Maximizing the Performance of Wind Turbines with Nonlinear Aeroservoelastic Power Flow Control

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***Abstract*— Maximum energy and power can be generated by a wind turbine by operating at the edge of dynamic stall. This paper applies a novel nonlinear power flow control technique to the nonlinear stall flutter problem that occurs when the wind turbine passes into dynamic stall. A nonlinear aerodynamic and structural model is developed that is representative of the first torsional mode of a nominal 5 MW rated power wind turbine blade. This model is analyzed using the power flow control technique to determine the limit cycle behavior of the nonlinear stall flutter condition of the first torsional mode. This model is further analyzed to determine the effectiveness of feedback control to generate nonlinear flutter suppression to ensure stability while maximizing the performance of the wind turbine. In addition, the limit cycle is shown to be a stability boundary for the nonlinear system.**

I. Introduction

Researchers have been attempting to maximize the energy and power output of wind turbines for many years. Maximum energy and power can be generated by a wind turbine by operating at the edge of dynamic stall. As a result, limit cycle behavior of a wind turbine induced by dynamic stall must be investigated in order to make progress toward the goal of maximizing the performance of wind turbines.

Many different fields of engineering have been performing research in the analysis of limit cycle behavior. Specific applications that relate to stall flutter of wind turbines include helicopter blades in forward flight, gas turbine compressor blades, and airplane wing flutter, all of which can be categorized as Limit-Cycle Oscillations (LCO). The analysis and control of LCO's continues to be a challenge and an ongoing area of research. For example, Gopinath, Beran, and Jameson [1] explore various methods in the computation of time-periodic solutions for autonomous systems. The goal was to determine the range of applicability of models of varying fidelity to the numerical prediction of LCOs and related evaluations. A simple aeroelastic model of an airfoil with nonlinear structural coupling was used to show the efficacy of the procedure. Several researchers are investigating cyclic methods to compute LCO's for potentially large, nonlinear, systems of equations. One such method by Hall, Thomas, and Clark [2] introduces a harmonic balance technique for modeling unsteady nonlinear flows in turbomachinery. Additional wing flutter LCO identification and control investigations by others are further discussed in the following references [3], [4], [5], [6], [7].

The goal of this paper is to utilize the results of refs. [8], [9], [10] to analyze the stall flutter of wind turbines and design nonlinear flutter suppression controllers to maximize the performance of wind turbines. To begin the analysis of the stall flutter of wind turbines, a line integral of the power flow for Hamiltonian systems is utilized to determine the onset of a limit cycle. In particular, the work per cycle [11], [12], [13], [14] defined by the line integral of the power flow

$$W_{cyclic} = \oint_{\tau} F \cdot \dot{x} dt = 0$$

is chosen since the time derivative of the Hamiltonian is the generalized power flow for natural systems [8].

A specific application of these concepts is the analysis of classical flutter. Classical flutter is a linear limit cycle that is a result of the coalescence of a bending mode and a torsional mode to produce a self-excited oscillation. Abramson [15] describes the existence of a flutter mode as

> *We see now that negative work is done on the wing by part of the torsional motion, by the flexural motion, and by the elastic restoring forces; positive work is done on the wing by part of the torsional motion. The motion will maintain itself (the condition for flutter) when the net positive work just balances the dissipation of energy due to all the damping forces. The magnitude of the positive work done by the additional lift due to the twist is directly dependent upon the phase relationship between the coupled torsional and flexural motions* ...

This concept of the existence of limit cycles based on power flows leads to a balance between positive work and energy dissipation due to damping in the stall flutter of wind turbines.

This paper applies a novel nonlinear power flow control technique to the nonlinear stall flutter problem that occurs when the wind turbine passes into dynamic stall. A nonlinear aerodynamic and structural model is developed that is representative of the first torsional mode of a nominal 5 MW rated power wind turbine [16] blade. This model is analyzed using the power flow control technique to determine the limit cycle behavior of the nonlinear stall flutter condition of the first torsional mode. This model is further analyzed to determine the effectiveness of feedback control to generate nonlinear

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flutter suppression to ensure stability while maximizing the performance of the wind turbine. In addition, the limit cycle is shown to be a stability boundary for the nonlinear system.

This paper is divided into three sections. Section 2 introduces a nonlinear stall flutter model, discusses the effects of additional nonlinearities, and presents the design and analysis of a PID controller based on power flow control concepts. The discussion begins with a linear flutter analysis of the first torsional mode of a 5MW wind turbine blade and progresses to a nonlinear stall flutter analysis and controller design. Nonlinear power flow control design allows the partitioning of the power flows of the nonlinear dynamical system into three categories: storage, generation, and dissipation. By identifying and balancing the power flows over a cycle, the system stability and performance characteristics can be determined. Further details using this technique are outlined in references [9], [10]. Finally, Section 3 summarizes the results with concluding remarks.

II. Nonlinear Stall Flutter Model

This section develops a single degree-of-freedom (DOF) model which is representative of the first torsional mode (on-the-order of 5.58 Hz) of a large wind turbine (5 MW) blade. Figure 1 depicts the simplified nonlinear model, where K is the torsional stiffness, K_{NL} the nonlinear torsional stiffness, C the torsional damping, C_{NL} the nonlinear torsional damping, I the wing (or cross-section of the blade) section torsional moment of inertia, and M_α, $M_{\dot\alpha}$ the aerodynamic moments. The equation of motion is derived from Lagrange's

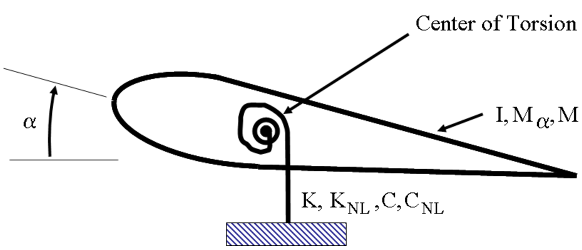

Fig. 1. Nonlinear flutter model

equation

$$\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot\alpha}\right)-\frac{\partial\mathcal{L}}{\partial\alpha}=Q_\alpha \tag{1}$$

where

$$\begin{aligned}
\mathcal{L} &= \mathcal{T}-\mathcal{V}\\
\mathcal{T} &= \tfrac{1}{2}I\dot\alpha^2\\
\mathcal{V} &= \tfrac{1}{2}K\alpha^2+\tfrac{1}{4}K_{NL}\alpha^4\\
Q_\alpha &= Q_{damp}+Q_{aero}+Q_{control}\\
Q_{damp} &= -C\dot\alpha-C_{NL}\text{sign}(\dot\alpha)\\
Q_{aero} &= M_\alpha(\alpha)+M_{\dot\alpha}(\dot\alpha,\alpha)\quad\text{and}\\
Q_{control} &= u=-K_P\alpha-K_I\textstyle\int_0^t\alpha d\tau-K_D\dot\alpha
\end{aligned}$$

where $\mathcal{L}$ is the Lagrangian, $\mathcal{T}$ the kinetic energy, $\mathcal{V}$ the potential energy, and Q_α the generalized forces. The controller input u consists of Proportional-Integral-Derivative (PID) control action where K_P is the proportional gain, K_I the integral gain, and K_D the derivative gain. The aerodynamic moments M_α and $M_{\dot\alpha}$ are generated based on the following nonlinear hysteresis logic (also shown visually in Fig. 2)

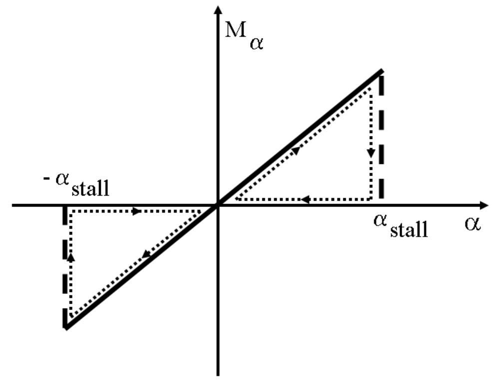

Fig. 2. Nonlinear hysteresis aerodynamic moment characteristic

$$M_\alpha(\alpha)=\begin{cases}\hat{C}_{M_\alpha}\alpha & \text{for}\quad |\alpha|<\alpha_{stall}\\ 0 & \text{for}\quad |\alpha|>\alpha_{stall}\\ 0 & \text{for}\quad \text{the return hysteresis cycle}\end{cases}$$

and

$$M_{\dot\alpha}(\dot\alpha,\alpha)=\begin{cases}\hat{C}_{M_{\dot\alpha}}\dot\alpha & \text{for}\quad |\alpha|<\alpha_{stall}\\ 0 & \text{for}\quad |\alpha|>\alpha_{stall}.\end{cases}$$

Applying (1) yields the equation of motion as

$$I\ddot\alpha+K\alpha+K_{NL}\alpha^3=$$
$$-C\dot\alpha-C_{NL}\text{sign}(\dot\alpha)+u+M_\alpha(\alpha)+M_{\dot\alpha}(\dot\alpha,\alpha)$$

which has many interesting properties given next. The next sections will discuss the following regions: i) linear, ii) nonlinear stall flutter, iii) further nonlinearities (such as cubic stiffness and Coulomb friction), and iv) conventional PID control design.

A. Linear Region

For $|\alpha|<\alpha_{stall}$, the model is linear with $K_{NL}=C_{NL}=0$ or

$$I\ddot\alpha+\left[C-\hat{C}_{M_{\dot\alpha}}\right]\dot\alpha+\left[K-\hat{C}_{M_\alpha}\right]\alpha=u$$

which produces typical linear aeroelastic behavior. Divergence occurs when

$$\hat{C}_{M_\alpha}\geq K\quad\text{for}\quad u=0$$

where

$$\hat{C}_{M_\alpha}=K_{M_\alpha}qA=K_{M_\alpha}A\left(\frac{1}{2}\rho V^2\right)$$

and q is dynamic pressure, ρ is density, A is the cross-sectional area, d is the cord length, and V is the free stream velocity. The angle of attack relative to the free stream velocity vector is α. Torsional flutter occurs when

$$\hat{C}_{M_{\dot\alpha}}\geq C\quad\text{for}\quad K-\hat{C}_{M_\alpha}>0\quad\text{and}\quad u=0$$

where

$$\hat{C}_{M_{\dot\alpha}}=K_{M_{\dot\alpha}}qAd.$$

Linear torsional divergence is typically determined and stiffness and/or mass balancing are added as needed. Linear torsional flutter rarely happens and is typically not of concern.

B. *Nonlinear Stall Flutter with Linear Dynamics*

When the motion reaches $|\alpha| > \alpha_{stall}$, the model becomes nonlinear where

$$I\ddot{\alpha} + C\dot{\alpha} + K\alpha = M_\alpha(\alpha) + M_{\dot{\alpha}}(\dot{\alpha}, \alpha)$$

with

$$\begin{aligned} \mathcal{H} &= \tfrac{1}{2}I\dot{\alpha}^2 + \tfrac{1}{2}K\alpha^2 \\ \dot{\mathcal{H}} &= \left[I\ddot{\alpha} + K\alpha\right]\dot{\alpha} = \left[-C\dot{\alpha} + M_\alpha(\alpha) + M_{\dot{\alpha}}(\dot{\alpha}, \alpha)\right]\dot{\alpha} \end{aligned}$$

which produces a limit cycle when

$$\oint_\tau \left[M_\alpha(\alpha) + M_{\dot{\alpha}}(\dot{\alpha}, \alpha)\right]\dot{\alpha}dt = \oint_\tau \left[C\dot{\alpha}\right]\dot{\alpha}dt.$$

C. *Nonlinear Stall Flutter with Nonlinear Dynamics*

The nonlinear stall flutter can be further modified by adding the nonlinear stiffness and damping

$$I\ddot{\alpha} + C\dot{\alpha} + C_{NL}\mathrm{sign}(\dot{\alpha}) + K\alpha + K_{NL}\alpha^3 = M_\alpha(\alpha) + M_{\dot{\alpha}}(\dot{\alpha}, \alpha)$$

with

$$\begin{aligned} \mathcal{H} &= \tfrac{1}{2}I\dot{\alpha}^2 + \tfrac{1}{2}K\alpha^2 + \tfrac{1}{4}K_{NL}\alpha^4 \\ \dot{\mathcal{H}} &= \left[I\ddot{\alpha} + K\alpha + K_{NL}\alpha^3\right]\dot{\alpha} \\ &= \left[-C\dot{\alpha} - C_{NL}\mathrm{sign}(\dot{\alpha}) + M_\alpha(\alpha) + M_{\dot{\alpha}}(\dot{\alpha}, \alpha)\right]\dot{\alpha} \end{aligned}$$

which produces a limit cycle when

$$\oint_\tau \left[M_\alpha(\alpha) + M_{\dot{\alpha}}(\dot{\alpha}, \alpha)\right]\dot{\alpha}dt = \oint_\tau \left[C\dot{\alpha} + C_{NL}\mathrm{sign}(\dot{\alpha})\right]\dot{\alpha}dt.$$

D. *Controller Design*

The nonlinear system can be modified by feedback control to meet several performance requirements. A PID controller is implemented to show the effects of feedback control. The model becomes

$$I\ddot{\alpha} + \left[K + K_P\right]\alpha + K_{NL}\alpha^3 =$$
$$-\left[C + K_D\right]\dot{\alpha} - C_{NL}\mathrm{sign}(\dot{\alpha}) + M_\alpha(\alpha) + M_{\dot{\alpha}}(\dot{\alpha}, \alpha) - K_I\int_0^t \alpha d\tau$$

with

$$\begin{aligned} \mathcal{H} &= \tfrac{1}{2}I\dot{\alpha}^2 + \tfrac{1}{2}\left[K + K_P\right]\alpha^2 + \tfrac{1}{4}\alpha^4 \\ \dot{\mathcal{H}} &= \left[I\ddot{\alpha} + (K + K_P)\alpha + K_{NL}\alpha^3\right]\dot{\alpha} \\ &= \Big[-(C + K_D)\dot{\alpha} - C_{NL}\mathrm{sign}(\dot{\alpha}) + M_\alpha(\alpha) \\ &\quad + M_{\dot{\alpha}}(\dot{\alpha}, \alpha) - K_I\textstyle\int_0^t \alpha d\tau\Big]\dot{\alpha} \end{aligned}$$

which produces a limit cycle when

$$\oint_\tau \left[M_\alpha(\alpha) + M_{\dot{\alpha}}(\dot{\alpha}, \alpha) - K_I\int_0^t \alpha d\tau_1\right]\dot{\alpha}dt =$$
$$\oint_\tau \left[(C + K_D)\dot{\alpha} + C_{NL}\mathrm{sign}(\dot{\alpha})\right]\dot{\alpha}dt.$$

Numerical simulations were performed for a representative 5 MW wind turbine blade first torsion mode. Three cases were considered. Case 1 represents a passivity PID control design, Case 2 represents a limit cycle oscillation for the system, and Case 3 is without control, where the aerodynamic loads drive the system unstable or generates more energy and power into the system then can be dissipated. These three cases are shown in a Hamiltonian 3D surface with the resulting paths (see Fig. 3). The corresponding phase plane plots are shown in Fig. 4. Case 1 shows a stable, damped response that occurs when the power flows due to the system damping (linear, nonlinear) and derivative PID control action are greater than the power flows due to the nonlinear aerodynamic loads and the integral PID control action. Case 2 identifies the existence of a limit cycle oscillation (LCO) which results in nonlinear stall flutter when the power flows due to the derivative PID control action and damping (linear, nonlinear) balance the power flows due to the nonlinear aerodynamic loads and the integral PID control action over a cycle. Case 3 shows an unstable response where the power flows of the aerodynamic loads and integral PID control action are greater than the power flows due to the derivative PID control action and damping (linear, nonlinear). Notice, the limit cycle shows power flowing into the first torsional mode up to stall and then the decay of this energy state due to the damping such that the net work over a cycle is zero. These numerical results

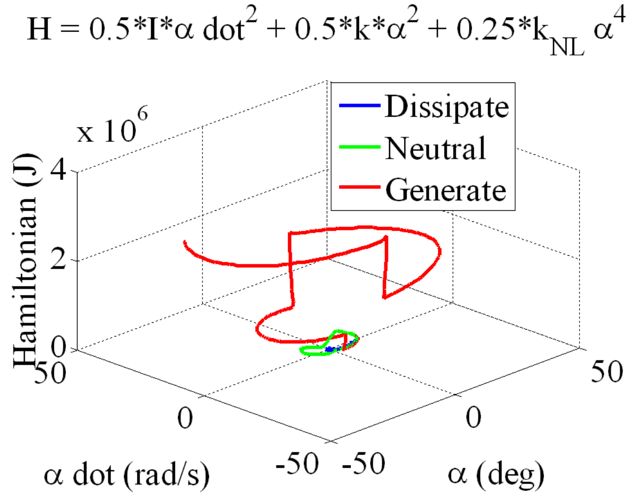

Fig. 3. **Nonlinear Stall Flutter with Nonlinear Dynamics and PID Control System Results: 3D Hamiltonian trajectory paths**

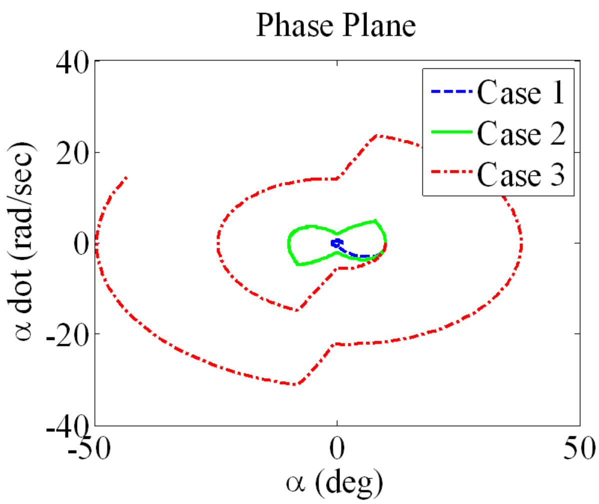

Fig. 4. **Nonlinear Stall Flutter with Nonlinear Dynamics and PID Control System Results: Phase Plane Plots**

are given for energy/power flows; Case 1 (see Fig. 8), Case 2 (see Fig. 9), and Case 3 (see Fig. 10). In addition, the numerical results for the nonlinear hysteretic aerodynamic moments are shown for Case 1 (see Figs. 11), Case 2 (see Fig. 12), and Case 3 (see Fig. 13). The corresponding control effort and acceleration responses are given for Case 1 in Figs. 14 and 15, respectively.

III. Summary and Conclusions

A nonlinear power flow control design methodology was applied to a nonlinear stall flutter problem (dynamic stall) that approximates the first torsional mode of a large wind turbine single blade (on-the-order of 5MW and larger). The methodology directly accommodated nonlinear structural and discontinuous aerodynamic models. The limit cycles (nonlinear stall flutter) were found by partitioning the power flows and identifying when the dissipation and generation balanced over a cycle subject to the storage (kinetic and potential energies) in the system. The limit cycles were shown to be stability boundaries. The flutter suppression control design was initially assessed by designing a PID controller. This initial first step starts the process of how to design a nonlinear flutter/dynamic stall controller that could be incorporated into the standard controllers for a typical 5MW turbine in below-, at-, and above-rated power conditions. The closer the wind turbine can safely operate to dynamic stall, the greater the energy that can be generated.

Acknowledgments

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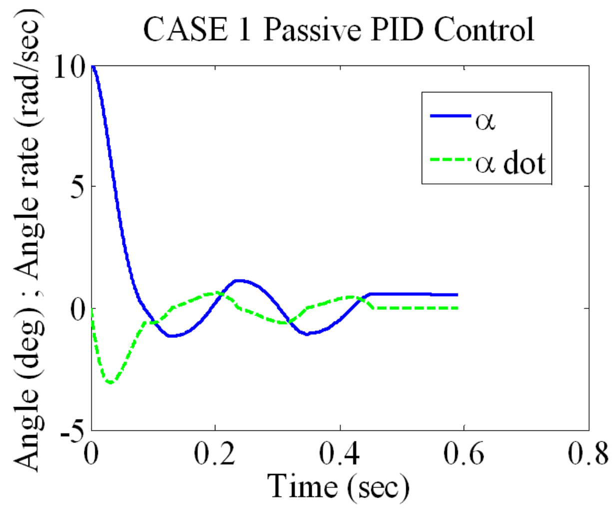

Fig. 5. **Nonlinear Stall Flutter with Nonlinear Dynamics Case 1 Dissipative PID Control Results**

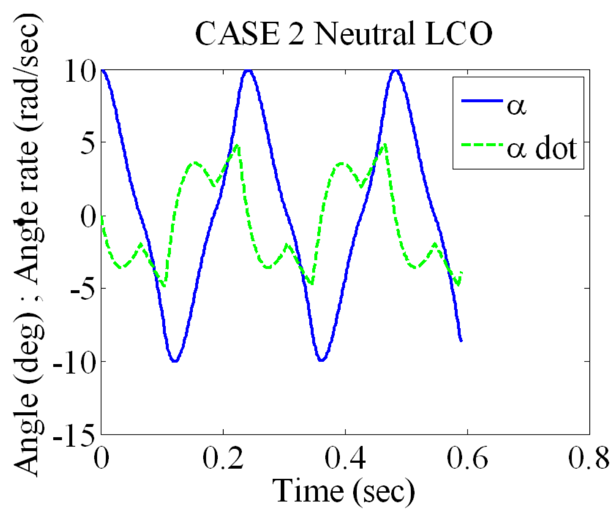

Fig. 6. **Nonlinear Stall Flutter with Nonlinear Dynamics Case 2 Neutral LCO Results**

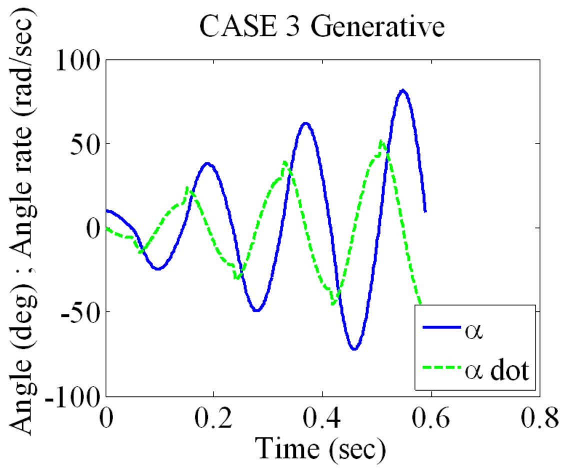

Fig. 7. **Nonlinear Stall Flutter with Nonlinear Dynamics Case 3 Generative Results**

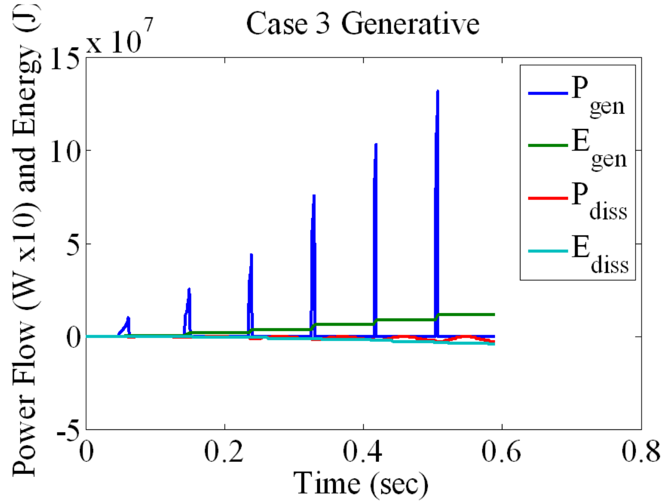

Fig. 10. **Power and Energy Flow Responses for Case 3 Generative Results**

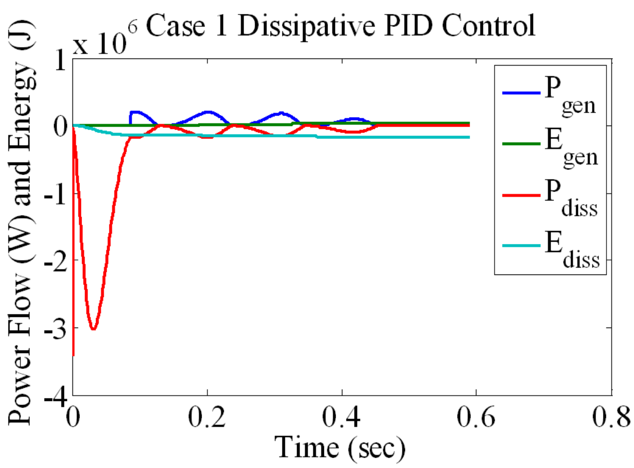

Fig. 8. **Power and Energy Flow Responses for Case 1 Dissipative PID Control Results**

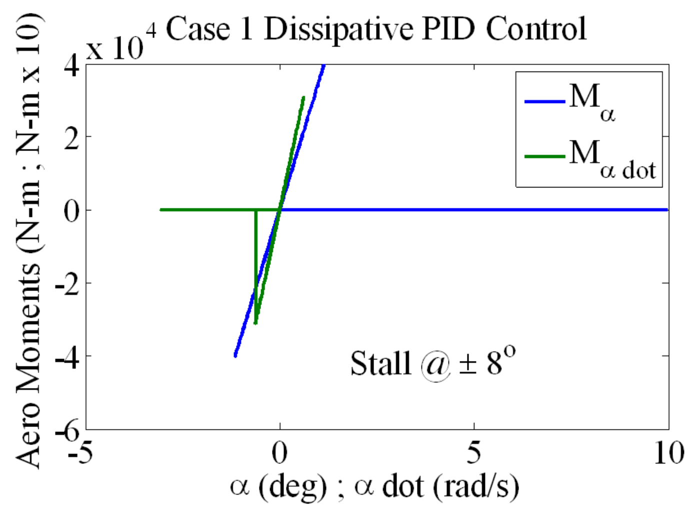

Fig. 11. **Aero Moment Responses for Case 1 Dissipative PID Control Results**

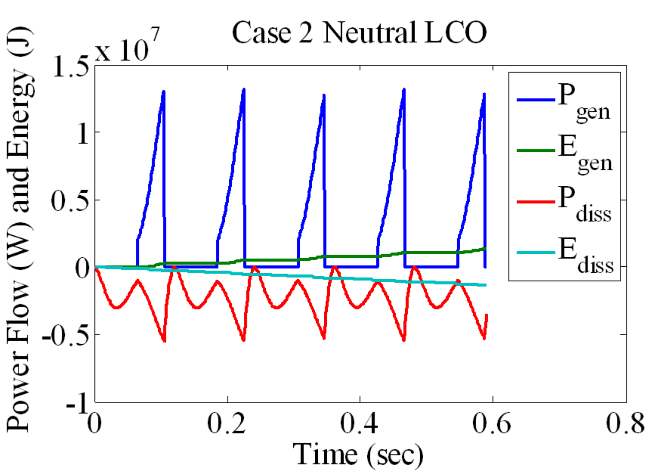

Fig. 9. **Power and Energy Flow Responses for Case 1 Neutral LCO Results**

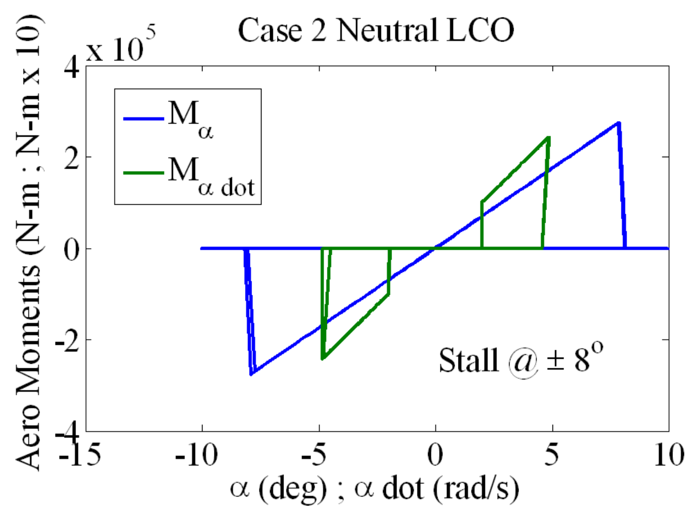

Fig. 12. **Aero Moment Responses for Case 2 Neutral LCO Results**

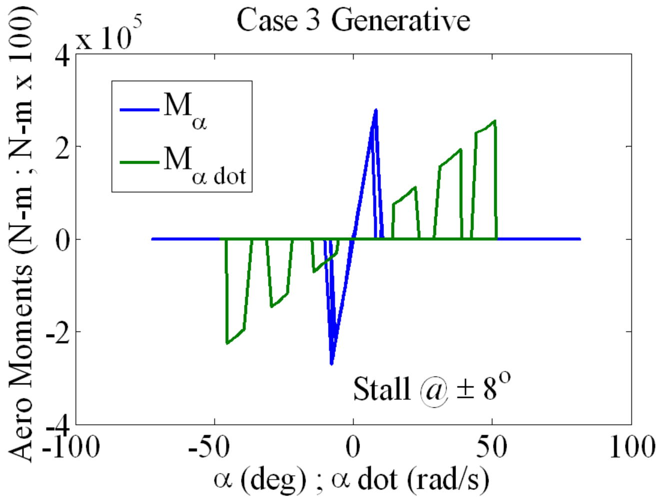

Fig. 13. **Aero Moment Responses for Case 3 Generative Results**

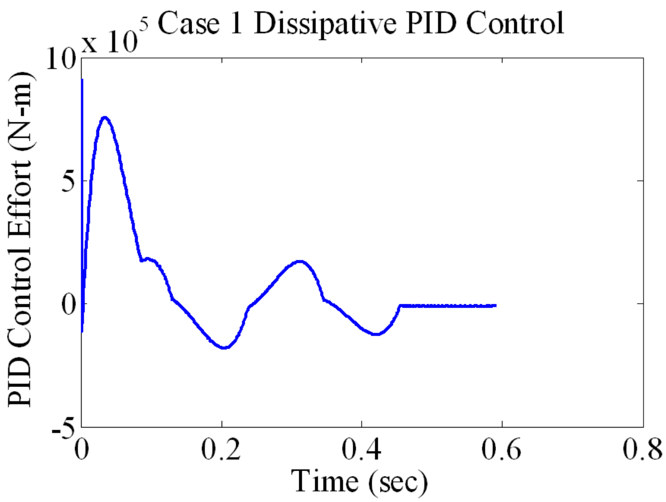

Fig. 14. **Control Effort for Case 1 Dissipative PID Control Results**

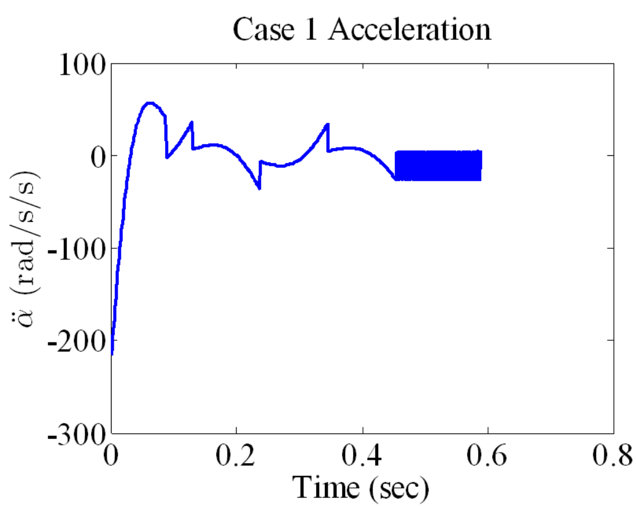

Fig. 15. **Acceleration for Case 1 Dissipative PID Control Results**