CDC '08 Workshop

Nonlinear Power Flow Control Design:

Utilizing Exergy, Entropy, Static and Dynamic Stability, and Lyapunov Analysis

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Goal of Workshop

- Present innovative control system design process based on R&D at SNL in the area of renewable energy electric grid integration (future electric power grid control, wind turbine load alleviation control, and novel micro-grid control design).
- The main result of this research is a unique set of criteria to design controllers for nonlinear systems with respect to both performance and stability, instead of following the typical linear controller zero-sum design trade-off process between stability and performance.
- This control design process combines concepts from thermodynamic exergy and entropy; Hamiltonian systems; Lyapunov's direct method and Lyapunov optimal analysis; static and dynamic stability analysis, electric AC power concepts including limit cycles; and power flow analysis. The thermodynamic concepts combined with Hamiltonian systems provide the theoretical foundations necessary to view control design as power flow control problem of balancing the power flowing into the system versus the power being dissipated within the system subject to power being stored in the system.
- Emphasis is placed on necessary design steps for which the concepts are introduced and explained with examples and case studies.

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Agenda

8:30 - 9:15	Welcome and Introduction	Charlie Hanley
9:15 - 10:15	Theory	Rush Robinett III
10:15 - 10:30	BREAK!	
10:30 - 12:00	Continue Theory/Discussions	Rush Robinett III David Wilson
12:00 - 1:00	Lunch!	
1:00 - 2:30	Case Studies 1 & 2	Rush Robinett III David Wilson
2:30 - 2:45	BREAK!	
2:45 - 4:15	Case Studies 3 & 4	Rush Robinett III David Wilson
4:15 - 4:30	Open Discussions	All
4:30	Adjourn!	

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Speakers
Energy Infrastructure Futures - Sandia National Labs

- **Charles J. Hanley** (Keynote Speaker) - Mr. Hanley is manager of Sandia's Photovoltaic Systems Evaluation Laboratory and the Distributed Energy Technologies Laboratory. He manages research and development focused on new photovoltaic components and systems with improved performance, higher reliability, and lower overall cost, in support of the US Department of Energy's efforts to make these technologies cost effective. This research includes; modernization of the electric grid through controls and communications, to support the eventual high penetration of renewable energy technologies into our infrastructure. Until 2002, Charlie managed Sandia's international renewable energy programs, through which he oversaw the implementation of more than 400 photovoltaic, wind, and passive solar energy systems in Latin America. He received his B.S. degree in Engineering Science from Trinity University in San Antonio, TX, and his M.S. degree in Electrical Engineering from Rensselaer Polytechnic Institute, in Troy, N.Y.
- **Rush D. Robinett III -** has three degrees in Aerospace Engineering from Texas A&M University (B.S. - 1982, Ph.D.- 1987) and The University of Texas at Austin (M.S. - 1984). He has authored over 100 technical articles including two books and holds 7 patents. He began working on ballistic missile defense at Sandia National Laboratories in 1988. In 1995, he was promoted to Distinguished Member of Technical Staff. In 1996, he was promoted to technical manager of the Intelligent Systems Sensors and Controls Department within the Robotics Center. In 2002, he was promoted to Deputy Director of the Energy and Transportation Security Center where he is developing distributed power and transportation infrastructures focusing on exergy, entropy and information metrics.
- **David G. Wilson -** has three degrees in Mechanical Engineering from Washington State University (B.S. - 1982, M.S. 1984) and The University of New Mexico (Ph.D. - 2000). He has authored over 50 technical articles including two books. He is currently a Principal Member of Technical Staff at Sandia National Laboratories, Energy Systems Analysis Department. He has over 20 years of research and development engineering experience in energy systems, robotics, automation, and space and defense projects. His current areas of research include: design, analysis, and implementation of nonlinear/adaptive control; distributed decentralized control architectures; exergy/entropy and collective control for dynamical systems.

Workshop Notes and References

- Viewgraph Handouts (booklet)
- Several conference and journal papers included:

1) R.D. Robinett III and D.G. Wilson, *What is a Limit cycle?,* International Journal of Control, Vol. 81, No. 12, Dec. 2008, pp. 1886-1900.
2) R.D. Robinett III and D.G. Wilson, *Exergy and Irreversible Entropy Production Thermodynamic Concepts for Nonlinear Control Design*, accepted for publication, International Journal of Exergy, February 2008.
3) R.D. Robinett III and D.G. Wilson, *Collective Plume Tracing: A Minimal Information Approach to Collective Control*, accepted for publication, International Journal of Robust and Nonlinear Control, Nov. 2008.
4) R.D. Robinett III and D.G. Wilson, *Nonlinear Power Flow Control Applied to Power Engineering*, SPEEDAM 2008, International Symposium on Power Electronics, Electrical Drives, Automation and Motion, Ischia, Italy, June 2008.
5) R.D. Robinett III and D.G. Wilson, *Collective Systems: Physical and Information Exergies*, SNL, SAND2007-2327 Report, March 2007.
6) R.D. Robinett III and D.G. Wilson, *Exergy and Entropy Thermodynamic Concepts for Control System Design: Slewing Single Axis*, AIAA Guidance, Navigation, and Control Conference and Exhibit, Keystone, Co., August 2006.
7) R.D. Robinett III and D.G. Wilson, *Exergy and Entropy Thermodynamic Concepts for Nonlinear Control Design*, Proceedings of IMECE2006-15205, 2006 ASME International Mechanical Engineering Congress and Exposition, Nov. 2006, Chicago, Illinois.
8) R.D. Robinett III and D.G. Wilson, *Exergy and Irreversible Entropy Production Thermodynamic Concepts for Control System Design: Robotic Servo Applications*, Proceedings of the 2006 IEEE International Conference on Robotics and Automation, Orlando, Florida, May 2006.
9) Robinett, III, R.D., Wilson, D.G., and Reed, A.W., *Exergy Sustainability for Complex Systems*, No. 1616, InterJournal Complex Systems, New England Complex Systems Institute, 2006.

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Acknowledgments

- **Ben Schenkman**, Dept. 6336 – Power Engineering Modeling
- **Val Weekly**, Dept. 6330 – Workshop Coordinator

- *Innovative Control of a Flexible, Adaptive Energy Grid,* Sandia National Laboratories, LDRD Project, No. 09-1125, Oct. 2006 – Oct. 2009.

LABORATORY DIRECTED RESEARCH & DEVELOPMENT

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Outline

- Theory (Morning)
  - Thermodynamics
  - Mechanics
  - Stability
  - Advanced Control Design
- Case Studies (Afternoon)
  - Control Design Issues
  - Collective Plume Tracing
  - Nonlinear Aeroelasticity
  - Power Engineering

THERMODYNAMICS

- First Law - Energy
- Second Law – Stability/Entropy
- Equilibrium Thermodynamics - Reversible/Irreversible Paths
- Local Equilibrium – Non Equilibrium Thermodynamics
- Rate Equations – Energy, Entropy, and Exergy

Thermodynamics (cont.)

- First Law [Ref. Gyftopoulos] - A statement of the existence of a property called energy
  - Energy is a state function: path independent; E
  - Adiabatic work is path independent

$$E_2 - E_1 = -W_{12}$$

E_i – Energy at state i of system

W_{12} – Work done by system between states 1 and 2

[Ref. Gyftopoulos]: E.P. Gyftopoulos and G.P. Beretta, **Thermodynamics, Foundations and Applications**, MPC, NY, 1991.

Thermodynamics (cont.)

- Corollary of First Law

$$dE = \delta Q - \delta W$$

dE – change of state; path independent

δQ – flow of heat; path dependent

δW – work done by the system; path dependent

(conservation of energy)

Thermodynamics (Cont.)

- Second Law – [Ref. Gyftopoulos]: A statement of the existence of stable equilibrium states and of special processes that connect these states together
  - Equilibrium State: A state that does not change with time while the system is isolated from all other systems in the environment
  - Stable Equilibrium State: A finite change of state cannot occur, regardless of interactions that leave no net effects in the environment
  - Restate Second law: Among all the allowed states of a system with given values of energy, number of particles, and constraints, one and only one is a stable equilibrium state

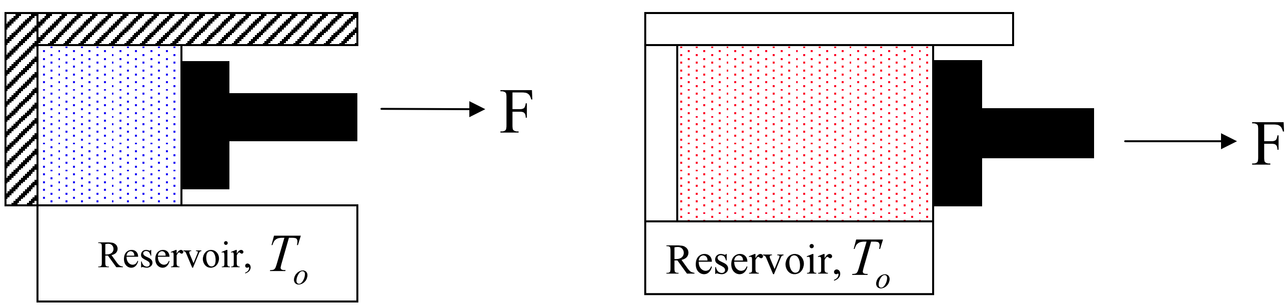
[Ref. Gyftopoulos]: E.P. Gyftopoulos and G.P. Beretta, **Thermodynamics, Foundations and Applications**, MPC, NY, 1991.

Thermodynamics (cont.)

– Reversible and Irreversible Processes: A process is reversible if the system and its environment can be restored to their initial states, except for differences of small order of magnitude than the maximum changes that occur during the process

Example 1: Expansion of Gas

Adiabatic Expansion – Irreversible



Reversible Expansion

Thermodynamics (cont.)

– Available work: Ω

$$\Omega_2 - \Omega_1 = -(W_{12})_{rev}; (W)_{12rev} \geq W_{12}$$

– Entropy: S

$$dS \equiv c_R (d\mathrm{E} - d\Omega)$$

– Adiabatic processes: Reversible:

$$(dS)_{REV} = 0$$

Irreversible:

$$(dS)_{irrev} \geq 0$$

Thermodynamics (cont.)

- Corollary: At a stable equilibrium state, the entropy will be at its maximum value for fixed values of energy, number of particles, and constraints

- Reversible process: Interacting systems quasi-statically pass only through stable equilibrium states

$$dQ = (\delta Q)_{REV} = TdS; \quad T - temperature$$

$$dS = \frac{dQ}{T}; \quad S = 0 = \oint \frac{dQ}{T}$$

Thermodynamics (cont.)

– Irreversible process [Ref. Prigogine]:

$$dS \geq \frac{\delta Q}{T}$$

$$dS = d_e S + d_i S$$

$d_e S$ - Entropy change due to exchange of energy and matter (entropy flux)

$d_i S$ - Entropy change due to irreversible processes

[Ref. Prigogine] D. Kondepudi and I. Prigogine, **Modern Thermodynamics: From Heat Engines to Dissipative Structures**, John Wiley & Sons, New York, N.Y., 1999.

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Thermodynamics (cont.)

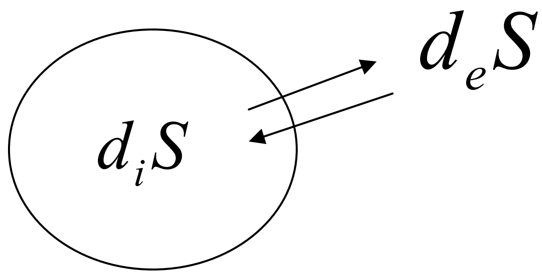


Figure 1- Entropy changes

$$d_i S = \sum_k F_k dX_k \geq 0$$

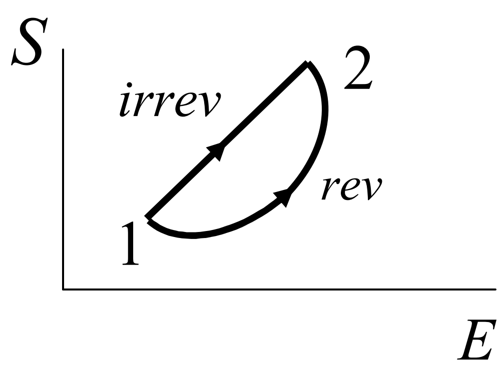
$F_k - k^{th}$ thermodynamics force

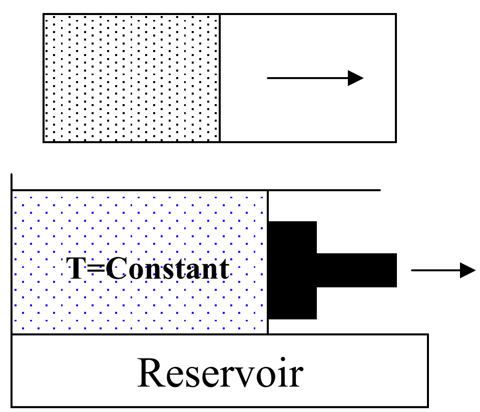
$X_k - k^{th}$ thermodynamics flow

Thermodynamics (cont.)

- Equilibrium Thermodynamics [Ref. Prigogine] – In classical thermodynamics it is assumed that every irreversible transformation that occurs in nature can also be achieved through a reversible process where

$$S_2 = S_1 + \int_1^2 \frac{dQ}{T}$$





[Ref. Prigogine] D. Kondepudi and I. Prigogine, **Modern Thermodynamics: From Heat Engines to Dissipative Structures**, John Wiley & Sons, New York, N.Y., 1999.

Thermodynamics (cont.)

- Reversible Cycle Processes:

$$dU = TdS - pd\overline{V}$$

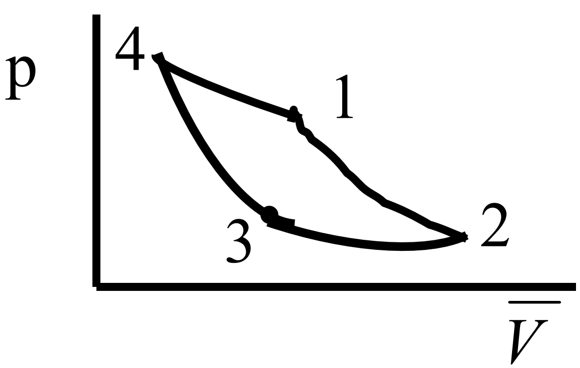
$$dU = dE \quad = \text{internal energy}$$

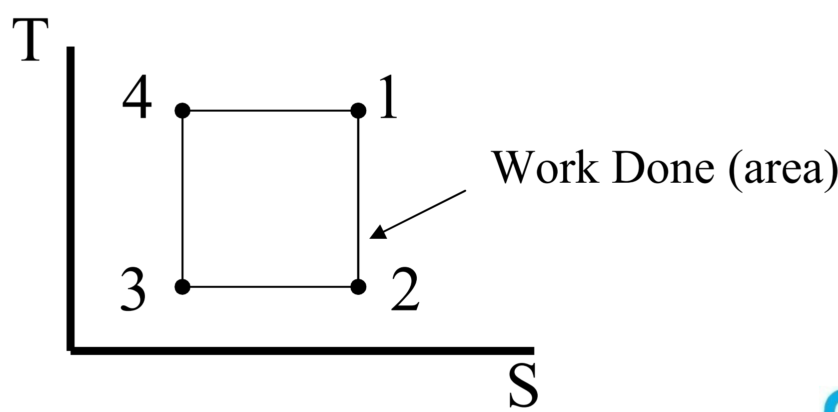
$$TdS = dQ$$

$$pd\overline{V} = dW \quad ; p - pressure, \overline{V} - volume$$

$$\oint dU = 0 = \oint TdS - \oint pd\overline{V}$$

$$\Rightarrow \oint TdS = \oint pd\overline{V}$$





- A uniform temperature exists throughout the system.

Thermodynamics (cont.)

- Local Equilibrium – Thermodynamic quantities are well-defined concepts locally (i.e., within each elemental volume)
  - Temperature is not uniform, but is well defined locally
  - Non-equilibrium systems: define thermodynamic quantities in terms of densities
  - Thermodynamic variables become functions of position and time
- Rate Equations
  - Energy: $\dot{E} = \sum_j \dot{Q}_j + \sum_k \dot{W}_k + \sum_l \dot{m}_l (h_l + ke_l + pe_l + \ldots)$
  - Entropy: $\dot{S} = \dot{S}_e + \dot{S}_i = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_k \dot{m}_k s_k + \dot{S}_i$

$$\dot{S}_i = \sum_l F_l \dot{X}_l \geq 0$$

  - Exergy (Available work)

$$\dot{\Xi} = \dot{E} - T_o \dot{S} = \sum_j \left(1 - \frac{T_o}{T_j} \dot{Q}_j\right) + \sum_k (\dot{W}_k - p_o \dot{\bar{V}}) + \sum_l \dot{m}_l \zeta_l^{Flow} - T_o \dot{S}_i$$

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MECHANICS

- Work, Energy, and Power
- Energy Diagrams and Phase Planes
- Hamiltonian Mechanics
- Connections between Thermodynamics and Hamiltonian Mechanics
- Line Integrals and Limit Cycles

Mechanics (cont.)

- Work, Energy, and Power
  - Work: $dW = \underline{F} \cdot d\underline{r}$

    $W -$ Work

    $\underline{F} -$ Force Vector

    $\underline{r} -$ Position Vector

  - Power: $P = \frac{d}{dt}W = \underline{F} \cdot \frac{d}{dt}\underline{r} = \underline{F} \cdot \underline{\dot{r}}$

    $P -$ Power

    $\underline{\dot{r}} -$ Velocity Vector

  - Kinetic Energy: $\underline{F} = m\underline{\ddot{r}}$ (Newton's 2nd law)

$$dW = \underline{F} \cdot d\underline{r} = m\underline{\ddot{r}} \cdot d\underline{r} = m\underline{\dot{r}} \cdot d\underline{\dot{r}} = d\left(\frac{1}{2}m\underline{\dot{r}} \cdot \underline{\dot{r}}\right) = dT$$

$T = \frac{1}{2}m\underline{\dot{r}} \cdot \underline{\dot{r}} -$ Kinetic Energy

Mechanics (cont.)

– Potential Energy:

$$\underline{F}(\underline{r}) \cdot d\,\underline{r} = -dV(\underline{r})$$

$$V(\underline{r}_1) - V(\underline{r}_2) = \int_{\underline{r}_1}^{\underline{r}_2} \underline{F}(\underline{r}) \cdot d\,\underline{r}$$

V – Potential Energy

$\Rightarrow$ Conservative and path independent force field

$$\oint \underline{F} \cdot d\underline{r} = 0$$

$$\frac{dT}{dt} = \underline{F} \cdot \underline{r} = -\frac{dV}{dt} \Rightarrow \qquad \frac{d}{dt}(T + V) = 0$$

$$\Rightarrow T + V = E = \text{Constant}$$

E – Total Energy (stored energy)

Mechanics (cont.)

– Time Dependent Potential Field: $\underline{F}(\underline{r},t)\cdot d\underline{r} = -\,dV(\underline{r},t)$

$$\frac{d}{dt}(T+V) = \frac{\partial V}{\partial t}$$

Mechanics (cont.)

-Non-conservative Forces:

$$\underline{F} = \underline{F}_c + \underline{F}_{NC}$$

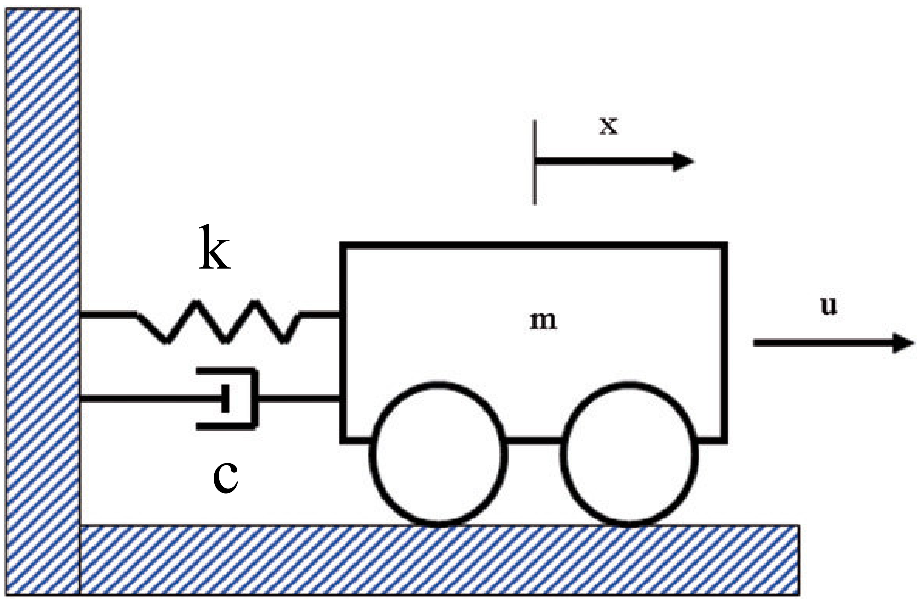
$\underline{F}_c$ – Conservative Force (Potential Field)

$\underline{F}_{NC}$ – Non-Conservative Force

$$\frac{dE}{dt} = \frac{d}{dt}(T + V) = \frac{\partial V}{\partial t} + \underline{F}_{NC} \cdot \underline{\dot{r}}$$

Mechanics (cont.)

- Energy Diagrams and Phase Planes
  - Mass, Spring, Damper System



$$T = \frac{1}{2} m \dot{x}^2$$

$u-$ Control Force

$$V = \frac{1}{2} k x^2$$

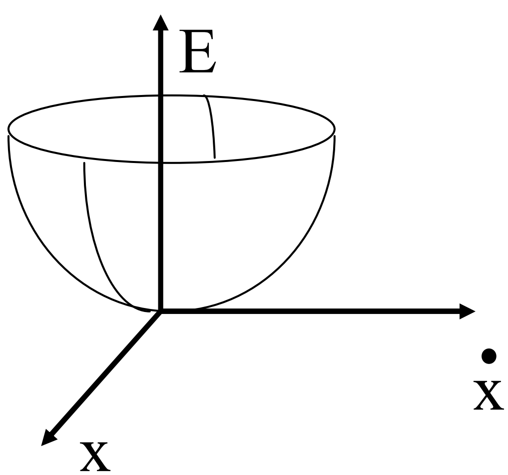
$c\dot{x}-$ Damping Force

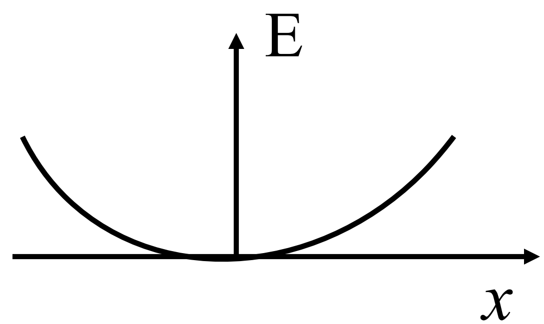
$$F = u - c\dot{x}$$

$$m\ddot{x} + kx = u - c\dot{x}$$

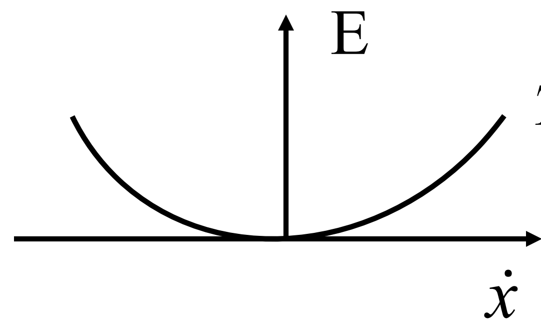
Mechanics (cont.)

a) Set $u = c\dot{x} = o$: conservative system, $E = T + V =$ Constant



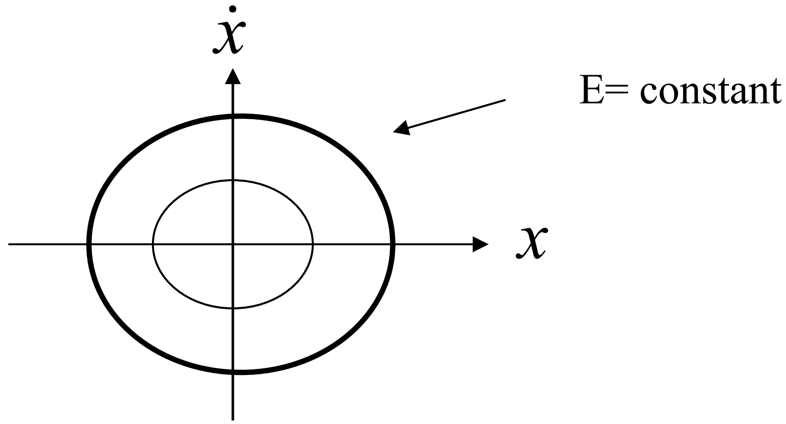


$V = \frac{1}{2} k x^2$



$T = \frac{1}{2} m \dot{x}^2$

Mechanics (cont.)

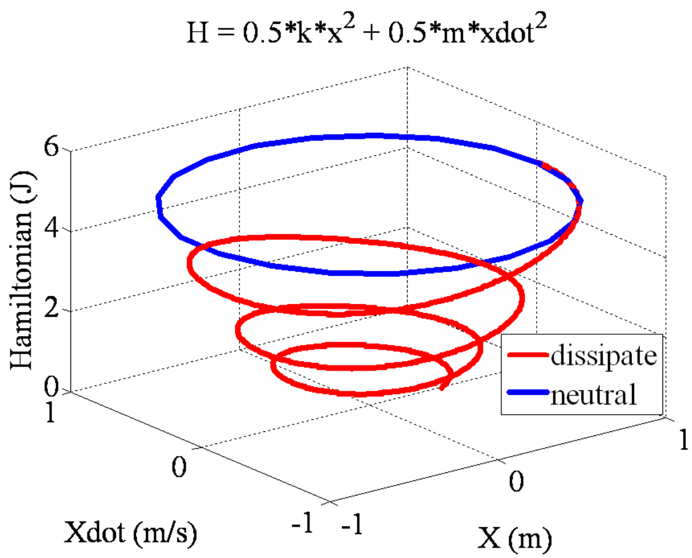


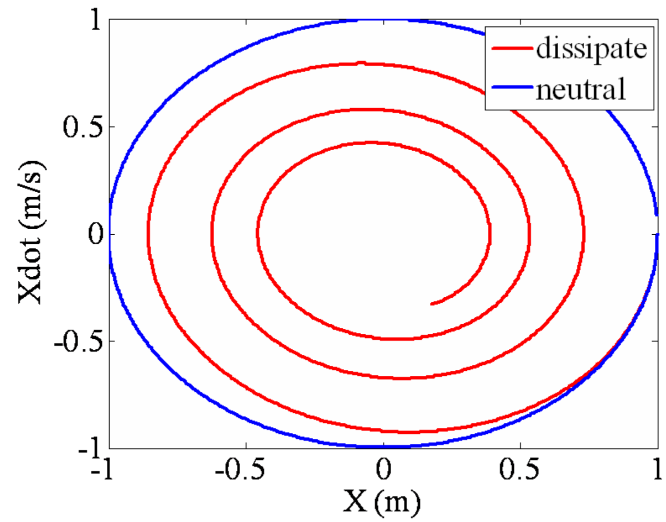
$\Rightarrow$ The system path is constrained to the energy storage surface along a constant energy orbit

$$E = \frac{1}{2} m \dot{x}_o^2 + \frac{1}{2} k x_o^2$$

Mechanics (cont.)

b) Set $u=0$ and $c>0$





⇒ The system path is constrained to the energy storage surface as it spirals down into the minimum energy state

Mechanics (cont.)

- Hamiltonian Mechanics
  - Lagrangian: $L = T(\underline{q}, \underline{\dot{q}}, t) - V(\underline{q}, t)$

    $t-$ Time Explicitly

    $\underline{q}-$ N Dimensional Generalized Coordinate Vector

    $\underline{\dot{q}}-$ N Dimensional Generalized Velocity Vector

  - Equations of Motion:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \underline{\dot{q}}}\right) - \frac{\partial L}{\partial \underline{q}} = \underline{Q}$$

$\underline{Q}-$ Generalized Force Vector

  - Hamiltonian: $H \equiv \sum_{j=1}^{N} \frac{\partial L}{\partial \dot{q}_i} \dot{q}_i - L(\underline{q}, \underline{\dot{q}}, t) = H(\underline{q}, \underline{\dot{q}}, t)$

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Mechanics (cont.)

– Canonical Momentum: $p_i = \frac{\partial L}{\partial \dot{q}_i}$

$$H(\underline{q}, \underline{p}, t) = \sum_{i=1}^{N} p_i \dot{q}_i - L(\underline{q}, \underline{\dot{q}}, t)$$

– Hamilton's Canonical Equations of Motion:

$$\dot{q}_j = \frac{\partial H}{\partial p_j}$$

$$\dot{p}_j = -\frac{\partial H}{\partial q_j} + Q_j$$

– Time Derivative of the Hamiltonian:

$$\dot{H} = \sum_{j=1}^{N} \left(\dot{p}_j \dot{q}_j + p_j \ddot{q}_j - \frac{\partial L}{\partial t} - \frac{\partial L}{\partial q_j} \dot{q}_j - \frac{\partial L}{\partial \dot{q}_j} \ddot{q}_j \right)$$

$$= \sum_{j=1}^{N} Q_j \dot{q}_j - \frac{\partial L}{\partial t}$$

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Mechanics (cont.)

– For most natural systems:

$$\frac{\partial L}{\partial t} = 0$$

$$\boxed{\dot{H}(\underline{q}, \underline{\dot{q}}) = \sum_{j=1}^{N} Q_j \dot{q}_j}$$

Power (work/energy) flow equation

Mechanics (cont.)

- Connections between Thermodynamics and Hamiltonian Mechanics
  - Conservative Mechanical Systems:

a) $\dot{H} = 0$ and $H = \text{Constant}$

b) Conservative Force (storage device)

$$\oint \underline{F} \cdot d\underline{r} = \oint \underline{F} \cdot \dot{\underline{r}} dt = \oint \underline{Q} \cdot \dot{\underline{q}} dt = 0$$

For any closed path

Mechanics (cont.)

– Reversible Thermodynamic Systems:

$$dS = \frac{dQ}{T}$$

$$\oint dS = \oint \frac{dQ}{T} = 0$$

$$\oint dS = \oint [d_e S + d_i S] = \oint [\dot{S}_e + \dot{S}_i] dt = 0$$

$$\Rightarrow \dot{S}_e = \frac{\dot{Q}}{T} \quad \text{and} \quad \dot{S}_i = 0 \quad \text{(Second Law)}$$

– Irreversible Thermodynamic Systems:

for $\oint dS = \oint [\dot{S}_e + \dot{S}_i] dt = 0$

then $\dot{S}_e \le 0$ and $\dot{S}_i \ge 0$

Mechanics (cont.)

– Hamiltonian is stored exergy since potential and kinetic energies are available work; Exergy rate relationship:

$$\dot{H} = \sum_k Q_k \dot{q}_k = \underline{F}_{NC} \cdot \underline{\dot{q}}$$

$$\dot{\Xi} = \dot{W} - T_o S_i = \sum_{j=1}^{N} Q_j \dot{q}_j - \sum_{l=N+1}^{M+N} Q_l \dot{q}_l = \underline{F}_{NC} \cdot \underline{\dot{q}}$$

a) $\dot{W}$ – Power flowing in (N generators)

b) $T_o \dot{S}_i$ - Power Dissipation (M Dissipators)

c) $\dot{S}_i = \sum_k F_k \dot{X}_k = \frac{1}{T_o} \sum_k Q_k \dot{q}_k \geq 0$ (Assuming Local Equilibrium)

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Mechanics (cont.)

d) Assumptions applied to exergy rate equation:

$$\dot{Q}_i \cong 0$$

$$1 - \frac{T_o}{T_j} \cong 0$$

$$p_o \dot{\bar{V}} = 0$$

$$\sum_k \dot{m}_k \zeta_k^{FLOW} = 0$$

e) A conservative system is equivalent to a reversible system when

$$\dot{H} = 0 \quad \text{and} \quad \dot{S}_e = 0$$

then $\dot{S}_i = 0$ and $\dot{W} = 0$

Mechanics (cont.)

- Line Integrals and Limit Cycles
  - Cyclic Equilibrium Thermodynamics:

$$\oint dU = \oint TdS - \oint p d\overline{V} = 0$$

$$\oint dE = \oint TdS - \oint dW = 0$$

$$\Rightarrow \boxed{\oint p d\overline{V} = \oint TdS}$$

$$\Rightarrow \boxed{\oint dW = \oint TdS}$$

  - Cyclic Non-Equilibrium Thermodynamics with Local Equilibrium

$$\oint \dot{H}dt = \oint [\dot{W} - T_o \dot{S}_i]dt = 0$$

$$\Rightarrow \boxed{\oint \dot{W}dt = \oint T_0 \dot{S}_i dt}$$

$$\Rightarrow \oint [\sum_{j=1}^{N} Q_j \dot{q}_j]dt = \oint [\sum_{k=N+1}^{M+N} Q_k \dot{q}_k]dt$$

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Mechanics (cont.)

– Sorting Power Terms:

a) Conservative Terms

$$\oint \underline{F}_c \cdot \underline{\dot{q}} dt = 0 \Rightarrow \text{Potential functions}$$

b) Generator Terms

$$\int_c \underline{F}_{NC} \cdot \underline{\dot{q}} dt > 0 \Rightarrow \int_c [\sum_{j=1}^{N} Q_j \dot{q}_j] dt > 0$$

c) Dissipator Terms

$$\int_c \underline{F}_{NC} \cdot \underline{\dot{q}} dt < 0 \Rightarrow \int_c [\sum_{k=N+1}^{N+M} Q_k \dot{q}_k] dt < 0$$

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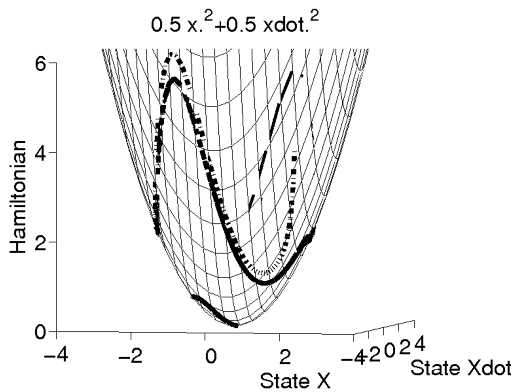
Mechanics (cont.)

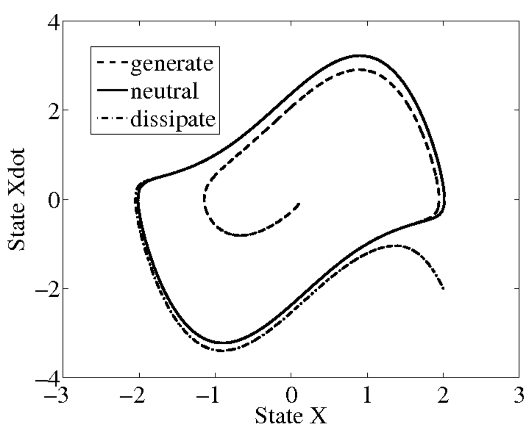
– Limit Cycles:

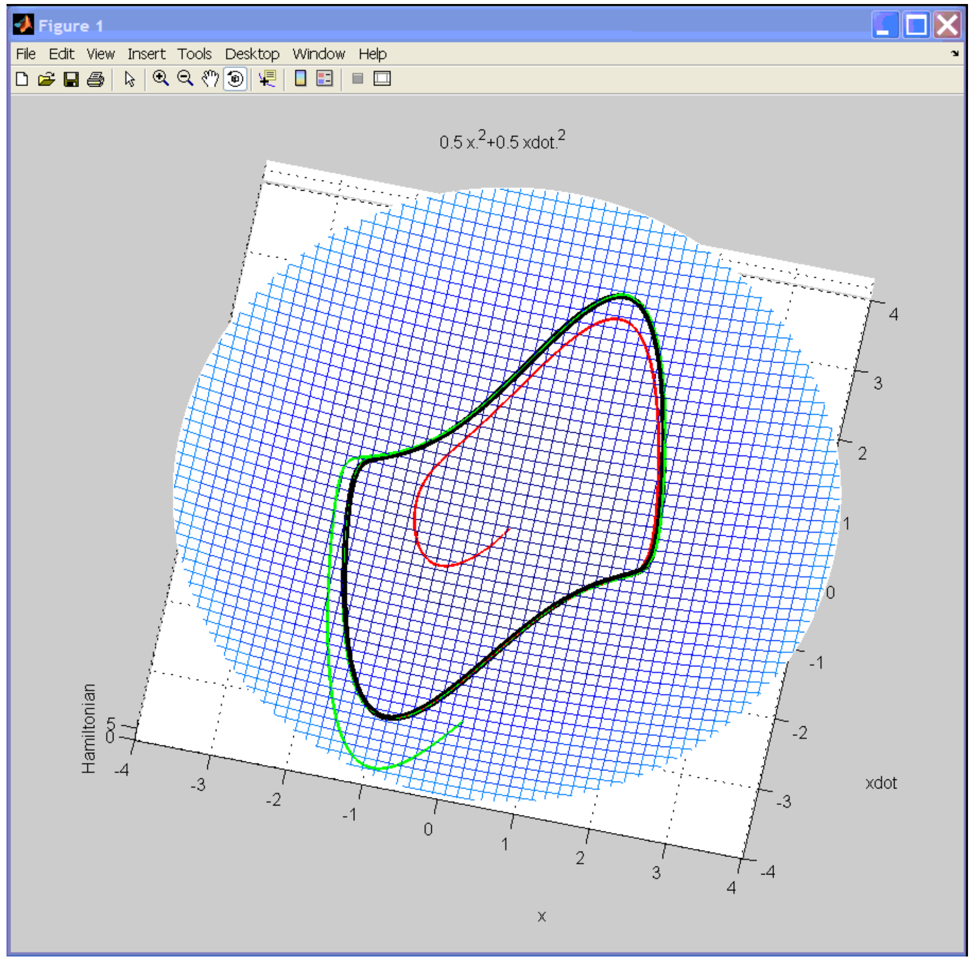
$$\boxed{\oint \dot{W}dt = \oint T_o \dot{S}_i dt}$$

$$\oint \left[\sum_{j=1}^{N} Q_j \dot{q}_j \right] = \oint \left[\sum_{k=N+1}^{N+M} Q_k \dot{q}_k \right] dt$$

Mechanics (cont.)
van der Pol Oscillator: Limit Cycle Identification







-Dissipative

-Neutral

-Generative

-Hamiltonian surface

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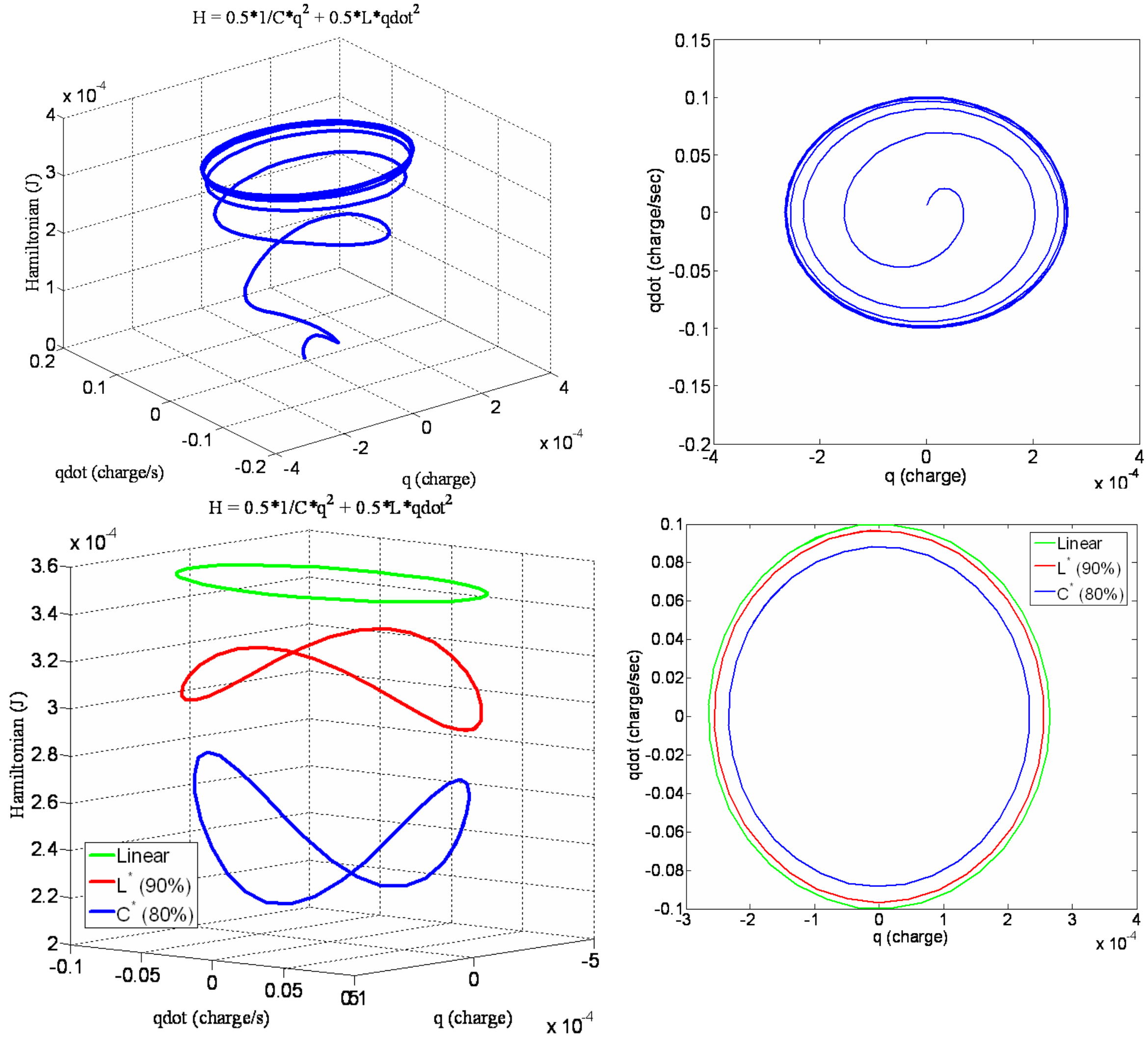
Mechanics (cont.)
Linear Limit Cycles: Power Grid Applications

- **Goal Power Engineering** maximize real power flow: match frequency and phase of applied voltage and resulting current
- **Exergy/Entropy Control Design** identifies stability boundaries and improves system performance with nonlinear feedback by providing robustness to disturbances and adaptation to parameter changes
- **Current Power Engineering** solutions "open loop" – subject to changing load variations and delays due to manual operator set point updates and approvals
- **Simple RLC circuit** – Driven with 60 Hz sinusoidal input demonstrates variations and changes in load and frequency



Exergy/Entropy Distributed Control: Predicts Performance and Stability for Power System

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STABILITY

- Static and Dynamic Stability
- Eigenanalysis
- Lyapunov Analysis
- Energy Storage Surface and Power Flow: Necessary and Sufficient Conditions for Stability

Stability (cont.)

- Static and Dynamic Stability
  - Static Stability: If the forces and moments on a body caused by a disturbance tend <u>initially</u> to return (move) the body towards (away from) its equilibrium state, the body is statically stable (unstable) [Refs. Anderson, Robinett]

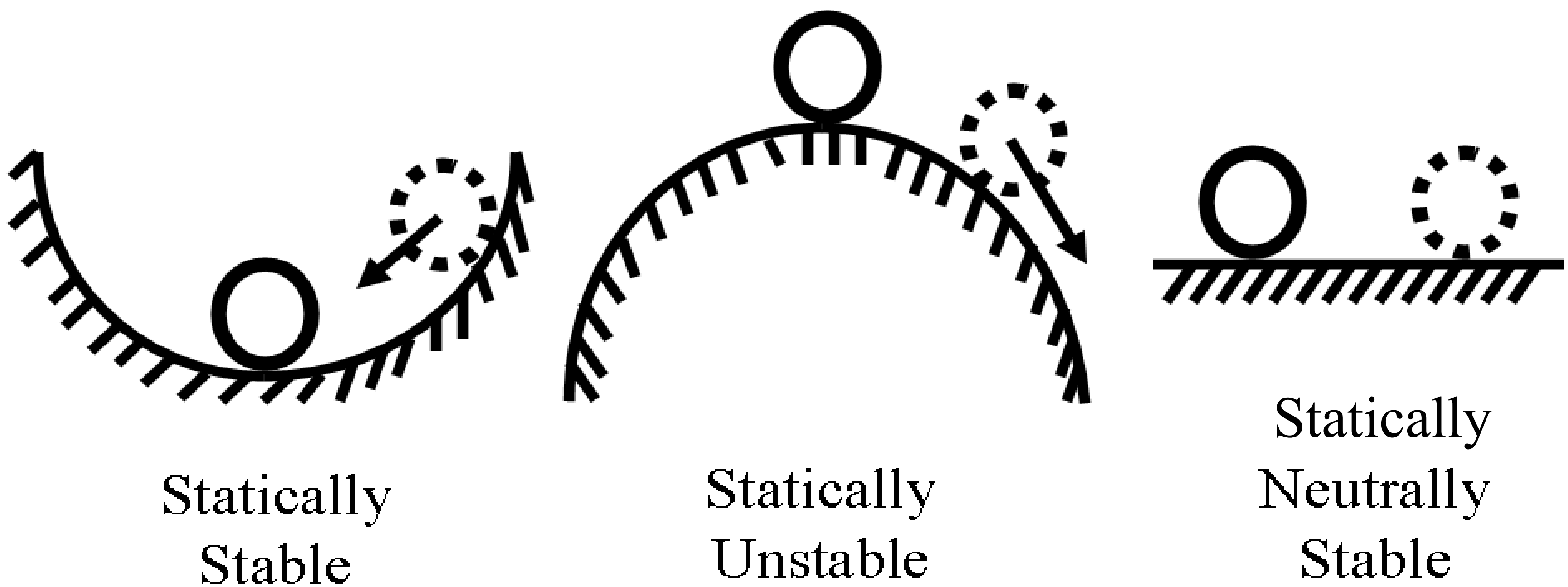
a) Equilibrium State- An unaccelerated motion wherein the sums of the forces and moments on the body are zero $(\dot{x}_e, x_e)$

b) Neutral Stability - Occurs when the body is disturbed and the sums of the forces and moments on the body remain zero

[Ref. Anderson] J.D. Anderson, Jr., **Introduction to Flight**, McGraw-Hill, 1978.

[Ref. Robinett] R.D. Robinett III, *A Unified Approach to Vehicle Design, Control, and Flight Path Optimization*, PhD Dissertation, Texas A&M University, 1987.

Stability (cont.)



Stability (cont.)

c) The Energy Storage Surface is

$$H = T + V = E$$

d) A system is statically stable if

$$H > 0 \quad \text{for} \quad V(x) > 0 \; \forall x \neq x_e, \dot{x} \neq \dot{x}_e$$

$$H(\dot{x}_e, x_e) = 0$$

Statically unstable if

$$V(x) < 0$$

And Neutrally stable if

$$V(x) = 0$$

Stability (cont.)

- Dynamic Stability: Defined in terms of the time history of the motion of a body after encountering a disturbance
  - A body is dynamically stable (unstable) if, out of its own accord, it eventually returns to (deviates from) and remains at (away from) its equilibrium state over a period of time [Refs. Anderson, Robinett1]
    a) A dynamically neutral stable body occurs when a limit cycle exists [Refs. Robinett2, Robinett3]
    b) A dynamically stable body must always be statically stable, but static stability is <u>not</u> sufficient to ensure dynamic stability [Refs. Abramson, Anderson, Robinett1] Therefore, static stability is a necessary condition for stability

[Ref. Anderson] J.D. Anderson, Jr., **Introduction to Flight**, McGraw-Hill, 1978.

[Ref. Robinett1] R.D. Robinett III, *A Unified Approach to Vehicle Design, Control, and Flight Path Optimization*, PhD Dissertation, Texas A&M University, 1987.

[Ref. Robinett2] R.D. Robinett III, *What is a Limit Cycle?,* Int'l Journal of Control, Vol. 81, No. 12, Dec. 2008, pp. 1886-1900.

[Ref. Robinett3] R.D. Robinett III and D.G. Wilson, *Collective Systems: Physical and Information Exergies*, Sandia National Laboratories, SAND2007-2327 Report, March 2007

[Ref. Abramson] H.N. Abramson, **Introduction to the Dynamics of Airplanes**, Ronald Press, 1958.

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Stability (cont.)

c) The time derivative of the energy storage surface defines the power flow into, dissipated within, and stored in the system

d) A system is dynamically stable if

$$\dot{H}_{AVE} = \frac{1}{\tau_c}\int_o^{\tau_c} \dot{H}dt < 0$$

dynamically unstable if

$$\dot{H}_{AVE} = \frac{1}{\tau_c}\int_o^{\tau_c} \dot{H}dt > 0$$

and dynamically neutral stable if (Limit Cycle)

$$\dot{H}_{AVE} = \frac{1}{\tau_c}\oint_{\tau_c} \dot{H}dt = 0$$

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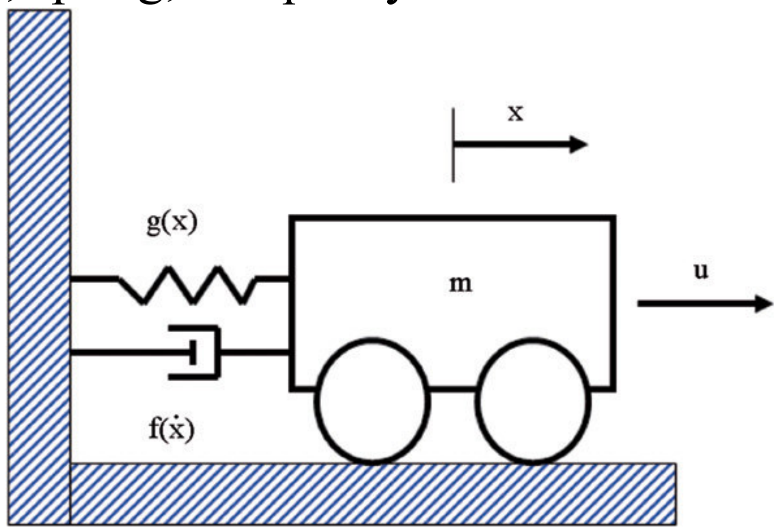
Stability (cont.)

- Eigenanalysis: The goal is to relate/ extend eigenanalysis of linear systems to nonlinear systems via the energy storage surface, power flow, and limit cycles
  - Mass, spring, damper system



$$m\ddot{x} + g(x) = -f(\dot{x}) + u$$

$$H = E = \frac{1}{2}m\dot{x}^2 + V(x); \quad g(x) = \frac{\partial V(x)}{\partial x}$$

Stability (cont.)

a) Linearized system

$$V(x) = \frac{1}{2}kx^2 > 0;\ f(\dot{x}) = c\dot{x};\ m, c, k > 0$$

$$H = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 > 0$$

$$m\ddot{x} + kx = -c\dot{x} + u$$

$$\dot{H} = [m\ddot{x} + kx]\dot{x} = [u - c\dot{x}]\dot{x}$$

b) Assume $u = c\dot{x} = 0, \dot{H} = 0$, Eigenvalue Problem

$$m\ddot{x} + kx = 0$$

$$\omega^2 = \frac{k}{m}$$

Undamped natural frequency (eigenvalue) of a statically stable and dynamically neutral stable system

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Stability (cont.)

c) Assume $u = -K_D \dot{x}$ and $c > 0$

$$H = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 > 0$$

$$\dot{H} = [m\ddot{x} + kx]\dot{x} = -[c + K_D]\dot{x}^2$$

This system is statically stable and dynamically stable if $c + K_D > 0$

Dynamically unstable $c + K_D < 0$

Dynamically neutral stable if $c + K_D = 0$

d) Extend results to nonlinear systems:

$$H = \frac{1}{2} m \dot{x}^2 + V(x) = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 + \frac{1}{4} k_{NL} x^4 > 0$$

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Stability (cont.)

This system is statically stable and dynamically neutral stable (limit cycle) if

$$\dot{H}_{AVE} = \frac{1}{\tau_c} \oint_{\tau_c} [m\ddot{x} + g(x)]\dot{x}dt = \frac{1}{\tau_c} \oint_{\tau_c} [u - f(\dot{x})]\dot{x}dt = 0$$

with nonlinear frequency content of

$$m\ddot{x} + g(x) = 0$$

and dynamically stable if

$$\dot{H}_{AVE} = \frac{1}{\tau_c} \int_o^{\tau_c} [u - f(\dot{x})]\dot{x}dt < 0$$

and dynamically unstable if

$$\dot{H}_{AVE} = \frac{1}{\tau_c} \int_o^{\tau_c} [u - f(\dot{x})]\dot{x}dt > 0$$

Stability (cont.)

- Lyapunov Analysis: The goal is to relate static and dynamic stability, energy storage surface, and power flow to Lyapunov Analysis
  - The definition of static stability is equivalent to the requirement placed on a Lyapunov function: must be positive definite

$$\mathscr{V}(x,\dot{x}) = H > 0$$

  - A statically unstable system is unstable in the sense of Lyapunov. Assume $u = c\dot{x} = 0$ and $V(x) = -\frac{1}{2}kx^2$ for $k > 0;$ pick Lagrangian

$$\mathscr{V} = L = T - V = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 > 0$$

$$\dot{\mathscr{V}} = [m\ddot{x} + kx]\dot{x} = 2kx\dot{x} > 0$$

for $m\ddot{x} - kx = 0$ which is unstable

Stability (cont.)

– The definition of dynamic stability is equivalent to the requirements placed on a Lyapunov function. A system is asymptotically stable if [Ref. Robinett1]

$$\mathscr{V} = H > 0$$

$$\left(\dot{\mathscr{V}}\right)_{ave} = \frac{1}{\tau_c}\int_o^{\tau_c} [u - f(\dot{x})]\dot{x}dt < 0$$

– And unstable if

$$\mathscr{V} = H > 0$$

$$\left(\dot{\mathscr{V}}\right)_{ave} = \frac{1}{\tau_c}\int_o^{\tau_c} [u - f(\dot{x})]\dot{x}dt > 0$$

The limit cycle defines the stability boundary between asymptotically stable and unstable [Ref. Robinett2]

[Ref. Robinett1] R.D. Robinett III and D.G. Wilson, *Exergy and Irreversible Entropy Production Thermodynamic Concepts for Nonlinear Control Design,* accepted for publication, International Journal of Exergy, Feb. 2008.

[Ref. Robinett2] R.D. Robinett III and D.G. Wilson, *What is a Limit Cycle*?, International Journal of Control, Vol. 81, No. 12, Dec. 2008, pp. 1886-1900.

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Stability (cont.)

- Energy Storage Surface and Power Flow: Necessary and Sufficient Conditions for Stability
  - Energy Storage Surface: Total energy; stored exergy; paths/trajectories are constrained to this surface; infrastructure
    a) Determines static stability which is a necessary condition for stability
    b) Proportional feedback affects static stability; assume

$V(x) = -\frac{1}{2}kx^2 + \frac{1}{2}k_{NC}x^4$ for $k, k_{NL} > 0$ and $u = -K_p x, \ c = 0$

$$m\ddot{x} - kx + k_{NL}x^3 = u = -K_p x$$

$$m\ddot{x} + [K_p - k]x + k_{NL}x^3 = 0$$

$$H = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}[K_p - k]x^2 + \frac{1}{4}k_{NC}x^4$$

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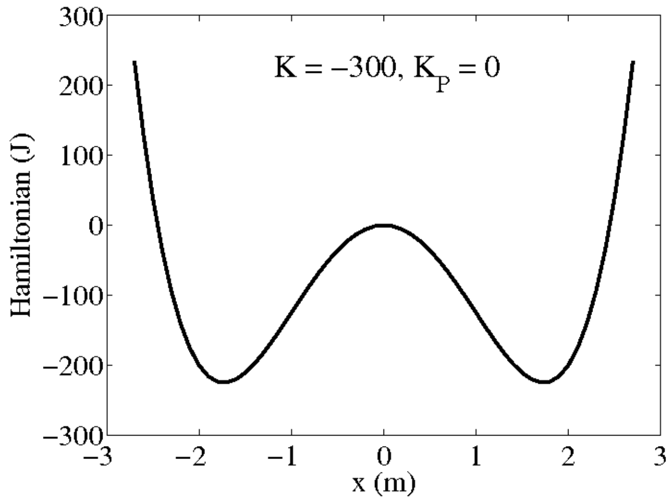
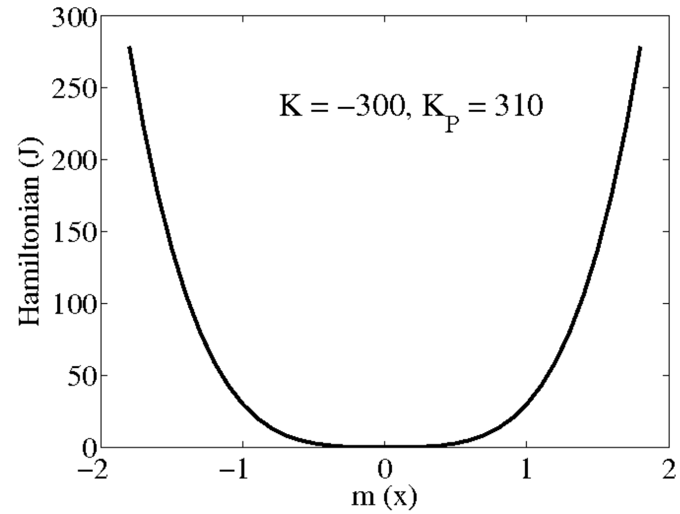
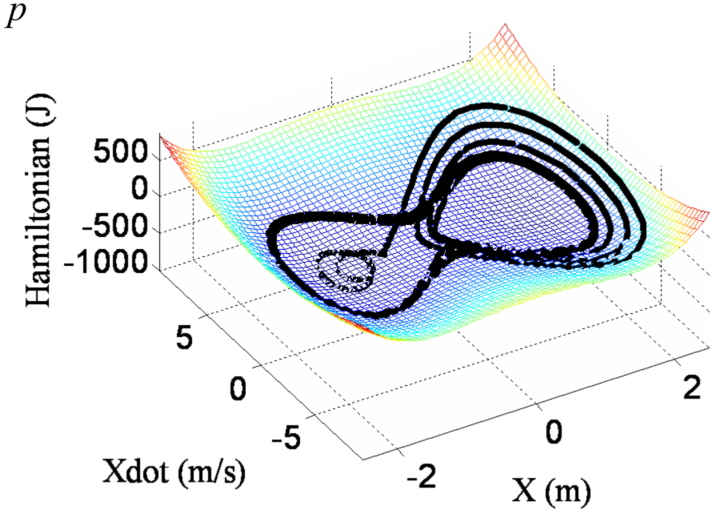
Stability (cont.)

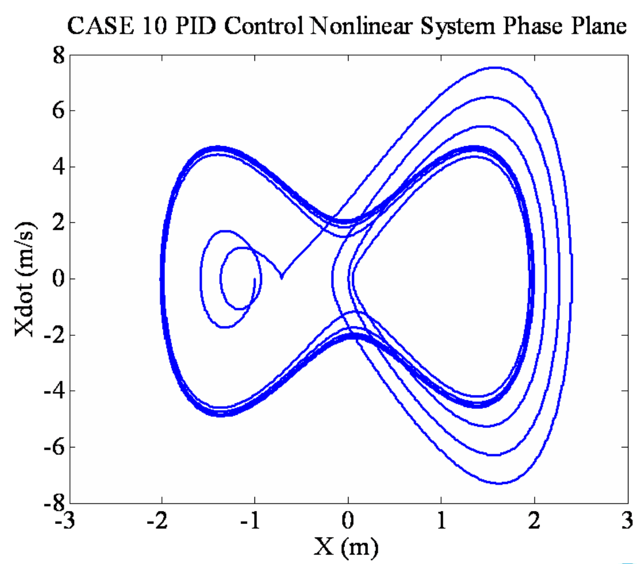




$k > K_p$

$H = 0.5*(K_p+k)*x^2 + 0.5*m*xdot^2+0.25*k_{NL}*x^4$





Stability (cont.)

- Power Flow: Exergy Flow; 3 types– into, dissipated within, and stored in the system; determines the path/trajectory across the energy storage surface

  a) Determines the dynamic stability which is a sufficient condition for stability

  b) Sort Power Terms:

$$\dot{H} = \dot{W} - T_o \dot{S}_i$$

  c) Derivative Feedback: $u = -K_D \dot{x}$

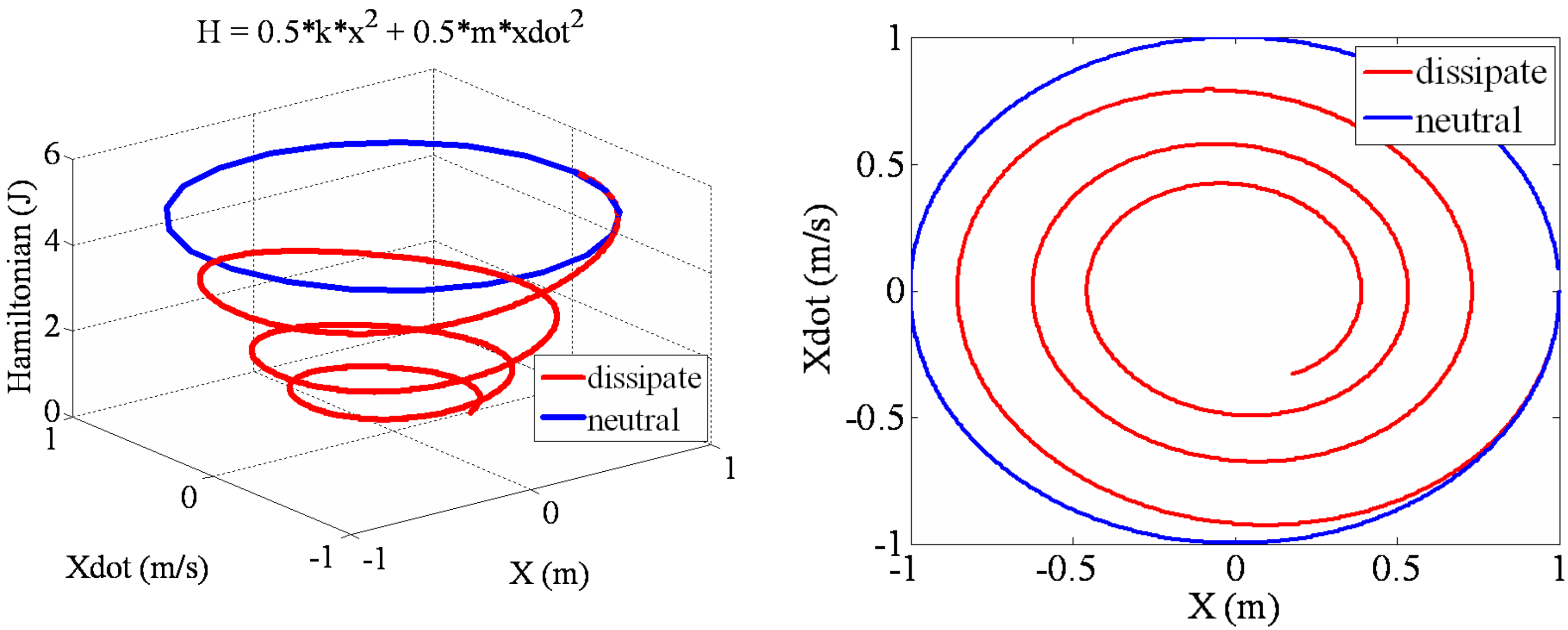
$$\Rightarrow T_o \dot{S}_i = K_D \dot{x}^2$$

  d) Integral Feedback: $u = -K_I \int x dt$

$$\Rightarrow \dot{W} = -K_I \int x dt \dot{x}$$

Stability (cont.)

$$m\ddot{x} + kx = -c\dot{x}$$



$$c > 0, \quad \int_o^{\tau_c} \dot{W} dt < \int_o^{\tau_c} T_o \dot{S}_i dt$$

Dynamically Stable

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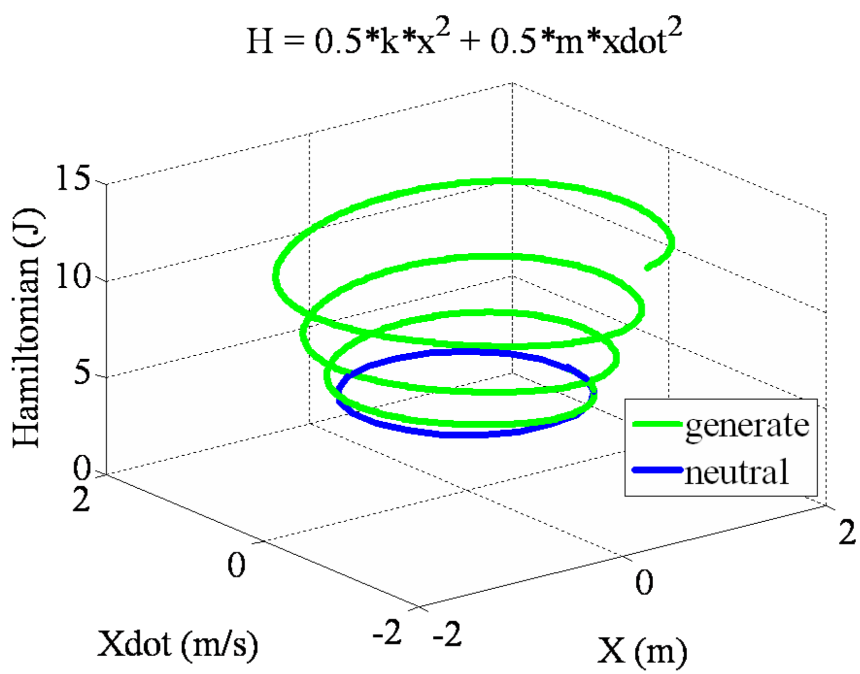
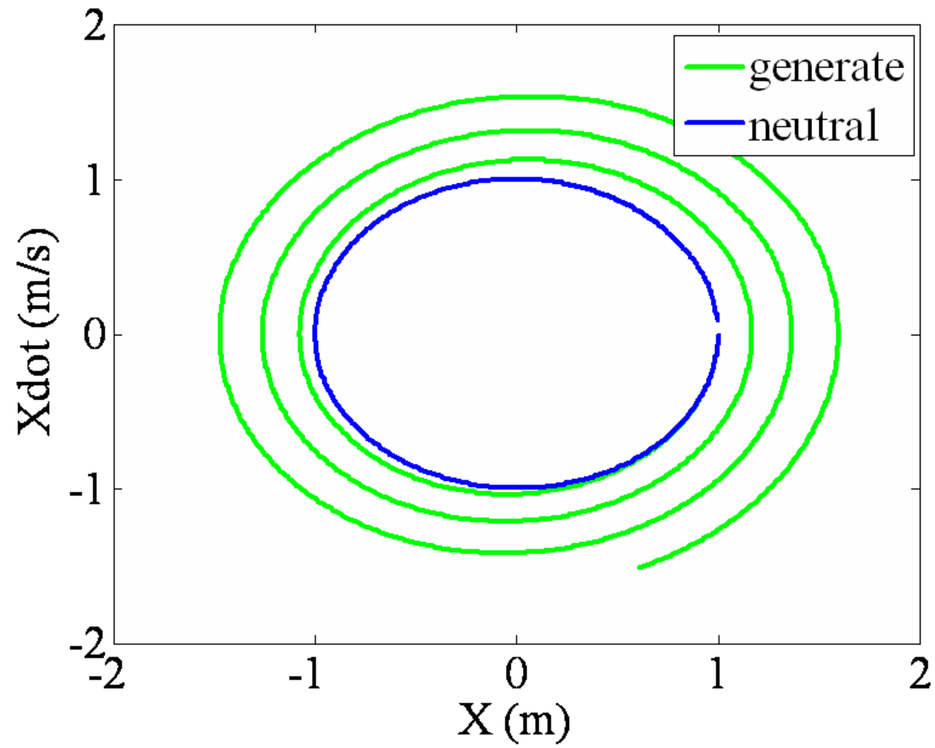
Stability (cont.)

$$m\ddot{x} + kx = c\dot{x}$$





$$c > 0, \quad \int_o^{\tau_c} \dot{W} dt > \int_o^{\tau_c} T_o \dot{S}_i dt$$

Dynamically Unstable

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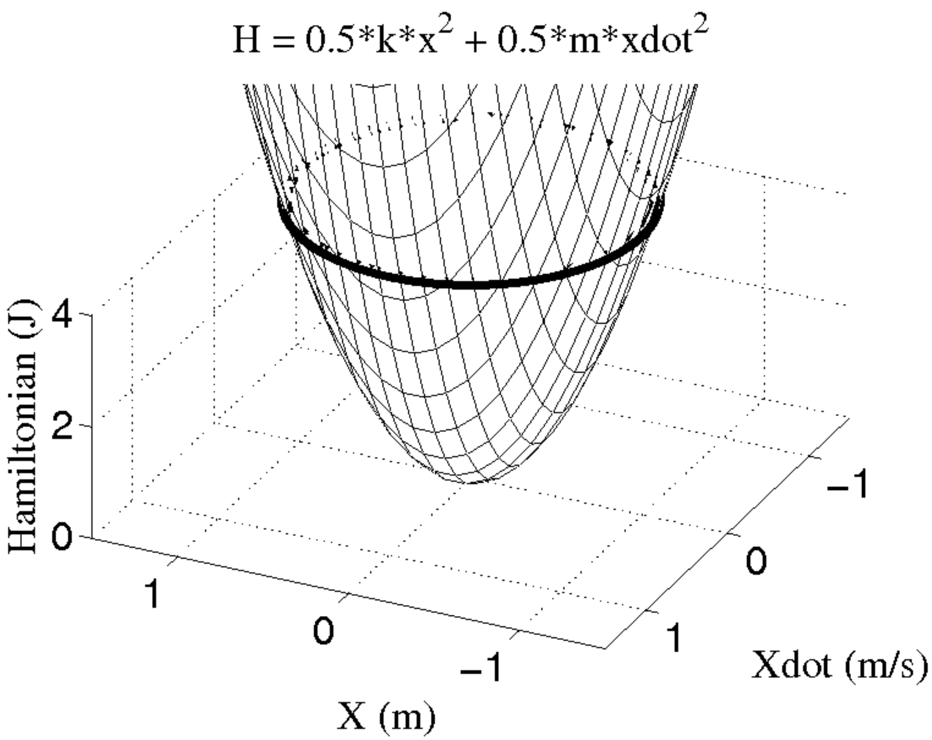
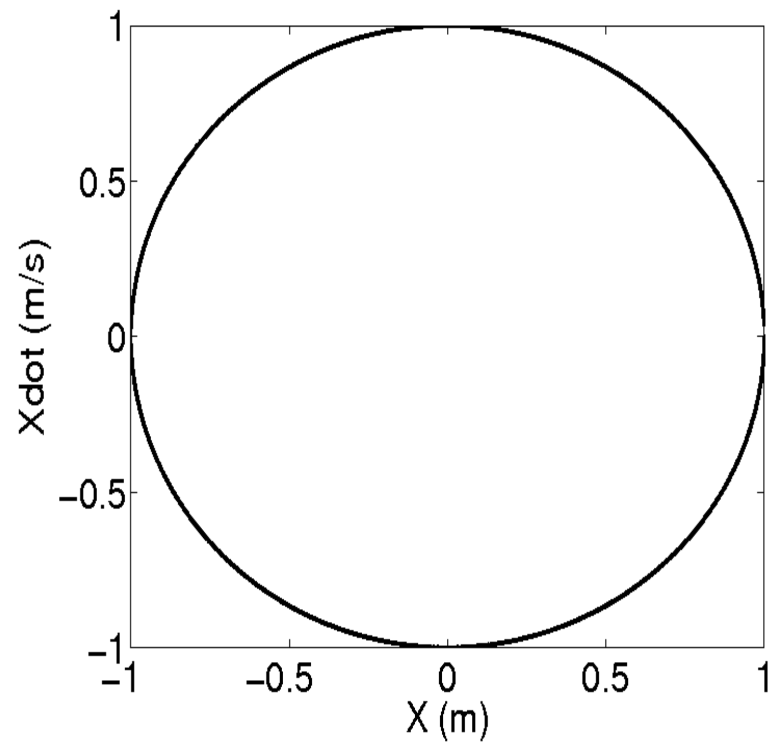
Stability (cont.)

$$m\ddot{x} + kx = 0$$





$$c = 0,\ \int_o^{\tau_c} \dot{W} dt = \int_o^{\tau_c} T_o \dot{S}_i dt$$

Dynamically Neutral Stable

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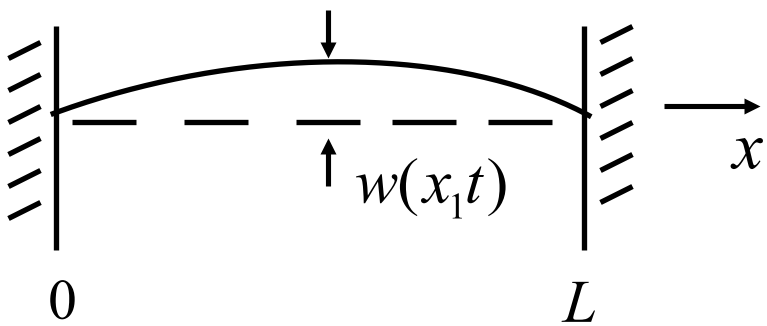
ADVANCED CONTROL DESIGN

- Distributed Parameter/PDE's
- Fractional Calculus
- Optimal
- Robust/ Tracking
- Adaptive/Tracking

Advanced Control Design (cont.)

- Distributed Parameter/PDE's
  - Vibrating String



$\tau =$ Tension in String

$\sigma(x) =$ Mass/unit length

$$\dot{w} = \frac{\partial w}{\partial t}$$

$$w_x = \frac{\partial w}{\partial x}$$

$$w(0,t) = w(L,t) = 0$$

Advanced Control Design (cont.)

a) $$T = \frac{1}{2}\int_0^L \sigma(x)\dot{w}^2 dx$$

$$V = \frac{1}{2}\tau\int_0^L w_x^2 dx$$

b) Non-conservative Distributed Force

$$\Rightarrow W = \int_0^L F(x,t) w dx = work$$

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Advanced Control Design (cont.)

c) Extended Hamilton's Principle

$$\bar{I} = \int_o^t [T - V + W]dt$$

$$= \int_o^t \int_o^L \left[\frac{1}{2}(\sigma\dot{w}^2 - \tau w_x^2) + Fw\right]dxdt$$

$$= \int_0^t \int_0^L f dxdt$$

$$\frac{\partial f}{\partial w} - \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial w_x}\right) - \frac{\partial}{\partial t}\left(\frac{\partial f}{\partial w_t}\right) = 0$$

$$F + \tau w_{xx} - \sigma\ddot{w} = 0$$

$$\Rightarrow \boxed{\sigma\ddot{w} - \tau w_{xx} = F;\ w(0,t) = w(L,t) = 0}$$

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Advanced Control Design (cont.)

d) Static Stability

$$H = T + V = \int_o^L \frac{1}{2}(\sigma \dot{w}^2 + \tau w_x^2)dx > 0$$

for $\sigma, \tau > 0$

e) Dynamic Stability

$$\dot{H} = \int_o^L [\sigma \dot{w}\ddot{w} + \tau w_x \dot{w}_x]dx$$

$$= \int_o^L [\sigma \ddot{w} - \tau w_{xx}]\dot{w}\,dx$$

$$= \int_o^L F\dot{w}\,dx$$

$$\Rightarrow \boxed{\dot{H}_{AVE} = \frac{1}{\tau_c}\oint_{\tau_c}\left[\int_o^L F\dot{w}dx\right]dt = 0;\quad \text{Limit Cycle}}$$

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Advanced Control Design (cont.)

- Controller: $F = K_p w_{xx} - K_I \int wdt - K_D \dot{w}$

$$H_c = T + V = \int_0^L \frac{1}{2}[\sigma \dot{w}^2 + (\tau + K_p) w_x^2] dx > 0$$

$$\sigma \ddot{w} - (\tau + K_p) w_{xx} = -K_I \int wdt - K_D \dot{w}$$

$$\dot{H}_c = \int_0^L [\sigma \dot{w} \ddot{w} + (\tau + K_p) w_x \dot{w}_x] dx$$

$$= \int_0^L [\sigma \ddot{w} - (\tau + K_p) w_{xx}] \dot{w}\, dx$$

$$= \int_0^L [-K_I \int wdt - K_D \dot{w}] \dot{w}\, dx$$

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Advanced Control Design (cont.)

- Fractional Calculus
  - Differintegral: Derivative and Integral to Arbitrary order

    a) Oldham and Spanier [Ref. Oldham]:

$$\frac{d^q f}{[d(x-a)]^q} = \lim_{N\to\infty}\left\{\frac{\left[\frac{x-a}{N}\right]^{-q}}{\Gamma(-q)}\sum_{j=0}^{N-1}\frac{\Gamma(j-q)}{\Gamma(j+1)} f\left(x - j\left[\frac{x-a}{N}\right]\right)\right\}$$

q= Arbitrary real number

$\Gamma(j+1) =$ Gamma Function

b) Podlubny [Ref. Podlubny]:

$$_{a}D_t^q f(t) = \lim_{\substack{h\to o\\ nh=t-a}} h^{-q}\sum_{r=o}^{n}(-1)^r \binom{q}{r} f(t-rh)$$

h = small change in t

$$\binom{q}{r} = \frac{q(q-1)(q-2)\cdots(q-r+1)}{r!}$$

[Ref. Podlubny] I. Podlubny, **Fractional Differential Equations**, Academic Press, N.Y., 1999.

[Ref. Oldham] K.B. Oldham and J. Spanier, **The Fractional Calculus**, Academic Press, N.Y., 1974.

Advanced Control Design (cont.)

c) First Order Fractional Difference approximation

$$ {}_{o}\widetilde{D}_{t}^{q} f(t) = h^{-q} \sum_{k=o}^{[t/h]} w_{k}^{(q)} f(t-kh) $$

$$ w_{k}^{(q)} = (-1)\binom{q}{k} \quad \text{for} \quad k = 0,1,2,\ldots $$

$$ w_{o}^{(q)} = 1 \text{ and } \quad w_{k}^{(q)} = (1 - \frac{q+1}{k}) w_{k-1}^{(q)} \quad \text{for} \quad k = 1,2,3,\ldots $$

d) Inhomogeneous Bagley-Torvik Equation

$$ A\ddot{y}(t) + B\,{}_{o}D_{t}^{3/2} y(t) + Cy(t) = u(t), \forall t > 0 $$

$$ y(0) = \dot{y}(0) = 0 $$

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Advanced Control Design (cont.)

e) Generalized to

$$M\ddot{x}(t) + K_{q\,o}D_t^q x(t) + Kx(t) = u(t), \forall t > 0$$
$$x(0) = \dot{x}(0) = 0$$

f) First-Order Approximation/numerical algorithm

$$x_o = x_1 = 0$$

$$x_m = \frac{h_2(u_m - Kx_{m-1}) + M(2x_{m-1} - x_{m-2})}{M + K_q h^{(2-q)}} + \frac{-K_q h^{(2-q)} \sum_{j=1}^{m} w_j^{(q)} x_{m-j}}{M + K_q h^{(2-q)}}$$

For $m = 2,3,\ldots$

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Advanced Control Design (cont.)

g) Sort power terms:

$$M\ddot{x} + C\dot{x} + Kx = u$$

$$u = -K_p x - K_I \int x dt - K_D \dot{x}$$

$$H = M\dot{x}^2 + \frac{1}{2}(K + K_p)x^2 > 0$$

$$\dot{H} = \frac{1}{2}[M\ddot{x} + (K + K_p)x]\dot{x}$$

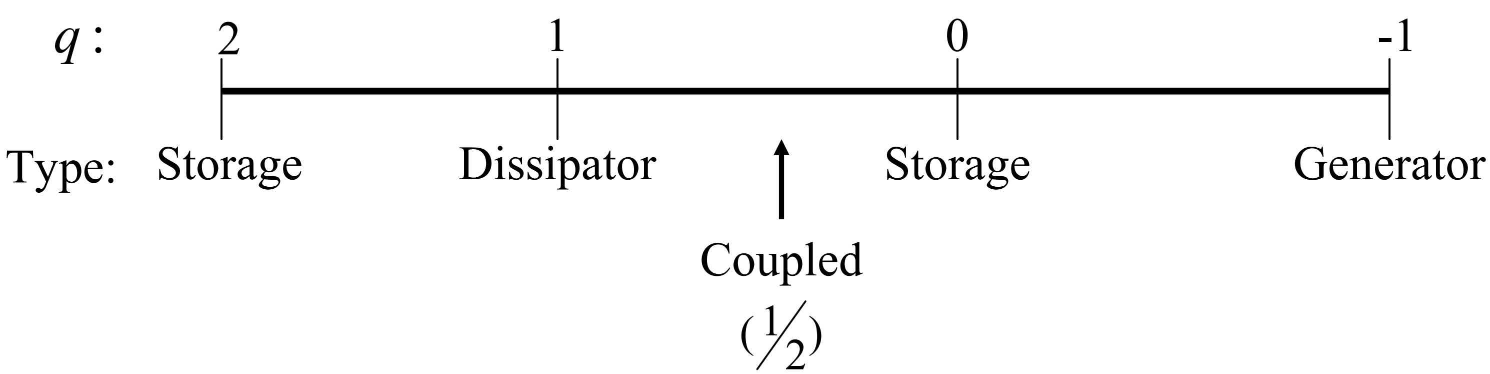
$$= [-K_I \int x dt - K_D \dot{x}]\dot{x}$$

Advanced Control Design (cont.)

Storage: $\dfrac{d^2 x}{dt^2}$ and $\dfrac{d^o x}{dt^o}$

Dissipator: $\dfrac{d^1 x}{dt^1}$

Generator: $\dfrac{d^{-1} x}{dt^{-1}}$



Advanced Control Design (cont.)

$$u_q = -K_{q\,o} D_t^q x(t) \quad \begin{cases} q = 0,2: & \text{Storage} \\ 2 > q > 0: & \text{Dissipator} \\ 0 > q \geq -1: & \text{Generator} \end{cases}$$

h) Lag – Stabilized Controller [Ref. Robinett]

$$M\ddot{x} + Kx = u(t) = K_p x(t - \tau_d)$$

For $K_p, \tau_d > 0$

$\Rightarrow$ Equivalent to positive proportional and negative derivative feedback

$\Rightarrow$ Shifted Sinusoids

$$u \cong K_p x - K_D \dot{x}$$

[Ref. Robinett] R.D. Robinett III, B.J. Peterson, J.C. Fahrenholtz, *Lag-Stabilized Force Feedback Damping*, Journal of Intelligent and Robotic Systems, Vol. 21, Issue 3, March 1998, PP. 277-285.

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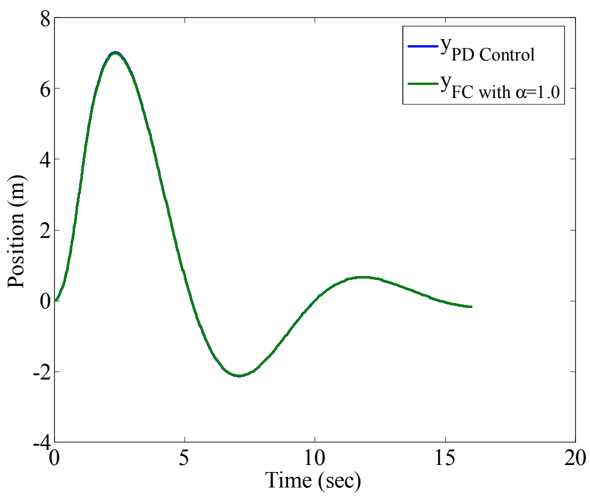
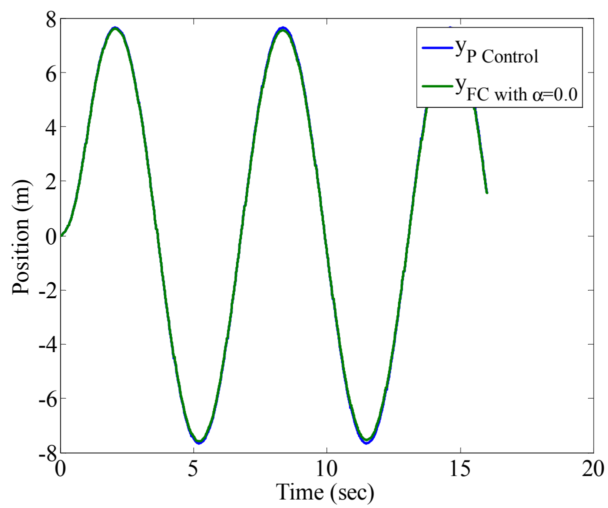
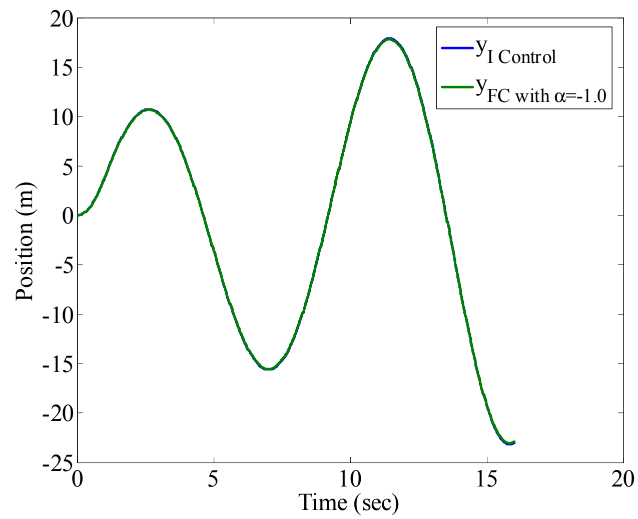
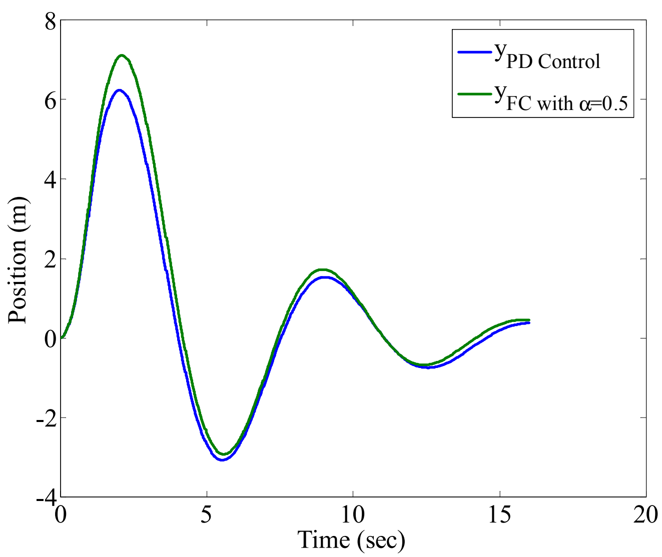
Advanced Control Design (cont.)
The Fractional Calculus Approach

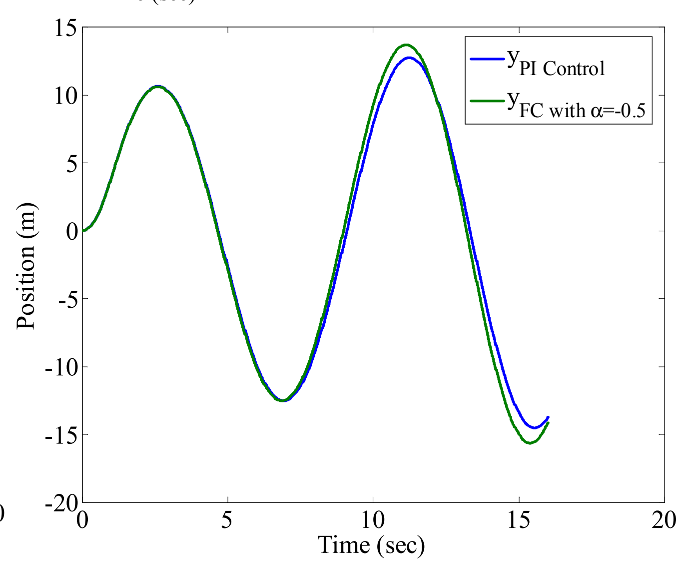
- **Fractional calculus compared with PID type controllers**











$$\alpha = q$$

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Advanced Control Design (cont.)

- Optimal [Ref. Ogata] -

$$\dot{\underline{x}} = \underline{f}(\underline{x},\underline{u}); \quad \underline{u} = \underline{g}(\underline{x})$$

$$\dot{\underline{x}} = \underline{f}(\underline{x},\underline{g}(\underline{x})) \rightarrow$$ Asymptotically Stable to $\underline{x}=\underline{0}$

$$J = \int_o^\infty L(\underline{x},\underline{u})dt$$

$$\widetilde{H}(\underline{x},\underline{u}) = \dot{\mathscr{V}} + L(\underline{x},\underline{u}) \geq 0$$

$$\min_{\underline{u}} \widetilde{H}(\underline{x},\underline{u}) = \min_{\underline{u}}[\dot{\mathscr{V}} + L(\underline{x},\underline{u})] = \dot{\mathscr{V}}\Big|_{\underline{u}=\underline{u}_1} + L(\underline{x},\underline{u}) = 0$$

$$\Rightarrow \dot{\mathscr{V}}\Big|_{\underline{u}=\underline{u}_1} = -L(\underline{x},\underline{u}_1)$$

Note:
$$\mathscr{V}(\underline{x}(\infty)) = 0 = \mathscr{V}(\underline{x}(0)) - \int_o^\infty L(\underline{x}(t),\underline{u}_1(t))dt$$

$$\mathscr{V}(\underline{x}(0)) = \int_o^\infty L(\underline{x}(t)),\underline{u}_1(t))dt$$

[Ref. Ogata] K. Ogata, **Modern Control Engineering**, Prentice-Hall, Inc., N.J., 1970.

Advanced Control Design (cont.)

a) Example #1: $m\ddot{x} + c\dot{x} + kx = u$

$$J = -H = -\frac{1}{2}[m\dot{x}^2 + kx^2]_o^\infty = \frac{1}{2}[m\dot{x}_o^2 + kx_o^2]$$

$$= \int_o^\infty -[m\ddot{x} + kx]\dot{x}dt = \int_o^\infty -[u - c\dot{x}]\dot{x}dt$$

$$\dot{\mathscr{V}} = [u - c\dot{x}]\dot{x} < 0 \Rightarrow \boxed{u < c\dot{x}}$$

$$\mathscr{V} = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$$

Advanced Control Design (cont.)

b) Example #2: $m\ddot{x} + kx = u$

$$J = \int_o^\infty [k_1\dot{x}^2 + k_2 x^2 + k_3 u^2]dt$$

$$\dot{\mathscr{V}} = -L(x,u) = -k_1\dot{x}^2 - k_2 x^2 - k_3 u^2$$

$$\mathscr{V} = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 \Rightarrow \dot{\mathscr{V}} = [m\ddot{x} + kx]\dot{x} = u\dot{x}$$

$$-u\dot{x} = k_1\dot{x}^2 + k_2 x^2 + k_3 u^2$$

$$* k_1 = k_2 = 0: \quad u(k_3 u + \dot{x}) = 0, u = -\frac{1}{k_3}\dot{x}$$

$$* u_{1,2} = \frac{1}{2k_3}[-\dot{x} \pm (\dot{x}^2 - 4k_3\langle k_1\dot{x}^2 + k_2 x^2\rangle)^{1/2}]$$

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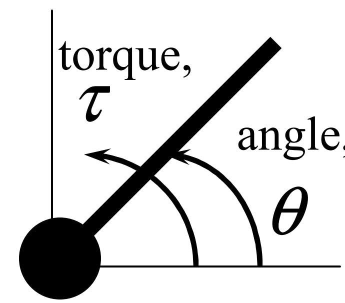
Advanced Control Design (cont.)

– Fixed Final Time: $J = \int_o^{t_f} \frac{1}{2}\left[(ux_2)^2 + u^2\right] dt$

$$I\ddot{\theta} = \tau = u; \dot{\underline{x}} = \underline{f}(\underline{x},u); \widetilde{H} = L + \underline{\lambda}^T \underline{f}; \underline{x} = (\theta, \dot{\theta})^T$$

$$\dot{\underline{x}} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} \dot{\theta} \\ \ddot{\theta} \end{pmatrix} = \begin{pmatrix} x_2 \\ u/I \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 1/I \end{Bmatrix} u = A\underline{x} + Bu$$



I, inertia

Horizontal Link

$$\widetilde{H} = \frac{1}{2}(ux_2)^2 + \frac{1}{2}u^2 + \lambda_1 x_2 + \lambda_2 \frac{u}{I}$$

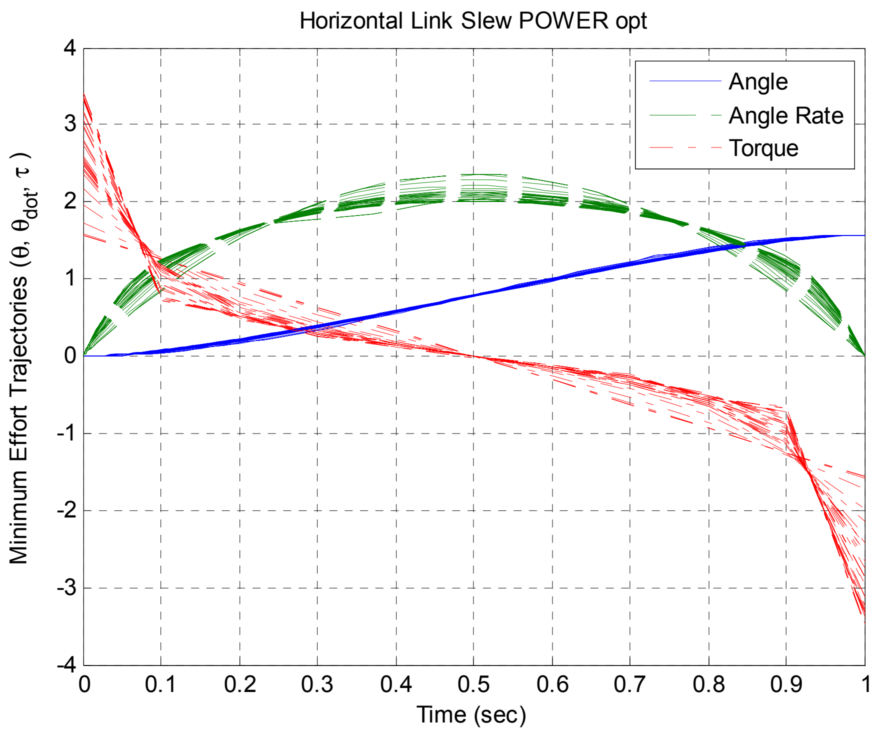
$$0 = \frac{\partial \widetilde{H}}{\partial u} = ux_2^2 + u + \frac{1}{I}\lambda_2 \Rightarrow u = -\lambda_2 / I(x_2^2 + 1)$$

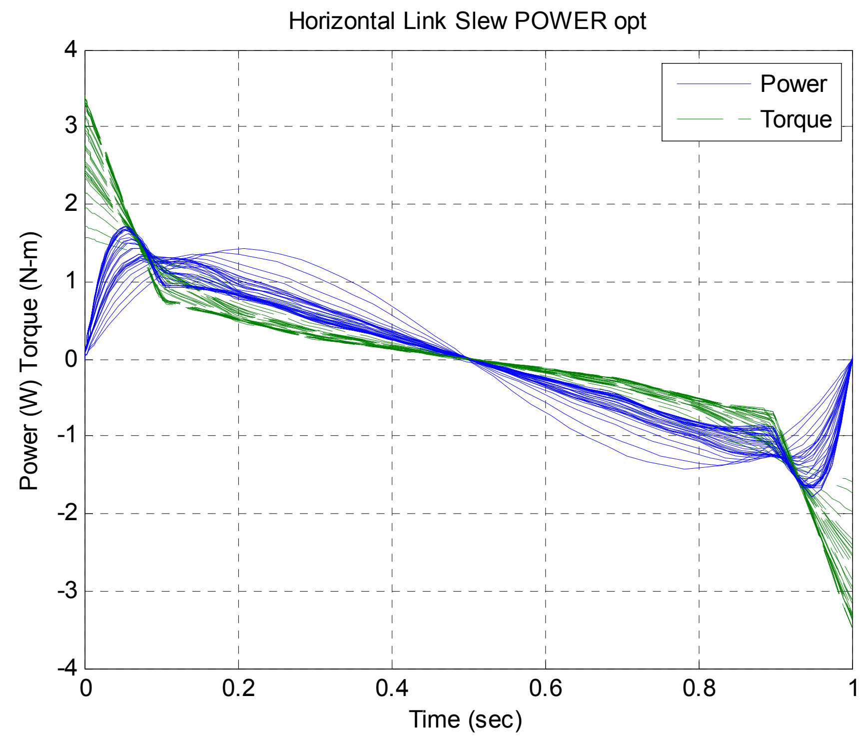
$$\begin{pmatrix} \dot{\lambda}_1 \\ \dot{\lambda}_2 \end{pmatrix} = \begin{pmatrix} -\partial \widetilde{H} / \partial x_1 \\ -\partial \widetilde{H} / \partial x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ -u^2 x_2 - \lambda_1 \end{pmatrix} \Rightarrow \begin{matrix} \lambda_1 = \lambda_{1_0} = const. \\ \dot{\lambda}_2 = -u^2 x_2 - \lambda_{1_o} \end{matrix}$$

$$\dot{x}_2 = \frac{1}{I}u = {}^{-\lambda_2}\!\big/_{I^2(x_2^2+1)}$$

Advanced Control Design (cont.)

- Homotopy from i) min. energy to ii) hybrid min. energy plus min. power, to iii) min. power cost functions





$$J = \phi(\xi) = \frac{1}{2}\int_{o}^{t_f} \left[W_1 \alpha (u(\xi) x_2)^2 + W_2 (1-\alpha) u^2(\xi) \right] dt$$

$$\underline{\dot{x}} = A\underline{x} + Bu$$

$$\Psi_1(\xi) = \theta(t_f) - \theta_{desired} = 0$$

$$\Psi_2(\xi) = \dot{\theta}(t_f) = 0$$

$$\theta(t_0) = \dot{\theta}(t_0) = 0$$

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Advanced Design Control (cont.)

- Robust / Tracking
  - Nonlinear Model: $m\ddot{x} + g(x) = -f(\dot{x}) + u; \quad g(x) = \frac{\partial V(x)}{\partial x}$

a) Hamiltonian: $H = \frac{1}{2} m(\dot{x} - \dot{x}_R)^2 + V(x - x_R)$

$$\dot{H} = \left[m(\ddot{x} - \ddot{x}_R) + g(x - x_R) \right](\dot{x} - \dot{x}_R)$$

$$= \left[-m\ddot{x}_R + g(x - x_R) - g(x) - f(\dot{x}) + u \right](\dot{x} - x_R)$$

$$u = u_{REF} + \Delta u$$

$$u_{REF} = \hat{m}\ddot{x}_R - \hat{g}(x - x_R) + \hat{g}(x) + \hat{f}(\dot{x}); \quad \hat{g}(x) = \frac{\partial \hat{V}(x)}{\partial x}$$

$$\Delta u = -\hat{g}(x - x_R) - K_I \int (x - x_R) dt - K_D (\dot{x} - \dot{x}_R)$$

Robustness/Tracking addressed in the RLC example

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Advanced Design Control (cont.)

– Static Stability: $\hat{H} = \frac{1}{2} m(\dot{x} - \dot{x}_R)^2 + V(x - x_R) + \hat{V}(x - x_R) > 0$

– Dynamic Stability: $\dot{\hat{H}} = [m(\ddot{x} - \ddot{x}_R) + g(x - x_R) + \hat{g}(x - x_R)](\dot{x} - \dot{x}_R)$

$$= [-m\ddot{x}_R + g(x - x_R) + \hat{g}(x - x_R) - g(x) - f(\dot{x}) + u](\dot{x} - \dot{x}_R)$$

$$= [\langle \hat{m} - m \rangle \ddot{x}_R + \langle g(x - x_R) - \hat{g}(x - x_R) \rangle + \langle \hat{g}(x) - g(x) \rangle + \langle \hat{f}(\dot{x}) - f(\dot{x}) \rangle$$

$$- K_I \int (x - x_R) dt - K_D (\dot{x} - \dot{x}_R) \;](\dot{x} - \dot{x}_R)$$

– Robustness:

$$[K_D(\dot{x} - \dot{x}_R)^2]_{AVE} > \{ \; [-K_I \int (x - x_R) dt + \langle \hat{m} - m \rangle \ddot{x}_R + \langle g(x - x_R) - \hat{g}(x - x_R) \rangle + \langle \hat{g}(x) - g(x) \rangle$$

$$+ \langle \hat{f}(\dot{x}) - f(\dot{x}) \rangle \;](\dot{x} - \dot{x}_R) \; \}_{AVE}$$

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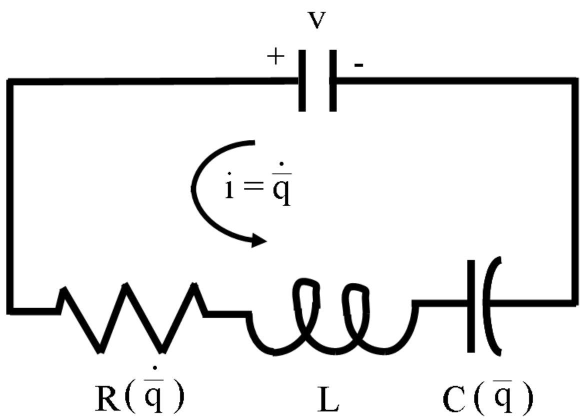
Advanced Design Control (cont.)
Adaptive/Tracking: Simple RLC Model



EOM:

$$L\ddot{\bar{q}} + R(\dot{\bar{q}}) + \frac{1}{C}(\bar{q}) = v = v_0 \cos \Omega t$$

$$\dot{\bar{q}} R(\dot{\bar{q}}) > 0 \text{ for } \dot{\bar{q}} \neq 0$$

$$\bar{q} C(\bar{q}) > 0 \text{ for } \bar{q} \neq 0$$

Hamiltonian and time derivative:

$$H = \frac{1}{2} L \dot{\bar{q}}^2 + \int_0^{\bar{q}} C(y)dy$$

$$\dot{H} = [L\ddot{\bar{q}} + C(\bar{q})]\dot{\bar{q}} = [v - R(\dot{\bar{q}})]\dot{\bar{q}} = vi - R(i)i$$

Goal:

Design applied voltage that creates desired limit cycle (e.g., 60 Hz)

$$\oint [L\ddot{\bar{q}}\dot{\bar{q}} + C(\bar{q})\dot{\bar{q}}]dt = 0$$

Where storage terms matched

$$\oint [vi - R(i)i]dt = 0$$

vi maximally used by load

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Advanced Control Design (cont.)

- Goal: 60Hz Limit Cycle

$$\oint_{\tau_c} \left[L\ddot{\bar{q}} + C(\bar{q})\right]\dot{\bar{q}}\,dt = \oint_{\tau_c} \left[v - R(\dot{\bar{q}})\right]\dot{\bar{q}}\,dt = 0$$

for matched storage terms and maximal real power

c) Specific Nonlinearities:

$$L\ddot{\bar{q}} + \left[\frac{1}{C}\bar{q} + \frac{1}{C_{NL}}\bar{q}^3\right] = v - R\dot{\bar{q}} - R_{NL}\,sign(\dot{\bar{q}})$$

d) Tracking Hamiltonian with controller and information potentials:

$$\overline{H} = \frac{1}{2}L(\dot{\bar{q}} - \dot{\bar{q}}_R)^2 + \frac{1}{2}\left(\frac{1}{C} + K_{CAP}\right)(\bar{q} - \bar{q}_R)^2 + \frac{1}{4}\frac{1}{C_{NL}}(\bar{q} - \bar{q}_R)^4 + \frac{1}{2}\tilde{\Phi}^T\Gamma^{-1}\tilde{\Phi}$$

$$= T + V + V_{cap} + V_I$$

Advanced Design Control (cont.)
Adaptive PID Control Strategy: Improve Performance

$$H = T + V + V_{cap} + V_I$$

$$H = \frac{1}{2}L\dot{e}^2 + \frac{1}{2}\frac{1}{C}e^2 + \frac{1}{2}K_{cap}e^2 + \frac{1}{2}\tilde{\Phi}^T\Gamma^{-1}\tilde{\Phi}$$

$$\dot{H} = [-L\ddot{\bar{q}}_R - \frac{1}{C}\bar{q}_R + K_{cap}e - R\dot{\bar{q}} + v] + \tilde{\Phi}\Gamma^{-1}\dot{\tilde{\Phi}}$$

$$e = (\bar{q} - \bar{q}_R)$$

Controller selected as:

$$v = v_R + \Delta v$$

$$v_R = \hat{L}\ddot{\bar{q}}_R + \hat{R}\dot{\bar{q}} + \frac{\hat{1}}{C}\bar{q}_R$$

Advanced Design Control (cont.)
One Wants to Design for Ideal 60 Hz Condition

- Ideal

$$\omega^2 = \frac{1}{LC} = \Omega^2$$

- Then

$$v_R \dot{\bar{q}}_R = R\dot{\bar{q}}^2$$

- Requires feedback control if

$$\omega^2 = \frac{1}{LC} \neq \Omega^2$$

- To maintain a power factor of 1

- Select feedback portion as

$$\Delta v = -K_{cap} e - K_{gen} \int_0^t e\, d\tau - K_{diss} \dot{e}$$

Advanced Design Control (cont.)
Incorporate into Hamiltonian Time Derivative

$$\dot{H} = [-K_{gen}\int_0^t e\,d\tau - K_{diss}\dot{e}]\dot{e} + \tilde{\Phi}^T[Y^T\dot{e} + \Gamma^{-1}\dot{\tilde{\Phi}}]$$

Where

$$Y\tilde{\Phi} = (\hat{L} - L)\ddot{\bar{q}}_R + (\frac{\hat{1}}{C} - \frac{1}{C})\bar{q}_R + (\hat{R} - R)\dot{\bar{q}}$$

$$\tilde{\Phi}^T = [(\hat{L} - L) \quad (\frac{\hat{1}}{C} - \frac{1}{C}) \quad (\hat{R} - R)]$$

$$\dot{\tilde{\Phi}}^T = \dot{\hat{\Phi}}^T = [\dot{\hat{L}} \quad \frac{\dot{\hat{1}}}{C} \quad \dot{\hat{R}}]$$

$$Y = [\ddot{\bar{q}}_R \quad \bar{q}_R \quad \dot{\bar{q}}]$$

Advanced Design Control (cont.)
Identify Adaptive Parameter Update Equations

$$\dot{\hat{L}} = -\gamma_1 \ddot{\bar{q}}_R \dot{e}$$

$$\frac{\dot{\hat{1}}}{C} = -\gamma_2 \bar{q}_R \dot{e}$$

$$\dot{\hat{R}} = -\gamma_3 \dot{\bar{q}} \dot{e}$$

$$\dot{e} = (\dot{\bar{q}} - \dot{\bar{q}}_R)$$

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Advanced Design Control (cont.)
Perfect Parameter Matching: Nonlinear Stability Boundary

$$\oint_{\tau}[-K_{gen}\int_{0}^{t} e\,d\tau']\dot{e}\,dt < K_{diss}\oint_{\tau}\dot{e}^{2}\,dt$$

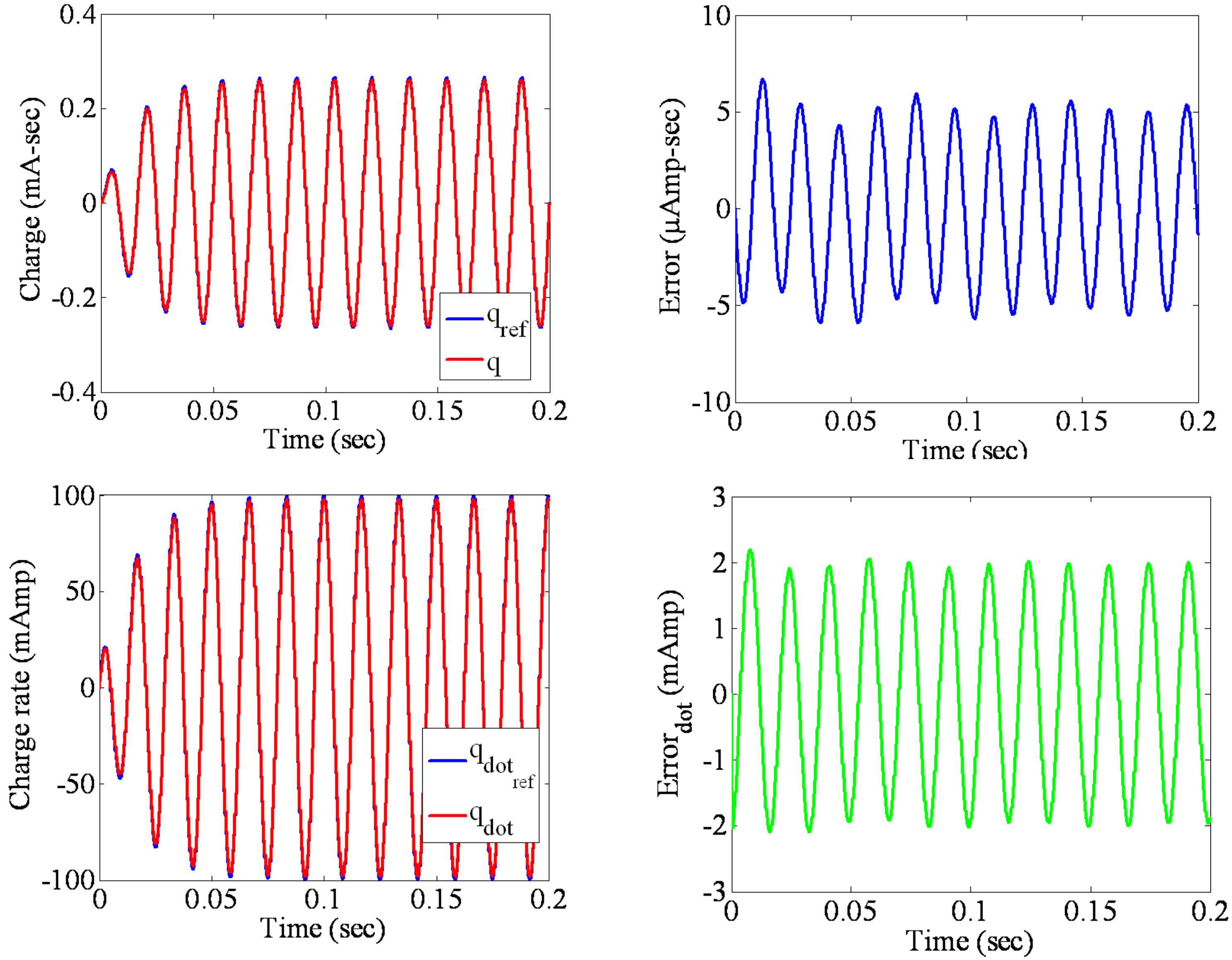
Results in asymptotically stable (passivity) solution

Advanced Design Control (cont.)
Robustness/Tracking vs Adaptive/Tracking

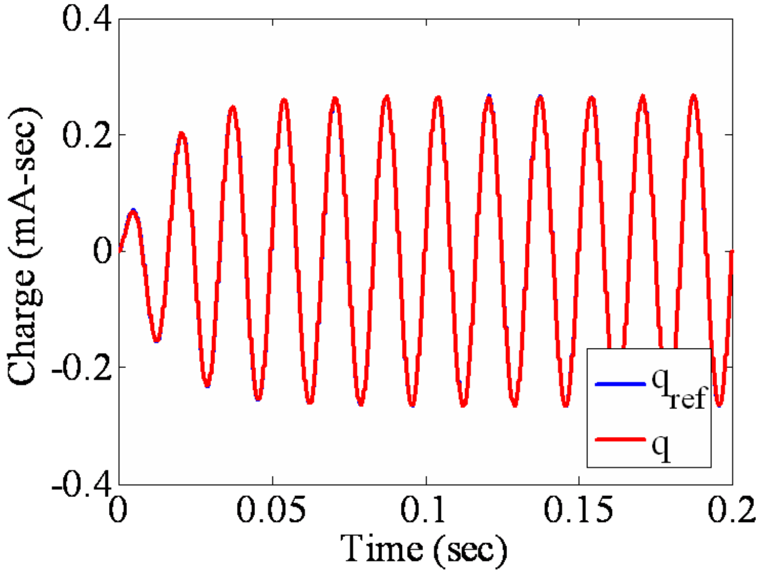
Gain	Robustness	Adaptive
K_cap	10000	750
K_diss	500	10
K_gen	5e6	5e6

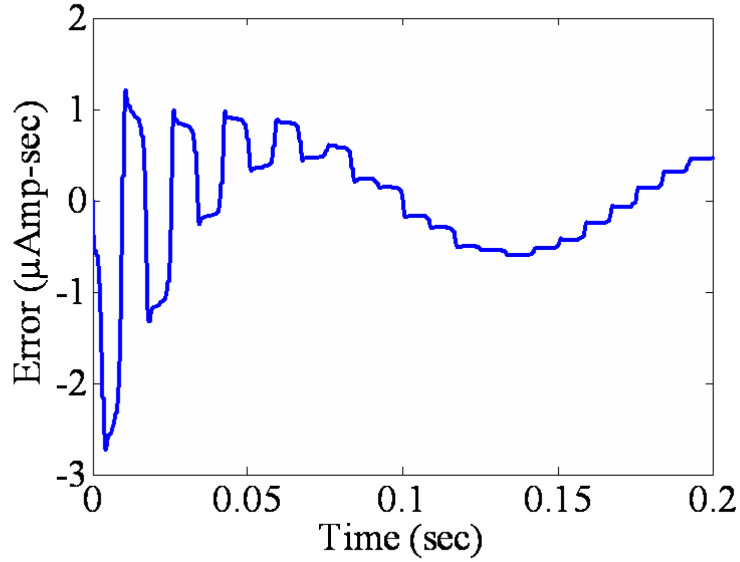
Higher controller gains achieved Robustness at expense of increased power usage

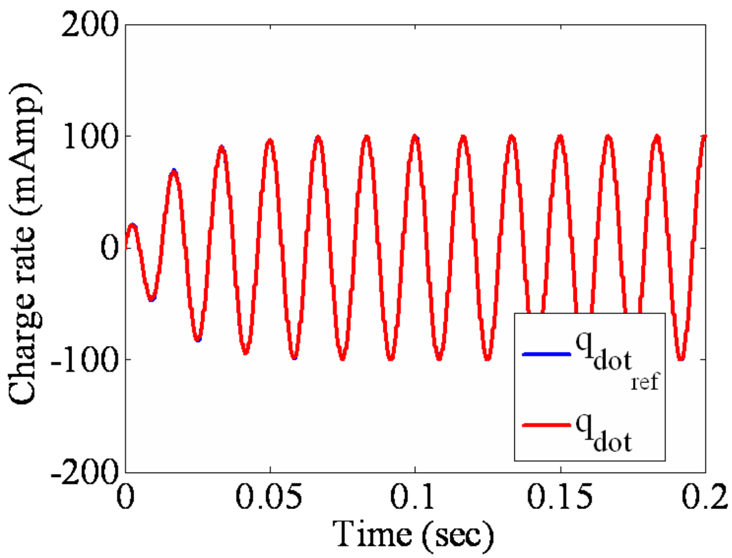
Advanced Design Control (cont.)
Linear RLC Network w/ PID Numerical Results

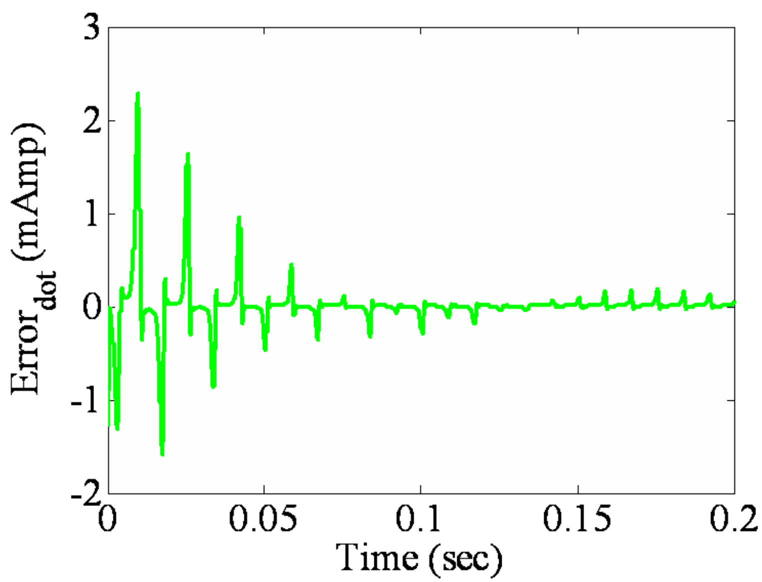


Advanced Design Control (cont.)
Linear RLC Network w/ PID/Adaptive Numerical Results









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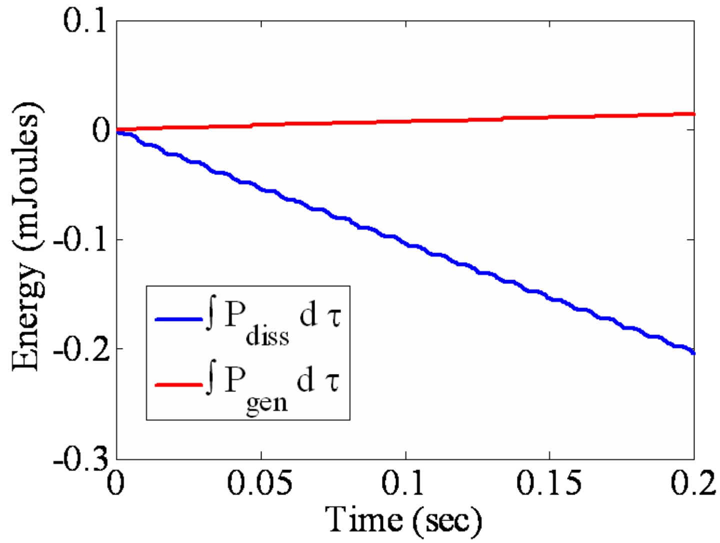
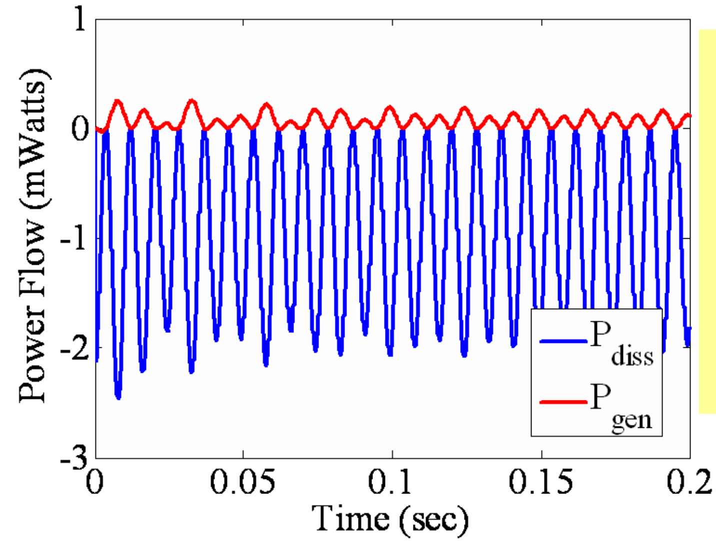
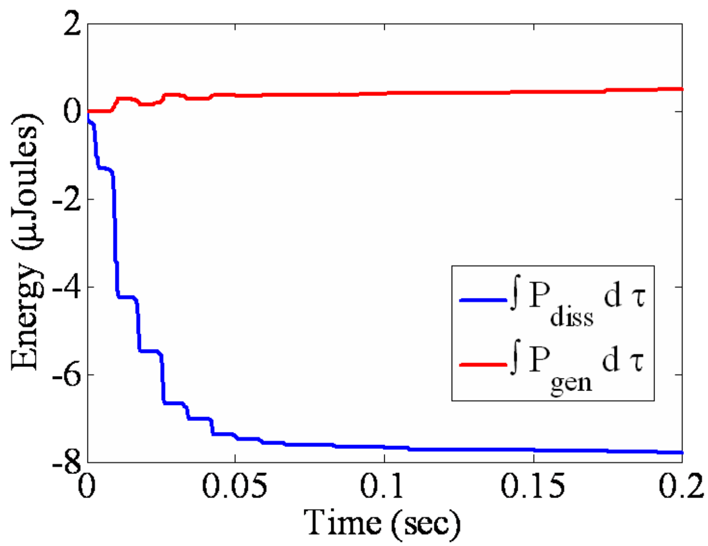
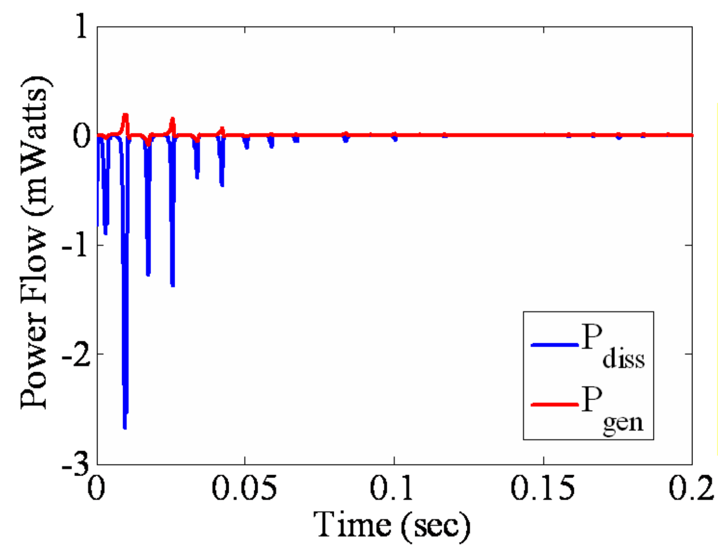
Advanced Design Control (cont.)
Linear RLC Network Energy & Power Flow Results





PID Controller





PID/Adaptive

PID /adaptive with information flow more efficient than PID with physical flow alone

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Advanced Design Control (cont.)
Next Model Includes Nonlinearities

$$L\ddot{\bar{q}} + R\dot{\bar{q}} + \frac{1}{C}\bar{q} + \frac{1}{C_{NL}}\bar{q}^3 + R_{NL}\,sign(\dot{\bar{q}}) = v$$

ADDED: Nonlinear Capacitance and Nonlinear Resistance

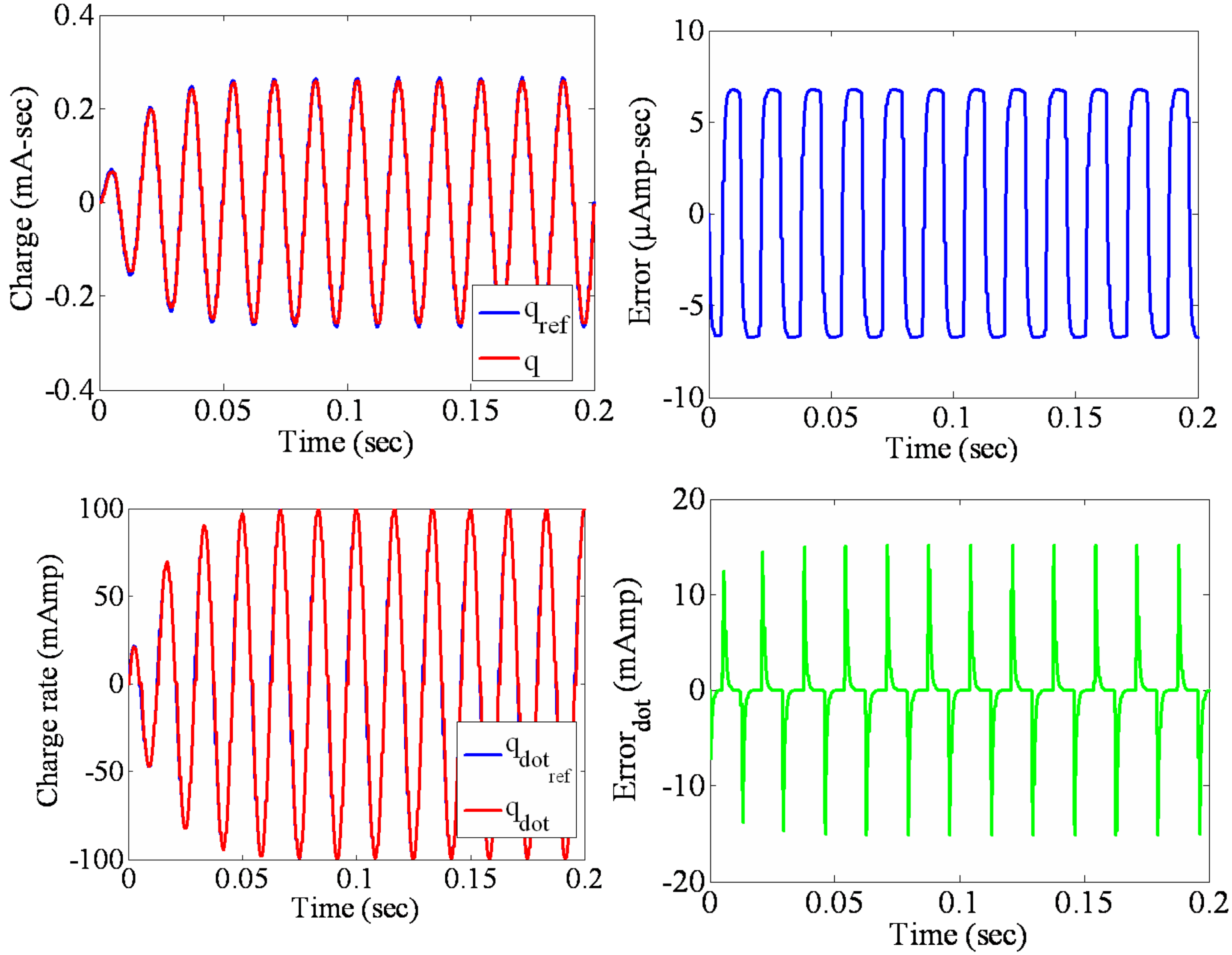
$$v_{ref} = \hat{L}\ddot{\bar{q}}_R + \frac{\hat{1}}{C}\bar{q}_R + \frac{1}{C_{NL}}(\bar{q}^3 - e^3) + \hat{R}\dot{\bar{q}} + \hat{R}_{NL}\,sign(\dot{\bar{q}})$$

$$\frac{\dot{\hat{1}}}{C_{NL}} = -\gamma_4[\bar{q}^3 - \dot{e}^3]\dot{e}$$

$$\dot{\hat{R}}_{NL} = -\gamma_5\,sign(\dot{\bar{q}})\dot{e}$$

$$e = (\bar{q} - \bar{q}_R) \quad \dot{e} = (\dot{\bar{q}} - \dot{\bar{q}}_R)$$

Advanced Design Control (cont.)
Nonlinear RLC Network w/ PID Numerical Results



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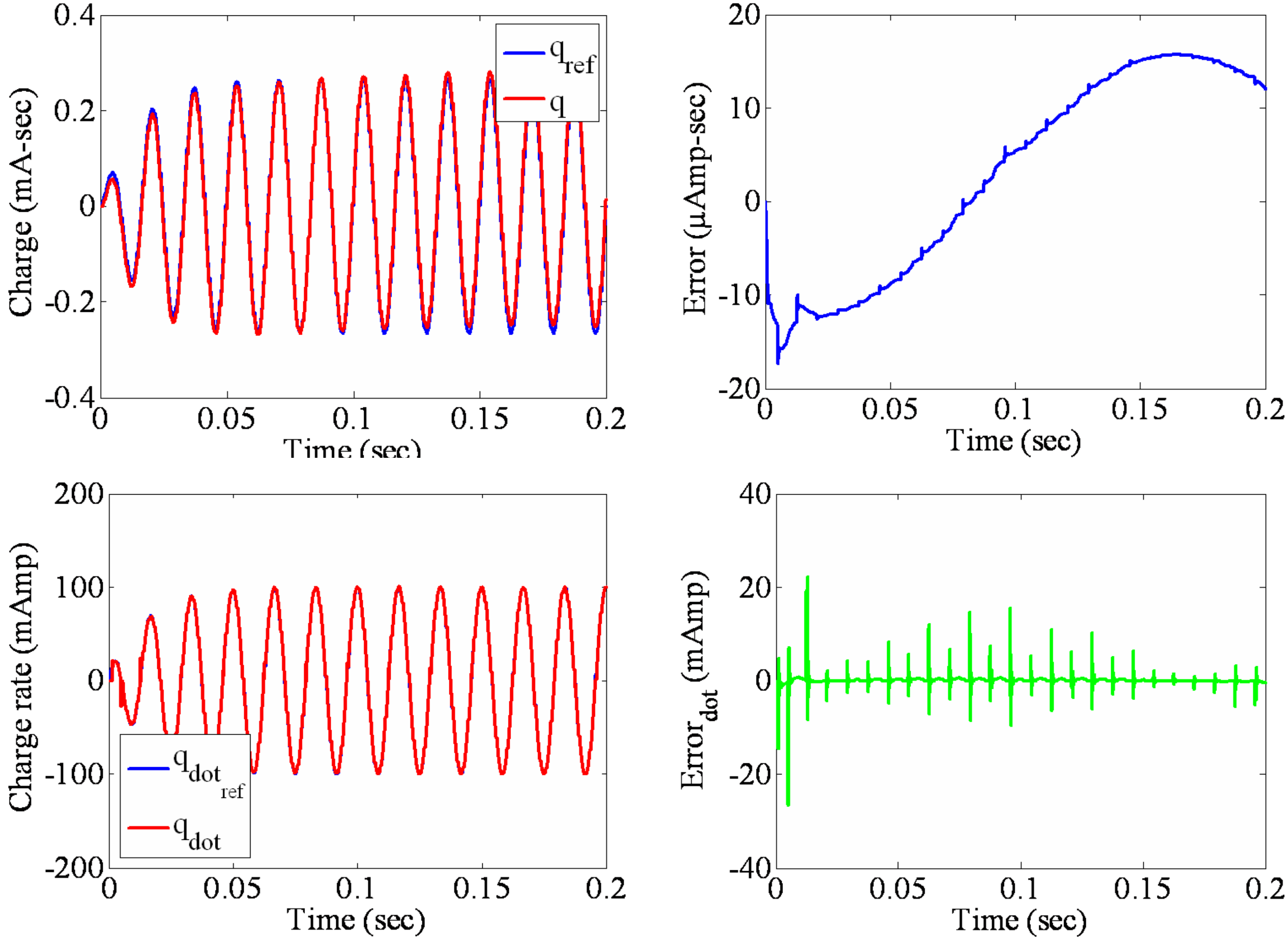
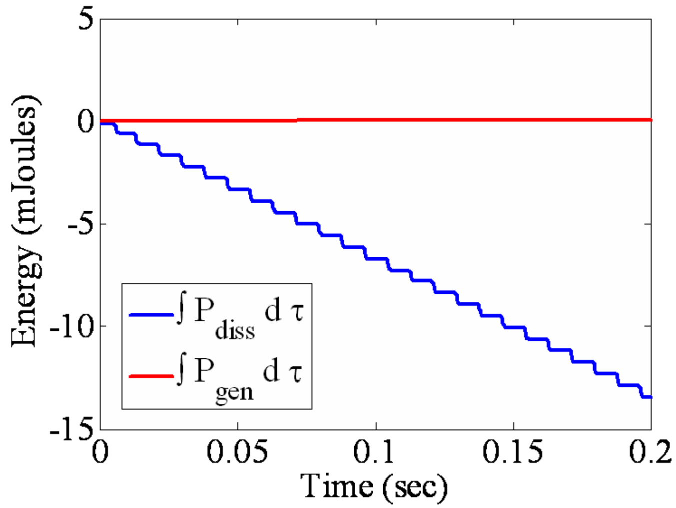
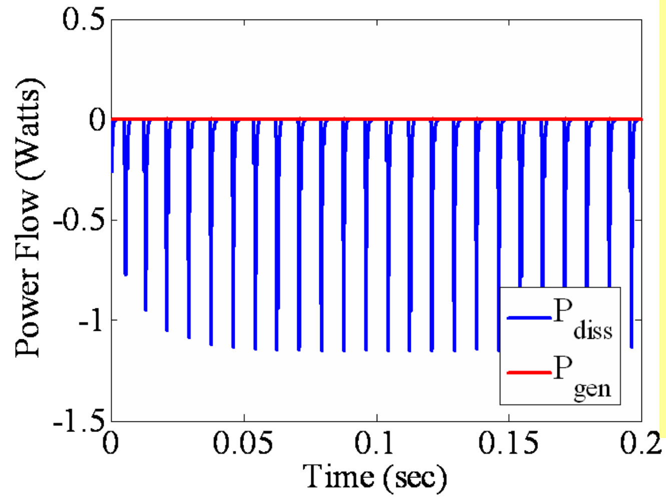
Advanced Design Control (cont.)
Nonlinear RLC Network w/ PID/Adaptive Numerical Results



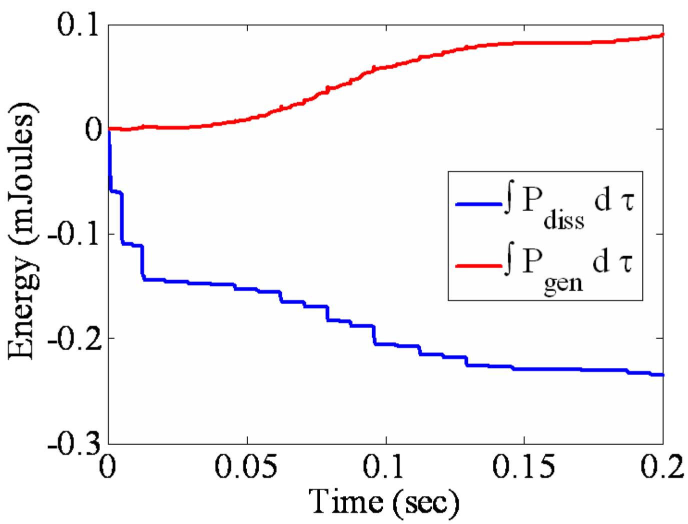
Advanced Design Control (cont.)
Nonlinear RLC Network Energy & Power Flow Results

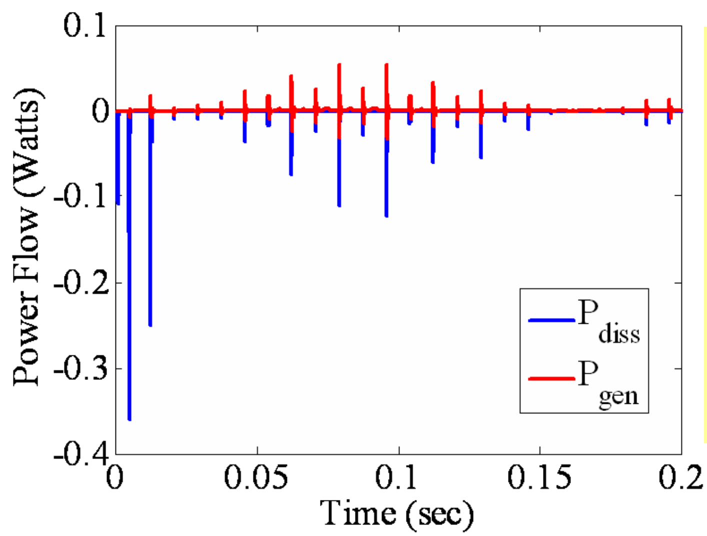
PID Controller





PID/Adaptive





PID /adaptive with information flow more efficient than PID with physical flow alone

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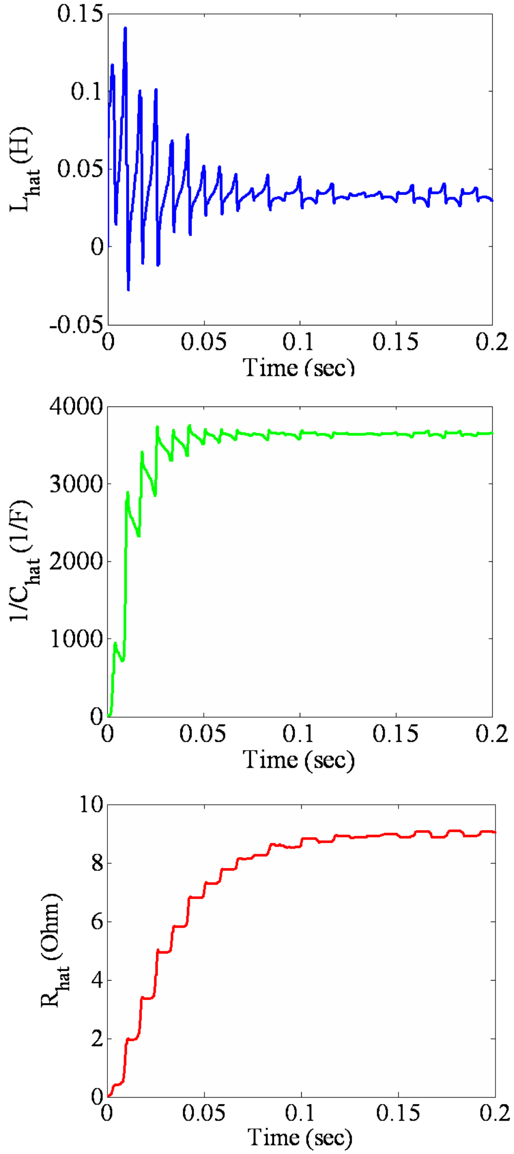
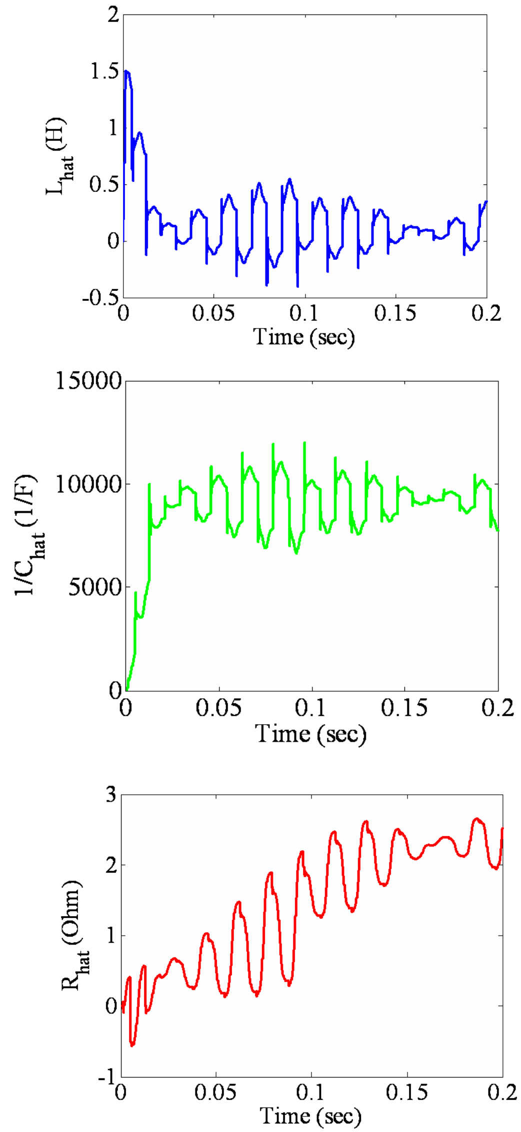
Advanced Design Control (cont.)
PID/Adaptive Parameter Update Responses



Linear RLC Model



Nonlinear RLC Model

Advanced Design Control (cont.) Adaptive/Tracking Conclusions

- Applied new nonlinear power flow control design to power engineering
- Used to design adaptive PID control architecture
- The power flow and energy responses demonstrated the required energy consumption of each controller
- Adaptive PID control showed to be more efficient than just PID control alone
- Future work to extend apply to power grid control problems

Case Study #1:
Control Design Issues

Case Study #1:
Control Design Issues

1) **Sinusoid Damping/ Nonlinear Feedback**

i.) Model $m\ddot{x} + kx = u = \sin(\dot{x})$

$$\ddot{x} - \sin(\dot{x}) + x = 0$$

$$H = \frac{1}{2}\dot{x}^2 + \frac{1}{2}x^2 > 0 \Rightarrow$$ Statically Stable

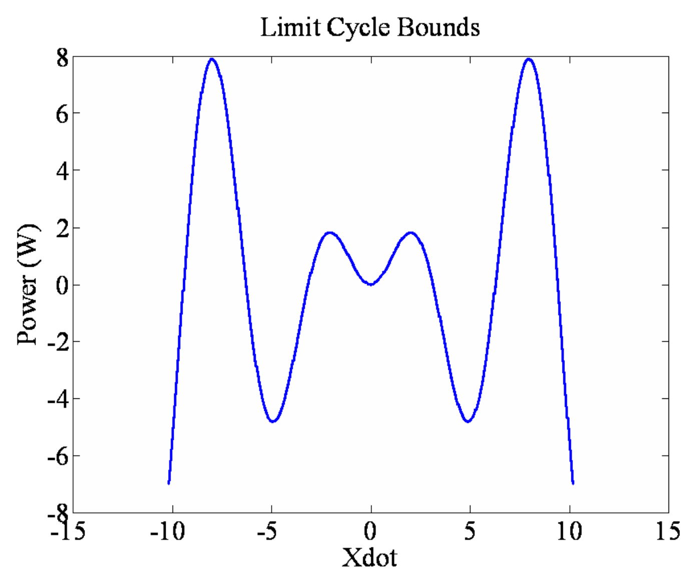
$$\dot{H} = [m\ddot{x} + x]\dot{x} = [\sin(\dot{x})]\dot{x}$$

Limit Cycles:

$$\dot{H}_{cyclic} = \oint_{\tau} \sin(\dot{x})\dot{x}dt = 0$$

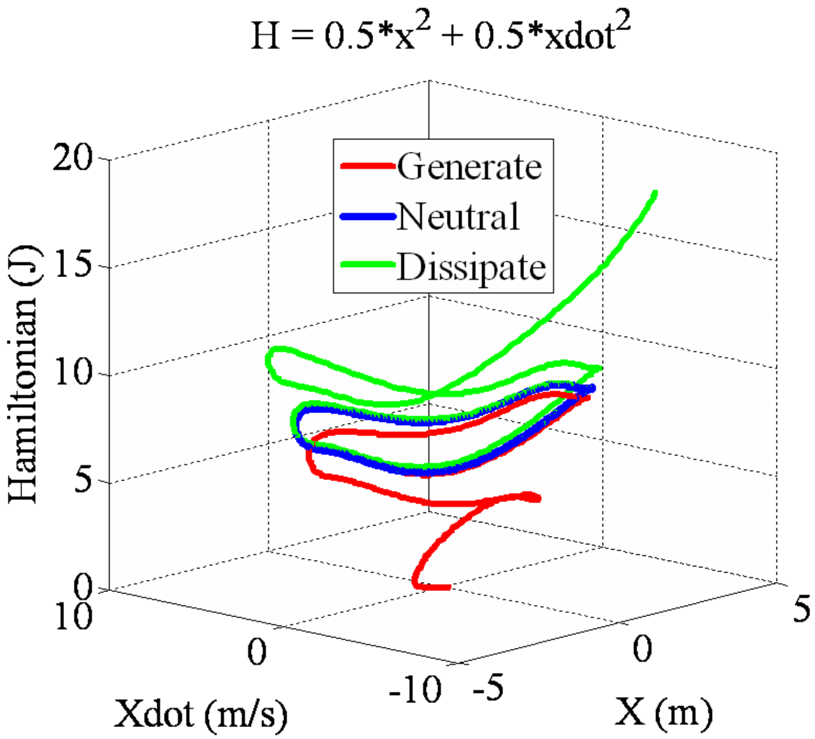
Where $\dot{x} = \pm n\pi$

for $n = 1,3,\ldots$

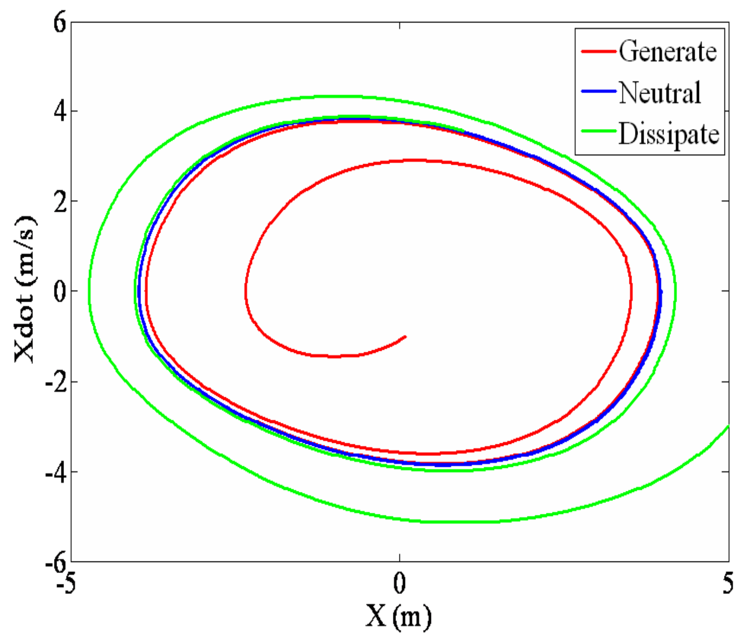


Case Study #1: Control Design Issues

ii.) Nonlinear second-order model – responses for initial limit cycle



3D Hamiltonian



2D Phase Plane

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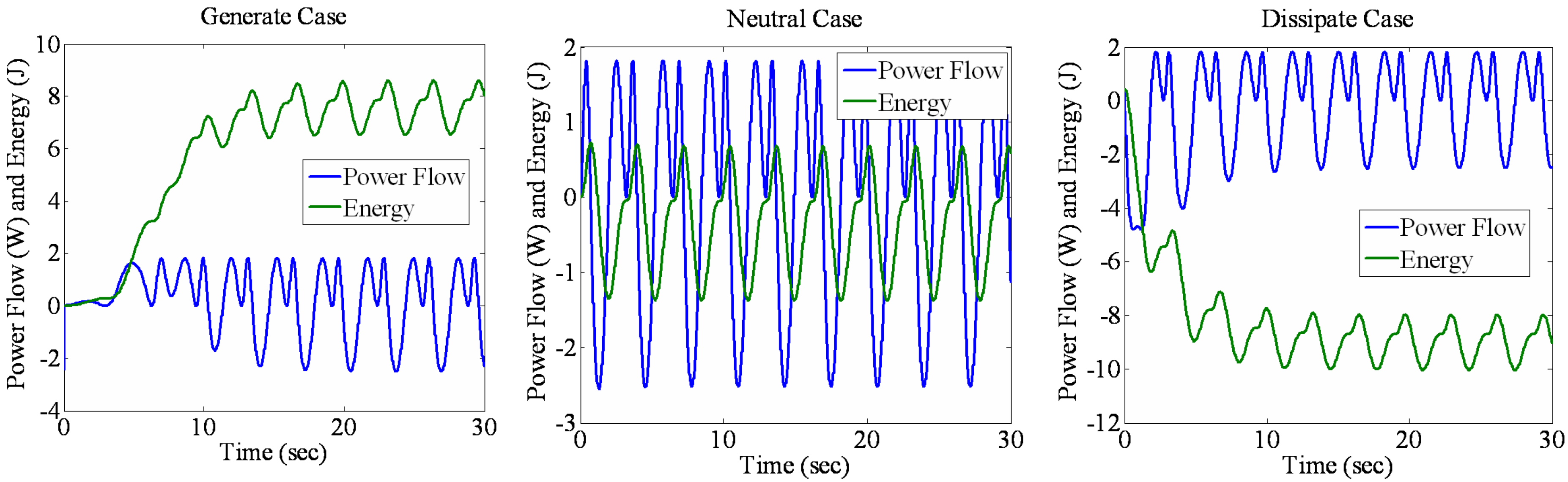
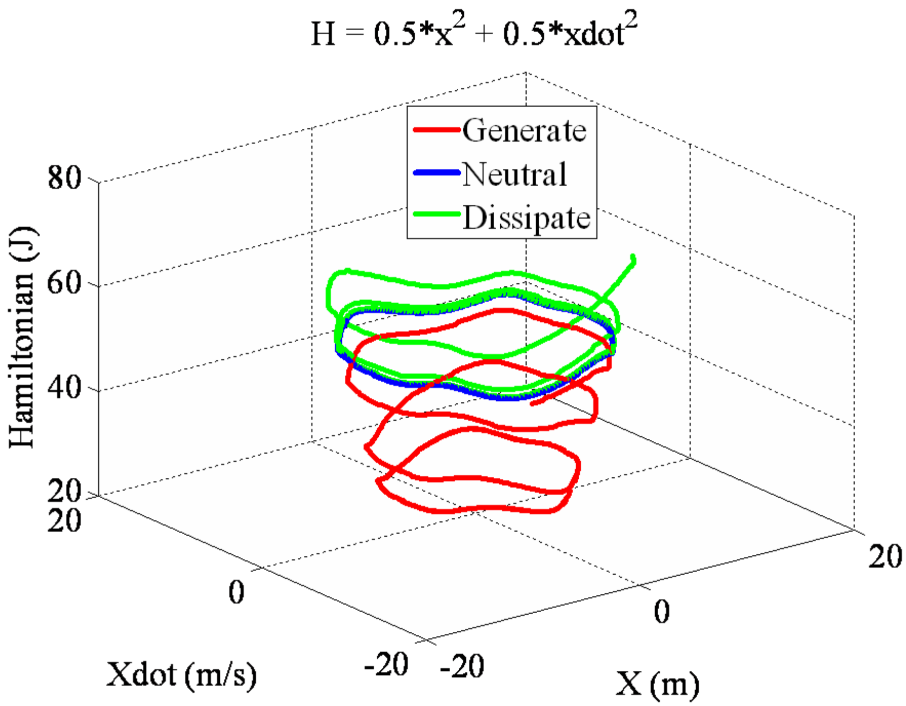
Case Study #1: Control Design Issues

- Dynamically unstable, neutral stable, stable

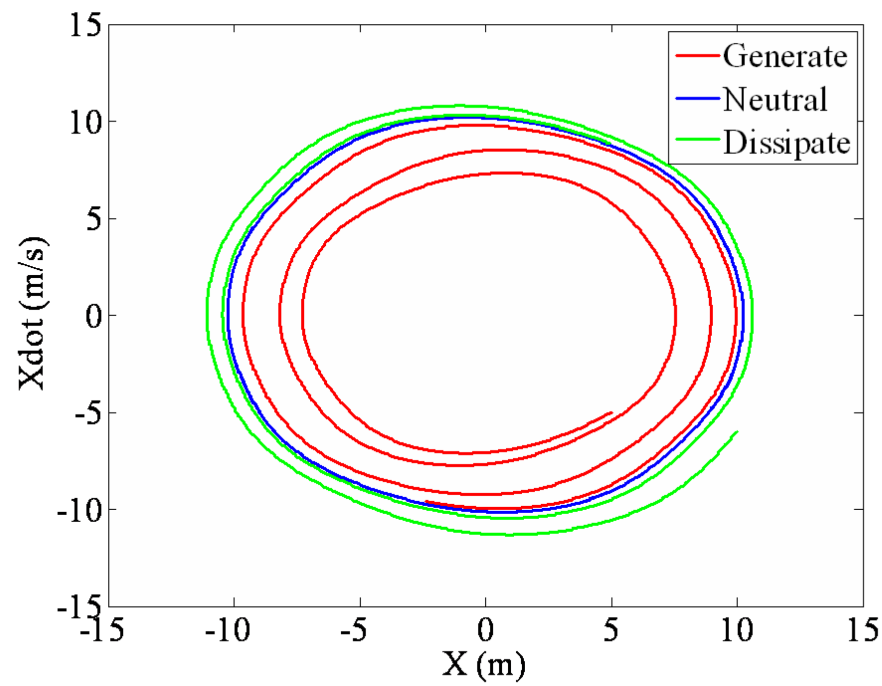


Case Study #1: Control Design Issues

- Nonlinear second-order model – responses for second limit cycle



3D Hamiltonian



2D Phase Plane

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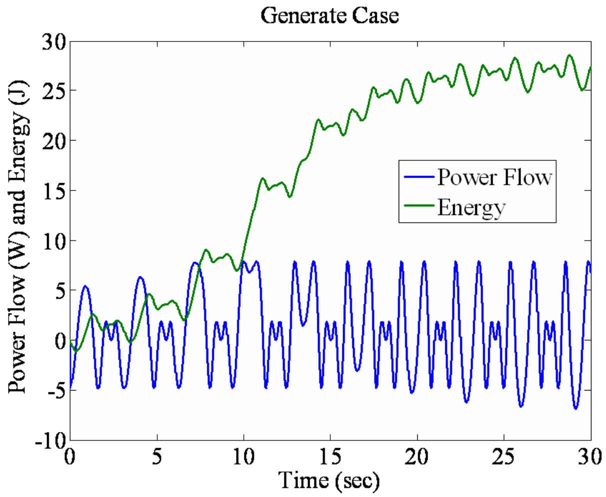
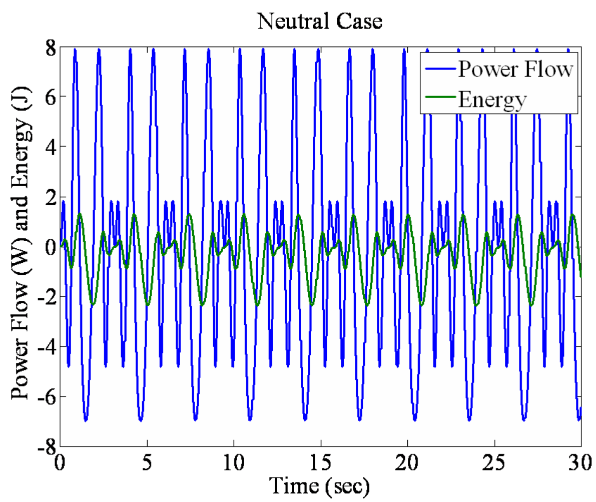
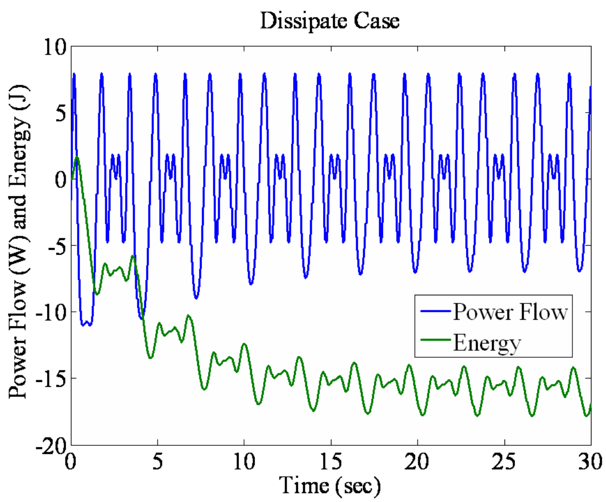
Case Study #1:
Control Design Issues

- Dynamically unstable, neutral stable, stable (second limit cycle)







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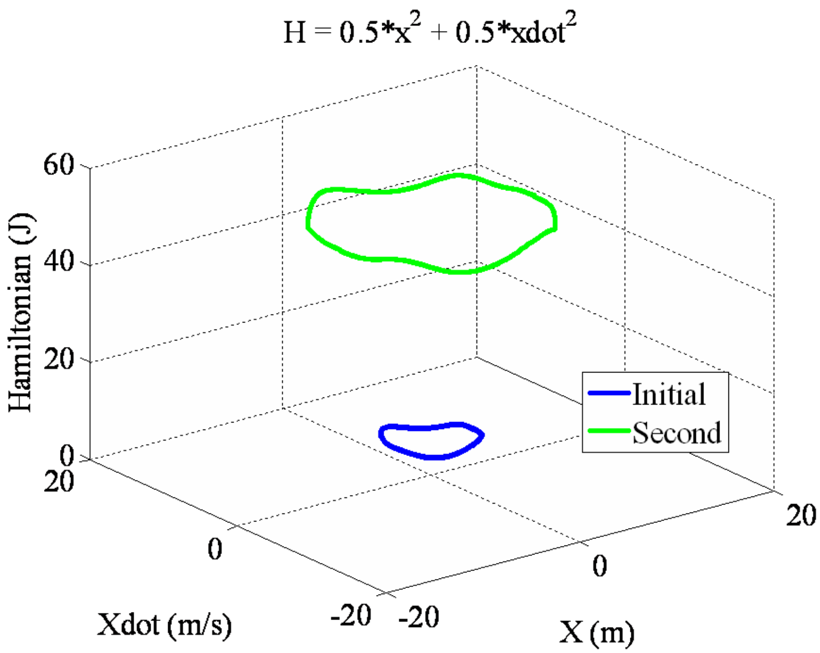
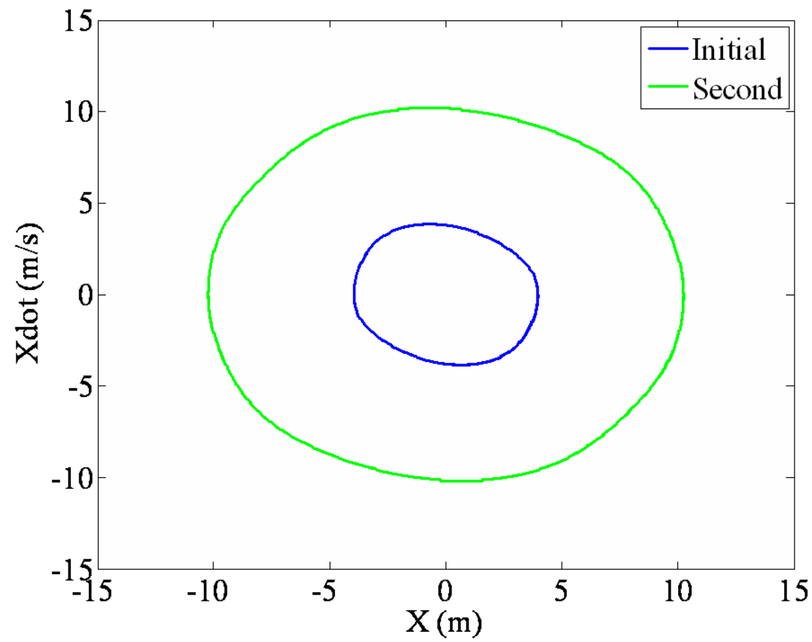
Case Study #1: Control Design Issues

• Nonlinear second-order model – responses for BOTH limit cycles



3D Hamiltonian

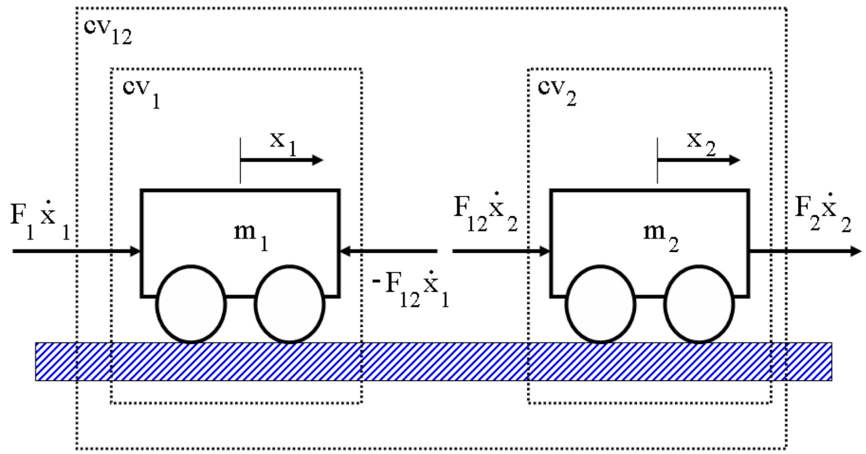


2D Phase Plane

Case Study #1: Control Design Issues

2) **Multiple Input Multiple Output (MIMO)**

i.) model:



$$m_1\ddot{x}_1 = F_1 - F_{12} \qquad H_1 = \frac{1}{2}m_1\dot{x}_1^2$$

$$m_2\ddot{x}_2 = F_2 + F_{12} \qquad H_2 = \frac{1}{2}m_2\dot{x}_2^2$$

Case Study #1: Control Design Issues

$$\dot{H}_1 = m_1\ddot{x}_1\dot{x}_1 = [F_1 - F_{12}]\dot{x}_1$$

$$\dot{H}_2 = m_2\ddot{x}_2\dot{x}_2 = [F_2 + F_{12}]\dot{x}_2$$

$$H_{12} = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_1\dot{x}_2^2$$

$$\dot{H}_{12} = [m_1\ddot{x}_1]\dot{x}_1 + [m_2\ddot{x}_2]\dot{x}_2 = F_1\dot{x}_1 + F_2\dot{x}_2 + F_{12}(\dot{x}_2 - \dot{x}_1)$$

$$H_{IN} = \sum_{i=1}^{N} \frac{1}{2}m_i\dot{x}_i^2$$

$$\dot{H}_{IN} = \sum_{i=1}^{N} F_i\dot{x}_i + \sum_{j=1}^{N-1} F_{j(j+1)}[\dot{x}_{j+1} - \dot{x}_j]$$

Case Study #1:
Control Design Issues

ii.) Decoupled (Neutral Stability) – Linear System Controller Design

$$M\ddot{x} = F = \begin{Bmatrix} F_1 - F_{12} \\ F_2 + F_{12} \end{Bmatrix} = -Kx - C\dot{x} + u$$

$$M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}, K = \begin{bmatrix} k_{11} & -k_{12} \\ -k_{12} & k_{22} \end{bmatrix}, C = \begin{bmatrix} c_{11} & -c_{12} \\ -c_{12} & c_{22} \end{bmatrix}$$

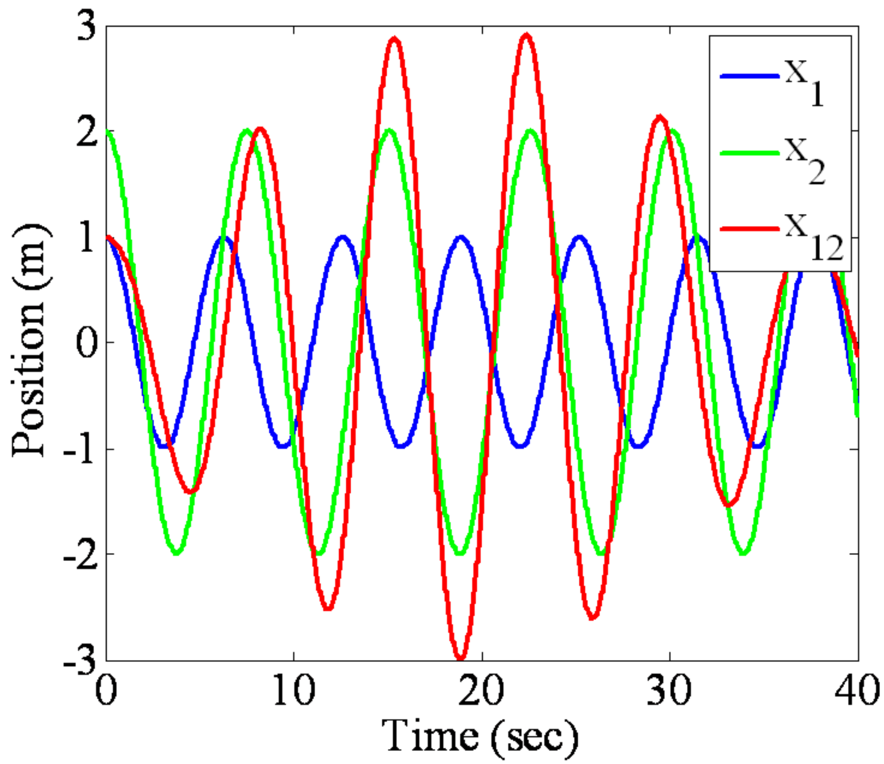
Decoupling controller defined as:

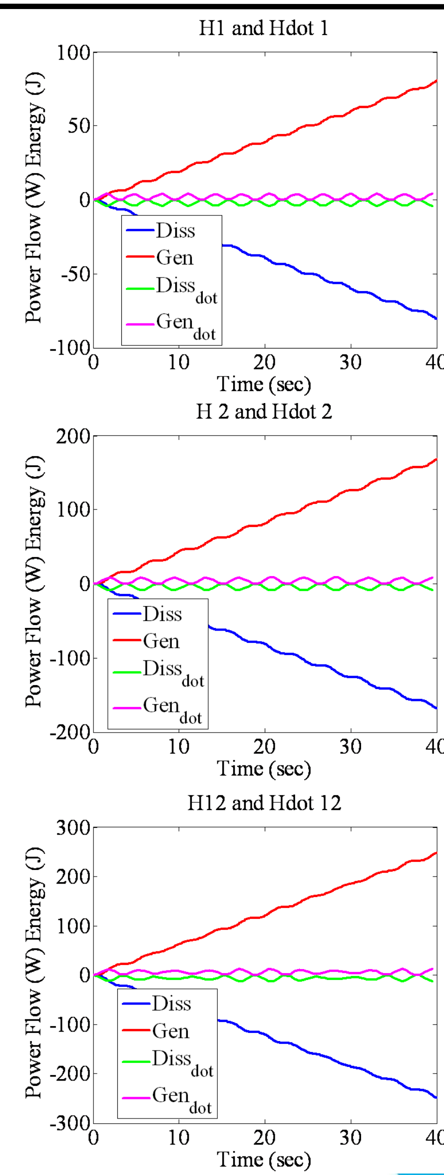
$$u = -K_P x - K_I \int_0^t x d\tau - K_D \dot{x} - \begin{Bmatrix} -\hat{F}_{12} \\ \hat{F}_{12} \end{Bmatrix}$$

$$K_P = \begin{bmatrix} K_{P_1} & k_{12} \\ k_{12} & K_{P_2} \end{bmatrix}, K_D = \begin{bmatrix} K_{D_1} & c_{12} \\ c_{12} & K_{D_2} \end{bmatrix}, K_I = \begin{bmatrix} K_{I_1} & 0 \\ 0 & K_{I_2} \end{bmatrix}$$

Case Study #1: Control Design Issues

- Decoupled (Neutral Stability)





$$\omega_i^2 = \frac{k_{ii} + K_{P_i}}{m_i} = \frac{K_{I_i}}{c_{ii} + K_{D_i}} \; for \qquad i = 1,2$$

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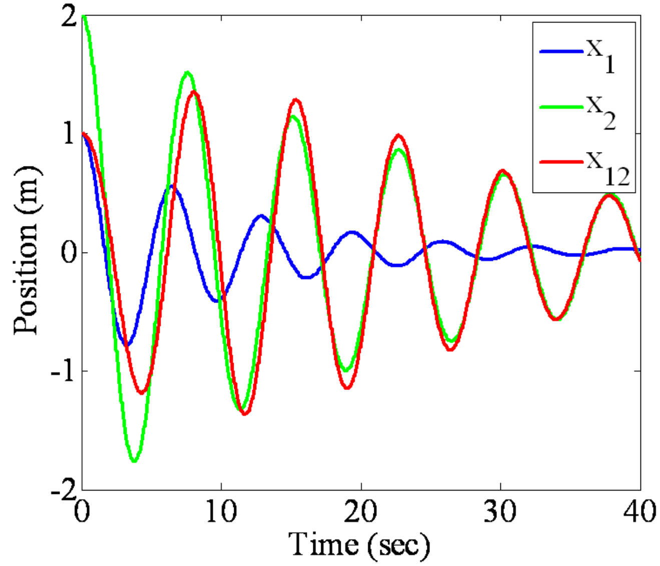
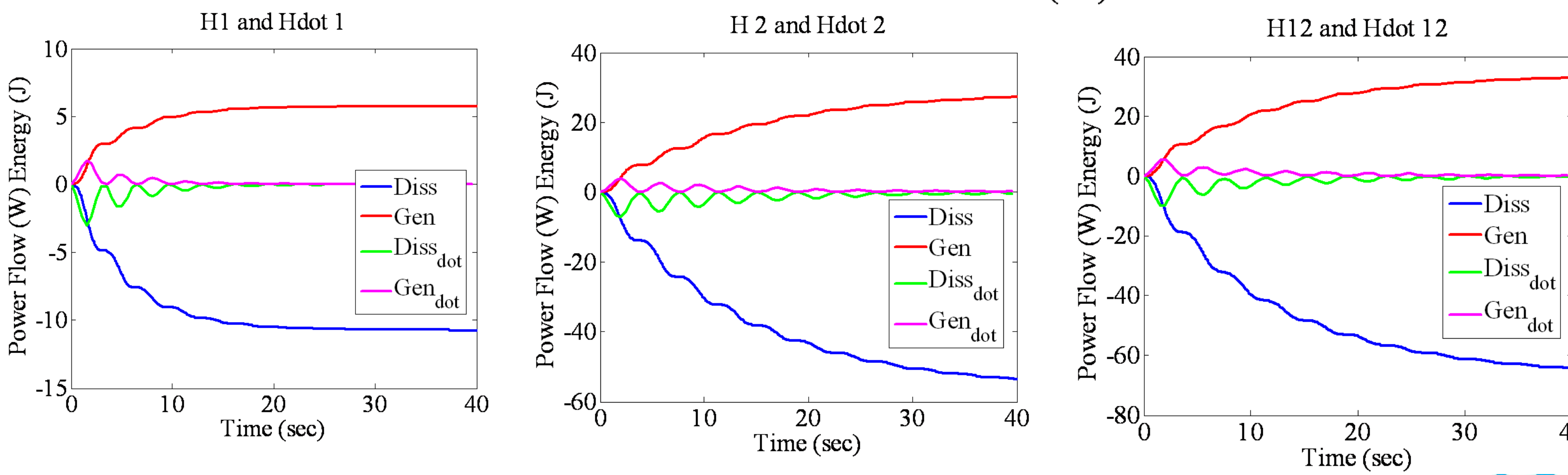
Case Study #1:
Control Design Issues

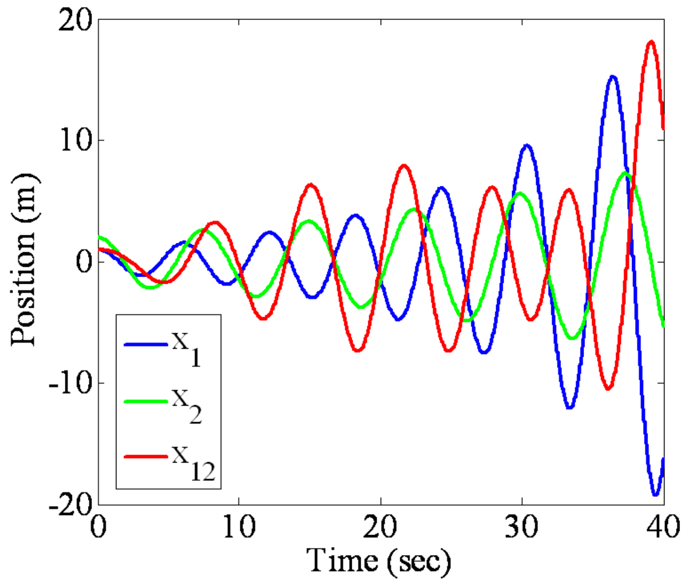
- Decoupled
  (Asymptotically Stable)

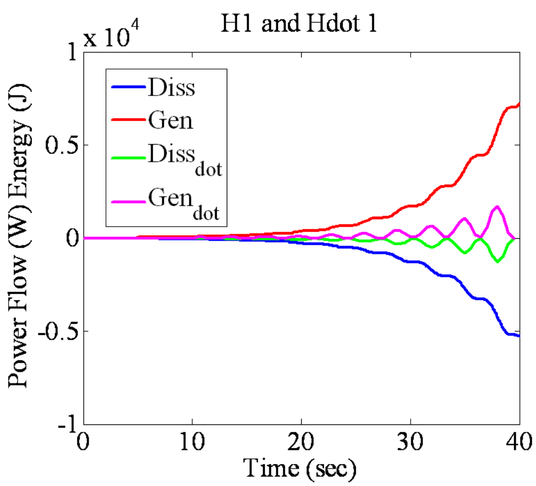


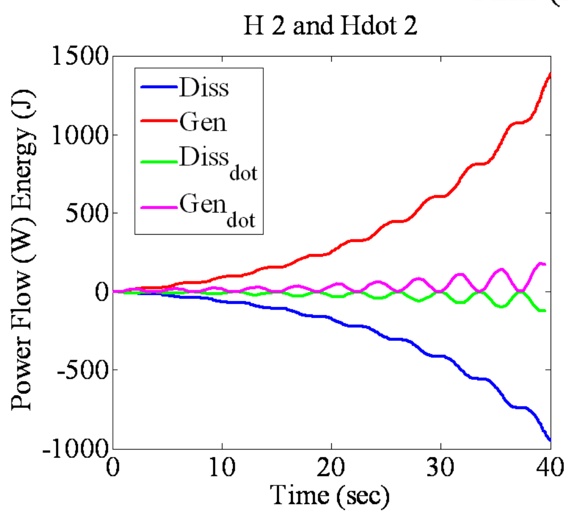


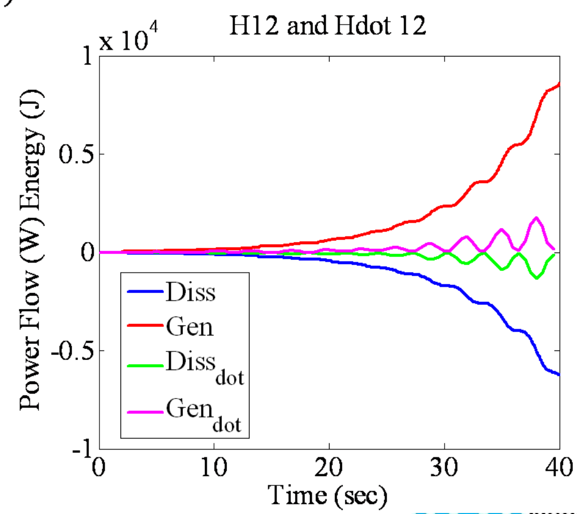
Case Study #1: Control Design Issues

- Decoupled (Unstable)



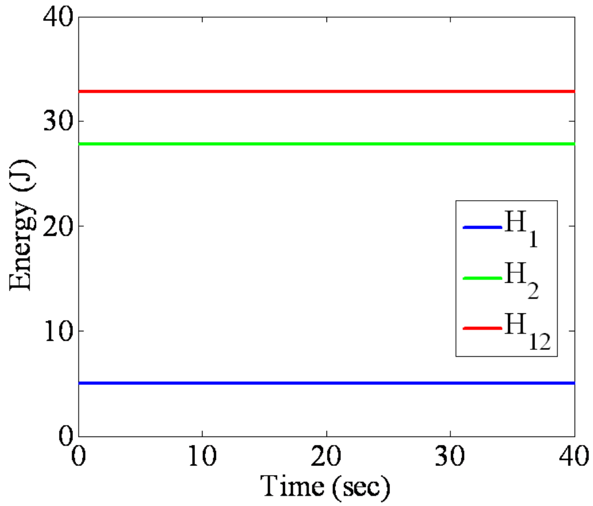


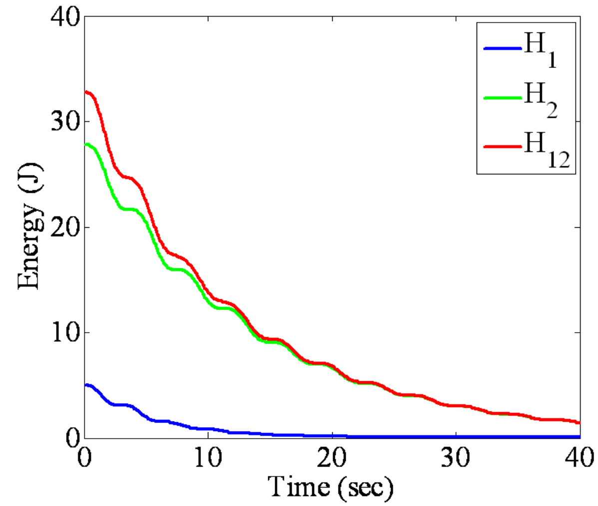


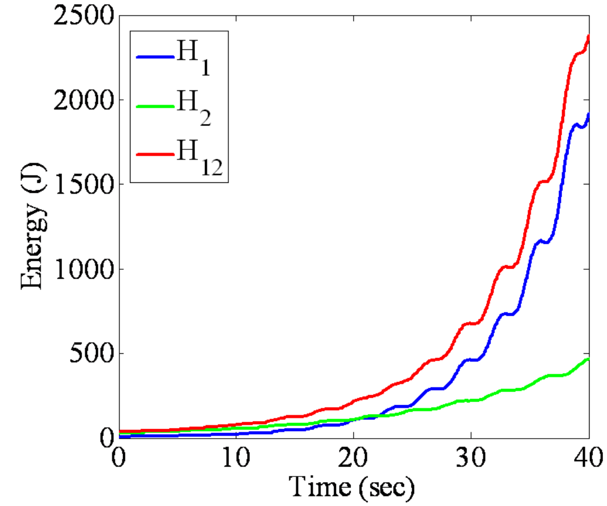


Case Study #1: Control Design Issues

- Decoupled Energy Responses (neutral, stable, unstable)

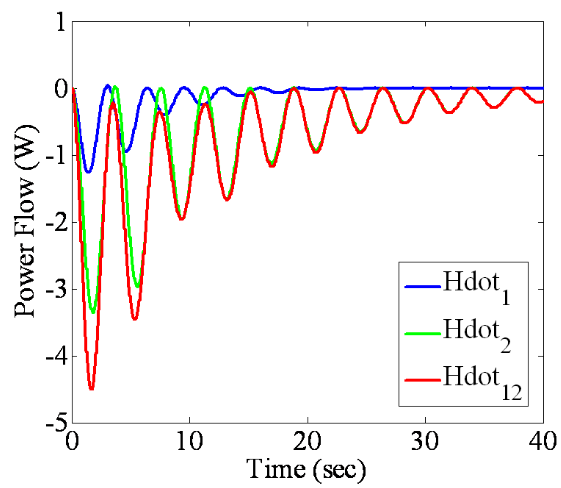


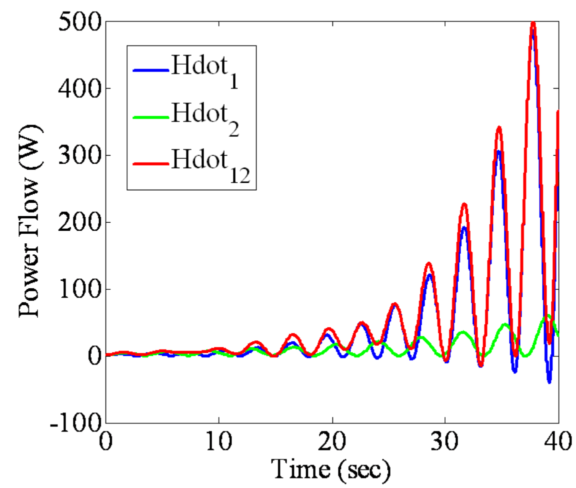




- Decoupled Power Flow Responses (neutral, stable, unstable)

Zero power flow for dynamically neutral stability decoupled control case





Case Study #1: Control Design Issues

iii.) Coupled – Linear System Controller Design from combined Hamiltonian

$$H_{12} = \frac{1}{2}\dot{x}^T M\dot{x} + \frac{1}{2}x^T[K + K_P]x$$

$$\dot{H}_{12} = \dot{x}^T[M\ddot{x} + [K + K_P]x] = \dot{x}^T\left[-[C + K_D]\dot{x} - K_I\int_0^t x d\tau\right]$$

Which matches up eigenvalues of system when (coupled integral gain matrix)

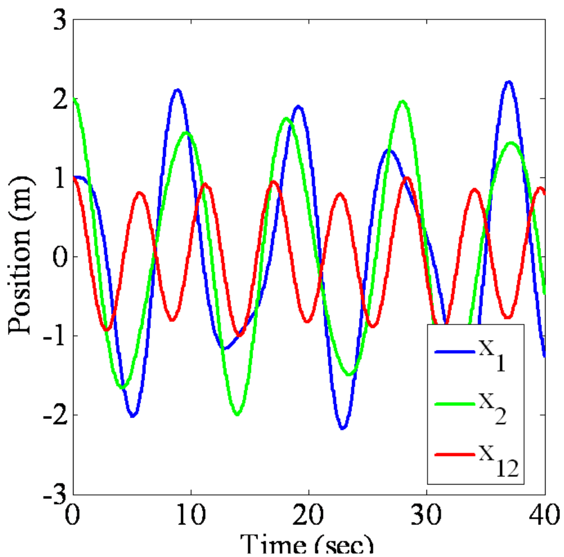
$$K_I = [C + K_D]M^{-1}[K + K_P]$$

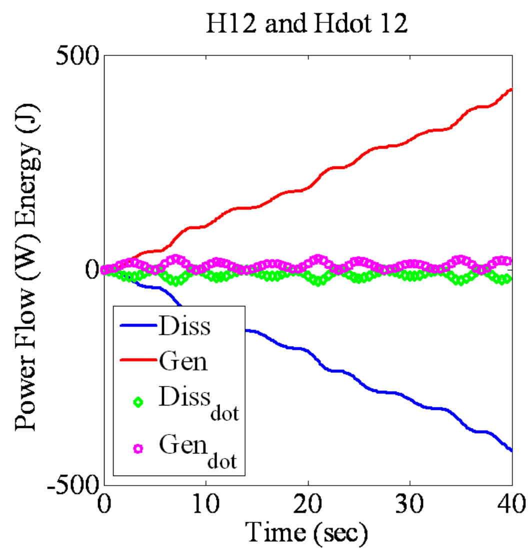
Eigenvalues and eignevectors:

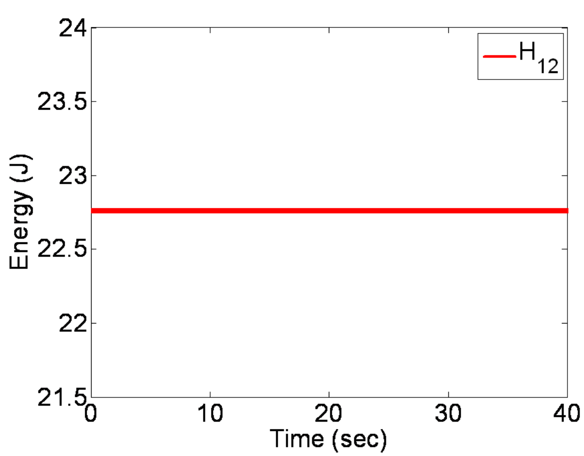
$$\left[[K + K_P] - \omega^2 M\right]\Phi = \left[K_I - \omega^2[C + K_D]\right]\Phi = 0$$

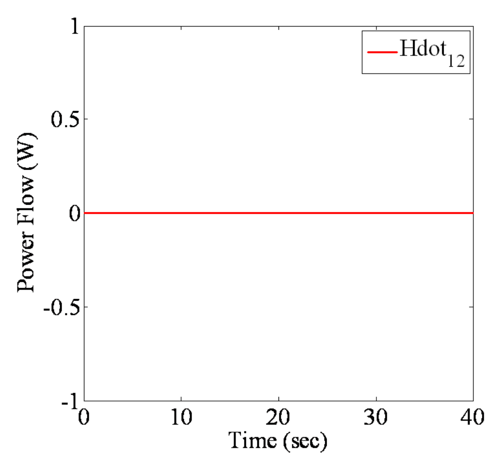
Case Study #1: Control Design Issues

• System effectively decoupled into eigenspace:

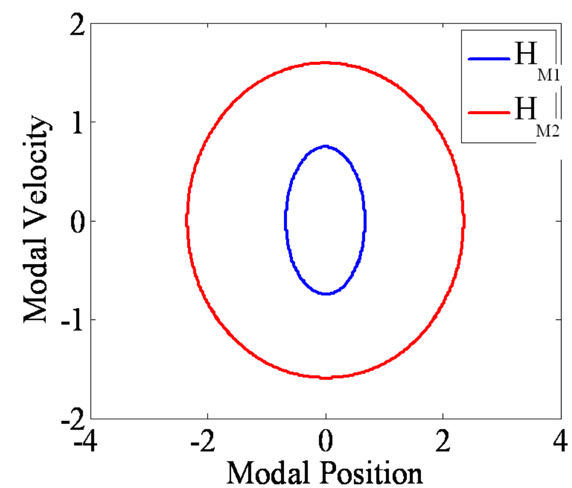








Decoupled in phase plane:



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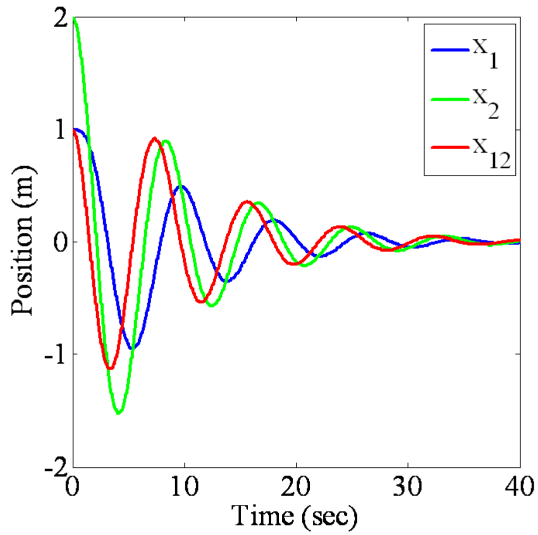
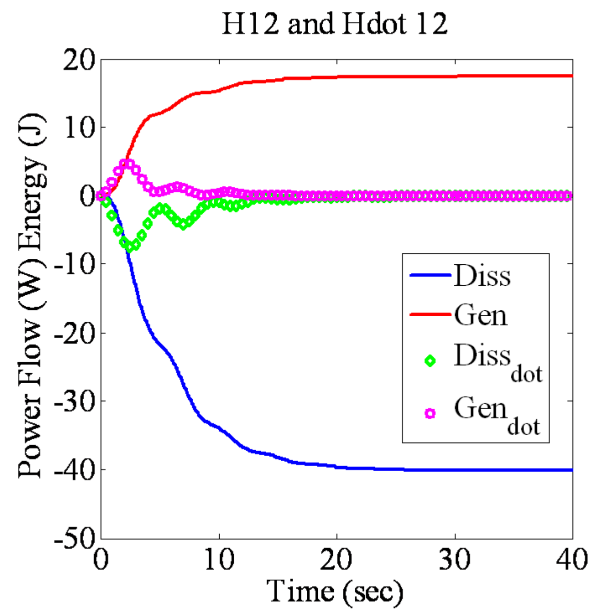
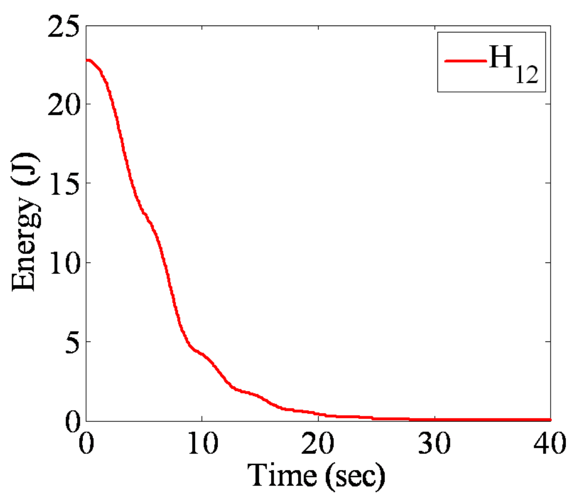
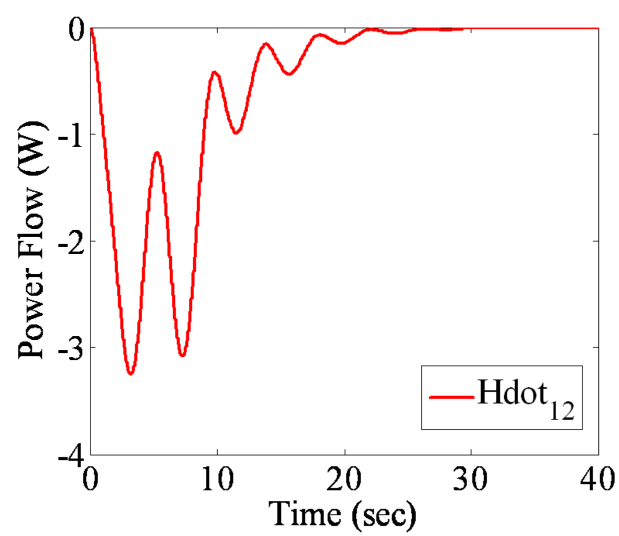
Case Study #1:
Control Design Issues

- Similar results (Dissipative):









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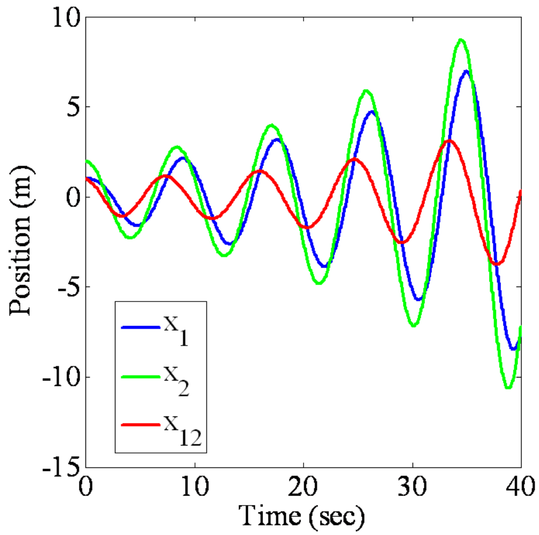
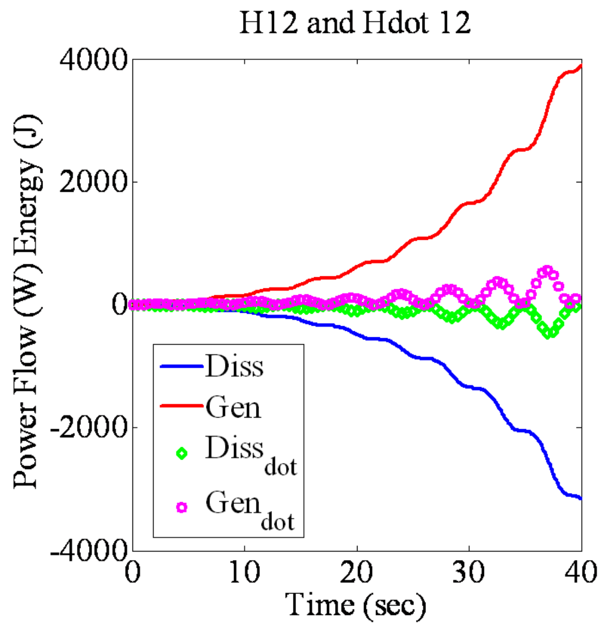
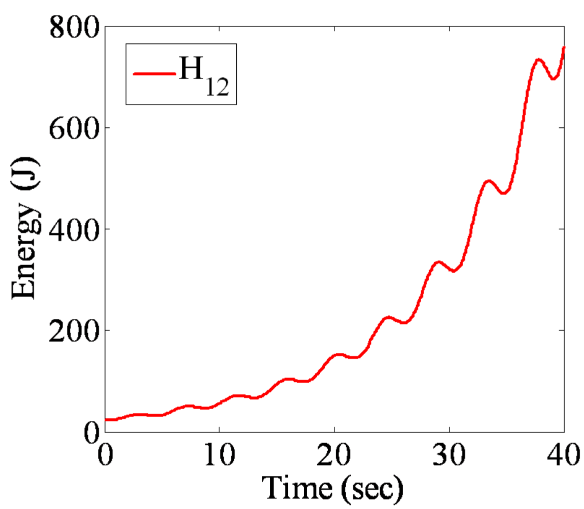
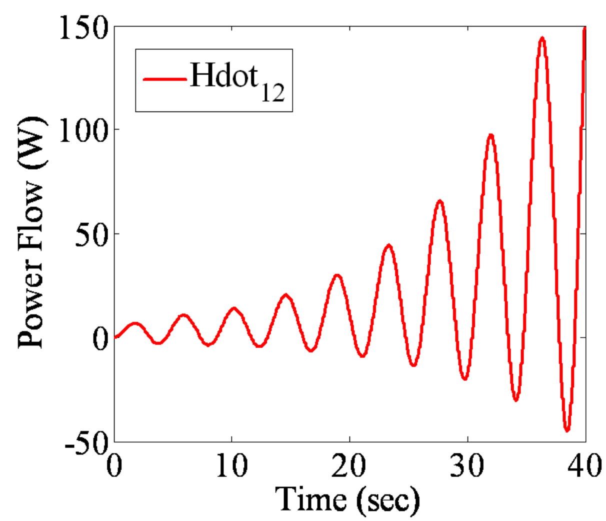
Case Study #1: Control Design Issues

- Similar results (Generative):









Case Study #1: Control Design Issues

iv.) Coupled – Linear System Controller Design where

$$m_1 > m_2 \qquad u = \begin{bmatrix} u_1 & 0 \end{bmatrix}^T \; and \quad F_2 = 0$$

With a single PID controller

$$u_1 = -K_{P_1} x_1 - K_{I_1} \int_0^t x_1 d\tau - K_{D_1} \dot{x}_1$$

The coupled Hamiltonian

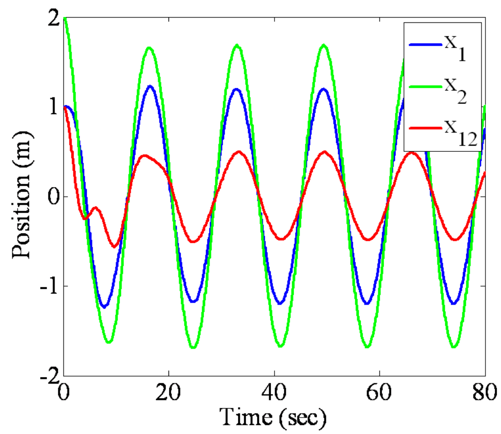
$$H_{12} = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2}\left(k_1 + K_{P_1}\right) x_1^2 + \frac{1}{2} k_{12} \left(x_2 - x_1\right)^2$$

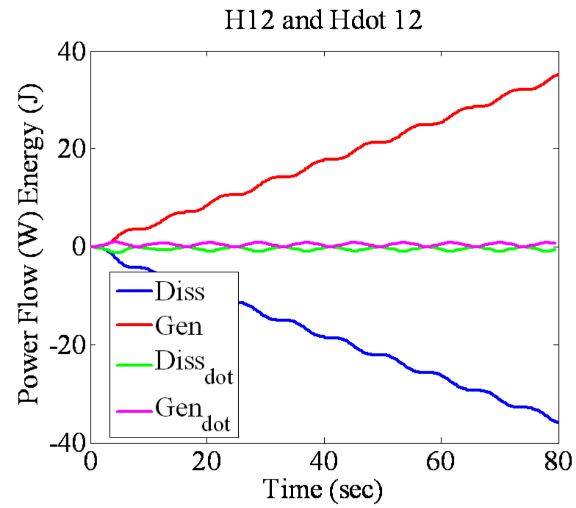
Resulting time derivative

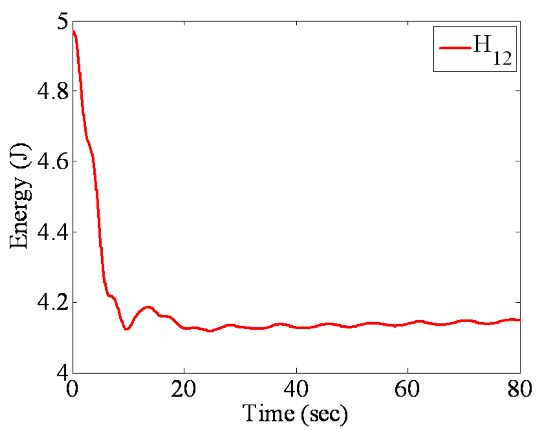
$$\dot{H}_{12} = \left[-\left(c_1 + K_{D_1}\right) \dot{x}_1 - K_{I_1} \int_0^t x_1 d\tau \right] \dot{x}_1 - c_{12} \left(\dot{x}_2 - \dot{x}_1\right)^2$$

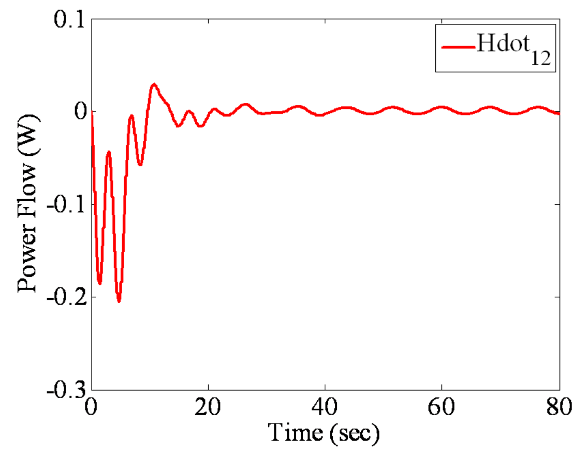
Case Study #1: Control Design Issues

- Coupled (Neutral)









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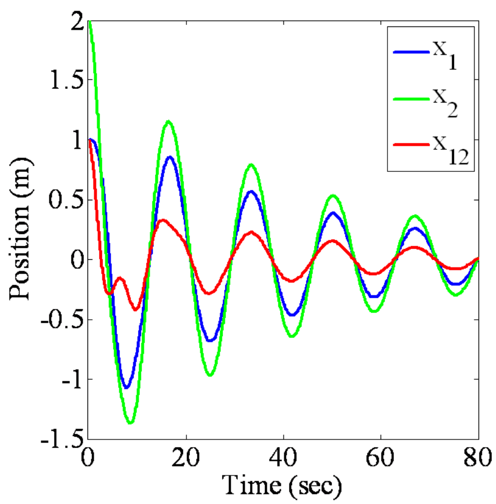
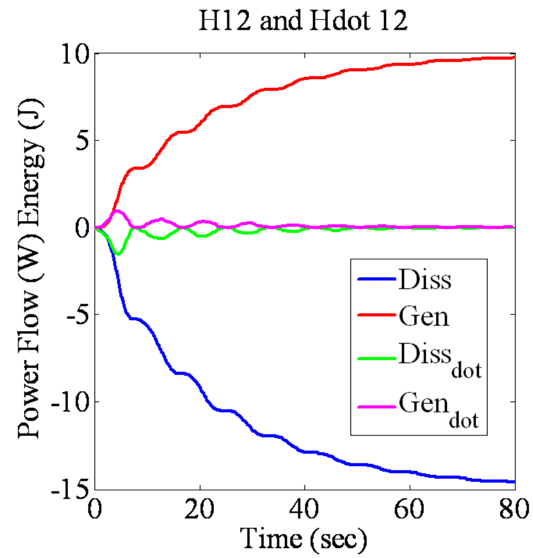
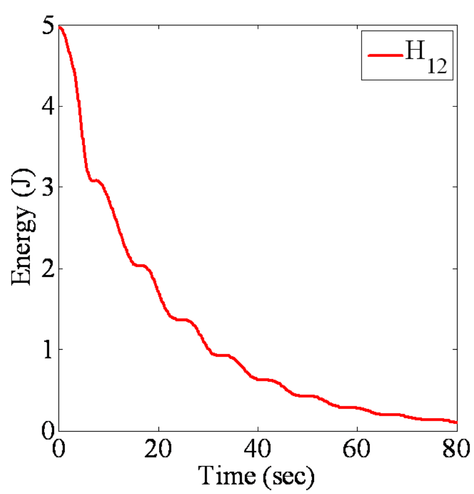
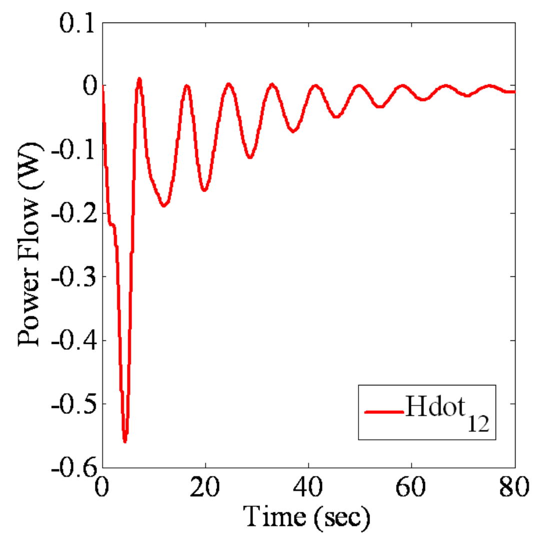
Case Study #1: Control Design Issues

- Coupled (Asymptotically Stable)



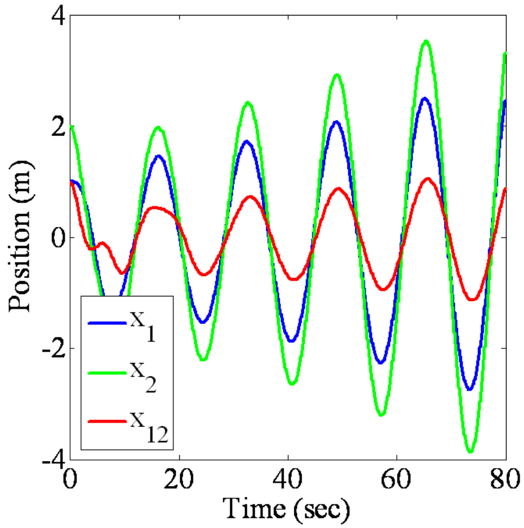


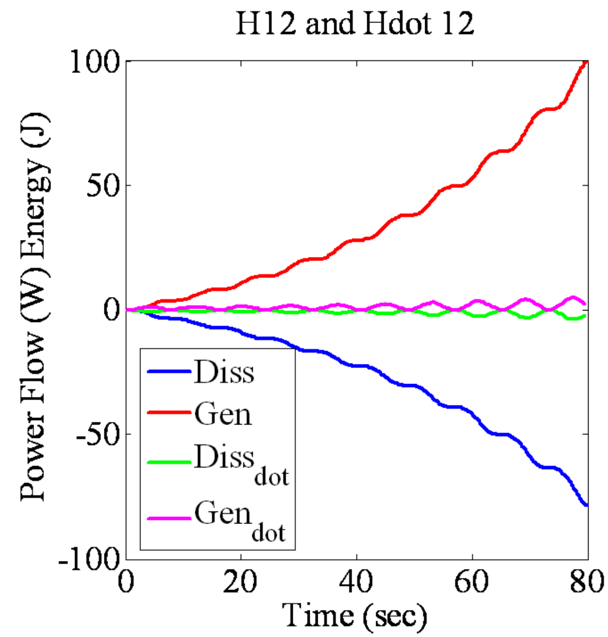


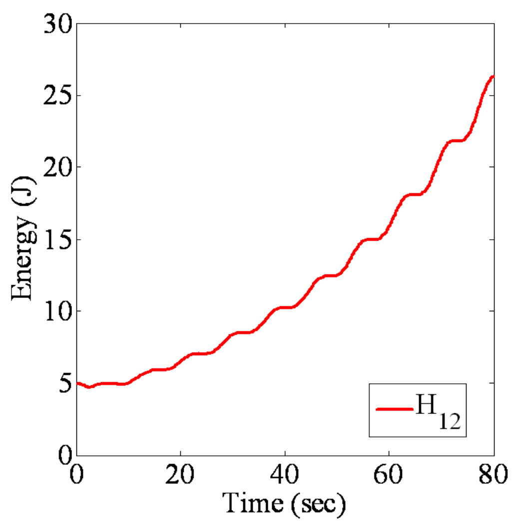


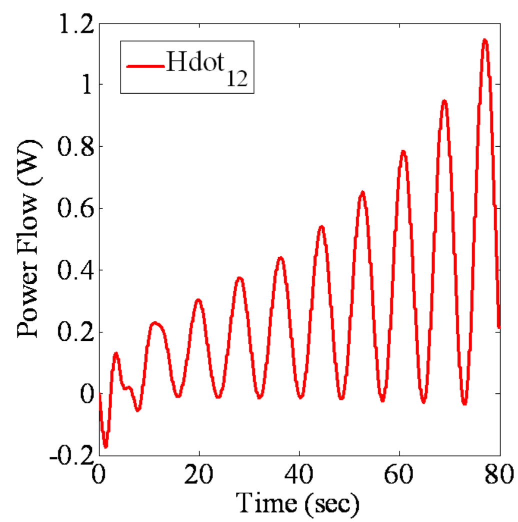
Case Study #1: Control Design Issues

- Coupled (Unstable)









Case Study #1: Control Design Issues

v.) van der Pol / Nonlinear Controller:

$$M\ddot{x} + Kx = -f(x,\dot{x}) + \Delta u$$

1. Nonlinear MIMO system
2. Van der Pol damping treated as controller

$$f(x,\dot{x}) = \begin{Bmatrix} (K_{G_1} - K_{D_1} x_1^2)\dot{x}_1 - \left(K_{G_{12}} - K_{D_{12}} \Delta x^2\right)\Delta\dot{x} \\ (K_{G_2} - K_{D_2} x_2^2)\dot{x}_2 + \left(K_{G_{12}} - K_{D_{12}} \Delta x^2\right)\Delta\dot{x} \end{Bmatrix}$$

$$\Delta x = x_2 - x_1$$

$$\Delta u = -\begin{Bmatrix} -\hat{F}_{12} \\ \hat{F}_{12} \end{Bmatrix}$$

3. Combined Hamiltonian $\quad H_{12} = \frac{1}{2}\dot{x}^T M\dot{x} + \frac{1}{2}x^T K x$
4. Time derivative $\quad \dot{H}_{12} = \dot{x}^T\left[M\ddot{x} + Kx\right] = \dot{x}^T\left[-f(x,\dot{x})\right]$
5. Neutral stability condition $\quad \oint_\tau \dot{H}_{12}\,dt = 0 = \oint_\tau \dot{x}^T\left[-f(x,\dot{x})\right]dt$

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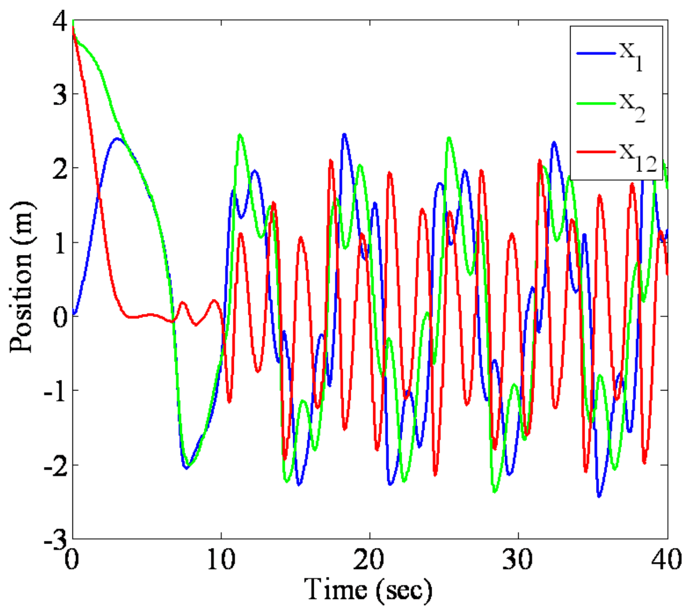
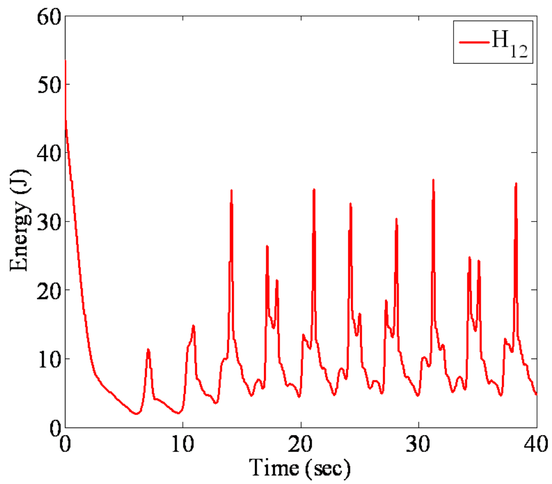
Case Study #1: Control Design Issues

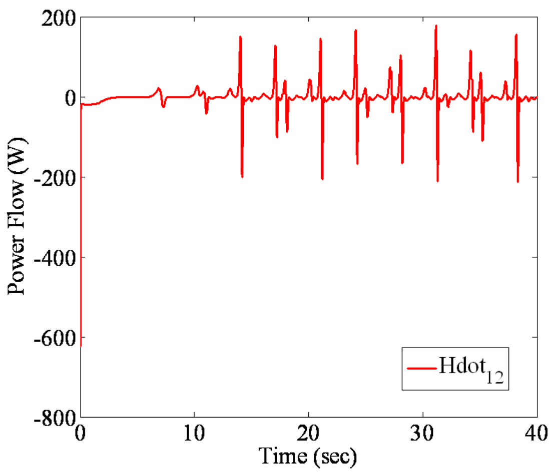
- Two-body dynamically neutral stability coupled control van der Pol system



•Positions

•Energy and power flow





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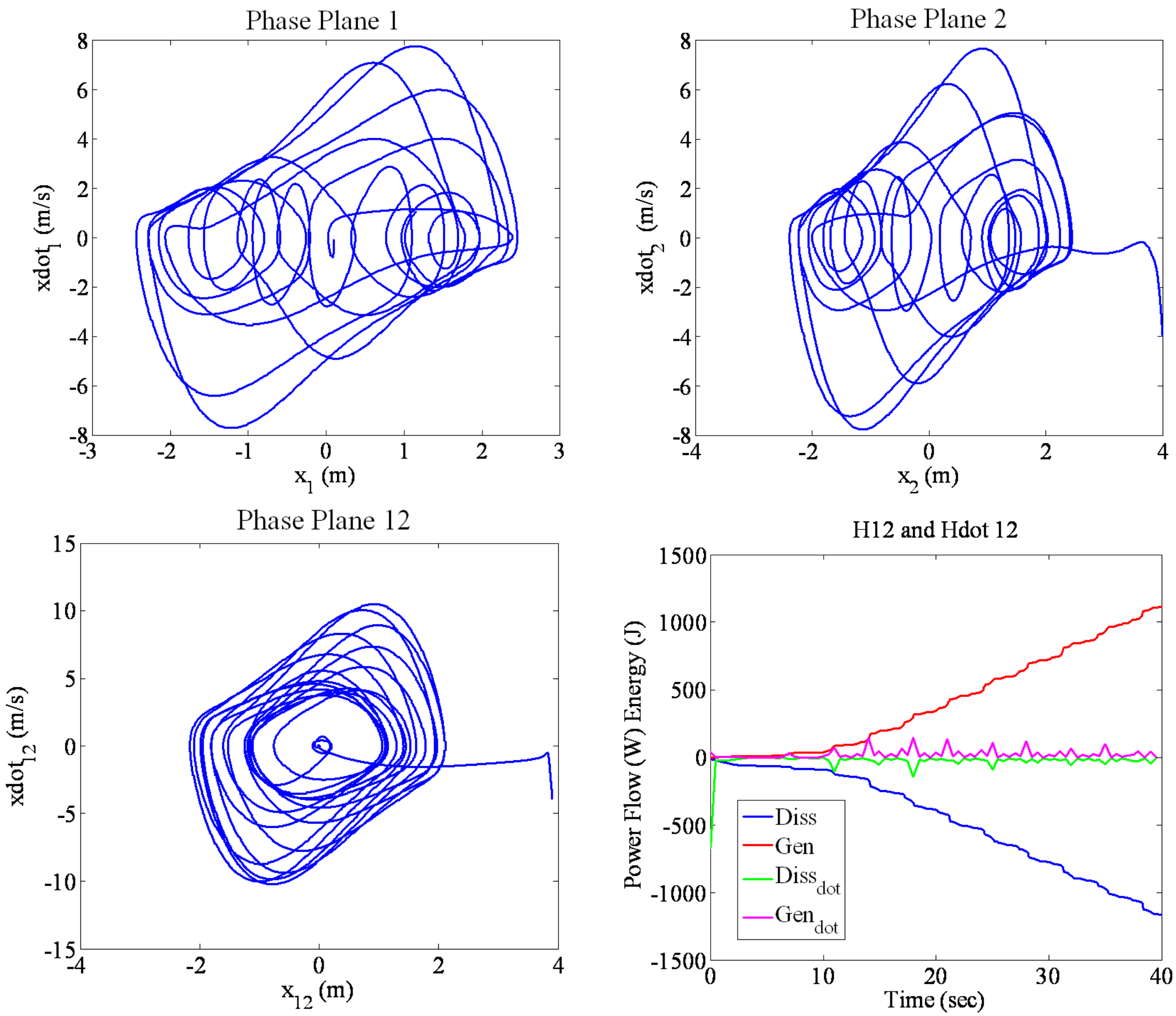
Case Study #1: Control Design Issues

- Two-body dynamically neutral stability coupled control van der Pol system



- Phase plane projections
- Power flow partitioned into dissipation and generation for coupled 12 system

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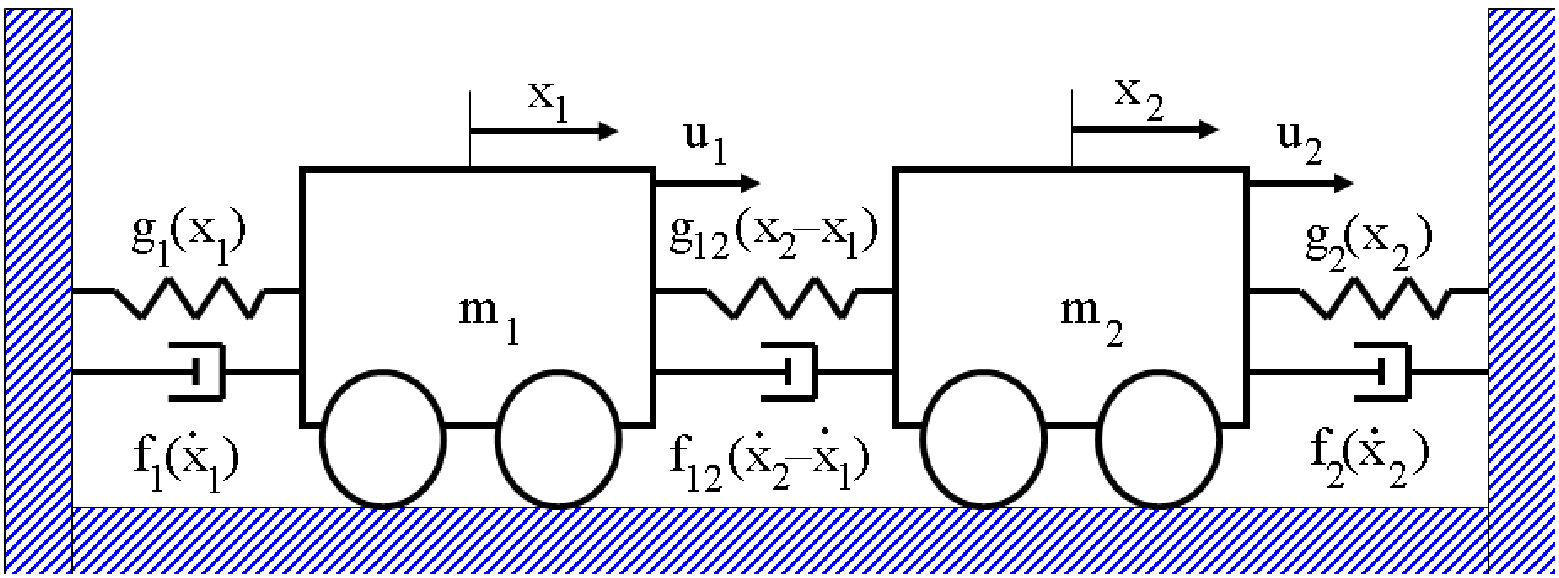
Case Study #1: Control Design Issues

3) Noncollocated Control

i.) Model



Case Study #1:
Control Design Issues

$$m_1\ddot{x}_1 = -g_1(x_1) - f_1(\dot{x}_1) - g_{12}(x_1 - x_2) - f_{12}(\dot{x}_1 - \dot{x}_2) + u_1$$

$$m_2\ddot{x}_2 = -g_{12}(x_2 - x_1) - f_{12}(\dot{x}_2 - \dot{x}_1) + u_2$$

$$g_2(x_2) = f_2(\dot{x}_2) = 0$$

$$H_1 = \frac{1}{2}m_1\dot{x}_1^2 + V_1(x_1)$$

$$\dot{H}_1 = [m_1\ddot{x}_1 + g_1(x_1)]\dot{x}_1 = [-f_1(\dot{x}_1) - f_{12}(\dot{x}_1 - \dot{x}_2) - g_{12}(x_1 - x_2) + u_1]\dot{x}_1$$

$$H_2 = \frac{1}{2}m_2\dot{x}_2^2$$

$$\dot{H}_2 = [m_2\ddot{x}_2]\dot{x}_2 = [-f_{12}(\dot{x}_2 - x_1) - g_{12}(x_2 - x_1) + u_2]\dot{x}_2$$

Case Study #1:
Control Design Issues

$$H_{12} = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 + V_1(x_1) + V_{12}(x_1 - x_2)$$

$$\dot{H}_{12} = [m_1\ddot{x}_1 + g_1(x_1)]\dot{x}_1 + [m_2\ddot{x}_2]\dot{x}_2 + g_{12}(x_1 - x_2)(\dot{x}_1 - \dot{x}_2)$$

$$= [m_1\ddot{x}_1 + g_1(x_1) + g_{12}(x_1 - x_2)]\dot{x}_1 + [m_2\ddot{x}_2 + g_{12}(x_2 - x_1)]\dot{x}_2$$

$$= [-f_1(\dot{x}_1) - f_{12}(\dot{x}_1 - \dot{x}_2) + u_1]\dot{x}_1 + [-f_{12}(\dot{x}_2 - \dot{x}_1) + u_2]\dot{x}_2$$

$$= [-f_1(\dot{x}_1) + u_1]\dot{x}_1 + [u_2]\dot{x}_2 + [-f_{12}(\dot{x}_1 - \dot{x}_2)](\dot{x}_1 - \dot{x}_2)$$

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Case Study #1: Control Design Issues

ii.) Design Controller

A) Pick $V_1(x_1) > 0$ and $V_{12}(x_1 - x_2) > 0$

$\Rightarrow$ Drive to $(0,0)$!!

$\Rightarrow$ Statically Stable

Case Study #1:
Control Design Issues

B) Pick

$$u_1 = -K_{p_1} x_1 - K_{I1} \int x_1 dt - K_{D1} \dot{x}_1$$
$$u_2 = -K_{p_2} x_2 - K_{I2} \int x_2 dt - K_{D2} \dot{x}_2$$

Collocated Control

$$H_{12} = \frac{1}{2} m_1 x_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} K_{p_1} x_1^2 + \frac{1}{2} K_{p_1} x_2^2 + V_1(x_1) + V_{12}(x_1 - x_2)$$

$$\dot{H}_{12} = \lfloor m_1 \ddot{x}_1 + g_1(x_1) + K_{p_1} x_1 \rfloor \dot{x}_1 + \lfloor m_2 \ddot{x}_2 + K_{p_2} x_2 \rfloor \dot{x}_2 + g_{12}(x_1 - x_1)(\dot{x}_1 - \dot{x}_2)$$

$$= \lfloor m_1 \ddot{x}_1 + g_1(x_1) + g_{12}(x_1 - x_2) + K_{p_1} x_1 \rfloor x_1 + \lfloor m_2 \ddot{x}_2 + g_{12}(x_2 - x_1) + K_{p_2} x_2 \rfloor \dot{x}_2$$

$$= \left[-f_1(\dot{x}_1) - K_{D_1} \dot{x}_1 - K_I \int x_1 dt \right] \dot{x}_1 + \left[-K_{D_2} \dot{x}_2 - K_{I_2} \int x_2 dt \right] \dot{x}_2$$

$$+ \left[-f_{12}(\dot{x}_1 - \dot{x}_2) \right] (\dot{x}_1 - \dot{x}_2)$$

Case Study #1: Control Design Issues

C) Pick

$$u_1 = -K_{p_1} x_2 - K_{I_1} \int x_2 dt - K_{D_1} \dot{x}_2; \quad u_2 = 0\}$$

Noncollocated Control

$$H_{12} = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + V_1(x_1) + V_{12}(x_1 - x_2)$$

$$\dot{H}_{12} = [m_1 \ddot{x}_1 + g_1(x_1) + g_{12}(x_1 - x_2)]\dot{x}_1 + [m_2 \ddot{x}_2 + g_{12}(x_2 - x_1)]\dot{x}_2$$

$$= [-f_1(\dot{x}_1) - f_{12}(\dot{x}_1 - \dot{x}_2) + u_1]\dot{x}_1 + [-f_{12}(\dot{x}_2 - \dot{x}_1)]\dot{x}_2$$

$$= [-f_1(\dot{x}_1) + u_1]\dot{x}_1 + [-f_{12}(\dot{x}_1 - \dot{x}_2)](\dot{x}_1 - \dot{x}_2)$$

$$= \left[-f_1(\dot{x}_1) - K_{p_1} x_2 - K_{I_1} \int x_2 dt - K_{D_1} \dot{x}_2\right]\dot{x}_1 + [-f_{12}(\dot{x}_1 - \dot{x}_2)](\dot{x}_1 - \dot{x}_2)$$

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Case Study #1: Control Design Issues

* $$\Delta x = x_2 - x_1 \Rightarrow \quad x_2 = x_1 + \Delta x$$

$$H_{12} = \left[-f_1(\dot{x}_1) - K_p(x_1 + \Delta x) - K_{I_1}\int (x_1 + \Delta x)dt - K_{D_1}(\dot{x}_1 + \Delta\dot{x}) \right]\dot{x}_1$$

$$+ \left[-f_{12}(\Delta\dot{x}) \right]\Delta\dot{x}$$

$$= \left[-f_1(\dot{x}_1) - K_{p_1}x_1 - K_{I_1}\int x_1 dt - K_{D_1}\dot{x}_1 \right]\dot{x}_1 + \left[-f_{12}(\Delta\dot{x}) \right]\Delta\dot{x}$$

$$+ \left[-K_{p_1}\Delta x - K_{I_1}\int \Delta x dt - K_{D_1}\Delta\dot{x} \right]\dot{x}_1$$

$$\Rightarrow H_{12} = \frac{1}{2}m_1 x_1^2 + \frac{1}{2}m_2\dot{x}_2^2 + V_1(x_1) + V_{12}(x_1 - x_2) + \frac{1}{2}K_{p_1}x_1^2$$

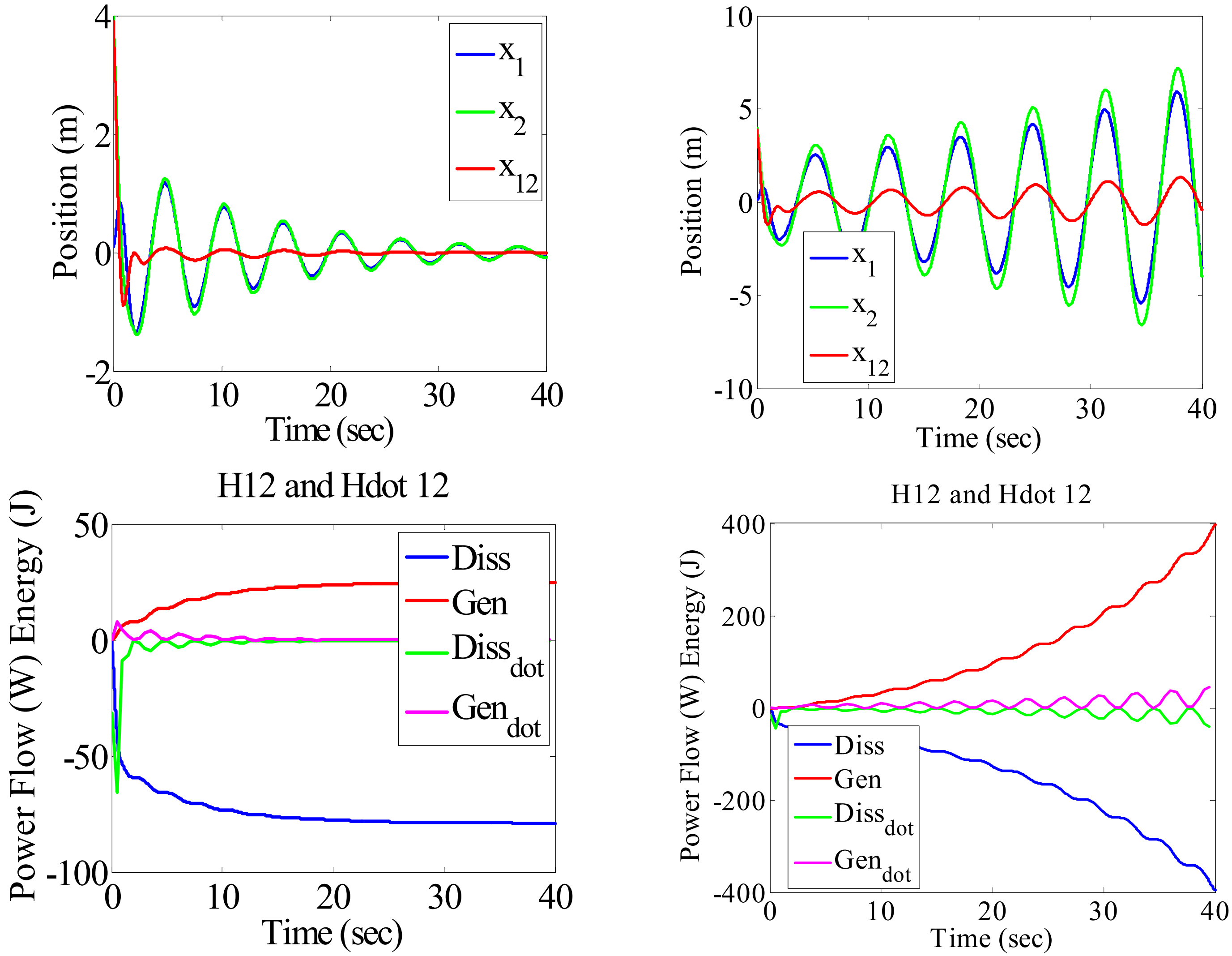
$$\dot{H}_{12} = \left[-f_1(\dot{x}_1) - K_{D_1}\dot{x}_1 - K_{I_1}\int x_1 - \underbrace{\left\langle K_{p_1}\Delta x + K_{I_1}\int \Delta x dt + K_{D_1}\Delta\dot{x} \right\rangle}_{\Delta u_{\Delta x}} \right]\dot{x}_1$$

$$+ \left[-f_{12}(\Delta\dot{x}) \right]\Delta\dot{x}$$

How big is this?

Case Study #1:
Control Design Issues

iii.) Collocated vs. NonCollocated (Unstable)



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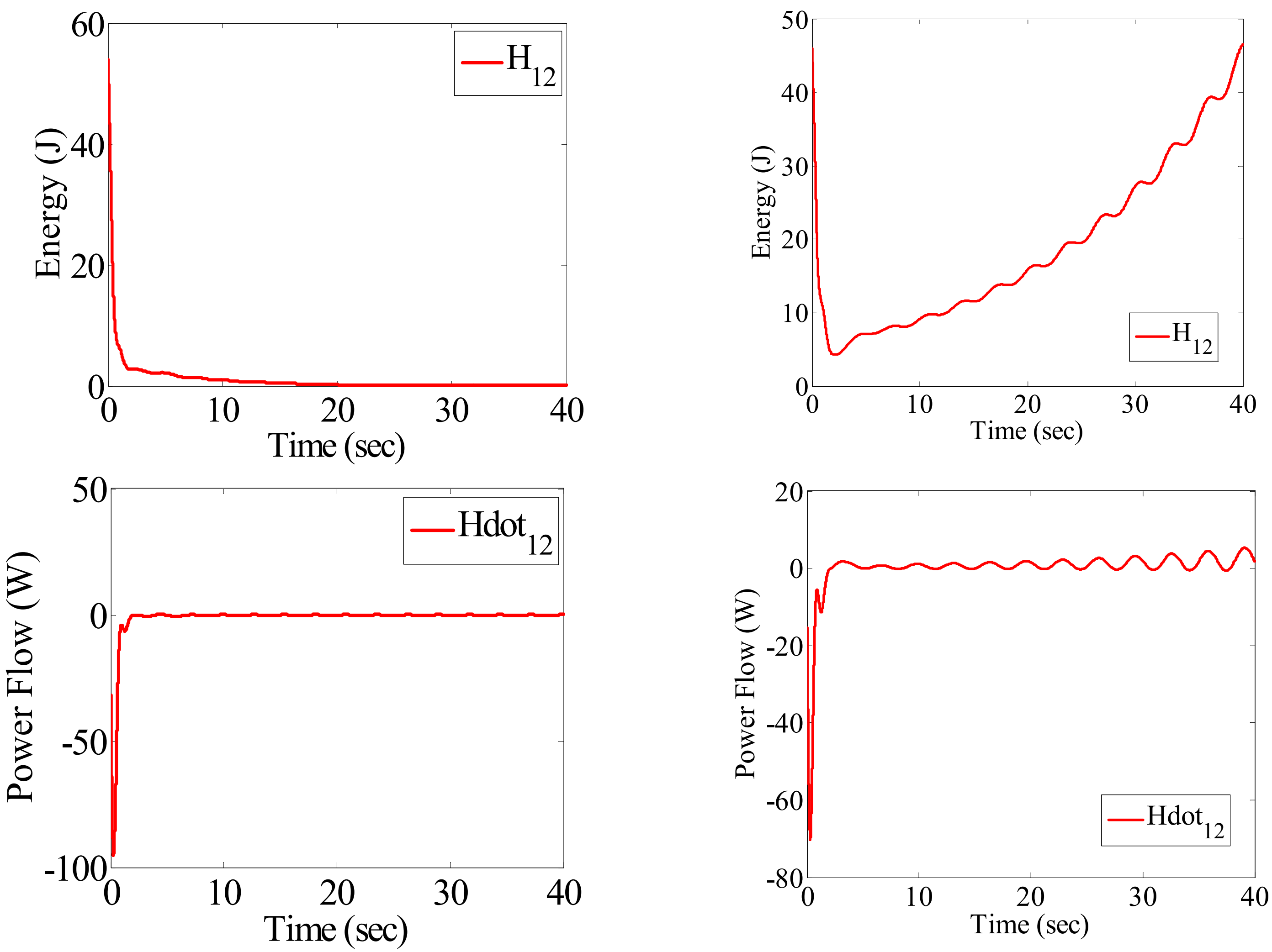
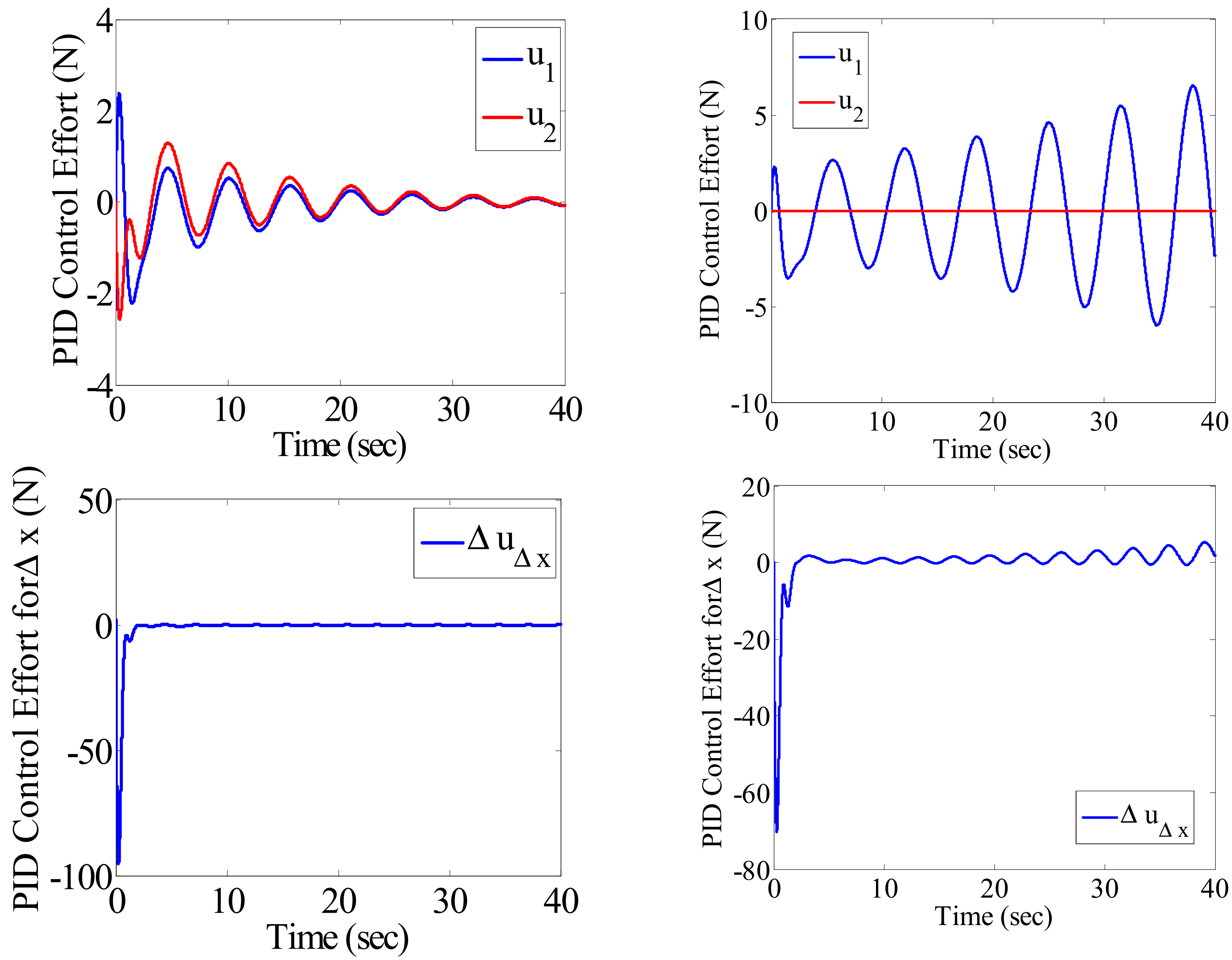
Case Study #1:
Control Design Issues

iii.) Collocated vs. NonCollocated (Unstable)



Case Study #1: Control Design Issues

iii.) Collocated vs. NonCollocated (Unstable)



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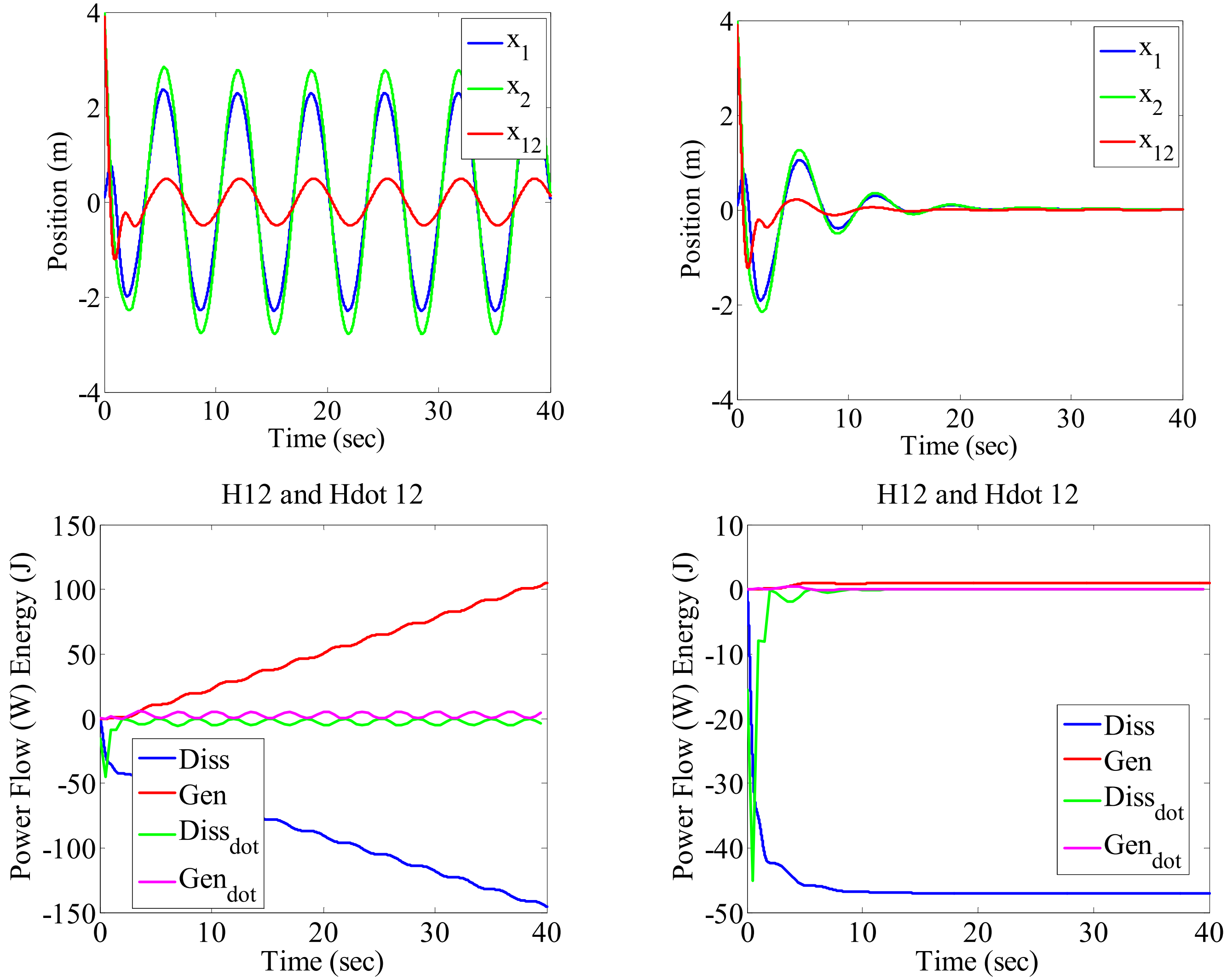
Case Study #1: Control Design Issues

iii.) NonCollocated (Neutral) vs. NonCollocated (Stable)



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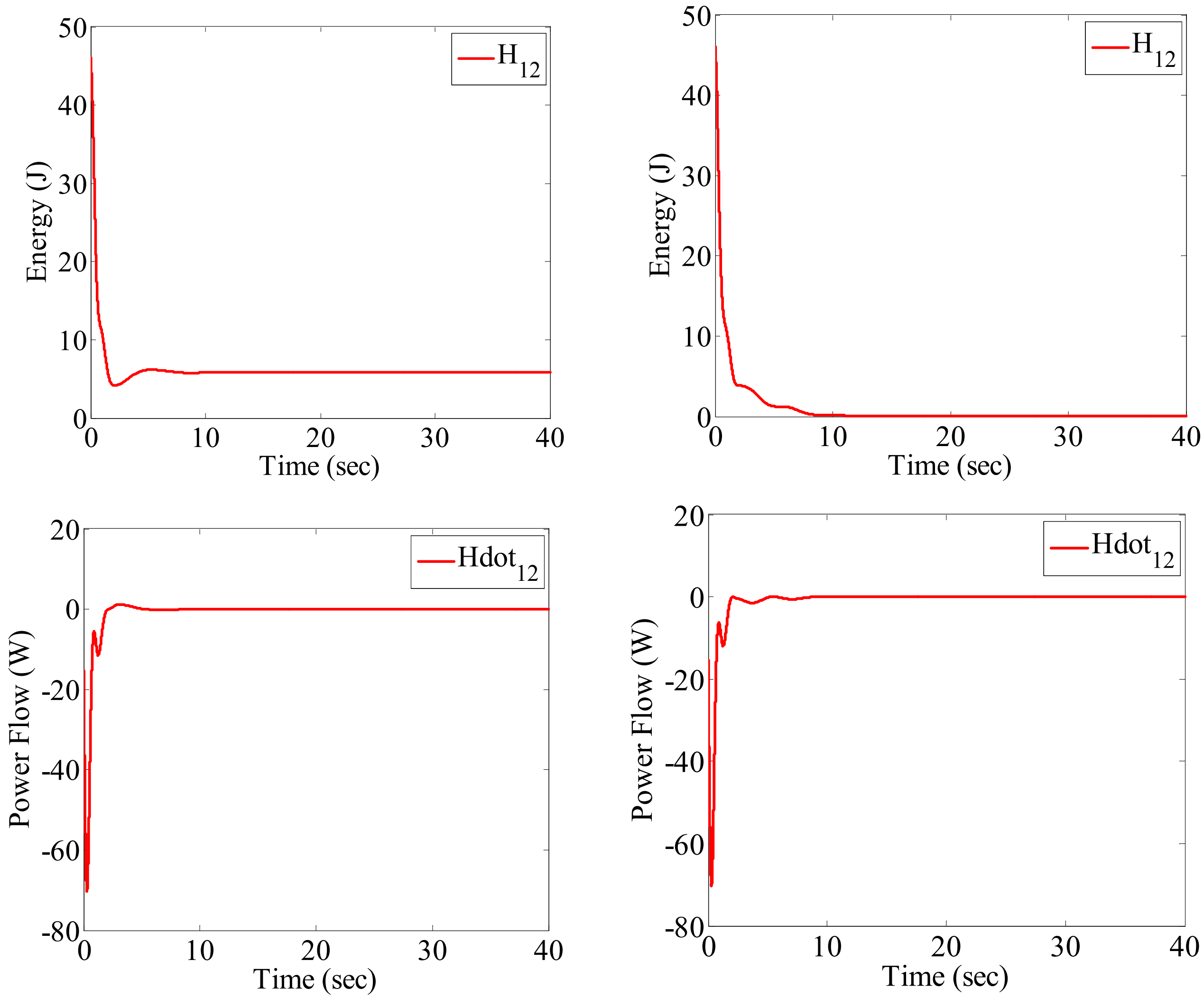
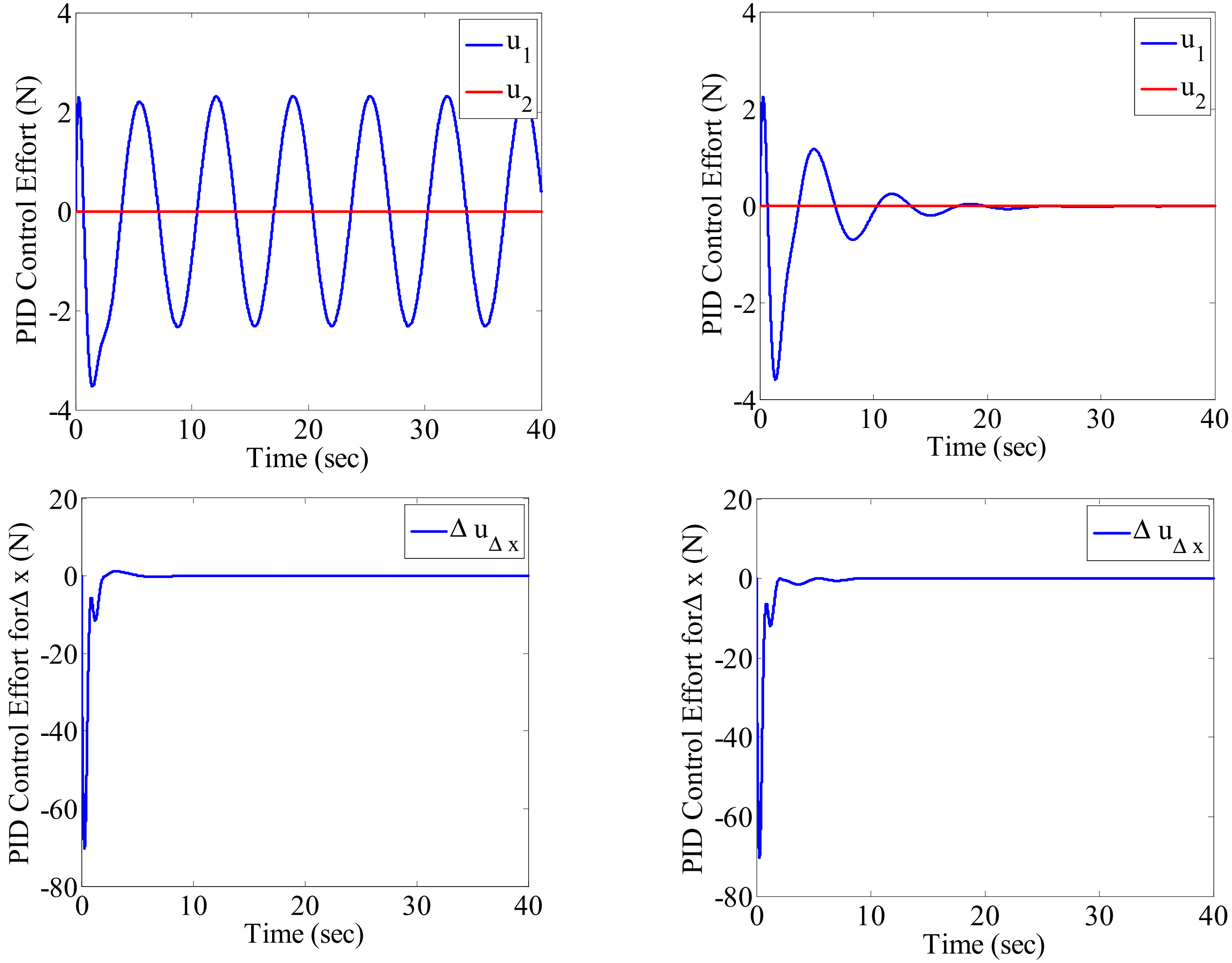
Case Study #1:
Control Design Issues

iii.) NonCollocated (Neutral) vs. NonCollocated (Stable)



Case Study #1:
Control Design Issues

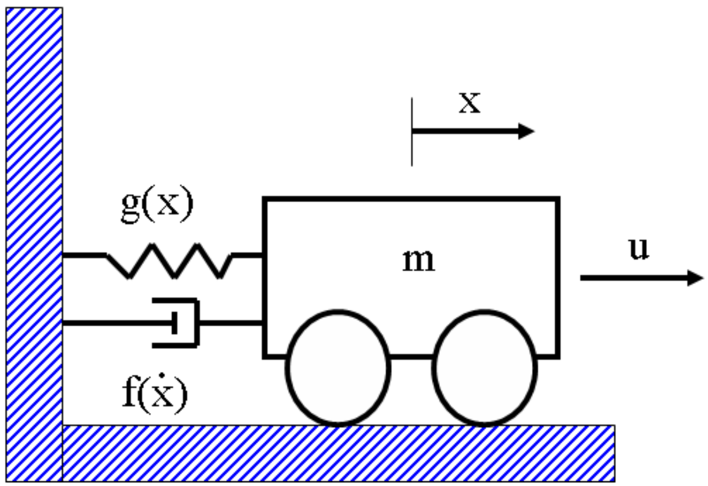
iii.) NonCollocated (Neutral) vs. NonCollocated (Stable)



Case Study #1:
Control Design Issues

4) Stable / Unstable Systems

i.) Model : Nonlinear Linear Oscillator



Equation of motion:

$$m\ddot{x} + g(x) = u - f(\dot{x})$$

$$H = \frac{1}{2} m\dot{x}^2 + V(x)$$

$$\dot{H} = [m\ddot{x} + g(x)]\dot{x} = [u - f(\dot{x})]\dot{x}$$

Case Study #1:
Control Design Issues

ii.) Pick Functions:

$$V(x) = \frac{1}{2}kx^2 + \frac{1}{4}k_{NL}x^4$$

$$f(\dot{x}) = c\dot{x} + c_{NL}\,sign(\dot{x})$$

$$u = -K_p x - K_I \int x\,dt - K_D \dot{x}$$

$$m\ddot{x} + \left(k + K_p\right)x + k_{NL}x^3 = -\left(c + K_D\right)\dot{x} - c_{NL}\,sign(\dot{x}) - K_I \int x\,dt$$

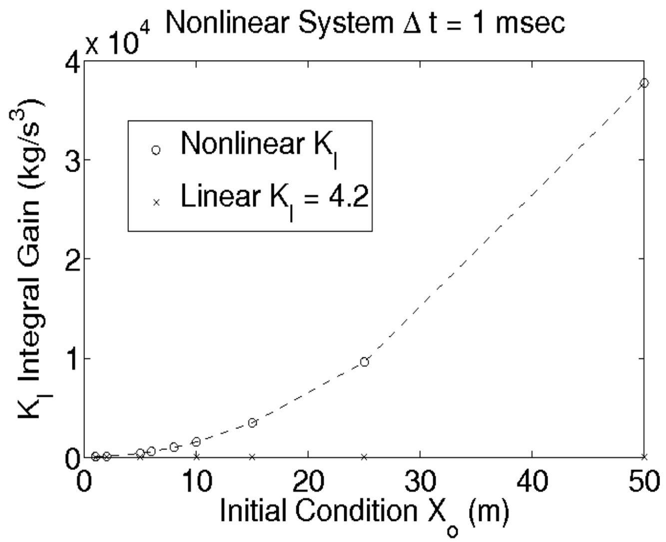
Case Study #1: Control Design Issues

iii.) Evaluate Control Design

A. Static Stability: $H = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}(k+K_p)x^2 + \frac{1}{4}k_{NL}x^4$

$\Rightarrow$ Stable for $m, k_{NL} > 0$ and $k + K_p > 0$

$\Rightarrow$ Pick $k + K_p < 0$: Unstable at $(0,0)$



Gain Scheduling

B. Dynamic Stability:

$$\dot{H} = \left[-(c+K_D)\dot{x} - c_{NL}sign(\dot{x}) - K_I \int xdt\right]\dot{x}$$

Limit cycle:

$$\oint_{\tau_c} \left[(c+K_D)\dot{x} + c_{NL}sign(\dot{x})\right]\dot{x} = -\oint_{\tau_c} \left[K_I \int xdt\right]\dot{x}dt$$

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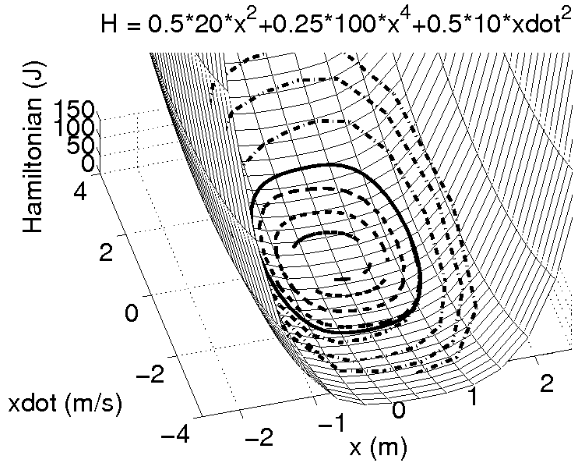
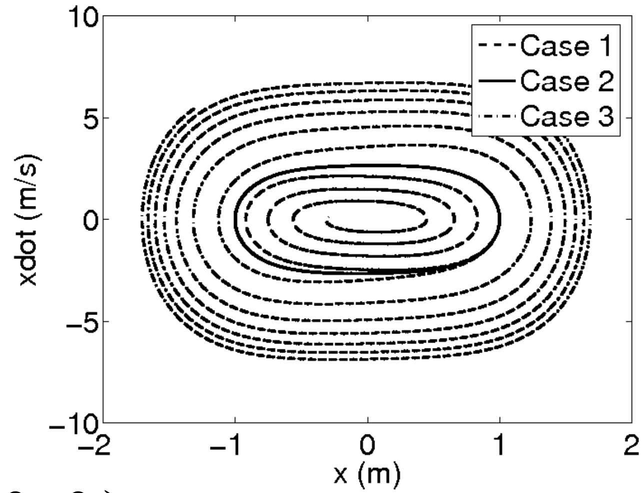
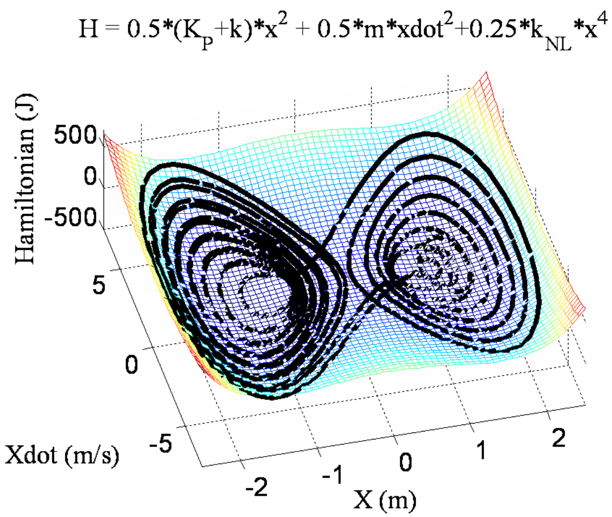
Case Study #1: Control Design Issues

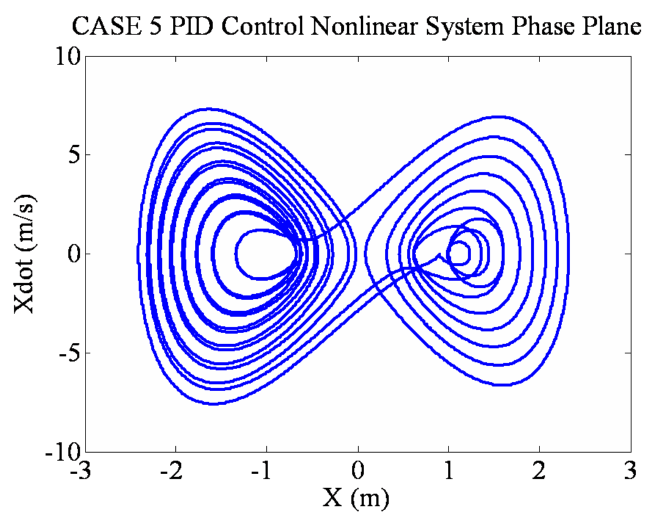
$\Rightarrow$ Stable for $k + K_p > 0$





$\Rightarrow k + K_p < 0$: Unstable at $(0,0)$





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Case Study #1: Control Design Issues

5) Many popular coupled dynamical systems: (robots, spacecraft, etc.) can be modeled as

$$\tau = M(\underline{q})\ddot{\underline{q}} + C(\underline{q},\dot{\underline{q}}) + G(\underline{q})$$

Work rate of the gyroscopic or centripetal/Coriolis terms are zero [Ref. Slotine, Robinett]

$\tau =$ External torque

$\underline{q} =$ Generalized position

$\dot{\underline{q}} =$ Generalized velocity

$M(\underline{q}) =$ Symmetric mass matrix

$C(\underline{q},\dot{\underline{q}}) =$ Centripetal/Coriolis vector

$G(\underline{q}) =$ Gravitational vector

Dynamical systems in this form can be shown to be designed as decoupled SISO systems

[Ref. Slotine] J..-J.E. Slotine and W. Li, **Applied Nonlinear Control**, Prentice-Hall, Inc., N.J., 1991

[Ref. Robinett] R.D. Robinett, G.G. Parker, H. Schaub, J.L. Junkins, *Lyapunov Optimal Saturated Control for Nonlinear Systems*, J. Guidance, Control, and Dynamics, Vol. 20, No. 6, pp. 1083-1088, Nov.-Dec., 1997

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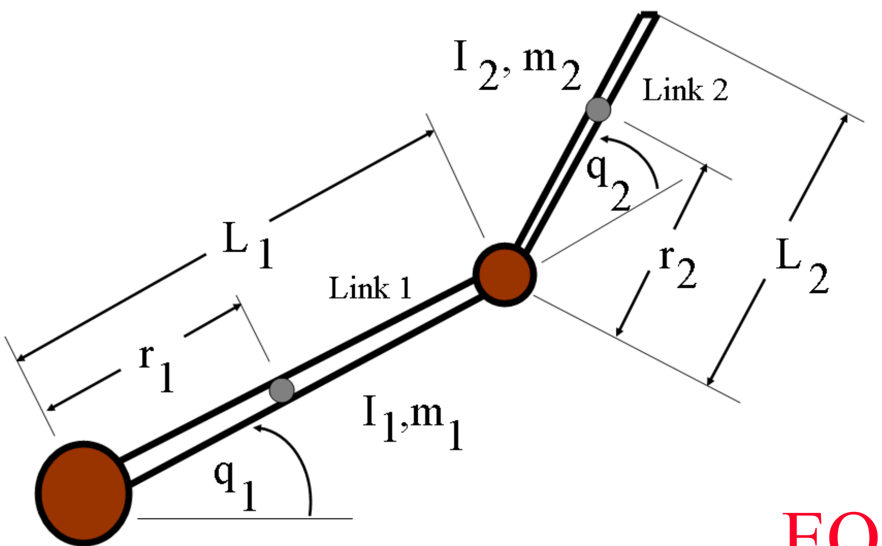
Case Study #1: MIMO 2 DOF Planar Robot w/ PID Control System



EOM and PID Control Law:

$$H(q)\ddot{q} + C(q,\dot{q}) = \tau$$

$$\tau = \hat{H}\ddot{q}_{ref} + \hat{C}\dot{q}_{ref} - K_P\tilde{q} - K_I\int_0^t \tilde{q}\, d\bar{\tau} - K_D\dot{\tilde{q}}$$

$$\tilde{q} = q_{ref} - q$$

Case Study #1: Lyapunov Function/ Hamiltonian Based on Error Energy

$$\mathscr{V} = \Delta H = \frac{1}{2}\dot{\tilde{q}}^T H \dot{\tilde{q}} + \frac{1}{2}\tilde{q}^T K_P \tilde{q}$$

$$\dot{\mathscr{V}} = -\dot{\tilde{q}}^T K_D \dot{\tilde{q}} - \dot{\tilde{q}}^T K_I \int_0^t \tilde{q}\, d\bar{\tau}$$

(assumes perfect parameter matching)

$$\left[\Delta \dot{W}\right]_{ave} = \left[T_o \Delta \dot{S}_i\right]_{ave}$$

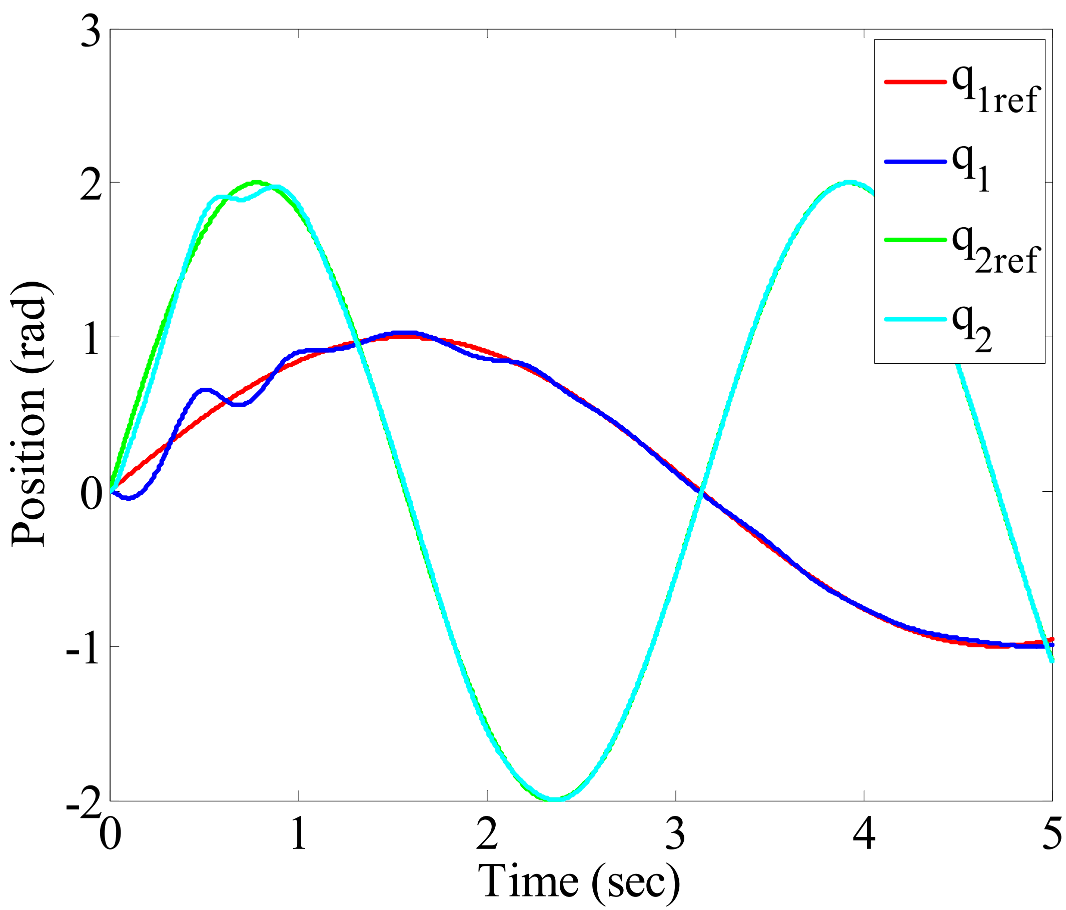
$$[\dot{\tilde{q}}^T K_D \dot{\tilde{q}}]_{ave} = [-\dot{\tilde{q}}^T K_I \int_0^t \tilde{q}\, d\bar{\tau}]_{ave}$$

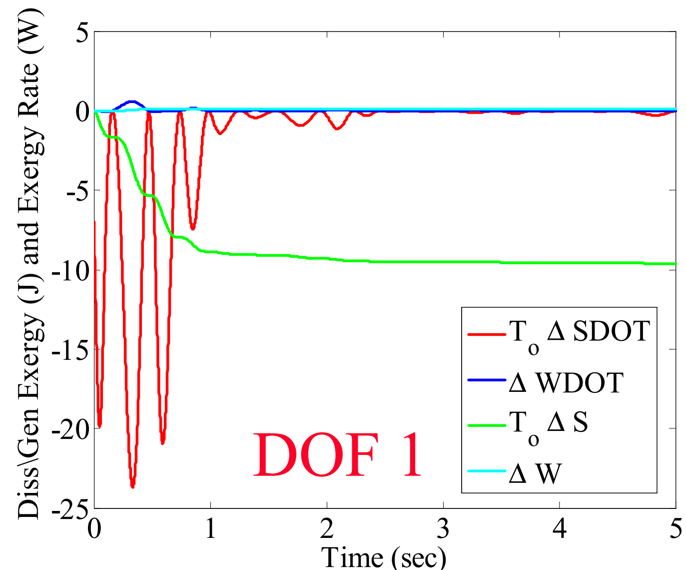
Applied per DOF

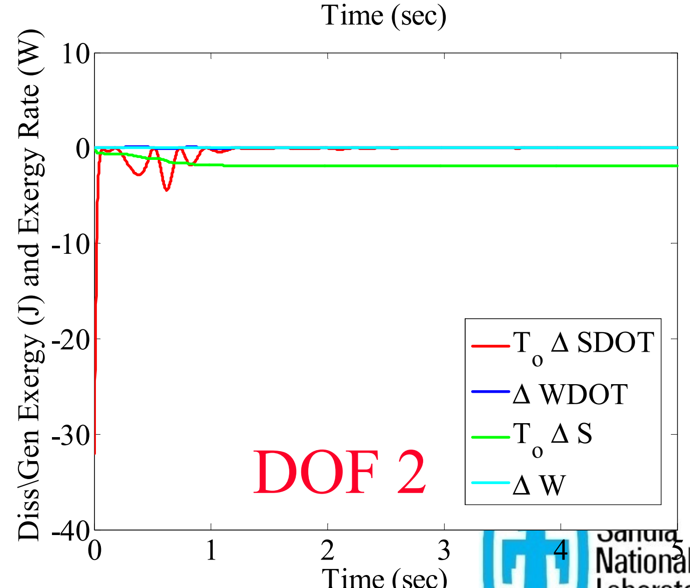
Nonlinear stability boundary

Case Study #1: Planar Robot w/ PID Control
Numerical Simulation Results - Case 1

DISSIPATIVE







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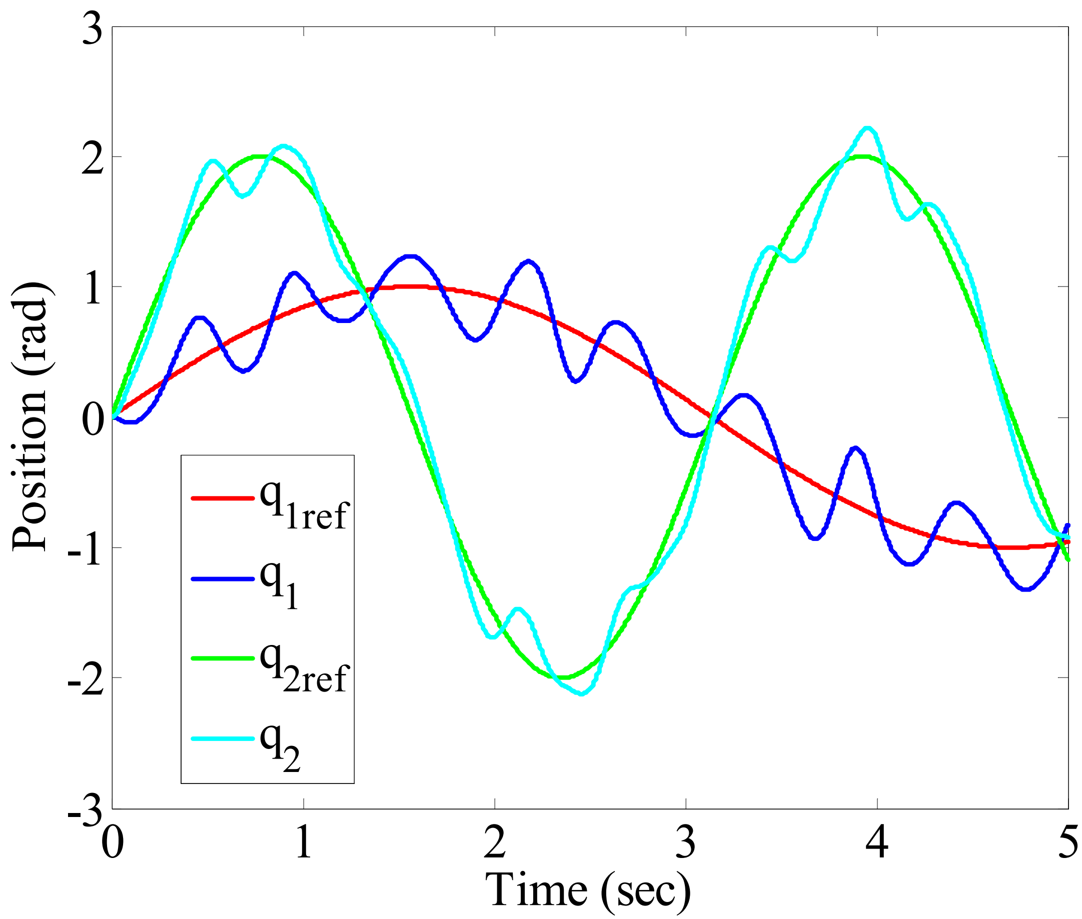
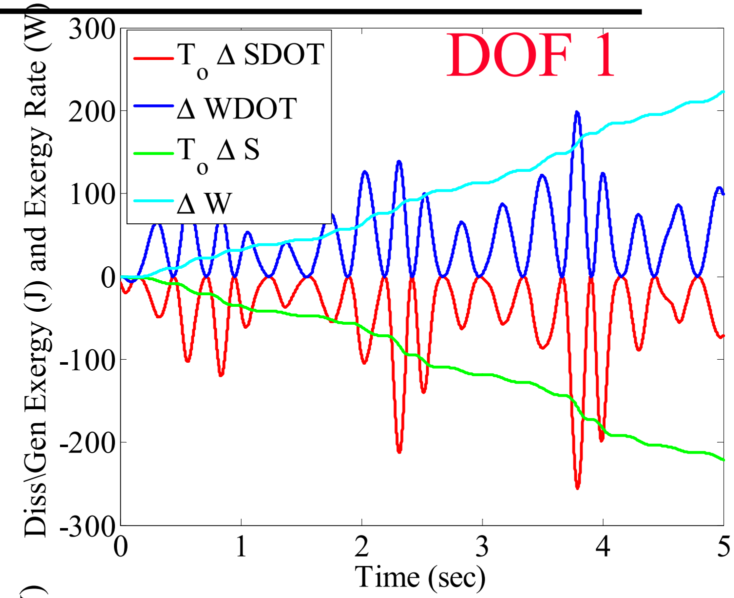
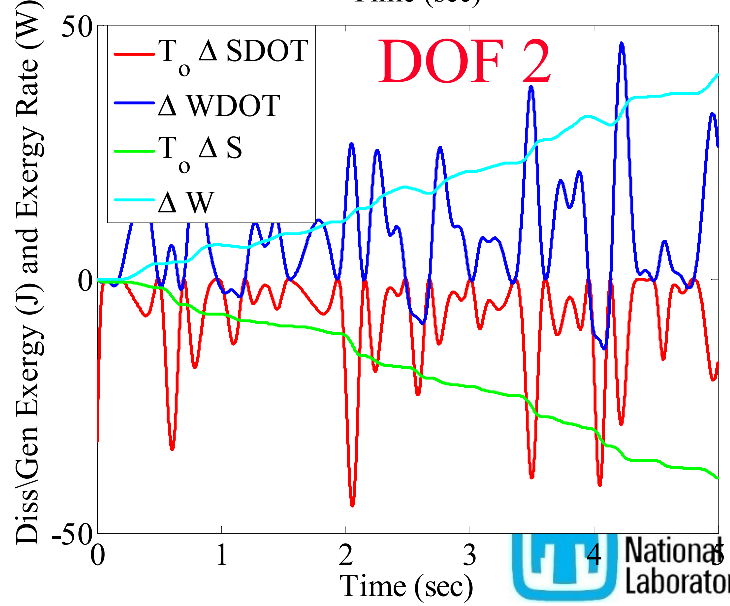
Case Study #1: Planar Robot w/ PID Control
Numerical Simulation Results - Case 2

NEUTRAL STABILITY







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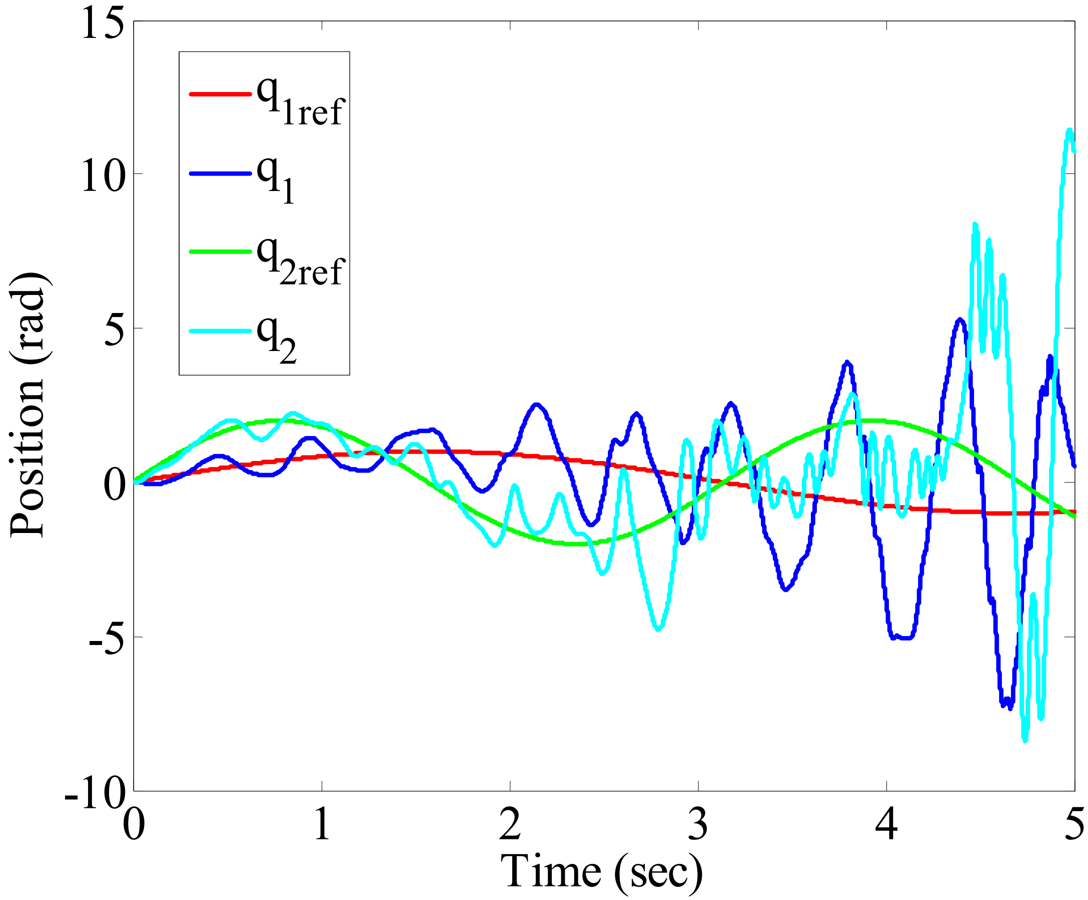
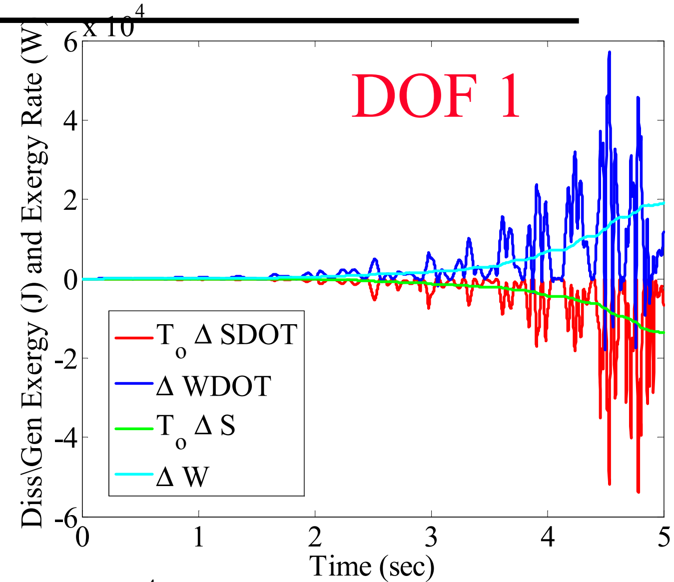
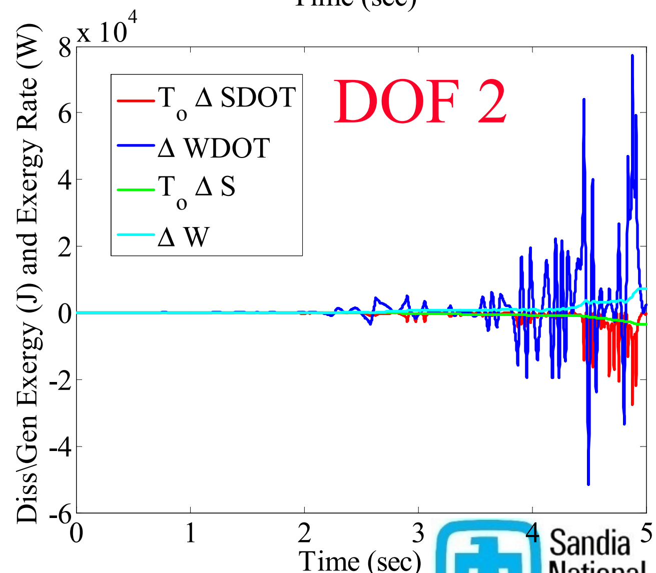
Case Study #1: Planar Robot w/ PID Control
Numerical Simulation Results - Case 3

GENERATIVE







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Case Study #2

Collective Plume Tracing: A Minimal Information Approach to Collective Control

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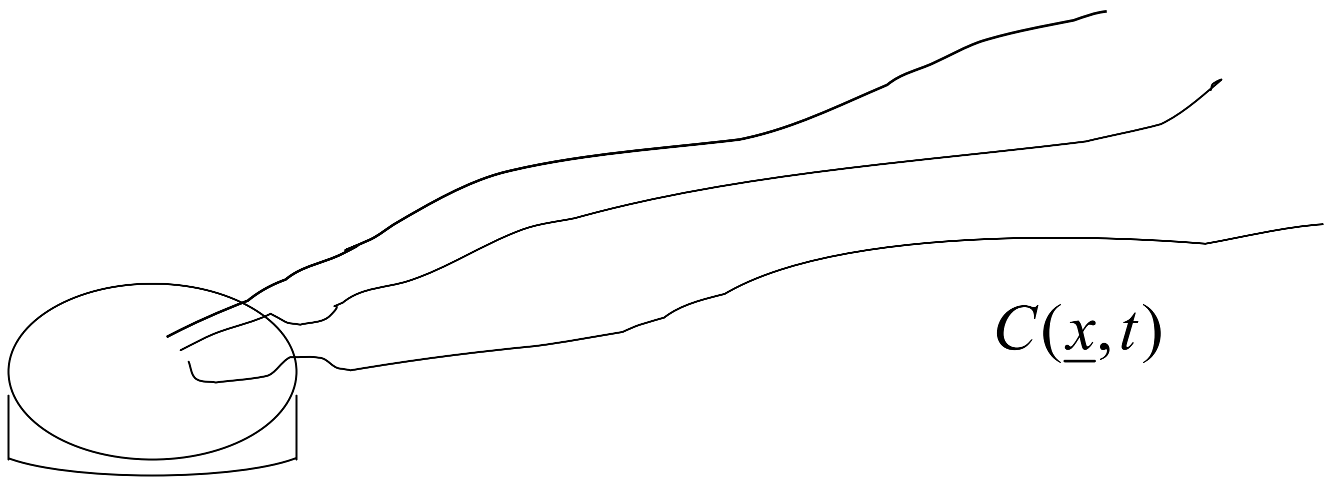
Case Study #2: Collective Plume Tracing

- **Problem**: Track a chemical plume to its source; find buried landmines



- **Characteristics**: 1- Nonlinear time/spatially varying plume field
  2-Complicated model-turbulence
  3-Difficult to correlate model to data in time/space
  4- Can't count on wind or water flow direction
  5- Sensor Latency

Solution ??

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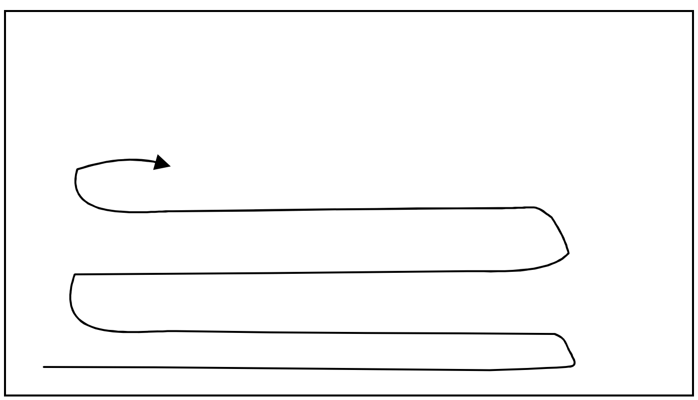
Case Study #2:
Collective Plume Tracing

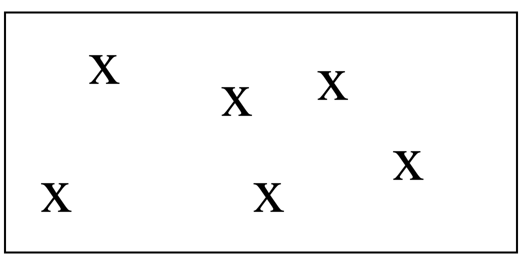
- **<u>Possible solution</u>**: Person with a chemical sensor and a GPS receiver

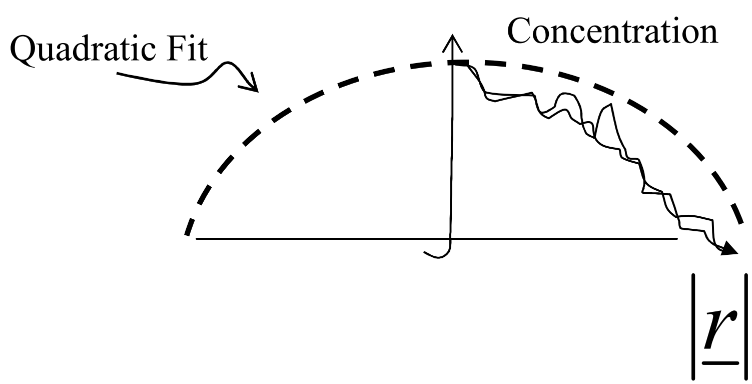
1. Time–spatial correlation issues
2. Sensor latency/dynamic range issues
3. No redundancy
4. Leads to exhaustive search



Case Study #2: Collective Plume Tracing

- **<u>Collective Solution</u>**: Team of simple robots performing a collective search
  - Distributed sensing
  - Decentralized control (optimization/Newton update)
  - Simple model of plume
  - Redundant
  - Independent of latency for stability (Performance?)
  - Minimize processing, memory, and communications
  - Beat down noise





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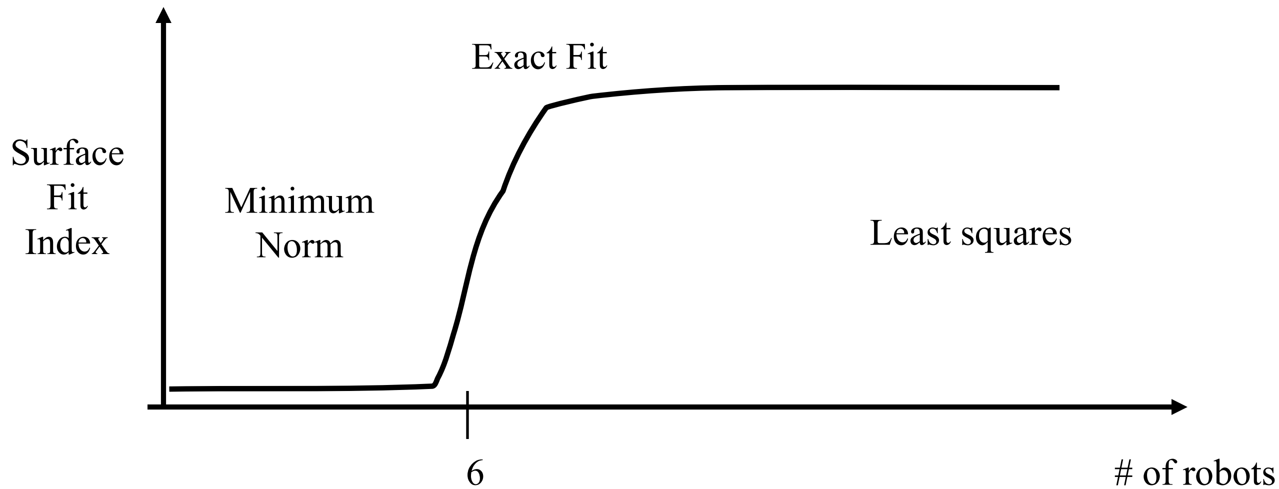
Case Study #2: Collective Plume Tracing

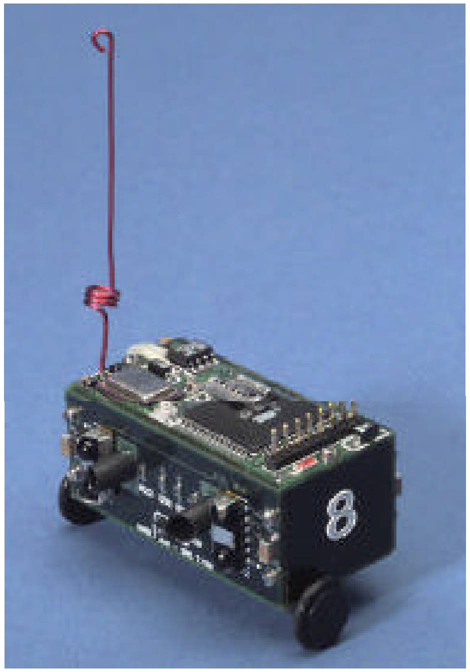
- **Emergent Behavior**: Quadratic fit w/o memory



- **Evaluate Resource Utilization**: Trade-off between processing, memory, and communications
  - **Baseline**: Minimize all three simultaneously
    - 8 bit processor, no memory, broadcast 3 words

Case Study #2:
Collective Plume Tracing

- Goal of DARPA Distributed Robotics [Ref. Byrne] Program: build smallest, dumbest robot that couldn't do anything other than random motion
- As a team of like robots - could solve complicated problem
- N land-based robots whose task to localize a source that emits a measurable scalar field F(x)
- Robots evaluate the field with a sensor and communicate locations and sensor measurements to neighboring robots
- Robots shared sensor information, but individually decided course of action based on their own estimate of plume field
- Robot samples its environment, broadcasts information to others



0.75x0.71x1.6 inches
Onboard temperature sensor
8-bit RISC processor
CSMA radio network
50 Kbits/sec data rate

[Ref. Byrne] Byrne, et. al., *Miniature Mobile Robots for Plume Tracking and Source Localization Research*, J. MicroMech., Vol. 1, No. 3, 2002.

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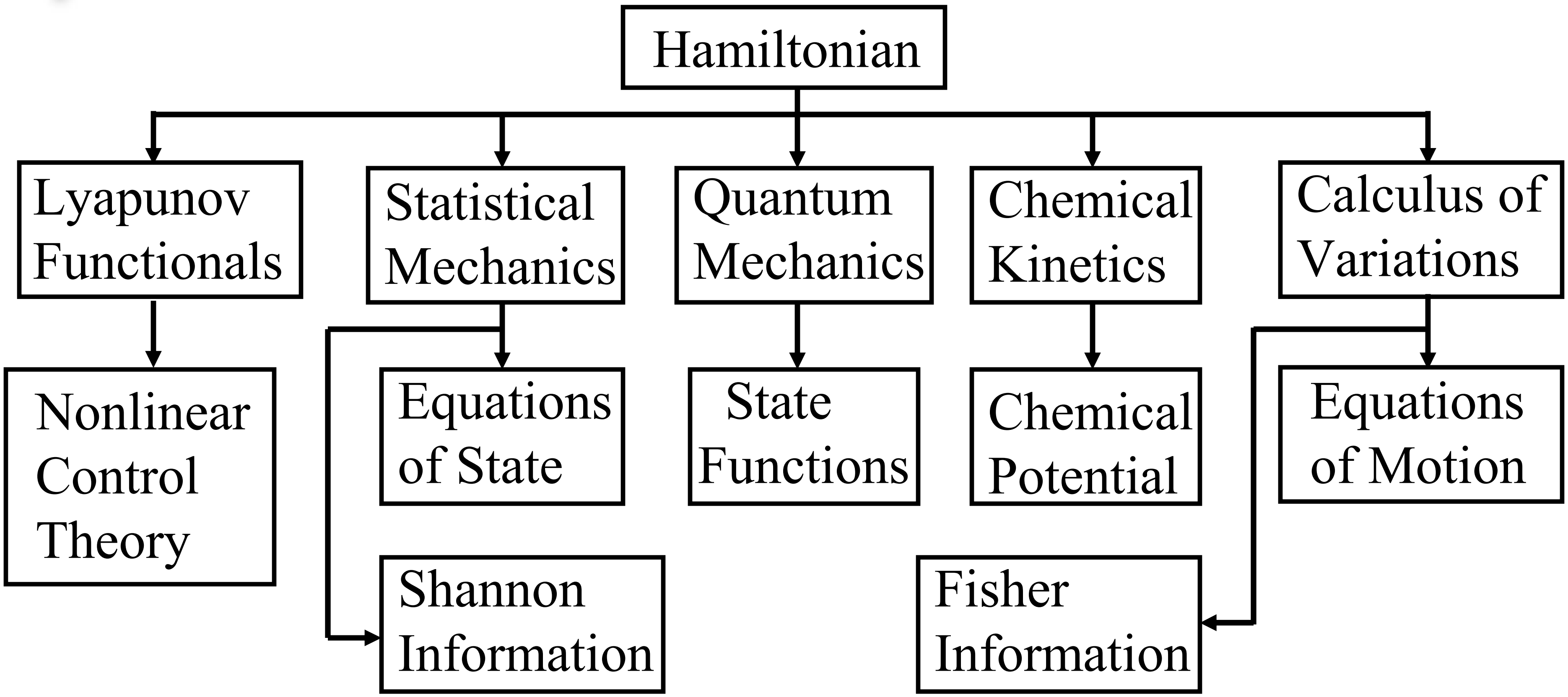
Case Study #2: Collective Plume Tracing



- Universal Mathematical Approach
  - Nonlinear control design via exergy/entropy thermodynamic concepts
  - Hamiltonian (universal function) is the key element
  - Each column has the ability to work on collective systems

Case Study #2: Collective Plume Tracing

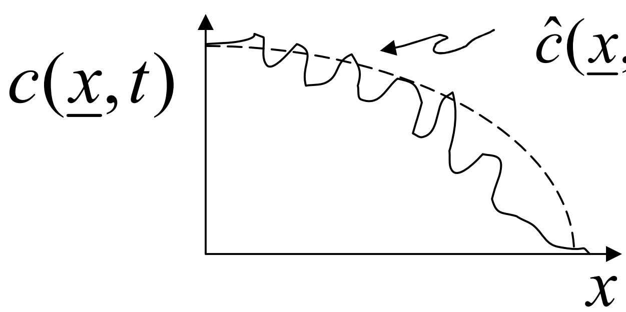
- **Kinematic Control** – No dynamics; assume tight autopilot loop; control velocity (speed)

  1- Need to measure plume field: $C(\underline{x},t)-$ concentration at a point in time

  2- Convert concentration measurements to range and bearing to target

  ⇒ Fit a quadratic surface to distributed sensor measurements

  ⇒ Perform decentralized Newton updates for feedback control



$$\hat{c}(\underline{x},t)=\hat{c}_o+\hat{c}_1^T\underline{x}+\frac{1}{2}\underline{x}+\hat{c}_2\underline{x}$$

$$\frac{\partial\hat{c}}{\partial\underline{x}}=0\Rightarrow\underline{x}_{k+1}=-\alpha\hat{c}_2^{-1}\hat{c}_1+x_k$$

  3- Shape the "virtual potential" with a minimum (maximum) at $\underline{x}^*=-\hat{c}_2^{-1}\hat{c}_1$ for Hamiltonian to meet static stability requirements for the i^{th} robot:

$$H_i=V_{c_i}(\underline{x}_i)=\frac{1}{2}\hat{c}_1^T\hat{c}_2^{-1}\hat{c}_1+\hat{c}_1^T\underline{x}_i+\frac{1}{2}\underline{x}_i^T\hat{c}_2\underline{x}_i>0$$

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Case Study #2: Collective Plume Tracing

And collective robots

$$H = \sum_{i=1}^{N} H_i = \sum_{i=1}^{N} V_{c_i} > 0$$

4-Design the power flow, time derivative of the Hamiltonian, to meet the dynamic stability requirements for the feedback controller

$$\dot{x}_i = -\hat{c}_2^{-1}\left[\hat{c}_1 + \hat{c}_2 \underline{x}_i\right] = -\hat{c}_2^{-1}\hat{c}_1 - \underline{x}_i = \underline{x}^* - \underline{x}_i \quad \text{(Linear tracker)}$$

$$\dot{H}_i = \underline{\dot{x}}_i^T\left[\hat{c}_1 + \hat{c}_2 \underline{x}_i\right] = -\left[\hat{c}_1 + \hat{c}_2 \underline{x}_i\right]^T \hat{c}_2^{-1}\left[\hat{c}_1 + \hat{c}_2 \underline{x}_i\right] < 0$$

$$\dot{H} = \sum_{i=1}^{N} \dot{H}_i < 0$$

$$\underline{x}_{cm} = \frac{1}{N}\sum_{i=1}^{N} \underline{x}_i \Rightarrow \underline{\dot{x}}_{cm} = \frac{1}{N}\sum_{i=1}^{N} \left[-\underline{x}_i + \underline{x}^*\right] = -\underline{x}_{cm} + \underline{x}^*$$

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Case Study #2: Collective Plume Tracing

- **Kinetic Control** – dynamics included: $m_i \ddot{\underline{x}}_i = \underline{u}_i$

1-Shape the Hamiltonian surface to meet the static stability requirements for the i^{th} robot

$$H_i = T_i + V_{c_i} = \frac{1}{2}\dot{\underline{x}}_i^T M_i \dot{\underline{x}}_i + V_{c_i} > 0$$

and the collective of robots

$$H = \sum_{i=1}^{N} H_i = \sum_{i=1}^{N} \left[T_i + V_{c_i}\right] > 0$$

2- Design the power flow, time derivative of the Hamiltonian, to meet the dynamic stability requirements for the feedback controller

$$\underline{u}_i = -\frac{\partial V_{c_i}}{\partial \underline{x}} - K_I \int \underline{x}_i dt - K_D \dot{\underline{x}}_i$$

$$\dot{H}_i = \dot{\underline{x}}_i^T \left[M_i \ddot{x}_i + \frac{\partial V_{c_i}}{\partial \underline{x}} \right] = \dot{\underline{x}}_i^T \left[-K_I \int \underline{x}_i dt - K_D \dot{\underline{x}}_i \right] < 0$$

$$\dot{H}_i = \sum_{i=1}^{N} \dot{H}_i < 0$$

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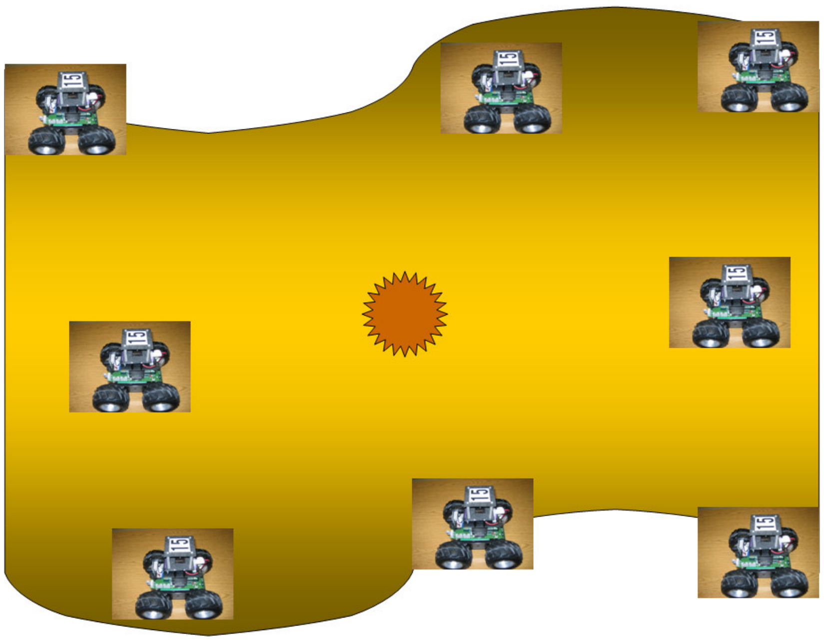
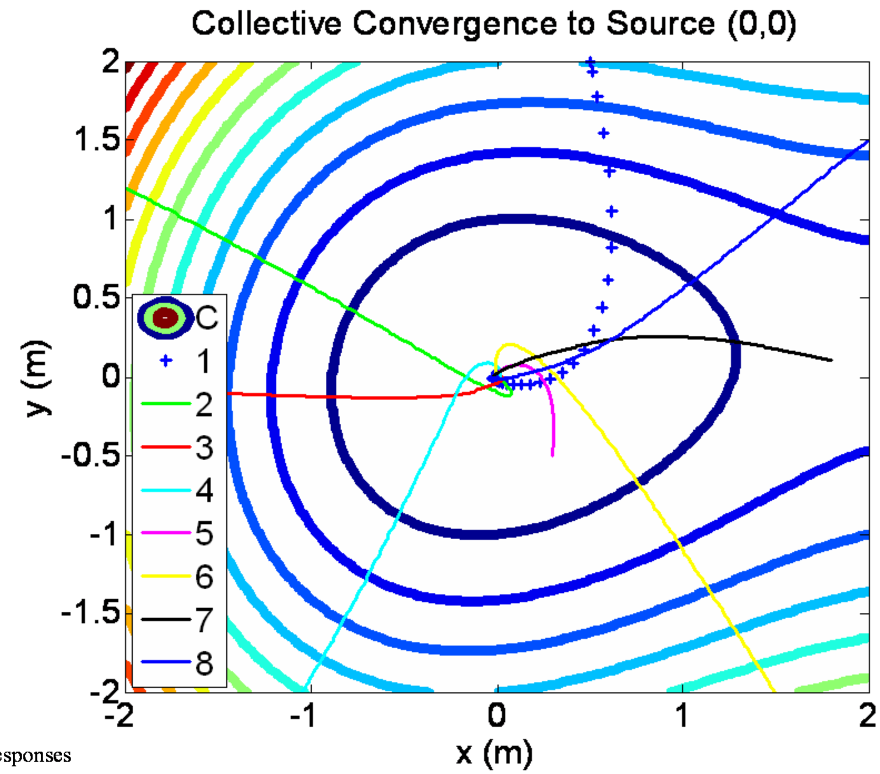
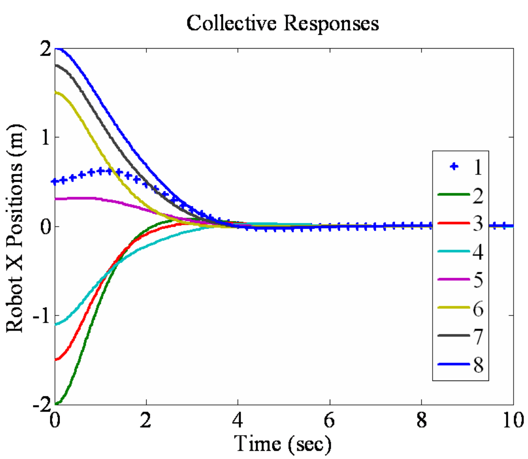
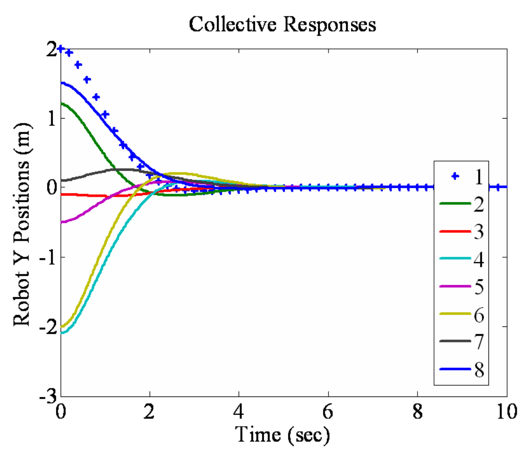
Case Study #2: Collective Robots
Numerical Results









Convergence:
Dissipative > Generative
for the collective

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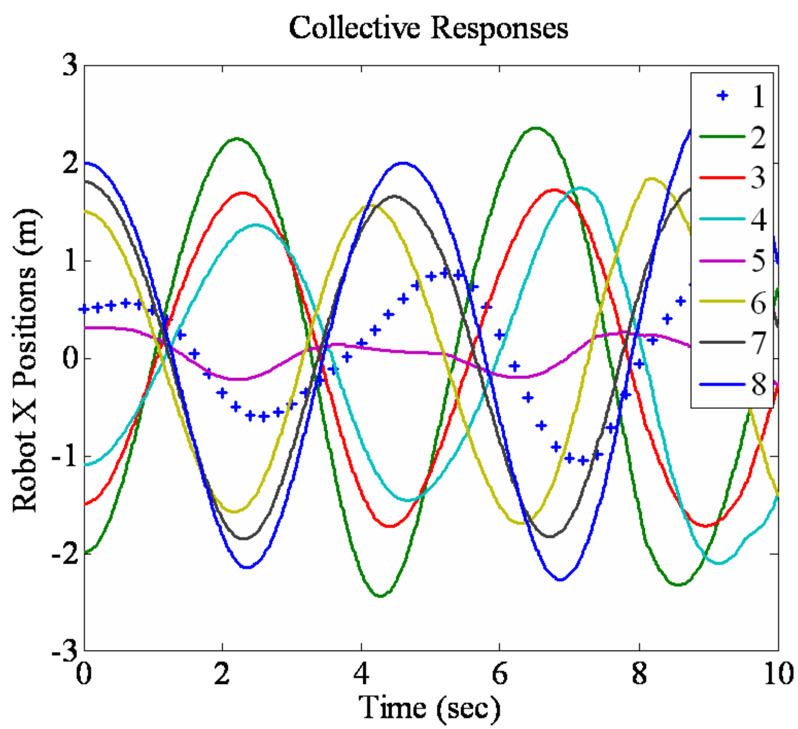
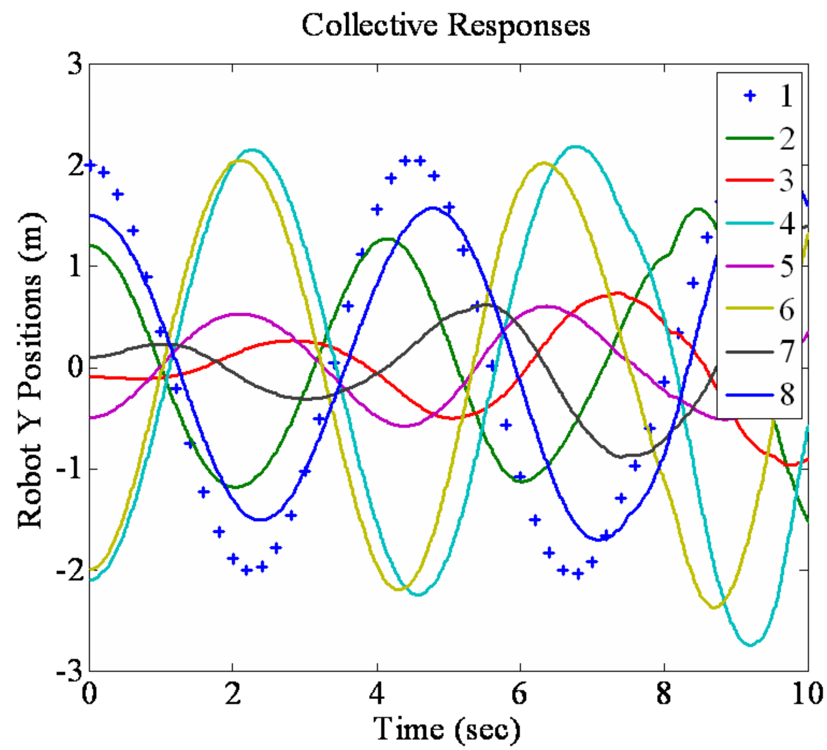
Case Study #2: Collective Robots Stability Boundary

- Collective robot oscillations at the limit cycle





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Case Study #2: Collective Plume Tracing

- **<u>Information Theory</u>** Applied To:

  1- Kinematic Control: Shannon information /entropy

  a) Fundamental trade-off: processing, memory, and communications

  $\Rightarrow$ minimize all three simultaneously

  $\Rightarrow$ 8 bit processor, no memory, 3 words to communicate

  b) Stability: latency independent due to no dynamics; stop until next update/command

  c) Performance: the limitation is the sensor update rate; 60 sec

  $\Rightarrow$ Channel/system capacity of an equiprobable source

$$C_{ave} = \frac{\text{information}}{\text{time}} = \frac{1}{\tau}\log_2 n \frac{bits}{\sec} = \dot{H}_{ave}$$

COMM. Link = 50 Kbits/sec

$$n = (T_{HIGH} - T_{LOW})/\Delta T = 40/0.5 = 80 bits; T - temperature$$

$$\Rightarrow C_{ave} = \frac{1}{60}\log_2(80) < 0.1\frac{bit}{\sec}$$

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Case Study #2: Collective Plume Tracing

- **<u>Kinetic Control</u>**: Fisher Information
  a) Fisher Information – Measure of how well the receiver can estimate the message from the sender.
     Shannon Entropy – Measure of the sender's transmission efficiency over a communications channel
  b) Hamiltonian is exergy; portion of energy that can do work; physical and information exergies
  c) Fisher Information:

$$I = \int \left(\frac{\partial \ln p(x)}{\partial x} \right)^2 p(x)dx = \int \frac{1}{p(x)} \left(\frac{\partial p(x)}{\partial x} \right)^2 dx = 4\int \left(\frac{\partial q(x)}{\partial x} \right)^2 dx$$

$q^2(x) = p(x) -$ "Real Amplitude" function of the probability density function

Case Study #2: Collective Plume Tracing

d) 'Mean Kinetic Energy" interpretation of Fisher Information

$$I = 4\int \dot{q}^2 dt = 4\int \frac{2}{m} T dt$$

From Quantum Mechanics in the "Classical Limit" of the expectation of the momentum squared

$$\left\langle p^2 \right\rangle = \frac{\hbar^2}{2m} \int \left| \frac{\partial \Psi(x,t)}{\partial x} \right|^2 dx$$

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Case Study #2: Collective Plume Tracing

$\hbar$ - Dirac's constant

$\Psi(x,t)$ - wave function

$\langle \ \rangle$ - expectation operator

The classical limit is reached when

$$\frac{d}{dt}\langle p\rangle_t = m\frac{d^2}{dt^2}\langle x\rangle_t = -\left\langle \frac{dV(x)}{dx}\right\rangle_t = \langle F(x)\rangle_t$$

Is equivalent to

$$m\frac{d^2}{dt^2}x_{classical} = -\frac{dV(x_{classical})}{dx_{classical}} = F(x_{classical})$$

Case Study #2: Collective Plume Tracing

Which occurs when

$$x_{classical} = \langle x \rangle$$

and

$$(\Delta x)^2 = \left\langle \left(x - \langle x \rangle\right)^2 \right\rangle \qquad \text{is small}$$

This occurs when

$$F(x) = F(\langle x \rangle) + (x - \langle x \rangle)F'(\langle x \rangle) + \frac{1}{2!}(x - \langle x \rangle)^2 F''(\langle x \rangle) + \ldots$$

which leads to

$$\langle F(x) \rangle \approx F(\langle x \rangle) + \langle x - \langle x \rangle \rangle F'(\langle x \rangle)$$

$$\approx F(\langle x \rangle)$$

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Case Study #2:
Collective Plume Tracing

e) Fisher Information Equivalency and Exergy

$$\tilde{H} = \tilde{T} + \tilde{V} + \tilde{V}_c + \tilde{V}_I = \sum_{i=1}^{N} \frac{1}{m_i} H_i$$

where $\tilde{T} = \sum_{i=1}^{N} \frac{1}{m_i} T_i$

$$\tilde{V} = \sum_{j=1}^{N} \frac{1}{m_j} V_j = \text{ potential}$$

$$\tilde{V}_c = \sum_{k=1}^{N} \frac{1}{m_k} V_{c_k} = \text{ control potential}$$

$$\tilde{V}_I = \sum_{l=1}^{N} \frac{1}{m_l} V_{I_l} = \text{ information potential}$$

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Case Study #2: Collective Plume Tracing

$$I + J = 8\int \tilde{H}dt = 8\int\left[\tilde{T} + \left(\tilde{V} + \tilde{V}_c + \tilde{V}_I\right)\right]dt$$

$$\dot{I} + \dot{J} = \tilde{H} > 0$$ (Static stability)

$$\ddot{I} + \ddot{J} = \dot{\tilde{H}} < 0$$ (Dynamic stability)

Where $J = 8\int\left(\tilde{V} + \tilde{V}_c + \tilde{V}_I\right)dt =$ Bound Fisher Information

[Ref. Frieden] B.R. Frieden, **Physics from Fisher Information: A Unification**, Cambridge University Press, 1999.

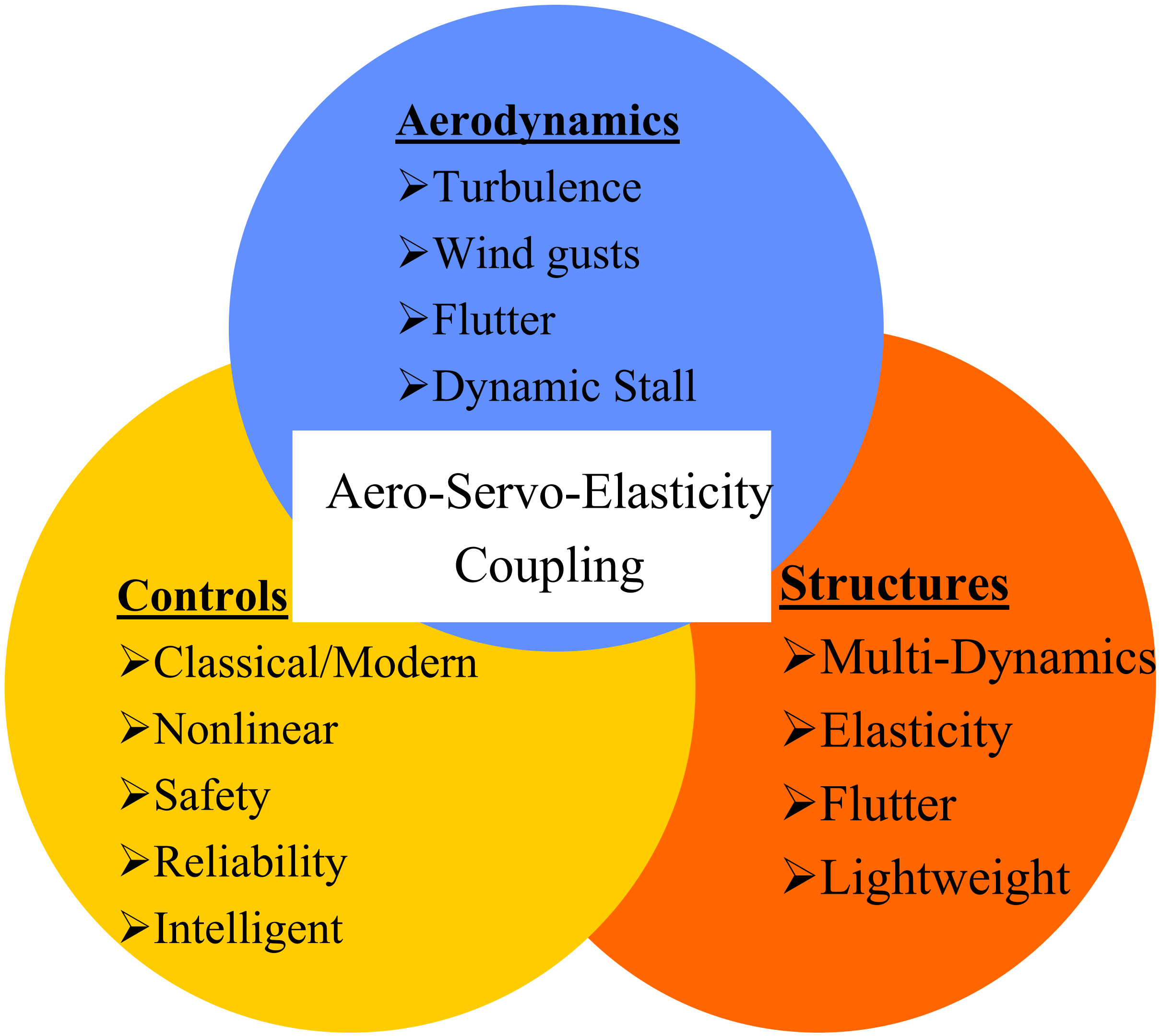
Case Study #2: Summary and Conclusions

- Analyze and design "emergent" behaviors to enable a team of simple robots to perform plume tracing with assistance of information theory
- Demonstrated fundamental nature of Hamiltonian function in design of collective systems
- Equivalences between physical and information-based exergies shown for Shannon information, Fisher information, and virtual fields
- Fisher Information Equivalency developed that can serve as an ideal optimization functional to measure performance and stability of collective system with respect to required information resources

Case Study #3

Nonlinear Aeroelasticity

Case Study #3:
Challenging Engineering Problem



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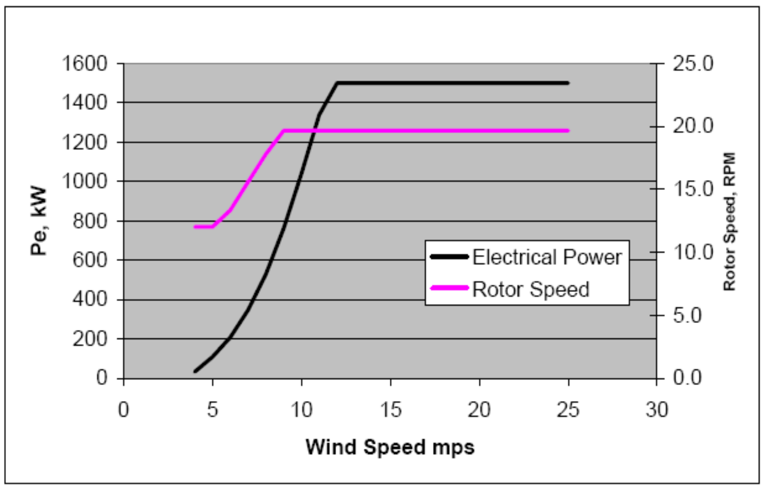
Case Study #3:
Challenging Engineering Problem

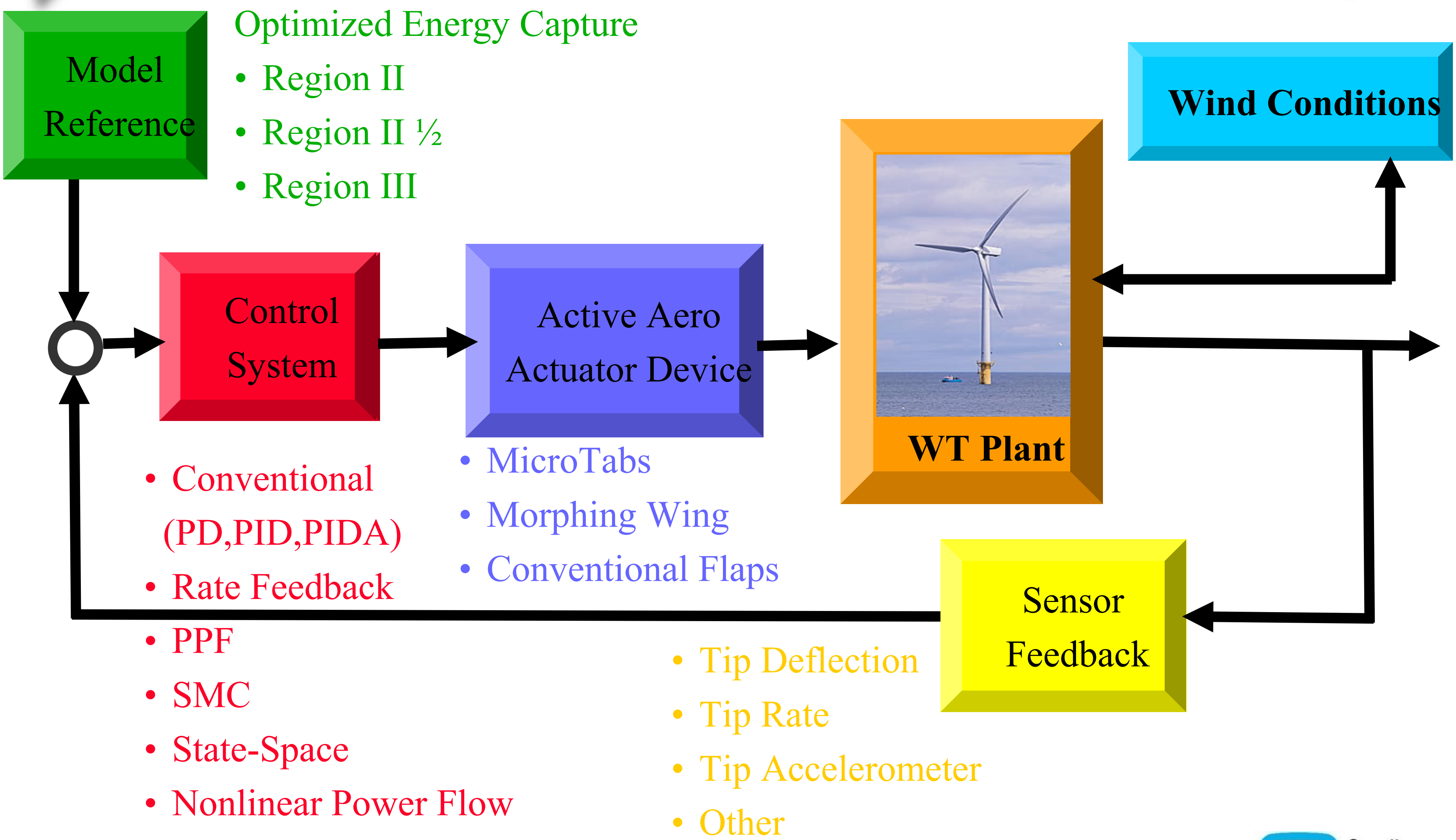
Exergy/Entropy Nonlinear Power Flow Control:

- Rigid Torque Controller Region II & II ½
- Hybrid Pitch and Active Aero Controller Region III
- Include: Dynamic Stall, Classical Flutter, etc.



WINDPACT 1.5 MW WT:
Region III starts around 12 m/s

Case Study #3:
Active Aero Load Control Architecture Analysis/Design



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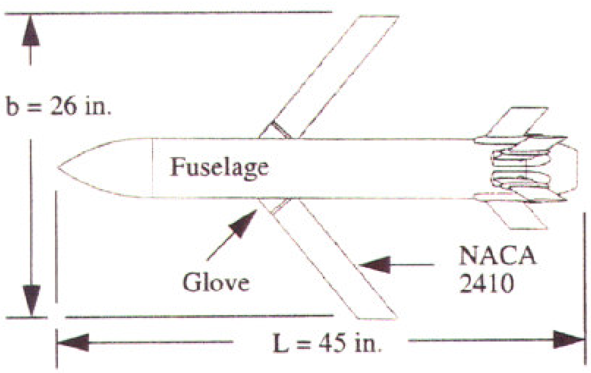
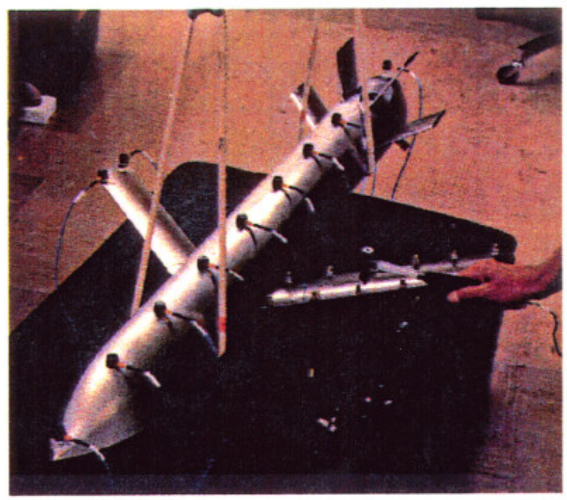
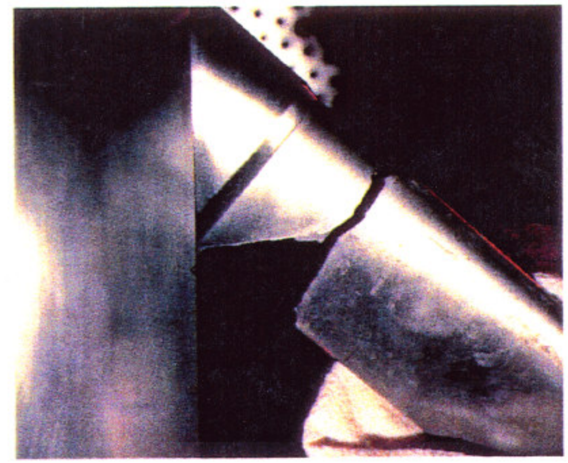
Case Study #3: Nonlinear Aeroelasticity

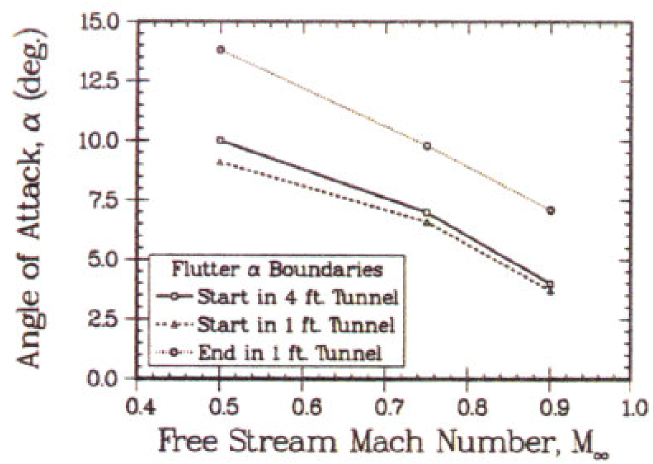
• Wind tunnel model experienced stall flutter over its entire flight envelope M=0.4-0.9; -10^{o} to -3^{o} and 10^{o} to 3^{o} angle of attack [Ref. Guiterrez]

•The wings broke and cracked at the 1/3 span position due to fatigue; first torsional mode ~ 450 Hz









[Ref. Guiterrez] W.T. Gutierrez, R. Tate, H. Fell, *Investigation of a Wind Tunnel Model High Aspect Ratio Wing Fracture*, AIAA 94-2558, 18th AIAA Aeropsace Ground Testing conference, June 1994, Colorado Springs, CO.

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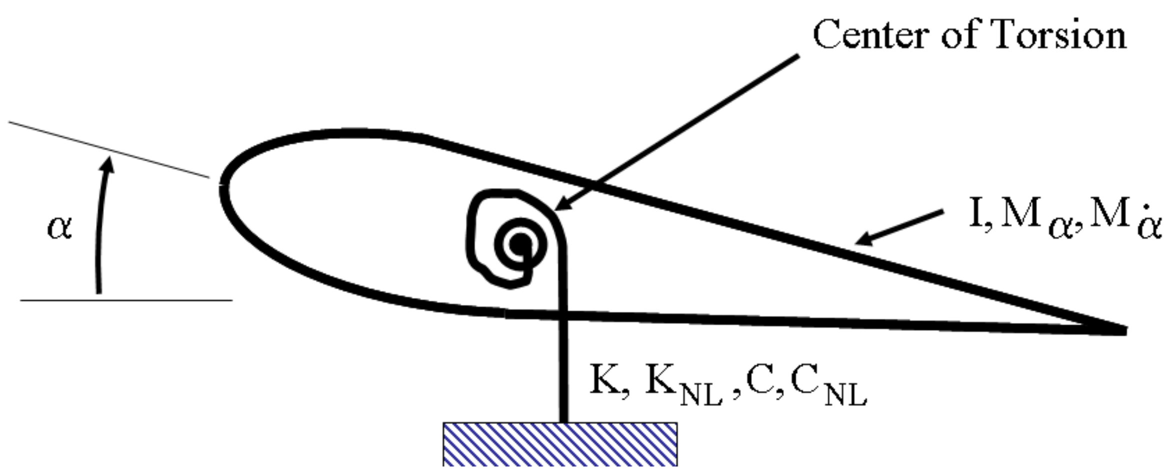
Case Study #3: Nonlinear Aeroelasticity

- Nonlinear aerodynamic and structural model
  - Nonlinear stall flutter model
  - Single DOF model



Case Study #3: Nonlinear Aeroelasticity

- Equations of motion derived from Lagrange's equation:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\alpha}}\right) - \frac{\partial L}{\partial \alpha} = Q_\alpha$$

$$where$$

$$L = T - V$$

$$T = \frac{1}{2} I \dot{\alpha}^2$$

$$V = \frac{1}{2} K \alpha^2 + \frac{1}{4} K_{NL} \alpha^4$$

$$Q_\alpha = Q_{damp} + Q_{aero} + Q_{control}$$

$$Q_{damp} = -C\dot{\alpha} - C_{NL} sign(\dot{\alpha})$$

$$Q_{aero} = M_\alpha(\alpha) + M_{\dot{\alpha}}(\dot{\alpha}, \alpha)$$

$$Q_{control} = u = -K_p \alpha - K_I \int_0^t \alpha d\tau - K_D \dot{\alpha}$$

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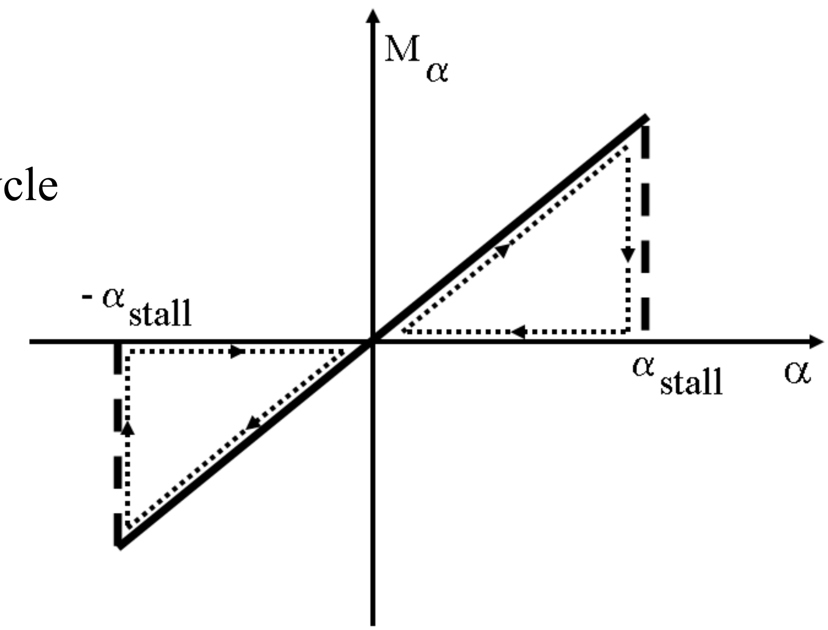
Case Study #3: Nonlinear Aeroelasticity

- The aerodynamic moments are generated based on the following hysteresis logic:

$$M_\alpha(\alpha) = \begin{cases} \hat{C}_{M_\alpha}\alpha & \text{for } |\alpha| < \alpha_{stall} \\ 0 & \text{for } |\alpha| > \alpha_{stall} \\ 0 & \text{for the return hysteresis cycle} \end{cases}$$

$$M_{\dot{\alpha}}(\dot{\alpha},\alpha) = \begin{cases} \hat{C}_{M_{\dot{\alpha}}}\dot{\alpha} & \text{for } |\alpha| < \alpha_{stall} \\ 0 & \text{for } |\alpha| > \alpha_{stall} \end{cases}$$



- Applying Lagrange's equation yields the EOM :

$$I\ddot{\alpha} + K\alpha + K_{NL}\alpha^3 = -C\dot{\alpha} - C_{NL}sign(\dot{\alpha}) + u + M_\alpha(\alpha) + M_{\dot{\alpha}}(\dot{\alpha},\alpha)$$

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Case Study #3: Nonlinear Aeroelasticity

A) Linear Region

For $|\alpha| < \alpha_{stall}$, the model is linear with $K_{NL} = C_{NL} = 0$ or

$$I\ddot{\alpha} + \left[C - \hat{C}_{M_{\dot{\alpha}}}\right]\dot{\alpha} + \left[K - \hat{C}_{M_{\alpha}}\right]\alpha = u$$

which produces typical linear aeroelastic behavior. Divergence occurs when

$$\hat{C}_{M_{\alpha}} \geq K \quad \text{for} \quad u = 0$$

where $\hat{C}_{M_{\alpha}} = K_{M_{\alpha}} qA = K_{M_{\alpha}} A\left(\frac{1}{2}\rho V^2\right)$

Torsional flutter occurs when $\hat{C}_{M_{\dot{\alpha}}} \geq C$ for $K - \hat{C}_{M_{\alpha}} > 0$

and $u = 0$

where $\hat{C}_{M_{\dot{\alpha}}} = K_{M\dot{\alpha}} qAd$.

Case Study #3: Nonlinear Aeroelasticity

B) Nonlinear Stall Flutter

When the motion reaches $|\alpha| > \alpha_{stall}$, the model becomes nonlinear where

$$I\ddot{\alpha} + C\dot{\alpha} + K\alpha = M_{\alpha}(\alpha) + M_{\dot{\alpha}}(\dot{\alpha},\alpha)$$

with

$$H = \frac{1}{2}I\dot{\alpha}^2 + \frac{1}{2}K\alpha^2$$

$$\dot{H} = [I\ddot{\alpha} + K\alpha]\dot{\alpha} = [-C\dot{\alpha} + M_{\alpha}(\alpha) + M_{\dot{\alpha}}(\dot{\alpha},\alpha)]\dot{\alpha}$$

which produces

$$\oint_{\tau} [M_{\alpha}(\alpha) + M_{\dot{\alpha}}(\dot{\alpha},\alpha)]\dot{\alpha}dt = \oint_{\tau} [C\dot{\alpha}]\dot{\alpha}dt$$

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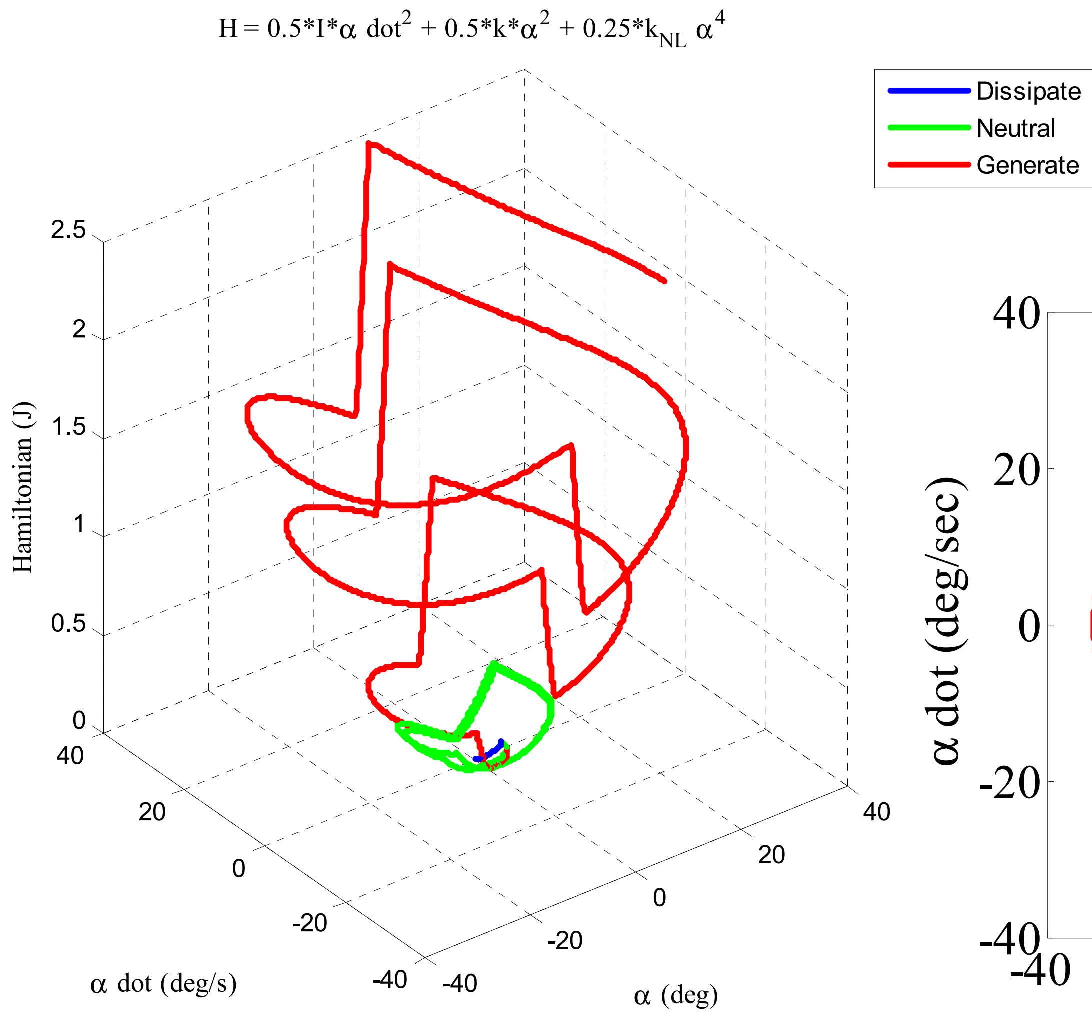
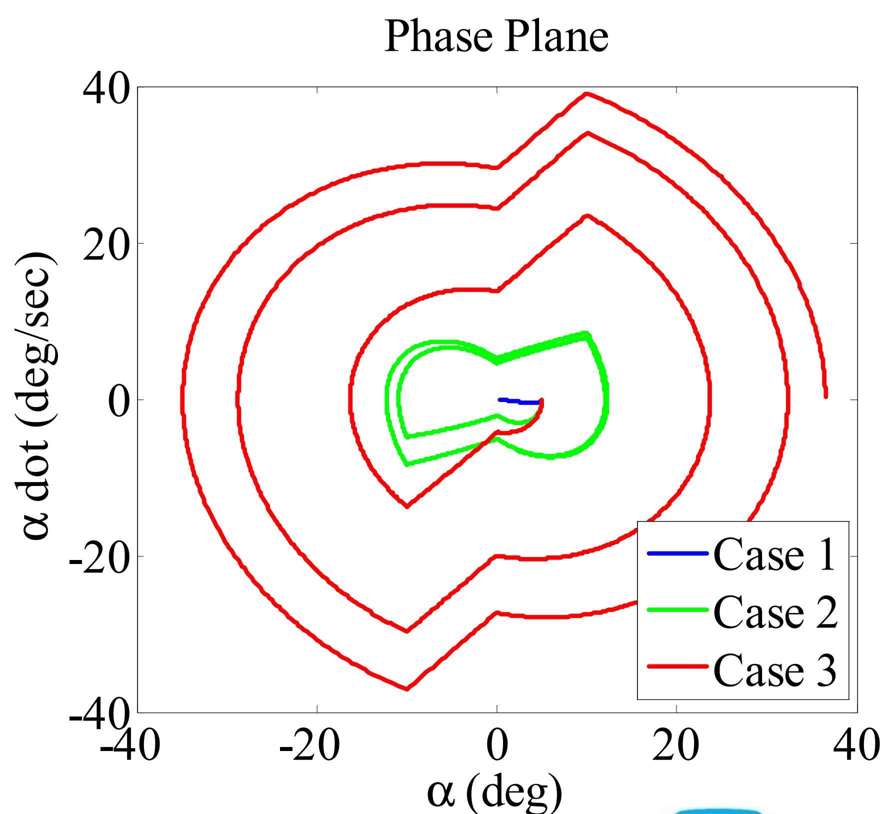
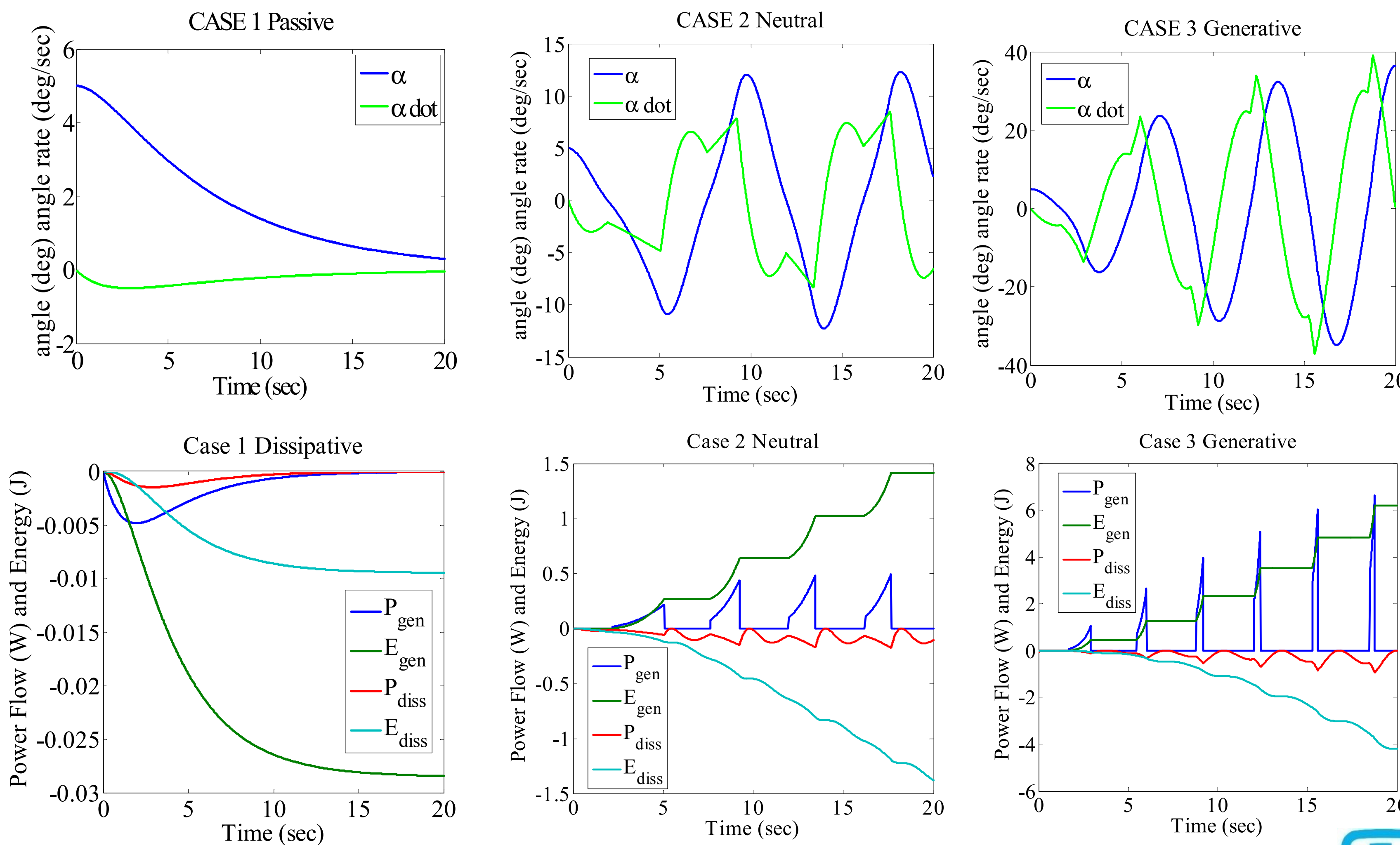
Case Study #3:
Nonlinear Stall Flutter Linear Dynamics Results





Case Study #3:
Nonlinear Stall Flutter Linear Dynamics Results



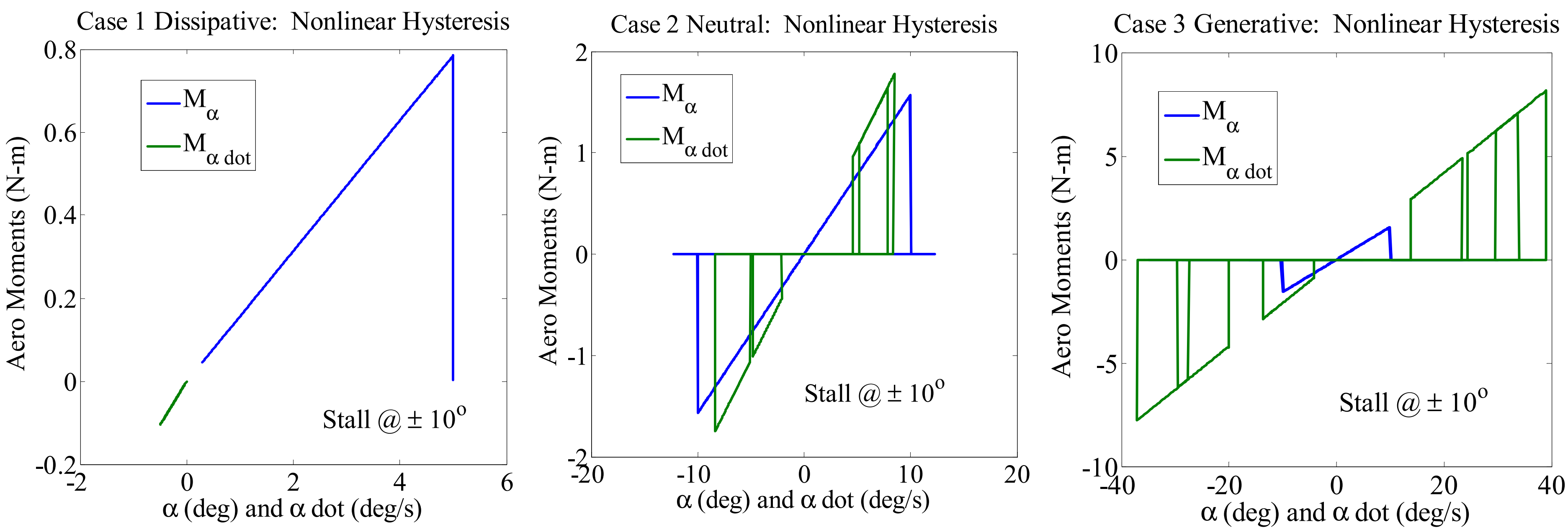
Responses

Power Flow

Sandia National Laboratories

Case Study #3:
Nonlinear Stall Flutter Linear Dynamics Results

Aero Moments



Case Study #3: Nonlinear Aeroelasticity

C) The nonlinear stall flutter can be further modified by adding the nonlinear stiffness and damping

$$I\ddot{\alpha} + C\dot{\alpha} + C_{NL}sign(\dot{\alpha}) + K\alpha + K_{NL}\alpha^3 = M_{\alpha}(\alpha) + M_{\dot{\alpha}}(\dot{\alpha},\alpha)$$

with

$$H = \frac{1}{2}I\dot{\alpha}^2 + \frac{1}{2}K\alpha^2 + \frac{1}{4}K_{NL}\alpha^4$$

$$\dot{H} = \left[I\ddot{\alpha} + K\alpha + K_{NL}\alpha^3\right]\dot{\alpha} = \left[-C\dot{\alpha} - C_{NL}sign(\dot{\alpha}) + M_{\alpha}(\alpha) + M_{\dot{\alpha}}(\dot{\alpha},\alpha)\right]\dot{\alpha}$$

which produces a limit cycle when

$$\oint_{\tau}\left[M_{\alpha}(\alpha) + M_{\dot{\alpha}}(\dot{\alpha},\alpha)\right]\dot{\alpha}dt = \oint_{\tau}\left[C\dot{\alpha} + C_{NL}sign(\dot{\alpha})\right]\dot{\alpha}dt$$

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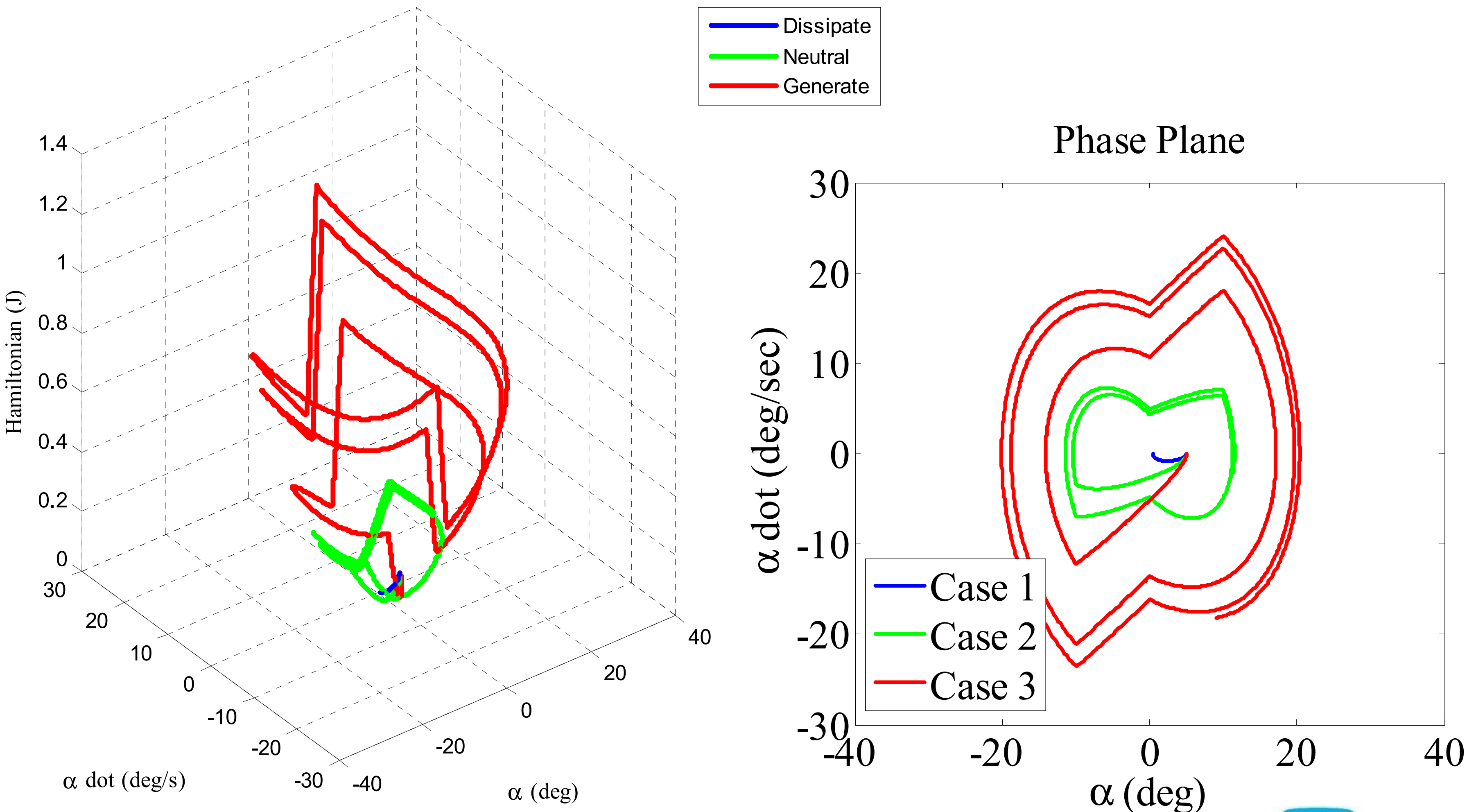
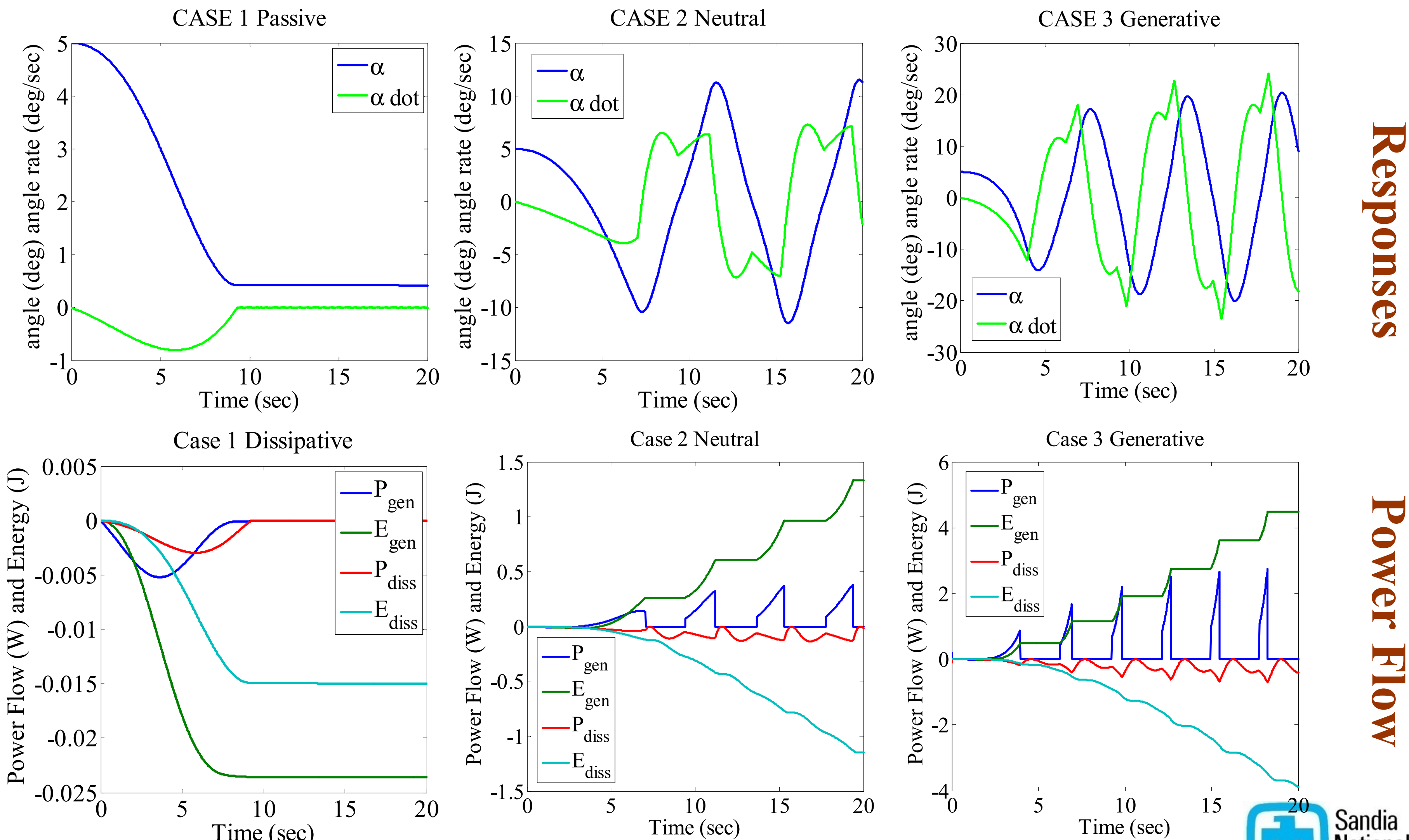
Case Study #3:
Nonlinear Stall Flutter NonLinear Dynamics Results

$$H = 0.5*I*\alpha\ dot^2 + 0.5*k*\alpha^2 + 0.25*k_{NL}\ \alpha^4$$

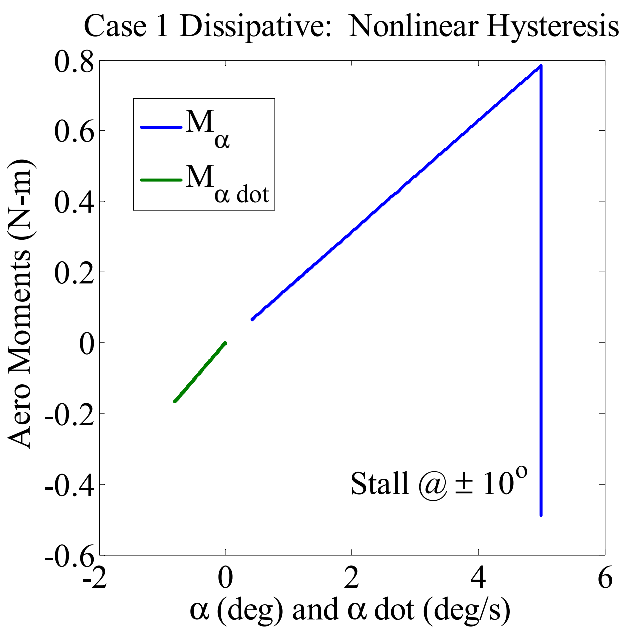


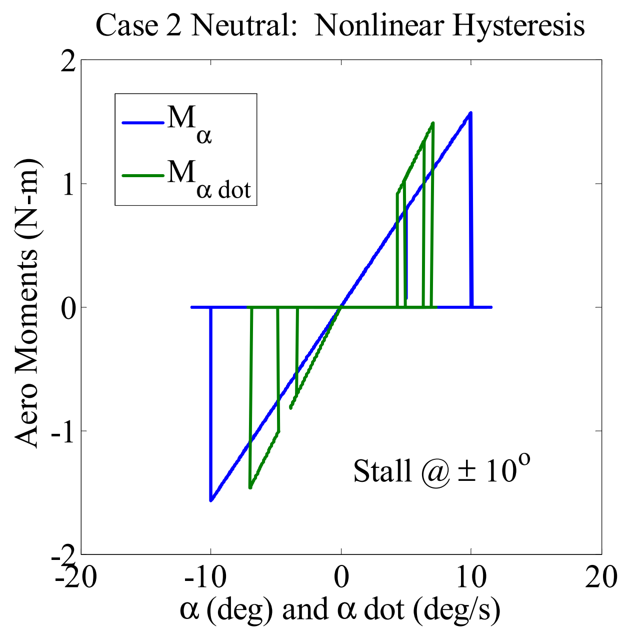
Case Study #3:
Nonlinear Stall Flutter NonLinear Dynamics Results

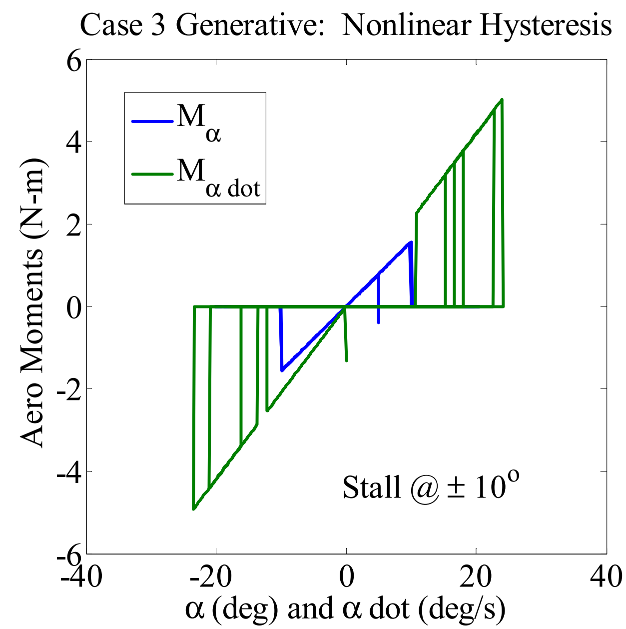


Case Study #3:
Nonlinear Stall Flutter Linear Dynamics Results

Aero Moments







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Case Study #3: Nonlinear Aeroelasticity

D) The nonlinear system can be modified by feedback control to meet several performance requirements. A PID controller is implemented to show the effects of feedback control. The model becomes

$$I\ddot{\alpha}+\left[K+K_p\right]\alpha+K_{NL}\alpha^3=-\left[C+K_D\right]\dot{\alpha}-C_{NL}sign(\dot{\alpha})+M_\alpha(\alpha)+M_{\dot{\alpha}}(\dot{\alpha},\alpha)-K_I\int_0^t \alpha d\tau$$

with

$$H=\frac{1}{2}I\dot{\alpha}^2+\frac{1}{2}\left[K+K_p\right]\alpha^2+\frac{1}{4}\alpha^4$$

$$\dot{H}=\left[I\ddot{\alpha}+(K+K_p)\alpha+K_{NL}\alpha^3\right]\dot{\alpha}$$

$$=\left[-(C+K_D)\dot{\alpha}-C_{NL}sign(\dot{\alpha})+M_\alpha(\alpha)+M_{\dot{\alpha}}(\dot{\alpha},\alpha)-K_I\int_0^t \alpha d\tau\right]\dot{\alpha}$$

which produces a limit cycle when

$$\oint_\tau \left[M_\alpha(\alpha)+M_{\dot{\alpha}}(\dot{\alpha},\alpha)-K_I\int_0^t \alpha d\tau_1\right]\dot{\alpha}dt=\oint_\tau \left[(C+K_D)\dot{\alpha}+C_{NL}sign(\dot{\alpha})\right]\dot{\alpha}dt$$

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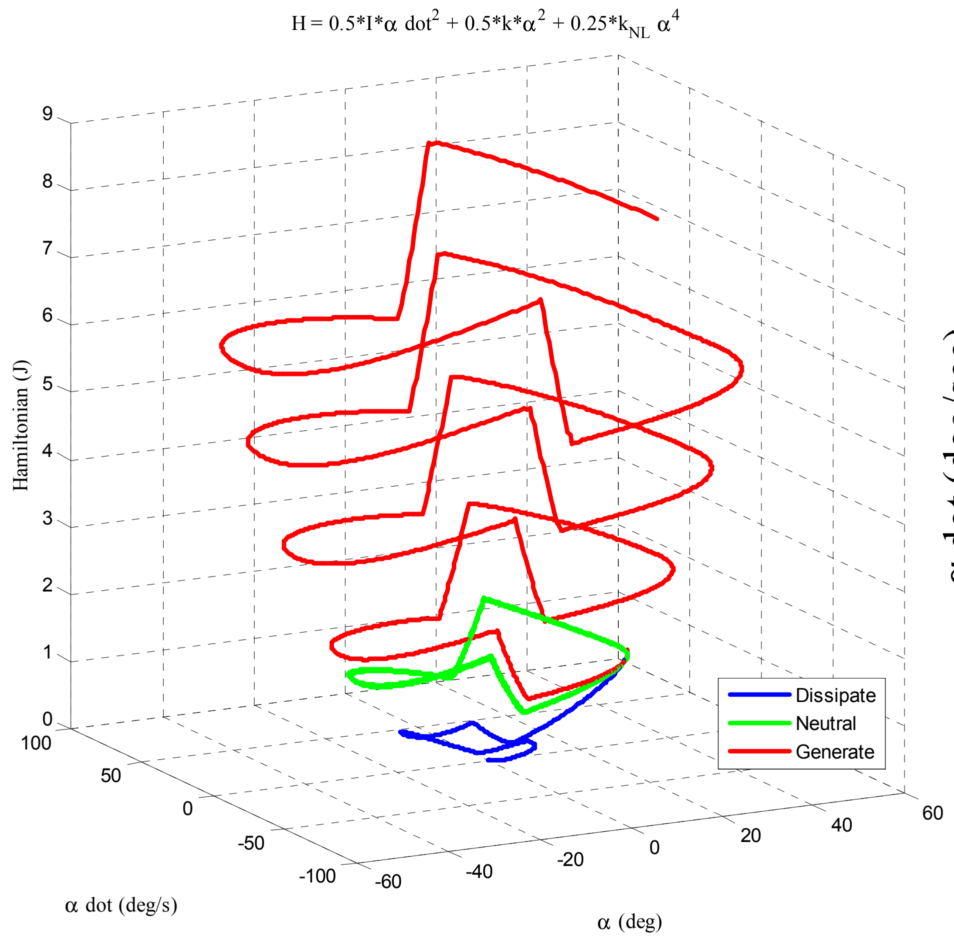
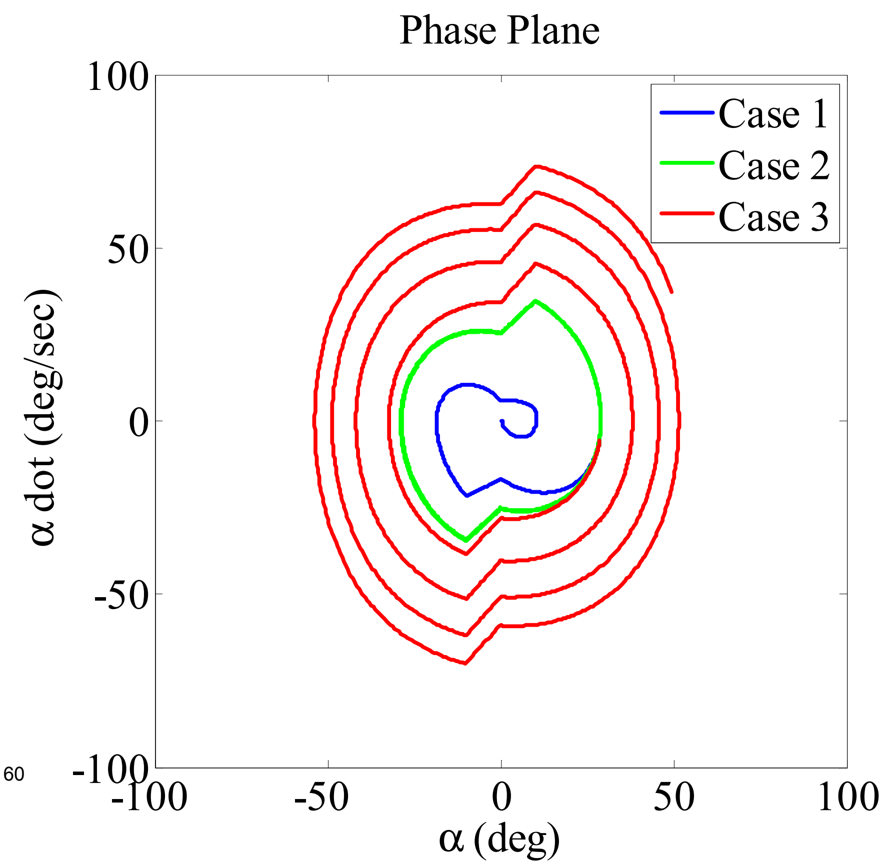
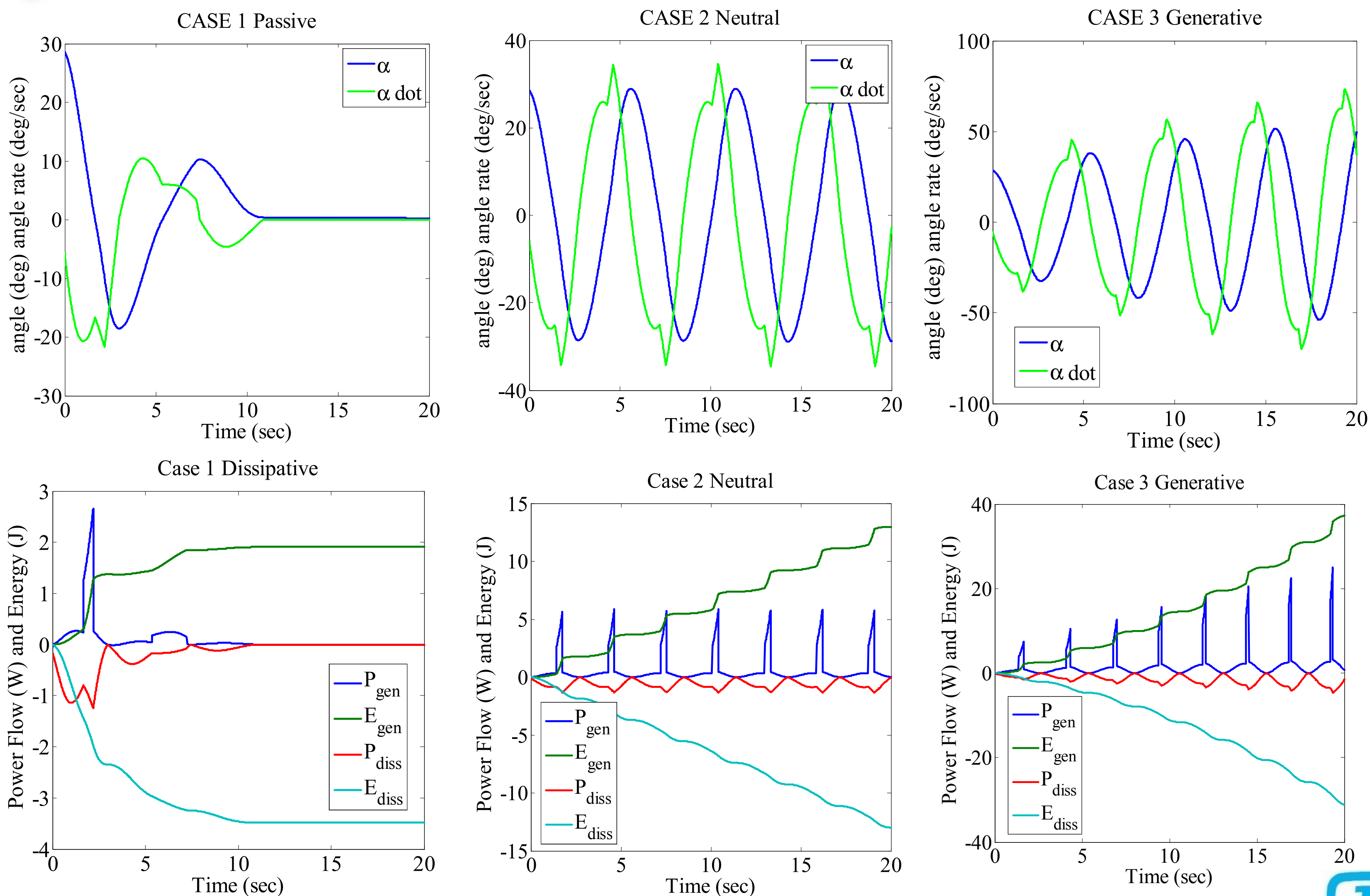
Case Study #3: Dynamic Stall Control System Results

- Nonlinear Power Flow Control Design Dynamic Stall: Limit Cycle Identification





Case Study #3:
Dynamic Stall Control System Results



Responses

Power Flow

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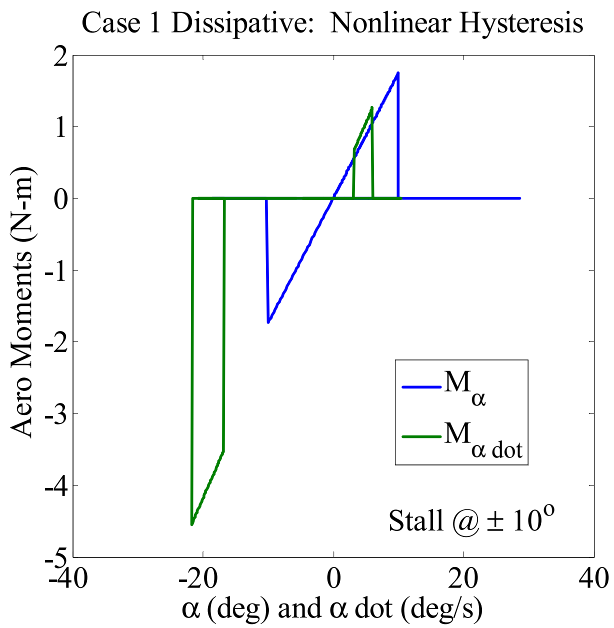
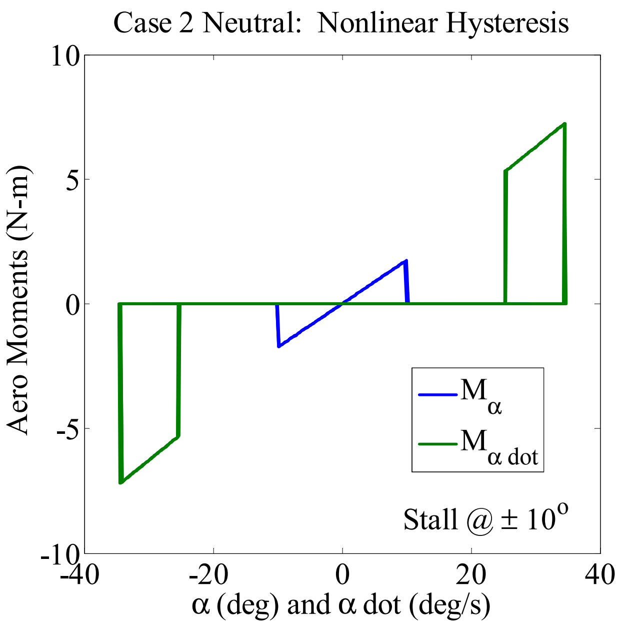
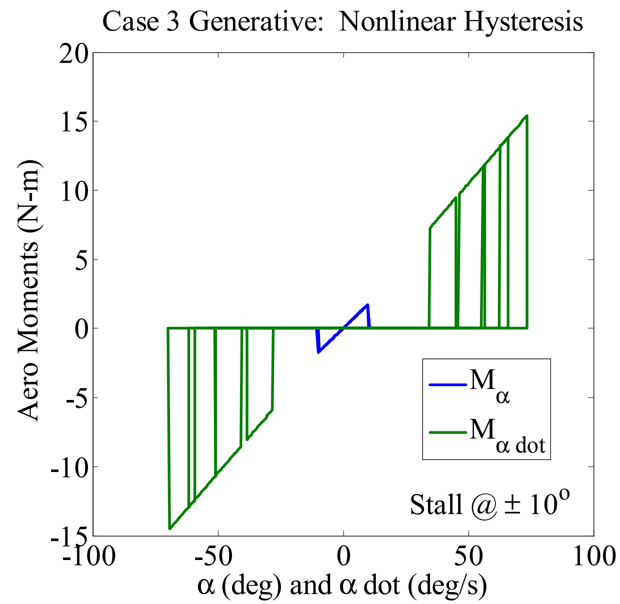
Case Study #3:
Dynamic Stall Control System Results

Aero Moments







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Case Study #3: Summary and Conclusions

- Nonlinear power flow control was applied to a nonlinear stall flutter problem (dynamic stall)
- Nonlinear structural and discontinuous aerodynamic models were directly accommodated
- The limit cycles were found by partitioning power flows
- The limit cycles were shown to be stability boundaries
- Flutter suppression controllers were initially assessed

Case Study #4

Power Engineering

Case Study #4:
Power Engineering - OMIB

I. Model (Simple one-machine infinite bus swing equation):

$$\dot{\delta} = \omega$$

$$\dot{\omega} = \frac{1}{M}\left[P_m - P_{MAX}\sin(\delta) - D\omega\right]$$

$$\ddot{\delta} = \dot{\omega}$$

$$\dot{\delta} = \omega$$

$$M\ddot{\delta} + P_{MAX}\sin(\delta) = P_m - D\dot{\delta}$$

II. Controller Design:

$$u = -K_P \sin(\delta - \delta_s) - K_D\dot{\delta} - K_I\int_0^t (\delta - \delta_s)d\tau$$

$$\delta_s = 1.0547$$

$$K_P = K_D = 0$$

$$H = \frac{1}{2}M\dot{\delta}^2 + P_{MAX}\left[1 - \cos(\delta)\right]$$

$$\dot{H} = \left[M\ddot{\delta} + P_{MAX}\sin(\delta)\right]\dot{\delta} = \left[-D\dot{\delta} + P_m + u\right]\dot{\delta}$$

$$\Rightarrow \dot{H} < 0$$

$$\Rightarrow \oint_\tau \left[-D\dot{\delta} + P_m + u\right]\dot{\delta} < 0$$

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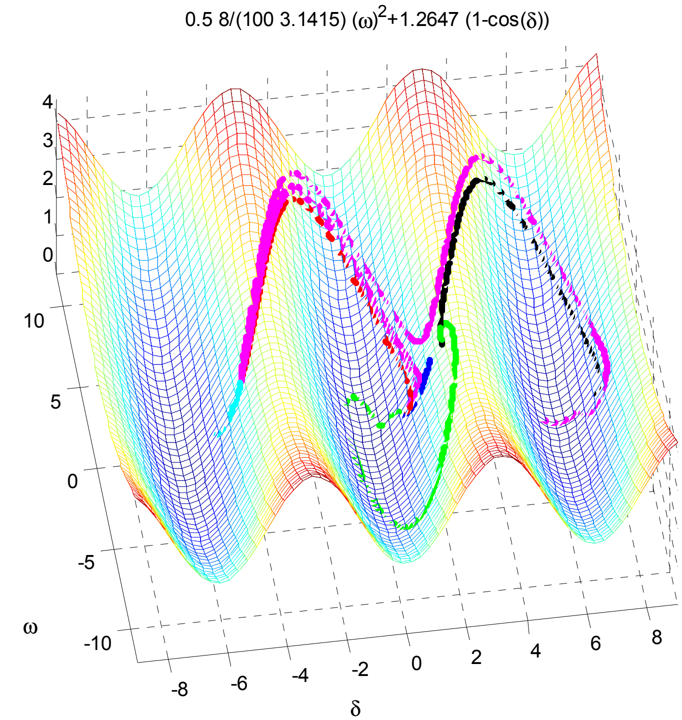
Case Study #4:
Power Engineering- OMIB Trajectories

- Dark blue (2.0865,0)
- Black (2.0865*1.2,0)
- Green (2.0865*1.2,0), K_I = 1.0
- Cyan (-4.197,0)
- Red (-4.196,0)
- Magenta (-4.197,0), K_I = 0.01
- Magenta (dash) (-4.197,0), K_I = 0.05



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Case Study #4:
Power Engineering - 2MIB

I. Model (Two-machine infinite bus swing equations):

$$\dot{\delta}_1 = \omega_1$$

$$\dot{\delta}_2 = \omega_2$$

$$\dot{\omega}_1 = \frac{1}{M_1}\left[P_{m_1} - C_{12}\sin\delta_{12} - C_{13}\sin\delta_1 - D_1\omega_1\right]$$

$$\dot{\omega}_2 = \frac{1}{M_2}\left[P_{m_2} - C_{12}\sin\delta_{21} - C_{23}\sin\delta_2 - D_2\omega_2\right]$$

II. Controller Design:

$$H_{12} = \frac{1}{2}M_1\dot{\delta}_1^2 + \frac{1}{2}M_2\dot{\delta}_2^2 + C_{13}\left[1-\cos\delta_1\right] + C_{23}\left[1-\cos\delta_2\right] + C_{12}\left[1-\cos\delta_{21}\right]$$

$$\dot{H}_{12} = \left[P_{m_1} - D_1\dot{\delta}_1 + u_1\right]\dot{\delta}_1 + \left[P_{m_2} - D_2\dot{\delta}_2 + u_2\right]\dot{\delta}_2$$

For:

$$M_1\ddot{\delta}_1 + C_{13}\sin\delta_1 = P_{m_1} - D_1\dot{\delta}_1 - C_{12}\sin(\delta_1 - \delta_2) + u_1$$

$$M_2\ddot{\delta}_2 + C_{23}\sin\delta_2 = P_{m_2} - D_2\dot{\delta}_2 - C_{12}\sin(\delta_2 - \delta_1) + u_2$$

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Case Study #4: Overview of Lanai "Like" Model

- Real World Microgrid – "Lanai and Kauai"
- "Like" Model of Real World Microgrid
- Advanced Controls on Distributed Generation

Lanai "Like" Model Parameter Characteristics:

- 12470 V, 5.5 MW peak load
- 4 Diesel Generators (2 – 2.2 MW, 2 – 1.0 MW) with controls
- 1.2 MW High Level PV controls
- Switchable Loads
- Distribution lines (series RL) calculated for 350kcmil under 5 miles
- 3-phase faults at each bus

Model Courtesy: Ben Schenkman

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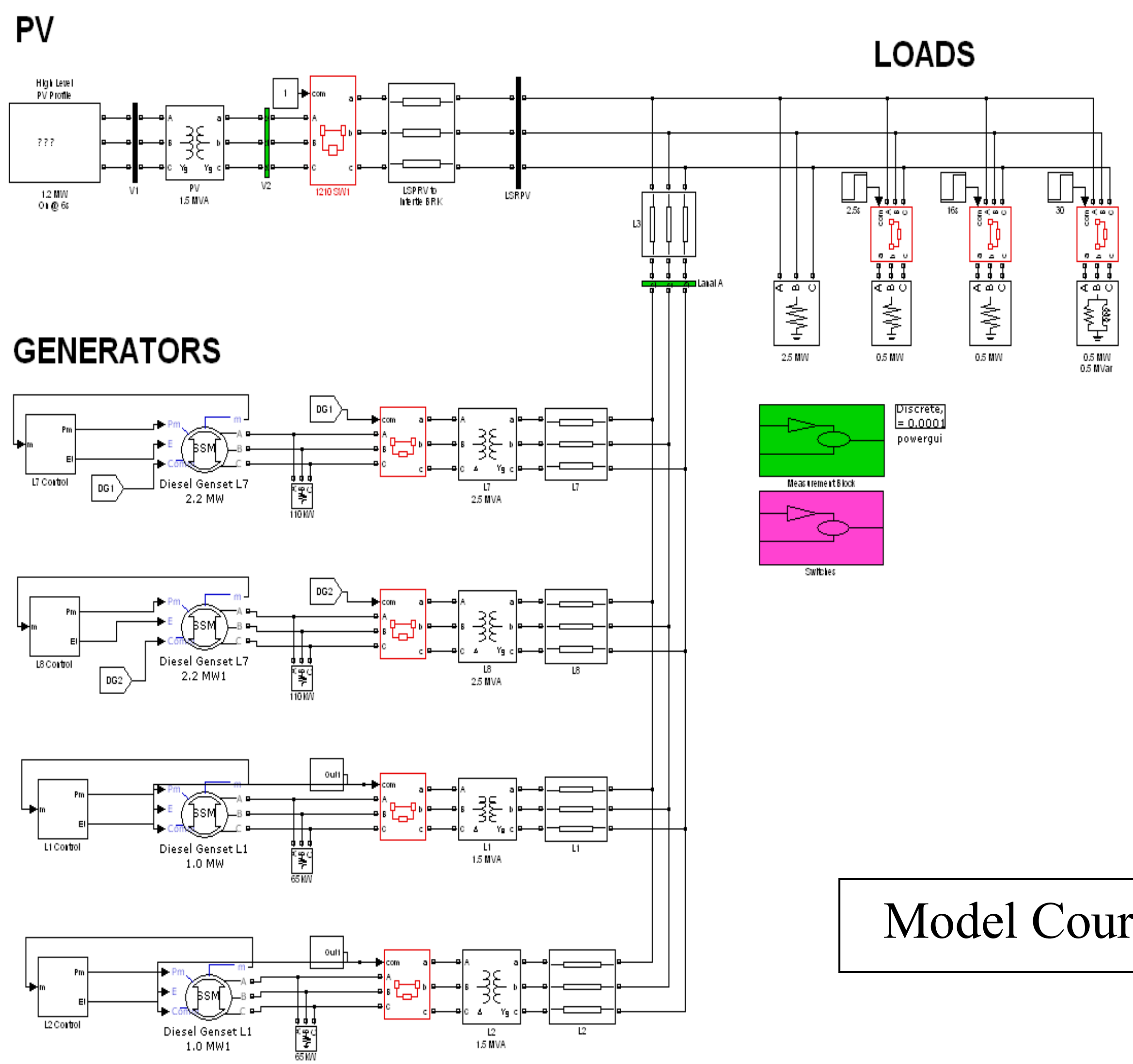
Case Study #4:
Lanai "Like" MicroGrid Model



Model Courtesy: Ben Schenkman

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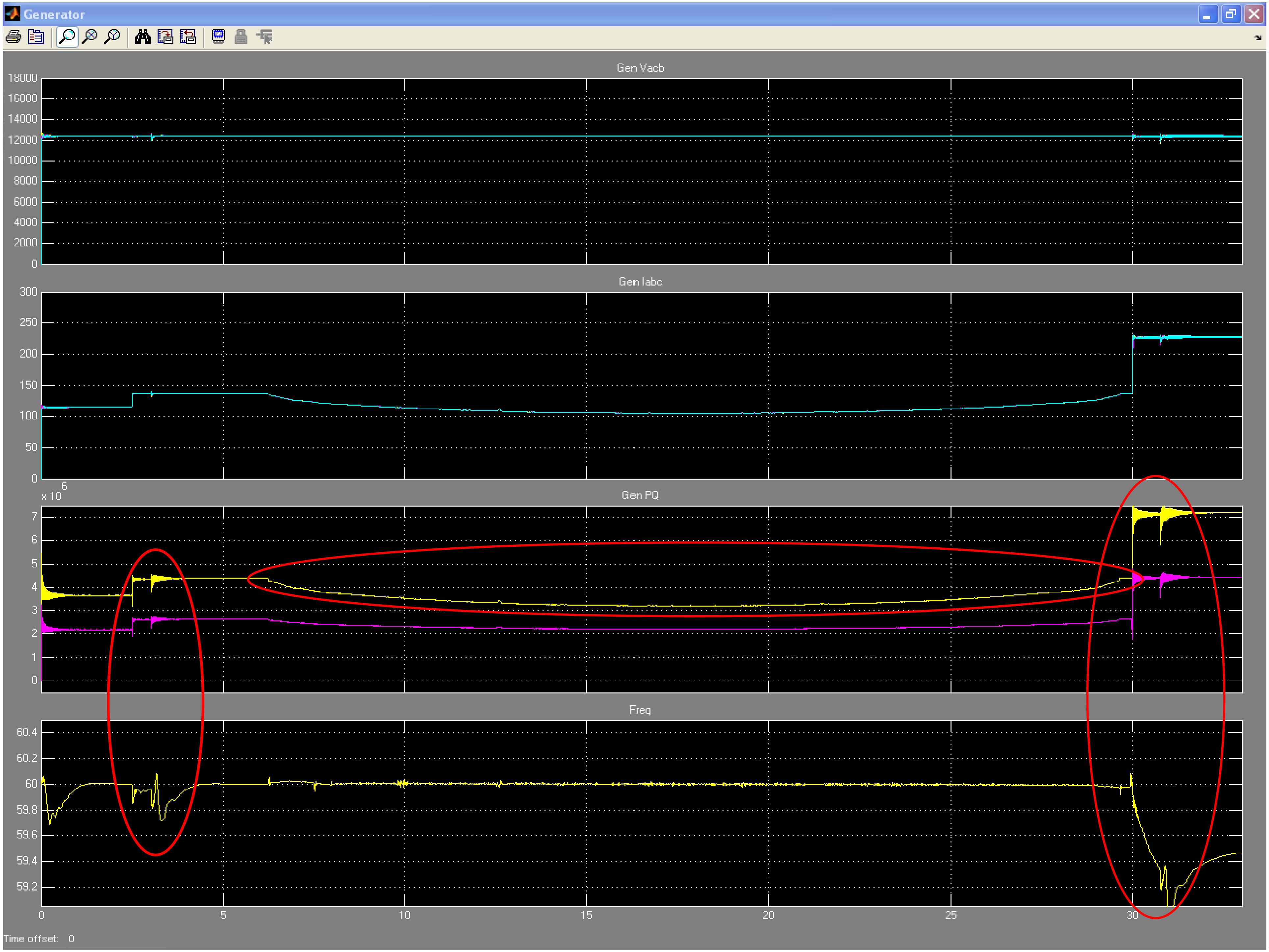
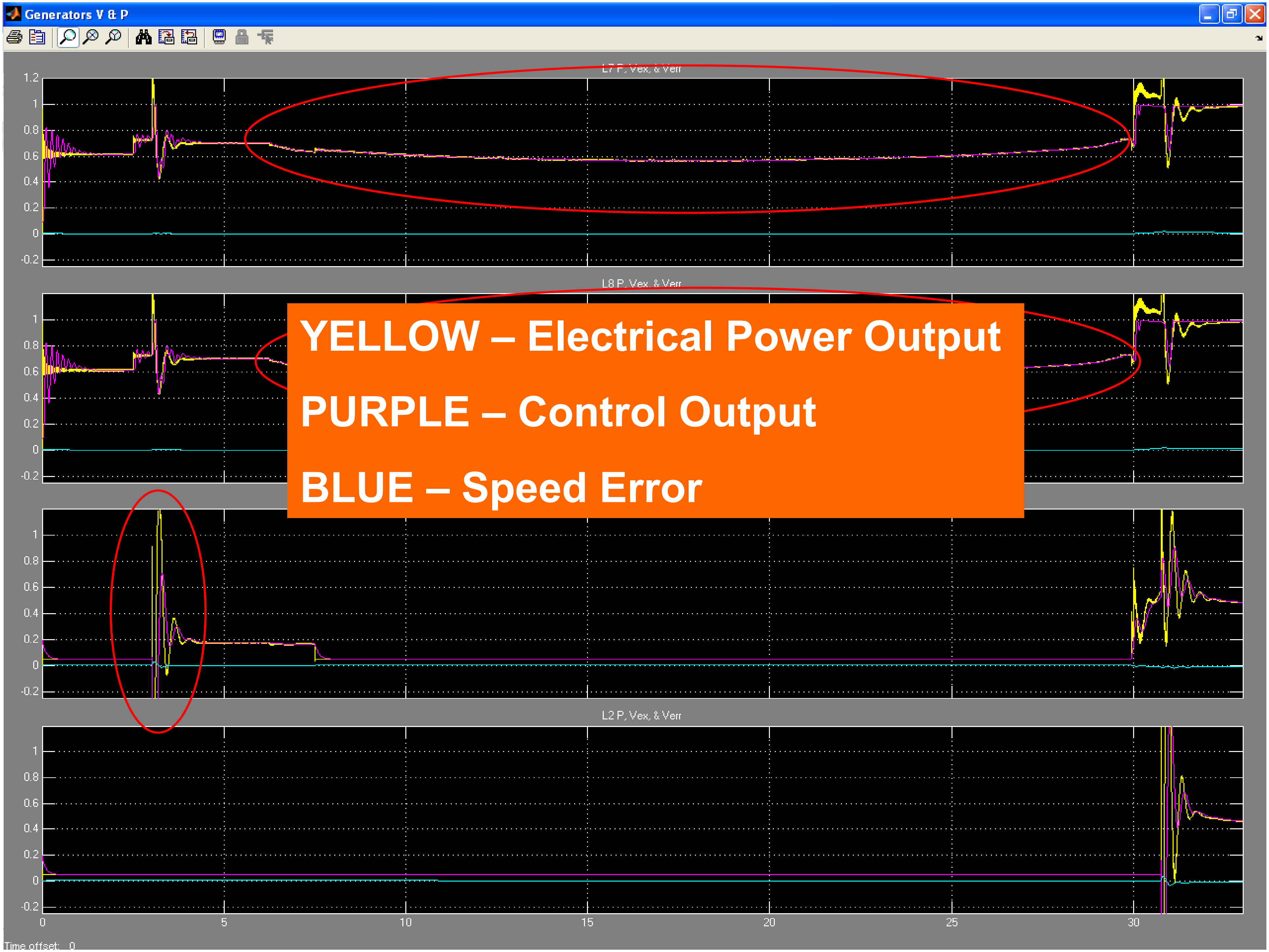

Generator
Gen Vacb
18000
16000
14000
12000
10000
8000
6000
4000
2000
0
Gen Iabc
300
250
200
150
100
50
0
x 10^6
Gen PQ
7
6
5
4
3
2
1
0
Freq
60.4
60.2
60
59.8
59.6
59.4
59.2
0
5
10
15
20
25
30
Time offset: 0


Generators V & P
L7 P, Vex, & Verr
L8 P, Vex, & Verr
L2 P, Vex, & Verr
Time offset: 0
YELLOW – Electrical Power Output
PURPLE – Control Output
BLUE – Speed Error

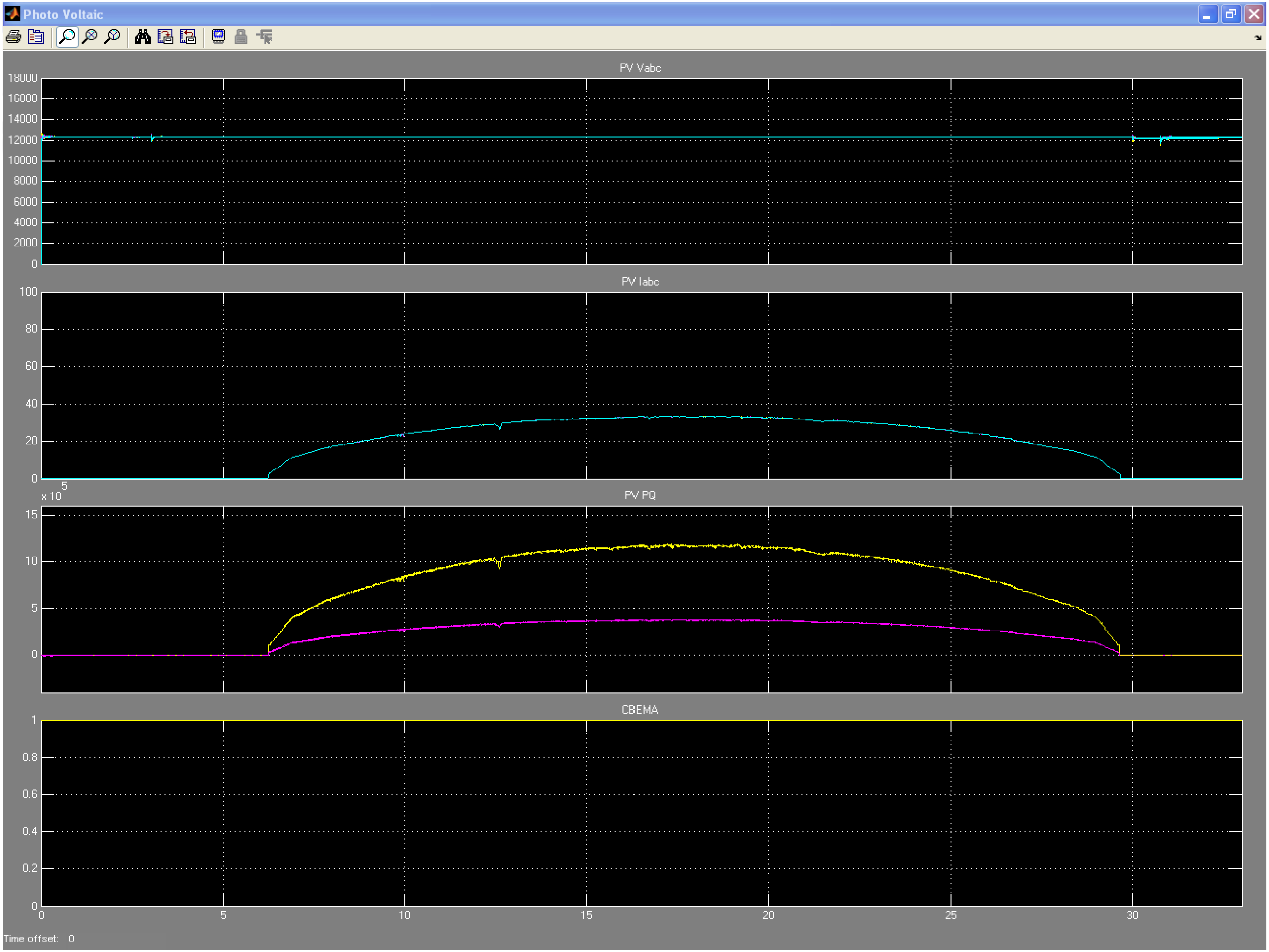
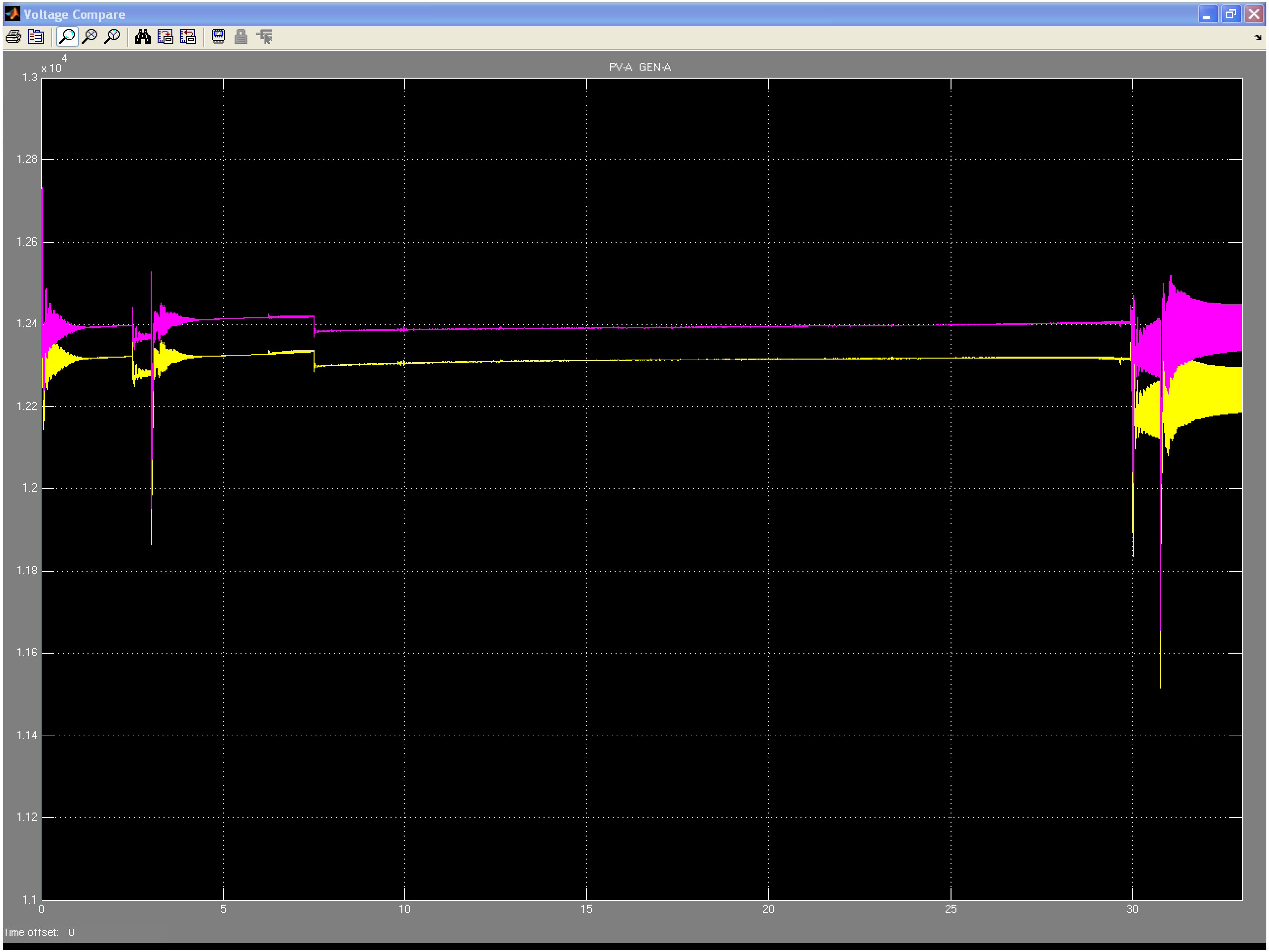


Photo Voltaic
PV Vabc
PV Iabc
PV PQ
x 10^5
CBEMA
Time offset: 0

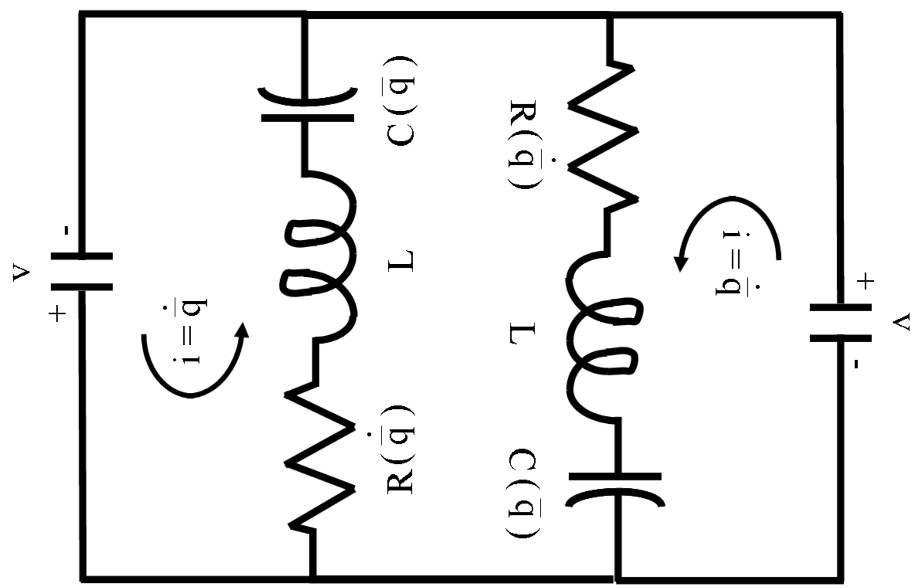

Voltage Compare
PV-A GEN-A
x 10^4
1.3
1.28
1.26
1.24
1.22
1.2
1.18
1.16
1.14
1.12
1.1
0
5
10
15
20
25
30
Time offset: 0

Case Study #4:
Lanai "Like" Control Model

•Nonlinear Power Flow Control Design:

$$\sum_{1}^{4}$$

generators

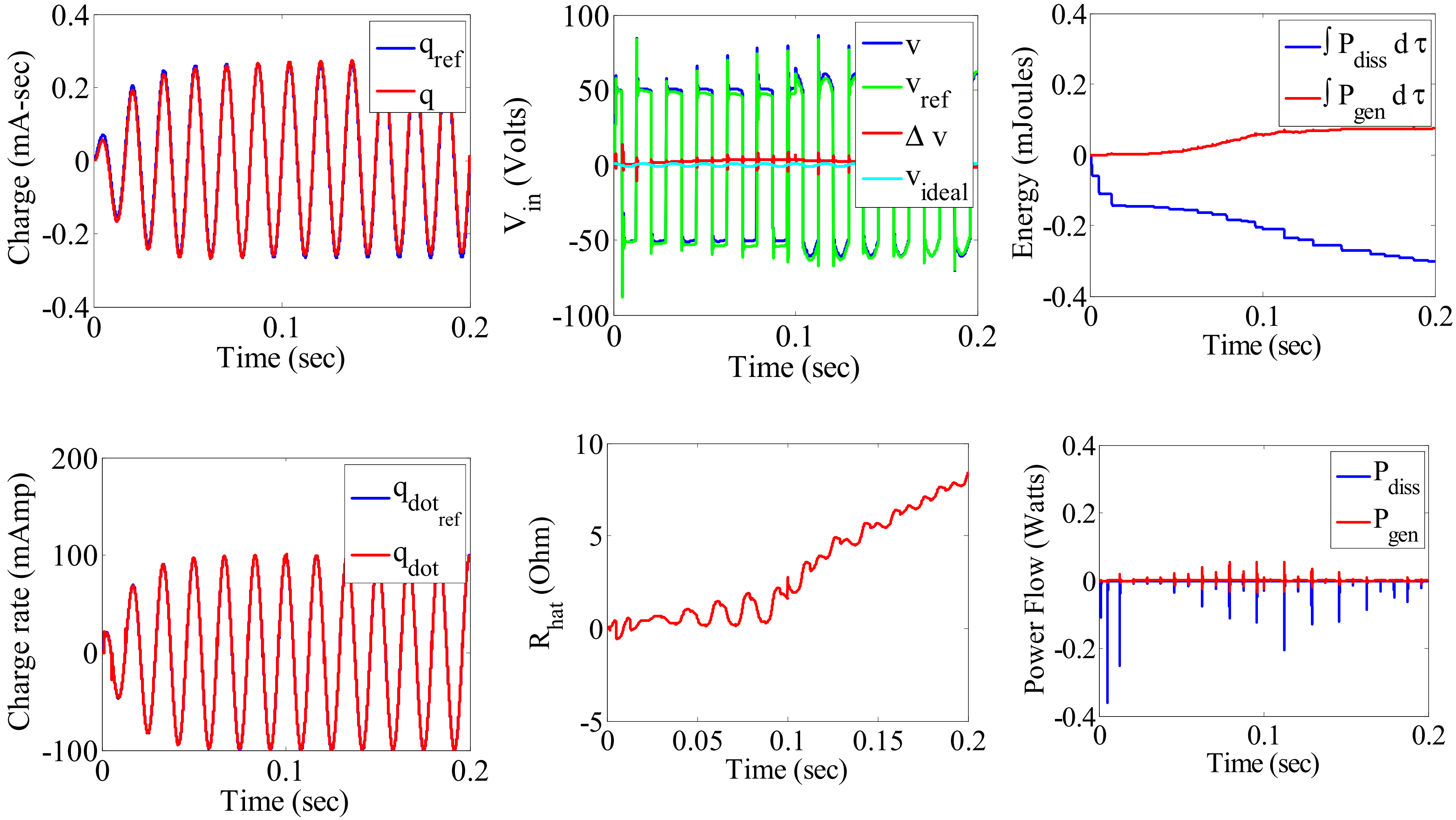


PV

Generation function (sun)

Loads setup similar to Lanai "like" model

Case Study #4:
Lanai "Like" Control Model



Case Study #4: Summary and Conclusions

- Investigated OMIB model – Limit cycles and control design options
- Investigated 2MIB model
- Identified microgrid model based on actual Lanai "like" electric power grid system
- Evaluated Exergy/entropy control design for simplified Lanai "like" model