Condensate and atomic kinetic energy of liquid ^{3}He-^{4}He mixtures

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Abstract

Liquid ^{3}He -^{4}He mixtures are the arch example of mixed Bose and Fermi liquids in nature. Taking advantage of the high energy neutrons available at the ISIS spallation source and the high resolution of the MARI instrument, we have measured the Bose-Einstein condensate fraction and the momentum distribution of ^{3}He and ^{4}He atoms in ^{3}He -^{4}He mixtures to new accuracy. We find that the condensate fraction increases only slightly above the pure ^{4}He value with increasing ^{3}He concentration x. This agrees with several calculations but is in contrast with the only single previous measurement of n_0 in ^{3}He -^{4}He mixtures, thus resolving a long-standing disagreement between theory and experiment. We also find that the observed ^{3}He $n(k)$ is not well fitted with a a Fermi step function alone. A high momentum tail in $n(k)$ is needed to get a good fit - a tail that is consistent with calculated tails in $n(k)$. The ^{3}He atomic kinetic energy, $\langle K_3 \rangle$, depends almost entirely on the tail. The tail is difficult to determine accurately in neutron scattering experiments. Thus $\langle K_3 \rangle$ is not a well determined quantity which explains why previous observed and calculated $\langle K_3 \rangle$ disagreed substantially. Measurements of the dynamics of single atoms and molecules in materials are ideally done at spallation neutron sources and the SNS will greatly expand opportunities in this field.

Introduction

To obtain the atomic kinetic energies and the ^{4}He condensate fraction, the single particle atomic momentum distribution, $n(k)$, must be measured. In Deep Inelastic Neutron Scattering measurements (DINS), the energy transfer E is Doppler broadened by the momentum distribution of the atoms in the liquid. If there is a condensate present, the fraction of atoms with zero momentum, n_0, contributes an unbroadened peak to $S(Q,\omega)$ at $\omega = \omega_R$, where $\omega_R = Q^2/2M$ is the free, stationary atom recoil frequency and M is the atomic mass. If the interaction of the recoiling atoms with their neighbours is neglected, n_0 can be extracted directly from $S(Q,\omega)$. For sufficiently large momentum transfers, $Q > 20$ Å^{-1}, the scattering involves only single-particle correlations, so that the scattering from individual ^{3}He and ^{4}He atoms can be considered to be independent. This is known as the impulse approximation (IA). In this IA, $S_{IA}(\mathbf{Q},\omega)$ depends only on a single 'y scaling'variable $y = (\omega - \omega_R)/v_R$ and is conveniently expressed as:

$$J_{IA}(y) = v_R S_{IA}(Q,\omega) = \int d\mathbf{k} n(\mathbf{k})\delta(y - k_Q) \qquad (1)$$

where $v_R = \hbar Q/M$ and $k_Q = \mathbf{k}\cdot\frac{\mathbf{Q}}{|Q|}$. $J_{IA}(y)$ is denoted the longitudinal momentum distribution. It follows that the recoil scattering from ^{3}He and ^{4}He will be centred at ω_R for each mass, so the signal from the two isotopes will be clearly separated (see Fig. 1). Fitting Eq. 1 to the data using an appropriate model of $n(\mathbf{k})$ for each isotope with adjustable parameters yields the condensate fraction and the respective kinetic energies.

Goals

The goal of this work is to determine the Bose-Einstein condensate fraction and the ^{4}He atomic kinetic energy as a function of ^{3}He concentration (x) by means of DINS. Another interesting aspect investigated in this work is the ^{3}He momentum distribution $n_3(k)$.
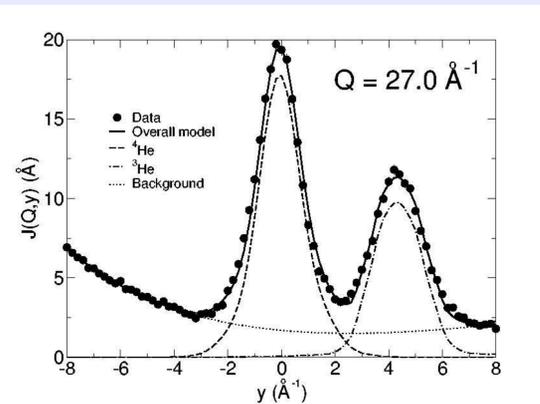
Data Analysis

FIGURE 1: MARI Data showing $J(Q,y)$ for the mixture ^{3}He-^{4}He at 10% ^{3}He concentration. The data is presented with the ^{4}He peak centered at $y_4 = 0$.

The total scattering function is a combination of the scattering intensities from both ^{3}He and ^{4}He atoms along their respective recoil lines. For a given Q, the scattering function is:

$$J_{Total_4}(Q,y_4) = \frac{Q}{m_4}S_{Total}(Q,\omega)$$
$$= \frac{Q}{m_4}S_3(Q,\omega) + \frac{Q}{m_4}S_4(Q,\omega)$$
$$= \frac{Q}{m_4}\frac{m_3}{Q}J_3(Q,y_4) + \frac{Q}{m_4}\frac{m_4}{Q}J_4(Q,y_4)$$
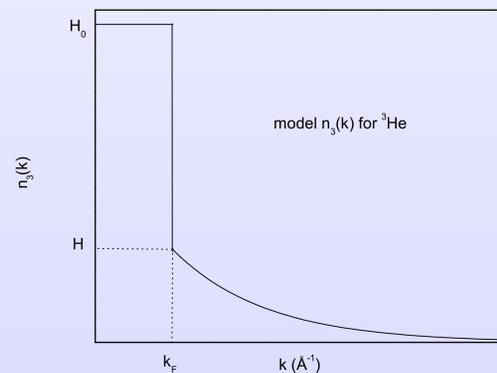$$= \frac{m_3}{m_4}J_3(Q,y_4) + J_4(Q,y_4)$$

FIGURE 2: Our proposed model of the Fermi momentum distribution, $n_3(k)$, for ^{3}He that accounts for the high energy tails is depicted above. The model has 3 adjustable paramters (a_F, H_0, H) defined below.

The ^{3}He momentum distribution $n_3(k)$ is modelled as follows:

$$\frac{H_0}{v_F} \qquad k < k_F$$
$$\frac{H}{v_F}e^{-a_F(k-k_F)} \qquad k > k_F$$

where k_F is the Fermi momentum and v_F the volume of the Fermi sphere. The step (below k_F) is represented by the parameter H_0 and the tail (above k_F) by the parameters H and a_F. The overall method consists of simultaneously fitting the two neutron scattering peaks. The ^{4}He peak is fitted using the convolution approach (CA) [1] and the ^{3}He peak using the derived $J_{IA_3}(y)$ from the model $n_3(k)$ above.

Results

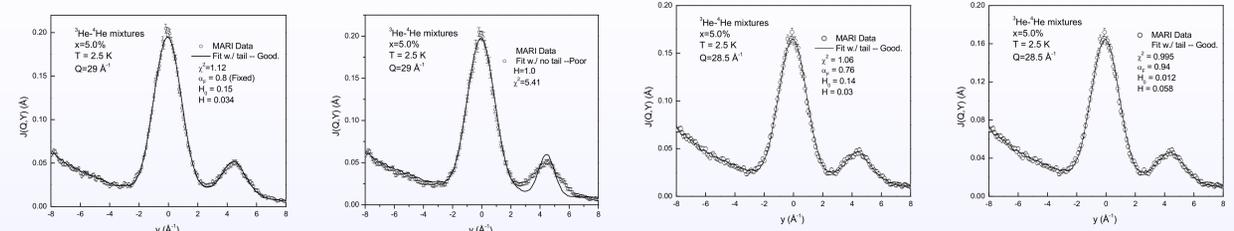FIGURE 3: Typical fit to data (x=5% at T=2.5 K). The figure on the left shows that a good fit to data is obtained if a calculated tail ($\alpha_F = a_F.k_F$) is assumed. The figure on the right shows that when a Fermi step function alone is assumed for $n(k)$, the fit to data is poor.

FIGURE 4: The same data set is shown in both graphs. The figures show that good fits to the same data can be obtained with different sets of parameters. This is found to be specifically true for $\alpha_F = a_F.k_F$. The high energy tails are difficult to detect from DINS data. In our reported analysis, the calculated α_F is assumed, and the observed $\langle K_3 \rangle$ agree to within 20% of the calculated $\langle K_3 \rangle$.
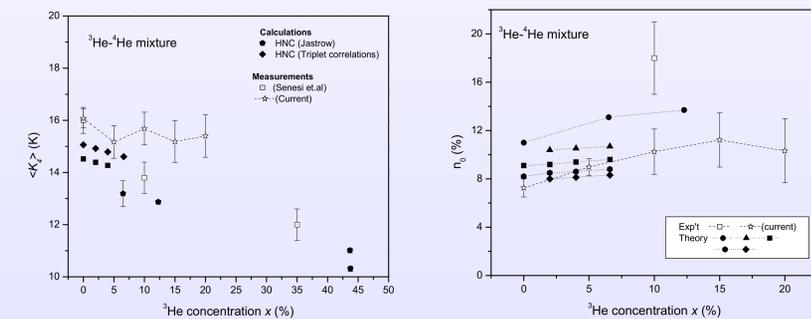

FIGURE 5: Left figure: ^{4}He kinetic energy $\langle K_4 \rangle$ as a function of x obtained from theory and experiments. The open stars represent the current data at T=2.5 K. Other experimental values of $\langle K_4 \rangle$ (open squares) are from reference [2]. The figure shows also VMC calculations [3] for ^{4}He (closed pentagons), HNC calculations using a Jastrow function [4] (closed diamonds), HNC including triplet correlations [4] (closed squares). Right figure: The condensate fraction, n_0, as a function of the ^{3}He concentration, from various experiments (open symbols) and calculations (close symbols). The present measurement is represented by the open stars; the open square is from [5]. The theoretical calculations are represented by the closed symbols as follows; circles: VMC [3], triangles: HNC(Jastrow [4]), squares: HNC(+triplet correlations) [4], hexagons: HNC(Lennard-Jones) [6], diamonds: HNC(Aziz potential) [4].

Conclusions

In summary, we have performed high resolution measurements of the momentum distribution of ^{3}He-^{4}He mixtures in both the superfluid and normal phases for ^{3}He concentrations x between 0 and 20% in steps of 5%. We find that the Bose-Einstein condensate fraction n_0 in the ^{3}He-^{4}He mixtures **increases only marginally with increasing** x, from 7.25±0.75% for the bulk data $(x = 0\%)$ to 11.2±1.85% for $x = 20\%$. In our ^{3}He momentum distribution model, $\langle K_3 \rangle$ **depends entirely (99%) on the high momentum tails**. Our result is to be compared with the calculations of Boronat and collaborators [4] which found that 92% of $\langle K_3 \rangle$ are due to k in $n(k)$ above k_F. We note that these tails are difficult to extract from DINS data, thus $\langle K_3 \rangle$ **alone is not a good description of** $n(k)$ and it should be abandoned as a comparison parameter of theory and experiment for isotopic helium mixtures. Instead, the momentum distributions $n(k)$ for ^{3}He obtained from experiment and the calculated $n(k)$ should be compared directly. The ^{4}He kinetic energy $\langle K_4 \rangle$ **is found to be largely independent of** x.

References

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