

Appendix B--Our Asymmetric Model of Price Movements

Since we have three things to explain (the farm, wholesale, and retail prices of beef), our model has to have three equations. One equation focuses on the general level of prices and the other two look at the spread between farm and wholesale prices and between wholesale and retail prices. Our three equations allow adjustment in the general price level, adjustment to the farm-wholesale spread, and adjustment to the wholesale-retail spread. All the adjustment equations compare where something is to where it should be and require prices to adjust to get them closer to where they should be. That is, our adjustment equations are based on the idea that there is a target value for the price level or some spread. The target level is a longrun value, that is, if current conditions were fixed forever, prices and spreads would eventually settle down to their targets. In our adjustment models, it is possible for prices or spreads to go immediately to their targets, or to go partially toward their targets, or even to over-shoot their targets. If the general price level is too low, having one or all of the prices increase could get the general price level to where it should be. If the farm-wholesale spread is too low, increasing the wholesale price or decreasing the farm price could make the spread closer to what it should be. Also, if the wholesale-retail spread needs to increase, either the retail price goes up or the wholesale price goes down.

Solving endogenous switching models such as our asymmetric model can be more complicated than solving symmetric models. In fact, there are special coherency conditions on the model's parameters. If these are not met, the model's equations may not have a solution in some cases or might have more than one solution in other cases. Gourieroux, Laffont, and Monfort (1980) have a complete explanation of why coherency is necessary and what the coherency conditions are. In our case, it turns out that we can impose coherency by forcing the parameters to take sensible values. For example, if the wholesale price follows the farm price, a set of sensible values would make the wholesale price go up (or at least not down) when the farm price goes up, and vice versa.

Price Level Equation

Our first equation is the price-level equation. We will explain our price-level equation by starting with a simple form, then adding complications until we reach the final functional form that we used.

One common hypothesis in this type of research is that prices are determined at the farm level and wholesale and retail prices follow the farm price. Suppose that this was the case. Suppose that f_t is the farm price in month t , (f_{t-1} is the previous month's farm price) and that F_t is the target farm price. To hit its target in month t , the farm price has to move from f_{t-1} to F_t . In partial adjustment models, the actual change in the price can be less than that needed to get the price to its target level. In our simple model, actual movement between months t and $t-1$, which is $f_t - f_{t-1}$, is given by the formula:

$$\alpha_{1,f}(f_t - f_{t-1}) = F_t - f_{t-1}. \quad (1)$$

Equation 1 is written in a slightly unusual way, but this specification simplifies the task of programing the estimation routine. The type of adjustment from equation 1 depends on the value of $\alpha_{1,f}$. If $\alpha_{1,f}$ is 1, there is complete adjustment. If the term is greater than one, a smaller change on the left-hand side balances a larger change on the right-hand side, resulting in partial adjustment. A value under 1 leads to overshooting. The sensible values for $\alpha_{1,f}$ are positive. A negative value means that prices adjust away from their equilibrium values.

To make equation 1 asymmetric, allow the value of $\alpha_{1,f}$ to vary depending on the sign of $(f_t - f_{t-1})$. This will give two values for the coefficient, $\alpha_{1,f}^+$ when the farm price increases and $\alpha_{1,f}^-$ when the farm price decreases. We now write equation 1 as:

$$\alpha^*_{1,f}(f_t - f_{t-1}) = G_t - f_{t-1} \quad (2)$$

where $\alpha^*_{1,f}$ is $\alpha^+_{1,f}$ or $\alpha^-_{1,f}$ depending on the direction that farm prices actually change.

When we set up our equations, we did not want to pre-specify that the farm price leads the other two prices. We did not, in fact, want to specify that any particular price leads the others. So we modified equation 2 to come up with our final form. Call the general target price level for month t , G_t , and assume that the general price level in month $t-1$ is some weighted average of the farm price, the wholesale price, w_{t-1} , and the retail price, r_{t-1} . To help ensure that the adjustment process is stable, we require that the weights on the lagged prices be positive. If prices are in general too low, we can get closer to the equilibrium price level by raising farm, wholesale, or retail prices. Our general price-level equation is written:

$$\alpha^*_{1,f}(f_t - f_{t-1}) + \alpha^*_{1,w}(w_t - w_{t-1}) + \alpha^*_{1,r}(r_t - r_{t-1}) = G_t - \beta_{1,f}f_{t-1} - \beta_{1,w}w_{t-1} - \beta_{1,r}r_{t-1},$$

where

$$\alpha^*_{1,i} \geq 0, i = \{f,w,r\} \text{ and } * = \{+, -\}$$

$$\alpha^*_{1,i} = \alpha^+_{1,i}, \text{ when } i_t - i_{t-1} > 0,$$

$$= \alpha^-_{1,i}, \text{ when } i_t - i_{t-1} < 0,$$

$$\sum \beta_{1,i} = 1, \text{ and}$$

$$\beta_{1,i} \geq 0, i = \{f,w,r\}. \quad (3)$$

We will leave the specification of G_t until later.

We would like to point out that equation 3 does not actually require that all prices increase when the general target price, G_t , is greater than the last month's price. However, if any price decreases, some other price would have to pick up the slack by increasing enough to offset the decrease in the other price and balance the right-hand side of equation 3. However, equation 3 does imply that if the last month's price level is too low, at least one price has to increase this month.

Farm-to-Wholesale Price Spread Equation

The next equation in our system is the farm-to-wholesale spread equation. Call the target spread in month t , $S_{2,t}$. The spread in the previous month can be calculated as $(w_{t-1} - f_{t-1})$. If the current target spread is higher than last month's spread, we can move closer to equilibrium if the farm price goes down or if the wholesale price goes up. We can write the farm-wholesale spread equation as:

$$\alpha^*_{2,w}(w_t - w_{t-1}) - \alpha^*_{2,f}(f_t - f_{t-1}) = S_{2,t} - w_{t-1} + f_{t-1} \quad (4)$$

All the α s in equation 4 are required to be positive. (When we estimated the model, it was easier to specify that the $\alpha^*_{2,f}$ be added on the left-hand side, but that their sign be negative.) If last month's spread was lower than this month's target, prices can adjust to get the spread closer to its target by dropping the farm price or raising the wholesale price.

As in equation 3, equation 4 does not require that farm price go down or that the wholesale price go up when the target spread is larger than the old spread. One price can move in the "wrong" direction, however. If one goes in the "wrong" direction, the other has to move even more in the "right" direction to correct. Equation 4 does require that either the wholesale price rise or the farm price fall when the target spread is larger than last month's spread. Again, we specified equation 4 so that it could handle a wide variety of cases. Equation 4 is valid if the farm price leads the wholesale price, if the farm price follows the wholesale price, and cases when their interactions are more complex.

Wholesale-Retail Price Spread Equation

The last equation is the wholesale-retail spread. This equation is similar to equation 4: the wholesale price replaces the farm price and the retail price the wholesale, and there is a different target spread, but otherwise the structure and interpretations are identical. We use the same type of sign constraints and our model is valid for all general types of wholesale-retail price interactions.

$$\alpha^*_{3,r} (r_t - r_{t-1}) - \alpha^*_{3,w} (w_t - w_{t-1}) = S_{3,t} - r_{t-1} + w_{t-1} \tag{5}$$

The target levels and spreads determine the longrun retail, wholesale, and farm prices, that is, if the target levels were fixed for a time, eventually all three prices would get to their respective target levels. We have not yet specified how these longrun targets are determined. As is the case for models of this type, these target levels are functions of other variables. As these variables change over time, the targets change over time as well. However, even though the targets change, we will refer to a variable's effect on the target level as its longrun effect. Because our model allows for partial adjustment, the shortrun effects of variables can vary from their longrun effects.

Targets

Our model allows changes in supply and demand conditions to affect price spreads in the short run but not in the long run. To account for this, the price-spread equations contain the change in the supply and demand variables, but not their lagged values. If the other variables were stabilized for a long period of time, that is, they did not change, their level would have no effect on the target price spread. Supply and demand variables affect the shortrun and longrun price level, so the price-level equation has both the changes and the lagged values. Also, no statistical model is perfect: minor variables could be missing, and even purely random effects could affect price levels and spreads. These random effects are modeled as error terms, which we designate as $e_{i,t}$. If we call the current value of the supply and demand variables, X_t , the change in these between month $t-1$ and t , dX_t , and the set of intercept, trend, and seasonal variables, I_t , then our equations for the longrun price level target and longrun spread targets can be written:

$$G_t = dX_t g_{1,dX} + X_{t-1} g_{1,X} + I_t g_{1,I} + e_{1,t} \tag{6}$$

$$S_{2,t} = dX_t g_{2,dX} + I_t g_{2,I} + e_{2,t} \text{ and} \tag{7}$$

$$S_{3,t} = dX_t g_{3,dX} + I_t g_{3,I} + e_{3,t} \tag{8}$$