

Appendix A—A Conceptual Model of the Agricultural Sector

Efficient modeling of the impacts of foreign animal diseases in the United States requires integrating a disease-spread model with an economic model. The general approach follows that of Jones (1981) and Sanyal and Jones (1982). The model and application are based on several key assumptions: Economic agents are assumed to be price-taking maximizers of well-defined objective functions. Consumers are assumed to select a consumption vector that maximizes a well-defined, homothetic utility function, given their income and prices. The utility maximization problem for a consumer gives a set of per capita demand functions. Producers (firms or farms) are assumed to select an input-output vector that maximizes profits, subject to a well-defined, constant-returns-to-scale production function. There are four types of inputs, or factors of production: (1) mobile among production activities (e.g., labor, fuel) and in perfectly elastic supply, so the price is treated as given, (2) sector-specific intermediate goods (e.g., hogs for pork), (3) sector-specific primary factors (e.g., physical and human capital), and (4) land, which is mobile across crop production. Land used in livestock production is treated as land used for forages and pasture. Thus, the model has the structure of a Ricardo-Viner or Specific-Factors model, where perfect competition prevails.

Final Consumer Demand

Based on the above assumptions, the demands for final goods are straightforward. Let $\mathbf{DP}_t$ be a column vector of per capita consumption for final goods in quarter t . Final goods in the model are beef, pork, poultry meat, lamb and sheep meat, eggs, milk, wheat, coarse grains, rice, and soybean oil. $\mathbf{DP}_t$ is a vector of per capita demand functions, $DP_i(\dots)$, each of which depends on a vector of retail prices for the final goods, $\mathbf{PR}_t$, and a scalar, per capita income, y_t :

$$(1) \mathbf{DP}_t = \mathbf{DP}_t(\mathbf{PR}_t, y_t).$$

Total consumption of final goods in the U.S. economy in quarter t depends on per capita consumption multiplied by the scalar population at time t , pop_t , and a column vector $\boldsymbol{\alpha}$, $0 \leq \alpha_j \leq 1$, of parameters that indicate the share of the population unafraid of a health risk associated with each final good. Thus, $\boldsymbol{\alpha}$ is a potential disease outbreak shock instrument in cases where consumers fear a health risk: If $\alpha_j = 1$, consumers do not fear a health risk. The vector of total consumption of final goods, $\mathbf{DF}_t$, is:

$$(2) \mathbf{DF}_t = \text{pop}_t * \boldsymbol{\alpha}_t * \mathbf{DP}_t.$$

Supplies of Final Goods

Meat, Milk, and Egg Production

Meat, milk, and eggs are produced by separate industries (sectors). Firms in the production of individual meats do not earn super-normal profits, so a zero-profit condition holds for each meat, as well as for milk and eggs. Production is assumed to occur at time t . Three types of production factors

are used. Prices for factors in perfectly elastic supply are exogenous and denoted with a column vector $\mathbf{W}_t$. Prices for the animal intermediate inputs are denoted by the vector $\mathbf{PA}_t$ while prices for the sector-specific primary factors are denoted as $\mathbf{R}_t$. Unit cost under the constant-returns-to-scale technology is a function of only factor prices, $\mathbf{CM}(\mathbf{W}_t, \mathbf{PA}_t, \mathbf{R}_t)$. Let $\mathbf{PM}_t$ be the wholesale prices of meats, milk, and eggs at time t . With perfect competition and constant returns to scale, the zero-profit conditions are:

$$(3) \mathbf{CM}(\mathbf{W}_t, \mathbf{PA}_t, \mathbf{R}_t) = \mathbf{PM}_t.$$

Determining production of meats, milk, and eggs requires inclusion of factor-market-clearing conditions. Define $\mathbf{AM}$ as the matrix of per unit demands for factors of production in the meat, milk, and egg industries, with $\mathbf{QM}_t$ the column vector of outputs of meats, milk, and eggs at time t . Let the column vector $\mathbf{Z}$ consist of a partition $[\mathbf{K}_t, \mathbf{DA}_t]^T$ where $\mathbf{K}_t$ indicates primary, sector-specific factors in fixed supply and $\mathbf{DA}_t$ denotes the derived demands for animals at time t . Given the above assumptions, $\mathbf{AM}$ depends only on factor prices, so the factor-market-clearing conditions are:

$$(4) \mathbf{AM}(\mathbf{W}_t, \mathbf{PA}_t, \mathbf{R}_t) * \mathbf{QM}_t = \mathbf{Z}.$$

The vertical linkages to animal agriculture are established through $\mathbf{Z}$ in equation (4), along with $\mathbf{K}_t$, through the derived demand for animals for slaughter ($\mathbf{DA}_t$). In general, these linkages proceed from slaughter, net of trade, back through livestock inventories, into the derived demands for feedstuffs. In this way, not only are linkages established for final products and the livestock sectors, but linkages are also established to crops and their factors of production. Because each type of animal has unique features, each type is presented individually.

Beef Cattle

The beef cattle market clears via a market price, $P_{c,t}$, determined by a market-clearing identity. That identity requires that beef animals slaughtered at time t , $D_{c,t}$, equal finished cattle from calves born five quarters previously, $Sc_{f,t}$, and dairy cows culled, Sdc_t , plus imports of cattle for slaughter, Mc_t , less cattle exported, $X_{c,t}$:

$$(5) D_{c,t} = Sc_{f,t} + Sdc_t + Mc_t - X_{c,t}.$$

Beef cattle born five quarters previously are finished at time t and have moved through a production process. Animals born in quarters $t-3$ and $t-4$ are in the grower and backgrounding stages at time t , denoted $Sc_{g,t}$ and $Sc_{b,t}$. These cattle will be finishing at time $t+1$ and $t+2$. Animals in their preweaning stage at time t , $Sc_{w,t}$ are linked to quarter $t-2$ births and will be finished at time $t+3$. The post-birth stage, $Sc_{p,t}$, is linked to calves born at $t-1$ that will be finished at time $t+4$. A disease outbreak means a policy of stamping out, or possibly natural deaths, so a mortality rate parameter, $\lambda_{c,j}$, $0 \leq \lambda_{c,j} \leq 1$, $j = f, g, b, w, p$, is inserted to reflect a reduction in cattle numbers. Thus, the flow of market cattle through the production stages is captured by the following equations:

$$(6) Sc_{f,t} = \lambda_{c,f} Sc_{g,t-1}$$

$$(7) Sc_{g,t} = \lambda_{c,g} Sc_{b,t-1}$$

$$(8) Sc_{b,t} = \lambda_{c,b} Sc_{w,t-1}$$

$$(9) Sc_{w,t} = \lambda_{c,w} Sc_{p,t-1}$$

Calves born in the quarter reflect the beef cow inventory, Ic_t , and since disease outbreaks can affect the rate of abortions and calf mortality, the term $\lambda_{c,p}$ is included to measure calves lost (equation 10):

$$(10) Sc_{p,t} = Sc_p(Ic_t, \lambda_{c,p}).$$

Cattle trade is linked to U.S. market prices for cattle, to trade policy, and to any disease outbreaks. Trade policy intervention is modeled as a specific trade intervention, t_c , with trade determined by the U.S. domestic price less the specific trade intervention. Because an animal disease outbreak can disrupt trade, parameters γm_c and γx_c , ranging from 0 to 1, are used to indicate the severity of trade restrictions. Thus, trade behavior is described as:

$$(11) Mc_t = Mc(Pc_t - t_c) * \gamma m_c;$$

$$(12) Xc_t = Xc(Pc_t - t_c) * \gamma x_c.$$

Each stage has unique derived demands for feed. There are four types of feed available in the model, wheat (w), coarse grains (g), soybean meal (sm), and forages and pasture (fo). Use of feed ingredient i ($= w, g, sm, fo$) by fed cattle at stage j at time t is given by $as_{i,j,t}$, which is a function of feed prices. Total demand, $DSc_{i,j,t}$, is the per animal use multiplied by the number of cattle consuming feed in each stage:

$$(13) DSc_{i,j,t} = as_{i,j,t}(Pw_t, Pg_t, Pfo_t, Psm_t) * Sc_{j,t}$$

for $i = w, g, fo, sm$ and $j = f, g, b, w, p$.

The decision to hold an animal for breeding at time t reflects two effects. One is the expected relative profitability of producing calves for future sale plus the utility cow value at time $t+9$ compared with selling the cow for slaughter at time t (Rosen, 1989). Thus, beef cow inventory at t , (equation 14) is partly explained by the expected return to retaining a heifer for breeding at time t , Rc_t^e , where the “e” indicates expectations relative to the current cattle price, Pc_t . The expected return is the utility value of the cow plus the values of two future calves. Another factor is the previous quarter’s inventory, since cow inventories cannot be immediately rebuilt. The coefficient on the lagged inventory controls the speed of adjustment. Also, any disease-induced losses, γc , must be recognized. Thus:

$$(14) Ic_t = Ic(Rc_t^e / Pc_t, Ic_{t-1}) * \gamma c.$$

Cows have unique feed requirements. Define the per cow feed ingredient requirement at time t as $ac_{i,t}$. These per cow demands depend on the feed ingredient prices and season of the year. Demand for feed ingredient i by cows at time t , $DC_{i,t}$, are the per cow ingredient demands multiplied by the inventory:

$$(15) Dc_{i,t} = ac_{i,t}(Pw_t, Pg_t, Pfo_t, Psm_t)*Ic_t$$

where $i = w, g, fo, sm$.

Replacement heifers at time t , Hc_t , affect cow inventory eight quarters in the future. The decision to raise a replacement heifer is based on the expected return, Rc^e , at time $t+16$ from a retained heifer, balanced against the expected market value of a heifer fed for five quarters. As in other cases, a disease outbreak could result in a loss of replacement heifers, λh , so total replacement heifer inventory entering at t is:

$$(16) Hc_t = Hc(Rc^e_{t+16}/Pc^e_{t+5}, \lambda h).$$

Each replacement heifer will have quarterly feed demands based on the season. Let $ah_{i,t}$ be the per heifer feed use of feed i . The feed use of each ingredient at time t by heifers, $DH_{i,t}$, will depend on the prevailing prices and the number of heifers:

$$(17) Dh_{i,t} = ah_{i,t}(Pw_t, Pg_t, Pfo_t, Psm_t)*Hc_t$$

where $i = w, g, fo, sm$.

The number of bulls, Bc_t , is exogenous, as these inventories vary little. Each bull consumes feed based on the season (quarter):

$$(18) Db_{i,t} = ab_{i,t}(Pw_t, Pg_t, Pfo_t, Psm_t)*Bc_t.$$

Swine Production

The swine component is similar to that for beef, but requires fewer stages and quarters. Slaughter hogs are assumed to be produced two quarters after farrowing. The market price for hogs, Phg_t , is determined where slaughter, $Dhgt$, equals market hogs, $Shgt$, from the pig crop produced two quarters earlier, PIG_{t-2} , plus imports of slaughter hogs, $Mhgs_t$, less exports of hogs for slaughter, $Xhgs_t$:

$$(19) Dhgt = Shgt + Mhgs_t - Xhgs_t.$$

Trade of slaughter hogs depends on the market price of hogs, a specific trade intervention, t_{hg} , and parameters ranging from 0 to 1 that indicate the restrictiveness of trade following a disease outbreak, γm_{hg} and γx_{hg} :

$$(20) Mhgs_t = Mhgs(Phg_t - t_{hg})*\gamma m_{hg};$$

$$(21) Xhgs_t = Xhgs(Phg_t - t_{hg})*\gamma x_{hg}.$$

Hogs for market at time t are the pigs at time $t-2$ plus feeder pigs coming in from Canada at time $t-1$, $Mhgfd_{t-1}$, less any exports of feeder pigs, $Xhgfd_{t-1}$. Because a disease outbreak could affect the number of animals available for slaughter, a scalar, λs , $0 \leq \lambda s \leq 1$, is introduced:

$$(22) Shgt = Mhgfd_{t-1} + \lambda s*PIG_{t-2} - Xhgfd_{t-1}.$$

Trade of feeders follows the same specification as for slaughter hogs:

$$(23) \text{Mhgfd}_t = \text{Mhgfd}(\text{Phg}_t - t_{hg}) * \gamma m_{hg},$$

$$(24) \text{Xhgfd}_t = \text{Xhgfd}(\text{Phg}_t - t_{hg}) * \gamma x_{hg}.$$

Gestation for hogs is about 4 months, so pigs born in period t depend on the numbers of sows in period $t-1$, Isw_{t-1} , a variable, α s, to account for increased abortions, and the relative price of future hogs compared with the market value of a sow last quarter ($\text{Phg}_{t+2}^e / \text{Psw}_{t-1}$). Thus, the pig crop starting on feed at time t is given as:

$$(25) \text{PIG}_t = \text{PIG}((\text{Phg}_{t+2}^e / \text{Psw}_{t-1}), \text{Isw}_{t-1}, \alpha s).$$

Sow numbers depend on the expected return for breeding a sow, Rsw_t^e , piglets plus sow cull value, vs. the market value of a sow at time t , Psw_t . The expected return includes the market value of a sow plus the value of four litters of pigs. Another influence is the effect of disease on sow inventories, γ s, $0 \leq \gamma \leq 1$. Also, because inventory adjusts slowly to new desired levels, lagged inventory is included. Thus:

$$(26) \text{Isw}_t = \text{Isw}(\text{Rsw}_t^e / \text{Psw}_{t-4}, \gamma s, \text{Isw}_{t-1}).$$

As with cattle, swine at each stage have unique feed requirements. Per animal feed demands depend on the relative prices of wheat, coarse grains, and soybean meal. Forage and pasture are not used for hogs. Total demands are found by multiplying per animal feed use by the number of animals at each point in the production process. Let $\text{ahg}_{i,j,t}(\text{Pw}_t, \text{Pg}_t, \text{Psm}_t)$ be the per animal derived demand for feed i , $i = w, g, sm$, for market hogs in quarter t , stage j , where $j = f$ for market hogs and $j = p$ for pigs starting on feed.

Per unit use in each stage is a time-weighted average for two production substages linked to weight. Pigs starting on feed are comprised of two substages. Production substage 1 lasts about 40 days and brings the pig to a weight of 60 pounds. Substage 2 raises the weight to 120 pounds and covers about 45 days. Thus, feed demands for pigs starting on feed (births plus net imports), $\text{Dhg}_{i,p,t}$, are:

$$(27) \text{Dhg}_{i,p,t} = \text{ahg}_{i,p,t}(\text{Pw}_t, \text{Pg}_t, \text{Psm}_t) * \text{Shg}_{i,p,t}$$

where $\text{Sgh}_{i,j,t} = (\text{PIG}_t + \text{Mhgfd}_t - \text{Xhgfd}_t)$
for $i = w, g, sm$.

Market hogs are composed of the third and fourth substages. Substage 3 lasts for about 40 days, and the hog achieves a weight of 180 pounds. Substage 4 takes the animals to market weight of about 250 pounds and lasts about 40 days. Thus, for market animals at time t there are derived demands for each feed ingredient, $\text{Dhg}_{i,f,t}$:

$$(28) \text{Dhg}_{i,f,t} = \text{ahg}_{i,f,t}(\text{Pw}_t, \text{Pg}_t, \text{Psm}_t) * \text{Shg}_t,$$

$i = w, g, sm$.

Sows in a given quarter have different per unit feed demands. Denote per sow feed use of ingredient i as $asw_{i,t}(Pw_t, Pg_t, Psm_t)$. Thus, total feed use by sows at time t is, Dsw_{it} :

$$(29) Dsw_{i,t} = asw_{i,t}(Pw_t, Pg_t, Psm_t) * Isw_t.$$

Dairy Cattle

The structure of the dairy animal and feed allocation differs from other live-stock feed allocations, since the model determines milk production using the zero-profit and sector-specific, factor-market-clearing conditions. Both milk output and dairy cattle being milked are determined simultaneously. The decision to determine milk output directly and convert that output into dairy cows reflects the way cost data are reported. Production costs for milk include the feed costs and not the heifer cost, whereas meat cost data include the animal but not the feed. Disease outbreaks are reflected in reduced milk output, which translates into reduced dairy cattle inventory. Thus, current milk output, qmk_t , determines the size of the dairy herd, Id_t . Further, because inventories of dairy cattle are slow to adjust, lagged inventory is included:

$$(30) Id_t = Id(qmk_t, Id_{t-1}).$$

Quarterly feeding for dairy cows is tied to the dairy herd. Dairy cows eat wheat, coarse grains, soybean meal, and forage and pasture. Denote the per cow use of feed ingredient i as $ad_{i,t}(Pw_t, Pg_t, Pfo_t, Psm_t)$. For quarter t , the demand for feed ingredient i by animals, $Dd_{i,t}$, is:

$$(31) Dd_{i,t} = ad_{i,t}(Pw_t, Pg_t, Pfo_t, Psm_t) * Id_t.$$

The replacement decision, Rd_t , depends on the expected return from milk production and calves over the next 16 quarters, Rmk_t^e , relative to the expected slaughter value of the animal in five quarters, Pc_{t+5}^e . Also, disease could exogenously cut replacement numbers by γd , $0 \leq \gamma d \leq 1$:

$$(32) Rd_t = Rd(Rmk_t^e / Pc_{t+5}^e, \gamma d).$$

As replacement heifers move through the system, feed consumption varies. Let $ard_{i,t}(Pw_t, Pg_t, Pfo_t, Psm_t)$ be the per animal use of feedstuff i in quarter t . Total feed use for all replacement dairy heifers in quarter t for feedstuff i , $Drd_{i,t}$, is:

$$(33) Drd_{i,t} = ard_{i,t}(Pw_t, Pg_t, Pfo_t, Psm_t) * Rd_t.$$

Slaughter of dairy cattle at time t , Sdc_t , is determined by the inventory plus imports, Mdc_t , less dairy cattle exported, Xdc_t :

$$(34) Sdc_t = Id_t + Mdc_t - Xdc_t.$$

Dairy cattle trade is tied to the market price of cattle, a specific trade intervention, t_{dc} , and disease parameters that range from 0 to 1, γm_{dc} and γx_{dc} :

$$(35) Mdc_t = Mdc(Pc_t - t_c) * \gamma m_{dc};$$

$$(36) Xdc_t = Xdc(Pc_t - t_c) * \gamma_{dc}$$

Poultry Meat

Due to its fast production process, poultry is relatively simple to model. For broilers, hatch-to-kill represents one quarter. To account for the time to hatch, lagged output is included, but with a very fast quarter-to-quarter adjustment. The meat model determines poultry meat production, qpm_t , via the zero-profit and sector-specific, factor-market-clearing conditions. Meat production is linked directly to bird numbers. Disease effects enter via the poultry meat production. Birds produced in a specific quarter, t , require a certain amount of feed of type i per ton $apm_{i,t}(Pw_t, Pg_t, Psm_t)$. Birds are assumed not to consume forage and pasture. Thus, total demand for feedstuff i , $Dpm_{i,t}$, is:

$$(37) Dpm_{i,t} = apm_{i,t}(Pw_t, Pg_t, Psm_t) * qpm_t$$

$i = w, g, sm.$

Layers

The model also determines egg production, qe_t , using the zero-profit and sector-specific, factor-market-clearing conditions. The number of layers and feed use are known from egg production. Disease affects egg production (layer numbers). Layers respond more slowly than broilers, so lagged production is included with a stronger effect. With $al_{i,t}(Pw_t, Pg_t, Psm_t)$ the per unit use of feed, the total demand for feed of kind i , $Dl_{i,t}$, is:

$$(38) Dl_{i,t} = al_{i,t}(Pw_t, Pg_t, Psm_t) * qe_t$$

$i = w, g, sm.$

Lambs and Sheep

The equations describing lambs and sheep are structured like those for beef cattle. Market lambs move through several distinct production stages. Lambs in the finishing stage at time t , $Slb_{f,t}$, move to slaughter. These animals were in the growing stage, $Slb_{g,t}$, in the previous quarter, so current slaughter lambs are those previous grower lambs adjusted for the effects of any disease outbreak, $\lambda lb_{f,t}$, $0 \leq \lambda lb_{f,t} \leq 1$. Current-period grower lambs were backgrounding lambs in the previous quarter, again adjusted for disease effects. Backgrounders in the current quarter, $Slb_{b,t}$, were the lamb crop in the previous quarter, $Slb_{p,t-1}$. Thus, the flow of market lambs is described by:

$$(39) Slb_{f,t} = \lambda lb_{f,t} * Slb_{g,t-1}$$

$$(40) Slb_{g,t} = \lambda lb_{g,t} * Slb_{b,t-1}$$

$$(41) Slb_{b,t} = \lambda lb_{b,t} * Slb_{p,t}$$

The lamb crop is tied to the ewe inventory, Iew_t , and a disease shock:

$$(42) Slb_{p,t} = Slb(Iew_t) * \lambda lb_{p,t}$$

Market equilibrium for slaughter lambs determines the price, Plb_t , by equating the demand for slaughter lambs, Dlb_t , to the supply of finished lambs plus net imports, $Mlb_t - Xlb_t$:

$$(43) Dlb_t = Slb_{f,t} + Mlb_t - Xlb_t.$$

Trade depends on the price of lambs, trade policy, t_{lb} , and the effects of any disease on trade. Those effects are described by parameters, m_{lb} and x_{lb} , respectively, ranging from 0 to 1, which reflect the restrictiveness of trade after an outbreak. The behavioral equations are:

$$(44) Mlb_t = Mlb(Plb_t - t_{lb}) * \gamma m_{lb},$$

$$(45) Xlb_t = Xlb(Plb_t - t_{lb}) * \gamma x_{lb}.$$

Ewe inventory at time t depends on the expected value of holding a ewe for breeding, Rew_t^e , relative to the current market price of a ewe, Pew_t , ewe inventory one quarter previously, and a disease shock, λew_t . The expected return includes the slaughter value of a ewe plus the value of 2 lambs. Thus, the ewe inventory is given by:

$$(46) Iew_t = Iew((Rew_t^e / Pew_t), Iew_{t-1}) * \lambda ew_t.$$

Inventories of replacement ewes, Ier_t , depend on the expected return to a ewe over the period $t+4$ to $t+12$, Rew_{t+4}^e , relative to the current market price of a slaughter ewe, the number of ewes one quarter previously, and a disease shock, λer_t :

$$(47) Ier_t = Ier((Rew_{t+4}^e / Pew_t), Ier_{t-1}) * \lambda er_t.$$

Rams, Irm_t , are treated as exogenous.

Sheep and lambs use all feedstuffs: wheat, coarse grains, soybean meal, and forage and pasture. The mix of feeds varies by production stage. Let $alb_{i,j,t}$ be the use of feed i , $i = w, g, sm, f$, by an individual lamb at production stage j , $j = f, g, b, p$, at time t . With constant returns to scale, the per animal feed demands depend only on feedstuff prices. Consequently, the total feed demands, $Dlb_{i,j,t}$, are:

$$(48) Dlb_{i,j,t} = alb_{i,j,t}(Pw_t, Pg_t, Psm_t, Pfo_t) * Slb_{j,t}$$

where $i = w, g, sm, fo$, and $j = f, g, b, p$.

Ewes, replacements, and rams have a similar structure. Let $aew_{i,t}$ be the per ewe use of feed i , with $aer_{i,t}$ and $arm_{i,t}$ the same for replacements and rams, respectively. These depend on feedstuff prices and season. Thus, the total demand for feed i by ewes, $Dew_{i,t}$, is:

$$(49) Dew_{i,t} = aew_{i,t}(Pw_t, Pg_t, Psm_t, Pfo_t) * Iew_t$$

for $i = w, g, sm, fo$.

The feed demands for replacements, $Der_{i,t}$, are:

(50) $Der_{i,t} = aer_{i,t}(Pw_t, Pg_t, Psm_t, Pfo_t)*Ier_t$,
for $i = w, g, sm, fo$.

The feed demands by rams, $Drm_{i,t}$, are:

(51) $Drm_{i,t} = arm_{i,t}(Pw_t, Pg_t, Psm_t, Pfo_t)*Irm_t$,
for $i = w, g, sm, fo$.

Crops

The previous discussion identified demands for feedstuffs. The crop agriculture relationships are now developed. Crops included are wheat, coarse grains, soybeans, rice, and forage and pasture. In addition to the feedstuff demands, there are demands arising from final (retail) demands. The focus here is the supply side. Crop production occurs at set times and then becomes carryin stock in subsequent quarters until a new crop is harvested. Another key feature is that due to the dynamics, production decisions are made well before harvest, based on expectations of returns for the crops. Finally, except for forage and pasture, all of the crops included in the model are program crops. This means the influence of the various U.S. Government price and income supports must be incorporated.

Crop production (acreage) decisions are made quarters ahead of production because input supplies must be ordered and land leases arranged. The acreage allocation is based on expected returns for crops at harvest, with expected returns being the previous harvest prices plus appropriate government payments. The computations are done in quarter 1 so that an acreage allocation consistent with one crop cycle can be imposed. Since there are both winter and spring crops in the model, this is a simplification of the actual decision process.

Soybeans and rice are spring crops. They are planted in the second quarter of the current year and harvested in quarters 3 (rice) or 4 (soybeans). Coarse grains are an aggregate of crops: corn, sorghum, millet, barley, rye, and oats. Coarse grains are dominated by corn and sorghum, which are planted in quarter 2 and harvested in quarter 4. Oats and rye are minor crops, harvested in the third quarter. Barley, also a minor crop, consists of both winter and spring barley, which are assumed to be harvested in quarters 2 and 3, based on earlier production decisions.

Wheat poses a larger problem as a major crop with both spring and winter production. Spring wheat is harvested in quarter 3 after being planted in quarter 2. Winter wheat is planted in the fourth quarter of the previous year and harvested beginning in the second quarter of the current year (May, June, and July). For this model the assumption is that winter wheat is harvested in quarter 2. The acreage (production) decision for that harvest is assumed to be made in the first quarter of the year, based on returns to second-quarter wheat in the previous year, to create a consistent use of land. This is because the decision to plant winter wheat the previous fall requires arranging inputs earlier in the year and constrains cropping decisions in the spring.

Modeling of forage and pasture poses problems similar to wheat. Production occurs in quarters 2 and 3. Forage and pasture acreage is assumed to be determined in quarter 1, based on the quarter 2 and 3 prices of the previous year.

To determine crop production, return to the structure outlined at the beginning of this section. Crop production for quarter t is based on the vector of expected crop returns calculated in the first quarter, $\mathbf{P}^e_1$, via zero-profit conditions:

$$(52) \text{Ct}(\mathbf{W}, \mathbf{R}^e, \tau) = \mathbf{P}^e_1,$$

where $\mathbf{R}^e$ is the vector of expected returns to physical and human capital and $\mathbf{P}^e_1$ is the vector of expected returns for crops in the first quarter of the year. Since the mobile input is in perfectly elastic supply, $\mathbf{W}$ is exogenous. The return to land, τ , captures the negotiation process between farmer and landlord for land rent in the upcoming crop season. Thus, the expected return to physical and human capital, $\mathbf{R}^e$, is determined by the expected zero-profit condition.

Expected returns for crops consist of several parts and vary depending on market conditions. The price expected in quarter 1 is that prevailing in the harvest quarter of the previous year. The return reflects U.S. Government payments. There are several payments to include, and there is debate about how they affect production—the decoupling issue (Goodwin and Mishra). If the loan rate (LR) exceeds the quarterly market price, the farmer is assumed to receive loan deficiency payments (LDPs) equal to the difference. The payments are made on the full amount of production. Direct payment rates (DPs) are established by law. Total payments are the rate multiplied by 85 percent multiplied by program yield and base area. Additionally, the 2002 Farm Act provides for countercyclical payments (CCPs), calculated from an announced target price (TP). The payment rate is the difference between the target price, less any direct payment, and the market price when the market price is above the loan rate. If LDPs are paid, they are not adjusted by the .85 used in the CCP adjustment, but instead, the full LDP is added to the market price. The payments are 85 percent of the payment rate times the eligible production, which is program yield, times crop base acreage. The expected return is the expected price on the previous crop plus CCP payments, LDPs, and direct payments.

Loan deficiency payments are coupled payments. A critical issue is whether direct payments and CCPs are decoupled or not. Returns to human and physical capital and to land cannot be adequately modeled without including these payments, so they are reflected in the model and affect the dynamics of the model solutions. The payments are modeled to affect relative per acre returns among program crops. Since forage and pasture are not program crops, there is no direct price adjustment, but there is a relative price effect.

Sector-specific, factor-market-clearing conditions, using expected rent and factor prices in quarter 1, determine crop output for quarter t :

$$(53) \mathbf{a}_K(\mathbf{W}, \mathbf{R}^e, \tau) * \mathbf{Q}_t = \mathbf{K}_1.$$

Land is mobile among the crops. Its return is determined in quarter 1 by the demand and supply for land for the upcoming crops in period t :

$$(54) \text{at}(\mathbf{W}, \mathbf{R}^e, \boldsymbol{\tau}) * \mathbf{Q}_t = \mathbf{T},$$

where $\mathbf{T}$ is total land available for crops in t .

While crop output is determined based on the expected returns to sector-specific factors, actual returns to the sector-specific factors, $\mathbf{R}_t$, can differ from expected returns because actual returns to crop production differ from expected returns. The actual market prices, $\mathbf{Pm}_t$, are determined in the market-clearing identities. Once the crop market prices are known, the LDP and CCP payment rates and total payments can be calculated for the crop produced at time t . The actual return to the program crop, $\mathbf{P}_t$, is found with the addition of the payments:

$$(55) \mathbf{P}_t = \mathbf{Pm}_t + 0.85 * \mathbf{DP} * (\mathbf{y} * \mathbf{A}) / \mathbf{Q} + \mathbf{Z}_1 + \mathbf{Z}_2,$$

$$\text{where } \mathbf{Z}_1 = \begin{cases} 0.85 * (\mathbf{TP}_t - \mathbf{Pm}_t) (\mathbf{y} * \mathbf{A}) / \mathbf{Q}, & \text{if } \mathbf{Pm}_t < \mathbf{TP}_t \\ \mathbf{0}, & \text{if } \mathbf{Pm}_t > \mathbf{TP}_t \end{cases}$$

$$\mathbf{Z}_2 = \begin{cases} (\mathbf{LR}_t - \mathbf{Pm}_t) & \text{if } \mathbf{Pm}_t < \mathbf{LR}_t \\ \mathbf{0}, & \text{if } \mathbf{Pm}_t > \mathbf{LR}_t \end{cases}$$

where $\mathbf{y}$ is a vector of program yields established by rules set by the U.S. Government, $\mathbf{A}$ is a vector of base acreages, and $\mathbf{Q}$ is a vector of quarterly production.

The return to forage and pasture is the market price, since there is no program. Thus, the zero-profit condition determining the actual return to physical and human capital in period t , $\mathbf{R}_t$, is:

$$(56) \mathbf{C}(\mathbf{W}, \mathbf{R}_t, \boldsymbol{\tau}) = \mathbf{P}_t.$$

Supply in a given quarter is any production in that quarter, $\mathbf{Q}_t$, plus carryin stocks, $\mathbf{I}_{t-1}$. To identify carryin stocks, carryover stock must be identified. Carryin stocks are the previous period's carryover stock, $\mathbf{I}_{t-1}$, so defining carryover stocks with behavioral equations completes the supply side supply of crop i in period t . Carryout stocks are determined by a price-speculative motive, $\mathbf{Pm}_{t+1}^e / \mathbf{Pm}_t$:

$$(57) \mathbf{I}_t = \mathbf{I}(\mathbf{Pm}_{t+1}^e / \mathbf{Pm}_t).$$

Soybean Complex

The soybean complex is included for two reasons. First, soybean meal is a major feedstuff, and its use is affected by any disease outbreak. Second, soybeans compete with other crops for acreage. Since the supply describes soybean production and the demands are for soybean meal for feed and soybean oil for food, the crushing of soybeans into the joint products, soybean meal and soybean oil, must be modeled. This is done by specifying a derived demand for soybeans for crushing, $\mathbf{Dsb}_t$, which is a function of the current period crushing margin, $\mathbf{SPD}_t$:

$$(58) \mathbf{Dsb}_t = \mathbf{Dsb}(\mathbf{SPD}_t).$$

The crushing margin is the value of the joint products, given by their yields multiplied by their prices, $\mathbf{Psm}$ and $\mathbf{Pso}$, less the price of soybeans, $\mathbf{Psb}$.

Let β_m and β_o be the yields of soybean meal and soybean oil from a ton of soybeans. The margin is defined as:

$$(59) SPD_t = \beta_m * Psm_t + \beta_o * Pso_t - Psb_t.$$

Outputs of soybean meal, qsm , and soybean oil, qso , are the yields multiplied by the crush:

$$(60) qsm_t = \beta_m * Dsb_t,$$

$$(61) qso_t = \beta_o * Dsb_t.$$

Closure

Model closure requires domestic and international market-clearing relationships for quantities and prices. Exports, X_t , and imports, M_t , depend on prices and trade interventions, and in some cases on the disease outbreak. For many agricultural goods, the United States is an exporter and does not intervene in the market. Let tx be a vector of specific trade interventions. To allow effects from disease-related trade restrictions, like a ban on beef exports, define λx as a vector of parameters ranging from 0 to 1, where 0 implies an export ban and 1 is no restriction. So the excess demand faced by the United States is:

$$(62) X_t = X(Pm_t - tx) * \lambda x.$$

While many agricultural goods are imported into the United States without restriction, beef and dairy products are subject to tariff-rate quotas, TRQs. A tariff-rate quota is a stepped tariff where import volumes below the quota require payment of a lower tariff than import volumes above the quota. To illustrate, the below-quota tariff for beef is \$0.04 per kilogram (\$88.18 per ton), while the over-quota tariff is 26 percent. Let beef imports be Mb_t , the U.S. domestic beef price be Pb_t , and the world beef price be PWb_t . Thus, the policy is modeled as:

$$(63) \text{ If } Mb_t < \text{Quota, then } Pb_t = PWb_t + 88.18, \\ \text{ If } Mb_t > \text{Quota, then } Pb_t = PWb_t * (1 + 0.26), \\ \text{ If } Mb_t = \text{Quota, then } Pb_t \text{ clears the domestic market-given Quota.}$$

To facilitate model solution, it is assumed that the quotas are not filled, and the below-quota specific intervention applies, tm . Quota underfill seems to be more common for U.S. beef imports than quota overfill. When an intervention is applied, it is deducted from the U.S. domestic price so that trade reacts to the “world” or border price. Disease-related trade restrictions are allowed through a parameter, λm .

The remaining imports are explained by an excess supply to the United States:

$$(64) M_t = M(Pm_t - tm) * \lambda m.$$

Market-clearing identities can be written using the matrix notation:

$$(65) M_t = DF_t + DD_t + X_t + I_t - Q_t - I_{t-1},$$

where **DD** is column vector of derived demands for feedstuffs and animals.

Completing the model requires vertically linking the prices. This improves the numerical accounting of the impacts, but does not affect model response to shocks. There are three levels to prices: farm prices for crops and live-stock, **Pm**, wholesale prices for meats, milk, and eggs, **PM**, and retail prices for all final goods, **PR**. These levels are linked by marketing margins calculated by the Economic Research Service of the U.S. Department of Agriculture (ERS). The farm-to-wholesale margin is denoted SPDW, while the wholesale-to-retail margin is denoted SPDR. Thus, price linkage equations are:

$$(66) \text{PM}_t = \text{Pm}_t + \text{SPDW}_t,$$

$$(67) \text{PR}_t = \text{PM}_t + \text{SPDR}_t.$$

Differential Transformation of the Conceptual Model

A numerical solution of the integrated epidemiological and economic agricultural sector model is facilitated by a total logarithmic differential version of the equations presented in the preceding section. The logarithmic differential version is advantageous because the differential version is driven by elasticities, which are easier to obtain than specific functional forms and are also more intuitive than partial derivatives. The logarithmic differential version can also be applied to observed historical data, avoiding the need to forecast future exogenous variable values. The base data can also be updated as new values become available.

Final Consumer Demand

The final demand system in general functional form is given by equations (1) and (2). Substituting equation (1) into equation (2) and logarithmically differentiating gives a 10-equation system:

$$(68) \text{dln}(\text{DF}_t) = \text{dln}(\text{pop}_t) + \text{dln}(\alpha_t) + \epsilon * \text{dln}(\text{PR}_t) + \epsilon_y \text{dln}(y),$$

where ϵ is a 10 X 10 matrix of own- and cross-price elasticities and ϵ_y is a vector of income elasticities.

Supplies of Final Goods

Meat, Milk, and Egg Production

Meat, milk, and egg production are described by the zero-profit equations (3) and the sector-specific, factor-market-clearing conditions (4). There are six of each type. Totally differentiating the zero-profit conditions at time t , applying the envelope property, and with quantity normalization on the unit isoquant, the percentage change in the wholesale price is a linear combination of the factor-price changes. Let Θ be the 6x13 matrix of unit revenue shares and Ω^T the 13x1 column vector of factor-price changes, $\Omega = [\text{dlnW}_t, \text{dlnPA}_t, \text{dlnR}_t]$. Total differentiation of equation (3) gives:

$$(69) \Theta * \Omega^T = \text{dln}(\text{PM}_t).$$

With the mobile factor price, $\mathbf{W}$, exogenous, the mobile factor-market-clearing identity is dropped so equation (4) can be partitioned into two sets of equations. Define $\mathbf{QM}$ as a 6x1 column vector of beef, pork, poultry meat, lamb and sheep meat, milk, and egg production. Let $\mathbf{AK}$ be the 6x1 column vector of per unit use of physical and human capital. Thus, part of equation (4) can be written in differential form:

$$(70) \text{dln}(\mathbf{QM}_t) + \text{dln}(\mathbf{AK}_t) = \text{dln}(\mathbf{K}_t).$$

The second part of equation (4) gives the derived demand for animals for slaughter—beef cattle, swine, lambs and sheep, broilers, and for production inventory—dairy cows and layers. The 6x1 column vector, $\mathbf{AA}$, gives per unit derived demands, and $\mathbf{DA}$ gives the total derived demand. Thus, the factor demands are:

$$(71) \text{dln}(\mathbf{DA}_t) = \text{dln}(\mathbf{QM}_t) + \text{dln}(\mathbf{AA}_t).$$

Completing this part of the model requires specifying the changes in per unit factor uses. Let $\mathbf{AW}$ be a 6x1 column vector of per unit demand for the mobile factor. Logarithmic differentiation links changes in the ratio of per unit factor use to changes in factor prices via the matrix of Morishima elasticities of substitution between mobile factors and capital, σ_w , and between animals and capital, σ_a , under constant returns to scale (e.g., Chambers, 1988, p. 96):

$$(72) \text{dln}(\mathbf{AW}_t) - \text{dln}(\mathbf{AK}_t) = -\sigma_w^*(\text{dln}(\mathbf{W}_t) - \text{dln}(\mathbf{R}_t)),$$

$$(73) \text{dln}(\mathbf{AA}_t) - \text{dln}(\mathbf{AK}_t) = -\sigma_a^*(\text{dln}(\mathbf{PA}_t) - \text{dln}(\mathbf{R}_t)).$$

Also it can be shown that for movements around the unit isoquant:

$$(74) \Theta^* \text{dln}(\mathbf{A}^T) = 0,$$

where $\mathbf{A} = [\mathbf{AW}_t, \mathbf{AA}_t, \mathbf{AK}_t]$.

Animal Inventories

Differentiation of the animal inventory is straightforward. One element of $\mathbf{DA}_t$ is the demand for cattle for slaughter at time t , Dc_t . Equation (5) for beef cattle becomes:

$$(75) Dc_t * \text{dln}(Dc_t) = Sc_{f,t} * \text{dln}(Sc_{f,t}) + Sdc_t * \text{dln}(Sdc_t) + Mc_t * \text{dln}(Mc_t) - Xc_t * \text{dln}(Xc_t).$$

Cattle slaughtered at time t depend on the flow of animals through the different stages of production, as given by equations (6) through (9). Differentiating equations (6) through (9) gives:

$$(76) \text{dln}(Sc_{f,t}) = \text{dln}(Sc_{g,t-1}) + \text{dln}(\lambda_{c,f}),$$

$$(77) \text{dln}(Sc_{g,t}) = \text{dln}(Sc_{b,t-1}) + \text{dln}(\lambda_{c,g}),$$

$$(78) \text{dln}(Sc_{b,t}) = \text{dln}(Sc_{w,t-1}) + \text{dln}(\lambda_{c,b}),$$

$$(79) \text{dln}(Sc_{w,t}) = \text{dln}(Sc_{p,t-1}) + \text{dln}(\lambda_{c,w}).$$

Calves entering production depends on cow inventories, equation (10), which, after some manipulation, becomes:

$$(80) \ln(Sc_{p,t}) = \eta_c * \ln(Ic_t) + \eta_{ac} * \ln(\lambda_{c,p}),$$

where in equation (80) and in what follows, η_{ij} is the percent response of i with respect to a 1-percent change in j (i.e., elasticities). Cow inventory, equation (14), depends on breeding and replacement decisions:

$$(81) \ln(Ic_t) = \eta_{pe} * (\ln(Rc_t^e) - \ln(Pc_t)) + \eta_{hc} * \ln(Ic_{t-1}) + \eta_{\gamma} * \ln(\gamma_c).$$

The change in replacement heifers is determined by the replacement decision based on expected relative prices:

$$(82) \ln(Hc_t) = \eta_{hp} * (\ln(Pc_{t+16}^e) - \ln(Pc_{t+5}^e)) + \eta_{\lambda h} * \ln(\lambda_h).$$

Swine follow the same pattern. The demand for market hogs, Dhg_t , is one element in **DA**. The change in demand for market hogs equals the changes in domestic supply plus imports less exports:

$$(83) Dhg_t * \ln(Dhg_t) = Shg_t * \ln(Shg_t) + Mhgs_t * \ln(Mhgs_t) - Xhgs_t * \ln(Xhgs_t).$$

The supply of market hogs equals feeder pig imports and farrowings, adjusted for deaths, less feeder pig exports:

$$(84) Shg_t * \ln(Shg_t) = Mhgfd_{t-1} * \ln(Mhgfd_{t-1}) + \lambda_s * \text{PIG}_{t-2} * (\ln(\lambda_s) + \ln(\text{PIG}_{t-2})) - Xhgfd_{t-1} * \ln(Xhgfd_{t-1}).$$

The change in the pig crop depends on the expected value of market hogs in two quarters, relative to the market value of a sow last quarter and the sow inventory:

$$(85) \text{PIG}_t * \ln(\text{PIG}_t) = \eta_{phg} * (\ln(\text{Phg}_{t+2}^e) - \ln(\text{Psw}_{t-1})) + \eta_{\text{PIG}} * (\ln(\text{Isw}_t) + \eta_{\alpha s} * \ln(\alpha_s)).$$

Sow inventory depends on the expected relative returns:

$$(86) \ln(\text{Isw}_t) = \eta_{\text{IS}} * (\ln(\text{Rsw}_t^e) - \ln(\text{Psw}_{t-1})) + \eta_{\gamma s} * \ln(\gamma_s) + \eta_{\text{sw}} * \ln(\text{Isw}_{t-1}).$$

Milk production is determined by disease impacts on milk output as reflected in equation (30), due to the way cost data for milk are reported. Based on changes in milk output, the change in dairy cow inventory is determined:

$$(87) \ln(Id_t) = \eta_{mk} * \ln(qmk_t) + \eta_{\text{Id}} * \ln(Id_{t-1}).$$

The breeding/replacement decision depends on relative expected returns:

$$(88) \ln(Rd_t) = \eta_{\text{Rd}} * (\ln(\text{Rmk}_t^e) - \ln(\text{Pc}_{t+5}^e)) + \eta_{\gamma d} * \ln(\gamma_d).$$

Cull of dairy cows, Sdc , is determined by the residual of the inventory plus imports less exports:

$$(89) Sdc_t * \ln(Sdc_t) = Id_t * \ln(Id_t) + Mdc_t * \ln(Mdc_t) - Xdc_t * \ln(Xdc_t).$$

Lamb and sheep inventory is described by equations (46) and (47). Differentiating equation (46) gives the change in ewe inventory:

$$(90) \text{dln}(Iew_t) = \eta_{ew_p} * (\text{dln}(Rew_t^e - \text{dln}(Pew_t)) + \eta_{ew_I} * \text{dln}(Iew_{t-1}) + \text{dln}(\lambda_{ew_t}).$$

The change in replacement ewes comes from differentiating equation (47):

$$(91) \text{dln}(Ier_t) = \eta_{er_p} * (\text{dln}(Rew_{t+4}^e - \text{dln}(Pew_t)) + \eta_{er_I} * \text{dln}(Ier_{t-1}) + \text{dln}(\lambda_{er_t}).$$

Once ewe inventory is determined, the flow of market lambs can be found, as in equations (39) through (42). Equation (42) gives the slaughter lamb crop, which, when differentiated, becomes:

$$(92) \text{dln}(Slb_{p,t}) = \eta_{lb_I} * \text{dln}(Iew_t) + \text{dln}(\lambda_{lb_{p,t}}).$$

Subsequently, lambs move through the remaining stages of production:

$$(93) \text{dln}(Slb_{b,t}) = \text{dln}(Slb_{p,t-1}) + \text{dln}(\lambda_{lb_{b,t}}),$$

$$(94) \text{dln}(Slb_{g,t}) = \text{dln}(Slb_{b,t-1}) + \text{dln}(\lambda_{lb_{g,t}}),$$

$$(95) \text{dln}(Slb_{f,t}) = \text{dln}(Slb_{g,t-1}) + \text{dln}(\lambda_{lb_{f,t}}).$$

With the change in supply of finished slaughter lambs determined, the change in the market price for lambs is determined using the differential of the market-clearing identity, equation (43):

$$(96) \text{dln}(Dlb_t) = \text{dln}(Slb_{f,t}) + \text{dln}(Mlb_t) - \text{dln}(Xlb_t).$$

Feed Demands

The feed demands reflect the age distribution and flow of animals. Beef cattle generate a large set of feed demands. Finished beef cattle slaughtered in period t are the outcome of a process beginning five quarters previously. Thus, for market cattle there are five temporal demands for each feed ingredient—wheat, coarse grains, soybean meal, forage, and pasture. Let subscript i denote the feed ingredient ($i = w, g, sm, fo$) and j ($j = f, g, b, w, p$) denote the stage. Recall that $as_{i,j,t}$ is the per animal feed use at time t . This follows equation (13), and totally differentiating it gives:

$$(97) \text{dln}(DSc_{i,j,t}) = \text{dln}(as_{i,j,t}) + \text{dln}(Sc_{j,t}),$$

$i = w, g, sm, fo$, and $j = f, g, b, w, p$.

Because the per unit feed demands are responsive to changes in relative feed prices, the percentage changes in the derived demands for feeds use Morishima elasticities of substitution, $\sigma_{i,g,c}$, where i gives feedstuff, wheat, soybean meal, or forage/pasture, g indicates coarse grains, and c indicates beef cattle:

$$(98) \text{dln}(a_{i,c,t}) - \text{dln}(a_{g,c,t}) = -\sigma_{i,g,c} * (\text{dln}(P_{i,t}) - \text{dln}(P_{g,t})),$$

$i = w, sm, fo$,

$$(99) \sum_i \theta_{i,c} * d\ln(a_{i,c,t}) = 0,$$

$i = w, g, sm, fo.$

The cow inventory at time t also has unique feed demands, as in equation (15). Thus, the change in feed demand for feedstuff i by cows at time t is:

$$(100) d\ln(Dc_{i,t}) = d\ln(ac_{i,t}) + d\ln(Ic_t),$$

$i = w, g, sm, fo.$

Changes in relative prices alter the per cow mix of feedstuff according to the Morishima elasticities of substitution:

$$(101) d\ln(ac_{i,t}) - d\ln(ac_{g,t}) = -\sigma_{i,g,c} * (d\ln(P_{i,t}) - d\ln(P_{g,t})),$$

$i = w, sm, fo,$

$$(102) \sum_i \theta_{i,c} * d\ln(ac_{i,t}) = 0,$$

$i = w, g, sm, fo.$

Feed use by replacement heifers at time t varies by per unit use, $ah_{i,k}$, and hence by feedstuff prices, changes in replacement heifer numbers:

$$(103) d\ln(Dh_{i,t}) = d\ln(ah_{i,t}) + d\ln(Hc_t),$$

$i = w, g, sm, fo,$

$$(104) d\ln(ah_{i,t}) - d\ln(ah_{g,t}) = -\sigma_{i,g,c} * (d\ln(P_{i,t}) - d\ln(P_{g,t})),$$

$i = w, sm, fo,$

$$(105) \sum_i \theta_{i,c} * d\ln(ah_{i,t}) = 0,$$

$i = w, g, sm, fo.$

Feed use by bulls for ingredient i at time t is similar:

$$(106) d\ln(Db_{i,t}) = d\ln(ab_{i,t}) + d\ln(Bc_t),$$

$i = w, g, sm, fo,$

$$(107) d\ln(ab_{i,t}) - d\ln(ab_{g,t}) = -\sigma_{i,g,c} * (d\ln(P_{i,t}) - d\ln(P_{g,t})),$$

$i = w, sm, fo,$

$$(108) \sum_i \theta_{i,c} * d\ln(ab_{i,t}) = 0,$$

$i = w, g, sm, fo.$

Slaughter hogs go through two cycles per quarter. At time t , there is one group of hogs ready for slaughter, $j = f$, and another just beginning the process, $j = p$. The change in demand for feedstuff i by hogs ready for slaughter depends on the change in per hog use, the disease effects, and any changes in the supply of market hogs:

$$(109) d\ln(Dhg_{i,j,t}) = d\ln(ahg_{i,j,t}) + d\ln(Shg_{j,t}),$$

$i = w, g, sm,$

Changes in per unit feed use are linked to change in feed prices:

$$(110) d\ln(ahg_{i,j,t}) - d\ln(adhg_{i,g,t}) = -\sigma_{i,g,s} * (d\ln(P_{i,t}) - d\ln(P_{g,t})),$$

$i = w, sm,$

$$(111) \sum_i \theta_{i,s} * d\ln(\text{ahg}_{i,j,t}) = 0,$$

$i = w, g, \text{sm}.$

The changes in the demands for feed ingredients for breeding hogs take the form:

$$(112) d\ln(D\text{sw}_{i,t}) = d\ln(\text{asw}_{i,t}) + d\ln(I\text{sw}_t),$$

$i = w, g, \text{sm},$

$$(113) d\ln(\text{asw}_{i,t}) = -\sigma_{i,g,s} * (d\ln(P_{i,t}) - d\ln(P_{g,t})),$$

$i = w, \text{sm},$

$$(114) \sum_i \theta_{i,s} * d\ln(\text{asw}_{i,t}) = 0,$$

$i = w, g, \text{sm}.$

Changes in feed demand for ingredient i by dairy cows depend on a change in the per unit demand, driven by feed price changes and changes in inventory:

$$(115) d\ln(Dd_{i,t}) = d\ln(ad_{i,t}) + d\ln(Id_t),$$

$i = w, g, \text{sm}, \text{fo},$

$$(116) d\ln(ad_{i,j,t}) - d\ln(ad_{g,t}) = -\sigma_{i,g,d} * (d\ln(P_{i,t}) - d\ln(P_{g,t})),$$

$i = w, \text{sm}, \text{fo},$

$$(117) \sum_i \theta_{i,d} * d\ln(ad_{i,t}) = 0,$$

$i = w, g, \text{sm}, \text{fo}.$

Replacement dairy heifers also generate feed demands:

$$(118) d\ln(Drd_{i,t}) = d\ln(\text{ard}_{i,t}) + d\ln(Rd_t),$$

$i = w, g, \text{sm}, \text{fo},$

$$(119) d\ln(\text{ard}_{i,t}) - d\ln(\text{ard}_{g,t}) = -\sigma_{i,g,d} * (d\ln(P_{i,t}) - d\ln(P_{g,t})),$$

$i = w, \text{sm}, \text{fo},$

$$(120) \sum_i \theta_{i,d} * d\ln(\text{ard}_{i,t}) = 0,$$

$i = w, g, \text{sm}, \text{fo}.$

Given their shorter production cycle, changes in demands for feed ingredients for broilers and layers are straightforward:

$$(121) d\ln(D\text{pm}_{i,t}) = d\ln(\text{apm}_{i,t}) + d\ln(\text{qpm}_t),$$

$i = w, g, \text{sm},$

$$(122) d\ln(\text{apm}_{i,t}) - d\ln(\text{apm}_{g,t}) = -\sigma_{i,g,\text{pm}} * (d\ln(P_{i,t}) - d\ln(P_{g,t})),$$

$i = w, \text{sm},$

$$(123) \sum_i \theta_{i,\text{pm}} * d\ln(\text{apm}_{i,t}) = 0,$$

$i = w, g, \text{sm}.$

$$(124) d\ln(Dl_{i,t}) = d\ln(al_{i,t}) + d\ln(qe_t),$$

$i = w, g, \text{sm},$

$$(125) \ln(al_{i,t}) - \ln(al_{g,t}) = -\sigma_{i,g,l} * (\ln(P_{i,t}) - \ln(P_{g,t})),$$

$i = w, sm,$

$$(126) \sum_i \theta_{i,l} * \ln(al_{i,t}) = 0,$$

$i = w, g, sm.$

Feed demand by lambs and sheep arise from market lambs, ewe inventory, and replacement ewes. For market lambs, there are four stages of production, denoted by j , $j = f, g, b, p$. The change in the demand for feed ingredient i by market lambs at stage j is:

$$(127) \ln(Dlb_{i,j,t}) = \ln(alb_{i,j,t}) + \ln(Slb_{j,t}),$$

$i = w, g, sm, fo,$

$$(128) \ln(alb_{i,j,t}) - \ln(alb_{g,j,t}) = -\sigma_{i,g,lb} * (\ln(P_{i,t}) - \ln(P_{g,t})),$$

$i = w, sm, fo,$

$$(129) \sum_i \theta_{i,lb} * \ln(alb_{i,j,t}) = 0,$$

$i = w, g, sm, fo, \text{ and } j = f, g, sm, fo.$

Ewe inventory, replacement ewes, and rams have a similar structure. Differentiating equation (49), feed demand by ewes, gives:

$$(130) \ln(Dew_{i,t}) = \ln(aew_{i,t}) + \ln(Iew_t),$$

$i = w, g, sm, fo,$

$$(131) \ln(aew_{i,t}) - \ln(aew_{g,t}) = -\sigma_{i,g,lb} * (\ln(P_{i,t}) - \ln(P_{g,t})),$$

$i = w, sm, fo,$

$$(132) \sum_i \theta_{i,lb} * \ln(aew_{i,t}) = 0,$$

$i = w, g, sm, fo.$

Feed demands by replacements are expressed by equation (50). Differentiation gives:

$$(133) \ln(Der_{i,t}) = \ln(aer_{i,t}) + \ln(Ier_t),$$

$i = w, g, sm, fo,$

$$(134) \ln(aer_{i,t}) - \ln(aer_{g,t}) = -\sigma_{i,g,lb} * (\ln(P_{i,t}) - \ln(P_{g,t})),$$

$i = w, sm, fo,$

$$(135) \sum_i \theta_{i,lb} * \ln(aer_{i,t}) = 0,$$

$i = w, g, sm, fo.$

Differentiation of the feed demands by rams has the same pattern:

$$(136) \ln(Drm_{i,t}) = \ln(arm_{i,t}) + \ln(Irm_t),$$

$i = w, g, sm, fo,$

$$(137) \ln(arm_{i,t}) - \ln(arm_{g,t}) = -\sigma_{i,g,lb} * (\ln(P_{i,t}) - \ln(P_{g,t})),$$

$i = w, sm, fo,$

$$(138) \sum_i \theta_{i,lb} * \ln(arm_{i,t}) = 0,$$

$i = w, g, sm, fo.$

Crop Production

The next component of the model consists of logarithmic differentiation of the crop production structure. Differentiating equation (52) determines the change in expected returns for crop i :

$$(139) \theta_{L_i} * d\ln(W_i) + \theta_{K_i} * d\ln(R^*_i) + \theta_{T_i} * d\ln(\tau_i) = d\ln(P^*_i),$$

$i = w, g, r, sb, fo.$

Differentiating the sector-specific, factor-market-clearing conditions for crop i , equation (53) determines the changes in production of crop i :

$$(140) d\ln(a_{K_{i,t}}) + d\ln(q_{i,t}) = d\ln(K_i),$$

$i = w, g, r, sb, fo,$

$$(141) d\ln(a_{L_{i,t}}) - d\ln(a_{K_{i,t}}) = -\sigma_{L,K_i}(d\ln(W_i) - d\ln(R^*_i)),$$

$i = w, g, r, sb, fo,$

$$(142) d\ln(a_{T_{i,t}}) - d\ln(a_{K_{i,t}}) = -\sigma_{T,K_i} * (d\ln(\tau) - d\ln(R^*_i)),$$

$i = w, g, r, sb, fo,$

$$(143) \theta_{L_i} * d\ln(a_{L_{i,t}}) + \theta_{K_i} * d\ln(a_{K_{i,t}}) + \theta_{T_i} * d\ln(a_{T_{i,t}}) = 0,$$

$i = w, g, r, sb, fo.$

Factoring in the change in land allocation, which determines the land rent, differentiating equation (54), and letting λ_{T_i} be the share of land used by crop i , yields:

$$(144) \sum_i \lambda_{T_i} * (d\ln(q_{i,t}) + d\ln(a_{T_{i,t}})) = d\ln(T_t),$$

$i = w, g, r, sb, fo.$

The logarithmic differential of actual returns to crop i , $i = w, g, r, sb, fo$, is:

$$(145) d\ln(P_{i,t}) = (Pm_{i,t}/P_{i,t}) * d\ln(Pm_{i,t}) + (0.85 * y_i * A_i / (q_i * P_{i,t})) * (d\ln(DP_i) + d\ln(y_i) + d\ln(A_i) - d\ln(q_i) + Z_{1,i,t} * d\ln(Z_{1,i,t}) + Z_{2,i,t} * d\ln(Z_{2,i,t})),$$

where

$$Z_{1,i,t} * d\ln(Z_{1,i,t}) = \begin{cases} [0.85 * y_i * A_i / q_i] * [d\ln(y_i) + d\ln(A_i) - d\ln(q_i) + (TP_{i,t} / (TP_{i,t} - Pm_{i,t})) * d\ln(TP_{i,t}) - (Pm_{i,t} / (TP_{i,t} - Pm_{i,t})) * d\ln(Pm_{i,t})], & \text{when } Pm_{i,t} < TP_{i,t}, \text{ or} \\ 0, & \text{when } Pm_{i,t} \geq TP_{i,t}, \end{cases}$$

$$Z_{2,i,t} * d\ln(Z_{2,i,t}) = \begin{cases} [LR_{i,t} / (LR_{i,t} - Pm_{i,t})] * d\ln(LR_{i,t}) - [Pm_{i,t} / (LR_{i,t} - Pm_{i,t})] * d\ln(Pm_{i,t}), & \text{when } Pm_{i,t} < LR_{i,t}, \\ 0, & \text{when } Pm_{i,t} \geq LR_{i,t}. \end{cases}$$

The changes in the actual returns to sector-specific factors for crops are found by differentiating equation (56) and using the actual return as determined above:

$$(146) \theta_{L_i} * \ln(W_i) + \theta_{K_i} * \ln(R_{i,t}) + \theta_{T_i} * \ln(\tau) = \ln(P_{i,t}).$$

Total crop supply in a quarter includes carryin stocks from the previous quarter. Ending stocks are given by equation (57). Totally differentiating that equation gives:

$$(147) \ln(I_{i,t}) = \varepsilon_p * (\ln(P_{i,t+1}^*) - \ln(P_{m_{i,t}})),$$

$i = w, g, r, sb, fo,$

where ε_p is the elasticity of expected returns to price speculation.

Soybean Complex

Soybean crushing depends on the margin, which depends on the prices of soybean meal, soybean oil, and soybeans. Assuming meal and oil yields are constant, differentiating the crush demand and the margin identity gives:

$$(148) \ln(D_{sb,t}) = \varepsilon_m * \ln(SPD_t),$$

$$(149) \ln(SPD_t) = (P_{sm_t}/SPD_t) * \ln(P_{sm_t}) + (P_{so_t}/SPD_t) * \ln(P_{so_t}) - (P_{sb_t}/SPD_t) * \ln(P_{sb_t}).$$

The changes in supplies of meal and oil are obtained from changes in the crush:

$$(150) \ln(q_{sm_t}) = \ln(D_{sb_t}),$$

$$(151) \ln(q_{so_t}) = \ln(D_{sb_t}).$$

Closure

Closure requires logarithmically differentiating the remaining equations. The excess demand and excess supply equations include trade policy interventions. Since several commodities do not have trade interventions, the logarithmic change is not defined. Thus, trade policy interventions are treated as specific (per unit) policies, and the differential form differs from the other equations:

$$(152) \ln(X_t) = \varepsilon_x * [P_{m_t} - tx]^{-1} * (\ln(P_{m_t}) - dtx) + \ln(\lambda_x),$$

$$(153) \ln(M_t) = \eta_m * [P_{m_t} - tm]^{-1} * (\ln(P_{m_t}) - dtm) + \ln(\lambda_m),$$

where ε_x and η_m are the matrices of excess demand and excess supply elasticities facing the United States.

Each commodity has a market-clearing condition, as given by equation (65). Totally differentiating that identity in column vectors gives:

$$(154) M_t * \ln(M_t) = DF_t * \ln(DF_t) + DD_t * \ln(DD_t) + X_t * \ln(X_t) + I_t * \ln(I_t) - Q_t * \ln(Q_t) - I_{t-1} * \ln(I_{t-1}),$$

where **DD** is a vector of derived demands for animals and for feed ingredients.

Margin-markup equations (66) and (67) become:

$$(155) \ln(P_{M_{i,t}}) = (P_{m_{i,t}}/P_{M_{i,t}}) * \ln(P_{m_{i,t}}) + (SPDW_{i,t}/P_{M_{i,t}}) * \ln(SPDW_{i,t}),$$

$$(156) \ln(P_{R_{i,t}}) = (P_{M_{i,t}}/P_{R_{i,t}}) * \ln(P_{M_{i,t}}) + (SPDR_{i,t}/P_{R_{i,t}}) * \ln(SPDR_{i,t}).$$