18	12541**4	51175171**2	N Exceeds M
12541xQ	5x151x640261xP	3xP	
41**18	12541**4	51175171**4	N Exceeds M
12541xQ	5x151x640261xP	5x11x41x101xQ(comp)QHNPFLT	31,000,001
41**18	12541**4	51175171**6	N Exceeds M
12541xQ	5x151x640261xP	43x117670lxP	
41**18	12541**4	51175171**I	Proposition 5
12541xQ	5x151x640261xP		
41**18	12541**6	377063084752085l329**2	N Exceeds M
12541xQ	71x14533xP	127xP	
41**18	12541**6	3770630847520851329**J	N Exceeds M
12541xQ	71x14533xP		
41**18	12541**10		N Exceeds M
12541xQ	11x23x2003xP		
41**18	12541**K		N Exceeds M
12541xQ	Q is composite and QHNPFLT 100,000,000 (Apply Prop 1)		

In the last case of Lemma 4.3 it is assumed that 41**18||N. With this

S(41**18) = 12541 x Q

also divides N. The number Q is not a perfect square and there is no prime factor F of Q such that for some natural number X(Mod 4) = 1, F**X||N. Also, since Q has no prime factor less than its cube root, then Q**2 divides N by Proposition 1.

Block 617

This block, labelled Block 617, is used in Theorem 2, and Lemma 5.4
Each case or subcase is eliminated for the reason indicated.

```
617**X                        Block     5233   617**Z 145159381141**X Proposition 11
2x3x103                                          P    2x17x31x1439xP
617**Y 3931**Y    23743**Y N  Exceeds   M      617**Z 145159381141**2 Proposition  1
 97xP  3x7x31xP    3x37xP                         P       3x126691xP
617**Y 3931**Y    23743**4 N  Exceeds   M      617**Z 145159381141**4 N  Exceeds   M
 97xP  3x7x31xP 11x11x251xP                       P       5x11x401xP
617**Y 3931**Y    23743**6 N  Exceeds   M      617**Z 145159381141**C Proposition  5
 97xP  3x7x31xP 10732597xQ  composite             P     QHNPFLT 39,318,217
617**Y 3931**Y    23743**10N  Exceeds   M      617**6        29387639**2 N  Exceeds   M
 97xP  3x7x31xP  67x199xQ(comp)      10m7x29387639xP   193x373xP
617**Y 3931**Y    23743**A N  Exceeds   M      617**6        29387639**4 N  Exceeds   M
 97xP  3x7x31xP                         7x29387639xP  151x19551xP
617**Y 3931**4             N  Exceeds   M      617**6        29387639**D N  Exceeds   M
 97xP  5x11xP                           7x29387639xP
617**Y 3931**6             N  Exceeds   M      617**10                   N  Exceeds   M
 97xP  29x569415lxP                            11x67xP
617**Y 3931**B             N  Exceeds   M      617**E                    N  Exceeds   M
 97xP
```

S(145159381141**2) = 3 x 126691 x 5543999 6877924151. To show that the number Q = 5543999 6877924151 is a prime number, for each prime factor P of Q-1 we find a prime Px which is relatively prime to Q such that both of the following are true.

Px**(Q-1) [Mod Q] is 1 and Px**[(Q-1)/P] [Mod Q] is not 1

(See Table V below)

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	3	1	5543999 6877924150
3	3	1	14 5159381141
5	3	1	1148970 1196656523
7	3	1	4486048 3158680645
52799997026593	3	1	2278936 2586405657

TABLE V

Lemma 4.4 If N is an odd perfect number less than M, then there is no X such that X(Mod 4) = 1 for which 409**X||N.

S(409**1) = 2 x 5 x 41

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

41**Y			Lemma 4.1
41**Z			Lemma 4.2
41**6			Lemma 4.3
41**10			Lemma 4.3
41**12			Lemma 4.3
41**16			Lemma 4.3
41**18			Lemma 4.3
41**22	28429**2		Proposition 7
28429xQ	3x7x43x895057		
41**22	28429**4		Corollary 3.1
28429xQ	11x31x41x10211xP		
41**22	28429**6	527940254843336209169517811**2	Proposition 7
28429xQ	P	3x7xQ	
41**22	28429**6	527940254843336209169517811**A	Proposition 5
28429xQ	P		
41**22	28429**B		N Exceeds M
28429xQ	Q is composite and has no prime factor less than 255,249,631.		
41**28	248879**2		N Exceeds M
59x349x248879xQ	373xP		
41**28	248879**J		N Exceeds M
59x349x248879xQ	Q is composite and QHNPFLT 86,858,771		
41**30			N Exceeds M
373x5563013xQ	Q is composite and QHNPFLT 219,249,361		
41**H			Proposition 5

Lemma 4.5 If N is an odd perfect number less than M, then for no Y such that Y(Mod 3) = 2 and X such that X(Mod 4) = 1 will both of the following happen simultaneously..

(A) 409**Y||N (B) 55897**X||N

Note S(409**2) = 3 x 55897 S(55897**1) = 2 x 19 x 1471

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

1471**Y	877**Y	991**Y				Proposition 6
3x823xP	3x7x37xP	3x7x13x13x277				
1471**Y	877**Y	991**Z				Proposition 9
3x823xP	3x7x37xP	5xP				
1471**Y	877**Y	991**6	11078936989**Y	31**Y	331**Y	Proposition 6
3x823xP	3x7x37xP	85581973xP	3x13x31x31xQ	3x331	3x7x5233	QHNPFLT 4m
1471**Y	877**Y	991**6	11078936989**4			N Exceeds M
3x823xP	3x7x37xP	85581973xP	401x20411x62171xP			
1471**Y	877**Y	991**6	11078936989**A			Proposition 5
3x823xP	3x7x37xP	85581973xP				
1471**Y	877**Y	991**10				Block 23
3x823xP	3x7x37xP	11x23xP				
1471**Y	877**Y	991**12	11779**Y	207409**Y	91334821**Y	N Exceeds M
3x823xP	3x7x37xP	11779xP	3x223xP	3x157xP	3x7xP	
1471**Y	877**Y	991**12	11779**Y	207409**Y	91334821**4	Proposition 7
3x823xP	3x7x37xP	11779xP	3x223xP	3x157xP	5x61x44221xQ	
1471**Y	877**Y	991**12	11779**Y	207409**Y	91334821**B	N Exceeds M
3x823xP	3x7x37xP	11779xP	3x223xP	3x157xP		
1471**Y	877**Y	991**12	11779**Y	207409**4		Proposition 8B
3x823xP	3x7x37xP	11779xP	3x223xP	11x61xQ		
1471**Y	877**Y	991**12	11779**Y	207409**6		N Exceeds M
3x823xP	3x7x37xP	11779xP	3x223xP	631x4241903x22150339xP		
1471**Y	877**Y	991**12	11779**Y	207409**C		Proposition 5
3x823xP	3x7x37xP	11779xP	3x223xP			
1471**Y	877**Y	991**12	11779**4			N Exceeds M
3x823xP	3x7x37xP	11779xP	11x2621x5011xP			
1471**Y	877**Y	991**12	11779**D			N Exceeds M
3x823xP	3x7x37xP	11779xP				
1471**Y	877**Y	991**16				N Exceeds M
3x823xP	3x7x37xP	103x18803x34273xQ	Q is composite			
1471**Y	877**Y	991**E				Proposition 5
3x823xP	3x7x37xP					
1471**Y	877**Z			203447171**Y	110908033**Y	N Exceeds M
3x823xP	41x71xP			7x53314123xP	3xP	

```
  1471**Y   877**Z                  203447171**Y  110908033**4 Proposition 8B
  3x823xP   41x71xP                 7x53314123xP    11x6691xQ        10m
  1471**Y   877**Z                  203447171**Y  110908033**6 N  Exceeds   M
  3x823xP   41x71xP                 7x53314123xP       701xP
  1471**Y   877**Z                  203447171**Y  110908033**F Proposition  5
  3x823xP   41x71xP                 7x53314123xP
  1471**Y   877**Z                  203447171**4               Proposition 8E
  3x823xP   41x71xP               5x11x1291x78191xQ
  1471**Y   877**Z                  203447171**G               N  Exceeds   M
  3x823xP   41x71xP
  1471**Y   877**6                84406199767**2               N  Exceeds   M
  3x823xP 29x379x491xP             3x73x3511xP
  1471**Y   877**6                84406199767**4               N  Exceeds   M
  3x823xP 29x379x491xP                  P
  1471**Y   877**6                84406199767**H               Proposition  5
  3x823xP 29x379x491xP
  1471**Y   877**10                                            Block       23
  3x823xP    23xQ  Q is composite and    QHNPFLT  32,349,901
  1471**Y   877**12                                            Proposition  1
  3x823xP  79x1249xQ  Q  is composite  and  QHNPFLT 100,000,000
  1471**Y   877**16                                            N  Exceeds   M
  3x823xP 1667x5237xQ(composite) Q  has no prime factor less than 20,000,000
  1471**Y   877**I                                             Proposition  5
  3x823xP
  1471**Z                             2941691**2               Proposition  7
5x461x691xP                             7x307xP
  1471**Z                             2941691**4               Proposition 8E
5x461x691xP                             5x11xP
  1471**Z                             2941691**6               N  Exceeds   M
5x461x691xP                             9241xQ(composite)  QHNPFLT 17,799,979
  1471**Z                             2941691**J               Proposition  5
5x461x691xP
  1471**6                   1448349368557553911**2             Proposition  1
    7xP                       3x19x43x127x181xQ(composite)  QHNPFLT 25 million
  1471**6                   1448349368557553911**K             Proposition  5
    7xP
  1471**10     2333**Y          777889**Y                      N  Exceeds   M
67x67x2333xP     7xP       3x79x181x2053x6871
  1471**10     2333**Y          777889**4                      N  Exceeds   M
67x67x2333xP     7xP     41x2178131x24714721xP
  1471**10     2333**Y          777889**L                      N  Exceeds   M
67x67x2333xP     7xP
  1471**10     2333**Z       31049861**Y                       N  Exceeds   M
67x67x2333xP 31x41x751xP          P   P = 964093899169183
  1471**10     2333**Z       31049861**4                       Corollary  3.1
67x67x2333xP 31x41x751xP         5xQ
  1471**10     2333**Z       31049861**6                       N  Exceeds   M
67x67x2333xP 31x41x751xP       29xQ   QHNPFLT 17,799,979  Q is composite
  1471**10     2333**Z       31049861**R                       Proposition  5
67x67x2333xP 31x41x751xP
```

1471**10 67x67x2333xP	2333**6 43x757x15720811x315235663	N Exceeds M
1471**10 67x67x2333xP	2333**10 11xQ Q is composite and QHNPFLT 13,374,901	N Exceeds M
1471**10 67x67x2333xP	2333**S	N Exceeds M
1471**12 P	102718202448455962998021356542005747073**T	Proposition 5
1471**U		Proposition 5

S(1471**6) = 7 x 144834936 8557553911. To show that Q = 144834936 8557553911 is a prime number, for each prime factor P of Q-1 we find a prime Px which is relatively prime to Q such that both of the following are true.

Px**(Q-1) [Mod Q] is 1 and Px[(Q-1)/P] [Mod Q] is not 1

(See Table VI below)

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	3	1	144834936 8557553910
3	3	1	78322585 9443062967
5	3	1	19492322 3349569101
7	11	1	3183010111
61	3	1	125188675 9267784021
1193	3	1	107329979 8968101829
501443443	3	1	47413544 0035801620

TABLE VI

Also, S(777889**4) = 41 x 89 3073540598 3106440381 = 41 x Q. To imply that Q is composite, it is sufficient to make the following statement.

5**(Q-1) [Mod Q] = 50 5973577769 5987234412.

Lemma 4.6 Let N be an odd perfect number less than M and suppose 71**Y||N where Y(Mod 3) = 2. Except possibly when both 5113**X||N and 2557**Y||N, the prime 5113 does not divide N.

Note S(71**2) = 5113

Block 28885322563

28885322563**2	N Exceeds M	28885322563**A	Proposition 5
3x31xP			
28885322563**4	N Exceeds M		

Block 1049590933663

1049590933663**2	N Exceeds M	1049590933663**A	Proposition 5
3x103x853x4153xP			
1049590933663**4	N Exceeds M		

Q Q is composite and Q has no prime factor less than 10,000,000

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(2557**6) = 743 x 112210253 x 3353767033

Possibilities And Reasons By Which They May Be Excluded

5113**X	2557**Z	323683781**2	1047711904061397 41**2	N Exceeds M
2x2557	11x12011xP	P	13xP	
5113**X	2557**Z	323683781**2	104771190406139741**A	Proposition 5
2x2557	11x12011xP	P		
5113**X	2557**Z	323683781**4		N Exceeds M
2x2557	11x12011xP	5x11xQ	Q is composite and QHNPFLT 10 million	
5113**X	2557**Z	323683781**B		Proposition 5
2x2557	11x12011xP			
5113**X	2557**6	3353767033**2	112210253**Y	See Block
2x2557		3x19xP	349x1249x28885322563	
5113**X	2557**6	3353767033**2	112210253**CC	Proposition 1
2x2557		3x19xP P = 197329005526164739		
5113**X	2557**6	3353767033**4		N Exceeds M
2x2557		19681x43541xQ	QHNPFLT 10,000,000 (composite)	
5113**X	2557**6	3353767033**C		Proposition 5
2x2557				
5113**X	2557**10	237997**Y	4723777**2	N Exceeds M
2x2557	23x237997x1360943xP	3x7x571xP	3x7x7x13159xP	
5113**X	2557**10	237997**Y	4723777**4	N Exceeds M
2x2557	23x237997x1360943xP	3x7x571xP	11x71x409291xP	
5113**X	2557**10	237997**Y	4723777**D	N Exceeds M
2x2557	23x237997x1360943xP	3x7x571xP		

```
5113**X          2557**10      237997**4                            Proposition  1
2x2557    23x237997x1360943xP    71xP
5113**X          2557**10      237997**6                            N  Exceeds   M
2x2557    23x237997x1360943xP  29x43xP
5113**X          2557**10      237997**E                            Proposition  5
2x2557    23x237997x1360943xP
5113**X          2557**12         53**Y                             Proposition  1
2x2557             53xP           7x409
5113**X          2557**12         53**4                             N  Exceeds   M
2x2557             53xP         11x131xP
5113**X          2557**12         53**6                             N  Exceeds   M
2x2557             53xP           29xP
5113**X          2557**12         53**F                             N  Exceeds   M
2x2557             53xP
5113**X          2557**G                                            Proposition  5
2x2557
5113**Y       8715961**X        92723**Y          8597647453**2  See       Block
  3xP           2x47xP              P        3x61x384847x1049590933663
5113**Y       8715961**X        92723**Y          8597647453**4  N  Exceeds   M
  3xP           2x47xP              P        (composite)  Q  QHNPFLT 10 million
5113**Y       8715961**X        92723**Y          8597647453**H  Proposition  5
  3xP           2x47xP              P
5113**Y       8715961**X        92723**4   31**Y        331**Y  Proposition 8D
  3xP           2x47xP          11x31xQ    3x331       3x7x5233
5113**Y       8715961**X        92723**6                         N  Exceeds   M
  3xP           2x47xP            7xP
5113**Y       8715961**X        92723**10                        N  Exceeds   M
  3xP           2x47xP          67x199x295417xQ(composite) QHNPFLT 13,374,901
5113**Y       8715961**X        92723**I                         Proposition  5
  3xP           2x47xP
5113**Y       8715961**Y        3617523089023**2      249421**X  Proposition  1
  3xP           3x7xP             3x7x249421xQ        2x311x401
5113**Y       8715961**Y        3617523089023**2      249421**2  N  Exceeds   M
  3xP           3x7xP             3x7x249421xQ 3x7x13x13x13x13x103723
5113**Y       8715961**Y        3617523089023**2      249421**4  Proposition  7
  3xP           3x7xP             3x7x249421xQ         5xQ
5113**Y       8715961**Y        3617523089023**2      249421**J  N  Exceeds   M
  3xP           3x7xP             3x7x249421xQ(composite) QHNPFLT 15,000,000
5113**Y       8715961**Y        3617523089023**K                 Proposition  5
  3xP           3x7xP
5113**Y       8715961**4      5**X                               Proposition  1
  3xP            5xQ      Q  is composite  and  QHNPFLT  1,050,000,000
5113**Y       8715961**4      5**KK                              Block        5
  3xP            5xQ      Q  is composite  and  QHNPFLT  1,050,000,000
5113**Y       8715961**6                                         Proposition  1
  3xP             Q       QHNPFLT  60  million   Q  is composite
5113**Y       8715961**L                                         Proposition  5
  3xP
5113**Z  13080080081**X                                          Block       23
11x4751xP 2x3x23x94783189
```

```
   5113**Z  13080080081**2 21261152595806269**X      42254623**2 Proposition  1
  11x4751xP    13x619xP       2x5x7x139x51713xP            3xP
   5113**Z  13080080081**2 21261152595806269**X      42254623**4 N  Exceeds   M
  11x4751xP    13x619xP       2x5x7x139x51713xP       11x431xQ  QHNPFLT 3m
   5113**Z  13080080081**2 21261152595806269**X      42254623**R N  Exceeds   M
  11x4751xP    13x619xP       2x5x7x139x51713xP
   5113**Z  13080080081**2 21261152595806269**2                  N  Exceeds   M
  11x4751xP    13x619xP        3x13x3176149xQ QHNPFLT 9,176,161
   5113**Z  13080080081**2 21261152595806269**S                  Proposition  5
  11x4751xP    13x619xP
   5113**Z  13080080081**4                                       Corollary  3.1
  11x4751xP 5x31x41x2371xQ
   5113**Z  13080080081**T                                       Proposition  5
  11x4751xP
   5113**6        10768274427527**2                              N  Exceeds   M
113x14686393xP 397x2647x3769x17107xP
   5113**6        10768274427527**AA                             Proposition  5
113x14686393xP
   5113**10                                                      N  Exceeds   M
    4049xP
   5113**12                                                      Proposition  1
      Q       Q has no prime factor less than   190,000,000  Q is composite
   5113**BB                                                      Proposition  5
```

S(13080080081**2) = 13 x 619 x 2126115 2595806269. The entries in Table VII below are used to show that Q = 2126115 2595806269 is a prime number.

Note: Q - 1 = 2**2 x 3 x 1249 x 1511 x 4007 x 234293

P	Px	Px(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	7	1	2126115 2595806268
3	5	1	2126113 9515726187
1249	3	1	1924658 1152993523
1511	3	1	1631407 8840705089
4007	3	1	1849212 6416520216
234293	3	1	1283978 2177032754

TABLE VII

Also, S(3617523089023**2) = 3 x 7 x 249421 x 249844798 6416182833. To imply that Q = 249844798 6416182833 is composite, it is sufficient to state that 5**(Q-1) [Mod Q] = 110525623 0426907586 and not 1.

Block 5233

In the use of Block 5233, it is assumed that for some X(Mod 4)=1 and for some P other than one within the block, it is true that P**X||N. It is further assumed that 331**Y||N.

```
 5233**Y       42073**Y          75931**Y                                 Proposition  6
3x7x31xP      3x19x409xP     3x7x7x19x43x61x787
 5233**Y       42073**Y          75931**4                                 Proposition  7
3x7x31xP      3x19x409xP      5x11x751x1171xP
 5233**Y       42073**Y          75931**6   31**Y    331**Y               N  Exceeds   M
3x7x31xP      3x19x409xP     (comp) 71xQ    3x331    3x7x5233 QHNPFLT 17,799,979
 5233**Y       42073**Y          75931**10                                N  Exceeds   M
3x7x31xP      3x19x409xP        23x127139xQ(comp)    QHNPFLT 13,374,901
 5233**Y       42073**Y          75931**A                                 Proposition  5
3x7x31xP      3x19x409xP
 5233**Y       42073**4                                                   Proposition  6
3x7x31xP        11xQ    QHNPFLT  100,000,000
 5233**Y       42073**6                                                   Block      631
3x7x31xP       29x631x1220423xP
 5233**Y       42073**10                                                  N  Exceeds   M
3x7x31xP      23x67x881xQ   Q is composite and QHNPFLT 10,000,000
 5233**Y       42073**B                                                   Proposition  5
3x7x31xP
 5233**Z    41213191**Y             183067**Y     10001107**2             N  Exceeds   M
2351x7741xP 3x199x3217x4831xP       3x1117xP      3x7x79x42409x1421647
 5233**Z    41213191**Y             183067**Y     10001107**4             Proposition  1
2351x7741xP 3x199x3217x4831xP       3x1117xP      41x101x421xQ(comp) QHNPFLT 20mi
 5233**Z    41213191**Y             183067**Y     10001107**C             N  Exceeds   M
2351x7741xP 3x199x3217x4831xP       3x1117xP
 5233**Z    41213191**Y             183067**4                             Proposition  1
2351x7741xP 3x199x3217x4831xP         11xQ  Q is composite and   QHNPFLT 67,000,001
 5233**Z    41213191**Y             183067**6                             N  Exceeds   M
2351x7741xP 3x199x3217x4831xP     43x449x757xP
 5233**Z    41213191**Y             183067**D                             Proposition  5
2351x7741xP 3x199x3217x4831xP.
 5233**Z    41213191**4                                                   Proposition  9
2351x7741xP 5x1184731xQ  QHNPFLT 5,389,991
 5233**Z    41213191**6                                                   N  Exceeds   M
2351x7741xP   17389xQ  Q is composite and    QHNPFLT 17,799,979
 5233**Z    41213191**E                                                   Proposition  5
2351x7741xP
 5233**6                                                                  N  Exceeds   M
3718919xP
 5233**10         23**2              79**2                                N  Exceeds   M
   23xQ           7x79             3x7x7xP
 5233**10         23**2              79**F                                N  Exceeds   M
   23xQ           7x79
 5233**10         23**4                                                   N  Exceeds   M
   23xQ             P
```

5233**10	23**G	N Exceeds M
	23xQ(composite) Q has no prime factor less than 15,106,213	
5233**12		N Exceeds M
274301x39587159xQ		
5233**H		Proposition 5

S(183067**6) = 43 x 449 x 757 x 25754 4383915864 8011791483. To show that the number Q = 25754 4383915864 8011791483 is a prime number, for each prime factor P of Q-1 = 2 x 7 x 67 x 211 x 1301268120 7159773299, we find a prime Px which is relatively prime to Q such that both of the following are true.

Px**(Q-1) [Mod Q] is 1 and Px**[(Q-1)/P] [Mod Q] is not 1

However, we must first show that this largest factor of Q-1 is a prime number. Let Q1 = 1301268120 7159773299 and Q1 - 1 = 2 x 2579 x 252281 5278627331. It will be left to the reader to verify that Q1-1 is the product of three distinct prime factors. The entries in the table below are used to show that Q1 is a prime number.

P	Px	Px**(Q1-1) [Mod Q]	Px**[(Q1-1)/P] [Mod Q1]
2	17	1	1301268120 7159773298
2579	3	1	650657920 3143222811
2522815278627331	3	1	432393646 2497680216

TABLE VIII

The entries in Table IX are now used to determine that Q is prime.

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	5	1	25754 4383915864 8011791483
7	3	1	613522 0753761763
67	3	1	21802 7533998333 5267673233
211	3	1	19104 3799053477 3419042289
13012681207159773299	3	1	14450 6223313607 7072180186

TABLE IX

Lemma 4.7 If N is an odd perfect number less than M, then 71 does not divide N unless all three of (A), (B) and (C) happen simultaneously or one of (D) and (E) is true.

(A) 71**Y||N (B) 5113**X||N (C) 2557**Y||N

(D) 71**12||N (E) 71**18||N

Block 47

```
 47**Y     61**A                       Bk61     47**12                       PR1
 37x61                                 53x2237x14050609x71265169
 47**Z     31**Y     331**Y             PR6     47**16                       PR1
11x31xP    3x331     3x7x5233                3571x10099xQ(comp)QHNPFLT 1573,749,841
 47**6                                  PR1     47**18                       PR1
  43xP                                          419xQ  Q is composite  QHNPFLT  64m
 47**10                                 PR1     47**B                        N>M
134707xP
```

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

```
  71**Y  Note the exception above                                   Lemma       4.6
  5113
  71**Z    2221**X     211**Y                                       Proposition 8E
5x11x211xP 2x11x101   3x13x31x37
  71**Z    2221**X     211**Z         292661**Y     85650753583**2  Proposition 8E
5x11x211xP 2x11x101    5x1361xP          P               3xQ
  71**Z    2221**X     211**Z         292661**Y     85650753583**4  N  Exceeds    M
5x11x211xP 2x11x101    5x1361xP          P                 Q(composite) QHNPFLT 10m
  71**Z    2221**X     211**Z         292661**Y     85650753583**A  Proposition   5
5x11x211xP 2x11x101    5x1361xP          P
  71**Z    2221**X     211**Z         292661**4                     N  Exceeds    M
5x11x211xP 2x11x101    5x1361xP 5x191x241x8741xP
  71**Z    2221**X     211**Z         292661**6                     Proposition   1
5x11x211xP 2x11x101    5x1361xP 4344397x5082071xP
  71**Z    2221**X     211**Z         292661**B                     Proposition   5
5x11x211xP 2x11x101    5x1361xP
  71**Z    2221**X     211**6          41233879**Y                  Proposition   7
5x11x211xP 2x11x101   7x307189xP         3x241xP
  71**Z    2221**X     211**6          41233879**4                  Corollary   3.1
5x11x211xP 2x11x101   7x307189xP         11x31xQ
  71**Z    2221**X     211**6          41233879**6                  N  Exceeds    M
5x11x211xP 2x11x101   7x307189xP         7x379xQ(composite) QHNPFLT 17,799,979
```

71**Z	2221**X	211**6	41233879**C		Proposition 5
5x11x211xP	2x11x101	7x307189xP			
71**Z	2221**X	211**10	2069**Y	611833**2	Proposition 7
5x11x211xP	2x11x101	23x2069xP	7xP	3x307x577xP	
71**Z	2221**X	211**10	2069**Y	611833**D	N Exceeds M
5x11x211xP	2x11x101	23x2069xP	7xP		
71**Z	2221**X	211**10	2069**Z	18333775916461**2	Proposition 8A
5x11x211xP	2x11x101	23x2069xP	P	3x19x709xQ	
71**Z	2221**X	211**10	2069**Z	18333775916461**E	Proposition 5
5x11x211xP	2x11x101	23x2069xP	P		
71**Z	2221**X	211**10	2069**6	186419270607899071**2	Proposition 8A
5x11x211xP	2x11x101	23x2069xP	421xP	3x37x157xQ	
71**Z	2221**X	211**10	2069**6	186419270607899071**P	Proposition 5
5x11x211xP	2x11x101	23x2069xP	421xP		
71**Z	2221**X	211**10	2069**10		Proposition 1
5x11x211xP	2x11x101	23x2069xP	11x67xP		
71**Z	2221**X	211**10	2069**12		Proposition 1
5x11x211xP	2x11x101	23x2069xP	26833xP		
71**Z	2221**X	211**10	2069**G		Proposition 5
5x11x211xP	2x11x101	23x2069xP			
71**Z	2221**X	211**12			N Exceeds M
5x11x211xP	2x11x101	131x37181xP			
71**Z	2221**X	211**16	239**2		N Exceeds M
5x11x211xP	2x11x101	137x239xP	19x3019		
71**Z	2221**X	211**16	239**4		N Exceeds M
5x11x211xP	2x11x101	137x239xP	P		
71**Z	2221**X	211**16	239**H		N Exceeds M
5x11x211xP	2x11x101	137x239xP			
71**Z	2221**X	211**18			Proposition 1
5x11x211xP	2x11x101	723901xQ	Q is composite and QHNPFLT 20,000,000		
71**Z	2221**X	211**22			N Exceeds M
5x11x211xP	2x11x101	277x783151xQ	Q is composite and QHNPFLT 100,000,000		
71**Z	2221**X	211**I			Proposition 5
5x11x211xP	2x11x101				
71**Z	2221**Y				Proposition 7
5x11x211xP	3x7xP				
71**Z	2221**4	4868776221241**X		124146469**2	Proposition 8E
5x11x211xP	5xP	2x19609xP		3x31x79x2179xP	
71**Z	2221**4	4868776221241**X		124146469**4	N Exceeds M
5x11x211xP	5xP	2x19609xP		211xQ(composite) QHNPFLT 10m	
71**Z	2221**4	4868776221241**X		124146469**J	N Exceeds M
5x11x211xP	5xP	2x19609xP			
71**Z	2221**4	4868776221241**2			Proposition 7
5x11x211xP	5xP	3x7xQ			
71**Z	2221**4	4868776221241**K			Proposition 5
5x11x211xP	5xP				
71**Z	2221**6	1183576566098753**1			Proposition 11
5x11x211xP	71x1429xP	2x3x3x53x103x257x3659x12809			
71**Z	2221**6	1183576566098753**L			N Exceeds M
5x11x211xP	71x1429xP				

```
  71**Z   2221**10            5034283**2                               Proposition  7
5x11x211xP 23x6689x5034283xP 3x7x37x67xQ
  71**Z   2221**10            5034283**L                               N  Exceeds   M
5x11x211xP 23x6689x5034283xP
  71**Z   2221**12                                                     N  Exceeds   M
5x11x211xP  859xP
  71**Z   2221**R                                                      Proposition  5
5x11x211xP
  71**6      21020917**X   10510459**Y         431311**Y               N  Exceeds   M
 7x883xP        2xP        3x431311xP          3x19x127x733x35059
  71**6      21020917**X   10510459**Y         431311**4               Proposition  7
 7x883xP        2xP        3x431311xP             5xQ
  71**6      21020917**X   10510459**Y         431311**6               N  Exceeds   M
 7x883xP        2xP        3x431311xP           29x43xP
  71**6      21020917**X   10510459**Y         431311**S               Proposition  5
 7x883xP        2xP        3x431311xP
  71**6      21020917**X   10510459**4            31**Y     331**Y    Block      5233
 7x883xP        2xP      11x31x246781x2558321xP  3x331     3x7x5233
  71**6      21020917**X   10510459**6                                 N  Exceeds   M
 7x883xP        2xP      7x71x71x87151xP
  71**6      21020917**X   10510459**T                                 Proposition  5
 7x883xP        2xP
  71**6      21020917**Y   97292389**1                                 Proposition  7
 7x883xP     3x1513921xP 2x5x13x29x131xP
  71**6      21020917**Y   97292389**2     25813566589**1              Proposition  7
 7x883xP     3x1513921xP   3x31x3943xP        2x5x7xQ
  71**6      21020917**Y   97292389**2     25813566589**2              N  Exceeds   M
 7x883xP     3x1513921xP   3x31x3943xP  3x61x1483x94531xP
  71**6      21020917**Y   97292389**2     25813566589**U              N  Exceeds   M
 7x883xP     3x1513921xP   3x31x3943xP
  71**6      21020917**Y   97292389**V                                 Proposition  5
 7x883xP     3x1513921xP
  71**6      21020917**4      702721**X          351361**2             N  Exceeds   M
 7x883xP  11x1291x702721xP      2xP               3x433xP
  71**6      21020917**4      702721**X          351361**4             N  Exceeds   M
 7x883xP  11x1291x702721xP      2xP              5x55061xP
  71**6      21020917**4      702721**X          351361**VV            N  Exceeds   M
 7x883xP  11x1291x702721xP      2xP
  71**6      21020917**4      702721**Y  2970313**X 1485157**2         N  Exceeds   M
 7x883xP  11x1291x702721xP 3x151x367xP     2xP   3x7x127x277xP
  71**6      21020917**4      702721**Y  2970313**X 1485157**W         N  Exceeds   M
 7x883xP  11x1291x702721xP 3x151x367xP     2xP
  71**6      21020917**4      702721**Y  2970313**Y 26980924429**1 Proposition  7
 7x883xP  11x1291x702721xP 3x151x367xP  3x109xP        2x5xQ
  71**6      21020917**4      702721**Y  2970313**Y 26980924429**B N  Exceeds   M
 7x883xP  11x1291x702721xP 3x151x367xP  3x109xP
  71**6      21020917**4      702721**Y  2970313**4                    N  Exceeds   M
 7x883xP  11x1291x702721xP 3x151x367xP    11xQ(composite) QHNPFLT 10m
  71**6      21020917**4      702721**Y  2970313**6                    N  Exceeds   M
 7x883xP  11x1291x702721xP 3x151x367xP   3851xP
```

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 71**6       21020917**4      702721**Y  2970313**BB                   Proposition  5
7x883xP  11x1291x702721xP 3x151x367xP
 71**6       21020917**4      702721**4                                N  Exceeds   M
7x883xP  11x1291x702721xP      5xP
 71**6       21020917**4      702721**CC                               N  Exceeds   M
7x883xP  11x1291x702721xP
 71**6       21020917**6                                               N  Exceeds   M
7x883xP      7x127x1723xQ     Q  is composite and QHNPFLT 17,799,979
 71**6       21020917**DD                                              N  Exceeds   M
7x883xP
 71**10                    143554218709131407**2  3463**2              N  Exceeds   M
  23xP                           7x3463xP         3x61xP
 71**10                    143554218709131407**2  3463**EE             N  Exceeds   M
  23xP                           7x3463xP
 71**10                    143554218709131407**FF                      Proposition  5
  23xP
 71**12                                                                Note Exception
   Q      Q  is composite and has no prime factor less than  3,202,878,952
 71**16                                                                N  Exceeds   M
 239xP
 71**18                                                                Note Exception
   Q     Q has no prime factor less than  1,042,749,451
 71**22                  242329**X                                     Block        47
 47x47x242329xP            2x5x11xP
 71**22                  242329**2                                     Block        47
 47x47x242329xP             3x13xP
 71**22                  242329**4                                     Block        47
 47x47x242329xP          31x7228681xP
 71**22                  242329**HH                                    Block        47
 47x47x242329xP
 71**28                                                                N  Exceeds   M
 59x233x55217x78823x151381xQ Q is composite and   QHNPFLT  35,271,773
 71**GG                                                                Proposition  5
```

From time to time, we shall state that S(71**12) is composite and has no prime factor less than 3,202,878,952. Whenever we assume that 71**12||N, it goes without saying that S(71**12) has a prime factor P such that P x S(71**12) divides N. In particular, if it is known that S(71**12) has no prime factor which appears to an odd power in the prime factorization of N, then N is divisible by [S(71**12)]**2.

Lemma 4.8 If N is an odd perfect number less than M, then for no Y such that Y(Mod 3) = 2 does 409**Y||N.

Block 937

937**X		Prop 1	937**4	91525691**E	N>M
2x7x67			4831xP		
937**Y	2929691**X	Prop 7	937**6		N>M
3xP	2x5xP		22751xP		
937**Y	2929691**Y	N > M	937**10		N>M
3xP	3x61x127x139x163xP		353xQ Q is composite QHNPFLT 10m		
937**Y	2929691**4	N > M	937**F		N>M
3xP	131x1181x1721xP				
937**Y	2929691**D	N > M			

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(409**2) = 3 x 55897

Possibilities And Reasons By Which They May Be Excluded

55897**X					Lemma 4.5
55897**Y	158791**Y	2823283**Y	543237901**X	1798801**Y	N Exceeds M
3x7x937xP	3x13x229xP	3x67x73xP	2x151xP	3x7x73xP	
55897**Y	158791**Y	2823283**Y	543237901**X	1798801**4	Proposition 9
3x7x937xP	3x13x229xP	3x67x73xP	2x151xP		
55897**Y	158791**Y	2823283**Y	543237901**X	1798801**6	N Exceeds M
3x7x937xP	3x13x229xP	3x67x73xP	2x151xP (comp)197xQ QHNPFLT 17,799,979		
55897**Y	158791**Y	2823283**Y	543237901**X	1798801**B	Proposition 5
3x7x937xP	3x13x229xP	3x67x73xP	2x151xP		
55897**Y	158791**Y	2823283**Y	543237901**Y		N Exceeds M
3x7x937xP	3x13x229xP	3x67x73xP	3x3967xP		
55897**Y	158791**Y	2823283**Y	543237901**4		Proposition 9
3x7x937xP	3x13x229xP	3x67x73xP			
55897**Y	158791**Y	2823283**Y	543237901**D		Proposition 5
3x7x937xP	3x13x229xP	3x67x73xP			
55897**Y	158791**Y	2823283**4			N Exceeds M
3x7x937xP	3x13x229xP	61xP			
55897**Y	158791**Y	2823283**6			N Exceeds M
3x7x937xP	3x13x229xP	7xQ Q is composite QHNPFLT 17,799,979			
55897**Y	158791**Y	2823283**F			Proposition 5
3x7x937xP	3x13x229xP				
55897**Y	158791**4				Proposition 9
3x7x937xP	5x11xQ				
55897**Y	158791**6				Block 937
3x7x937xP	29x43x337xQ Q is composite and QHNPFLT 70,999,741				
55897**Y	158791**G				Proposition 5
3x7x937xP					

55897**4	31**Y	331**Y	1567693701851**2	200881**X	Proposition 6
31x200881xP	3x331	3x7x5233	7x7x3547x118891xP	2x11x23x397	
55897**4	31**Y	331**Y	1567693701851**2	200881**Y	N Exceeds M
31x200881xP	3x331	3x7x5233	7x7x3547x118891xP	3x7x7xP	
55897**4	31**Y	331**Y	1567693701851**2	200881**4	Proposition 9
31x200881xP	3x331	3x7x5233	7x7x3547x118891xP		
55897**4	31**Y	331**Y	1567693701851**2	200881**H	N Exceeds M
31x200881xP	3x331	3x7x5233	7x7x3547x118891xP		
55897**4	31**Y	331**Y	1567693701851**I		N Exceeds M
31x200881xP	3x331	3x7x5233			
55897**6					Lemma 4.7
71xQ	Q is composite and has no prime factor less than 372,599,981				
55897**10					N Exceeds M
463x727x6669059xQ	Q is composite and QHNPFLT 59,499,901				
55897**H					Proposition 5

In Case 9 of Lemma 4.8 it is assumed that 2823283**4||N. This implies that S(2823283**4) = 61 x 10415 6882162080 4982606221 also divides N. To show that Q = 10415 6882162080 4982606221 is a prime number, for each prime factor P of Q - 1 = 2**2 x 5 x 7 x 19 x 74297 x 736973 x 7151275007, we find a prime Px which is relatively prime to Q such that both of the following are true.

Px**(Q-1) [Mod Q] is 1 and Px[(Q-1)/P] [Mod Q] is not 1

(See Table X below)

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	3	1	10415 6882162080 4982606220
5	3	1	2250418240 5617406187
7	3	1	8914 3435109781 3304575280
19	3	1	1844 6051047061 0144125488
74297	3	1	13 5292703882 6739096689
736973	3	1	4773 4874621610 7788186261
7151275007	3	1	7754 4443909597 1435386794

TABLE X

In Case 13 of this same lemma it is assumed that 158791**6||N. This implies that S(158791**6) = 29 x 43 x 337 x 381471 1363222668 7349228423 also divides N. To imply that Q = 381471 1363222668 7349228423 is a composite number, we state that 5**(Q-1) [Mod Q] = 321304 4282769646 0697633077.

Block 5

5**Y				Corollary 3.1
31				
5**Z	71**Y	5113**X	2557**Y	Proposition 7
11x71	5113	2x2557	3x7x13x13x19x97	
5**Z	71**12			N Exceeds M
11x71	Q	Q is composite and QHNPFLT 3,202,878,952		
5**Z	71**18			N Exceeds M
11x71	Q	Q is composite and QHNPFLT 1,042,749,451		
5**6	19531**Y	127159831**Y	763058587**2	Proposition 7
P	3xP	3x7063513xP	3x7x193xQ	
5**6	19531**Y	127159831**Y	763058587**4	Corollary 3.1
P	3xP	3x7063513xP	11x31x151xQ	
5**6	19531**Y	127159831**Y	763058587**A	Proposition 5
P	3xP	3x7063513xP		
5**6	19531**Y	127159831**4		N Exceeds M
P	3xP	5x251x267431xQ(composite) QHNPFLT 59,499,901		
5**6	19531**Y	127159831**6		Proposition 7
P	3xP	7x43x337xQ (composite) QHNPFLT 13,599,979		
5**6	19531**Y	127159831**B		Proposition 5
P	3xP			
5**6	19531**4	32009891**Y		N Exceeds M
P	5x191x4760281xP	7x283x468913xP		
5**6	19531**4	32009891**4		Corollary 3.1
P	5x191x4760281xP	5x31xQ		
5**6	19531**4	32009891**6		N Exceeds M
P	5x191x4760281xP	22247xP		
5**6	19531**4	32009891**C		Proposition 5
P	5x191x4760281xP			
5**6	19531**6	631**Y		Proposition 7
P	7x631xP	3x307xP		
5**6	19531**6	631**Z	46601**X	Proposition 7
P	7x631xP	5x11x41x1511xP	2x3x3x3xP	
5**6	19531**6	631**Z	46601**Y	N Exceeds M
P	7x631xP	5x11x41x1511xP	7x13xP	
5**6	19531**6	631**Z	46601**4	Corollary 3.1
P	7x631xP	5x11x41x1511xP	5x11x31x61x71xP	
5**6	19531**6	631**Z	46601**6	N Exceeds M
P	7x631xP	5x11x41x1511xP	3697xQ(comp) QHNPFLT 13,599,979	
5**6	19531**6	631**Z	46601**10	N Exceeds M
P	7x631xP	5x11x41x1511xP	23x28183x129449xP	
5**6	19531**6	631**Z	46601**D	Proposition 5
P	7x631xP	5x11x41x1511xP		
5**6	19531**6	631**6		N Exceeds M
P	7x631xP	7x6032531xP		
5**6	19531**6	631**10		N Exceeds M
P	7x631xP	89xP		
5**6	19531**6	631**12		Proposition 1
P	7x631xP	131x443x26339x103091xQ(comp)QHNPFLT 975,249,991		

5**6	19531**6	631**E			N Exceeds M
P	7x631xP				
5**6	19531**10				Proposition 1
P	23x23x4159xQ	Q is composite and QHNPFLT 12,375,001			
5**6	19531**F				Proposition 5
P					
5**10	12207031**2				Proposition 7
P	3x7x1041757xP				
5**10	12207031**4	33899939683211028917156831**2			N Exceeds M
P	5x131xP	163xQ Q is composite			
5**10	12207031**4	33899939683211028917156831**E			Proposition 5
P	5x131xP				
5**10	12207031**6	37871**Y			N Exceeds M
P	37871xP	P			
5**10	12207031**6	37871**4			N Exceeds M
P	37871xP	5x11x181xP			
5**10	12207031**6	37871**6			N Exceeds M
P	37871xP	7x113x4733x9198197xP			
5**10	12207031**6	37871**10			N Exceeds M
P	37871xP	23x199xP			
5**10	12207031**6	37871**F			Proposition 5
P	37871xP				
5**10	12207031**FF				Proposition 5
P					
5**12	305175781**X	43609**Y	633929497**Y	1433991301**Y	N Exceeds M
P	2x3499x43609	3xP	3x19x127x38713xP	3x19x12301x779869xP	
5**12	305175781**X	43609**Y	633929497**Y	1433991301**4	N Exceeds M
P	2x3499x43609	3xP	3x19x127x38713xP	5xP	
5**12	305175781**X	43609**Y	633929497**Y	1433991301**G	Proposition 5
P	2x3499x43609	3xP	3x19x127x38713xP		
5**12	305175781**X	43609**Y	633929497**4		Proposition 8E
P	2x3499x43609	3xP	11x131x241x1901xQ		
5**12	305175781**X	43609**Y	633929497**H		Proposition 5
P	2x3499x43609	3xP			
5**12	305175781**X	43609**4	2819051**Y		N Exceeds M
P	2x3499x43609	11x11701x9967721xP	7x151xP		
5**12	305175781**X	43609**4	2819051**4		Corollary 3.1
P	2x3499x43609	11x11701x9967721xP	5x11x31x61x941xQ		
5**12	305175781**X	43609**4	2819051**6		N Exceeds M
P	2x3499x43609	11x11701x9967721xP	127x197x281x1938889xP		
5**12	305175781**X	43609**4	2819051**I		Proposition 5
P	2x3499x43609	11x11701x9967721xP			
5**12	305175781**X	43609**6	17137**Y		N Exceeds M
P	2x3499x43609	17137xQ	3x13x31xP		
5**12	305175781**X	43609**6	17137**Z		N Exceeds M
P	2x3499x43609	17137xQ	9421xP		
5**12	305175781**X	43609**6	17137**6		N Exceeds M
P	2x3499x43609	17137xQ	7x15569x19447x74761xP		
5**12	305175781**X	43609**6	17137**10		N Exceeds M
P	2x3499x43609	17137xQ	23x661xP		

```
5**12     305175781**X     43609**6              17137**12        N  Exceeds   M
  P       2x3499x43609      17137xQ                  131xQ(composite)  14.6m
5**12     305175781**X     43609**6              17137**J         Proposition  5
  P       2x3499x43609      17137xQ  Q is composite    QHNPFLT 62,599,979
5**12     305175781**X     43609**10                              N  Exceeds   M
  P       2x3499x43609      67xP
5**12     305175781**X     43609**K                               Proposition  5
  P       2x3499x43609
5**12     305175781**Y                      12340172617**X        N  Exceeds   M
  P       3x271x9283xP                       2x6170086309
5**12     305175781**Y                      12340172617**2        N  Exceeds   M
  P       3x271x9283xP                           3x181xP
5**12     305175781**Y                      12340172617**4        N  Exceeds   M
  P       3x271x9283xP                  11x521x3691x9081881xQ    10m
5**12     305175781**Y                      12340172617**L        Proposition  5
  P       3x271x9283xP
5**12     305175781**4                                            N  Exceeds   M
  P       5x11x3011xQ    Q  is composite '    QHNPFLT 25,000,000
5**12     305175781**R                                            Proposition  5
  P
5**16     466344409**X                                            Proposition 11
409xP       2x5x7xP
5**16     466344409**2                                            N  Exceeds   M
409xP         3x19xP    P = 3815387864419363
5**16     466344409**4                                            N  Exceeds   M
409xP     11x11x4931x6673411xQ  Q is composite and  QHNPFLT  25,000,000
5**16     466344409**S                                            Proposition  5
409xP
  5**18      3981071**Y      1219148483701**X                     Corollary  3.1
191x6271xP     13xP                2x31xP
  5**18      3981071**Y      1219148483701**2                     N  Exceeds   M
191x6271xP     13xP          3x61507x9181327x877327920409
  5**18      3981071**Y      1219148483701**4                     N  Exceeds   M
191x6271xP     13xP            5x11x32371xP
  5**18      3981071**Y      1219148483701**T                     Proposition  5
191x6271xP     13xP
  5**18      3981071**4                                           N  Exceeds   M
191x6271xP    5x9341xP
  5**18      3981071**6     71**Y       5113**X     2557**Y       Proposition  7
191x6271xP    71x127xP      5113        2x2557  3x7x13x13x19x97
  5**18      3981071**6     71**12                                N  Exceeds   M
191x6271xP    71x127xP        Q      QHNPFLT  3,202,878,952
  5**18      3981071**6     71**18                                N  Exceeds   M
191x6271xP    71x127xP        Q      QHNPFLT    1,042,749,451
  5**18      3981071**U                                           Proposition  5
191x6271xP
  5**22      332207361361**1        612928711**2                  N  Exceeds   M
  8971xP         2x271xP                3xQ(composite) QHNPFLT 10,000,000
  5**22      332207361361**1        612928711**4                  N  Exceeds   M
  8971xP         2x271xP             5x296741xP
```

5**22	332207361361**1	612928711**V	Proposition 5
8971xP	2x271xP		
5**22	332207361361**2		N Exceeds M
8971xP	3x19x97x27751xP		
5**22	332207361361**4		Corollary 3.1
8971xP	5x31x37441xQ		
5**22	332207361361**W		N Exceeds M
8971xP			
5**28		22125996444329**1	Proposition 11
59x35671xP		2x3x3x3x3x3x5x11069xP	
5**28		22125996444329**2	Corollary 3.1
59x35671xP		31x67xQ	
5**28		22125996444329**AA	Proposition 5
59x35671xP			
5**30		625552508473588471**2	N Exceeds M
1861xP		3x13xQ(Composite)	QHNPFLT 7,080,001
5**30		625552508473588471**BB	Proposition 5
1861xP			
5**CC			N Exceeds M

Note: S(3981071**4) = 5 x 9341 x 53 7819351636 2980863601. To show that Q = 53 7819351636 2980863601 is a prime number, for each prime factor P of Q - 1 = 2**4 x 3 x 5**2 x 17 x 29 x 1063831 x 8545463391, we find Px which is relatively prime to Q such that both of the following are true.

Px**(Q-1) [Mod Q] is 1 and Px**[(Q-1)/P] [Mod Q] is not 1

(See Table XI below)

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	11	1	53 7819351636 2980863600
3	3	1	2 7897502592 3416150204
5	5	1	6309570090 2098020911
17	3	1	18 8719206988 9954304396
29	3	1	51 6696613165 2198903115
1063831	3	1	17 5020473527 0759498799
8545463391	3	1	1 7057193856 0644376668

TABLE XI

In showing that S(332207361361**2) is the product of five distinct prime factors, consider the fact that

71927 2201686876 = 2**2 x 3**3 x 71 x 93801799907.

Lemma 4.9 If N is an odd perfect number less than M, then for no Z such that Z(Mod 5) = 4 does 409**Z||N.

Note S(409**4) = 71 x 8971 x 44041

Block 44041

The following block of sub-cases is provided for use in this lemma. It is labeled Block 44041. Each sub-case leads to the contradiction as indicated.

For this block

S(92364463**2) = 3 x 7 x 13 x 31 x 5851 x 9001 x 19141
S(92364463**4) = 11 x 11 x 104711x1833431xP

44041**X	61**Y		PR6	44041**Y	92364463**2		N>M
2x19x19xP	3x13x97			3x7xP	(See above)		
44041**X	61**Z		PR7	44041**Y	92364463**4		N>M
2x19x19xP	5x131xP			3x7xP	11x11xQ		
44041**X	61**6	52379047267**2	PR1	44041**Y	92364463**6		N>M
2x19x19xP	P	3x6619231xP		3x7xP	Q(comp)	QHNPFLT	10m
44041**X	61**6	52379047267**A	PR1	44041**Y	92364463**C		PR5
2x19x19xP	P			3x7xP			
44041**X	61**10		N>M	44041**4			N>M
2x19x19xP	199x859xP			5xP			
44041**X	61**12		N>M	44041**6			N>M
2x19x19xP	187123xP			18397x49057xP			
44041**X	61**16		N>M	44041**10			N>M
2x19x19xP	103xP			67x199xP			
44041**X	61**18		N>M	44041**D			PR5
2x19x19xP	229xP						
44041**X	61**B		N>M				
2x19x19xP							

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

8971**Y	14143**Y	3509449**X		Proposition 7
3x7x271xP	3x19xP	2x5x5x7x37x271		
8971**Y	14143**Y	3509449**Y	4105411931017**2	Block 44041
3x7x271xP	3x19xP	3xP	3x67xQ	Q is composite 10m
8971**Y	14143**Y	3509449**Y	4105411931017**A	Proposition 5
3x7x271xP	3x19xP	3xP		
8971**Y	14143**Y	3509449**4		Proposition 8B
3x7x271xP	3x19xP	11x61x1231x6011x8831x318211xP		

```
 8971**Y      14143**Y          3509449**6                       N  Exceeds   M
3x7x271xP      3x19xP              43xP
 8971**Y      14143**Y          3509449**B                       Proposition  5
3x7x271xP      3x19xP
 8971**Y      14143**Z      20248001**X                          Proposition 11
3x7x271xP    20248001xP  2x3x3x3x257x1459
 8971**Y      14143**Z      20248001**2 1976126401**X 988063201**2Proposition  1
3x7x271xP    20248001xP 7x37x73x73x7243xP    2xP       3x19x4711081xP
 8971**Y      14143**Z      20248001**2 1976126401**X 988063201**4Proposition  7
3x7x271xP    20248001xP 7x37x73x73x7243xP    2xP          5x11xQ
 8971**Y      14143**Z      20248001**2 1976126401**X 988063201**CProposition  5
3x7x271xP    20248001xP 7x37x73x73x7243xP    2xP
 8971**Y      14143**Z      20248001**2 1976126401**D            N  Exceeds   M
3x7x271xP    20248001xP 7x37x73x73x7243xP
 8971**Y      14143**Z      20248001**4                          Proposition  7
3x7x271xP    20248001xP     5x11x4931xQ
 8971**Y      14143**Z      20248001**E                          Block     44041
3x7x271xP    20248001xP
 8971**Y      14143**6                                           Block     44041
3x7x271xP 29x3347x3739xP
 8971**Y      14143**10                                          N  Exceeds   M
3x7x271xP     114203xQ  Q is composite and  QHNPFLT  13,374,901
 8971**Y      14143**12                                          N  Exceeds   M
3x7x271xP       313xP
 8971**Y      14143**F                                           Proposition  5
3x7x271xP
 8971**Z                 18246664520771**2                       Block     44041
 5x71xP         181x241x829x868531x10600625756047
 8971**Z                 18246664520771**G                       Proposition  5
 5x71xP
 8971**6                  1888079002013**1                       N  Exceeds   M
 452873x609673xP        2x3x73x4310682653
 8971*6                   1888079002013**H                       N  Exceeds   M
 452873x609673xP
 8971**10         71**Y    5113**X     2557**Y                   Block     44041
   P
 8971**10         71**12                                         Block     44041
   P
 8971**10         71**18                                         Block     44041
   P
 8971**12                                                        N  Exceeds   M
13x131x17681xQ  Q is composite and  QHNPFLT 46,191,601
 8971**I                                                         Proposition  5
```

Lemma 4.10 If N is an odd perfect number less than M, then it is not true that 409**6||N.

```
         Sub-Block  881527
881527**Y     259030244419**A      N Exceeds M    881527**6     N Exceeds M
  3xP                                               617xP
881527**4                          N Exceeds M    881527**B     Proposition 5
11x521x3214091xP
         Note   S(409**6)  =  6133 x 15919 x 48063373
```

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

```
48063373**X    24031687**Y    218379397**Y   29061280470727**2  Block  881527
   2xP          3x881527xP     3x547xP          3x49681xP
48063373**X    24031687**Y    218379397**Y   29061280470727**A  Proposition  5
   2xP          3x881527xP     3x547xP
48063373**X    24031687**Y    218379397**4                      N  Exceeds  M
   2xP          3x881527xP     11x6991xP
48063373**X    24031687**Y    218379397**6                      N  Exceeds  M
   2xP .        3x881527xP     2801x3347xP
48063373**X    24031687**Y    218379397**B                      Proposition  5
   2xP          3x881527xP
48063373**X    24031687**4      120418316715577381**2           N  Exceeds  M
   2xP    11x491x691x742151xP          7xQ(composite)  QHNPFLT 18,000,031
48063373**X    24031687**4      120418316715577381**C           Proposition  5
   2xP    11x491x691x742151xP
48063373**X    24031687**6                                      Proposition  1
   2xP         7x29x17627x65563x71261x378883xQ  Q is composite;  QHNPFLT 31m
48063373**X    24031687**D                                      Proposition  5
   2xP
48063373**Y 15484511869**X                                      Proposition 11
3x223x223xP   2x5x673xP
48063373**Y 15484511869**Y    79923369278895461677**1           N  Exceeds  M
3x223x223xP       3xP                   2xP
48063373**Y 15484511869**Y    79923369278895461677**2           N  Exceeds  M
3x223x223xP       3xP            3x31x1033x766039xQ(composite)  QHNPFLT 7m
48063373**Y 15484511869**Y    79923369278895461677**E           Proposition  5
3x223x223xP       3xP
48063373**Y 15484511869**4                                      N  Exceeds  M
3x223x223xP   131x4271xQ  Q is composite and  QHNPFLT  10,000,000
48063373**Y 15484511869**F                                      Proposition  5
3x223x223xP
48063373**4        15919**Y                           331**Y   N  Exceeds  M
  41x61xQ          3x331xP                           3x7x5233
```

```
48063373**4       15919**4             31**Y           331**Y  N  Exceeds   M
  41x61xQ     31x151x340801xP          3x331         3x7x5233
48063373**4       15919**6                                     N  Exceeds   M
  41x61xQ          7xP
48063373**4       15919**10                                    N  Exceeds   M
  41x61xQ          23x67xQ  QHNPFLT 12,375,001
48063373**4       15919**G                                     N  Exceeds   M
  41x61xQ      Q  has no prime factor less than  43,000,001 and is composite
48063373**6                                                    N  Exceeds   M
  7x29xQ  Q is composite and  Q  has no prime factor less than  22,599,991
48063373**G                                                    Proposition  5
```

In Case 11 of Lemma 4.10 it is assumed that 7992336927889546l677**1||N. Then

S(79923369278895461677**1) = 2 x Q

divides N. To show that Q = 3996168463 9447730839 is a prime number, for each prime factor of 2 x 9 x 19 x 43 x 75431 x 36 0246160933, we find a prime Px which is relatively prime to Q such that both of the following are true.

Px**(Q-1) [Mod Q] is 1 and Px**[(Q-1)/P] [Mod Q] is not 1

(See Table XII below)

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	3	1	3996168463 9447730838
3	7	1	2622528319 5090855820
19	3	1	149358436 2420911800
43	3	1	2027145237 0652330242
75431	3	1	438678048 6835777527
360246160933	3	1	2451827715 8637313389

TABLE XII

Theorem 4 If N is an odd perfect number less than M, then the prime 409 does not divide N.

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(409**6) = 6133 x 15919 x 48063373

Possibilities And Reasons By Which They May Be Excluded

409**X	where	X(Mod 4) = 1		Lemma 4.4
409**Y	where	Y(Mod 3) = 2		Lemma 4.8
409**Z	where	Z(Mod 5) = 4		Lemma 4.9
409**6				Lemma 4.10
409**10	425243941566841**1		2126219707833421**2	N Exceeds M
23x2311x5809387xP	2xP		3x7x31x109x304807x80227x578353xP	
409**10	425243941566841**1		2126219707833421**F	Proposition 5
23x2311x5809387xP	2xP			
409**10	425243941566841**2		31**Y 331**Y	N Exceeds M
23x2311x5809387xP	3x31x1521769xP		3x331 3x7x5233	
409**10	425243941566841**A			N Exceeds M
23x2311x5809387xP				
409**12		1608751**Y		Proposition 1
10193x1608751xP		3x7x163xP	where P = 756085711	
409**12		1608751**4		N Exceeds M
10193x1608751xP		5x1021x1471xP		
409**12		1608751**6		N Exceeds M
10193x1608751xP		43x1289x47419xP		
409**12		1608751**10		Proposition 5
10193x1608751xP		11x23x67x230143x327889xP		
409**12		1608751**B		Proposition 5
10193x1608751xP				
409**16				N Exceeds M
17x103x307x443x3163x43283x47363x55217x2190654lx329083009				
409**18		59699**2	274156177**C	N Exceeds M
59699x11459737xQ		13xP		
409**18		59699**D		N Exceeds M
59699x11459737xQ	Q is composite and	QHNPFLT 32,757,001		
409**E				Proposition 5

In determining the prime factorization of S(1608751**4) as being

5 x 1021 x 1471 x 89196367 9285453711

we may consider the fact that 89196367 9285453710 in its prime factorization form is 2 x 3 x 5 x 13 x 228708 6357142189.

Lemma 5.1 Let N be an odd perfect number less than M. Then, not all three of the following can happen simultaneoulsy.

(A) 5419**Y||N (B) 1009**Y||N (C) 9181**X||N

```
Note  S(5419**2)  =  3 x 31 x 313 x 1009
      S(1009**2)  =  3 x 37 x 9181
      S(9181**1)  =  2 x 4591
      S(31**2)    =  3 x 331            S(331**2)  =  3 x 7 x 5233
```

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

```
4591**Y 55333**Y 2780923**Y 135676061629**2 201858430533847**2 Proposition  6
3x127xP 3x367xP    3x19xP   3x1321x23011xP          3x13xQ
4591**Y 55333**Y 2780923**Y 135676061629**2 201858430533847**A Proposition  5
3x127xP 3x367xP    3x19xP   3x1321x23011xP
4591**Y 55333**Y 2780923**Y 135676061629**4                    Proposition 8B
3x127xP 3x367xP    3x19xP       11x101xQ
4591**Y 55333**Y 2780923**Y 135676061629**B                    Proposition  5
3x127xP 3x367xP    3x19xP
4591**Y 55333**Y 2780923**4                                    Proposition  1
3x127xP 3x367xP     41xQ         Q  is composite  and QHNPFLT  110,000,000
4591**Y 55333**Y 2780923**6                                    N  Exceeds   M
3x127xP 3x367xP  113x127x9857x1027530lxQ(composite) QHNPFLT 70,000,000
4591**Y 55333**Y 2780923**C                                    Proposition  5
3x127xP 3x367xP
4591**Y 55333**Z                                               Proposition 8D
3x127xP 11x251x3191x46411x22926121
4591**Y 55333**6                                               N  Exceeds   M
3x127xP 43x617x135913x141667xP
4591**Y 55333**10                                              N  Exceeds   M
3x127xP  23x89xQ  Q is composite and  QHNPFLT  13,374,901
4591**Y 55333**D                                               Proposition  5
3x127xP
4591**Z                                                        Proposition 8E
5x11xP
4591**6                                                        Block     5233
254322041x36825984292217
4591**10                                                       Block     5233
881x2113651xQ  Q is composite and has no prime factor less than  54,499,901
4591**12                                                       N  Exceeds   M
 521xP
4591**E                                                        Proposition  5
```

Lemma 5.2 Let N be an odd perfect number less than M. Then, not both of the following can happen simultaneously.

A) 5419**Y1||N B) 1009**Y2||N
where Y1(Mod 3) = 2 = Y2(Mod 3)

Note S(5419**2) = 3 x 31 x 313 x 1009 S(1009**2) = 3 x 37 x P

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(9181**2) = 3 x 7 x 7 x 13 x 31 x 1423

Possibilities And Reasons By Which They May Be Excluded

```
   9181**X          where  X(Mod 4)  =  1                        Lemma        5.1

   9181**Y          where  Y(Mod 3)  =  2                        Proposition  6

   9181**Z          where  Z(Mod 5)  =  4                        Corollary  3.1
  5x11311xP
   9181**6                     19937168287**Y                    N  Exceeds    M
   113x17431x15251867xP        3x541x25981x46153x204245563
   9181**6                     19937168287**4                    Proposition  6
   113x17431x15251867xP            11xQ
   9181**6                     19937168287**A                    Proposition  5
   113x17431x15251867xP
   9181**10                        635471**Y                     Block       23
  23x635471xQ                        7xP
   9181**10                        635471**4                     Corollary  3.1
  23x635471xQ                      5x31x61xQ
   9181**10                        635471**C                     Block       23
  23x635471xQ  Q is composite and QHNPFLT  10,000,000
   9181**12                                                      N  Exceeds    M
   157x521x241229xQ  Q is composite and  QHNPFLT  15,624,961
   9181**C                                                       Proposition  5
```

For Case 2, let P1=3, P2=7, P3=13, P4=31, and P5=37. Now consider the fraction

$$F = \prod_{i=1}^{5} Pi^{**}Ei / S(Pi^{**}Ei)$$

where E1 > 3. If E1 = 4 (or E2 = 2), then 11 (or 19) divides N. Otherwise, both E1 > 4 and E2 > 2. In any of these cases F < 1/2 and Proposition 6 applies.

Lemma 5.3 Let N be an odd perfect number less than M. Then, not both of the following can happen simultaneously.

(A) 5419**Y||N (B) 1009**Z||N
Note S(5419**2) = 3x31x313x1009 S(1009**4) = 1037517185381

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

```
 1037517185381**X  172919530897**2                               Proposition  1
     2x3xP               3x13xQ  Q is composite and  QHNPFLT  10,000,000
 1037517185381**X  172919530897**4                               Proposition 8D
     2x3xP               11x61xQ
 1037517185381**X  172919530897**B                               Proposition  5
     2x3xP
 1037517185381**2                           127992782604139**2  N  Exceeds   M
 727x1399x8269xP Q = 469774,661989,273213  3x43x61x109x109x373xQ comp  34.8m
 1037517185381**2                           127992782604139**F  Proposition  5
 727x1399x8269xP
 1037517185381**4                                                Corollary  3.2
     5x331xQ
 1037517185381**D                                                Proposition  5
```

Lemma 5.4 Let N be an odd perfect number less than M and let 5419**Y||N. Then no one of the following can happen.

(A) 1009**6||N (B) 1009**10||N (C) 1009**12||N

Note S(5419**2) = 3 x 31 x 313 x 1009

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(1009**6) = 7 x 29 x 5077297 x 1024823381

Possibilities And Reasons By Which They May Be Excluded

1009**6	1024823381**X				Proposition 8C
	2x3x11xP				
1009**6	1024823381**2	27647957545189**1			Proposition 7
	37987xP	2x5x281x7907xP			
1009**6	1024823381**2	27647957545189**2			N Exceeds M
	37987xP	3x7x112927x8845231xP			
1009**6	1024823381**2	27647957545189**A			Proposition 5
	37987xP				
1009**6	1024823381**4				Corollary 3.2
	5x541xQ				
1009**6	1024823381**B				Proposition 5
1009**10					Block 617
617x4647193x16441151xP					
1009**12	31**Y	331**Y	157**X		Block 5233
157xP	3x331	3x7x5233	2x79		
1009**12	31**Y	331**Y	157**Y	8269**X	Proposition 7
157xP	3x331	3x7x5233	3xP	2x5x827	
1009**12	31**Y	331**Y	157**Y	8269**Y	N Exceeds M
157xP	3x331	3x7x5233	3xP	3x7xP	
1009**12	31**Y	331**Y	157**Y	8269**4	N Exceeds M
157xP	3x331	3x7x5233	3xP	61xP	
1009**12	31**Y	331**Y	157**Y	8269**C	N Exceeds M
157xP	3x331	3x7x5233	3xP		
1009**12	31**Y	331**Y	157**4		N Exceeds M
157xP	3x331	3x7x5233	11x31xP		
1009**12	31**Y	331**Y	157**6		N Exceeds M
157xP	3x331	3x7x5233	12503xP		
1009**12	31**Y	331**Y	157**10		N Exceeds M
157xP	3x331	3x7x5233	28447xP		
1009**12	31**Y	331**Y	157**12		N Exceeds M
157xP	3x331	3x7x5233	13x245753x251057xP		
1009**12	31**Y	331**Y	157**D		N Exceeds M
157xP					

Block 398581 The block of sub-cases, Block 398581 is used only in Theorem 15 and Lemma 22.3. However, the block of sub-cases labeled Block 1621 is used more than once in Block 398581. Both blocks are given below. Each sub-case or sub-subcase leads to the contradiction that is listed.

1621**Y Proposition 6 1621**6 N Exceeds M
3x7x13xP 211x4105333x20957295829
1621**Z Proposition 6 1621**A N Exceeds M
5x11xP

398581**Y 32668561**Y 2309087647**Y 441105499**Y N Exceeds M
3x1621xP 3x7x13x1693xP 3x13x12097x25621xP 3x7x13xP
398581**Y 32668561**Y 2309087647**Y 441105499**4 N Exceeds M
3x1621xP 3x7x13x1693xP 3x13x12097x25621xP 751x1201xP
398581**Y 32668561**Y 2309087647**Y 441105499**A Proposition 5
3x1621xP 3x7x13x1693xP 3x13x12097x25621xP
Block 1621
398581**Y 32668561**Y 2309087647**4
3x1621xP 3x7x13x1693xP 11x41xQ Q is composite and QHNPFLT 10,000,000
398581**Y 32668561**Y 2309087647**B Proposition 5
3x1621xP 3x7x13x1693xP
Block 1621
398581**Y 32668561**4
3x1621xP 5xQ QHNPFLT 10,000,000
398581**Y 32668561**6 N Exceeds M
3x1621xP 140813xQ QHNPFLT 9,399,979 Also, Q is composite
398581**Y 32668561**C Proposition 5
3x1621xP
398581**4 2703853428809791**2 1866871**Y 331**Y Corollary 3.2
5x1866871xP 3xQ 3x19x331xP 3x7x5233
398581**4 2703853428809791**2 1866871**4 N Exceeds M
5x1866871xP 3xQ 5x41x2019041x1330609lx2205504671
398581**4 2703853428809791**2 1866871**6 N Exceeds M
5x1866871xP 3xQ 29x1093x8387x63337x85121xP
398581**4 2703853428809791**2 1866871**D Proposition 5
5x1866871xP 3xQ Q is composite and QHNPFLT 8,799,997
398581**4 2703853428809791**E Proposition 5
5x1866871xP
398581**6 47251**Y N Exceeds M
7x113x1093x47251xQ 3x13x241xP
398581**6 47251**Z N Exceeds M
7x113x1093x47251xQ 5x101x87679lxP
398581**6 47251**6 N Exceeds M
7x113x1093x47251xQ 7xP
398581**6 47251**10 N Exceeds M
7x113x1093x47251xQ 23x23x14851x238943xP
398581**6 47251**F Proposition 5
7x113x1093x47251xQ Q is composite QHNPFLT 22,139,377
398581**G Proposition 5

Theorem 5 The number 3 x 5419 cannot be a factor of an odd perfect number N that is less than M.

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(5419**2)=3x31x313x1009 S(5419**6)=7x29x211x1499x475483x829621381

Possibilities And Reasons By Which They May Be Excluded

5419**Y	31**Y	331**Y	1009**X	Corollary 3.1
3x31x313xP	3x331	3x7x5233	2x5x101	
5419**Y	31**Y	331**Y	1009**Y	Lemma 5.2
3x31x313xP	3x331	3x7x5233	3x37x9181	
5419**Y	31**Y	331**Y	1009**Z	Lemma 5.3
3x31x313xP	3x331	3x7x5233	P	
5419**Y	31**Y	331**Y	1009**6	Lemma 5.4
3x31x313xP	3x331	3x7x5233	7x29x5077297x1024823381	
5419**Y	31**Y	331**Y	1009**10	Lemma 5.4
3x31x313xP	3x331	3x7x5233	617x4647193x16441151xP	
5419**Y	31**Y	331**Y	1009**12	Lemma 5.4
3x31x313xP	3x331	3x7x5233	157xP	
5419**Y	31**Y	331**Y	1009**16	N Exceeds M
3x31x313xP	3x331	3x7x5233	137x443x1531xQ Q is composite and QHNPFLT 10m	
5419**Y	31**Y	331**Y	1009**A	Proposition 5
3x31x313xP	3x331	3x7x5233		
5419**4	836561914831**2		347568611538691**2	N Exceeds M
1031xP	3x109x6157549xP		3x7x7x61x2377xQ(Composite) QHNPFLT 13,104,853	
5419**4	836561914831**2		347568611538691**C	Proposition 5
1031xP	3x109x6157549xP			
5419**4	836561914831**4			Proposition 8A
1031xP	5x11x61x401xQ			
5419**4	836561914831**B			Proposition 5
5419**6	829621381**X		414810691**Y	N Exceeds M
	2xP		3x13x13x3559xP	
5419**6	829621381**X		414810691**4	Proposition 8E
	2xP		5x11xQ	
5419**6	829621381**X		414810691**D	Proposition 5
	2xP			
5419**6	829621381**2		154702548132607**2	N Exceeds M
	3x1483xP		3x19x67xP	
5419**6	829621381**2		154702548132607**E	Proposition 5
	3x1483xP			
5419**6	829621381**4			Proposition 7
	5xQ			
5419**6	829621381**F			Proposition 5

5419**10	33287**Y	14221**1	N Exceeds M
67x33287x274121xQ	7x11131xP	2x13x547	
5419**10	33287**Y	14221**G	N Exceeds M
67x33287x274121xQ	7x11131xP		
5419**10	33287**Z		N Exceeds M
67x33287x274121xQ	28631xP		
5419**10	33287**6		N Exceeds M
67x33287xQ	29x197x471451xP		
5419**10	33287**O		N Exceeds M
67x33287x274121x43038337x830088529041623897			
5419**12			N Exceeds M
53xQ Q is composite and Q has no prime factor less than 24,624,991			Proposition 5
5419**R			

In Theorem 5 we have S(33287**6) = 29 x 197 x 471451 x 50508003 0371606299. To show that Q = 50508003 0371606299 is a prime number, for each prime factor P of Q - 1 = 2 x 3**4 x 7 x 17 x 184631 x 141903661, we find a prime Px which is relatively prime to Q such that both of the following are true.

Px**(Q-1) [Mod Q] is 1 and Px**[(Q-1)/P] [Mod Q] is not 1

(See Table XIII below)

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	3	1	50508003 0371606298
3	5	1	32991150 4001228137
7	3	1	27376014 3838871657
17	3	1	20967250 5211821842
184631	3	1	12932004 6516091856
141903661	3	1	16300914 4018109801

TABLE XIII

Lemma 6.1 Let N be as odd perfect number less than M. Then, the following cannot happen.

262209281**X||N where X(Mod 4) = 1

Note S(262209281**1) = 2 x 3 x 3137 x 13931

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(194086693**2) = 3 x 12277 x 740989 x 1380277

Possibilities And Reasons By Which They May Be Excluded

13931**2	194086693**2	1380277**2	2773167361**Y			N Exceeds M
P	3x12277x740989xP	3x229xP	3x13x271xP			
13931**2	194086693**2	1380277**2	2773167361**4			N Exceeds M
P	3x12277x740989xP	3x229xP	5x17291xQ(comp)QHNPFLT25,000,001			
13931**2	194086693**2	1380277**2	2773167361**A			Proposition 5
P	3x12277x740989xP	3x229xP				
13931**2	194086693**2	1380277**4				N Exceeds M
P	3x12277x740989xP	31x131x13831x17161xP				
13931**2	194086693**2	1380277**6				N Exceeds M
P	3x12277x740989xP	71xP				
13931**2	194086693**2	1380277**B				Proposition 5
P	3x12277x740989xP					
13931**2	194086693**4	91411**Y	31**Y	331**Y		Proposition 8D
P	11x31x91411xPxQ	3x43xP	3x331	3x7x5233		
13931**2	194086693**4	91411**4	31**Y	331**Y		Proposition 8A
P	11x31x91411xPxQ	5x281xP	3x331	3x7x5233		
13931**2	194086693**4	91411**6				Proposition 1
P	11x31x91411xPxQ	15359x29947x198241xP				
13931**2	194086693**4	91411**C				Proposition 1
P	11x31x91411x29784lxQ(comp) QHNPFLT 55,000,001(Apply Proposition 1)					
13931**2	194086693**6					N Exceeds M
P	Q QHNPFLT 17,599,975 and is composite					
13931**2	194086693**D					Proposition 5
P						
13931**4						Proposition 8E
5x11x491xP						

13931**6	3658159**Y	33829**Y			N Exceeds M
7x3658159xP	3x139x379x2503xP	3x13x577xP			
13931**6	3658159**Y	33829**Z	31**Y	331**Y	N Exceeds M
7x3658159xP	3x139x379x2503xP	11x31x101x5021xP			
13931**6	3658159**Y	33829**6			Proposition 1
7x3658159xP	3x139x379x2503xP	281x90007x22993111x2577314730803			
13931**6	3658159**Y	33829**E			N Exceeds M
7x3658159xP	3x139x379x2503xP				
13931**6	3658159**4		31**Y	331**Y	Proposition 1
7x3658159xP	31x41x392261x8809051xP		3x331	3x7x5233	
13931**6	3658159**F				N Exceeds M
7x3658159xP					
13931**10					Proposition 1
23x23x23x67x6733xQ	QHNPFLT 55,566,721				
13931**12					N Exceeds M
131xP					
13931**G					Proposition 5

In Case 9 of Lemma 6.1 it is assumed that 194086693**4||N and that for some X(Mod 4) = 1 it is true that 262209281**X||N. As a consequence of these conditions, S(194086693**4) = 11 x 31 x 91411 x 297841 x Q divides N also. Since Q has no prime factor less than its cube root and Q has no factor F such that for some natural number X(Mod 4) = 1, F**X||N, it follows that Q**2 divides N by Proposition 1.

Lemma 6.2 If N is an odd perfect number less than M and 127**Z||N,then the number 262209281 does not divide N.

Note S(262209281**2) = 13 x 1231 x 4296301150081

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

262209281**X				Lemma 6.1
262209281**Y	4296301150081**1	123152587**2	13339102733461**2	N Exceeds M
13x1231xP	2x17443xP	3x379xP	3x43x17491x93979xP	
262209281**Y	4296301150081**1	123152587**2	13339102733461**A	Proposition 5
13x1231xP	2x17443xP	3x379xP		
262209281**Y	4296301150081**1	123152587**4		N Exceeds M
13x1231xP	2x17443xP	Q	Q is composite and QHNPFLT25,000,001	
262209281**Y	4296301150081**1	123152587**B		N Exceeds M
13x1231xP	2x17443xP			
262209281**Y	4296301150081**2			Theorem 5
13x1231xP	3x61x5419xQ	Q is composite	QHNPFLT 1,000,000	
262209281**Y	4296301150081**C			Proposition 5
13x1231xP				
262209281**4	14621**X			Proposition8E
5x11x421x14621xQ	2x3x2437			
262209281**4	14621**Y	1644525251**Y		Proposition 7
5x11x421x14621xQ	13xP	3x7x4507x2857420699		
262209281**4	14621**Y	1644525251**4		N Exceeds M
5x11x421x14621xQ	13xP	5x11x2671x41281xP		
262209281**4	14621**Y	1644525251**D		Proposition 5
5x11x421x14621xQ	13xP			
262209281**4	14621**4	940739291**E		N Exceeds M
5x11x421x14621xQ	5x9716251xP			
262209281**4	14621**6			N Exceeds M
5x11x421x14621xQ	127x6802951xP			
262209281**4	14621**F			N Exceeds M
5x11x421x14621x6579701x477869921x4440731591				
262209281**G				Proposition 5

Block 162709

This block is used in Lemma 6.3.

162709**X	307**Y			Proposition 7
2x5x53x307	3x43x733			
162709**X	307**Z	5231**Y	3909799**Y	Proposition 7
2x5x53x307	1051x5231xP	7xP	3x2671xP	
162709**X	307**Z	5231**Y	3909799**4	N Exceeds M
2x5x53x307	1051x5231xP	7xP	11x11551x2396621xP	
162709**X	307**Z	5231**Y	3909799**A	N Exceeds M
2x5x53x307	1051x5231xP	7xP		
162709**X	307**Z	5231**4		N Exceeds M
2x5x53x307	1051x5231xP	5x601xP		
162709**X	307**Z	5231**6		N Exceeds M
2x5x53x307	1051x5231xP	71xP		
162709**X	307**Z	5231**B		N Exceeds M
2x5x53x307	1051x5231xP			
162709**X	307**6			N Exceeds M
2x5x53x307	659xP			
162709**X	307**C			N Exceeds M
2x5x53x307				
162709**2	8824793797**X	2409829**Y		N Exceeds M
3xP	2x1831xP	3x7x65029xP		
162709**2	8824793797**X	2409829**D		N Exceeds M
3xP	2x1831xP			
162709**2	8824793797**Y	425557298187947929**1		Proposition 7
3xP	3x61xP	2x5xQ		
162709**2	8824793797**Y	425557298187947929**2		N Exceeds M
3xP	3x61xP	3x13x31x157x12967xQ	Q is composite	11.2m
162709**2	8824793797**Y	425557298187947929**E		Proposition 5
3xP	3x61xP			
162709**2	8824793797**F			N Exceeds M
3xP				
162709**4				N Exceeds M
P	P= 700888562389531127981			
162709**G				N Exceeds M

Lemma 6.3 If N is an odd perfect number less than M, then 127**6||N is a false statement.

Note S(127**6) = 7 x 43 x 86353 x 162709

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(86353**4) = 11 x 281 x 1021 x 1964041 x 8970971

Possibilities And Reasons By Which They May Be Excluded

86353**X	162709**I where I(Mod 4) is not equal to 1			Block 162709
2x43117				
86353**Y	2485642321**X	1242821161**2		N Exceeds M
3xP	2xP	3x1993xP		
86353**Y	2485642321**X	1242821161**4		Proposition 7
3xP	2xP	5x41x1231xQ Q is composite		
86353**Y	2485642321**X	1242821161**A		Proposition 5
3xP	2xP			
86353**Y	2485642321**2	2202194614231**2		N Exceeds M
3xP	3x61x15331xP	3x19x337xQ(composite) QHNPFLT 10,000,000		
86353**Y	2485642321**2	2202194614231**4		Proposition 7
3xP	3x61x15331xP	5xQ		
86353**Y	2485642321**2	2202194614231**B		Proposition 5
3xP	3x61x15331xP			
86353**Y	2485642321**4			Proposition 7
3xP	5x151x1471xQ			
86353**Y	2485642321**C			Proposition 5
3xP				
86353**Z	8970971**Y	31**Y	331**Y	Proposition 6
	7x13x19x31xP	3x331	3x7x5233	
86353**Z	8970971**4		1964041**X	N Exceeds M
	5x11x811xQ		2xP	
86353**Z	8970971**4		1964041**Y	Proposition 7
	5x11x811xQ		3x7x19x31x607xP	
86353**Z	8970971**4		1964041**4	N Exceeds M
	5x11x811xQ		5x41x131x431x16267271xP	
86353**Z	8970971**4		1964041**6	N Exceeds M
	5x11x811xQ		29x43x43x1583x26209x154267x343253xP	
86353**Z	8970971**4		1964041**D	Proposition 5
	5x11x811xQ Q is composite and QHNPFLT 25,000,001			
86353**Z	8970971**6			N Exceeds M
	29x617x1124131x16141189x85044793xP			
86353**Z	8970971**E			Proposition 5
86353**6				Block 162709
7x29x29x2339x316037xP				
86353**E				N Exceeds M

Theorem 6 Let N be an odd perfect number less than M. Then, the prime 127 cannot be a factor of N unless one of the following is true.

(A) 127**12||N (B) 127**16||N (C) 127**18||N

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(127**12) HNPFLT 411,429,721 S(127**18) HNPFLT 164,599,969

Possibilities And Reasons By Which They May Be Excluded

127**Y		Theorem 5
127**Z		Lemma 6.2
127**6		Lemma 6.3
127**10	47834644354838156839**2	N Exceeds M
23xP	3x7x193xQ Q is composite QHNPFLT 11,400,001	
127**10	47834644354838156839**A	Proposition 5
23xP		
127**22	1936934955557397191559102266683483741754688985 7**1	N Exceeds M
P	2x3x7x7x577x17477xQ QHNPFLT 600,000	
127**22	19369349555573971915591022666834837417546889857**B	Proposition 5
P		
127**C		Proposition 5

Lemma 7.1 If N is an odd perfect number less than M, then not all of the following are true simultaneously.

(A) The prime 5 divides N (B) 131**Y||N
(C) 17293**X||N

Note S(131**2) = 17293 S(17293**1) = 2 x 8647

Block 2081

2081**Y 7xP	618949**Y 3x7x7xP	Proposition 7	2081**4 5x31x275251x439781	Corollary 3.1
2081**Y 7xP	618949**A	N Exceeds M	2081**B	N Exceeds M

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

8647**Y 3x7x37x157x613			Proposition 7
8647**4 41x251x2081xP	261085291**2 3x31x571x9949xP		Corollary 3.1
8647**4 41x251x2081xP	261085291**4 5x11xQ Q is composite and QHNPFLT 10,000,000		Block 2081
8647**4 41x251x2081xP	261085291**A		Proposition 5
8647**6 43x9437x393017297xP	9437**Y 109xP	817123**Y 3x31x5737x1251433	Corollary 3.1
8647**6 43x9437x393017297xP	9437**Y 109xP	817123**4 326141xP P = 1366925586944435821	Proposition 1
8647**6 43x9437x393017297xP	9437**Y 109xP	817123**6 197xQ(comp) QHNPFLT 10,000,000	Proposition 1
8647**6 43x9437x393017297xP	9437**Y 109xP	817123**B	Proposition 5
8647**6 43x9437x393017297xP	9437**Z 4481x8081xP	219049261**2 3x7xQ	Proposition 7
8647**6 43x9437x393017297xP	9437**Z 4481x8081xP	219049261**4 5x41x661xQ(composite) QHNPFLT 30,000,000	Proposition 1
8647**6 43x9437x393017297xP	9437**Z 4481x8081xP	219049261**C	N Exceeds M
8647**6 43x9437x393017297xP	9437**6 7x197xQ Q is composite and QHNPFLT 32,599,981		Proposition 1
8647**6 43x9437x393017297xP	9437**10 991x59467x4574923xP		Proposition 1
8647**6 43x9437x393017297xP	9437**12 7333x144223x24963173xP		Proposition 1

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 8647**6              9437**D                                      Proposition  5
43x9437x393017297x2621373511
 8647**10            30493**2                                      Proposition 8E
11x30493xPxQ      3x43x97x74311
 8647**10            30493**4          12512290252905ll**E         N  Exceeds   M
11x30493xPxQ          691xP
 8647**10            30493**F                                      N  Exceeds   M
11x30493x1477081xQ(composite) has no prime factor less than 100,000,000
 8647**12                                                          N  Exceeds   M
 15497x98411xQ    Q  is composite and  QHNPFLT  210,000,000
 8647**G                                                           Proposition  5
```

In Lemma 7.1 we have S(817123**4) = 326141 x 136692558 6944435821. To show that Q = 136692558 6944435821 is a prime number, for each prime factor P of Q - 1 = 2**2 x 3**4 x 5 x 42649 x 101107 x 195677, we find a prime Px which is relatively prime to Q such that both of the following are true.

Px**(Q-1) [Mod Q] is 1 and Px**[(Q-1)/P] [Mod Q] is not 1

(See Table XIV below)

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	11	1	136692558 6944435820
3	5	1	17347283 1268111628
5	3	1	54558485 3524039867
42649	3	1	84519152 6889676081
101107	3	1	47918379 7266089506
195677	3	1	74286889 5932525244

TABLE XIV

Lemma 7.2 If N is an odd perfect number less than M, then not all of the following are true simultanously.

(A) 5 divides N (B) 131**Y||N (C) 17293**6||N

where Y(Mod 3) = 2.

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(17293**6) has no prime factor less than 504,749,911.

Possibilities And Reasons By Which They May Be Excluded

5**X	3**Y	13**Y	61**Y		Proposition 6
2x3	13	3x61	3x13x97		
5**X	3**Y	13**Y	61**Z		Proposition 6
2x3	13	3x61	5x131xP		
5**X	3**Y	13**Y	61**6		N Exceeds M
2x3	13	3x61	P		
5**X	3**Y	13**Y	61**10		N Exceeds M
2x3	13	3x61	199x859xP		
5**X	3**Y	13**Y	61**12		N Exceeds M
2x3	13	3x61	187123xP		
5**X	3**Y	13**Y	61**A		N Exceeds M
2x3	13	3x61			
5**X	3**Y	13**Z			N Exceeds M
2x3	13	30941			
5**X	3**Y	13**6		5229043**2	N Exceeds M
2x3	13	P		3x31x4051xP	
5**X	3**Y	13**6		5229043**C	N Exceeds M
2x3	13	P			
5**X	3**Y	13**10			N Exceeds M
2x3	13	23x419x859xP			
5**X	3**Y	13**12			N Exceeds M
2x3	13	53x264031xP			
5**X	3**Y	13**16			N Exceeds M
2x3	13	103x443xP			
5**X	3**Y	13**18			N Exceeds M
2x3	13	P			
5**X	3**Y	13**D			N Exceeds M
2x3	13				
5**X	3**Z				Block 11
2x3	11x11				
5**X	3**U	1093**Y		398581**2	N Exceeds M
2x3	1093	3xP		3x1621xP	
5**X	3**U	1093**Y		398581**4	N Exceeds M
2x3	1093	3xP		5x1866871xP	
5**X	3**U	1093**Y		398581**E	N Exceeds M
2x3	1093	3xP			

5**X	3**U	1093**4		Corollary 3.1
2x3	1093	11x31xP		
5**X	3**U	1093**6		Proposition 7
2x3	1093	7x29x14939xP		
5**X	3**U	1093**F		N Exceeds M
2x3	1093			
5**X	3**10			N Exceeds M
2x3	23x3851			
5**X	3**12		797161**2	N Exceeds M
2x3	P		3x61x151x22996651	
5**X	3**12		797161**G	N Exceeds M
2x3	P			
5**X	3**16			N Exceeds M
2x3	1871x34511			
5**X	3**18			N Exceeds M
2x3	1579x363889			
5**X	3**22			N Exceeds M
2x3	47xP			
5**X	3**28			N Exceeds M
2x3	59x28537xP			
5**X	3**30			N Exceeds M
2x3	683x102673xP			
5**X	3**36			N Exceeds M
2x3	13097927xP			
5**X	3**40			N Exceeds M
2x3	83x2526913xP			
5**X	3**42			N Exceeds M
2x3	431xP			
5**X	3**46			N Exceeds M
2x3	1223x21997x5112661xP			
5**X	3**H			N Exceeds M
2x3				
5**I				Block 5

Lemma 7.3 If N is an odd perfect number less than M, then not both of the following are true.

(A) 5 divides N and (B) 131**Y||N where Y(Mod3) = 2.

Note S(131**2) = 17293

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

17293**X				Lemma 7.1
17293**Y	7668337**X			Proposition 8F
3x13xP	2x23xP			
17293**Y	7668337**Y	801337**X		Proposition 6
3x13xP	3x24460537xP	2x59x6791		
17293**Y	7668337**Y	801337**Y	214047262969**1	Proposition 11
3x13xP	3x24460537xP	3xP	2x5x107xP	
17293**Y	7668337**Y	801337**Y	214047262969**2	Proposition 6
3x13xP	3x24460537xP	3xP	3xQ QHNPFLT	1,000,000
17293**Y	7668337**Y	801337**Y	214047262969**A	N Exceeds M
3x13xP	3x24460537xP	3xP		
17293**Y	7668337**Y	801337**4		Proposition 8A
3x13xP	3x24460537xP	11x2251xQ		
17293**Y	7668337**Y	801337**6	24460537**1	N Exceeds M
3x13xP	3x24460537xP	911xQ	2x3271x3739	
17293**Y	7668337**Y	801337**6	24460537**2	Corollary 3.2
3x13xP	3x24460537xP	911xQ	3x331x4591xP	
17293**Y	7668337**Y	801337**6	24460537**B	N Exceeds M
3x13xP	3x24460537xP	911x25150903xQ		
17293**Y	7668337**Y	801337**C		Proposition 5
3x13xP	3x24460537xP			
17293**Y	7668337**4	207331**2		N Exceeds M
3x13xP	207331xP	3x1531xP		
17293**Y	7668337**4	207331**4		Proposition 8A
3x13xP	207331xP	5x11x131x151x1181xQ		
17293**Y	7668337**4	207331**D		N Exceeds M
3x13xP	207331xP			
17293**Y	7668337**E			N Exceeds M
3x13xP				
17293**Z	232611722621**X	1435874831**Y	384481**Y	N Exceeds M
384481xP	2x3x3x3x3xP	P	3xP	
17293**Z	232611722621**X	1435874831**Y	384481**4	Proposition 8E
384481xP	2x3x3x3x3xP	P	5x11x1181xQ	
17293**Z	232611722621**X	1435874831**Y	384481**6	N Exceeds M
384481xP	2x3x3x3x3xP	P	P	

17293**Z	232611722621**X	1435874831**Y	384481**F	Proposition 5
384481xP	2x3x3x3x3xP	P		
17293**Z	232611722621**X	1435874831**4		N Exceeds M
384481xP	2x3x3x3x3xP	5x241xP		
17293**Z	232611722621**X	1435874831**G		Proposition 5
384481xP	2x3x3x3x3xP			
17293**Z	232611722621**2	239025974415379**2		Proposition 7
384481xP	61x373x9949xP	3x7x79x229xQ		
17293**Z	232611722621**2	239025974415379**H		Proposition 5
384481xP	61x373x9949xP			
17293**Z	232611722621**4			Corollary 3.1
384481xP	5x11x31xQ			
17293**Z	232611722621**I			Proposition 5
384481xP				
17293**6				Lemma 7.2
Q	Q is composite and has no prime factor less than 504,749,911			
17293**10	261581**1			Proposition 8E
11x261581xQ	2x3xP			
17293**10	261581**2		61**X	Corollary 3.1
11x261581xQ	61x307xP		2x31	
17293**10	261581**2		61**Y	Proposition 8A
11x261581xQ	61x307xP		3x13x97	
17293**10	261581**2		61**J	N Exceeds M
11x261581xQ	61x307xP			
17293**10	261581**K			N Exceeds M
11x261581xQ	Q is composite and QHNPFLT 59,499,901			
17293**L				Proposition 5

Block 571

Block 571 is to be used only when 5 divides N or when a few sufficiently small primes divide N. It is first used in Theorem 7.

```
          Block 13537
  13537**X      967**Y                        N>M   13537**2              N>M
   2x7xP        3x67xP                               3x523xP
  13537**X      967**4   875296605041**2  C 3.1   13537**4              N>M
   2x7xP          P           19x31xQ                48271xP
  13537**X      967**4   875296605042**A     N>M   13537**6              N>M
   2x7xP          P                                  29x43xP
  13537**X      967**6                        N>M   13537**B              N>M
   2x7xP     7x43x211xP
  13537**X      937**C                        N>M

 571**Y                                                Proposition  6
3x7x103x151
 571**Z         11631811**Y                            Proposition  7
5x1831xP        3x7x13x19x37x37x73x211x1237
 571**Z         11631811**4                            N  Exceeds   M
5x1831xP         5x11x11xP
 571**Z         11631811**6                            N  Exceeds   M
5x1831xP            127xQ  Q is composite and  QHNPFLT  10,000,000
 571**Z         11631811**A                            Proposition  5
5x1831xP
 571**6    41284013010997**X     2662454083**Y         Proposition  6
29x29xP        2x7753xP      3x7x7x13x1861x688249xP
 571**6    41284013010997**X     2662454083**4         N  Exceeds   M
29x29xP        2x7753xP           11x1961651xQ(composite)  QHNPFLT 10,000,000
 571**6    41284013010997**X     2662454083**B         Proposition  5
29x29xP        2x7753xP
 571**6    41284013010997**2                           N  Exceeds   M
29x29xP    3x13x43x271x311827x541141xP
 571**6    41284013010997**C                           Proposition  5
29x29xP
 571**10             419**Y                            Block    13537
  419xQ               13xP
 571**10             419**Z                            Corollary  3.1
  419xQ               31xP
 571**10             419**6                            N  Exceeds   M
  419xQ             7603xP
 571**10             419**10                           N  Exceeds   M
  419xQ            11x3719xP
 571**10             419**12                           N  Exceeds   M
  419xQ            11987xQ  Q  is composite and   QHNPFLT  24,624,991
 571**10             419**D                            N  Exceeds   M
  419xQ  Q is composite and QHNPFLT  100,000,000
```

571**12	4603**Y		N Exceeds M
79x4603x6543343xQ	3x7xP		
571**12	4603**4		N Exceeds M
79x4603xQ	11x911x208511x214891		
571**12	4603**6	693696942149**1	Proposition 1
79x4603x6543343xQ	36583x374879xP	2x3x5x5x53xP	
571**12	4603**6	693696942149**Y	N Exceeds M
79x4603x6543343xQ	36583x374879xP	13x127xQ(comp)	QHNPFLT 10,000,000
571**12	4603**6	693696942149**G	N Exceeds M
79x4603x6543343xQ	36583x374879xP		
571**12	4603**10		N Exceeds M
79x4603x6543343xQ	23x89x727xQ	Q is composite and QHNPFLT 37,125,001	
571**12	4603**E		N Exceeds M
79x4603x6543343x89218117xP			N Exceeds M
571**F			

For Case 19 of Block 571 it is assumed both that 571**12||N and that 693696942149**1||N. Since 693696942149 is not of the form 26n + 1, this prime cannot be a factor of S(571**12). Now that S(571**12) has no prime factor less than its cube root, is not a perfect square and has no prime factor F such that for some natural number X(Mod 4) = 1, F**X||N, then by Proposition 1 the square of S(571**12) divides N. Under these circumstances N exceeds M and we have our desired contradiction.

Theorem 7 The number 5 x 131 cannot be a factor of an odd perfect number N which is less than M unless 131**16||N.

61**X		Corollary 3.1	61**6	N Exceeds M
2x31			P	
61**Y		Proposition 7	61**10	N Exceeds M
3x13x97			199x859xP	
61**4	21491**2	N Exceeds M	61**12	N Exceeds M
5x131xP	P		187123xP	
61**4	21491**A	N Exceeds M	61**B	N Exceeds M
5x131xP				

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(131**6)=127x40100703931 S(131**10)=23x67x353x1453x15401x123210869

Possibilities And Reasons By Which They May Be Excluded

131**Y				Lemma 7.3
131**Z	973001**X			Proposition 11
	2x3x257xP			
131**Z	973001**Y	3832922749**X	528679**Y	Proposition 7
5x61xP	13x19xP	2x5x5x5x29xP	3x7x7x31x61334653	
131**Z	973001**Y	3832922749**X	528679**4	N Exceeds M
5x61xP	13x19xP	2x5x5x5x29xP	4831xP	
131**Z	973001**Y	3832922749**X	528679**6	N Exceeds M
5x61xP	13x19xP	2x5x5x5x29xP	7309xP	
131**Z	973001**Y	3832922749**X	528679**A	Proposition 5
5x61xP	13x19xP	2x5x5x5x29xP		
131**Z	973001**Y	3832922749**2		Proposition 8F
5x61xP	13x19xP	3xQ QHNPFLT 10,000,000		
131**Z	973001**Y	3832922749**4	813311**B	N Exceeds M
5x61xP	13x19xP	813311xQ(composite) QHNPFLT 25,000,001		
131**Z	973001**Y	3832922749**6		Proposition 5
5x61xP	13x19xP	71x36191xQ(comp) QHNPFLT 10,000,000		
131**Z	973001**Y	3832922749**C		Proposition 5
5x61xP	13x19xP			
131**Z	973001**4			Corollary 3.1
5x61xP	5x31x12601x19801xP			
131**Z	973001**6			Block 61
5x61xP	7x449xQ Q is composite and QHNPFLT 49,374,991			
131**Z	973001**D			Proposition 5
5x61xP				
131**6	40100703931**2	536022151933939852231**2		N Exceeds M
127xP	3xP	3xQ(composite) QHNPFLT 9,999,997		
131**6	40100703931**2	536022151933939852231**E		Proposition 5
127xP	3xP			

```
   131**6        40100703931**4                                            N  Exceeds    M
    127xP           5x199321xQ  Q is composite and   QHNPFLT 25,000,001
   131**6        40100703931**F                                            Proposition  5
    127xP
   131**10         123210869**X                                            Proposition 11
                    2x3x5xP
   131**10         123210869**2                                            Corollary  3.1
                  19x31xP
   131**10         123210869**4          71**Y     5113**X     2557**Y  Proposition  7
                     71xQ                5113      2x2557 3x7x13x13x19x97
   131**10         123210869**4          71**12                            N  Exceeds    M
                     71xQ
   131**10         123210869**4          71**18                            N  Exceeds    M
                     71xQ  Q is composite and  QHNPFLT 25,000,001
   131**10         123210869**6                                            N  Exceeds    M
                     P
   131**10         123210869**G                                            Proposition  5

   131**12                          25061845458479893445539**2           N  Exceeds    M
   13x79xP                              3x163x1723x54829xP
   131**12                          25061845458479893445539**H           Proposition  5
   13x79xP
   131**18                                                                 Block       571
    571xQ  Q is composite and has no prime factor less than 100 million
   131**22                                                                 N  Exceeds    M
 47x139x277xP
   131**I                                                                  Proposition  5
```

For the second case of Theorem 7 it is assumed that for some natural number X(Mod 4) = 1, 973001**X||N which implies that the prime 257 divides N. By Proposition 2, P = 257 necessarily must appear to an even power in the prime factorization of N. The prime 257 is of the form 2**n + 1 and except for a power greater than 255, only an odd power X(Mod 4) = 1 of a prime P will be such that S(P**X) is divisible by 257. We apply Proposition 11.

Theorem 8 If N is an odd perfect number less than M and if 61 divides N then either 61**X||N or 61**Y||N.

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

```
   61**4           131**16            21491**Y   461884573**1    N  Exceeds   M
5x131x21491           Q                  P     2x23x1583x6343
   61**4           131**16            21491**Y   461884573**A    N  Exceeds   M
5x131x21491           Q                  P
   61**4           131**16            21491**4                   Corollary  3.1
5x131x21491           Q                5x31xQ
   61**4           131**16            21491**6                   N  Exceeds   M
5x131x21491           Q      7x1009x3571x149381x209707xP
   61**4           131**16            21491**B                   Theorem      7
5x131x21491           Q     Q  is composite
   61**6     52379047267**2         6619231**Y   768670625449**1 Proposition  7
    P         3x6619231xP             3x19xP     2x5x5x7x13x811xP
   61**6     52379047267**2         6619231**Y   768670625449**2 N  Exceeds   M
    P         3x6619231xP             3x19xP       3x41641xP
   61**6     52379047267**2         6619231**Y   768670625449**C N  Exceeds   M
    P         3x6619231xP             3x19xP
   61**6     52379047267**2         6619231**4                   Proposition 8A
    P         3x6619231xP           5x11x101xQ
   61**6     52379047267**2         6619231**6                   N  Exceeds   M
    P         3x6619231xP      113x1933x42197xQ composite  QHNPFLT 63,000,000
   61**6     52379047267**2         6619231**D                   Proposition  5
    P         3x6619231xP
   61**6     52379047267**4                                      Proposition  1
    P              Q  Q is composite and    QHNPFLT  25,000,001
   61**6     52379047267**E                                      Proposition  5
    P
   61**10 4242586390571**2                             1130011**2 N  Exceeds   M
        7x13x19x241x39667x1130011x963689941               3x61xP
   61**10 4242586390571**2                             1130011**4 N  Exceeds   M
        7x13x19x241x39667x1130011x963689941        5x11x41x790351xP
   61**10 4242586390571**2                             1130011**R N  Exceeds   M
        7x13x19x241x39667x1130011x963689941
   61**10 4242586390571**F                                        Proposition  5
 199x859xP
   61**12 1442146675646079l**2        187123**Y                   N  Exceeds   M
  187123xP 3x73x367x1133191xP         3x19x4729xP
   61**12 1442146675646079l**2        187123**G                   N  Exceeds   M
  187123xP 3x73x367x1133191xP
   61**12 1442146675646079l**H                                    Proposition  5
  187123xP
```

61**16 36275943774350895510464675 9**I		Proposition 5
103xP		
61**18 60712781828773132166057742705 1**J		Proposition 5
229xP		
61**22 409578448863774427631697090276261 55549**1		N Exceeds M
47xP		
61**22 409578448863774427631697090276261 55549**K		Proposition 5
47xP		
61**28		N Exceeds M
12703x37991x59503651xQ	Q is composite and QHNPFLT 263,124,541	
61**L		Proposition 5

In Theorem 8 we have S(768670625449**2) = 3 x 41641 x 472974976 9289286337. To show that Q = 472974976 9289286337 is a prime number, for each prime factor P of Q - 1 = 2**6 x 3 x 7 x 83 x 18439 x 2299447187, we find a prime Px which is relatively prime to Q such that both of the following are true.

Px**(Q-1) [Mod Q] is 1 and Px**[(Q-1)/P] [Mod Q] is not 1

(See Table XV below)

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	5	1	472974976 9289286336
3	3	1	76 8670625449
7	5	1	27353810 0060377909
83	3	1	449833353 9338622424
18439	3	1	267822683 1780885345
2299447187	3	1	409957911 1588202156

TABLE XV

Lemma 9.1 If N is an odd perfect number less than M and if 3 divides N, then there is no X such that X(Mod 4) = 1 for which 8269**X||N and either

(A) 827**Y||N or (B) 827**Z||N

Note S(8269**1) = 2 x 5 x 827

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

```
827**Y      684757**2    156297611269**2                    Proposition  7
  P            3xP       3x7x5479x16381xQ
827**Y      684757**2    156297611269**4   131**16          N  Exceeds   M
  P            3xP           131xP
827**Y      684757**2    156297611269**A                    Proposition  5
  P            3xP
827**Y      684757**4        91141**Y     14551**Y 1907623**2 Proposition  7
  P         41x91141xP   3x37x37x139xP     3x37xP   3x7x13xP
827**Y      684757**4        91141**Y     14551**Y 1907623**4 Proposition 8E
  P         41x91141xP   3x37x37x139xP     3x37xP     11xQ
827**Y      684757**4        91141**Y     14551**Y 1907623**B N  Exceeds   M
  P         41x91141xP   3x37x37x139xP     3x37xP
827**Y      684757**4        91141**Y     14551**4          Proposition 8E
  P         41x91141xP   3x37x37x139xP 5x11x41x241xP
827**Y      684757**4        91141**Y     14551**C          N  Exceeds   M
  P         41x91141xP   3x37x37x139xP
827**Y      684757**4        91141**4                       N  Exceeds   M
  P         41x91141xP     5x89521xP
827**Y      684757**4        91141**6                       Proposition  7
  P         41x91141xP     7x29x71xQ
827**Y      684757**4        91141**D                       N  Exceeds   M
  P         41x91141xP
827**Y      684757**6                                       Block        5
  P         25253971xQ  QHNPFLT 48,878,971
827**Y      684757**E                                       Proposition  5
  P
827**Z 7677461821**2                                        Proposition  6
 .1xP    3x13x73xP
827**Z 7677461821**4              61**Y                     Proposition  6
 61xP        5xP                 3x13x97
827**Z 7677461821**F                                        Proposition  5
 61xP
```

Lemma 9.2 If N is an odd perfect number such that 3 divides N, then no one of the following can happen.

(A) 8269**X||N (B) 8269**Y||N (C) 8269**Z||N

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(8269**2) = 3 x 7 x 3256411 S(8269**4) = 61 x 76653970756001

Possibilities And Reasons By Which They May Be Excluded

8269**X	827**Y				Lemma 9.1
2x5xP	P				
8269**X	827**Z				Lemma 9.1
2x5xP	61xP				
8269**X	827**6				Proposition 7
2x5xP	7xQ				
8269**X	827**10	25833032065234l**2			Proposition 7
2x5xP	7129xP	7xQ			
8269**X	827**10	25833032065234l**A			Proposition 5
2x5xP	7129xP				
8269**X	827**12	23054903**Y			Proposition 7
2x5xP	23054903xQ	7x739xP			
8269**X	827**12	23054903**4			Proposition 8E
2x5xP	23054903xQ	11xQ			
8269**X	827**12	23054903**6			N Exceeds M
2x5xP	23054903xQ	Q Q is composite and QHNPFLT 32,599,981			
8269**X	827**12	23054903**I			Proposition 5
2x5xP	23054903xQ	Q is composite QHNPFLT 37,680,241			
8269**X	827**16				N Exceeds M
2x5xP	1100581xP				
8269**X	827**B				Proposition 5
2x5xP					
8269**Y	3256411**Y	446467**Y	58369**X		Corollary 3.1
	3x7x733x1543xP	3x31x36721xP	2x5x13xP		
8269**Y	3256411**Y	446467**Y	58369**Y	65581**X	Proposition 6
	3x7x733x1543xP	3x31x36721xP	3x17317xP	2x11x11xP	
8269**Y	3256411**Y	446467**Y	58369**Y	65581**2	Proposition 6
	3x7x733x1543xP	3x31x36721xP	3x17317xP	3x13x43x433xP	
8269**Y	3256411**Y	446467**Y	58369**Y	65581**4	Proposition 9
	3x7x733x1543xP	3x31x36721xP	3x17317xP	5xQ	
8269**Y	3256411**Y	446467**Y	58369**Y	65581**6	N Exceeds M
	3x7x733x1543xP	3x31x36721xP	3x17317xP	71x211x967xP	
8269**Y	3256411**Y	446467**Y	58369**Y	65581**B	N Exceeds M
	3x7x733x1543xP	3x31x36721xP	3x17317xP		
8269**Y	3256411**Y	446467**Y	58369**4		Proposition 6
	3x7x733x1543xP	3x31x36721xP	11xQ		
8269**Y	3256411**Y	446467**Y	58369**6		N Exceeds M
	3x7x733x1543xP	3x31x36721xP	556668lxP		

```
8269**Y      3256411**Y      446467**Y   58369**C            N  Exceeds   M
        3x7x733x1543xP  3x31x36721xP
8269**Y      3256411**Y      446467**4     1543**Y           Proposition  6
        3x7x733x1543xP     41xQ    3x13x13x37x127
8269**Y      3256411**Y      446467**4     1543**4           N  Exceeds   M
        3x7x733x1543xP     41xQ      11x2591xP
8269**Y      3256411**Y      446467**4     1543**6           N  Exceeds   M
        3x7x733x1543xP     41xQ        197xP
8269**Y      3256411**Y      446467**4     1543**D           N  Exceeds   M
        3x7x733x1543xP     41xQ  Q is composite and  QHNPFLT 49,000,001
8269**Y      3256411**Y      446467**6                       N  Exceeds   M
        3x7x733x1543xP  43x9619x28547xQ(composite)  QHNPFLT 100,000,000
8269**Y      3256411**Y      446467**E                       Proposition  5
        3x7x733x1543xP
8269**Y      3256411**4                                      Proposition  9
               5xQ
8269**Y      3256411**6                                      Theorem      6
          43x127xQ  Q is composite and  QHNPFLT  49,374,991
8269**Y      3256411**F                                      Proposition  5

8269**Z      76653970756001**1                               Proposition 11
 61xP   2x3x3x3x191x1097x2383x2843
8269**Z      76653970756001**2         822277**1       61**Y N  Exceeds   M
 61xP           7x822277xP             2x197x2087     3x13x97
8269**Z      76653970756001**2         822277**G             N  Exceeds   M
 61xP           7x822277xP
8269**Z      76653970756001**H                               Proposition  5
 61xP
```

Lemma 9.3 The number 3 x 8269 cannot be a factor of an odd perfect number N which is less than M.

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

8269**X where X(Mod 4) = 1	Lemma 9.2
8269**Y where Y(Mod 3) = 2	Lemma 9.2
8269**Z where Z(Mod 5) = 4	Lemma 9.2
8269**6 3197204941662418048422 91**A P	N Exceeds M
8269**10 23**Y 79**Y 43**Y 23x23xQ 7x79 7x7x7x43 3xP	Proposition 1
8269**10 23**Y 79**Y 43**4 23x23xQ 7x79 7x7x7x43 P	Proposition 1
8269**10 23**Y 79**Y 43**6 23x23xQ 7x79 7x7x7x43	Proposition 1
8269**10 23**Y 79**Y 43**R 23x23xQ 7x79 7x7x7x43	N Exceeds M
8269**10 23**Y 79**4 39449441**X 23x23xQ 7x79 P 2x3x1993x3299	Proposition 11
8269**10 23**Y 79**4 39449441**AA 23x23xQ 7x79 P	N Exceeds M
8269**10 23**Y 79**AAA 23x23xQ 7x79	Block 79
8269**10 23**4 292561**1 23x23xQ P	Proposition 11
8269**10 23**4 292561**B 23x23xQ P	N Exceeds M
8269**10 23**6 23x23xQ 29xP	N Exceeds M
8269**10 23**10 23x23xQ 11xP	N Exceeds M
8269**10 23**12 23x23xQ 769161xP	N Exceeds M
8269**10 23**16 23x23xQ 103xP	N Exceeds M
8269**10 23**F 23x23xQ Q is composite and QHNPFLT 100,000,000	N Exceeds M
8269**12 12143**Y 147464593**1 13x12143xP P 2x29x1087x2339	N Exceeds M
8269**12 12143**Y 147464593**G 13x12143xP P	N Exceeds M
8269**12 12143**H 13x12143xP	N Exceeds M
8269**I	Proposition 5

Theorem 9 If N is an odd perfect number less than M and if 157 divides N then for some X such that X(Mod 4) = 1, 157**X||N unless it is true that 157**16||N. S(157**16) HNPFLT 195,624,781

The following block of sub-cases is provided for use in this lemma.

Block 12503

12503**Y	166849**X	937**2	292969**2	PR6	12503**Y	166849**2	1325655031**C	
937xP	2x5x5x47x71	3xP	3x61x127xQ		937xP	3x7xP		
12503**Y	166849**X	937**2	292969**A	TH8	12503**Y	166849**4		N>M
937xP	2x5x5x47x71	3xP			937xP	9491xP		
12503**Y	166849**X	937**4		N>M	12503**Y	166849**D		N>M
937xP	2x5x5x47x71	8431xP			937xP			
12503**Y	166849**X	937**B		N>M	12503**4	465238870891**E		N>M
937xP	2x5x5x47x71				131x401xP			
12503**Y	166849**2	1325655031**2		N>M	12503**6			N>M
937xP	3x7xP	3x331x4447xP			7x2003xP			
12503**Y	166849**2	1325655031**4		PR7	12503**F			N>M
937xP	3x7xP	5x31xQ						

Note S(292969**2) = 3 x 61 x 127 x 139 x 163 x 163

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(157**2) = 3 x 8269 S(157**12) = 13 x 245753 x 251057 x P

Possibilities And Reasons By Which They May Be Excluded

157**Y					Lemma 9.3
157**Z	31**Y	331**Y	1793161**X		Proposition 6
11x31xP	3x331	3x7x5233	2x7x349x367		
157**Z	31**Y	331**Y	1793161**2	20070961**X	Proposition 6
11x31xP	3x331	3x7x5233	3x53401xP	2x241xP	
157**Z	31**Y	331**Y	1793161**2	20070961**2	Proposition 8B
11x31xP	3x331	3x7x5233	3x53401xP	3x19x1026391xP	
157**Z	31**Y	331**Y	1793161**2	20070961**4	Corollary 3.1
11x31xP	3x331	3x7x5233	3x53401xP	5xQ	
157**Z	31**Y	331**Y	1793161**2	20070961**6	N Exceeds M
11x31xP	3x331	3x7x5233	3x53401xP	7x127x127x2017xP	
157**Z	31**Y	331**Y	1793161**2	20070961**A	Proposition 5
11x31xP	3x331	3x7x5233	3x53401xP		
157**Z	31**Y	331**Y	1793161**4		Corollary 3.1
11x31xP	3x331	3x7x5233	5xQ		
157**Z	31**Y	331**Y	1793161**6		N Exceeds M
11x31xP	3x331	3x7x5233	1163xQ Q is composite and QHNPFLT 10,000,000		

```
157**Z     31**Y     331**Y     1793161**B                                     Proposition  5
11x31xP    3x331     3x7x5233
157**6        1205476469**X                                                     Proposition 8E
12503xP  2x3x3x5x11x19x19x3373
157**6        1205476469**2                                                     Block     12503
12503xP             P        P  =  1453173518518184431
157**6        1205476469**4                                                     Block     12503
12503xP     61x2131x44041xP
157**6        1205476469**C                                                     Proposition  5
12503xP
157**10       321910472390668481**X   28447**Y    20750263**2                   N  Exceeds   M
28447xP               2x3xP             3x13xP   3x7x367x73999xP
157**10       321910472390668481**X   28447**Y    20750263**4                   N  Exceeds   M
28447xP               2x3xP             3x13xP        521xP
157**10       321910472390668481**X   28447**Y    20750263**D                   N  Exceeds   M
28447xP               2x3xP             3x13xP
157**10       321910472390668481**X   28447**4                                  N  Exceeds   M
28447xP               2x3xP          151x12161xP
157**10       321910472390668481**X   28447**6                                  N  Exceeds   M
28447xP               2x3xP           4733x44101xP
157**10       321910472390668481**X   28447**10                                 N  Exceeds   M
28447xP               2x3xP           11x72689xP
157**10       321910472390668481**X   28447**E                                  Proposition  5
28447xP               2x3xP          P  =  536517453984447
157**10       321910472390668481**2                                             N  Exceeds   M
28447xP            19x984391xQ(composite)  QHNPFLT  8,799,997
157**10       321910472390668481**F                                             Proposition  5
28447xP
157**12                   245753**X                 251057**Y                   Proposition  6
                       3x3x3x3x37x41            7x7x7x7x103xP
157**12                   245753**X                 251057**4                   N  Exceeds   M
                       3x3x3x3x37x41              11x131xP
157**12                   245753**X                 251057**6                   N  Exceeds   M
                       3x3x3x3x37x41             617x19237xP
157**12                   245753**X                 251057**G                   Proposition  5
                       3x3x3x3x37x41
157**12                   245753**2                  61723**2                   N  Exceeds   M
                     7x7x19x1051x61723               3x7x19xP
157**12                   245753**2                  61723**4                   Proposition  6
                     7x7x19x1051x61723               31x41xQ
157**12                   245753**2                  61723**H                   N  Exceeds   M
                     7x7x19x1051x61723
157**12                   245753**4       31**Y         331**Y                  Block    251057
13x245753x251057xP          31xQ(comp)    3x331         3x7x5233 QHNPFLT 25,000,001
157**12                   245753**I                                             N  Exceeds   M
13x245753x251057xP
157**18                   234271**Y                                             N  Exceeds   M
234271xQ               3x7x22279xP
157**18                   234271**4                                             Corollary  3.1
234271xQ                  5x11x31xQ
```

157**18 234271**6 N Exceeds M
234271xQ Q Q is composite and QHNPFLT 49,374,991
157**18 234271**J Proposition 5
234271xQ Q is composite and Q has no prime factor less than 20,000,000
157**22 N Exceeds M
47x277x829x967xQ Q is composite and QHNPFLT 41,381,001
157**K Proposition 5

In Theorem 9 we have S(1205476469**2) = Q = 145317351 8518184431. To show that Q is a prime number, for each prime factor P of

Q - 1 = 2 x 3 x 5 x 11 x 19**2 x 3373 x 3616429407

we find a prime Px which is relatively prime to Q such that both of the following are true.

Px**(Q-1) [Mod Q] is 1 and Px**[(Q-1)/P] [Mod Q] is not 1

(See Table XVI below)

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	3	1	145317351 8518184430
3	53	1	1205476469
5	3	1	64044221 0372377390
11	3	1	40025959 3937789667
19	3	1	18975174 7262469384
3373	3	1	39770634 0050547040
3616429407	3	1	28701716 8045045180

TABLE XVI

Note: In order to determine that Q = 32191047 2390668481, we write

Q - 1 = 2**6 x 5 x 11 x 1847 x 49513718867

and consider the following

3**[(Q-1)/2] [Mod Q]	=	32191047 2390668480
3**[(Q-1)/5] [Mod Q]	=	8326263 4119340882
3**[(Q-1)/11] [Mod Q]	=	4723472 2182717920
3**[(Q-1)/1847] [Mod Q]	=	14639733 5466369025
3**[(Q-1)/49513718867] [Mod Q]	=	25985696 5282428230.

Since 3**(Q-1) [Mod Q] = 1

it follows that Q is a prime number.

Lemma 10.1 Let N is an odd perfect number less than M. Then there is no Y such that 39449441**Y||N when 79**Z||N.

Note S(39449441**2) = 19 x 271 x 349 x 6163 x 140521

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

140521**X	4133**A					Prop	11
2x17x4133							
140521**Y	6582097321**X	88643**B				Prop	11
3xP	2x137x271xP						
140521**Y	6582097321**2	864111418057**1	61**Y			Prop	6
3xP	3x7x61x39139xP	2x7xP	3x13x97				
140521**Y	6582097321**2	864111418057**2	31**Y	331**Y	61**X	Block	5233
3xP	3x7x61x39139xP	3x31x73x7535167xP	3x331	3x7x5233	2x31		
140521**Y	6582097321**2	864111418057**2	31**Y	331**Y	61**Y	Prop	6
3xP	3x7x61x39139xP	3x31x73x7535167xP	3x331	3x7x5233			
140521**Y	6582097321**2	864111418057**C				N Exceeds	M
3xP	3x7x61x39139xP						
140521**Y	6582097321**4	71**Y	5113**X	2557**Y		Prop	7
3xP	5x71x2066321xQ	5113	2x2557	3x7x13x13x19x97			
140521**Y	6582097321**4	71**12				N Exceeds	M
3xP	5x71x2066321xQ						
140521**Y	6582097321**4	71**18				N Exceeds	M
3xP	5x71x2066321xQ	QHNPFLT	10,000,000				
140521**Y	6582097321**D					Prop	5
3xP							
140521**4	131**16	6163**2	31**Y	331**Y		Prop	7
5x131x57301xP		3x19x31xP	3x331	3x7x5233			
140521**4	131**16	6163**E				Theorem	7
5x131x57301xP							
140521**6	265492638202994383806238463963**F					Prop	5
29xP							
140521**G						Prop	5

Lemma 10.2 Let N is an odd perfect number less than M. Then no one of the following is true.

(A) 79**Z||N (B) 79**6||N

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(79**4) = 39449441 S(79**6) = 281 x 337 x 1289 x 2017

Possibilities And Reasons By Which They May Be Excluded

79**Z	39449441**X 2x3x1993x3299					Proposition 11
79**Z	39449441**Y					Lemma 10.1
79**Z	39449441**4 5x11x61xQ	61**X 2x31				Corollary 3.1
79**Z	39449441**4 5x11x61xQ	61**Y 3x13x97				Proposition 6
79**Z	39449441**6 29x197x6813101xQ	QHNPFLT 12,600,043				N Exceeds M
79**Z	39449441**A					Proposition 5
79**6	1289**X 2x3x5xP	2017**Y 3x331x4099	331**Y 3x7x5233			Proposition 7
79**6	1289**X 2x3x5xP	2017**4 11x11x31x7151xP				Proposition 8E
79**6	1289**X 2x3x5xP	2017**6 7x29xQ				Proposition 7
79**6	1289**X 2x3x5xP	2017**10 23x683x16061xP				Proposition 8I
79**6	1289**X 2x3x5xP	2017**12 131xQ(composite)	131**16 QHNPFLT 15,624,961			Theorem 7
79**6	1289**X 2x3x5xP	2017**B				Proposition 5
79**6	1289**Y 127xP	13093**X 2xP	6547**Y 3x7x7x19xP	15349**Y 3x13x79xP	127**U U>11	Proposition 1
79**6	1289**Y 127xP	13093**X 2xP	6547**Y 3x7x7x19xP	15349**4 11x31xP		N Exceeds M
79**6	1289**Y 127xP	13093**X 2xP	6547**Y 3x7x7x19xP	15349**6 43xQ(composite) QHNPFLT 75m		Proposition 1
79**6	1289**Y 127xP	13093**X 2xP	6547**Y 3x7x7x19xP	15349**C		N Exceeds M
79**6	1289**Y 127xP	13093**X 2xP	6547**4 61x19681xP	61**Y 3x13x97		N Exceeds M
79**6	1289**Y 127xP	13093**X 2xP	6547**6 43x113x91309x179327x989952181			N Exceeds M
79**6	1289**Y 127xP	13093**X 2xP	6547**D			N Exceeds M

```
79**6      1289**Y     13093**Y              57146581**X  127**U   Proposition  1
            127xP        3xP                    2x23xP     U>11
79**6      1289**Y     13093**Y              57146581**Y           Theorem      6
            127xP        3xP                  3x7x73xP
79**6      1289**Y     13093**Y              57146581**E           N  Exceeds   M
            127xP        3xP
79**6      1289**Y     13093**4                                    N  Exceeds   M
            127xP      11x2371x7621x8311x17791
79**6      1289**Y     13093**6                                    N  Exceeds   M
            127xP        43xP
79**6      1289**Y     13093**F                                    N  Exceeds   M
            127xP
79**6      1289**Z    815891**Y          665678939773**X  82591**Y N  Exceeds   M
         41x82591xP      P               2x491297x677471 3x283x811xP
79**6      1289**Z    815891**Y          665678939773**X  82591**4 N  Exceeds   M
         41x82591xP      P               2x491297x677471  5x11x61x131x1871xP
79**6      1289**Z    815891**Y          665678939773**X  82591**6 N  Exceeds   M
         41x82591xP      P               2x491297x677471   421xQ(comp)  20.6m
79**6      1289**Z    815891**Y          665678939773**X  82591**G N  Exceeds   M
         41x82591xP      P               2x491297x677471
79**6      1289**Z    815891**Y          665678939773**2           N  Exceeds   M
         41x82591xP      P                3x13x264757xP
79**6      1289**Z    815891**Y          665678939773**4           N  Exceeds   M
         41x82591xP      P                     Q  Q is composite  10m
79**6      1289**Z    815891**2          665678939773**H           Proposition  5
         41x82591xP      P
79**6      1289**Z    815891**4                                    Corollary  3.1
         41x82591xP    5x31xQ
79**6      1289**Z    815891**6                                    N  Exceeds   M
         41x82591xP    71x197xQ  Q is composite and QHNPFLT 49,374,991
79**6      1289**Z    815891**I                                    Proposition  5
         41x82591xP
79**6      1289**6    2333727793859393**1     93029091679**2       N  Exceeds   M
           7x281xP      2x3x37x113xP          3x271x1146679xP
79**6      1289**6    2333727793859393**1     93029091679**4       N  Exceeds   M
           7x281xP      2x3x37x113xP            31x92791xP
79**6      1289**6    2333727793859393**1     93029091679**J       Proposition  5
           7x281xP      2x3x37x113xP
79**6      1289**6    2333727793859393**2                          N  Exceeds   M
           7x281xP    13x19x151x538789xQ Q  is composite and QHNPFLT 16,236,991
79**6      1289**6    2333727793859393**K                          Proposition  5
           7x281xP
79**6      1289**10               2017**1            1009**Y       Proposition  1
             Q                      2xP              3x37xP
79**6      1289**10               2017**1            1009**U       N  Exceeds   M
             Q                      2xP
79**6      1289**10               2017**2  4099**Y    331**Y       N  Exceeds   M
             Q                    3x331xP   3x7xP    3x7x5233
79**6      1289**10               2017**2  4099**V    331**Y       N  Exceeds   M
             Q                    3x331xP            3x7x5233
```

79**6	1289**10	2017**L		N Exceeds M
	Q Q is composite and	QHNPFLT 37,125,001		
79**6	1289**12	2861**1		N Exceeds M
	2861xQ	2x3x3x3xP		
79**6	1289**12	2861**2	430957**X	N Exceeds M
	2861xQ	19xP	2x11x19xP	
79**6	1289**12	2861**2	430957**Y	N Exceeds M
	2861xQ	19xP	3x7x859x1831x5623	
79**6	1289**12	2861**2	430957**R	N Exceeds M
	2861xQ	19xP		
79**6	1289**12	2861**4		N Exceeds M
	2861xQ	5x92941xP		
79**6	1289**12	2861**S		N Exceeds M
	2851xQ Q is composite and	QHNPFLT 31,624,711		
79**6	1289**16			N Exceeds M
	137x152423xP			
79**6	1289**T			Proposition 5

Lemma 10.3 Let N be an odd perfect number less than M. Then, it is not true that 79**12||N.

Note S(79**12) = 13 x Q where Q has no prime factor less than 23,133,464,641 and is composite

Proof. All possibilities and the reasons by which they may be excluded are listed below. The details are included elsewhere herein.

13**X	7**Y	19**Y	127**Y		Theorem	6
13**X	7**Y	19**Y	127**Z		Theorem	6
13**X	7**Y	19**Y	127**6		Theorem	6
13**X	7**Y	19**Y	127**10		Theorem	6
13**X	7**Y	19**Y	127**12		Proposition	1
13**X	7**Y	19**Y	127**16		Proposition	1
13**X	7**Y	19**Y	127**18		Proposition	1
13**X	7**Y	19**Y	127**A		Theorem	6
13**X	7**Y	19**Z	151**Y	1093**Y	Proposition	1
13**X	7**Y	19**Z	151**Y	1093**Z	Proposition	1
13**X	7**Y	19**Z	151**Y	1093**6	Proposition	1
13**X	7**Y	19**Z	151**Y	1093**B	Proposition	1

13**X	7**Y	19**Z	151**4			Block	151
13**X	7**Y	19**Z	151**C	See Block 151	for details	N Exceeds	M
13**X	7**Y	19**6	70841**Y	39103**Y	67411**D	Proposition	1
13**X	7**Y	19**6	70841**Y	39103**E		Proposition	1
13**X	7**Y	19**6	70841**4	61**Y		Proposition	6
13**X	7**Y	19**6	70841**F			Proposition	1
13**X	7**Y	19**10		62060021**2		Proposition	6
13**X	7**Y	19**10		62060021**G		Proposition	1
13**X	7**Y	19**12		133338869**H		Proposition	1
13**X	7**Y	19**16		99995282631947**I		Proposition	1
13**X	7**Y	19**18		10991220309223964384022l**J		Proposition	1
13**X	7**Y	19**22				N Exceeds	M
13**X	7**Y	19**28	233**K			Proposition	11
13**X	7**Y	19**L				Proposition	1
13**X	7**Z	2801**Y	4933**Y	331**Y	127**Y	Theorem	6
13**X	7**Z	2801**Y	4933**Y	331**Y	127**Z	Theorem	6
13**X	7**Z	2801**Y	4933**Y	331**Y	127**6	Theorem	6
13**X	7**Z	2801**Y	4933**Y	331**Y	127**R	Proposition	1
13**X	7**Z	2801**Y	4933**4			Proposition	6
13**X	7**Z	2801**Y	4933**6	3221**X		Proposition	11
13**X	7**Z	2801**Y	4933**6	3221**S		Block	3221
13**X	7**Z	2801**Y	4933**T			Proposition	1
13**X	7**Z	2801**4	6294091**2			Proposition	7
13**X	7**Z	2801**4	6294091**U			Proposition	1
13**X	7**Z	2801**6		2884629032993**V		Proposition	1
13**X	7**Z	2801**AA				Proposition	1
13**X	7**U	4733**Y				Proposition	1
13**X	7**U	4733**4				Proposition	1
13**X	7**U	4733**6				Proposition	1
13**X	7**U	4733**BB				N Exceeds	M
13**X	7**10		293459**2	31089033**2		Proposition	1
13**X	7**10		293459**2	31089033**CC		Proposition	1
13**X	7**10		293459**4			N Exceeds	M
13**X	7**12		16148168401**2			N Exceeds	M
13**X	7**12		16148168401**DD			Proposition	2
13**X	7**16		2767631689**2			N Exceeds	M
13**X	7**16		2767631689**EE			N Exceeds	M
13**X	7**18		4534166740403**FF			Proposition	1
13**X	7**22		31479823396757**GG			Proposition	1
13**X	7**HH					N Exceeds	M
13**II	all other cases are eliminated by the Propositions and					N Exceeds	M

Theorem 10 If N is an odd perfect number less than M and if 79 divides N, then either 79**Y||N or 79**18||N.

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(79**10) = 5479 x 1750258119644519

Possibilities And Reasons By Which They May Be Excluded

79**Z				Lemma	10.2
79**6				Lemma	10.2
79**10	1750258119644519**2	5479**2	2647**2	N Exceeds	M
	7x61x379x134161xQ	3x19x199xP	3x127x18397		
79**10	1750258119644519**2	5479**2	2647**A	N Exceeds	M
	7x61x379x134161xQ	3x19x199xP			
79**10	1750258119644519**2	5479**4	901331344494641**1	Proposition	6
	7x61x379x134161xQ	P	2x3x17x23xP		
79**10	1750258119644519**2	5479**4	901331344494641**B	N Exceeds	M
	7x61x379x134161xQ	P			
79**10	1750258119644519**2	5479**6		N Exceeds	M
	7x61x379x134161xQ	547xP			
79**10	1750258119644519**2	5479**C		N Exceeds	M
	7x61x379x134161xQ Q is composite and QHNPFLT 14,183,821				
79**10	1750258119644519**D			Proposition	5
79**12				Lemma	10.3
13xQ	Q has no prime factor less than 23,133,464,641 and Q is composite				
79**16	290194897**X			Proposition	11
103x290194897xQ	2x7x167xP				
79**16	290194897**2			N Exceeds	M
103x290194897xQ	3x39518767xP				
79**16	290194897**4			N Exceeds	M
103x290194897xQ	11x71x34141x73755x16023991xP				
79**16	290194897**E			N Exceeds	M
103x290194897xQ	Q is composite and has no prime factor less than 510,374,851				
79**22	139**Y	499**Y	109**X	Proposition	7
47x139xQ	3x13x499	3x7x109x109	2x5x11		
79**22	139**Y	499**Y	109**2	N Exceeds	M
47x139xQ	3x13x499	3x7x109x109	3x7xP		
79**22	139**Y	499**Y	109**4	N Exceeds	M
47x139xQ	3x13x499	3x7x109x109	31x191xP		
79**22	139**Y	499**Y	109**D	N Exceeds	M
47x139xQ	3x13x499	3x7x109x109			
79**22	139**Y	499**4		N Exceeds	M
47x139xQ	3x13x499	11x101xP			
79**22	139**Y	499**6		N Exceeds	M
47x139xQ	3x13x499	4831x13567xP			

79**22	139**Y	499**E	N Exceeds M
47x139xQ	3x13x499		
79**22	139**4	9170881**1	N Exceeds M
47x139xQ	41xP	2x7x19x23xP	
79**22	139**4	9170881**F	N Exceeds M
47x139xQ	41xP		
79**22	139**6	87683177**1	Proposition 11
47x139xQ	29x2857xP	2x3x11x17x17x4597	
79**22	139**6	87683177**G	N Exceeds M
47x139xQ	29x2857xP		
79**22	139**10		N Exceeds M
47x139xQ	199xP		
79**22	139**12		N Exceeds M
47x139xQ	6449x205661xP		
79**22	139**16		N Exceeds M
47x139xQ	Q Q is composite and QHNPFLT 105,624,571		
79**22	139**18		N Exceeds M
47x139xQ	419x303431xP		
79**22	139**H		N Exceeds M
47x139xQ	Q is composite and QHNPFLT 100,000,000		
79**I			Proposition 5

Lemma 11.1 If N is an odd perfect number less than M and if 1093**X||N for some X such that X(Mod 4) = 1, then for no Y such that for Y(mod 3) = 2 will 613**Y||N.

Note S(1093**1) = 2 x 547   S(613**2) = 3 x 7 x 17923

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(17923**2) = 3 x 13 x 31 x 265717

Possibilities And Reasons By Which They May Be Excluded

```
  17923**Y 31**Y 331**Y  265717**Y 3362180467**2 502086025579**2 Block      5233
           3x331  3x7xP    3x7xP  3x7x13x82471xP       3xP
  17923**Y 31**Y 331**Y  265717**Y 3362180467**2 502086025579**4 N  Exceeds   M
           3x331  3x7xP    3x7xP  3x7x13x82471xP    20551xQ (composite) 25m
  17923**Y 31**Y 331**Y  265717**Y 3362180467**2 502086025579**A Proposition  5
           3x331  3x7xP    3x7xP  3x7x13x82471xP
  17923**Y 31**Y 331**Y  265717**Y 3362180467**4                 N  Exceeds   M
           3x331  3x7xP    3x7xP  181x12451x139599lxP
  17923**Y 31**Y 331**Y  265717**Y 3362180467**B                 Proposition  5
           3x331  3x7xP     3x7xP
  17923**Y 31**Y 331**Y  265717**4                               Proposition  6
           3x331  3x7xP  31x41x101xQ
  17923**Y 31**Y 331**Y  265717**6                               Block      5233
           3x331  3x7xP      Q  Q is composite and    QHNPFLT  49,375,001
  17923**Y 31**Y 331**Y  265717**C                               Proposition  5
           3x331  3x7xP
  17923**4  27308381**2                                          Proposition 8B
           7x13x23743xP        P  =  345155611
  17923**4  27308381**4                                          Proposition  9
11x343540871xP
  17923**4  27308381**D                                          N  Exceeds   M
11x343540871xP
  17923**6                     28769600332597803883**E           Theorem      0
 43x127x211xP
  17923**10                                                      N  Exceeds   M
23x23x1013x4423x6491x102433xP
  17923**F                                                       Proposition  5
```

Lemma 11.2 If N is an odd perfect number less than M and if 1093**X||N for some X such that X(Mod 4) = 1, then for no Z such that Z(Mod 5) is equal to 4 will 613**Z||N when the number 21 divides N.

Note S(613**4) = 131 x 20161 x 53551

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

53551**Y	955921051**2	85584439140289**2	31**Y	331**Y	Proposition 1
3xP	3x3559xP	3x19x31x9319xQ(com)	3x331	3x7xP	10.8mil
53551**Y	955921051**2	85584439140289**A			Proposition 5
3xP	3x3559xP				
53551**Y	955921051**4		131**16		Theorem 7
3xP	5x101x191x1238101xQ	Q is composite			
53551**Y	955921051**B				Proposition 5
3xP					
53551**4	131**16				Proposition 9
5x11x121021xP					
53551**6		62841393017789667289**2			N Exceeds M
7x43x197x6329xP		3xQ(composite)		QHNPFLT 8.8 million	
53551**6		62841393017789667289**C			Proposition 5
7x43x197x6329xP					
53551**10					N Exceeds M
617x13597xQ	Q is composite and	QHNPFLT 59,499,901			
53551**D					Proposition 5

Note: S(53551**6) = 7 x 43 x 197 x 6329 x 6284139301 7789667289 where

Q - 1 = 6284139301 7789667288 = 2**3 x 3 x 7 x 31063 x 1204 1847562057

11**[(Q-1)/2] [Mod Q]	=	6284139301 7789667288
5**[(Q-1)/3] [Mod Q]	=	6098780575 8840658834
3**[(Q-1)/7] [Mod Q]	=	53551
3**[(Q-1)/31063] [Mod Q]	=	4968218542 0457529255
3**[(Q-1)/12041847562057] [Mod Q]	=	2822507997 3201669207

Theorem 11 If N is an odd perfect number less than M and if 1093**X||N for some X such that X(Mod 4) = 1, then the number 3 x 7 x 613 cannot divide N.

Block 23431

23431**Y 3x7x37x661xP	1069**Y 3x13x139xP	N>M	23431**4 5x167191xP	PR9
23431**Y 3x7x37x661xP	1069**4 39251xP	N>M	23431**B	N>M
23431**Y 3x7x37x661xP	1069**A	N>M		

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(613**6) = 43 x 71 x 55721 x 312410911

Possibilities And Reasons By Which They May Be Excluded

613**Y	where Y(Mod 3) = 2			Lemma 11.1
613**Z	where Z(Mod 5) = 4			Lemma 11.2
613**6	312410911**2 3x163xP	199592183280697**2 3x7x7xP	P**A	Proposition 5
613**6	312410911**2 3x163xP	199592183280697**B		Proposition 5
613**6	312410911**4			Proposition 9
613**6	312410911**C			Proposition 5
613**10 2332903x7221259xP		445455365264339**Y 29947xQ	29947**Y 3x211xP	N Exceeds M
613**10 2332903x7221259xP		445455365264339**Y 29947xQ	29947**4 11x4451xP	N Exceeds M
613**10 2332903x7221259xP		445455365264339**Y 29947xQ	29947**6 7x31319x184997xP	Proposition 1
613**10 2332903x7221259xP		445455365264339**Y 29947xQ Q is composite and QHNPFLT 10 million	29947**D	N Exceeds M
613**10 2332903x7221259xP		445455365264339**E		Proposition 5
613**12 15263x42017xQ		15263**Y 61x163xP	61**Y 3x13x97	Block 23431
613**12 15263x42017xQ		15263**4 181x211x881xP		N Exceeds M
613**12 15263x42017xQ		15263**6 43xQ(composite) and QHNPFLT 110,000,000		Proposition 1
613**12 15263x42017xQ Q is composite and QHNPFLT 150,000,000		15263**F		N Exceeds M

613**16	1123**Y	Proposition 11
17x1123x72504389xQ	3x127x3313	
613**16	1123**G	Proposition 11
17x1123x72504389xQ	Q has no prime factor less than 72,504,389	
613**18		N Exceeds M
1103x2053x2538097xQ	Q is composite and QHNPFLT 25,000,000	
613**H		Proposition 5

In Theorem 11, whenever we assumed that 613**12||N, we also assumed that

S(613**12) = 15263 x 42017 x Q

divides N. Here, Q = 43971 3502256042 0452546651. To imply that Q is composite, it is sufficient to state the fact that 5**(Q-1)[Mod Q] is equal to

7946 1885125571 7211577331

and hence is not equal to 1.

In Theorem 11, whenever we assumed that 613**16||N we simultaneously assumed that S(613**16) = 17 x 1123 x Q also divides N. One basic assumption for Theorem 11 is that for some natural number X(Mod 4) = 1 1093**X||N. The prime 17 is of the form 2**n + 1. Proposition 2 tells us that 17 must appear to an even power in the prime factorization of N. Hence, for some prime P(Mod 17) = 1 and natural number W(Mod 17) = -1 the prime 17 will appear only as a factor of S(P**W) and will appear to the first power. In applying Proposition 11, the smallest P is 103 and the smallest W is 16. The condition for P greater than or equal to 103 and W greater than or equal to 16, P**W||N easily brings about the contradiction that N exceeds M.

Theorem 12 If N is an odd perfect number less than M and if 29**Y||N, then it is not true that 67**16||N whenever the prime 7 divides N.

Note S(29**2) = 13 x 67 and S(67**16) = 239 x 443 x 647 x Q

where Q = 11070911xP

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

239**Y	3019**Y				Proposition 6
19x3019	3x7x7x13x13x367				
239**Y	3019**Z	855641**X	142607**Y		Proposition 11
19x3019	11x2551x3461xP	2x3xP	P		
239**Y	3019**Z	855641**X	142607**A		Proposition 1
19x3019	11x2551x3461xP	2x3xP			
239**Y	3019**Z	855641**Y			N Exceeds M
19x3019	11x2551x3461xP	73x139x919x78511			
239**Y	3019**Z	855641**Z			N Exceeds M
19x3019	11x2551x3461xP	5x241xP			
239**Y	3019**Z	855641**B			N Exceeds M
19x3019	11x2551x3461xP				
239**Y	3019**6	71**Y	5113**X	2557**Y	Proposition 6
19x3019	71x11257x43933xP	5113	2x2557	3x7x13x13x19x97	
239**Y	3019**6	71**12			Proposition 1
19x3019	71x11257x43933xP				
239**Y	3019**6	71**18			Proposition 1
19x3019	71x11257x43933xP				
239**Y	3019**10				N Exceeds M
19x3019	23x6271xP				
239**Y	3019**12				N Exceeds M
19x3019	1301x1327x115831xQ	Q is composite and	QHNPFLT	30,000,000	
239**Y	3019**C				Proposition 5
19x3019					
239**Z	3276517921**X	1638258961**2			N Exceeds M
P	2xP	3x139x3559x942301xP			
239**Z	3276517921**X	1638258961**4			Corollary 3.1
P	2xP	5x11x31xQ			
239**Z	3276517921**X	1638258961**D			Proposition 5
P	2xP				
239**Z	3276517921**2	58666684033**X			Proposition 11
P	3x67x283x3217xP	2x7x7x227x443xP			
239**Z	3276517921**2	58666684033**2	62852942701**1		Proposition 6
P	3x67x283x3217xP	3x13x43x313x104323xP	2x11xP		
239**Z	3276517921**2	58666684033**2	62852942701**J		N Exceeds M
P	3x67x283x3217xP	3x13x43x313x104323xP			
239**Z	3276517921**2	58666684033**E			Proposition 5
P	3x67x283x3217xP				

```
239**Z    3276517921**4              21491**Y                  N  Exceeds   M
  P       5x41x21491xQ                421xP
239**Z    3276517921**4              21491**4                  Corollary  3.1
  P       5x41x21491xQ          5x31x1361081xP
239**Z    3276517921**4              21491**6                  N  Exceeds   M
  P       5x41x21491xQ     7x1009x3571x149381x209707xP
239**Z    3276517921**4              21491**F                  N  Exceeds   M
  P       5x41x21491xQ  Q is composite and  QHNPFLT  25,000,001
239**Z    3276517921**G                                        Proposition  5
  P
239**6              921960493427**2                            N  Exceeds   M
7x29xP                37x12799xP
239**6              921960493427**4                            N  Exceeds   M
7x29xP              71x23971x170194lxQ  QHNPFLT 9,000,001
239**6              921960493427**H                            Proposition  5
7x29xP
239**10                                                        N  Exceeds   M
 23xP
239**12                                                        N  Exceeds   M
23167xP
239**16                                                        N  Exceeds   M
 17xQ  Q  is composite and    QHNPFLT 133,624,591
239**18                                                        N  Exceeds   M
2625839xP
239**I                                                         Proposition  5
```

In Theorem 12 we have S(239**12) = 23167 x 15056 5668459085 2648797983. To show that Q = 15056 5668459085 2648797983 is a prime number, for each prime factor P of Q - 1 = 2 x 3**3 x 107 x 372263433 8601727417, we find a prime Px which is relatively prime to Q such that both of the following are true.

Px**(Q-1) [Mod Q] is 1 and Px**[(Q-1)/P] is not 1

In doing this we use the following facts

```
3**[(Q-1)/2] [Mod Q]               =     15056 5668459085 2648797982
5**[(Q-1)/3] [Mod Q]               =      7138 5785255021 7220572442
3**[(Q-1)/7] [Mod Q]               =     13731 7778872684 3084528039
3**[(Q-1)/107] [MOd Q]             =       723 0814561590 2603529782
3**[(Q-1)/3722634338601722417] [Mod Q]  =    1013 4256433333 2858923993
```

Block 761 The following block of sub-cases is provided for use in Lemma 13.1, Lemma 13.2, Lemma 13.8 and Lemma 19.1. Each subcase leads to the contradiction as indicated.

```
761**X                     Theorem      6   761**Z    67164484721**4 N  Exceeds  M
2x3x127             Q   is composite         5xP      5x11x661x9851xQ(composite)
761**Y      579883**Y      Proposition  6   761**Z    67164484721**B Proposition 5
579883        3x79xP                         5xP
761**Y      579883**Z      N  Exceeds   M   761**6  28059705349957**1N  Exceeds  M
579883   (comp)211xQ    QHNPFLT 22 million 29x239xP   2x103x5023xP
761**Y      579883**6      N  Exceeds   M   761**6  28059705349957**2N  Exceeds  M
579883      468623xQ    QHNPFLT 4 million  29x239xP 3x37x73x13478329x7209154292510161
761**Y      579883**A      Proposition  5   761**6  28059705349957**CN  Exceeds  M
579883                                     29x239xP
761**Z 67164484721**X      Proposition  7   761**10                  N  Exceeds  M
 5xP      2x3xP                           23x67x11551xP
761**Z 67164484721**2      N  Exceeds   M   761**D
 5xP   13x155413x554503x4026658309
```

Block175897 The following block of sub-cases is provided for use in Lemma 13.3 and Lemma 13.4. Each sub-case leads to the contradiction as indicated.

```
175897**Y  3304489**X     Proposition  7     175897**6       37409**X Proposition 7
3x3121xP   2x5x7x47207                   7x71x127x37409xP 2x3x5x29xP
175897**Y  3304489**Y     Proposition  6     175897**6       37409**Y N  Exceeds  M
3x3121xP   3x109x139xP                   7x71x127x37409xP 10753xP
175897**Y  3304489**4     Proposition  6     175897**6       37409**4 N  Exceeds  M
3x3121xP      61xP                       7x71x127x37409xP   11xP
175897**Y  3304489**6     Proposition  6     175897**6       37409**6 N  Exceeds  M
3x3121xP     113xP                       7x71x127x37409xP   7xP
175897**Y  3304489**A     Proposition  5     175897**6       37409**B N  Exceeds  M
3x3121xP                                 7x71x127x37409xP
175897**Z       61**X     Proposition  1    175897**C                 N  Exceeds  M
  61xQ           2xP
175897**Z       61**Y     Proposition  6    175897**C                 N  Exceeds  M
  61xQ      Q  is composite  and  QHNPFLT  10,000,000
```

Lemma 13.1 If N is an odd perfect number less than M and if both 67**Z||N and 26881**X||N, then 7 x 13 x 29 cannot divide N.

Note S(67**4) = 761 x 26881 S(26881**1) = 2 x 13441

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

13441**Y 3x37xP	1627693**Y 3x7x37xP	3409763359**2 3x7x67x73xP	Proposition 6
13441**Y 3x37xP	1627693**Y 3x7x37xP	3409763359**4 124721xQ(composite) QHNPFLT 16,000,001	N Exceeds M
13441**Y 3x37xP	1627693**Y 3x7x37xP	3409763359**A	Proposition 5
13441**Y 3x37xP	1627693**4 151x1091xQ Q is composite and QHNPFLT 50,000,001		Proposition 1
13441**Y 3x37xP	1627693**6 2003xP		Block 761
13441**Y 3x37xP	1627693**B		Proposition 5
13441**4 5xP	6528127566670081**2 3x709xQ		Proposition 7
13441**4 5xP	6528127566670081**C		Proposition 5
13441**6 7x76819xQ	76819**2 3x67x2203xP		Proposition 1
13441**6 7x76819xQ	76819**D Q is composite and has no prime factor less than 100,000,000		Proposition 1
13441**10 23x89x929897x5662933xQ(composite) QHNPFLT 47,124,991			N Exceeds M
13441**E			N Exceeds M

Note: S(13441**6) = 7 x 76819 x 1096617341 9001404059.

"Q = 1096617341 9001404059 is composite" is implied by the fact that

5**(Q-1) [Mod Q] = 928895480 9960284664.

Lemma 13.2 If N is an odd perfect number less than M and if 67**Z||N, then the number 7 x 13 x 29 cannot divide N.

Note S(67**4) = 761 x 26881

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

26881**X		Lemma 13.1
26881**Y 3x43xP	5601667**Y 3x19x2287xP	Proposition 6
26881**Y 3x43xP	5601667**4 11x521xQ	Proposition 6
26881**Y 3x43xP	5601667**6 7xQ QHNPFLT 50,000,000	Proposition 1
26881**Y 3x43xP	5601667**A	Proposition 5
26881**4 5x31xP		Corollary 3.1
26881**6 7xQ	Q is composite and has no prime factor less than 49,374,991	Block 761
26881**10 67xQ	Q is composite and has no prime factor less than 47,124,991	Block 761
26881**B		Proposition 5

Note: S(26881**6) = 7 x 539002 3289258691 1014992641.

"Q = 539002 3289258691 1014992641 is composite" is implied by the fact that

5**(Q-1) [Mod Q] = 154422 7725587319 6336668178.

Lemma 13.3 If N is an odd perfect number less than M and also if 67**6||N and either 522061**X||N or 522061**Y||N then 7 x 29 cannot divide N.

Note S(67**6) = 175897 x 522061

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

522061**X 2xP	261031**Y 3xP	22712481331**2 3x3825817xP	44945241607543**2 3x7x7x322633xP	Block 175897
522061**X 2xP	261031**Y 3xP	22712481331**2 3x3825817xP	44945241607543**A	Proposition 5
522061**X 2xP	261031**Y 3xP	22712481331**4 5xQ		Proposition 9
522061**X 2xP	261031**Y 3xP	22712481331**B		Proposition 5
522061**X 2xP	261031**4 5x971x1021x7151xP	130974955021**2 3xQ		Proposition 7
522061**X 2xP	261031**4 5x971x1021x7151xP	130974955021**4 5x11x31xQ		Corollary 3.1
522061**X 2xP	261031**4 5x971x1021x7151xP	130974955021**C		Proposition 5
522061**X 2xP	261031**6 7x43x31151xP	33737717566401346905161987**D		Proposition 5
522061**X 2xP	261031**E			Proposition 5
522061**Y 3xP	90849403261**X 2xP	45424701631**2 3xQ		Proposition 1 and Block 175897 Q is composite and QHNPFLT 12,799,999
522061**Y 3xP	90849403261**X 2xP	45424701631**4 5xQ		Proposition 9
522061**Y 3xP	90849403261**X 2xP	45424701631**E		Proposition 5
522061**Y 3xP	90849403261**2 3x41479xP	66327652329858859**2 3x7x7x1069x6679xP		N Exceeds M
522061**Y 3xP	90849403261**2 3x41479xP	66327652329858859**F		Proposition 5
522061**Y 3xP	90849403261**4 5x18691xQ			Proposition 9
522061**Y 3xP	90849403261**G			Proposition 5

Lemma 13.4 If N is an odd perfect number less than M and also if 67**6||N then 7 x 13 x 29 cannot divide N.

Note S(67**6) = 175897 x 522061

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

```
 522061**X                                                          Lemma     13.3

 522061**Y                                                          Lemma     13.3

 522061**4 175897**X                                                Block      571
  5x571xQ   2x37xP
 522061**4 175897**A                                                Block   175897
 5x571x1326814lx1960964108711
 522061**6 175897**X      2377**Y (Also see 4.1 for details)        Proposition  6
  7x43xQ    2x37xP         3x7xP
 522061**6 175897**X      2377**4        170351**Y  433128859**2 Proposition  6
  7x43xQ    2x37xP   401x170351x467531      67xP       3x7xQ
 522061**6 175897**X      2377**4        170351**Y  433128859**4 N  Exceeds   M
  7x43xQ    2x37xP   401x170351x467531      67xP     11x4751xQ(comp)QHNPFLT 10m
 522061**6 175897**X      2377**4        170351**Y  433128859**B Proposition  5
  7x43xQ    2x37xP   401x170351x467531      67xP
 522061**6 175897**X      2377**4      - 170351**4                 N  Exceeds   M
  7x43xQ    2x37xP   401x170351x467531  5x11x41x241xP
 522061**6 175897**X      2377**4        170351**C                 N  Exceeds   M
  7x43xQ    2x37xP   401x170351x467531
 522061**6 175897**X      2377**6                                  Proposition  1
  7x43xQ    2x37xP        2213x2927149x27856823471
 522061**6 175897**X      2377**10                                 N  Exceeds   M
  7x43xQ    2x37xP   11x23x199x10781x6963023xP
 522061**6 175897**X      2377**12                                 N  Exceeds   M
  7x43xQ    2x37xP         4993xP
 522061**6 175897**X      2377**D                                  Proposition  5
  7x43xQ    2x37xP
 522061**6 175897**E                                               Block   175897
  7x43xQ    Q  is composite and has no prime factor less than  43,599,991
 522061**F                                                         Proposition  5
```

Lemma 13.5 If N is an odd perfect number less than M and if both 29**Y||N and 67**22||N, then the prime 7 cannot divide N.

Note S(29**2) = 13 x 67 S(67**22) is composite >401,299,861

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

7**Y 3x19		Proposition 6
7**Z 2801	2801**1 2x3xP	Proposition 1
7**Z 2801	2801**2 37x43x4933	N Exceeds M
7**Z 2801	2801**A	N Exceeds M
7**6 29x4733	4733**1 2x3x3x263	Proposition 11
7**6 29x4733	4733**2 P	N Exceeds M
7**6 29x4733	4733**B	N Exceeds M
7**10 1123xP		Proposition 1
7**12 P		Proposition 1
7**16 14009xP		Proposition 1
7**18 419xP		Proposition 1
7**22 47x3083xP		Proposition 1
7**B		Proposition 1

Lemma 13.6 If N is an odd perfect number less than M and also if 29**Y||N then, the prime 7 cannot divide N.
Note S(29**2) = 13 x 67

Block 34171

34171**Y 3x7x7x163xP				PR6	34171**10 N>M 23x14323xQ(comp) QHNPFLT 10,000,000
34171**4 5x11x41xP				N>M	34171**A PR5
34171**6 71x127xP	71**Y 5113	5113**X 2x2557	2557**Y 3x7x13x13x19x97	PR6	
34171**6 71x127xP	71**12			N>M	
34171**6 71x127xP	71**18			N>M	

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

67**Y 3x7x7x31	31**Y 3x331	331**Y 3x7x5233		Proposition 6
67**Z 761x26881				Lemma 13.2
67**6 175897xP				Lemma 13.4
67**10 11x89xP	1890149702927663**2 7x7x34171xQ Q is composite QHNPFLT 9,999,997			Block 34171
67**10 11x89xP	1890149702927663**A			Proposition 5
67**12 79x157x5279xP	126867415853933**X 2x3x3x7xP	157**16		N Exceeds M
67**12 79x157x5279xP	126867415853933**2 163xP1	157**X	P1**G	Proposition 5
67**12 79x157x5279xP	126867415853933**2 163xP	157**16		N Exceeds M
67**12 79x157x5279xP	126867415853933**C			Proposition 5
67**16 239x443x647x11070911xP				Theorem 12

```
67**18      7514105974000646025234004270923 97**1    97259**2   N  Exceeds    M
  P                   2x97259xP                  43x2389xP
67**18      75141059740006460252340042709 2397**1    97259**4   Proposition  1
  P                   2x97259xP                2311x297277lxP
67**18      75141059740006460252340042709 2397**1    97259**6   Proposition  1
  P                   2x97259xP                7x211x18131xQ(composite) 64m
67**18      75141059740006460252340042709 2397**1    97259**D   N  Exceeds    M
  P                   2x97259xP
67**18      75141059740006460252340042709 2397**E               Proposition  5
  P
67**22                                                           Lemma      13.5
  Q    Q  is composite and has no prime factor less than  401,299,861
67**F                                                            Proposition  5
```

In Lemma 13.6 we have S(67**10) = 11 x 89 x 189014 9702927663. To show that Q = 189014 9702927663 is a prime number, for each prime factor P of

Q - 1 = 189014 9702927662 = 2 x 11**2 x 781 0535962511

we find a prime Px which is relatively prime to Q such that both of the following are true.

Px**(Q-1) [Mod Q] is 1 and Px**[(Q-1)/P] [Mod Q] is not 1

The following facts are used to produce the desired results.

5**[(Q-1)/2] [Mod Q]	=	189014 9702927662
3**[(Q-1)/11] [Mod Q]	=	300763
3**[(Q-1)/7810535962511] [Mod Q]	=	122649 3705408260

Lemma 13.7 If 7 is a factor of an odd perfect number N less than M, then there is no Z such that Z(Mod 5) = 4 and at the same time it is true that 29**Z||N.

Note S(29**4) = 732541

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

732541**X					Proposition 11
2x47xP					
732541**Y	15349897**X	247579**Y	208759**Y	31**Y	Block 5233
3x43x271xP	2x31xP	3x97x1009xP	3x31x769xP		
732541**Y	15349897**X	247579**Y	208759**4		N Exceeds M
3x43x271xP	2x31xP	3x97x1009xP	P		
732541**Y	15349897**X	247579**Y	208759**6		Block 5233
3x43x271xP	2x31xP	3x97x1009xP	80683xP		
732541**Y	15349897**X	247579**Y	208759**A		Proposition 5
3x43x271xP	2x31xP	3x97x1009xP			
732541**Y	15349897**X	247579**4			N Exceeds M
3x43x271xP	2x31xP	521x12871591x560256646231			
732541**Y	15349897**X	247579**6			Block 5233
3x43x271xP	2x31xP	113x10151xP			
732541**Y	15349897**X	247579**B			Proposition 5
3x43x271xP	2x31xP				
732541**Y	15349897**Y	4091671**Y	3507600181**X		Proposition 11
3x43x271xP	3x97x197887xP	3x37x43xP	2x89xP		
732541**Y	15349897**Y	4091671**Y	3507600181**Y		N Exceeds M
3x43x271xP	3x97x197887xP	3x37x43xP	3x79xP		
732541**Y	15349897**Y	4091671**Y	3507600181**4		Proposition 9
3x43x271xP	3x97x197887xP	3x37x43xP	5x1181x1871xQ		
732541**Y	15349897**Y	4091671**Y	3507600181**C		Proposition 5
3x43x271xP	3x97x197887xP	3x37x43xP			
732541**Y	15349897**Y	4091671**4			Proposition 9
3x43x271xP	3x97x197887xP	5xQ			
732541**Y	15349897**Y	4091671**6			N Exceeds M
3x43x271xP	3x97x197887xP	197x50989xP			
732541**Y	15349897**Y	4091671**D			Proposition 5
3x43x271xP	3x97x197887xP				
732541**Y	15349897**4				Block 43
3x43x271xP	Q	Q is composite and QHNPFLT 25,000,001			
732541**Y	15349897**6				N Exceeds M
3x43x271xP	29x953x967x10067xP				
732541**Y	15349897**E				Proposition 5
3x43x271xP					

732541**4	2617241417881**X	14380447351**2	Proposition 7
5x45121x487681xP	2x7x13xP	3x7x7x67xQ	
732541**4	2617241417881**X	14380447351**4	N Exceeds M
5x45121x487681xP	2x7x13xP	5x271x9851xQ(composite)	QHNPFLT25,000,000
732541**4	2617241417881**X	14380447351**F	Proposition 5
5x45121x487681xP	2x7x13xP		
732541**4	2617241417881**2		Proposition 7
5x45121x487681xP	3xQ		
732541**4	2617241417881**G		N Exceeds M
5x45121x487681xP			
732541**6			N Exceeds M
P			
732541**H			Proposition 5

Note: S(208759**4) = Q = 18 9925339619 0032164881 where

Q - 1 = 2**7 x 3 x 5 x 11 x 1543 x 5828 0473527503

Also,

23**[(Q-1)/2] [Mod Q]	=	18 9925339619 0032164880
3**[(Q-1)/3] [Mod Q]	=	6 0674475614 8457707165
5**[(Q-1)/5] [Mod Q]	=	909778 4039789479
3**[(Q-1)/11] [Mod Q]	=	17 0923184169 6619070582
3**[(Q-1)/1543] [Mod Q]	=	15 7059097076 1445265936
3**[(Q-1)/58280473527503] [Mod Q]	=	18 5102607624 2856351533

Lemma 13.8 Let N is an odd perfect number less than M; then it is not true that 29**6||N.

Note S(29**6) = 7 x 88009573

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

88009573**X	830279**A		Proposition 11
2x53xP			
88009573**Y	320374117039**2	113490363435541**X	N Exceeds M
3x8059xP	3x7x43066201xP	2x67xP	
88009573**Y	320374117039**2	113490363435541**2	N Exceeds M
3x8059xP	3x7x43066201xP	3x2665261xQ(composite)	QHNPFLT 8.8m
88009573**Y	320374117039**2	113490363435541**B	Proposition 5
3x8059xP	3x7x43066201xP		
88009573**Y	320374117039**4	61**X	Proposition 6
3x8059xP	11x61x211x1201x17891xQ	2x31	QHNPFLT 4,000,000
88009573**Y	320374117039**4	61**Y	Proposition 6
3x8059xP	11x61x211x1201x17891xQ	3x13x97	
88009573**Y	320374117039**C		Proposition 5
3x8059xP			
88009573**4			Block 761
11x761xP			
88009573**6		71**Y 5113**X 2557**Y	Proposition 6
7x29x29x43x71x22807xQ		5113 2x2557 3x7x13x13x19x97	
88009573**6		71**12	N Exceeds M
7x29x29x43x71x22807xQ			
88009573**6		71**18	N Exceeds M
7x29x29x43x71x22807xQ	Q is composite and QHNPFLT 10,000,000		
88009573**D			Proposition 5

Theorem 13 The number 7 x 29 cannot be a factor of an odd perfect number N which is less than M.

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

29**X			Proposition 7
2x3x5			
29**Y			Lemma 13.6
13x67			
29**Z			Lemma 13.7
732541			
29**6			Lemma 13.8
7xP			
29**10	18944890940537**X		Proposition 11
23xP	2x3x3x37x3359xP		
29**10	18944890940537**2	42391**Y	Proposition 6
23xP	7x13x42391x603793xP	3x37x43xP	
29**10	18944890940537**2	42391**Z	N Exceeds M
23xP	7x13x42391x603793xP	5x589811xP	
29**10	18944890940537**2	42391**6	N Exceeds M
23xP	7x13x42391x603793xP	113x197xP	
29**10	18944890940537**2	42391**A	N Exceeds M
23xP	7x13x42391x603793xP		
29**10	18944890940537**B		Proposition 5
23xP			
29**12	4748492087**2	1117740400785709**1	Proposition 11
521x148123xP	20173xP	2x5x13x43x59x137x167x167x887	
29**12	4748492087**2	1117740400785709**2	N Exceeds M
521x148123xP	20173xP	3x67x367x1063xP	
29**12	4748492087**2	1117740400785709**C	Proposition 5
521x148123xP	20173xP		
29**12	4748492087**4		N Exceeds M
521x148123xP	11xP		
29**12	4748492087**D		Proposition 5
521x148123xP			
29**16	33505187587603**2	5090660731996044113653**1	Proposition 11
3911x1977917xP	3x7x10501xP	2x17x37x17599x49921xP	
29**16	33505187587603**2	5090660731996044113653**2	N Exceeds M
3911x1977917xP	3x7x10501xP	3x67x2089x906391x1006231xQ(comp) 7m	
29**16	33505187587603**2	5090660731996044113653**E	Proposition 5
3911x1977917xP	3x7x10501xP		
29**16	33505187587603**F		Proposition 5
3911x1977917xP			

```
   29**18          15719338060016381330 9**1                    Proposition  7
 1386659xP            2x3x5x127x921667xP
   29**18          157193380600163813309**2                     N  Exceeds   M
 1386659xP                      Q    QHNPFLT 8,799,997
   29**18          157193380600163813309**G                     Proposition  5
 1386659xP
   29**22                                                       Block        7
     Q    Q  is composite  and has no prime factor less than 684,774,631
   29**28                        84449**X                       Proposition  7
59x16763x84449x2428577xPxQ       2x3x5x5xP
   29**28                        84449**2              17791**2 Proposition  6
59x16763x84449x2428577xPxQ     31x67x193xP             3x7x19xP
   29**28                        84449**2              17791**H N  Exceeds   M
59x16763x84449x2428577xPxQ     31x67x193xP
   29**28                        84449**4 202632669662828351**I N  Exceeds   M
59x16763x84449x2428577xPxQ       251xP
   29**28                        84449**J                       N  Exceeds   M
59x16763x84449x2428577x14111459x58320973xP
   29**30                             7**J                      Block        7
     Q    Q  is composite  and has no prime factor less than 74,499,000
   29**K                                                        Proposition  5
```

Note: S(1117740400785709**2) = 3 x 67 x 367 x 1063 x 159 3259480686 6956129671.

Let Q = 159 3259480686 6956129671.
     Q - 1 = 2 x 3**3 x 5 x 11 x 61 x 8794278 7475117051.

```
    3**[(Q-1)/2] [Mod Q]                          =  159 3259480686 6956129670
    3**[(Q-1)/3] [Mod Q]                          =             111774 0400785709
    3**[(Q-1)/5] [Mod Q]                          =   30 8306860315 6582323626
    3**[(Q-1)/11] [Mod Q]                         =  132 1142282728 6564643293
    3**[(Q-1)/61] [Mod Q]                         =  115 0785897049 0541689920
    3**[(Q-1)/87942787475117051] [Mod Q]          =  121 9416645908 4434599169
```

Also, 8794278 7475117050 = 2 x 5 x 5 x 7 x 31 x 810 5326034573. It is left to the reader to show that 8794278 7475117051 is a prime number.

In Block 613 of the following lemma we have S(613**12) given as follows.

S(613**12) = 15263 x 42017 x 43971 3502256042 0452546651

To imply that Q = 43971 3502256052 0452546651 is composite, it is sufficient to state the fact that 5**(Q-1) [Mod Q] = 8649 9253068479 8071454124.

Lemma 14.1 If N is an odd perfect number less than M and if 1093**X||N, where X(Mod 4) = 1, then for no Y such that Y(Mod 3) = 2 is it true that 547**Y||N.

Note S(547**2) = 3 x 163 x 613

```
          Block  613
613**Y                    Lemma     11.1       613**12               N  Exceeds  M
3x7xP                                      15263x42017xQ(composite) QHNPFLT 150m
613**Z                    Lemma     11.2       613**16               Theorem     0
131x20161xP                              17x1123x72504389xQ
613**6                    N  exceeds  M        613**18               Theorem     0
43x71x55721xP                           1103x2053x2538097xQ(composite)QHNPFLT 25m
613**10                   N  exceeds  M        613**A                Proposition 5
2332903x7221259xP
```

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(163**4) = 11 x 31 x 1301 x 1601 S(163**6) = 18871143464293

Possibilities And Reasons By Which They May Be Excluded

```
  163**Y                                                           Theorem      11
 3x7x19x67
  163**Z             31**Y       331**Y                            Theorem      11
11x31x1301xP         3x331       3x7x5233
  163**6             18871143464293**2     2293204797253**A        N  Exceeds    M
   P             3x13x1291x49633x62143xP
  163**6             18871143464293**B                             Proposition   5
   P
  163**10     579189657172292026411**2                             N  Exceeds    M
  23xP           3x199x619x687433xP
  163**10     579189657172292026411**C                             Proposition   5
  23xP
  163**12                   10865297**2                            Proposition   1
 10865297xQ                   7x181xP
  163**12                   10865297**D                            Proposition   1
 10865297xQ   Q is composite and QHNPFLT 71,624,791
  163**16                                                          N  Exceeds    M
 239x398311x409429x95196329xP
  163**18                                                          Block       613
  665723xQ        Q is composite and QHNPFLT  100,000,000
  163**22                                                          Block       613
    Q             Q is composite and QHNPFLT  100,000,000
  163**E                                                           Proposition   5
```

Lemma 14.2 If N is an odd perfect number less than M and if 3 divides N and also 1093**X||N, then it is not true that 547**Z||N.

Note S(1093**1) = 2 x 547   S(547**4) = 431 x 208097431

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(208097431**2) = 3 x 13 x 19 x 43 x 79 x 163 x 4027 x P

Possibilities And Reasons By Which They May Be Excluded

208097431**2	26209**Y 3xP	228979297**2 3xP	17477172894531169**2 3xQ(composite)	N Exceeds M QHNPFLT 16m
208097431**2	26209**Y 3xP	228979297**2 3xP	17477172894531169**A	Proposition 5
208097431**2	26209**Y 3xP	228979297**4 41x688411xQ	4027**2 3x7x229xP	Proposition 6
208097431**2	26209**Y 3xP	228979297**4 41x688411xQ(composite)	4027**B QHNPFLT 19,000,001	N Exceeds M
208097431**2	26209**Y 3xP	228979297**6 7xQ		Proposition 6
208097431**2	26209**Y 3xP	228979297**C		Proposition 5
208097431**2	26209**4 41xP	11508918571937341**2 3x7x13xQ		Proposition 6
208097431**2	26209**4 41xP	11508918571937341**D		Proposition 5
208097431**2	26209**6 7x113x3557x33937x107339xP			Proposition 6
208097431**2	26209**10 23xP			N Exceeds M
208097431**2	26209**F			Proposition 5
208097431**4 5x11xQ				Proposition 8E
208097431**6 347509xP				N Exceeds M
208097431**G				Proposition 5

Theorem 14 If N is an odd perfect number less than M and if 3 divides N then, there is no X such that X(Mod 4) = 1 and at the same time 1093**X||N.

Note S(1093**1) = 2 x 547

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

547**Y		Lemma 14.1
547**Z		Lemma 14.2
547**6 7x29xP		Theorem 13
547**10 23x10847xP	9630092768572369626677**2 7x7x4789xP	N Exceeds M
547**10 23x10847xP	9630092768572369626677**A	Proposition 5
547**12 13x313x82889xQ(composite)	Q has no prime factor less than 150,000,000	Proposition 1
547**16 52837xQ(composite)	Q has no prime factor less than 120,624,691	Proposition 1
547**18 P		N Exceeds M
547**J		Proposition 5

Note: S(547**12) = 13 x 313 x 82889 x 21313 6588241187 1928705741.

To imply that Q = 21313 6588241187 1928705741 is a composite number it is sufficient to state the following fact.

5**(Q-1) [Mod Q] = 19813 6650253648 3781696571.

It is assumed in the sixth case of Theorem 14 that for some X(Mod 4) =1 both 1093**X||N and 547**12||N which implies that S(547**12) divides N where S(547**12) = 13 x 313 x 82889 x Q. Since no prime factor P of Q occurs to an odd power in the prime factorization of N and Q has no prime factor less than its cube root, then Q**2 divides N.

Lemma 15.1 If N is an odd perfect number less than M and if 3 divides N, then neither of the following conditions holds.

(A) 1093**10||N (B) 1093**12||N

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

1093**10	2608387**Y		272878729**X			Proposition 11
23x6491x2608387xP	3x8311xP		2x5x17xP			
1093**10	2608387**Y		272878729**Y			N Exceeds M
23x6491x2608387xP	3x8311xP		3x7x19x109x157x367x709x41911			
1093**10	2608387**Y		272878729**E			N Exceeds M
23x6491x2608387xP	3x8311xP					
1093**10	2608387**4		37861**X			Proposition 1
23x6491x2608387xP	31x37861xQ		2xP			
1093**10	2608387**4	31**Y	37861**Y		331**Y	N Exceeds M
23x6491x2608387xP	31x37861xQ	3x331	3x37x1201xP		3x7x5233	
1093**10	2608387**4		37861**4			Corollary 3.1
23x6491x2608387xP	31x37861xQ		5xQ			
1093**10	2608387**4		37861**A			N Exceeds M
23x6491x2608387xP	31x37861x4143641xP					
1093**10	2608387**6					N Exceeds M
23x6491x2608387xP	463x84589xQ	Q is composite and QHNPFLT 50,000,000				
1093**10	2608387**B					Proposition 5
23x6491x2608387xP						
1093**12	937**X		67**Y	31**Y	331**Y	Block 5233
13x131x937xQ	2x7x67		3x7x7xP	3x331	3x7x5233	
1093**12	937**X		67**4			N Exceeds M
13x131x937xQ	2x7x67		761xP			
1093**12	937**X		67**6			N Exceeds M
13x131x937xQ	2x7x67		175897xP			
1093**12	937**X		67**F			N Exceeds M
13x131x937xQ	2x7x67	see 4.8 and 9.0				
1093**12	937**Y		292969**X			Proposition 11
13x131x937xQ	3xP		2x5xP			
1093**12	937**Y		292969**Y			N Exceeds M
13x131x937xQ	3xP		3x61x127x139x163x163			
1093**12	937**Y		292969**4			N Exceeds M
13x131x937xQ	3xP		131x1181x1721xP			
1093**12	937**Y		292969**C			N Exceeds M
13x131x937xQ	3xP					

1093**12	937**Z	N Exceeds M
13x131x937xQ	8431xP	
1093**12	937**6	N Exceeds M
13x131x937xQ	22751xP	
1093**12	937**D	N Exceeds M
13x131x937x58782569xQ	Q is composite and QHNPFLT 177,000,000	

For Case 4 of Lemma 15.1 it is assumed that for some natural number X(Mod 4) = 1, 37861**X||N. It is further assumed that 2608387**4||N. From the latter assumption, we have S(2608387**4) = 31 x 37861 x Q dividing N. Since Q

(A) has no prime factor less than its cube root,

(B) is not a perfect square,

and

(C) has no prime factor which occurs to an odd power in the prime factorization of N,

we can use Proposition 1 to imply that Q**2 divides N.

Theorem 15 The number 3 x 1093 cannot be a factor of an odd perfect number N less than M.

(A)	1621**Y 3x7x13xP	PR 6
(B)	1621**Z 5x11x125613804731	PR8E
(C)	1621**6 211x4105333x20957295829	N> M
(D)	1621**10 12893xQ Q is composite and QHNPFLT 10,000,000	N> M
(E)	1621**12 Q QHNPFLT 14,625,001	N> M
(F)	1621**16 103x239x1361xQ Q is composite and QHNPFLT 133,642,591	N> M

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(1093**2) = 3 x 398581 S(1093**4) = 11 x 31 x 4189129561

Possibilities And Reasons By Which They May Be Excluded

1093**X 2x547	where X(Mod 4) = 1				Theorem 14
1093**Y 3xP	398581**X 2x17x19xP				Prop 11
1093**Y 3xP	398581**Y 3x1621xP	32668561**X 2x19x43xP	19993**Y 3x73xP	1825297**Y 3x326863x3397663	N Exceeds M
1093**Y 3xP	398581**Y 3x1621xP	32668561**X 2x19x43xP	19993**Y 3x73xP	1825297**AA	N Exceeds M
1093**Y 3xP	398581**Y 3x1621xP	32668561**X 2x19x43xP	19993**4	4330075325201**2 19x37x686551x2333719xP	N Exceeds M
1093**Y 3xP	398581**Y 3x1621xP	32668561**X 2x19x43xP	19993**4 36901xP	4330075325201**A	N Exceeds M
1093**Y 3xP	398581**Y 3x1621xP	32668561**X 2x19x43xP	19993**6 7x43xQ(composite) QHNPFLT 60,499,993	1621**B	Block 1621
1093**Y 3xP	398581**Y 3x1621xP	32668561**X 2x19x43xP	19993**10 23x419xQ(composite) QHNPFLT13,374,901		N Exceeds M
1093**Y 3xP	398581**Y 3x1621xP	32668561**X 2x19x43xP	19993**C		Prop 5
1093**Y 3xP	398581**Y 3x1621xP	32668561**Y 3x7x13x1693xP	2309087647**2 3x13x12097x25621x441105499		N Exceeds M
1093**Y 3xP	398581**Y 3x1621xP	32668561**Y 3x7x13x1693xP	2309087647**4 11x41xQ QHNPFLT 10,000,000		Prop 8B
1093**Y 3xP	398581**Y 3x1621xP	32668561**Y 3x7x13x1693xP	2309087647**D		Prop 5
1093**Y 3xP	398581**Y 3x1621xP	32668561**4 5xQ(com)Q has no prime factor less than 10m			N Exceeds M

```
1093**Y        398581**Y     32668561**6                                   N Exceeds M
  3xP           3x1621xP       140813xQ QHNPFLT 10,000,000
1093**Y        398581**Y     32668561**E                                   Prop       5
  3xP           3x1621xP
1093**Y        398581**4      2703853428809791**2                          N Exceeds M
  3xP         5x1866871xP            3xQ   Q is composite and QHNPFLT 8,799,997
1093**Y        398581**4      2703853428809791**F                          Prop       5
  3xP         5x1866871xP
1093**Y        398581**6                  47251**Y                         Prop       6
  3xP   7x113x1093x47251xQ              3x13x241xP
1093**Y        398581**6                  47251**4                         Prop       9
  3xP   7x113x1093x47251xQ      5x101x876791x11258116711
1093**Y        398581**6                  47251**6                         N Exceeds M
  3xP   7x113x1093x47251xQ                  7xP
1093**Y        398581**6                  47251**10                        N Exceeds M
  3xP   7x113x1093x47251xQ        23x23x14851x238943xP
1093**Y        398581**6                  47251**G                         Prop       5
  3xP   7x113x1093x47251xQ      Q is composite  and  QHNPFLT 22,139,377
1093**Y        398581**H                                                   Prop       5
  3xP
1093**Z      4189129561**X          7247629**Y      2501339684251**I      Block  5233
11x31xP    2x17x17x7247629            3x7xP
1093**Z      4189129561**X          7247629**4      31**Y      331**Y     Prop       6
11x31xP    2x17x17x7247629           11x41xQ  QHNPFLT  1,000,000
1093**Z      4189129561**X          7247629**6                            Prop       6
11x31xP    2x17x17x7247629           29x71xQ
1093**Z      4189129561**X          7247629**J                            Prop       5
11x31xP    2x17x17x7247629
1093**Z      4189129561**2          31**Y          331**Y  902418613**X  Prop       1
11x31xP    3x163x39767719xP         3x331         3x7x5233     2xP
1093**Z      4189129561**2          31**Y          331**Y  902418613**2  N Exceeds M
11x31xP    3x163x39767719xP         3x331         3x7x5233 3x691x6607xP
1093**Z      4189129561**2          31**Y          331**Y  902418613**4  N Exceeds M
11x31xP    3x163x39767719xP         3x331         3x7x5233       P
1093**Z      4189129561**2          31**Y          331**Y  902418613**R  N Exceeds M
11x31xP    3x163x39767719xP         3x331         3x7x5233
1093**Z      4189129561**4          31**Y          331**Y                 Prop       9
11x31xP          5x71xQ             3x331         3x7x5233
1093**Z      4189129561**K          31**Y          331**Y                 Prop       5
11x31xP                             3x331         3x7x5233
1093**6                                                                   Theorem  13
7x29x14939xP
1093**10                                                                  Lemma  15.1
23x6491x2608387xP
1093**12                                                                  Lemma  15.1
13x131x937xQ  Q is composite and QHNPFLT 24,492,391
1093**16                                                                  N Exceeds M
    Q   Q is composite and  QHNPFLT  133,624,591
1093**L                                                                   Prop       5
```

Lemma 16.1 Let X(Mod 4) = 1, Y(Mod 3) = 2, and Z(Mod 5) = 4. The number 3 x 7 x 19**4 x 911 x 151 cannot be a factor of an odd perfect number which is less than M.

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(151**2) = 3 x 7 x 1093 S(151**4) = 5 x 104670301

Possibilities And Reasons By Which They May Be Excluded

151**Y	where Y(Mod 3) = 2		Theorem 15
151**Z	where Z(Mod 5) = 4		Prop 9
151**6	7960598843**2	63371133947133537493**1	Prop 11
1499xP	P	2x113x1493x1736347xP	
151**6	7960598843**2	63371133947133537493**2	N Exceeds M
1499xP	P	3x103x1009x694951xQ(comp) QHNPFLT	8,799,997
151**6	7960598843**2	63371133947133537493**A	Prop 5
1499xP	P		
151**6	7960598843**4	1499**B	N Exceeds M
1499xP	31x41xQ(composite)	Q has no prime factor less than	10,000,000
151**6	7960598843**C		Prop 5
1499xP			
151**10	18145704541823**2	14864609**1	Prop 7
23x14864609xP	19x397xQ	2x3x5x467x1061xP	
151**10	18145704541823**2	14864609**2	N Exceeds M
23x14864609xP	19x397xQ	7x7x7x2221xP	
151**10	18145704541823**2	14864609**F	N Exceeds M
23x14864609xP	19x397xQ	Q is composite and QHNPFLT 8,799,997	
151**10	18145704541823**D		Prop 5
23x14864609xP			
151**12	1414519880078598368963321713**1	31**Y 331**Y	Block 5233
P	2x7x29x31xQ Q is composite	3x331 3x7x5233	48,499,999
151**12	1414519880078598368963321713**E		Prop 5
P			
151**16			N Exceeds M
148768021xQ			
151**18			N Exceeds M
3041xP			
151**22			N Exceeds M
599x9109xQ	Q is composite and has no prime factor less than		110,000,000
151**G			Prop 5

In Case 3 of Lemma 16.1 we have assumed that 633711339471335374 93**1||N. This implies that 1493 divides N. By Proposition 2, 1493 must appear to an even power in the prime factorization of N. Other than an odd power, for a prime P and an exponent W to exist such that S(P**W) to be divisible by 1493, W must be greater than 371. Therefore, we use Proposition 11 to get our contradiction.

A similar condition exists in Case 8 of this same lemma. Here, we assume that 14864609**1||N. This implies that the prime 467 divides N. Except for an odd power, if 467 divides S(P**W) where P is a prime and W is a natural number, then W must be greater than 231.

In the same case, Case 8, it is assumed that 18145704541823**2||N. This implise that S(18145704541823**2) = 19 x 397 x 436 5194131236 2985027271 also divides N. Let Q = 436 5194131236 2985027271. To imply that Q is composite, it is sufficient to give the following fact.

3**(Q-1) [Mod Q] = 16 9613163840 3509321137

Lemma 16.2 If N is an odd perfect number less than M and if 19**28||N then the number 3 x 7 cannot be a factor of n.

313**X 2x157	157**16	N>M	313**Y 3x181x181	181**A	N>M
313**Y 3x181x181	181**X 2x7x13	PR6	313**4 11xP		P8B
313**Y 3x181x181	181**Y 3x79x139	N>M	313**6 29xP		PR6
313**Y 3x181x181	181**4 5x11xP	PR9	313**10 199xP		N>M
313**Y 3x181x181	181**6	N>M	313**B		N>M

Note The number 59 x 233 divides S(19**28)

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

59**Y 3541	3541**X 2x7x11x23	Proposition 8B
59**Y 3541	3541**2 3x19x19x37x313	Block 313
59**Y 3541	3541**4 5x427001xP	Proposition 7
59**Y 3541	3541**A	N Exceeds M
59**Z 11x41x151x181		Proposition 8B
59**6 43x281x757xP	4691**Y 97x103x2203	N Exceeds M
59**6 43x281x757xP	4691**Z 5x11x71x881xP	Proposition 7
59**6 43x281x751xP	4691**6 7x43x953x3966803x9366668683	N Exceeds M
59**6 43x281x757xP	4691**10 199x727x2003x394241xP	N Exceeds M
59**6 43x281x757xP	4691**B	N Exceeds M
59**10 23x67x419xP	805243954219**2 3x13x19x4987xP	Proposition 6
59**10 23x67x419xP	805243954219**4 Q Q is composite and QHNPFLT 5,000,000	N Exceeds M
59**10 23x67x419xP	805243954219**C	Proposition 5

```
   59**12          18098732357953867292241**1                         Proposition 11
    P                    2x11x37x97x257xP
   59**12          18098732357953867292241**D                         N  Exceeds   M
    P
   59**16          3613532049623638287855531**E                       N  Exceeds   M
 137x443xP
   59**18                                                             N  Exceeds   M
571x183503xQ  Q is composite and QHNPFLT 30,000,000
   59**22                                                             N  Exceeds   M
47x829x6763xP
   59**28                                                             N  Exceeds   M
29x80986039xQ  QHNPFLT  263,124,541
   59**J                                                              Proposition  5
```

Note: S(59**12) = Q = 18 0987323579 5386729241. To show that Q is a prime number, for each prime factor P of

Q - 1 = 2**3 x 3**2 x 5 x 7 x 13 x 59 x 163 x 1741 x 3541 x 931837

we find a prime Px which is relatively prime to Q such that both of the following are true.

Px**(Q-1) [Mod Q] is 1 and Px**[(Q-1)/P] is not 1

(See Table XVII below)

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	11	1	18 0987323579 5386729240
3	3	1	8 2325625593 1456499817
5	3	1	12 6473291469 1659547327
7	3	1	16 5901648337 5297101592
13	3	1	59
59	3	1	4 3235847993 5144715425
163	3	1	1 8898618933 8734310611
1741	3	1	2 9196712962 6625115322
3541	3	1	6 6083874801 2094089956
931837	3	1	10 2531185617 4651866583

TABLE XVII

Lemma 16.3 If N is an odd perfect number less than M, and if 19**6||N or 19**10||N, then 3 x 7 cannot divide N,

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

19**6	70841**X	11807**A			Proposition 11
701xP	2x3xP				
19**6	70841**Y	39103**Y	67411**Y	1514770111**Y	N Exceeds M
701xP	128341xP	3x7561xP	3xP	3x70051xP	
19**6	70841**Y	39103**Y	67411**Y	1514770111**4	Proposition 9
701xP	128341xP	3x7561xP	3xP	5x14461xQ	
19**6	70841**Y	39103**Y	67411**Y	1514770111**6	Proposition 5
701xP	128341xP	3x7561xP	3xP	7x2857x3011x32327xQ	QHNPFLT 25.7m
19**6	70841**Y	39103**Y	67411**Y	1514770111**B	Proposition 5
701xP	128341xP	3x7561xP	3xP		
19**6	70841**Y	39103**Y	67411**4		Proposition 9
701xP	128341xP	3x7561xP	5x11x79631xP	P=4715032190881	
19**6	70841**Y	39103**Y	67411**6	128341**X	N Exceeds M
701xP	128341xP	3x7561xP	7x71xQ	2xP	
19**6	70841**Y	39103**Y	67411**6	128341**C	N Exceeds M
701xP	128341xP	3x7561xP	7x71xQ	QHNPFLT 10,000,000	
19**6	70841**Y	39103**Y	67411**D		N Exceeds M
701xP	128341xP	3x7561xP			
19**6	70841**Y	39103**4			Proposition 8B
701xP	128341xP	11x5734474lxP			
19**6	70841**Y	39103**6	128341**X	64171**Y	Proposition 6
701xP	128341xP	7x43x7547xP1xP	2xP	3x7x7x13x79x27277	
19**6	70841**Y	39103**6	128341**X	64171**4	Proposition 9
701xP	128341xP	7x43x7547xP1xP	2xP	5xQ	
19**6	70841**Y	39103**6	128341**X	64171**E	N Exceeds M
701xP	128341xP	7x43x7547xP1xP	2xP		
19**6	70841**Y	39103**6	128341**Y	549051341**F	N Exceeds M
701xP	128341xP	7x43x7547xP1xP	3xP		
19**6	70841**Y	39103**6	128341**Z		Proposition 7
701xP	128341xP	7x43x7547xP1xP	5x11x11x61x541x1058821xP		
19**6	70841**Y	39103**6	128341**6		N Exceeds M
701xP	128341xP	7x43x7547xP1xP	29xQ	Q is composite and	QHNPFLT 10m
19**6	70841**Y	39103**6	128341**G		Proposition 5
701xP	128341xP	7x43x7547x1300467lxP			
19**6	70841**Y	39103**10			N Exceeds M
701xP	128341xP	23x89xQ	Q is composite and QHNPFLT 13,374,901		
19**6	70841**Y	39103**H			Proposition 5
701xP	128341xP				

19**6	70841**4			Proposition 9
701xP	5x61x71xP			
19**6	70841**6			Theorem 15
701xP	7x29x6301xP			
19**6	70841**10			Proposition 8B
701xP	11x67x2003x134047xP			
19**6	70841**I			Proposition 5
701xP				
19**10	62060021**X	104281**2	179743**Y	Proposition 6
104281xP	2x3x3x47x109x673	3x7x43x67xP	3x7x31xP	
19**10	62060021**X	104281**2	179743**4	Proposition 8B
104281xP	2x3x3x47x109x673	3x7x43x67xP	11xQ	
19**10	62060021**X	104281**2	179743**6	N Exceeds M
104281xP	2x3x3x47x109x673	3x7x43x67xP	Q QHNPFLT	5,199,979
19**10	62060021**X	104281**2	179743**J	Proposition 5
104281xP	2x3x3x47x109x673	3x7x43x67xP		
19**10	62060021**X	104281**4		Proposition 9
104281xP	2x3x3x47x109x673	5x41x3181xP		
19**10	62060021**X	104281**6		N Exceeds M
104281xP	2x3x3x47x109x673	P		
19**10	62060021**X	104281**L		Proposition 5
104281xP	2x3x3x47x109x673			
19**10	62060021**2	17748600316039**2	104281**X	Block 5233
104281xP	7x31xP	3xP	2x23x2267	
19**10	62060021**2	17748600316039**2	104281**O	Block 104281
104281xP	7x31xP	3xP		
19**10	62060021**2	17748600316039**R		N Exceeds M
104281xP	7x31xP			
19**10	62060021**4			Proposition 9
104281xP	5x71x251xQ	QHNPFLT 10,000,000	and is composite	
19**10	62060021**S			Proposition 5

Note: S(17748600316039**2) = 3 x 1050042 7105950581 3093655187

If Q = 1050042 7105950581 3093655187 we use the following information to show that Q is prime.

Q - 1 = 2 x 3**2 x 7 x 19 x 53 x 127 x 95471 x 465929 x 146491361.

3**[(Q1-)/2] [Mod Q]	=	1050042 7105950581 3093655186
3**[(Q-1)/7] [Mod Q]	=	264121 7712751639 2901447390
3**[(Q-1)/19] [Mod Q]	=	300530 0259162918 8283629930
3**[(Q-1)/53] [Mod Q]	=	108799 4838789566 2723530143
3**[(Q-1)/127] [Mod Q]	=	705022 0978623168 1621913969
3**[(Q-1)/95471] [Mod Q]	=	141177 4316611021 7513723595
3**[(Q-1)/465929] [Mod Q]	=	230058 2905249245 7950009864
3**[(Q-1)/146491361] [Mod Q]	=	195998 1043162446 2048676662
3**(Q-1) [Mod Q]	=	1

Block 433   This block is used in Lemmas 17.1, 17.2, and 19.2.

In using this block, it is assumed that 3 x 7 x 43 x 433 x 631 divides N and that 733**X||N.

433**Y	1693**Y			Proposition 6
3x37xP	3x13x151x487			
433**Y	1693**Z	10012471081**2	47737893884701799983**Y	Proposition 6
3x37xP	821xP	3x7xP	3x13x61xQ	
433**Y	1693**Z	10012471081**2	47737893884701799983**A	Proposition 5
3x37xP	821xP	3x7xP		
433**Y	1693**Z	10012471081**4		Proposition 7
3x37xP	821xP	5xQ		
433**Y	1693**Z	10012471081**B		Proposition 5
3x37xP	821xP			
433**Y	1693**6	5700731**Y		N Exceeds M
3x37xP	43x337x7673x37171xP	43x13933xP		
433**Y	1693**6	5700731**4		Proposition 7
3x37xP	43x337x7673x37171xP	5x11x811xQ		
433**Y	1693**6	5700731**6		N Exceeds M
3x37xP	43x337x7673x37171xP	7x211xQ	QHNPFLT 25,749,991	
433**Y	1693**6	5700731**C		Proposition 5
3x37xP	43x337x7673x37171xP			
433**Y	1693**10			Proposition 1
3x37xP	89xP			
433**Y	1693**12			N Exceeds M
3x37xP	154571xQ Q is composite and QHNPFLT 60,000,000			
433**Y	1693**D			Proposition 5
3x37xP				
433**Z	1768661**Y	13660102627**Y	5360731**2	N Exceeds M
11x1811xP	229xP	3x3691x5360731xP	3x31x8821xP	
433**Z	1768661**Y	13660102627**Y	5360731**4	Proposition 7
11x1811xP	229xP	3x3691x5360731xP	5x9311xP	
433**Z	1768661**Y	13660102627**Y	5360731**E	N Exceeds M
11x1811xP	229xP	3x3691x5360731xP		
433**Z	1768661**Y	13660102627**F		N Exceeds M
11x1811xP	229xP			
433**Z	1768661**4			Proposition 7
11x1811xP	5x11xP			
433**Z	1768661**6			N Exceeds N
11x1811xP	20483xP			
433**Z	1768661**G			Proposition 5
11x1811xP				
433**6	1706822489**Y	15332857950857769**1		Proposition 9
743x5209xP	19xP	2x5x11xQ		
433**6	1706822489**Y	15332857950857769**H		N Exceeds M
743x5209xP	19xP			

433**6	1706822489**4	N Exceeds M
743x5209xP	11xP	
433**6	1706822489**I	Proposition 5
743x5209xP		
433**10		N Exceeds M
947xP		
433**12	79**Y	N Exceeds M
79x473201xP	3x7x7xP	
433**12	79**18	N Exceeds M
79x473201xP		
433**16		N Exceeds M
Q	QHNPFLT 100,000,000 and Q is composite	
433**18		N Exceeds M
2243xQ	QHNPFLT 25,000,000 and Q is composite	
433**J		Proposition 5

Note: S(433**10) = 947 x Q = 947 x 2452 0715250499 4694307128. To show that Q is a prime number, for each prime factor P of

Q - 1 = 2**3 x 3**2 x 179**2 x 29283 x 3 6365621933

we find a prime Px which is relatively prime to Q such that both of the following are true.

Px**(Q-1) [Mod Q] is 1 and Px**[(Q-1)/P] [Mod Q] is not 1

(See Table XVIII below)

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	13	1	2452 0715250499 4694307128
3	7	1	93 7072915972 0387776753
179	3	1	1972 2775403150 9642204911
29283	3	1	437 0267565143 3611650372
36365621933	3	1	728 6906179220 2260800781

TABLE XVIII

Theorem 16 The number 3 x 7 x 19 cannot be a factor of an odd perfect number that is less than M unless both 19**Y||N and one of the following are true.

(A) 127**12||N (B) 127**16||N (C) 127**18||N

Block 149

149**X 2x3x5x5			Proposition 7	149**6 P	N Exceeds M
149**Y 7x31xP	31**Y 3x331	331**Y 3x7x5233	N Exceeds M	149**10 67xP	N Exceeds M
149**4 251x691xP			N Exceeds M	149**A	N Exceeds M

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

19**Y 3x127	127**R where R is not 12, 16, or 18			Theorem 6
19**Z 151x911				Lemma 16.1
19**6 701xP				Lemma 16.3
19**10 104281xP				Lemma 16.3
19**12 599x29251xP	133338869**X 2x3x3x5x7x113x1873			Proposition 9
19**12 599x29251xP	133338869**2 56094673xP	29251**Y 3x193xP	1477807**Y 3x7x47977xP	N Exceeds M
19**12 599x29251xP	133338869**2 56094673xP	29251**Y 3x193xP	1477807**A	N Exceeds M
19**12 599x29251xP	133338869**2 56094673xP	29251**4 5x41xP		Proposition 9
19**12 599x29251xP	133338869**2 56094673xP	29251**6 43x66683xP		N Exceeds M
19**12 599x29251xP	133338869**2 56094673xP	29251**B		N Exceeds M
19**12 599x29251xP	133338869**4 11xP			Proposition 8B
19**12 599x29251xP	133338869**C			N Exceeds M
19**16 3044803xP	99995282631947**2 67x87403x1311127xQ	3044803**2 3xP	3090276117871**2 3x73x6337x206623x1094833xP	Proposition 1
19**16 3044803xP	99995282631947**2 67x87403x1311127xQ	3044803**2 3xP	3090276117871**D	Proposition 5

```
  19**16      99995282631947**2 3044803**4                     Proposition 8B
3044803xP    67x87403x1311127xQ  11x631xP
  19**16      99995282631947**2 3044803**E                     N  Exceeds   M
3044803xP    67x87403x1311127x10050613x129574807
  19**16      99995282631947**F                                Proposition  5
3044803xP
  19**18      1099122030922396438402221**1                     Theorem      0
    P                2xQ   QHNPFLT     40,000,000 and is composite
  19**18      1099122030922396438402221**2                     Proposition  6
    P              3x13x157x183361xQ
  19**18      1099122030922396438402221**G                     Proposition  5
    P
  19**22                                                       N  Exceeds   M
277x2347xQ     Q is composite and has no prime factor less than  141,000,000
  19**28                                                       Lemma     16.2
 59x233xQ      Q is composite and has no prime factor less than  100,000,000
  19**30      24327031889148383810359338159515180970l**1       N  Exceeds   M
    P          2x24229x32579x327689x886799x1085785lxP
  19**30      24327031889148383810359338159515180970l**H       Proposition  5
    P
  19**36                                                       Block      149
  149xQ           Q  is composite and has no prime factor less than  10,000,000
  19**I                                                        Proposition  5
```

Note: S(1099122030922396438402221**1) = 2 x Q.

To imply that Q is composite, it is sufficient to state the fact that

5**(Q-1) [Mod Q] = 403 4687182244 7320262683.

Although Q is known to be composite, it is not a perfect square, is relatively prime to 1099122030922396438402221 and therefore has no prime factor which appears to an odd power in the prime factorization of N. Since Q also has no prime factor less than its cube root, Proposition 1 applies here. However, since the prime 19 must be a factor of several even powers of several primes, we also apply Theorem 0.

Note: S(149**10) = 67 x 8104242624 5204504653 where

Q - 1 = 8104242624 5204504652 = 2**2 x 3 x 11 x 13 x 67 x 70488 8374953941

To show that Q is a prime number, for each prime factor P of Q - 1, we find a prime Px which is relatively prime to Q such that both of the following are true.

Px**(Q-1) [Mod Q] is 1 and Px**[(Q-1)/P] [Mod Q] is not 1

(See Table XIX below)

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	5	1	8104242624 5204504652
3	7	1	2444572354 0615603937
11	3	1	3307949
13	3	1	7846126543 3206918279
67	3	1	2602246336 6343047976
704888374953941	3	1	2481454778 8748297385

TABLE XIX

We shall use Table XX to show that S(19**18) is prime.

Let Q = S(19**18). Q - 1 = 2**2 x 3**2 x 5 x 7**3 x 523 x 29989 x 236377.

P	Px	Px**(Q-1) [Mod Q]	Px[(Q-1)P] [Mod Q]
2	17	1	1099 1220309223 9643840221
3	3	1	939 3716597315 6843965488
5	3	1	534 8582110965 3110703282
7	3	1	35 8415916432 5276843247
523	3	1	494 8225399037 2660143566
29989	3	1	193 1820492248 4805069022
236377	3	1	552 2105447217 8982100116

TABLE XX

In the lemma that follows, we have S(326202527951**2) = 643 x Q in which Q = 1 6548691950 5364134971. To imply that Q is composite, it is sufficient to state the fact that 5**(Q-1) [Mod Q] = 1341770682 3517226723

Lemma 17.1 If N is an odd perfect number less than M and if 307**Y||N for Y(mod 3) = 2, and also 733**X||N for X(Mod 4) = 1, we now show that the number 3 x 7 x 43 x 433 cannot be a factor of N.

Note S(307**2) = 3 x 43 x 733 S(733**1) = 2 x 367

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

367**Y	3463**Y	61**Y			Proposition 6
3x13xP	3x61x65551	3x13x97			
367**Y	3463**4				Proposition 6
3x13xP	11x4271x19121xP				
367**Y	3463**6				Block 433
3x13xP	197x5237x142419xP				
367**Y	3463**10				Block 23
3x13xP	23x98737xQ	Q is composite and QHNPFLT			13,374,901
367**Y	3463**12				N Exceeds M
3x13xP	6449xQ	Q is composite QHNPFLT 100,000,000			
367**Y	3463**A				Proposition 5
3x13xP					
367**Z					Proposition 6
11x281xP					
367**6					Block 433
113x233437xP					
367**10	1783**2				Block 433
1783xQ	3x829x1279				
367**10	1783**4	31**Y	331**Y	326202527951**2	Block 5233
1783xQ	31xP	3x331	3x7x5233	643xQ Q is composite	12m
367**10	1783**4	31**Y	331**Y	326202527951**4	Proposition 9
1783xQ	31xP	3x331	3x7x5233	5x11x241xQ	
367**10	1783**4	31**Y	331**Y	326202527951**B	Proposition 5
1783xQ	31xP	3x331	3x7x5233		
367**10	1783**C				Block 433
1783xQ	Q is composite and QHNPFLT 100,000,000 (apply Proposition 1)				
367**12					N Exceeds M
53xP					
367**16					N Exceeds M
239x86837xP					
367**18					N Exceeds M
Q	Q is composite and Q has no prime factor less than 100,000,000				
367**D					Proposition 5

Lemma 17.2 If N is an odd perfect number less than M and if 307**Y||N for Y(mod 3) = 2, then the number 3 x 7 x 43 x 631 x 433 cannot be a factor of N. We note that S(307**2) = 3 x 43 x 733.

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(733**2) = 3 x 19 x 9439 S(733**4) = 5641 x 51245141

Possibilities And Reasons By Which They May Be Excluded

733**X					Lemma 17.1
733**Y	9439**Y	29701387**Y	42008210448817**1		N Exceeds M
3x19x9439	3xP	3x7xP	2x67x9857xP		
733**Y	9439**Y	29701387**Y	42008210448817**2		N Exceeds M
3x19x9439	3xP	3x7xP	3x7x19xQ	QHNPFLT	9,999,997
733**Y	9439**Y	29701387**Y	42008210448817**S		Proposition 5
3x19x9439	3xP	3x7xP			
733**Y	9439**Y	29701387**4			N Exceeds M
3x19x9439	3xP	31xQ	QHNPFLT 10,000,000		
733**Y	9439**Y	29701387**6			N Exceeds M
3x19x9439	3xP	281xQ			
733**Y	9439**Y	29701387**A			Proposition 5
3x19x9439	3xP				
733**Y	9439**Z	2452398401**X			N Exceeds M
3x19x9439	1181x2741xP	2x3x1063xP			
733**Y	9439**Z	2452398401**2			N Exceeds M
3x19x9439	1181x2741xP	619x3480619xP			
733**Y	9439**Z	2452398401**4			Proposition 9
3x19x9439	1181x2741xP				
733**Y	9439**Z	2452398401**U			Proposition 5
3x19x9439	1181x2741xP				
733**Y	9439**6				N Exceeds M
3x19x9439	5839xP				
733**Y	9439**10				Proposition 8B
3x19x9439	11xQ				
733**Y	9439**12				Proposition 6
3x19x9439	13xQ				
733**Y	9439**A				Proposition 5
3x19x9439					
733**Z	51245141**X	656989**Y			Proposition 6
5641xP	2x3x13xP	3x7x19x577xP			
733**Z	51245141**X	656989**4			Proposition 8B
5641xP	2x3x13xP	11xQ			
733**Z	51245141**X	656989**6			Proposition 1
5641xP	2x3x13xP	29x43x197x15289xQ	Q is composite	QHNPFLT 65,265,397	
733**Z	51245141**X	656989**B			Proposition 5
5641xP	2x3x13xP				

```
733**Z 51245141**2   143579119**2   302250313**1  18290017**2 Theorem      4
5641xP 18290017xP   3x139x163561xP   2x23x853xP 3x7x409x23773xP
733**Z 51245141**2   143579119**2   302250313**1  18290017**4 N  Exceeds   M
5641xP 18290017xP   3x139x163561xP   2x23x853xP    11x251xP
733**Z 51245141**2   143579119**2   302250313**1  18290017**C N  Exceeds   M
5641xP 18290017xP   3x139x163561xP   2x23x853xP
733**Z 51245141**2   143579119**2   302250313**2     163561**X Proposition  1
5641xP 18290017xP   3x139x163561xP      3xQ          2x7x7xP
733**Z 51245141**2   143579119**2   302250313**2     163561**2 N  Exceeds   M
5641xP 18290017xP   3x139x163561xP      3xQ             31xP
733**Z 51245141**2   143579119**2   302250313**2     163561**4 Proposition  7
5641xP 18290017xP   3x139x163561xP      3xQ       5x61x4651xQ
733**Z 51245141**2   143579119**2   302250313**2     163561**D N  Exceeds   M
5641xP 18290017xP   3x139x163561xP      3xQ(Composite) QHNPFLT 26,316,247
733**Z 51245141**2   143579119**2   302250313**E               N  Exceeds   M
5641xP 18290017xP   3x139x163561xP
733**Z 51245141**2   143579119**4                              N  Exceeds   M
5641xP 18290017xP    2791x21881x8192671xP
733**Z 51245141**2   143579119**6                              N  Exceeds   M
5641xP 18290017xP         Q                             QHNPFLT 4.3m
733**Z 51245141**2   143579119**F                              Proposition  5
5641xP 18290017xP
733**Z 51245141**4                                             Corollary  3.1
5641xP 5x31x181xQ
733**Z 51245141**6                                             N  Exceeds   M
5641xP 29x673x646549xP
733**Z 51245141**G                                             Proposition  5
5641xP
 733**6   8875104113**X                                        Proposition 11
8737x2003xP 2x3x41x353xP
 733**6   8875104113**2    305513679613**1                     Proposition  6
8737x2003xP 5689x45319xP  2x11x13x281x1283xP
 733**6   8875104113**2    305513679613**2        91493574241**X N  Exceeds  M
8737x2003xP 5689x45319xP 3x7x607x5749x13921xP        2x7xP
 733**6   8875104113**2    305513679613**2        91493574241**2 N  Exceeds  M
8737x2003xP 5689x45319xP 3x7x607x5749x13921xP  3x562711x1478287xP
 733**6   8875104113**2    305513679613**2        91493574241**4 Proposition 9
8737x2003xP 5689x45319xP 3x7x607x5749x13921xP     5x11x41xQ
 733**6   8875104113**2    305513679613**2        91493574241**H Proposition 5
8737x2003xP 5689x45319xP 3x7x607x5749x13921xP
 733**6   8875104113**2    305513679613**4                    N  Exceeds   M
8737x2003xP 5689x45319xP        31xQ(composite)   QHNPFLT  10,000,000
 733**6   8875104113**2    305513679613**I                    Proposition  5
8737x2003xP 5689x45319xP
 733**6   8875104113**4                                       N  Exceeds   M
8737x2003xP  31x20771xQ(composite)    QHNPFLT  10,000,000
 733**6   8875104113**J                                       Proposition  5
8737x2003xP
 733**10                                                      Block      433
 12084491xQ  Q is composite and   Q  has no prime factor less than  100,000,000
 733**12                                                      N  Exceeds   M
53x79x4759xP
 733**16       19381**S                                       N  Exceeds   M
 19381xQ  Q is composite and  QHNPFLT  131,624,881
 733**T                                                       Proposition  5
```

Lemma 17.3 The number 3 x 7 x 43 x 631 x 433 x 307 cannot be a factor of an odd perfect number N which is less than M.

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(307**4) = 1051 x 5231 x 1621

Possibilities And Reasons By Which They May Be Excluded

```
307**Y                                                          Lemma     17.2

307**Z   5231**Y   3909799**Y  1907716477**X    953858239**2 N  Exceeds    M
           7xP       3x2671xP        2xP        3x13x367x11953xP
307**Z   5231**Y   3909799**Y  1907716477**X    953858239**4 N  Exceeds    M
           7xP       3x2671xP        2xP         11x281x79861xQ(comp)  25m
307**Z   5231**Y   3909799**Y  1907716477**X    953858239**A Proposition   5
           7xP       3x2671xP        2xP
307**Z   5231**Y   3909799**Y  1907716477**2                 N  Exceeds    M
           7xP       3x2671xP  3x7x19x124291x73386315523
307**Z   5231**Y   3909799**Y  1907716477**4                 N  Exceeds    M
           7xP       3x2671xP        Q   Q  is composite  QHNPFLT 25,000,001
307**Z   5231**Y   3909799**Y  1907716477**B                 Proposition   5
           7xP       3x2671xP
307**Z   5231**Y   3909799**4                                N  Exceeds    M
           7xP     11x11551x5396621x12881177x26456251
307**Z   5231**Y   3909799**6                                N  Exceeds    M
           7xP    6959x624149xP
307**Z   5231**Y   3909799**C                                Proposition   5
           7xP
307**Z   5231**4                                             Proposition   7
         5x601xP
307**Z   5231**6    28862437308597030304 7**2  71**Y   5113**X Proposition 6
           71xP             13x37xQ           5113     2x2557
307**Z   5231**6    288624373085970303047**2  71**12         N  Exceeds    M
           71xP             13x37xQ            Q
307**Z   5231**6    288624373085970303047**2  71**18         N  Exceeds    M
           71xP             13x37xQ  Q is composite and QHNPFLT 9,999,991
307**Z   5231**6    288624373085970303047**D                 Proposition   5
           71xP
307**Z   5231**10     4643**2          19**Y  127**R         Block     3x127
       4643x42571xQ 7x19x223xP
307**Z   5231**10     4643**4                                N  Exceeds    M
       4643x42571xQ  41x131xP
307**Z   5231**10    4643**E                                 N  Exceeds    M
       4643x42571xQ  Q is composite and  QHNPFLT  13,374,901
307**Z   5231**F                                             N  Exceeds    M

307**6      1274564409623**2   659**Y                        Proposition   6
659xP            19x211xQ      13x33457
```

```
   307**6        1274564409623**2   659**4       31**Y      331**Y  N  Exceeds   M
   659xP             19x211xQ   31x6131x993821  3x331     3x7x5233
   307**6        1274564409623**2   659**6       71**Y     5113**X  Proposition  6
   659xP             19x211xQ    7x71x109789xP  5113       2x2557
   307**6        1274564409623**2   659**6       71**12             N  Exceeds   M
   659xP             19x211xQ    7x71x109789xP    Q
   307**6        1274564409623**2   659**6       71**18             N  Exceeds   M
   659xP             19x211xQ    7x71x109789xP    Q
   307**6        1274564409623**2   659**10                         N  Exceeds   M
   659xP             19x211xQ      199xP
   307**6        1274564409623**2   659**12                         N  Exceeds   M
   659xP             19x211xQ       2939xQ      Q  is composite and QHNPFLT 10m
   307**6        1274564409623**2   659**G                          N  Exceeds   M
   659xP             19x211xQ     Q  is composite  and QHNPFLT 35,400,001
   307**6        1274564409623**4                                   N  Exceeds   M
   659xP              7151xQ  Q is composite and QHNPFLT  4,000,000
   307**6        1274564409623**H                                   Proposition  5
   659xP
   307**10       12135499643725165489**1                            Proposition  9
 23x26731xP             2x5x71xQ
   307**10       12135499643725165489**2                            N  Exceeds   M
 23x26731xP              3x43xQ   QHNPFLT  9,099,997  Q is composite
   307**10       12135499643725165489**I                            Proposition  5
 23x26731xP
   307**12       203908875421708483655521**1                        Proposition 11
53x131x4967xP           2x3x3x17xP        P-1  =  53x73xQ
   307**12       203908875421708483655521**J                        N  Exceeds   M
53x131x4967xP
   307**16                          16661**X                        Proposition 11
 17x16661xQ                         2x3x2777
   307**16                          16661**2                        N  Exceeds   M
 17x16661xQ                         277605583
   307**16                          16661**4                        Proposition  9
 17x16661xQ                            5xQ
   307**16                          16661**K                        N  Exceeds   M
 17x16661xQ      Q  has no prime factor less than  36,000,000 and is comp
   307**18                                                          N  Exceeds   M
   731729x1716613xP
   307**R                                                           Proposition  5
```

Theorem 17 The number 3 x 7 x 43 x 631 cannot be a factor of an odd perfect number N that is less than M.

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(631**12) = 131 x 443 x 26339 x 103091 x Q QHNPFLT 945,999,991

Possibilities And Reasons By Which They May Be Excluded

631**Y				Lemma 17.3
3x307x433				
631**Z				Proposition 9
5x11x41x1511xP				
631**6	1497157061**X			Proposition 11
7x6032531xP	2x3x521xP			
631**6	1497157061**2	790165013221**1	6032531**2	N Exceeds M
7x6032531xP	103x27541xP	2x13xP	9319xP	
631**6	1497157061**2	790165013221**1	6032531**4	Proposition 9
7x6032531xP	103x27541xP	2x13xP	5xQ	
631**6	1497157061**2	790165013221**1	6032531**A	N Exceeds M
7x6032531xP	103x27541xP	2x13xP		
631**6	1497157061**2	790165013221**2		Block 6032531
7x6032531xP	103x27541xP	3x316903xQ is composite and QHNPFLT 2,799,997		
631**6	1497157061**2	790165013221**4		Proposition 9
7x6032531xP	103x27541xP			
631**6	1497157061**2	790165013221**B		Proposition 5
7x6032531xP	103x27541xP			
631**6	1497157061**4			Corollary 3.1
7x6032531xP	5x11x31x151xQ			
631**6	1497157061**C			Proposition 5
7x6032531xP				
631**10	112614002823949502163679249**1			Proposition 9
89xP	2x5x5x5x139xQ Q has no prime factor less than 30,000			
631**10	112614002823949502163679249**D			Proposition 5
89xP				
631**12	26339**Y	693769261**X		Proposition 1
131x443x26339xPxQ	P	2x29x709xP		
631**12	26339**Y	693769261**2		N Exceeds M
131x443x26339xPxQ	P	3x13x36855787xP		
631**12	26339**Y	693769261**4		Proposition 7
131x443x26339xPxQ	P	5x11xQ		
631**12	26339**Y	693769261**I		Proposition 5
131x443x26339xPxQ	P			
631**12	26339**4	89112666844321**1		Proposition 1
131x443x26339xPxQ	11x491xP	2x11x101xP		
631**12	26339**4	89112666844321**E		N Exceeds M
131x443x26339xPxQ	11x491xP			
631**12	26339**6			N Exceeds M
131x443x26339xPxQ	29x1009x5503xP			

631**12 26339**F		N Exceeds M
131x443x26339x103091xQ Q is composite and QHNPFLT 975,249,991		
631**16 2347**2		N Exceeds M
103x2347xQ 3x7x397x661		
631**16 2347**4		Proposition 6
103x2347xQ 11x41xP		
631**16 2347**G		N Exceeds M
103x2347xQ Q is composite and QHNPFLT 100,000,000		

For Case 14 of Theorem 17 it is assumed both

(A) that for some natural number X(Mod 4) = 1, 693769261**X||N

and

(B) that 631**12||N.

It follows that S(631**12) = 131 x 443 x 26339 x 103091 x Q divides N.

Since Q

(A) has no prime factor less than its cube root,

(B) is not a perfect square,

and

(C) has no prime factor which occurs to an odd power in the prime factorization of N

then by Proposition 1, Q**2 must divide N.

Lemma 18.1 Let N be an odd perfect number less than M. Then, not all three of the following can happen simultaneoulsy.

(A) 7**Z||N (B) 2801**Y||N (C) 4933**X||N

S(7**4) = 2801 S(2801**2) = 37 x 43 x 4933 S(4933**1) = 2xP

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

2467**Y	7489**Y	61**Y	Proposition 6
3x271xP	3x61xP	3x13x97	
2467**Y	7489**4		Proposition 6
3x271xP	11xP		
2467**Y	7489**6		Theorem 13
3x271xP	29xQ	QHNPFLT 20,000,000 and Q is composite	
2467**Y	7489**10	25741**Y	N Exceeds M
3x271xP	1321x25741xP	3x7x37xP	
2467**Y	7489**10	25741**4	Proposition 9
3x271xP	1321x25741xP	5x101xP	
2467**Y	7489**10	25741**A	N Exceeds M
3x271xP	1321x25741xP		
2467**Y	7489**12		N Exceeds M
3x271xP	13x53x161773x5187859xQ	QHNPFLT 5187859	
2467**Y	7489**B		N Exceeds M
3x271xP			
2467**Z	10177286401**2		Proposition 6
11x331xP	3x7x13x97x3313x545473x2164387		
2467**Z	10177286401**4		N Exceeds M
11x331xP	5x4021x1644691xQ(comp)	QHNPFLT 10,000,000	
2467**Z	10177286401**C		Proposition 5
11x331xP			
2467**6	5244722549705267119**2		N Exceeds M
43xP	3x73xQ	Q is composite and QHNPFLT 10 million	
2467**6	5244722549705267119**D		Proposition 5
43xP			
2467**10			N Exceeds M
23x89xP			
2467**12	79**Y		N Exceeds M
79xP	3x7x7x43		
2467**12	79**18		N Exceeds M
79xP			
2467**E			Proposition 5

Lemma 18.2 Let N be an odd perfect number that is less than M, and suppose that 43**Z||N. Then, the number 3 x 7 x 193 x 331 x 127**12 cannot divide N.

Note S(43**4) = 3500201

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

3500201**X	583367**Y	276456247**Y	17821801**2	N Exceeds M
2x3xP	1231xP	3x7x73x127x22027xP	3x7x31x271xP	
3500201**X	583367**Y	276456247**Y	17821801**4	Proposition 9
2x3xP	1231xP	3x7x73x127x22027xP	5xQ	
3500201**X	583367**Y	276456247**Y	17821801**6	Theorem 13
2x3xP	1231xP	3x7x73x127x22027xP	29x43x71xQ-composite	7.7m
3500201**X	583367**Y	276456247**Y	17821801**A	Proposition 5
2x3xP	1231xP	3x7x73x127x22027xP		
3500201**X	583367**Y	276456247**4		N Exceeds M
2x3xP	1231xP	17011x586541xQ(composite)	QHNPFLT 10,000,000	
3500201**X	583367**Y	276456247**B		Proposition 5
2x3xP	1231xP			
3500201**X	583367**4	10528717963958615161331**2		N Exceeds M
2x3xP	11xP	Q(comp) QHNPFLT 10,000,000		
3500201**X	583367**4	10528717963958615161331**C		Proposition 5
2x3xP	11xP			.
3500201**X	583367**6			Proposition 1
2x3xP	7x379x178627x327797xP			
3500201**X	583367**D			Proposition 5
2x3xP				
3500201**Y	6779972629**X			Proposition 7
13x139xP	2x5x19727xP			
3500201**Y	6779972629**2			Theorem 17
13x139xP	3x631xP			
3500201**Y	6779972629**4			N Exceeds M
13x139xP	186761x407821lxQ	QHNPFLT 10,000,000		
3500201**Y	6779972629**E			Proposition 5
13x139xP				
3500201**4				Proposition 7
5x689261xQ	Q is composite			
3500201**6				N Exceeds M
Q	Q is composite and Q has no prime factor less than 10,300,003			
3500201**F				Proposition 5

Lemma 18.3 The number 3 x 7 x 37 x 43 x 193 x 331 x 4933 x 127**12 cannot be a factor of an odd perfect number N that is less than M.

Block 193

193**X 331**Y 2x97 3x7x5233	Block	5233
193**Y 3x7xP	N Exceeds	M
193**4 P	N Exceeds	M
193**6 43xP	N Exceeds	M
193**10 23x67x27259x644557x2662289521	N Exceeds	M
193**12 131x65027x104287xP	N Exceeds	M
193**16 137x20707xQ composite 100m	N Exceeds	M
193**18 Q(comp) QHNPFLT 100,000,000	N Exceeds	M
193**22 54419xQ(comp) QHNPFLT 100,000,000	N Exceeds	M
193**A	Proposition	5

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(43**2) = 3x631 S(43**4) = 3500201 S(43**6) = 7x5839x158341

Possibilities And Reasons By Which They May Be Excluded

43**Y					Theorem 17
43**Z					Lemma 18.2
43**6	5839**Y 3xP	11366587**Y 3x67xP	642782643757**X 2x281xP		Proposition 11
43**6	5839**Y 3xP	11366587**Y 3x67xP	642782643757**2 3x7x19x31x193xQ		Proposition 6
43**6	5839**Y 3xP	11366587**Y 3x67xP	642782643757**4 241x6151xQ(comp) QHNPFLT 10m		N Exceeds M
43**6	5839**Y 3xP	11366587**Y 3x67xP	642782643757**A		Proposition 5
43**6	5839**Y 3xP	11366587**4 P	158341**X 2x41xP	1931**2 P	N Exceeds M
43**6	5839**Y 3xP	11366587**4 P	158341**X 2x41xP	1931**4	Proposition 9
43**6	5839**Y 3xP	11366587**4 P	158341**X 2x41xP	1931**B	N Exceeds M
43**6	5839**Y 3xP	11366587**4 P	158341**2 3xP		N Exceeds M
43**6	5839**Y 3xP	11366587**4 P	158341**4		Proposition 9
43**6	5839**Y 3xP	11366587**4 P	158341**C		N Exceeds M
43**6	5839**Y 3xP	11366587**6 7x2237327xQ	Q is composite QHNPFLT 14,837,355		N Exceeds M

```
   43**6          5839**Y        11366587**D                        Proposition  5
                    3xP
   43**6          5839**4       213567331**Y                        Proposition  6
               11x191x2591xP     3x13x73xP
   43**6          5839**4       213567331**4                        Proposition  9
               11x191x2591xP
   43**6          5839**4       213567331**6                        N  Exceeds   M
               11x191x2591xP       1583xP
   43**6          5839**4       213567331**E                        Proposition  5
               11x191x2591xP
   43**6          5839**6          158341**X                        N  Exceeds   M
                    7xP             2x41xP
   43**6          5839**6          158341**2                        N  Exceeds   M
                    7xP          3x8357343541
   43**6          5839**6          158341**4                        Proposition  9
                    7xP
   43**6          5839**6          158341**6                        N  Exceeds   M
                    7xP         7x113x6065221xP
   43**6          5839**6          158341**F                        Proposition  5
                    7xP
   43**6          5839**10                                          N  Exceeds   M
                  26489x515163xQ(composite)    QHNPFLT  13,374,901
   43**6          5839**12                                          N  Exceeds   M
                    Q    Q  is composite and QHNPFLT  100,000,000
   43**6          5839**G                                           Proposition  5

   43**10    3664405207**2                  182699505078481**1      Proposition 11
6038099xP     3x24499xP                    2x7x167x563x138798403
   43**10    3664405207**2                  182699505078481**H      N  Exceeds   M
6038099xP     3x24499xP
   43**10    3664405207**4                                          N  Exceeds   M
6038099xP      87041xQ(composite)    QHNPFLT  10,000,000
   43**10    3664405207**I                                          Proposition  5
6038099xP
   43**12    409110505781497806O1**1       31**Y    331**Y          Block     5233
    P                 2x7xQ  Q is composite and    QHNPFLT  10,080,000
   43**12    409110505781497806O1**J                                Block      193
    P
   43**16                                                           Block      193
   647xQ                        Q  is composite QHNPFLT 252,999,781
   43**18                                                           Block      193
   229x2699x4219x46399xP
   43**22                                                           Block      193
    Q(Composite)  Q has no prime factor less than  100,000,000
   43**28                                                           N  Exceeds   M
523x10499xQ  Q is composite and  QHNPFLT 263,124,541
   43**30                                                           N  Exceeds   M
   7069x37712369xP
   43**K                                                            Proposition  5
```

Lemma 18.4 Let N be an odd perfect number less than M. Then, not both of the following can happen simultaneously.

(A) 7**Z||N (B) 2801**Y||N

Note S(7**4) = 2801 S(2801**2) = 37 x 43 x 4933

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

4933**X			Lemma 18.1
4933**Y 3x193x127xP	331**Y 3x7x5233	127**Y	Theorem 6
4933**Y 3x193x127xP	331**Y 3x7x5233	127**Z	Theorem 6
4933**Y 3x193x127xP	331**Y 3x7x5233	127**6	Theorem 6
4933**Y 3x193x127xP	331**Y 3x7x5233	127**10	Theorem 6
4933**Y 3x193x127xP	331**Y 3x7x5233	127**A	Lemma 18.3
4933**4 11x31x7541xP	31**Y 3x331	331**Y 3x7x5233	Proposition 8D
4933**6 3221x360851xP	3221**X 2x3x3xP	179**B	Proposition 11
4933**6 3221x360851xP	3221**C		Block 3221
4933**10 Q Q is composite and has no prime factor less than 30,000,000			Block 43
4933**12 Q Q is composite and has no prime factor less than 30,000,000			N Exceeds M
4933**D			Proposition 5

Lemma 18.5 If N is an odd perfect number less than M, then for no Z such that Z(Mod 5) = 4 will 7**Z||N when 2801**10||N.

Note S(7**4) = 2801 and 23 x 1372537 divides S(2801**10)

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

1372537**X	686269**Y	61540027**Y				Block	23
2xP	3x2551xP	3x13x31x4999xP					
1372537**X	686269**Y	61540027**4				N Exceeds	M
2xP	3x2551xP	Q		QHNPFLT 4,000,000			
1372537**X	686269**Y	61540027**A				N Exceeds	M
2xP	3x2551xP						
1372537**X	686269**4	71**Y	5113**X	2557**Y		Proposition	2
2xP	71xP	5113	2x2557	3x7x13x13x19x97			
1372537**X	686269**4	71**12				N Exceeds	M
2xP	71xP						
1372537**X	686269**4	71**18				N Exceeds	M
2xP	71xP						
1372537**X	686269**B					N Exceeds	M
2xP							
1372537**Y	1313899**Y					Proposition	6
3x73x6547xP	3x19x67x1483x304813						
1372537**Y	1313899**4				61**X	Proposition	6
3x73x6547xP	11x11x61x1061x3511x3631x2985111321				2x31		
1372537**Y	1313899**4				61**Y	Proposition	6
3x73x6547xP	11x11x61x1061x3511x3631x2985111321				3x13x97		
1372537**Y	1313899**C					N Exceeds	M
3x73x6547xP							
1372537**4						N Exceeds	M
751x3041xP							
1372537**D						N Exceeds	M

Note:

S(1372537**4) = 751 x 3041 x 155396131 1679140731 = 751 x 3041 x Q
Q - 1 = 2 x 5 x 79 x 4261 x 46 1638027467

3**[(Q-1)/2] [Mod Q]	=	155396131 1679140730
3**[(Q-1)/5] [Mod Q]	=	52225618 3793434402
3**[(Q-1)/79] [Mod Q]	=	85311369 5671322422
3**[(Q-1)/4261] [Mod Q]	=	111918062 2552469778
3**[(Q-1)/461638027467] [Mod Q]	=	105660919 8839827670

Theorem 18 If N is an odd perfect number less than M, then it is not true that 7**Z||N.

Note S(7**4) = 2801

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(2801**1) = 2 x 3 x 467 S(2801**4) = 5 x 1956611 x P

Possibilities And Reasons By Which They May Be Excluded

2801**X 2x3xP					Proposition 11
2801**Y					Lemma 18.4
2801**4 5x1956611xP	6294091**Y 3xP				Proposition 7
2801**4 5x1956611xP	6294091**4 5x1963111x2692801xP	1956611**2 277x95479xP		144751**A	N Exceeds M
2801**4 5x1956611xP	6294091**4 5x1963111x2692801xP	1956611**4 5x61xP		61**X 2x31	Corollary 3.1
2801**4 5x1956611xP	6294091**4 5x1963111x2692801xP	1956611**4 5x61xP		61**Y 3x13x97	Proposition 7
2801**4 5x1956611xP	6294091**4 5x1963111x2692801xP	1956611**B			N Exceeds M
2801**4 5x1956611xP	6294091**6 P				N Exceeds M
2801**4 5x1956611xP	6294091**C				Proposition 5
2801**6 7x71x211x1597xP	2884629032993**1 2x3xP	480771505499**2 10099xP P = 22887537429473785399			N Exceeds M
2801**6 7x71x211x1597xP	2884629032993**1 2x3xP	480771505499**4 61x101x5403481xP		61**Y 3x13x97	N Exceeds M QHNPFLT
2801**6 7x71x211x1597xP	2884629032993**1 2x3xP	480771505499**D			Proposition 5
2801**6 7x71x211x1597xP	2884629032993**2 619xP	71**Y 5113	5113**X 2x2557	2557**Y 3x7x13x13x19x97	Proposition 6
2801**6 7x71x211x1597xP	2884629032993**2 619xP	71**12			Proposition 1
2801**6 7x71x211x1597xP	2884629032993**2 619xP	71**18			Proposition 1
2801**6 7x71x211x1597xP	2884629032993**E				N Exceeds M
2801**10 23x1372537x10196539x45437789xP					Lemma 18.5
2801**12 1483x8932457xQ	1483**Y 3xP	733591**2 3x51487xP			N Exceeds M
2801**12 1483x8932457xQ	1483**Y 3xP	733591**4			Proposition 9

2801**12 1483x8932457xQ	1483**Y 3xP	733591**F	N Exceeds M
2801**12 1483x8932457xQ	1483**4 11x64661xP		N Exceeds M
2801**12 1483x8932457xQ	1483**6 43x197x524231xP		N Exceeds M
2801**12 1483x8932457x22867937xQ	1483**G Q is composite and	QHNPFLT 100,000,000	N Exceeds M
2801**H			Proposition 5

Note: S(2884629032993**2) = 619 x Q = 619 x 134 4278620030 5355269097

To show that Q is a prime number, for each prime factor P of

Q - 1 = 2**3 x 3**2 x 7 x 179 x 463 x 4127 x 64451 x 1209931

we find a prime Px which is relatively prime to Q such that

Px**(Q-1) [Mod Q] = 1 and Px**[(Q-1)/P] [Mod Q] is not 1

(See Table XXI below)

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	5	1	134 4278620030 5355269096
3	3	1	288 4629032993
7	3	1	9 1569289226 6788050139
179	3	1	64 4281208866 8538953629
463	3	1	120 9762792127 6087230188
4127	3	1	67 5494443064 1871827689
64451	3	1	30 3992245192 0420339734
1209931	3	1	89 4058640190 5671941223

TABLE XXI

Lemma 19.1 There is no odd perfect number N less than M such that 7**10||N

Note S(7**10) = 1123 x 293459

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

In this lemma no Q has a prime factor less than 1,000,000.

Possibilities And Reasons By Which They May Be Excluded

293459**Y	310897033**X			Proposition 11
277xP	2x7x263xP			
293459**Y	310897033**2	139017523**2	1667353**X	N Exceeds M
277xP	3x139x1667353xP	3x523x1950391xP	2x13x13xP	
293459**Y	310897033**2	139017523**2	1667353**2	N Exceeds M
277xP	3x139x1667353xP	3x523x1950391xP	3x7x7x88069xP	
293459**Y	310897033**2	139017523**2	1667353**4	N Exceeds M
277xP	3x139x1667353xP	3x523x1950391xP	431xxQ(composite)	QHNPFLT31m
293459**Y	310897033**2	139017523**2	1667353**A	N Exceeds M
277xP	3x139x1667353xP	3x523x1950391xP		
293459**Y	310897033**2	139017523**4		N Exceeds M
277xP	3x139x1667353xP	131xQ Q is composite and QHNPFLT 31,000,000		
293459**Y	310897033**2	139017523**B		N Exceeds M
277xP	3x139x1667353xP			
293459**Y	310897033**4	160651**Y	8602968151**2	N Exceeds M
277xP	160651xQ	3xP	3x61xP	
293459**Y	310897033**4	160651**Y	8602968151**C	N Exceeds M
277xP	160651xQ	3xP		
293459**Y	310897033**4	160651**4		N Exceeds M
277xP	160651xQ	5x39371xP		
293459**Y	310897033**4	160651**6		N Exceeds M
277xP	160651xQ	7x29x2008973xQ	Q is composite and QHNPFLT10m	
293459**Y	310897033**4	160651**D		Proposition 5
277xP	160651x5492801xQ Q is composite and QHNPFLT 31,000,000			
293459**Y	310897033**E			Proposition 5
277xP				
293459**4	9875322243666178231**2			N Exceeds M
751xP	3x7x1303x36919xP			
293459**4	9875322243666178231**F			Proposition 5
751xP				
293459**6	8429**X			Proposition 7
491x8429xQ	2x3x5xP			
293459**6	8429**Y	262201**X	131101**Y	N Exceeds M
491x8429xQ	271xP	2xP	3x13x37x349xP	
293459**6	8429**Y	262201**X	131101**4	Corollary 3.1
491x8429xQ	271xP	2xP	5x11x31xQ	
293459**6	8429**Y	262201**X	131101**G	N Exceeds M
491x8429xQ	271xP	2xP		

293459**6	8429**Y	262201**GG	N Exceeds M
491x8429xQ	271xP		
293459**6	8429**4		N Exceeds M
491x8429xQ	11x331711xP		
293459**6	8429**6		N Exceeds M
491x8429xQ	7x617x13553x21520451xP		
293459**6	8429**H		N Exceeds M
491x8429xQ	Q is composite	QHNPFLT 15,000,000	
293459**I			Proposition 5

Note: (A)

S(8602968151**2) = 3 x 61 x Q = 3 x 61 x 40443202 7408324191

To show that Q is a prime number, for each prime factor P of

Q - 1 = 2 x 3 x 5 x 11 x 191 x 293 x 397 x 2153 x 25621

we find a prime Px which is relatively prime to Q such that

Px**(Q-1) [Mod Q] is 1 and Px**[(Q-1)/P] [Mod Q] is not 1

(See Table XXII below)

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	11	1	40443202 7408324190
3	11	1	8602968151
5	7	1	32713257 3378221021
11	3	1	304033 3737687272
191	3	1	6435580 4245085228
293	3	1	35642523 5800904853
397	3	1	21194091 1423666930
2153	3	1	8971593 4425939614
25621	3	1	35913545 1835776487

TABLE XXII

(B)

S(160651**4) = 5 x 39371 x Q = 5 x 39371 x 338368 5468044831

To show that Q is a prime number, for each prime factor P of

Q - 1 = 2 x 5 x 13 x 23 x 63377 x 17856121

we find a prime Px which is relatively prime to Q such that

Px**(Q-1) [Mod Q] is 1 and Px**[(Q-1)/P] [Mod Q] is not 1

(See Table XXIII below)

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	11	1	338368 5468044830
5	3	1	2 5808743801
13	3	1	64981 4042436731
23	3	1	10528 2863365435
63377	3	1	255578 7163128124
17856121	3	1	229153 1006199232

Block 3 x 127

3**Y	13**X	127**R		R not equal to 12, 16, or 18	Theorem	6
13	2x7					
3**Y	13**X	127**12		(See Lemma 19.9 for technique)	Theorem	0
13	2x7	Q				
3**Y	13**X	127**16			Theorem	0
13	2x7	Q	QHNPFLT 100,000,000			
3**Y	13**X	127**18			Theorem	0
13	2x7	Q	QHNPFLT 164,599,969			
3**Y	13**Y	61**X	127**12		Theorem	0
13	3x61	2x31	Q	QHNPFLT 382,179,981		
3**Y	13**Y	61**X	127**16		Theorem	0
13	3x61	2x31	Q			
3**Y	13**Y	61**X	127**18		Theorem	0
13	3x61	2x31	Q			
3**Y	13**Y	61**Y	97**X		Proposition	6
13	3x61	3x13x97	2x7x7			
3**Y	13**Y	61**Y	97**A		Block	97
13	3x61	3x13x97				
3**Y	13**Z	30941**X	127**R	R not equal to 12,16,18	Theorem	6
13	P					
3**Y	13**Z	30941**X	127**R	R = 12, 16, or 18	Theorem	0
13	P	2x3x3x3x3xP				
3**Y	13**Z	30941**Y	157**X		Theorem	0
13	P	157x433x14083	2x79			
3**Y	13**Z	30941**Y	157**16		N Exceeds	M
13	P	157x433x14083				
3**Y	13**Z	30941**4			Proposition	6
13	P	5x11xQ				
3**Y	13**Z	30941**B			N Exceeds	M
13	P					

3**Y	13**6	5229043**Y		N Exceeds M
13	P	3x31x4051xP		
3**Y	13**6	5229043**4		N Exceeds M
13	P	151x151841xP		
3**Y	13**6	5229043**C		N Exceeds M
13	P			
3**Y	13**10	18041**X		Theorem 0
13	23x419x859xP	2x3x31x97		
3**Y	13**10	18041**Y		N Exceeds M
13	23x419x859xP	7xP		
3**Y	13**10	18041**4		N Exceeds M
13	23x419x859xP	5x26801xP		
3**Y	13**10	18041**6		N Exceeds M
13		197x757x25384507xP		
3**Y	13**10	18041**D		N Exceeds M
13	23x419x859xP			
3**Y	13**12	1803647**2		N Exceeds M
13	53x264031xP	31xP		
3**Y	13**12	1803647**E		N Exceeds M
13	53x264031xP			
3**Y	13**16		15798461357509**1	Theorem 0
13	103x443xP		2x5x13x73xP	
3**Y	13**16		15798461357509**2	N Exceeds M
13	103x443xP		3xP	
3**Y	13**16		15798461357509**F	N Exceeds M
13	103x443xP			
3**Y	13**18			N Exceeds M
13	P	P = 1218266908646205O9223		
3**Y	13**22			N Exceeds M
13	1381xP	P = 2519545342349331183143		
3**Y	13**28			N Exceeds M
13	1973x2843x3539xP			
3**Y	13**G			N Exceeds M
13				
3**Z	11**Y			Proposition 6
11x11	7x19			
3**Z	11**Z	5**X		Proposition 6
11x11	5xP			
3**Z	11**Z	5**H		Block 3221
11x11	5xP			
3**Z	11**I			Block 11
11x11				
3**J	Details are found elsewhere within			Block 3

TABLE XXIII

Lemma 19.2 There is no odd perfect number N less than M such that 7**12||N

Note S(7**12) = 16148168401

Block 17971

17971**Y	415669**Y	N Exceeds M	17971**Z	N Exceeds M
3x7x37xP	3x7x49801xP		5xP	
17971**Y	415669**4	N Exceeds M	17971**6	N Exceeds M
3x7x37xP	3671x16668401xP	10,000,000	29x127xP	
17971**Y	415669**6	N Exceeds M	17971**10	N Exceeds M
3x7x37xP	16073xP		23x67x199x1409x77023xQ(comp)	QHNPFLT 13m
17971**Y	415669**10	Proposition 8B	17971**12	N Exceeds M
3x7x37xP	11x23x89xQ(comp)	QHNPFLT13,374,901	79x9049x4267927xQ	QHNPFLT 30m
17971**Y	415669**A	Proposition 5	17971**B	Proposition 5
3x7x37xP				

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

16148168401**X	110563**Y	226741**Y	Block 17971
2x103x709xP	3x17971xP	3x7x7x7x7x1867xP	
16148168401**X	110563**Y	226741**4	Proposition 9
2x103x709xP	3x17971xP	5x11xQ	
16148168401**X	110563**Y	226741**6	N Exceeds M
2x103x709xP	3x17971xP	197x94907xQ	Q is composite and QHNPFLT 5.2m
16148168401**X	110563**Y	226741**A	Proposition 5
2x103x709xP	3x17971xP		
16148168401**X	110563**4	709**Y 23971**2	N Exceeds M
2x103x709xP	P	3x7xP 3x4909x39019	
16148168401**X	110563**4	709**Y 23971**B	N Exceeds M
2x103x709xP	P	3x7xP	
16148168401**X	110563**4	709**4	N Exceeds M
2x103x709xP	P	11x103231xP	
16148168401**X	110563**4	709**6	N Exceeds M
2x103x709xP	P	43xP	
16148168401**X	110563**4	709**C	N Exceeds M
2x103x709xP	P		
16148168401**X	110563**6		Proposition 1
2x103x709xP	113x140533x36107xP		
16148168401**X	110563**D		Proposition 5
2x103x709xP			
16148168401**Y	20074160332810089 7**1		Proposition 11
3x433xP	2x7x7x4966373xP		

16148168401**Y 3x433xP	200741603328100897**2 3x1327xQ Q is composite and QHNPFLT	N Exceeds M 10,000,000
16148168401**Y 3x433xP	200741603328100897**E	Proposition 5
16148168401**4 5x61xP	61**X 2x31	Corollary 3.1
16148168401**4 5x61xP	61**Y 3x13x97	Proposition 7
16148168401**F		Proposition 5

Note: S(16148168401**2) = 3 x 433 x Q = 3 x 433 x 20074160 3328100897

To show that Q is a prime number, for each prime factor P of

Q - 1 = 2**6 x 167 x 277279 x 67736689

we find a prime Px which is relatively prime to Q such that

Px**(Q-1) [Mod Q] is 1 and Px[(Q-1)/P] [Mod Q] is not 1

(See Table XXIV below)

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/] [Mod Q]
2	5	1	20074160 3328100896
167	3	1	1308210 6750267048
277279	3	1	9021779 5162060286
67736689	3	1	8498101 9409247286

TABLE XXIV

Lemma 19.3 There is no odd perfect number N less than M such that 7**16||N

Note S(7**16) = 14009 x 27676311689

Block 14009

The following block is used in Lemma 19.3. Except where indicated otherwise each subcase is eliminated by N being greater than M.

14009**X			PR9	14009**Y	125731**6	1499**4	N>M
2x3x5x467				7x223xP	29x1499xP	11x131xP	
14009**Y	125731**Y	4618291**Y	N>M	14009**Y	125731**6	1499**B	N>M
7x223xP	3x7x163xP	3x3373x15451xP		7x223xP	29x1499xP		
14009**Y	125731**Y	4618291**4	PR7	14009**Y	125731**C		N>M
7x223xP	3x7x163xP	5x31x41x71x751xQ		7x223xP			
14009**Y	125731**Y	4618291**6		14009**Z	113990869481**X		N>M
7x223xP	3x7x163xP	9526973xQ		337901xP	2x3x7x31xP		
14009**Y	125731**Y	4618291**A	PR5	14009**Z	113990869481**D		N>M
7x223xP	3x7x163xP			337901xP			
14009**Y	125731**Z		PR1	14009**6			N>M
7x223xP	5x101x96911x5106325320721			7834583xP			
14009**Y	125731**6	1499**2	N>M	14009**E			N>M
7x223xP	29x1499xP	199x11299					

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

2767631689**X	276763169**Y	Corollary 3.1
2x5xP	7x19x31x8209xP	
2767631689**X	276763169**4	Block 14009
2x5xP	71x2621x345221x9327281xP	
2767631689**X	276763169**A	Block 14009
2x5xP		
2767631689**2	34199239**2	N Exceeds M
3x7x7x7x109x751x2659xP	3x7x13x43x103x139x1567x4441	
2767631689**2	34199239**4	N Exceeds M
3x7x7x7x109x751x2659xP	31xP	
2767631689**2	34199239**6	N Exceeds M
3x7x7x7x109x751x2659xP	71x211x5923x32999xQ(composite) QHNPFLT 10,000,000	
2767631689**2	34199239**B	Proposition 5
3x7x7x7x109x751x2659xP		
2767631689**4		Block 14009
11x5228501xQ QHNPFLT 31,000,000		
2767631689**D		Proposition 5

Lemma 19.4 There is no odd perfect number N less that M such that either of the following is true.

(A) 7**18||N (B) 7**22||N

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

7**18	4534166740403**2		4721697244987**2	N Exceeds M
419xP	31x199x229x373x8263xP		(comp)3x7x37x101173xQ	QHNPFLT 15m
7**18	4534166740403**2		4721697244987**A	Proposition 5
419xP	31x199x229x373x8263xP			
7**18	4534166740403**B			Proposition 5
419xP				
7**22	31479823396757**1	157**16	826632619**2	N Exceeds M
47x3083xP	2x3x11x577xP		3x157x859x1117x3769xP	
7**22	31479823396757**1		826632619**C	N Exceeds M
47x3083xP	2x3x11x577xP			
7**22	31479823396757**2		9428917993254432971089 57**1	Block 5233
47x3083xP	1051xP		2x31xP	
7**22	31479823396757**2		942891799325443297108957**D	N Exceeds M
47x3083xP	1051xP			
7**22	31479823396757**E			Proposition 5
47x3083xP				

Note: S(942891799325443297108957**1) = 2 x 31 x Q where

Q = 152 0793224718 4569308209

To show that Q is a prime number, for each prime pactor P of

Q - 1 = 2**4 x 3 x 13 x 19 x 11261 x 34603 x 3291859321

we find a prime Px which is relatively prime to Q such that

Px**(Q-1) [Mod Q] is 1 and Px**[(Q-1)/P] [Mod Q] is not 1

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [MOd Q]
2	7	1	152 0793224718 4569308208
3	3	1	67 6415048376 5070419028
13	3	1	146 8293968270 9290941674
19	3	1	123 2445093642 6410964346
11261	3	1	19 3935560893 7317955615
34603	3	1	118 2380616463 1869761013
3291859321	3	1	82 7131109217 6998413945

TABLE XXV

Lemma 19.5 If N is an odd perfect number less than M, then for no y such that Y(Mod 3) = 2 will 59**Y||N when 7**28||N.

Note S(7**28) = 59 x 9095,778971,223671,544739 where the larger factor is composite and has no prime factor less than 100,000,000.

S(59**2) = 3541

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

3541**X	23**Y	79**Y		Proposition 6
2x7x11x23	7x79	3x7x7xP		
3541**X	23**Y	79**16		N Exceeds M
2x7x11x23	7x79			
3541**X	23**A			Block 23
2x7x11x23				
3541**Y	19**Y	127**R	R = 12, 16, or 18	Block 3x127
3x19x19x37x313				
3541**4	73659281**X			Proposition 7
5x427001xP	2x3x1013x12119			
3541**4	73659281**Y			N Exceeds M
5x427001xP	P	P = 5425689751096243		
3541**4	73659281**4			N Exceeds M
5x427001xP	5x11x421x5441xP			
3541**4	73659281**6			N Exceeds M
5x427001xP	Q	QHNPFLT 10,500,000 Q is composite		
3541**4	73659281**B			Proposition 5
5x427001xP				
3541**6	42759529**X	16901**Y		N Exceeds M
42759529xP	2x5x11x23xP	P	P = 258660703	
3541**6	42759529**X	16901**4	944362901**1	Proposition 7
42759529xP	2x5x11x23xP	5x11x157099lxP	2x3x7x29x251x3089	
3541**6	42759529**X	16901**4	944362901**2	N Exceeds M
42759529xP	2x5x11x23xP	5x11x157099lxP	19x751xP	
3541**6	42759529**X	16901**4	944362901**F	N Exceeds M
42759529xP	2x5x11x23xP	5x11x157099lxP		
3541**6	42759529**X	16901**6		Proposition 1
42759529xP	2x5x11x23xP	29x3119089xP		
3541**6	42759529**X	16901**C		N Exceeds M
42759529xP	2x5x11x23xP			
3541**6	42759529**2			N Exceeds M
42759529xP	3x5953xP			
3541**6	42759529**4			N Exceeds M
42759529xP	10291181xQ	QHNPFLT 13,000,000		

```
   3541**6        42759529**6                                       N  Exceeds   M
 42759529xP         7x337xQ   QHNPFLT 5,199,979 and  P  = 46115329121443
   3541**6        42759529**D                                       N  Exceeds   M
 42759529xP                   P  = 46115329121443
   3541**10                                                         Proposition  1
      Q   Q is composite and QHNPFLT 13,374,901
   3541**12                                                         N  Exceeds   M
     79xQ  QHNPFLT 10,000,000 and   Q is composite
   3541**E                                                          Proposition  5
```

Note: S(73659281**2) = Q =542568 9751096243

To show that Q is a prime number, for each prime factor P of

Q - 1 = 2 x 3 x 1013 x 12119 x 73659281

we find a prime Px which is relatively prime to Q such that

Px**(Q-1) [Mod Q] is 1 and Px**[(Q-1)/P] [Mod Q] is not 1

(See Table XXVI below)

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	3	1	542568 9751096242
3	5	1	542568 9751096242
1013	3	1	345896 7271365595
12119	3	1	274136 0673210795
73659281	3	1	481347 3410581147

TABLE XXVI

Lemma 19.6 There is no odd perfect number N less than M such that 7**28||N

Note S(7**28) = 59 x 9095,778971,223671,544739 which is composite

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

59**Y	where Y(Mod 3) = 2			Lemma 19.5
59**Z	In using Block 151, we shall use small primes.			Block 151
11x41x151xP				
59**6	4691**Y			Block 103
43x281x757xP	97x103xP			
59**6	4691**Z	140785321**X		Corollary 3.1
43x281x757xP	5x11x71x881xP	2x31x1187xP		
59**6	4691**Z	140785321**Y		Proposition 7
43x281x757xP	5x11x71x881xP	3x1021xP		
59**6	4691**Z	140785321**4	71**Y 2557**Y	Proposition 7
43x281x757xP	5x11x71x881xP	5x393121xQ	5113 3x7x13x13x19x97	
59**6	4691**Z	140785321**4	71**12	N Exceeds M
43x281x757xP	5x11x71x881xP	5x393121xQ		
59**6	4691**Z	140785321**4	71**18	N Exceeds M
43x281x757xP	5x11x71x881xP	5x393121xQ(composite)	QHNPFLT 25,000,000	
59**6	4691**Z	140785321**6		N Exceeds M
43x281x757xP	5x11x71x881xP	29x379x4327x1279727xQ		
59**6	4691**Z	140785321**A		Proposition 5
43x281x757xP	5x11x71x881xP			
59**6	4691**6			N Exceeds M
43x281x757xP	7x43x953x3966803xP			
59**6	4691**10			N Exceeds M
43x281x757xP	199x727x2003x394241xP			
59**6	4691**B			N Exceeds M
43x281x757xP				
59**10	805243954219**2	175467775137640243**C		Block 23
23x67x419xP	3x13x19x4987xP			
59**10	805243954219**4			N Exceeds M
23x67x419xP	Q Q is composite and has no prime factor less than 25m			
59**10	805243954219**D			Proposition 5
23x67x419xP				
59**12	180987323579538672924l**1			Proposition 11
P	2x11x37x97x257x89190560937307			
59**12	180987323579538672924l**2		61**X	Proposition 1
P	3x61x5347xQ		2x31	
59**12	180987323579538672924l**2		61**Y	Proposition 6
P	3x61x5347xQ(composite)		3x13x97	

59**12	180987323579538672924l**E	Proposition 5
P		
59**16	361353204962363828785531**F	N Exceeds M
137x443xP		
59**18	183503**Y	N Exceeds M
571x183503xQ	32869xP	
59**18	183503**4	N Exceeds M
571x183503xQ	101x661xP	
59**18	183503**6	N Exceeds M
571x183503xQ	29x211x337x8387x3359287xP	
59**18	183503**GG	Proposition 5
571x183503xQ	Q is composite and QHNPFLT 100,000,000	
59**22		N Exceeds M
47x829x6763xP		
59**28		N Exceeds M
29x80986039xQ	QHNPFLT 263,124,541	
59**H		Proposition 5

Note: S(59**12) = Q = 18 0987323579 5386729241.

We use the following information to show that Q is a prime number.

Q - 1 = 2**3 x 3**2 x 5 x 7 x 13 x 59 x 163 x 1741 x 3541 x 931837.

11**[(Q-1)/2] [Mod Q]	=	18 0987323579 5386729240
3**[(Q-1)/3] [Mod Q]	=	8 2325625593 1456499817
3**[(Q-1)/5] [Mod Q]	=	12 6473291469 1659547327
3**[(Q-1)/7] [Mod Q]	=	16 5901648337 5297101592
3**[(Q-1)/13] [Mod Q]	=	59
3**[(Q-1)/59] [Mod Q]	=	4 3235847993 5144715425
3**[(Q-1)/163] [Mod Q]	=	1 8898618933 8734310611
3**[(Q-1)/1741] [Mod Q]	=	2 9196712962 6625115322
3**[(Q-1)/3541] [Mod Q]	=	6 6083874801 2094089956
3**[(Q-1)/931837] [Mod Q]	=	10 2531185617 4651866583

Lemma 19.7 There is no odd perfect number N less than M such that either 7**30||N or 7**36||N.

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

7**30 311x21143xP	3999088279399464409**1 2x5x43x89x3917x26677735199		Proposition 11
7**30 311x21143xP	3999088279399464409**2 3x7x61x463xQ	Q is composite and QHNPFLT	N Exceeds M 10,000,000
7**30 311x21143xP	3999088279399464409**A		Proposition 5
7**36 223x2887xP	4805345109492315767981401**1 2x107x157x2269x6726821xP	157**X	Proposition 2
7**36 223x2887xP	4805345109492315767981401**1 2x107x157x2269x6726821xP	157**16	N Exceeds M
7**36 223x2887xP	4805345109492315767981401**2 3x7x5659xP		N Exceeds M
7**36 223x2887xP	4805345109492315767981401**B		Proposition 5

For the second case of Lemma 19.8 one assumption is that 48037081**X||N for some X[Mod 4] = 1. This implies that S(48037081**1) = 2 x 2393 x 10037 divides N. By Proposition 2, the prime 2393 must appear to an even power in the prime factorization of N. Except for an odd power, only for a power W greater than 297 can there be a prime P such that S(P**W) is divisible by 2393. We may apply Proposition 11 here.

For Case 11 of Lemma 19.8 it will be assumed that for some X[Mod 4] = 1, 11411291417**X||N. With this assumption it is implied that 2 x 3 x 281 x P divides N, where P = 6768263. It is left to the reader to show that because of this P, N necessarily exceeds M.

Lemma 19.8 There is no odd perfect number N less than M such that 7**40||N

Note The prime 83 divides S(7**40)

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

83**Y	See Lemma 17.1 for details			Block 367
19x367				
83**Z	48037081**X			Proposition 11
P	2x2393xP			
83**Z	48037081**2	17293183**2	719657023**2	N Exceeds M
. P	3x2269x19603x17293183	3x138517xP	3x43x433x292021xP	
83**Z	48037081**2	17293183**2	719657023**A	N Exceeds M
P	3x2269x19603x17293183	3x138517xP		
83**Z	48037081**2	17293183**B		N Exceeds M
P	3x2269x19603x17293183			
83**Z	48037081**4			Corollary 3.1
P	5x11x11x31x211xQ			
83**Z	48037081**C			N Exceeds M
P				
83**6	11411291417**X			Proposition 11
29xP	2x3x281x6768263			
83**6	11411291417**2			N Exceeds M
29xP	709x13597x155841x86123299			
83**6	11411291417**4			N Exceeds M
29xP	11x41x721111x5963011xQ	QHNPFLT 19,999,991		
83**6	11411291417**D			Proposition 5
29xP				
83**10	15991874273**X			N Exceeds M
13003x75527xP	2x3x41x65007619			
83**10	15991874273**2			N Exceeds M
13003x75527xP	7x20323x487387xP			
83**10	15991874273**4	61**X		Proposition 1
13003x75527xP	61xP	2x31		
83**10	15991874273**4	61**Y		N Exceeds M
13003x75527xP	61xP	3x13x97		
83**10	15991874273**E			Proposition 5
13003x75527xP				
83**12		43580447**Y		N Exceeds M
1249x1396513x1423319xP		67x414553xP		
83**12		43580447**4		N Exceeds M
1249x1396513x1423319xP		11x61x150571xP		
83**12		43580447**6		N Exceeds M
1249x1396513x1423319xP		7xQ(composite)	QHNPFLT 5,199,979	

```
    83**12                              43580447**F              Proposition  5
    1249x1396513x1423319xP
    83**16                                                       Theorem      4
409x1259x8161xQ  Q is composite and  Q has no prime factor less than 100,000,000
    83**18                                                       N  Exceeds   M
 6689x11933xQ  Q is composite and   QHNPFLT 28,000,000
    83**22                                                       N  Exceeds   M
     47xP
    83**G                                                        Proposition  5
```

Lemma 19.9 Let N be an odd perfect number and suppose 7**42||N. Then N is greater than M. We shall assume the contrary and show that every other possibility leads to a contradiction.

Let Q = S(7**42)

The following statements are proved easily.

A) Q is composite,
B) Q has no prime factor less than 1,712,380,573
C) Q is not a perfect cube,
D) Q is not a perfect square,
E) If P is prime and S(P**2) is divisible by 7**6, then either

P(Mod 117649) = 34967 or P(Mod 117649) = 82681, and

E) Q has no prime factor P less than 2 x 10**16 for which

P(Mod 117649) = 34967 or P(Mod 117649) = 82681.

Proof of Lemma 19.9

It follows from the above that Q may be written as a product of powers of primes in exactly one of the following forms

Q = P1 x P2 Q = P1 x P2**2 Q = P1 x P2 x P3

where for each form, Pi = Pj if and only if i = j

(I) If Q is of the first or third form, then the following cases exhaust all other possibilities.

Possibilities And Reasons By Which They May Be Excluded

Case A The prime 3 divides N

3**Y	13**X	where X(Mod 4) = 1			Proposition 1
3**Y	13**Y	61**X			Proposition 1
3**Y	13**Y	61**Y			Proposition 6
3**Y	13**Z	30941**Y	157**X		Proposition 1
3**Y	13**Z	30941**Y	157**16		N Exceeds M
3**Y	13**Z	30941**Z			Proposition 6
3**Y	13**Z	30941**6			N Exceeds M
3**Y	13**Z	30941**A			N Exceeds M
3**Y	13**U		5229043**Y	72577051**Y	N Exceeds M
3**Y	13**U		5229043**Y	72577051**4	Proposition 9
3**Y	13**U		5229043**Y	72577051**B	N Exceeds M

Theorem 19 The prime 7 cannot be a factor of an odd perfect number N that is less than M.

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

7**Y 3x19	19**Y	127**R R = 12, 16, or 18	Block 3x127
7**Y 3x19	19**G	For all cases other than above	Theorem 16
7**Z 2801			Theorem 18
7**W 29x4733			Theorem 13
7**10 1123xP			Lemma 19.1
7**12 P			Lemma 19.2
7**16 14009xP			Lemma 19.3
7**18 419xP			Lemma 19.4
7**22 47x3083xP			Lemma 19.4
7**28 59x127540261xP			Lemma 19.5
7**30 311x21143xP			Lemma 19.7
7**36 223x2887xP			Lemma 19.7
7**40 83x20515909xP			Lemma 19.8
7**42 Q	Q is composite and Q has no prime factor less than 5,324,380,573		Lemma 19.9
7**46 Q	Q is composite and Q has no prime factor less than 1,204,463,737		Lemma 19.9
7**52 8269x319591xP	8269**X 2x5x827		N Exceeds M
7**52 8269x319591xP	8269**A		N Exceeds M
7**58 459257xQ	459257**X 2x3x76543		N Exceeds M
7**58 459257xQ	459257**Y 119227xP		N Exceeds M
7**58 459257x134927809xQ	459257**B QHNPFLT 154,875,001		N Exceeds M
7**C			Proposition 5

Corollary 19.1 If N is an odd perfect number less than M, then 31 does not divide N.

Corollary 19.2 If N is an odd perfect number less than M, then 3 x 331 does not divide N.

Lemma 20.1 Let N be an odd perfect number less than M and suppose further that 3**Z||N and 11**Y||N where Z(Mod 5) = Y(Mod 5) = 4. Then the following is not true.

3221**10||N

We need consider only two sub-cases.

(1) 5**1||N and (2) 5**W||N where W is greater than 1. In the latter case, we get our contradiction from Prop 8E.

In the former case, we consider two sub-cases, the case for primes that are less than 1000 and that for primes greater than 1000.

(1) Other than the prime 11, the primes P less than 1000 for which there are natural numbers W such that S(P**W) is divisible by 3221 are the primes 89, 281, 491, 503, 541, 643, and 983. For each of the primes P = 89 and P = 491, W = 69 is the smallest W such that S(P**W) is divisible by 3221. For each of the primes P = 281 and P = 503, W = 91 is the smallest W such that S(P**W) is divisible by 3221. For each of the primes P = 647 and P = 983 W = 19 is the smallest such W. In each of these cases, P**W is greater than 10**50 and hence, by Prop. 5, P**W cannot be a factor of N.
(2) on the other hand the primes P greater than 1000 for each of which there exists a W less than 17 such that S(P**W) is divisible by 3221 and at the same time P**W is less than 10**50 are given in Table XXVII.

	P(Mod 3221)	Powers W of P Such That S(P**W) Is Divisible By 3221
(A)	11, 121, 1331, 1757	W(Mod 5) = 4
(B)	166, 476, 744, 1433, 1509, 2115	W(Mod 14) = 13
(C)	234, 2987	W(Mod 4) = 3
(D)	1106, 1712, 1788, 2477, 2745, 3055	W(Mod 7) = 6
(E)	1464, 1890, 3100	W(Mod 10) = 9
(F)	3220	W(Mod 4) = 1

TABLE XXVII

Block 3221

This block, labeled Block 3221, is used in Theorem 20 as well as in Lemma 23.7. In each sub-case where this block is used, it is assumed that for some z (where z(mod 5) = 4), 11**Z||N.

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Some of the details of this block have been used in Lemma 2.

Possibilities And Reasons By Which They May Be Excluded

3221**X	179**A				Proposition 11
2x3x3x179					
3221**Y	10378063**Y		46685826619**Y		N Exceeds M
P	3x769xP		3x13x601x1321x1609x109537xP		
3221**Y	10378063**Y		46685826619**4		N Exceeds M
P	3x769xP		11x31xP		
3221**Y	10378063**Y		46685826619**B		Proposition 5
P	3x769xP				
3221**Y	10378063**4	33151**Y	366340651**2	68409301**X	Proposition 8e
P	11x33151xP	3xP	3x3229x202519xP	2x13x43x43xP	
3221**Y	10378063**4	33151**Y	366340651**2	68409301**Y	Proposition 7
P	11x33151xP	3xP	3x3229x202519xP	3x7x5095939xP	
3221**Y	10378063**4	33151**Y	366340651**2	68409301**4	Proposition 8E
P	11x33151xP	3xP	3x3229x202519xP	5x11x61x131xQ	
3221**Y	10378063**4	33151**Y	366340651**2	68409301**6	N Exceeds M
P	11x33151xP	3xP	3x3229x202519xP	71x3137x1823683xQ(comp)	6m
3221**Y	10378063**4	33151**Y	366340651**2	68409301**C	Proposition 5
P	11x33151xP	3xP	3x3229x202519xP		
3221**Y	10378063**4	33151**Y	366340651**4		Proposition 8e
P	11x33151xP	3xP	5xQ		
3221**Y	10378063**4	33151**Y	366340651**D		Proposition 5
P	11x33151xP	3xP			
3221**Y	10378063**4	33151**4	31810898376456201755581**X		Proposition 1
P	11x33151xP	5xP	2x41x547xQ(composite)		QHNPFLT 3m
3221**Y	10378063**4	33151**4	31810898376456201755581**E		N Exceeds M
P	11x33151xP	5xP			
3221**Y	10378063**4	33151**6			N Exceeds M
P	11x33151xP	43x127x371071xP			
3221**Y	10378063**4	33151**F			N Exceeds M
P	11x33151xP	P = 31810898376456201755581			
3221**Y	10378063**6				N Exceeds M
P	29x113x127xP				
3221**Y	10378063**G				Proposition 5
P					

```
   3221**Z    1957650063931**2                                    Proposition  7
   5x11xP         3x7x13xP
   3221**Z    1957650063931**4                                    N  Exceeds   M
   5x11xP           5x41xQ   QHNPFLT  1,000,000
   3221**Z    1957650063931**H                                    N  Exceeds   M
   5x11xP
   3221**6                                  92204351**2           N  Exceeds   M
7x673x10333x248879xP                    19x1033x56263x7698853
   3221**6                                  92204351**4           N  Exceeds   M
7x673x10333x248879xP                            5xQ  Q is composite  QHNPFLT 10m
   3221**6                                  92204351**6           N  Exceeds   M
7x673x10333x248879xP                       7x379x342049xP
   3221**6                                  92204351**I           N  Exceeds   M
7x673x10333x248879xP
   3221**C   C > 9                                                N  Exceeds   M
```

Note: S(33151**6) = 43 x 127 x 371071 x Q where

Q - 1 = 2 x 3**2 x 7**2 x 85091 x 8727934883.

The entries in Table XXVIII may be used to show that Q is prime.

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	3	1	65503379 9688089346
3	3	1	43921029 9271710277
7	3	1	4833074 9420441813
45091	3	1	46663422 9915098353
8727934883	3	1	41328953 9457944450

TABLE XXVIII

Also, S(31810898376456201755581**1) = 2 x 41 x 547 x Q. To imply that Q is composite, it is sufficient to make the following statement.

5**(Q-1) [Mod Q] = 20002023 2803497708.

Theorem 20 The number 3**4 x 11 cannot be a factor of an odd perfect number N that is less than M.

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

11**Y 7x19			Proposition 6
11**Z 5x3221	3221**X		Block 3221
11**Z 5x3221	3221**Y		Block 3221
11**Z 5x3221	3221**Z		Block 3221
11**Z 5x3221	3221**6		Block 3221
11**Z 5x3221	3221**10		Lemma 20.1
11**Z 5x3221	Q 3221**12 P	Q is composite and QHNPFLT 100,000,000	N Exceeds M
11**Z 5x3221	3221**A		Proposition 5
11**6			Lemma 20.2
11**10			Lemma 20.2
11**12			Lemma 20.3
11**16			Lemma 20.3
11**18			Lemma 20.3
11**22			Lemma 20.4
11**28			Lemma 20.4
11**30			Lemma 20.4
11**36			Lemma 20.4
11**40 83x1231x27061x509221x14092193x29866451			N Exceeds M
11**42			Proposition 1
Q 11**D		Q is composite and QHNPFLT 200,000,000	N Exceeds M

Lemma 21.1 Suppose N is an odd perfect number less than M and that 13**Z||N. Then for no Y such that Y(Mod3) = 2 will 3**Y||N.

Block 4281671749

4281671749**1 2x5x5x5xP	Proposition 6	4281671749**A	N Exceeds M
4281671749**2 3x439xP	Proposition 1		

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(13**4) = 30941 S(191**6) = 127 x 197 x 10627 x 183569

Possibilities And Reasons By Which They May Be Excluded

30941**X 2x3x3x3x3xP	191**Y 7x13x13x31	31**Y 3x331	331**Y 3x7x5233			Theorem 19
30941**X 2x3x3x3x3xP	191**Z 5x11x1871xP					Proposition 8A
30941**X 2x3x3x3x3xP	191**6	183569**2 13xP		127**Y		Theorem 6
30941**X 2x3x3x3x3xP	191**6	183569**2 13xP		127**Z		Theorem 6
30941**X 2x3x3x3x3xP	191**6	183569**2 13xP		127**6		Theorem 6
30941**X 2x3x3x3x3xP	191**6	183569**2 13xP		127**10		Theorem 6
30941**X 2x3x3x3x3xP	191**6	183569**2 13xP		127**12		Proposition 1
30941**X 2x3x3x3x3xP	191**6	183569**2 13xP		127**16		Proposition 1
30941**X 2x3x3x3x3xP	191**6	183569**2 13xP		127**18		Proposition 1
30941**X 2x3x3x3x3xP	191**6	183569**2 13xP		127**A		Theorem 6
30941**X 2x3x3x3x3xP	191**6	183569**4 61x761x51249241x477307001		61**Y 3x13x97		N Exceeds M
30941**X 2x3x3x3x3xP	191**6	183569**6 7x1163x34469x4689413xP				N Exceeds M
30941**X 2x3x3x3x3xP	191**6	183569**B				Proposition 5
30941**X 2x3x3x3x3xP	191**10 3**8 Q 13xP	757**Y 3x13xP	14713**Y 3x19xP	3798019**2 3x1123xP		BK 4281671749
30941**X 2x3x3x3x3xP	191**10 3**8 Q 13xP	757**Y 3x13xP	14713**Y 3x19xP	3798019**4 11x1491571xP		N Exceeds M

```
  30941**X      191**10 3**8     757**Y     14713**Y    3798019**C  Proposition  5
2x3x3x3x3xP        Q    13xP     3x13xP      3x19xP
  30941**X      191**10 3**8     757**Y     14713**4                Proposition  1
2x3x3x3x3xP        Q    13xP     3x13xP     1774921xP
  30941**X      191**10 3**8     757**Y     14713**6                Proposition  1
2x3x3x3x3xP        Q    13xP     3x13xP  7673x8387xP
  30941**X      191**10 3**8     757**Y     14713**D                Proposition  1
2x3x3x3x3xP        Q    13xP     3x13xP
  30941**X      191**10 3**8     757**4                             N  Exceeds   M
2x3x3x3x3xP        Q    13xP     11x191x2521x62081
  30941**X      191**10 3**8     757**6     30629717273**E          Proposition  1
2x3x3x3x3xP        Q    13xP  7x87887xP
  30941**X      191**10 3**8     757**10                            Proposition  1
2x3x3x3x3xP        Q    13xP     89x6654649xP
  30941**X      191**10 3**8     757**F                             Proposition  1
2x3x3x3x3xP        Q    13xP
  30941**X      191**10 3**14                                       Theorem     20
2x3x3x3x3xP        Q    11x11x13x4561
  30941**X      191**10 3**20                                       Theorem     15
2x3x3x3x3xP        Q    13x1093x368089
  30941**X      191**10 3**26                                       Block      757
2x3x3x3x3xP        Q    13x109x433x757x8209
  30941**X      191**10 3**32                                       Block       23
2x3x3x3x3xP        Q    13x23x3851x2413941289
  30941**X      191**10 3**38                                       N  Exceeds   M
2x3x3x3x3xP        Q    13x13x313x6553x7333x797161
  30941**X      191**10 3**44                                       Theorem     20
2x3x3x3x3xP        Q    11x11x13x181x757x1621x4561x927001
  30941**X      191**10 3**50                                       N  Exceeds   M
2x3x3x3x3xP        Q    13x1871x34511xP
  30941**X      191**10 3**56                                       N  Exceeds   M
2x3x3x3x3xP        Q    13x1597x363889xQ
  30941**X      191**10 3**62                                       Block      757
2x3x3x3x3xP        Q    13x757x1093xQ
  30941**X      191**10 3**68                                       N  Exceeds   M
2x3x3x3x3xP        Q    13x47x1001523179xP
  30941**X      191**10 3**74                                       Theorem     20
2x3x3x3x3xP        Q    11x11x13x4561xQ
  30941**X      191**10 3**80                                       Block      757
2x3x3x3x3xP        Q   13x757x109x433x8209xQ
  30941**X      191**10 3**J                                        Proposition  1
2x3x3x3x3xP        Q   Q is composite and  QHNPFLT 187,549,891
  30941**X      191**12                                             Block      757
2x3x3x3x3xP     131x1483x9049x92041x301627xP
  30941**X      191**16                                             Block      757
2x3x3x3x3xP        Q   Q  is composite and has no prime factor less than 126m
  30941**X      191**18                                             Block      757
2x3x3x3x3xP      19xQ   QHNPFLT 146,799,967
  30941**X      191**G                                              Proposition  5
2x3x3x3x3xP
```

30941**Y 157x433x14083	157**X 2x79	79**Y 3x7x7xP		Theorem 19
30941**Y 157x433x14083	157**X 2x79	79**18	14083**Y 3x4591xP	Proposition 1
30941**Y 157x433x14083	157**X 2x79	79**18	14083**4 11x71xP	Proposition 1
30941**Y 157x433x14083	157**X 2x79	79**18	14083**6 P	Proposition 1
30941**Y 157x433x14083	157**X 2x79	79**18	14083**H	Proposition 1
30941**Y 157x433x14083	157**16			Block 14083
30941**4 5x11xQ	Q is composite and has no prime factor less than 5,381,991			Proposition 6
30941**6 7xQ				Theorem 19
30941**10 23x4027xP				N Exceeds M.
30941**I				Proposition 5

For Lemma 21.1 we assumed that for some Y(Mod 3) = 2, 3**Y||N. For most cases it is also assumed that for some X(Mod 4) = 1, 30941**X||N. Fortunately, it is true that S(30941**1) = 2 x 3 x 3 x 3 x 3 x 191 which means that for Y=2 3**Y||N is false.

To imply that S(191**10) = Q = 649 5512737881 4706256961 is composite, it is sufficient to state the fact that

5**(Q-1) [Mod Q] = 254 0631698054 0386537317.

Lemma 21.2 Let N be an odd perfect number less than M and let 3**2|N. No one of the following can happen under these conditions.
(A) 13**U||N (B) 13**10||N (C) 13**12||N (D) 13**16||N

Block 264031 This block is used only in Lemma 21.2. Except where possibly indicated otherwise, each subcase is eliminated because it contradicts N being less than M.

264031**Y 3x19x29683xP	41203**Y 3x19xP	29784709**X P8H 2x5x1399xP	264031**Y 3x19x29683xP	41203**Z 31xQ	Cor 19.1
264031**Y 3x19x29683xP	41203**Y 3x19xP	29784709**Y 3x967xP	264031**Y 3x19x29683xP	41203**6 7xQ Q is composite	Theorem 19
264031**Y 3x19x29683xP	41203**Y 3x19xP	29784709**4TH19 11x31xQ	264031**Y 3x19x29683xP	41203**10 Q(comp) QHNPFLT 10m	
264031**Y 3x19x29683xP	41203**Y 3x19xP	29784709**6 4327xP	264031**Y 3x19x29683xP	41203**12 18617xQ(composite)	
264031**Y 3x19x29683xP	41203**Y 3x19xP	29784709**A	264031**Y 3x19x29683xP 264031**C	41203**B	Prop 5

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(13**6) = 5229043 S(13**10) = 23 x 419 x 859 x 18041

Possibilities And Reasons By Which They May Be Excluded

13**U	5229043**Y 3x31x4051xP		Corollary 19.1
13**U	5229043**4 151x151841xP	32607907713428723311**2 3xQ(composite) QHNPFLT	N Exceeds M 8,799,991
13**U	5229043**4 151x151841xP	32607907713428723311**J	Proposition 5
13**U	5229043**6 7x29x239xQ		Theorem 19
13**U	5229043**A		Proposition 5
13**10	18041**X 2x3x31x97		Corollary 19.1
13**10	18041**Y 7xP		Theorem 19
13**10	18041**4 5x26801xP	790579404481**X 2x7x68281x827023	Proposition 7
13**10	18041**4 5x26801xP	790579404481**2 3x13x32443xP	N Exceeds M
13**10	18041**4 5x26801xP	790579404481**4 5x11xQ	Proposition 8E
13**10	18041**4 5x26801xP	790579404481**C	Proposition 5

```
    13**10          18041**6             9108709959749**1       Proposition  6
               197x757x25384507xP        2x3x5x5x5x853x2559
    13**10          18041**6             9108709959749**2       N  Exceeds   M
               197x757x25384507xP 277x236773x10723561x117967608271
    13**10          18041**6             9108709959749**D       Proposition  5
               197x757x25384507xP
    13**10          18041**E                                    N  Exceeds   M

    13**12        1803647**Y                                    Corollary 19.1
 53x264031xP   31x104940138847
    13**12        1803647**4        264031**F                   Block    264031
 53x264031xP         41xQ    Q is composite and QHNPFLT 24,000,000
    13**12        1803647**6                                    Block    264031
 53x264031xP      8737x2064077xQ
    13**12        1803647**G                                    Proposition  5
 53x264031xP
    13**16  15798461357509**1                                   Proposition  6
 103x443xP    2x5x13x73xP
    13**16  15798461357509**2                                   Block       103
 103x443xP        3xP
    13**16  15798461357509**H                                   Proposition  5
 103x443xP
```

Note: S(5229043**4) = 3151 x 151841 x 3260790771 3428723311

Let Q = 3260790771 3428723311. Q-1 = 2 x 3 x 5 x 223 x 397 x 731363 x P

To show that Q is a prime number, we use the entries in Table XXIX.

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	3	1	3260790771 3428723310
3	7	1	1770345601 4612370715
5	3	1	1254552033 9179449263
223	3	1	241635239 3215747241
397	3	1	2526279978 6529964260
731363	3	1	1243031563 9525332143
16787009	3	1	1902314868 9162164436

TABLE XXIX

Lemma 21.3 Let N be an odd perfect number less than M and let 3**2|N. Then no one of the following is then true.

(A) 13**18||N (B) 13**22||N (C) 13**28||N

(D) 13**30||N (E) 13**36||N (F) 13**42||N

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(13**28) = 1973 x 2843 x 3539 x P

Possibilities And Reasons By Which They May Be Excluded

13**18	121826690864620509223**2				Theorem 19
P	3x7xQ				
13**18	121826690864620509223**A				Proposition 5
P					
13**22	2519545342349331183143**2				Theorem 19
1381xP	7x13x127x199xQ				
13**22	2519545342349331183143**B				Proposition 5
1381xP					
13**28	2843**Y	9283**Y	22567**Y	169764019**Y	N Exceeds M
	13x67xP	3x19x67x22567	3xP	3x13x19x421xP	
13**28	2843**Y	9283**Y	22567**Y	169764019**A	N Exceeds M
	13x67xP	3x19x67x22567	3xP		
13**28	2843**Y	9283**Y	22567**4	61**X	Corollary 19.1
	13x67xP	3x19x67x22567	61x1744200lxP	2x31	
13**28	2843**Y	9283**Y	22567**4	61**Y	N Exceeds M
	13x67xP	3x19x67x22567	61x1744200lxP	3x13x97	
13**28	2843**Y	9283**Y	22567**B		N Exceeds M
	13x67xP	3x19x67x22567			
13**28	2843**Y	9283**4	513572619821**X		Proposition 1
	13x67xP	1446lxP	2x3x233x1549xP		
13**28	2843**Y	9283**4	513572619821**2		Theorem 19
	13x67xP	1446lxP	7xQ QHNPFLT 3,000,000		
13**28	2843**Y	9283**4	513572619821**C		N Exceeds M
	13x67xP	1446lxP			
13**28	2843**Y	9283**6			Theorem 19
	13x67xP	7x239xQ			
13**28	2843**Y	9283**D			N Exceeds M
	13x67xP				
13**28	2843**4	594110926489l**2			N Exceeds M
	1lxP	8017x96259x109363xP			
13**28	2843**4	594110926489l**E			Proposition 5
	1lxP				
13**28	2843**6				Theorem 19
	7x16927x21042lx21185895697				
13**28	2843**10				N Exceeds M
	727x1607xQ	QHNPFLT 13,374,901			

13**28	2843**F			N Exceeds M
13**30	81705090114313634085681563 69**1			Proposition 1
311x1117xP	2x3x5x61x199x1511x158371x162091xP			
13**30	81705090114313634085681563 69**G			Proposition 5
311x1117xP				
13**36	1481**X			Proposition 1
1481xQ	2x3x13x19			
13**36	1481**Y			Theorem 19
1481xQ	7xP			
13**36	1481**4	962816607761**1	5810531**2	N Exceeds M
1481xQ	5xP	2x3x27617xP	4705387x7175239	
13**36	1481**4	962816607761**1	5810531**4	N Exceeds M
1481xQ	5xP	2x3x27617xP	5xQ(comp)QHNPFLT 25m	
13**36	1481**4	962816607761**1	5810531**H	N Exceeds M
1481xQ	5xP	2x3x27617xP		
13**36	1481**4	962816607761**2		N Exceeds M
1481xQ	5xP	3403441x10815733xP		
13**36	1481**4	962816607761**I		N Exceeds M
1481xQ	5xP			
13**36	1481**6			N Exceeds M
1481xQ	953x2087x265007xP			
13**36	1481**10			N Exceeds M
1481xQ	23x397x80917xP			
13**36	1481**J			N Exceeds M
1481xQ	Q is composite and Q has no prime factor less than 200,000,000			
13**42	119627**2			Theorem 19
119627xP	7x7x1567xP			
13**42	119627**K			N Exceeds M
119627xP				

Note: S(13**22) = 1381 x 25 1954534234 9331183143

25 1954534234 9331183142 = 2 x 23 x 42899 x 2197469 x 581024467

5**[(Q-1)/2] [Mod Q]	=	25 1954534234 9331183142
3**[(Q-1)/23] [Mod Q]	=	13
3**[(Q-1)/42899] [Mod Q]	=	21 0129408142 4060774634
3**[(Q-1)/2197469] [Mod Q]	=	14 3359246199 6935994632
3**[(Q-1)/581024467] [Mod Q]	=	24 7339221807 1058502151

Lemma 21.4 Let N be an odd perfect number less than M. Then not both of the of the following can happen simultaneously.

(A) 13**Y1||N (B) 61**Y2||N

where Y1(Mod 3) = Y2(Mod 3) = 2

Block 567661

567661**X 2xP	283831**Y 3x7x30427xP	Theorem	19
567661**X 2xP	283831**4 5x631x2111xQ	Prop	6
567661**X 2xP	283831**6 1009x2801xP	N Exceeds	M
567661**X 2xP	283831**A	N Exceeds	M
567661**2 3x13x67x9967x12373	12373**X	Prop	11
567661**2	12373**2 3x7xP	Theorem	19
567661**2	12373**4 11x31x8831xP	Corol	19.1
567661**2	12373**B	N Exceeds	M
567661**4 5x41x211x271x10271xP		N Exceeds	M
567661**C		N Exceeds	M

Note S(13**2) = 3 x 61, S(61**2) = 3 x 13 x 97

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

97**X 2x7x7					Theorem 19
97**Y 3xP	3169**X 2x5x317				Proposition 11
97**Y 3xP	3169**Y 3xP	3348577**X 2xP	1674289**Y 3xP	934415109937**2 3x61x79x127x226903xP	N Exceeds M
97**Y 3xP	3169**Y 3xP	3348577**X 2xP	1674289**Y 3xP	934415109937**A	N Exceeds M
97**Y 3xP	3169**Y 3xP	3348577**X 2xP	1674289**B		N Exceeds M
97**Y 3xP	3169**Y 3xP	3348577**2 3xP	3737657091169**1 2x5x443x443714919		Proposition 6
97**Y 3xP	3169**Y 3xP	3348577**2 3xP	3737657091169**2 3x43xQ(comp)	QHNPFLT 1,000,000	Block 43
97**Y 3xP	3169**Y 3xP	3348577**2 3xP	3737657091169**B		Proposition 5
97**Y 3xP	3169**Y 3xP	3348577**4 86524531xP	86524531**2 3x7xQ		Theorem 19
97**Y 3xP	3169**Y 3xP	3348577**4 86524531xP	86524531**4		Proposition 8A

```
97**Y     3169**Y       3348577**4    86524531**F                N  Exceeds   M
 3xP       3xP          86524531xP
97**Y     3169**Y       3348577**6                               Theorem      19
 3xP       3xP        7x281x2129xQ    QHNPFLT  1,000,000
97**Y     3169**Y       3348577**G                               Proposition  5
 3xP       3xP
97**Y     3169**4                     1653850268201**1           Proposition 11
 3xP       61xP                 2x3x11x19x19x59x577x2039
97**Y     3169**4                     1653850268201**2  78049**X Proposition  7
 3xP       61xP                           78049xQ      2x5x5x7x223
97**Y     3169**4                     1653850268201**2  78049**2 N  Exceeds   M
 3xP       61xP                           78049xQ        3x97xP
97**Y     3169**4                     1653850268201**2  78049**4 N  Exceeds   M
 3xP       61xP                           78049xQ      11x23981xP
97**Y     3169**4                     1653850268201**2  78049**H N  Exceeds   M
 3xP       61xP                           78049xQ(com) QHNPFLT 10,200,001
97**Y     3169**4                     1653850268201**I           N  Exceeds   M
 3xP       61xP
97**Y     3169**6           71**Y     5113**X     2557**Y        Proposition  6
 3xP      71x491x8429xP     5113      2x2557      3x7x13x13x19x97
97**Y     3169**6           71**12                               N  Exceeds   M
 3xP      71x491x8429xP       Q
97**Y     3169**6           71**18                               N  Exceeds   M
 3xP      71x491x8429xP       Q
97**Y     3169**10                                               Block        23
 3xP      11x23xQ    QHNPFLT 13,000,000  Q is composite
97**Y     3169**12          53**X                                Proposition  1
 3xP        53xQ          2x3x3x3
97**Y     3169**12          53**2                                Theorem      4
 3xP        53xQ          7x409
97**Y     3169**12          53**4                                N  Exceeds   M
 3xP        53xQ        11x131x5581
97**Y     3169**12          53**J                                N  Exceeds   M
 3xP        53xQ  Q is composite  and  QHNPFLT 10,000,000
97**Y     3169**K                                                Proposition  5
 3xP
97**Z                                                            Corollary 19.1
11x31xP
97**6                   20241187**Y        1890771811**2         Block    567661
43x967xP                  3x72229xP   3x2017x6829x152407x567661
97**6                   20241187**Y        1890771811**4         Proposition  6
43x967xP                  3x72229xP            5xQ
97**6                   20241187**Y        1890771811**L         Proposition  5
43x967xP                  3x72229xP
97**6                   20241187**4                              Corollary 19.1
43x967xP                    31xQ
97**6                   20241187**6                              Theorem      19
43x967xP                    7xQ
97**6                   20241187**R                              Proposition  5
43x967xP
```

97**10			N Exceeds M
89xP			
97**12	8224356155341457**1		Proposition 11
53x79x20359xP	2x3x11x293x6301x67496441		
97**12	8224356155341457**S		N Exceeds M
53x79x20359xP			
97**16			N Exceeds M
P			
97**18	21433**X	1531**Y	N Exceeds M
21433xQ	2x7xP	3x19xP	
97**18	21433**X	1531**4	Proposition 9
21433xQ	2x7xP	5x691xP	
97**18	21433**X	1531**6	N Exceeds M
21433xQ	2x7xP	29x631x9247939x76148717	
97**18	21433**X	1531**T	N Exceeds M
21433xQ	2x7xP		
97**18	21433**Y		N Exceeds M
21433xQ	3x13x37x241xP		
97**18	21433**4		N Exceeds M
21433xQ	11x1051xP		
97**18	21433**6		N Exceeds M
21433xQ	357421xP		
97**18	21433**U		N Exceeds M
21433xQ	Q is composite and Q has no prime factor less than 100,000,000		
97**22			N Exceeds M
47x10021699xQ	Q has no prime factor less than 30,000,000		
97**S			Proposition 5

Lemma 21.5 Let N be an odd perfect number less than M and let 3**Y||N. It follows that the following is not true.

13**40||N

We observe that S(13**40) has no prime factor less than 100 million

Other than 3 and 13 it is a relatively easy matter to eliminate cases involving small primes to small powers. Otherwise, it would be necessary for N to have several other factors thereby making N greater than M contradicting our hypothesis.

Theorem 21 Let N be an odd perfect number less than M. Then it is not true that 3**Y||N.

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

13**X		Theorem 19
13**Y	61**X	Corollary 19.1
13**Y	61**Y	Lemma 21.4
13**Z		Lemma 21.1
13**U	where U(Mod 7) = 6	Lemma 21.2
13**10		Lemma 21.2
13**12		Lemma 21.2
13**16		Lemma 21.3
13**18		Lemma 21.3
13**22		Lemma 21.3
13**28		Lemma 21.3
13**30		Lemma 21.3
13**36		Lemma 21.3
13**40		Lemma 21.5
13**42		Lemma 21.3
13**A		Proposition 5

Lemma 22.1 Let N be an odd perfect number less than M. Then it is not true that 3**10||N if at the same time 3851**Y||N.

Note S(3**10) = 23 x 3851 S(3851**2) = 13 x 1141081

Block 2573621

2573621**Y 3673xP	1803301831**2 3x7x31xQ	TH19	2573621**4 5xQ	PR8E
2573621**Y 3673xP	1803301831**4 5xQ	PR8E	2573621**6 7xQ	TH19
2573621**Y 3673xP	1803301831**A	PR 5	2573621**B	PR 5

Block 2937190033

2937190033**2 3xP	156941**2 73xP	337405951**2 3x14173x40423xP	N> M	2937190033**2 3xP	156941**4 5xQ	PR6
2937190033**2 3xP	156941**2 73xP	337405951**4 5x11x41x211xQ	PR8E	2937190033**2 3xP	156941**B	N>M
2937190033**2	156941**2	337405951**A	PR 5	2937190033**C		N>M

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

1141081**X 2x337xP	1693**Y 3x13x151xP	487**Y	Theorem 19
1141081**X 2x337xP	1693**Y 3x13x151xP	487**Z 11x11x181xP	Block 2573621
1141081**X 2x337xP	1693**Y 3x13x151xP	487**6 29x156941xP	BK 2937190033
1141081**X 2x337xP	1693**Y 3x13x151xP	487**10 23x23x1336039xP	Block 23
1141081**X 2x337xP	1693**Y 3x13x151xP	487**12 157**16 53x157x29723279xP	N Exceeds M
1141081**X 2x337xP	1693**Y 3x13x151xP	487**16 1123x51307xP	N Exceeds M
1141081**X 2x337xP	1693**Y 3x13x151xP	487**F	N Exceeds M
1141081**X 2x337xP	1693**Z 821xP	10012471081**2 3x7xP	Theorem 19
1141081**X 2x337xP	1693**Z 821xP	10012471081**4 5xP	N Exceeds M
1141081**X 2x337xP	1693**Z 821xP	10012471081**G	Proposition 5

```
1141081**X      1693**6                    5700731**Y              Block 54243547
  2x337xP  43x337x7673x37171xP          43x13933x54243547
1141081**X      1693**6                    5700731**4              Proposition 8E
  2x337xP  43x337x7673x37171xP             5x11x811xQ(comp)  QHNPFLT  1,000,000
1141081**X      1693**6                    5700731**6              Theorem      19
  2x337xP  43x337x7673x37171xP              7x211xQ
1141081**X      1693**6                    5700731**H              Proposition   5
  2x337xP  43x337x7673x37171xP
1141081**X      1693**10                                           Block        23
  2x337xP         89xP
1141081**X      1693**12                                           N  Exceeds    M
  2x337xP       154571xQ   Q  is composite and   QHNPFLT 90,000,000
1141081**X      1693**I                                            Proposition   5
  2x337xP
1141081**Y                                                         Theorem      19
3x7x7x53233x166393
1141081**4                                                         Corollary 19.2
5x31x31x41x101x331xQ
1141081**6                                                         Block        23
    P
1141081**J                                                         Proposition   5
```

Note: S(2937190033**2) = 3 x Q = 3 x 287569509 7630577041 where

Q - 1 = 2**4 x 5 x 37 x 499 x 643 x 2606717257.

We can use the entries in Table XXX to show that Q is prime.

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	11	1	287569509 7630577040
5	7	1	210714756 9473173663
37	3	1	12258392 6983073295
499	3	1	207160002 0579886185
643	3	1	177957421 4213924188
2606717257	3	1	230707144 2956400554

TABLE XXX

Lemma 22.2 Let N be an odd perfect number less than M. Then it is not true that 3**10||N.

Note S(3**10) = 23 x 3851

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

3851**Y				Lemma 22.1
3851**Z 5x2289401xP	19218301**X 2x211x45541	2289401**Y 7x37xP		Proposition 7
3851**Z 5x2289401xP	19218301**X 2x211x45541	2289401**4 5x11xQ		Proposition 8E
3851**Z 5x2289401xP	19218301**X 2x211x45541	2289401**6 29x631x701xQ	QHNPFLT 10,000,000	N Exceeds M Q is composite
3851**Z 5x2289401xP	19218301**X 2x211x45541	2289401**A		Proposition 5
3851**Z 5x2289401xP	19218301**Y 3x7x103x303619xP			Proposition 7
3851**Z 5x2289401xP	19218301**4 5x11x9281xP			Proposition 8E
3851**Z 5x2289401xP	19218301**6 Q Q is composite and QHNPFLT 10,000,000			N Exceeds M
3851**Z 5x2289401xP	19218301**B			Proposition 5
3851**6 7x29x1373x1777973x6583580711				Theorem 19
3851**10 11xQ Q is composite and QHNPFLT 59,499,901				Block 23
3851**12 131x84449xP	84449**1 2x3x5x5xP			Proposition 11
3851**12 131x84449xP	84449**2 31x67x193xP			Corollary 19.1
3851**12 131x84449xP	84449**C			N Exceeds M
3851**D				Proposition 5

Lemma 22.3 If N is an odd perfect number less than M, then none of the following is true.

(A) 3**12||N (B) 3**16||N

Block 2049790620979

2049790620979**2 3x249811xP	Prop 1	2049790620979**A	N Exceeds M

Block 1871

1871**Y 7x157xP	Theorem 19	1871**10 11x23x67x89x18488779xP	Block 23
1871**Z 5x71x151xP	Prop 6	1871**12 53x10453x442807xQ Q is comp 95m	N Exceeds M
1871**6 911xP	N Exceeds M	1871**A	Prop 5

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(797161**2) = 3 x 61 x 151 x 22996651

Possibilities And Reasons By Which They May Be Excluded

3**12 P	797161**X 2xP				Block 398581
3**12 P	797161**Y 3x61x151xP	61**X 2x31			Corollary 19.1
3**12 P	797161**Y 3x61x151xP	61**Y 3x13x97	22996651**Y 3x43xP	4099581241957**1 2x2049790620979	See Block
3**12 P	797161**Y 3x61x151xP	61**Y 3x13x97	22996651**Y 3x43xP	4099581241957**2 3x31x7417xQ	Corollary 19.1 QHNPFLT 1,000,000
3**12 P	797161**Y 3x61x151xP	61**Y 3x13x97	22996651**Y 3x43xP	4099581241957**A	Proposition 5
3**12 P	797161**Y 3x61x151xP	61**Y 3x13x97	22996651**4 5xQ		Proposition 6
3**12 P	797161**Y 3x61x151xP	61**Y 3x13x97	22996651**6 4159xQ(composite) QHNPFLT 10,000,000		N Exceeds M
3**12 P	797161**Y 3x61x151xP	61**Y 3x13x97	22996651**B		Proposition 5
3**12 P	797161**4 5x2671x178601xP		169299992707751**2 13x31x5209xQ		Corollary 3.1
3**12 P	797161**4 5x2671x178601xP		169299992707751**C		Proposition 5
3**12 P	797161**6 7x547x78919x959957xQ				Theorem 19

```
3**12  797161**D                                                   Proposition  5
  P
3**16   34511**Y             4822039**Y     331**Y                 Corollary 19.2
1871xP   13x19xP               3x331xP    3x7x5233
3**16   34511**Y             4822039**4                            Block      1871
1871xP   13x19xP             1230941x15965351xP
3**16   34511**Y             4822039**6                            Block      1871
1871xP   13x19xP            281x121661xQ  Q is composite and QHNPFLT 10,000,000
3**16   34511**Y             4822039**E                            Proposition  5
1871xP   13x19xP
3**16   34511**4                                                   Corollary  3.1
1871xP  5x11x31x71x131x89451727381
3**16   34511**6                                                   Theorem     19
1871xP   7x197xQ        Q is composite
3**16   34511**10                                                  N  Exceeds   M
1871xP      Q  Q is composite and    QHNPFLT 13,374,901
3**16   34511**F                                                   Proposition  5
1871xP
```

```
Note:  (A) S(1871**6)  =  911 x Q  =  911 x 4711471 0726125407
           Q - 1 = 2 x 7 x 13 x 25887 2036956733
           5**[(Q-1)/2] [Mod Q]               =    4711471 0726125406
           3**[(Q-1)/7] [Mod Q]               =                  1871
           3**[(Q-1)/13] [Mod Q]              =    1234879 4575472346
           3**[(Q-1)/258872036956733] [Mod Q] =4208189 9192055858

       (B) Q1 = 25887 2036956733   and   Q1 - 1  =  2**2 x 31 x 987689 x P
           3**[(Q1-1)/2] [Mod Q1]              =  25887 2036956732
           3**[(Q1-1)/31] [Mod Q1]             =   1473 7222683932
           3**[(Q1-1)/978689] [Mod Q1]         =  25071 1376993168
           3**[(Q1-1)/P] [Mod Q1]              =   9808 0953452044
```

Lemma 22.4 If N is an odd perfect number less than M, then none of the following is true.

(A) 3**18||N (B) 3**22||N (C) 3**28||N

Block 852460489981

```
       852460489981**2       Prop      7    852460489981**A           Prop      5
     3x7x13x13x26701xQ     QHNPFLT  1,000,000
       852460489981**4       Cor     19.1
            5x31xQ
```

Block 77230798373051

```
       77230798373051**2     N Exceeds M      77230798373051**A       Prop      5
         3001x638767xP
```

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

```
   3**18  363889**X         36389**Y    926659**Y     1599181**Y See       Block
 1597xP     2x5xP            1429xP    3x178987xP   3x852460489981
   3**18  363889**X         36389**Y    926659**Y     1599181**A N  Exceeds   M
 1597xP     2x5xP            1429xP    3x178987xP
   3**18  363889**X         36389**Y    926659**4                See       Block
 1597xP     2x5xP            1429xP     41x701x332191x77230798373051
   3**18  363889**X         36389**Y    926659**6                N  Exceeds   M
 1597xP     2x5xP            1429xP    2549x15107xP
   3**18  363889**X         36389**Y    926659**L                Proposition  5
 1597xP     2x5xP            1429xP
   3**18  363889**X         36389**4                  10091**2 Proposition  7
 1597xP     2x5xP           701x881x3911x10091x71941   7x13x109xP
   3**18  363889**X         36389**4                  10091**4 Proposition 8E
 1597xP     2x5xP           701x881x3911x10091x71941  5x11x2711xQ
   3**18  363889**X         36389**4                  10091**6 N  Exceeds   M
 1597xP     2x5xP           701x881x3911x10091x71941     P
   3**18  363889**X         36389**4                  10091**C N  Exceeds   M
 1597xP     2x5xP           701x881x3911x10091x71941
   3**18  363889**X         36389**6                           Proposition  6
 1597xP     2x5xP         29x43x71x659xQ
   3**18  363889**X         36389**10                          Proposition 8E
 1597xP     2x5xP             11xQ
```

```
 3**18  363889**X          36389**D                                  Proposition  5
1597xP    2x5xP
 3**18  363889**Y         215143**Y              15428908531**Y Corollary 19.1
1597xP 3x193x1063xP          3xP                        3x31xQ
 3**18  363889**Y         215143**Y              15428908531**4 N  Exceeds   M
1597xP 3x193x1063xP          3xP                        5x11xP
 3**18  363889**Y         215143**Y              15428908531**E Proposition  5
1597xP 3x193x1063xP          3xP
 3**18  363889**Y         215143**4     4750445693592883451**2  N  Exceeds   M
1597xP 3x193x1063xP         11x41xP           67x751xP
 3**18  363889**Y         215143**4     4750445693592883451**P  Proposition  5
1597xP 3x19371063xP         11x41xP
 3**18  363889**Y         215143**6                             Proposition  1
1597xP 3x19371063xP         45767xQ  Q is composite and   QHNPFLT  10,000,000
 3**18  363889**Y         215143**G                             Proposition  5
1597xP 3x19371063xP
 3**18  363889**4         1578202927087387741l**2               N  Exceeds   M
1597xP   11x101xP                   751xQ  QHNPFLT  2,799,997
 3**18  363889**4         1578202927087387741l**H               Proposition  5
1597xP   11x101xP
 3**18  363889**6                                               Theorem     19
1597xP     7xQ
 3**18  363889**I                                               Proposition  5
1597xP
 3**22          1001523179**Y    61**X     31**Y          331**Y Theorem     19
  47xP              61xP         2x31      3x331       3x7x5233
 3**22          1001523179**Y    61**Y     16443420968455561**1  Proposition 11
  47xP              61xP        3x13x97    2x227x5505377x6578839
 3**22          1001523179**Y    61**Y     16443420968455561**2  N  Exceeds   M
  47xP              61xP        3x13x97       3x3373x120103xP
 3**22          1001523179**Y    61**Y     16443420968455561**J  Proposition  5
  47xP              61xP        3x13x97
 3**22          1001523179**4           181**X                   Theorem      0
  47xP             181xQ                2x7x13
 3**22          1001523179**4           181**Y                   Block       79
  47xP             181xQ              3x79x139
 3**22          1001523179**4           181**4                   Proposition  6
  47xP             181xQ                5x11xQ
 3**22          1001523179**4           181**6                   N  Exceeds   M
  47xP             181xQ              29x281xP
 3**22          1001523179**4           181**R                   N  Exceeds   M
  47xP             181xQ        Q  is composite  and  QHNPFLT 25,000,001
 3**22          1001523179**K                                    Proposition  5
  47xP
 3**28            20381027**Y                                    Theorem     19
59x28537xP    7x67x19687x44988319
 3**28            20381027**4         28537**X                   Theorem      0
59x28537xP             P              2x19x751
 3**28            20381027**4         28537**Y    52783**Y       N  Exceeds   M
59x28537xP             P            3x37x139xP  3x13x613xP
```

3**28	20381027**4	28537**Y	52783**4	N Exceeds M
59x28537xP	P	3x37x139xP	11x11x41x393871x3972475951	
3**28	20381027**4	28537**Y	52783**6 71**Y	Proposition 6
59x28537xP	P	3x37x139xP	71xQ	
3**28	20381027**4	28537**Y	52783**6 71**12	N Exceeds M
59x28537xP	P	3x37x139xP	71xQ	
3**28	20381027**4	28537**Y	52783**6 71**18	N Exceeds M
59x28537xP	P	3x37x139xP	71xQ QHNPFLT 10,000,000	
3**28	20381027**4	28537**Y	52783**S	N Exceeds M
59x28537xP	P	3x37x139xP		
3**28	20381027**4	28537**4		N Exceeds M
59x28537xP	P	11xP		
3**28	20381027**4	28537**6		.. xceeds M
59x28537xP	P	412651xP		
3**28	20381027**4	28537**10		N Exceeds M
59x28537xP	P	67x43627x511457xQ(comp) QHNPFLT 10,000,000		
3**28	20381027**4	28537**T		Proposition 5
59x28537xP	P			
3**28	20381027**6			N Exceeds M
59x28537xP	43x463x3557x351023x837313xQ Q is composite and QHNPFLT 10,000,000			
3**28	20381027**L			Proposition 5
59x28537xP				

Note: If Q = 1578202927 0873877411, then

Q - 1 = 2 x 5 x 7 x 37 x 281 x 53951 x 401936329.

The entries in table XXXI may be used to show that Q is prime.

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	7	1	1578202927 0873877411
5	3	1	363889
7	3	1	1552489750 6897364697
37	3	1	1207160200 3417471555
281	3	1	1148621200 4687582706
.3951	3	1	229823288 3700509585
401936329	3	1	805392846 2376044502

TABLE XXXI

Lemma 22.5 Let N be an odd perfect number less than M. Then, none of the following is true.

(A) 3**30||N (B) 3**36||N (C) 3**40||N (D) 3**42||N

Block 102673

102673**X 2x11x13x359	Prop 11	102673**6 197x421x88523xP	N Exceeds M
102673**Y 3x7x7x67xP	Theorem 19	102673**A	Prop 5
102673**4 431xQ Q has no prime factor less than 69,100,001 and is composite	N Exceeds M		

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

3**30 4404047**Y	683x102673xP 7x823x3366713137	Theorem 19
3**30 4404047**4	683x102673xP 131x211x461x2770771xP	Block 102673
3**30 4404047**6	683x102673xP 29x29xQ Q is composite and QHNPFLT 10,000,000	N Exceeds M
3**30 4404047**A	683x102673xP	N Exceeds M
3**36 17189128703**2 13097927**Y 782191**2	13097927xP 19x43x10760647xP 19x139x83047xP 3x7x211x2437xP	Theorem 19
3**36 17189128703**2 13097927**Y 782191**4	13097927xP 19x43x10760647xP 19x139x83047xP 5x11x881xQ	Proposition 8A
3**36 17189128703**2 13097927**Y 782191**6	13097927xP 19x43x10760647xP 19x139x83047xP 43x2689x6217x1973567xQ QHNPFLT 10m	N Exceeds M
3**36 17189128703**2 13097927**Y 782191**B	13097927xP 19x43x10760647xP 19x139x83047xP	N Exceeds M
3**36 17189128703**2 13097927**4	13097927xP 19x43x10760647xP 233881xP	Proposition 1
3**36 17189128703**2 13097927**6	13097927xP 19x43x10760647xP 43x239xQ(composite) QHNPFLT 2,000,000	N Exceeds M
3**36 17189128703**2 13097927**C	13097927xP 19x43x10760647x33608357287	Proposition 5
3**36 17189128703**4	13097927xP 151x4801xP	N Exceeds M
3**36 17189128703**D	13097927xP	Proposition 5
3**40 86950696619**2 2526913**X	83x2526913xP P 2x13x17x5717	Proposition 11

3**40 83x2526913xP	86950696619**2 P	2526913**Y 3x7x967x1033xP	Theorem 19
3**40 83x2526913xP	86950696619**2 P	2526913**4 11x101x211x358811xP	N Exceeds M
3**40 83x2526913xP	86950696619**2 P	2526913**6 197x659x2423x156577xQ(comp)	N Exceeds M QHNPFLT 10,000,000
3**40 83x2526913xP	86950696619**2 P	2526913**E	Proposition 5
3**40	86950696619**4 31x271x1091x34421xQ	Q is composite	Corollary 19.1
3**40	86950696619**F		Proposition 5
3**42 431xP		380808546861411923**2 7x19x79x13477xQ	Theorem 19
3**42 431xP		380808546861411923**G	Proposition 5

Note: S(102673**6) = 197 x 421 x 88523 x 1 5956487468 0157418893.

If Q = 1 5956487468 0157418893, then Q - 1 = 2**2 x 3**2 x 7 x 15787 x P.

The entries in Table XXXII are provided to show that Q is prime.

P	Px	Px**(Q-1) [Mod Q]	Px**[(Q-1)/P] [Mod Q]
2	5	1	1 5956487468 0157418892
3	5	1	1 3752642479 3423605205
7	3	1	1 1112838614 8097215041
15787	3	1	1 1205478280 4281903145
40108566994583	3	1	6311517122 4123106897

TABLE XXXII

Lemma 22.6 Let N be an odd perfect number less than M. Then, none of the following is true.

(A) 3**46||N (B) 3**52||N (C) 3**60||N (D) 3**66||N

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

3**46 1223x21997x5112661xP	96656723**2 7x31x157xP			Theorem 19
3**46 1223x21997x5112661xP	96656723**4 11xQ	Q is composite	QHNPFLT 10,000,001	N Exceeds M
3**46 1223x21997x5112661xP	96656723**A			N Exceeds M
3**52 107x24169xP	3747607031112307667**2 31xQ			Corollary 19.1
3**52 107x24169xP	3747607031112307667**B			Proposition 5
3**60 603901xP	105293313660391861035901**1 2x13x13x67x25117x85909xP	603901**Y 3x7x17366524843		Theorem 19
3**60 603901xP	105293313660391861035901**1 2x13x13x67x25117x85909xP	603901**4 5xQ		Proposition 6
3**60 603901xP	105293313660391861035901**1 2x13x13x67x25117x85909xP	603901**6 127x66977xP		N Exceeds M
3**60 603901xP	105293313660391861035901**1 2x13x13x67x25117x85909xP	603901**C		Proposition 5
3**60 603901xP	105293313660391861035901**2 3x7x2551xQ			Theorem 19
3**60 603901xP	105293313660391861035901**D			Proposition 5
3**66 221101xQ	221101**X 2x7x17x929			Proposition 11
3**66 221101xQ	221101**Y 3x37xP	440413273**X 2x7x7x19x236527		Theorem 19
3**66 221101xQ	221101**Y 3x37xP	440413273**Y 3x43x67xP	22441727580121**1 2xP	N Exceeds M
3**66 221101xQ	221101**Y 3x37xP	440413273**Y 3x43x67xP	22441727580121**E	N Exceeds M
3**66 221101xQ	221101**Y 3x37xP	440413273**F		N Exceeds M
3**66 221101xQ	221101**4 5x1601x21122911x14133498791			N Exceeds M
3**66 221101xQ	221101**6 Q Q is composite and QHNPFLT 7,000,000			Proposition 1
3**66 221101xQ	221101**G Q is composite and QHNPFLT 30,000,000			Proposition 5

Lemma 22.7 Let N be an odd perfect number less than M. Then, none of the following is true.

(A) 3**72||N (B) 3**82||N (C) 3**88||N (D) 3**96||N

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(3**72) = 11243x20149xQ S(3**82) = 167x12119xQ

Possibilities And Reasons By Which They May Be Excluded

3**72	20149**X			Proposition 8E
	2x5x5x13x31			
3**72	20149**2	128767**2		Theorem 19
	3x1051xP	3x7x631xP		
3**72	20149**2	128767**A		N Exceeds M
	3x1051xP			
3**72	20149**B			N Exceeds M
11243x20149xQ	Q is composite		QHNPFLT 1,000,000,000	
3**82	12119**2			Theorem 19
	7x13x43xP			
3**82	12119**4			N Exceeds M
	5701x16001x236485661			
3**82	12119**C			N Exceeds M
167x12119xQ	Q is composite		QHNPFLT 560,250,001	
3**88	179**Y			Theorem 19
179xQ	7x4603			
3**88	179**4			N Exceeds M
179xQ	11xP			
3**88	179**6			N Exceeds M
179xQ	71xP			
3**88	179**D			N Exceeds M
179xQ	Q is composite	QHNPFLT 100,000,000		
3**96	76631**Y			Theorem 19
76631xQ	7xQ			
3**96	76631**E			N Exceeds M
76631xQ	Q is composite	QHNPFLT 100,000,000		

Lemma 22.8 If N is an odd perfect number less than M, then 3**58||N is not true.

Proof

In the proof of this lemma, we need only show that under our hypothesis together with the fact that S(3**58) has no prime factor less than 10,000,000,000, it follows that no one of the following primes can be a factor of N.

5 7 11 13 19 23 31 41 43 47 59 61

Case 1 The prime 5 cannot divide N.

The following sub-cases are exhaustive.

Possibilities And Reasons By Which They May Be Excluded

(A)	5**X (for some P**Z, 5/S(P**Z))	Theorem 0
(B)	5**Y	Corollary 3.1
(C)	5**Z	Proposition 8E
(D)	5**A	Block 5

Case 2 The prime 7 cannot divide N by Theorem 19.

Case 3 The prime 11 cannot divide N by Theorem 20.

Case 4 The prime 13 cannot divide N.

(A)	13**X			Theorem 19
(B)	13**Y	61**X		Corollary 19.1
(C)	13**Y	61**Y		Lemma 21.4
(D)	13**Z	30941**X	See Block 191	N Exceeds M
(E)	13**Z	30941**A	See Lemma 21.1	N Exceeds M
(F)	13**U			Lemma 21.2
(G)	13**10			Lemma 21.2
(H)	13**12			Lemma 21.2
(I)	13**16			Lemma 21.2
(J)	13**18			Lemma 21.3
(K)	13**22			Lemma 21.3
(L)	13**R			N Exceeds M

Case 5 None of the primes 19, 23, 31, etc. can divide N.
(The details for this case are included elsewhere herein.)

Disallowing the primes 5, 7, 11, 13, 19, 23, etc., the number N must have at least 17 factors besides 3 and those of the number S(3**58). Each, except possibly one, must occur to an even power in the prime factorization of N. This leads to contradiction.

Theorem 22 Let N be an odd perfect number less than M. Then, it is not true that 3 divides N.

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

3**Y		Theorem 21
3**Z		Theorem 20
3**U		Theorem 15
3**V		Lemma 22.2
3**W		Lemma 22.3
3**16		Lemma 22.3
3**18		Lemma 22.4
3**22		Lemma 22.4
3**28		Lemma 22.4
3**30		Lemma 22.5
3**36		Lemma 22.5
3**40		Lemma 22.5
3**42		Lemma 22.5
3**46		Lemma 22.6
3**52		Lemma 22.6
3**58 Q	Q is composite and Q has no prime factor less than 10,000,000,000	Lemma 22.8
3**60		Lemma 22.6
3**66		Lemma 22.6
3**70 P	37547332574898624019733579791287 73**A	N Exceeds M

```
   3**72                                                              Lemma        22.7
   11243x20149xQ    QHNPFLT  1,000,000,000  Q is composite
   3**78            432853009**1        1169873**2                    Proposition  6
432853009xQ       2x5x37x1169873          13xQ
   3**78            432853009**1        1169873**B                    N  Exceeds    M
432853009xQ       2x5x37x1169873
   3**78            432853009**2                                      N  Exceeds    M
432853009xQ       3x3259x53959xP
   3**78            432853009**C                                      N  Exceeds    M
432853009xQ    Q  is composite
   3**82                                                              Lemma        22.7
167x12119xQ     QHNPFLT  560,250,001  Q is composite
   3**88                                                              Lemma        22.7
   179xQ  Q is composite
   3**96                                                              Lemma        22.7
  76631xQ    Q is composite
   3**100                                                             Lemma        22.7
 33034273xQ     Q is composite
   3**102  6957596529882152968992225251835887181478451547013**D       N  Exceeds    M
     P
   3**E                                                               Proposition  5
```

Lemma 23.1 The number 5**4 x 11 x 71 cannot be a factor of an odd perfect number N that is less than M.

Proof Suppose to the contrary, that N is an odd perfect number less than M and that 5**4 x 11 x 71 is a factor of N. Then by our Lemma 4.7, either all of the conditions in (A) are satisfied together or one of (B) and (C) is true.

(A) 71**Y||N 5113**X||N 2557**Y||N
(B) 71**12||N (C) 71**18||N
It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

```
(A)  71**Y     5113**X     2557**Y                                      Proposition 8B
     5113      2x2557   3x7x13x13x19x97

(B)  71**12      Note  S(71**12) HNPFLT 200,000,000

11**Y                                                                   Theorem      19
7x19
11**Z                                                                   Block      3221
5x3221
11**U        45319**Y                                                   Proposition 8E
43xP         3x127xP
11**U        45319**4                                                   Block       151
43xP          151xP
11**U        45319**6                                                   Theorem      19
43xP         7x953x4327x7841x108866969xP
11**U        45319**10                                                  N  Exceeds    M
43xP         23x38677xQ   Q is composite and  QHNPFLT  10,000,000
11**U        45319**A                                                   Proposition   5
43xP
11**10      1806113**X                                                  Theorem      22
15797xP      2x3x17xP
11**10      1806113**Y        171686630257**1                           Proposition 11
15797xP       19xP              2x13x7727xP
11**10      1806113**Y        171686630257**2                           Proposition   7
15797xP       19xP            3x7x19x34267xP
11**10      1806113**Y        171686630257**B                           N  Exceeds    M
15797xP       19xP
11**10      1806113**4                                                  N  Exceeds    M
15797xP      4051xQ  Q is composite and QHNPFLT 10,000,000
11**10      1806113**C                                                  N  Exceeds    M
15797xP
11**12                          3158528101**1        1579264051**2      Theorem      22
1093xP                                2xP                3x19xQ
11**12                          3158528101**1        1579264051**D      N  Exceeds    M
1093xP                                2xP
```

```
11**12                     3158528101**2                         Theorem      22
1093xP                         3xP
11**12                     3158528101**E                         N  Exceeds    M
1093xP
11**16               50544702849929377**1                        Proposition  1
  P                      2x23x4591xP
11**16               50544702849929377**F                        N  Exceeds    M
  P
11**18             6115909044841454629**1                        Corollary  3.1
  P                       2x5x31xQ
11**18             6115909044841454629**2                        Theorem      22
  P                         3xQ
11**18             6115909044841454629**G                        Proposition  5
  P
11**22                              3740221981231**2             Theorem      22
829x28878847xP           3x73x283x487x811x1692283x337708039
11**22                              3740221981231**A             Proposition  5
829x28878847xP
11**28                 3033096170499983889893760 43**B           Proposition  5
523xP
11**30      2428541**X                                           Proposition 11
            2x3**5x19x263
11**30      2428541**2      24472256503**2                       Theorem      19
              241xP         3x7x139x46147xP
11**30      2428541**2      24472256503**4                       Corollary 19.1
              241xP             31xQ
11**30      2428541**2      24472256503**A                       Proposition  5
              241xP
11**30      2428541**4                                           Proposition  6
         5x11x61x71x151xQ
11**30      2428541**6                                           N  Exceeds    M
              29**2xQ  Q is composite and  QHNPFLT  5,199,979
11**30      2428541**B                                           Proposition  5
2428541xQ   Q is composite and QHNPFLT 100,000,000
11**36            36855109**1           3685511**2               N  Exceeds    M
2591x36855109xQ     2x5xP                 7x61xP
11**36            36855109**1           3685511**4               N  Exceeds    M
2591x36855109xQ     2x5xP             5x11x13721xP
11**36            36855109**1           3685511**C               N  Exceeds    M
2591x36855109xQ     2x5xP
11**36            36855109**2                                    Proposition  7
2591x36855109xQ     3x7xQ
11**36            36855109**D                                    N  Exceeds    M
2591x36855109x136151713xP
```

```
(C)  71**18        See  Case(B)  above     Note  S(71**18)  HNPFLT 972,312,233
```

Lemma 23.2 If N is an odd perfect number less than M, then neither 5**U||N nor 5**V||N.

Block 4159

4159**Y 3x31x186037	C3.1	4159**6 7x421x1471x1005677x1187371529	TH19
4159**4 7299216748441**1 41xP 2x179x2239x9106241	N> M	4159**10 11xQ QHNPFLT 10,000,000	N> M
4159**4 7299216748441**2 41xP 3x7xP	TH19	4159**B	PR5
4159**4 7299216748441**A 41xP	N> M		

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(5**6) = 19531, S(5**10) = 12207031

Possibilities And Reasons By Which They May Be Excluded

5**U	19531**Y where Y(Mod 3) = 2 3x127159831			Theorem 22
5**U	19531**4 5x191x4760281xP	32009891**2 7x283x468913xP		Theorem 19
5**U	19531**4 5x191x4760281xP	32009891**4 5x31xQ		Corollary 3.1
5**U	19531**4 5x191x4760281xP	32009891**A		N Exceeds M
5**U	19531**6 7x631xP			Theorem 19
5**U	19531**10 23x23x4159xQ	4159**2 3x31x186037		Corollary 3.1
5**U	19531**10 23x23x4159xQ Q is composite and QHNPFLT 12,375,001	4159**D		Block 4159
5**U	19531**E			Proposition 5
5**V	12207031**2 3x7x1041757x6811369			Proposition 7
5**V	12207031**4 5x131xP	131**16		N Exceeds M
5**V	12207031**6 37871xP	37871**2 P	1434250513**X 2x23x31179359	N Exceeds M
5**V	12207031**6 37871xP	37871**2 P	1434250513**F	N Exceeds M
5**V	12207031**6 37871xP	37871**G		N Exceeds M
5**V	12207031**H			Proposition 5

Lemma 23.3 If N is an odd perfect number less than M, then, no one of the following is true.
(A) 5**W||N , (B) 5**16||N (C) 5**18||N
where W(Mod 13) = 12 Let X(Mod 4) = 1.

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(5**12) = 305175781 S(5**16) = 409 x 466344409

Possibilities And Reasons By Which They May Be Excluded

5**W	305175781**X	43609**Y		Theorem 22
	2x3499xP	3xP		
5**W	305175781**X	43609**4	2819051**2	Theorem 19
	2x3499xP	11x11701x9967721xP	7x151x7518497029	
5**W	305175781**X	43609**4	2819051**4	Corollary 3.1
	2x3499xP	11x11701x9967721xP	5x11x31x61x941xQ	
5**W	305175781**X	43609**4	2819051**6	N Exceeds M
	2x3499xP	11x11701x9967721xP	127x197x281x1938889xP	
5**W	305175781**X	43609**4	2819051**A	Proposition 5
	2x3499xP	11x11701x9967721xP		
5**W	305175781**X	43609**6	3499**2	Theorem 22
	2x3499xP	17137xQ		
5**W	305175781**X	43609**6	3499**4	N Exceeds M
	2x3499xP	17137xQ	P	
5**W	305175781**X	43609**6	3499**6	N Exceeds M
	2x3499xP	17137xQ	43x71x113x211x137831x182958749	
5**W	305175781**X	43609**6	3499**D	N Exceeds M
	2x3499xP	17137xQ Q is composite and QHNPFLT 50,000,000		
5**W	305175781**X	43609**10		N Exceeds M
	2x3499xP	67xP		
5**W	305175781**X	43609**E		Proposition 5
	2x3499xP			
5**W	305175781**2			Theorem 22
	3x271x9283xP	where P = 12340172617		
5**W	305175781**4	3011**2	9069133**X	N Exceeds M
	5x11x3011xQ	P	2xP	
5**W	305175781**4	3011**2	9069133**2	Proposition 8E
	5x11x3011xQ	P	3x37x199xP	
5**W	305175781**4	3011**2	9069133**4	N Exceeds M
	5x11x3011xQ	P	331xQ Q is composite QHNPFLT 1m	
5**W	305175781**4	3011**2	9069133**F	N Exceeds M
	5x11x3011xQ	P		
5**W	305175781**4	3011**4		Corollary 3.1
	5x11x3011xQ	5x31x41xP		
5**W	305175781**4	3011**6		Theorem 19
	5x11x3011xQ	7x29xQ QHNPFLT 2,000,000		
5**W	305175781**4	3011**G		N Exceeds M
	5x11x3011xQ(composite) Q has no prime factor less than 25,000,001			

5**W	305175781**H		Proposition 5
5**16 409xP			Theorem 4
5**18 191x6271xP	3981071**2 13x1219148483701	1219148483701**X 2x31xP	Corollary 3.1
5**18 191x6271xP	3981071**2 13x1219148483701	1219148483701**2 3x61507xQ	Theorem 22
5**18 191x6271xP	3981071**2 13x1219148483701	1219148483701**4 5x11x32371xP	N Exceeds M
5**18 191x6271xP	3981071**2 13x1219148483701	1219148483701**I	Proposition 5
5**18 191x6271xP	3981071**4 5x9341xP	5378193516362980863601**1 2x13x127x1471xP	N Exceeds M
5**18 191x6271xP	3981071**4 5x9341xP	5378193516362980863601**J	N Exceeds M
5**18 191x6271xP	3981071**6 71x127xP	6271**2 3x43x304897	Theorem 22
5**18 191x6271xP	3981071**6 71x127xP	6271**K	N Exceeds M
5**18	3981071**L		Proposition 5

Note: S(3981071**4) = 5 x 9341 x P where P is a prime number.

P - 1 = 2**2 x 3**2 x 5**2 x 17 x 29 x 1063831 x 2848487797.

11**[(P-1)/2] [Mod P]	=	53 7819351636 2980863600
3**[(P-1)/3] [Mod P]	=	2 7897502592 3416150204
5**[(P-1)/5] [Mod P]	=	6309570090 2098020911
3**[(P-1)/17] [Mod Q]	=	18 8719206988 9954304396
3**[(P-1)/29] [Mod P]	=	51 6696613165 2198903115
3**[(P-1)/1063831] [Mod P]	=	17 5020473527 0759498799
3**[(P-1)/2848487797] [Mod P]	=	32 8674432920 9467892268

TABLE XXXIII

Lemma 23.4 If N is an odd perfect number less than M, then no one of the following is true.
(A) 5**22||N (B) 5**28||N (C) 5**30||N

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(5**22) = 8971 x 332207361361 S(5**28) = 59x35671x22125996444329

Possibilities And Reasons By Which They May Be Excluded

5**22	332207361361**X 2x271xP	612928711**2 3xQ	Theorem 22
5**22	332207361361**X 2x271xP	612928711**4 5x296741xP	N Exceeds M
5**22	332207361361**X 2x271xP	612928711**D	Proposition 5
5**22	332207361361**2 3x19x97x27751xP		Theorem 22
5**22	332207361361**4 5x31x37441xQ		Corollary 3.1
5**22	332207361361**E		Proposition 5
5**28	22125996444329**1 2x3x3x3x3x3x5x11069x822599		Theorem 22
5**28	22125996444329**2 31x67xQ	QHNPFLT 50,000	Corollary 3.1
5**28	22125996444329**F		Proposition 5
5**30 1861xP		625552508473588471**2 3x13xQ	Theorem 22
5**30 1861xP		625552508473588471**G	Proposition 5

Lemma 23.5 If N is an odd perfect number less than M, then not both of the following can happen simultaneously.

5**36||N 149|N

Block 691

691**Y 3x19x8389		TH22	691**10	59951**B 59951x133717x183041x455489x37187767	N> M
691**Z 5x11x61xP	61**X 2x31	C3.1	691**12 Q	2861**X 2x3x3x3xP	TH22
691**Z 5x11x61xP	61**Y 3x13x97	PR8B	691**12 Q	2861**Y 19xP	N> M
691**6 29x211x88523xP	201261481**X 2x1971x33871	PR 1	691**12 Q	2861**4 5x92941xP	N> M
691**6 29x211x88523xP	201261481**2 3x97xQ	TH22	691**12 Q	2861**6 P	N> M
691**6 29x211x88523xP	201261481**4 5x11x11x48731x257921xP	N> M	691**12 Q	2861**C QHNPFLT 24,624,991	N> M
691**6 29x211x88523xP	201261481**A	N> M	6691**D		N> M

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Note S(149**1) = 2 x 3 x 5**2 S(149**2) = 7 x 31 x 103

Possibilities And Reasons By Which They May Be Excluded

149**X		Theorem 22
149**Y		Corollary 3.1
149**Z 251x691x2861		Block 691
149**6 P	11016462577051**2 3x13x61x14593xQ	Theorem 22
149**6 P	11016462577051**A	Proposition 5
149**10 67xP	81042426245204504653**1 2x7x31x43xQ	Theorem 19
149**10 67xP	81042426245204504653**B	N Exceeds M
149**12 P	120547934639675608922684101**1 2x11x19x59x311xP	Theorem 0
149**12 P	120547934639675608922684101**C	Proposition 5
149**16 P	594161965566636633381674084369388 01**1 2x23xQ QHNPFLT 600,000	Theorem 0
149**16 P	59416196556663663338167408436938801**D	Proposition 5
149**18 2053xP		N Exceeds M
149**22 47xP		N Exceeds M
149**E		Proposition 5

Lemma 23.6 If N is an odd perfect number less than M, then none of the following is true.

5**40||N   5**42||N

5**60||N   5**66||N   5**70||N

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

5**40 2238236241xP	5079304643216687969**1 2x3x5xQ	Theorem 22
5**40 2238236241xP	5079304643216687969**2 739x3301x9649xQ QHNPFLT 2,799,997	N Exceeds M
5**40 2238236241xP	5079304643216687969**A	Proposition 5
5**42 1644512641xP	1644512641**1 8447**2 2x311x313x8447 19xP	N Exceeds M
5**42 1644512641xP	1644512641**1 8447**4 2x311x313x8447 1601xP	N Exceeds M
5**42 1644512641xP	1644512641**1 8447**B 2x311x313x8447	N Exceeds M
5**42 1644512641xP	1644512641**2 3x7xQ	Proposition 7
5**42 1644512641xP	1644512641**C	N Exceeds M
5**60 8419xQ	8419**2 3x241xP	Theorem 22
5**60 8419xQ	8419**4 701055701**1 11x101x6451xP 2x3x3x3x3083xP	Proposition 8E
5**60 8419xQ	8419**4 701055701**D 11x101x6451xP	N Exceeds M
5**60 8419xQ	8419**E QHNPFLT 104,000,000	N Exceeds M
5**66 269x1609x26399xQ	1609**1 2x5x7xP	Theorem 19
5**66 269x1609x26399xQ	1609**2 3xP	Theorem 22
5**66. 269x1609x26399xQ	1609**F Q is composite and QHNPFLT 100,000,000	N Exceeds M
5**70	569**G	N Exceeds M

Lemma 23.7 If N is an odd perfect number less than M, then neither of the following is true.

(A) 5**52||N (B) 5**58||N

Proof for (A)

Suppose, to the contrary, that N is an odd perfect number less than M and that 5**52||N. It follows that S(5**52) also divides N. Since 5 is relatively prime to S(5**52), the product of 5**52 and S(5**52) divides N.

Case 1 The prime 3 divides N. Theorem 22 eliminates this.

Case 2 The prime 7 divides N. Theorem 19 eliminates this.

Case 3 The prime 11 divides N. The following possibilities are exhaustive.

Possibilities And Reasons By Which They May Be Excluded

(A)	11**Y			Theorem 19
(B)	11**Z			Block 3221
(C)	11**U	45319**Y		Theorem 22
(D)	11**U	45319**4		Block 151
(E)	11**U	45319**A		N Exceeds M
(F)	11**10	1806113**1		Theorem 22
(G)	11**10	1806113**2	171686630257**1	Prop 11
(H)	11**10	1806113**2	171686630257**B	N Exceeds M
(I)	11**10	1806113**C		N Exceeds M
(J)	11**12	3158528101**1	1579264051**2	Theorem 22
(K)	11**12	3158528101**1	1579264051**D	N Exceeds M
(L)	11**12	3158528101**E		N Exceeds M
(M)	11**16	50544702849929377**F		N Exceeds M
(N)	11**18	6115909044841454629**1		Cor 3.1
(O)	11**18	6115909044841454629**G		Prop 5
(P)	11**22	3740221981231**H		N Exceeds M
(Q)	11**28	30330961704999838898937 6043**I		Prop 5
(R)	11**J			N Exceeds M

Case 4 The prime 13 divides N. It is shown easily that this case may be eliminated.

Case 5 (3 x 7 x 11 x 13 , N) = 1

In as much as S(5**52) has no prime factor less than 5,986,343,641 and since for each prime factor P of S(5**52) and positive exponent E it follows that P**E/S(P**E) is greater than .999999, then disallowing the primes 3, 7, 11 and 13, N must have at least eleven prime factors besides 5 and the factors of S(5**52). This gives rise to a contradiction, that of M exceeding N.

Since the sum of the factors of S(5**58) has no prime factor less than 9,870,399,691 then, by similarity of argument, we can show easily that 5**58||N is not true.

Theorem 23 The prime 5 cannot be a factor of an odd perfect number N that is less than M.

Proof It is sufficient to show that, except for the conclusion, any other possibility leads to a contradiction.

Possibilities And Reasons By Which They May Be Excluded

5**X		Theorem 22
5**Y		Corollary 3.1
5**Z		Lemma 23.1
5**U		Lemma 23.2
5**V		Lemma 23.2
5**W		Lemma 23.3
5**16		Lemma 23.3
5**18		Lemma 23.3
5**22		Lemma 23.4
5**28		Lemma 23.4
5**30		Lemma 23.4
5**36		Lemma 23.5
5**40		Lemma 23.6
5**42		Lemma 23.6
5**46	1776358639400250464677810668945 31**A	Proposition 5
5**52		Lemma 23.7
5**58		Lemma 23.7
5**60		Lemma 23.6
5**66		Lemma 23.6
5**70		Lemma 23.6
5**B		Proposition 5

: any prime P and exponent E we have the following inequality.

P/(P+1) ≥ (P**E)/(S(P**E)) > (P-1)/P

Now, suppose to the contrary that N is an odd perfect number less than M, yet contains no prime factor less than 8. Then, N contains at least 27 distinct prime factors, each of which, except possibly one must appear to an even power in the prime factorization of N.

Within the proof of Theorem A, each of the following conditions holds.

(A) The following primes cannot appear to the second power in the prime factorization of N.

11, 13, 19, 23, 37, 43, 47, 53, 67, 71, 73, 79,

97, 103, 107, 109, 127, 137, 139, 149, 151, 157, 163, 179,

181, 191, 193, 199, 211, 223, 229, 233, 241, 263, 271, 277,

283, 307, 313, 317, 331, 337, 347, 349, 359, 367, 373, 379.

(B) The prime 11 cannot appear to the fourth power in the prime factorization of N.

(C) The following primes can appear to no power in the prime factorization of N.

31, 61, 409.

The product of any collection of even powers of 26 or more primes that are not excluded by the above conditions is greater than 10**100.

Hence, if N is an odd perfect number less than M and contains no prime factor less than 8, then N is also greater than M. This is a contradiction.