

An Approach to the Fatigue Analysis of Vertical Axis Wind Turbine Blades

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AN APPROACH TO THE FATIGUE ANALYSIS
OF VERTICAL AXIS WIND TURBINE BLADES*

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Abstract

A cursory analysis of the stress history of wind turbine blades indicates that a single stress level at each wind speed does not adequately describe the blade stress history. A statistical description is required. Blade stress data collected from the DOE/ALCOA Low Cost experimental turbines indicate that the Rayleigh probability density function adequately describes the distribution of vibratory stresses at each wind speed. The Rayleigh probability density function allows the distribution of vibratory stresses to be described by the RMS of the stress vs. time signal. With the RMS stress level described for all wind speeds, the complete stress history of the turbine blades is known. Miner's linear cumulative damage rule is used as a basis for summing the fatigue damage over all operating conditions. An analytical expression is derived to predict blade fatigue life. Input to the blade life expression includes a basic blade S-N curve, RMS stress vs. wind speed data, the probability density function of vibratory stress and the probability density function which describes the wind speed distribution. The implications of the assumptions and the limitations of this approach are discussed.

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Introduction

The cost effective production of energy with a wind turbine relies on the assumption that the high initial cost of building the wind turbine will be offset by low cost operation over a long life. The fatigue life of major components, such as the blades, must be adequate to allow production of enough energy to balance the initial investment. Oscillating stresses are inherent in vertical axis wind turbine (VAWT) operation, the two main causes being the blade cycling through up-wind and down-wind orientations, and the aerodynamic forces changing as the induced angle of attack of the blades changes. The magnitudes of these vibratory stresses in the blades can be reduced by analysis of the resonant modes and frequencies of the operating turbine and judicious design to keep the inherent periodic loads from exciting any resonance. However, the vibratory stresses can never be removed completely. The effect of these oscillating stresses on blade life must be assessed before a wind turbine can be labeled cost effective and projected to last for a specified amount of time.

The first step in predicting the fatigue life of a wind turbine blade is to locate the weakest parts of the blade either due to degraded fatigue properties (at weldments or mechanical fasteners) or high stress levels (where the energy producing loads are removed from the blades). Predicting the life of these critical areas requires sufficient data to:

- A. Describe the fatigue life as a function of vibratory stress level.
- B. Describe the vibratory stress levels as a function of wind speed.
- C. Determine the wind speed distribution characteristic of the wind turbine site.

The emphasis of this report is on part B as well as on a method of combining these three areas of knowledge into an over-all fatigue life estimate. Superficial examination of the stress history of wind turbine blades is not sufficient for a

fatigue life prediction, although it will show if there is short term danger of blade failure. One 'quick look' procedure used by Sandia Laboratories during initial checkout of experimental wind turbines monitors the stresses over a five second interval. The maximum and minimum stresses during this time are determined and used to calculate the vibratory (maximum minus minimum divided by two) stresses. A plot of the vibratory stresses vs wind speed for a single strain gage location is shown in Figure 1. It is evident that a single stress level at each wind speed does not describe the stress history. A statistical distribution of stress levels is more likely to provide an accurate picture of the stress history.

Once the stress history has been described, a method of assessing the damage done during operation at varying stress levels must be implemented. The cumulative damage rule presented by Miner [11] provides a simple and direct means of combining the effects of a wide range of applied stresses. Though Miner's Rule is somewhat simplistic, there is no evidence that a better damage rule exists, and it is still the most commonly used damage rule for fatigue analysis.

METHOD OF FATIGUE ANALYSIS

1. Miner's Rule

Miner's linear cumulative damage rule uses the assumption that fatigue life is exhausted at a constant rate at each cyclic stress level. Therefore the total amount of life exhausted is equal to the summation of the portions of life exhausted at each stress level. In equation form this is:

$$D = \sum_{i=1}^k \frac{n_i}{N_i} \quad (1)$$

where:

- n_i = number of applied cycles at the i^{th} stress level
- N_i = number of cycles to failure at the i^{th} stress level
- D = total damage (all life is exhausted when $D = 1$)

This simple approach has several shortcomings which must be recognized and understood when using it in a fatigue analysis. The references cited provide good interpretations of some of the capabilities and limitations of Miner's Rule. For example, [3] furnishes the analyst with prior knowledge that life predictions made using Miner's Rule with random loading (which is the load environment experienced by wind turbine blades) may be non-conservative. Miner's Rule is, however, still the best tool available for correlating the actual loading condition with constant amplitude fatigue life data (S-N data). In view of the negligible effect of the sequence of cyclic stress levels upon fatigue life in a random load environment [8,14], the primary concern here is with the characterization of the vibratory stress level distribution.

2. The Distribution of Vibratory Stresses

Many random processes have a Gaussian or Normal distribution about the mean. The distribution of peak values for a Gaussian process depends upon the frequency content. If a process is wide band Gaussian (composed of frequencies over a wide band of the spectrum), the peaks will also be Gaussian. If it is narrow band Gaussian (composed of frequencies over a narrow band of the spectrum), the peaks will have a Rayleigh distribution. Examples of wide and narrow band signals are shown in Figure 2.

The mean stress level in VAWT blades is dominated by centrifugal and gravitational loads and is therefore a function of operating speed only. The frequency content of the stresses in VAWT blades has a lower bound of the rotational speed of the turbine and is otherwise limited to the first few harmonics of this fundamental frequency. The VAWT blade stress history is more nearly narrow band than wide band, although it is not likely to ever be perfectly narrow band. The more narrow band the stress signal, the more closely the peaks of a Gaussian signal will approach a Rayleigh distribution.

In order to apply this peak distribution to fatigue analysis, a closer look must be taken at determining what stress events provide a good measure of fatigue damage. The

stress excursions between adjacent peaks and valleys may be a better measure than peak stresses. A comparative study of cycle counting methods by Fritz [7] indicates that the most promising method is one that accounts for stress ranges between peaks and valleys, as well as the mean of each stress excursion. For a narrow band process as in Figure 2, the distribution of peaks and excursions are roughly equivalent because each peak is followed by a valley of about equal magnitude producing an excursion equal to twice the peak. With a wide band process this equivalence does not exist. Therefore, the peak distribution is a useful measure of fatigue damage for a narrow band Gaussian process, but not for a wide band process. For VAWT blades where the stress is close to, but not exactly, narrow band the peak-excursion relationship is not obvious.

The distribution of vibratory stresses was determined using short (30-40 seconds) time series records of operating stresses in the blades of the DOE/ALCOA 17m Low Cost VAWT. Vibrational frequencies of 1, 2, and 3 times the operating speed are present in the stress signal, with the three per revolution frequency dominating. To avoid counting the small amplitude cycles while missing the large ones, and to filter the digital noise in the signal, a method of cycle counting was adopted wherein the stress excursions below a threshold value are ignored. The effect of the threshold is to approximate the raw signal in Figure 3 with the dashed straight lines superimposed on it. A vibratory stress level is defined as peak minus valley divided by two. (The actual signal is strain gage output and will have units of microstrain. Since the stress state is nearly uniaxial, the stresses are just the strains multiplied by Youngs Modulus.)

Figures 4-6 are representative histograms of vibratory stresses, obtained by the above method. A Rayleigh probability density is superimposed on the histograms. Chi-square goodness-of-fit tests lead to acceptance of the Rayleigh distribution hypothesis at the 0.05 level of significance.

The reason that a time series which is not completely narrow band contains a distribution of vibratory stresses that fits the Rayleigh distribution so well is a result of the method of counting the vibratory stresses. The method used here ignores cycles below the threshold. Any frequency component that does not produce amplitudes of vibration large enough to exceed the threshold amplitude, will be completely filtered. Since adjacent peaks and valleys are used in calculating the vibratory stresses, the higher frequencies receive the priority treatment. In this way, this method acts as a filter that passes only the highest frequencies that produce stress amplitudes above the threshold. If the threshold is set at a level below which negligible fatigue damage is accumulated, this method will produce the distribution most useful in fatigue analysis. Care must be taken, however, to assure that the vibratory stress counting method is not missing relatively large amplitude, low-frequency vibrations. In VAWT application, this means checking the relative magnitudes of the power spectral densities of the first few harmonics of the fundamental rotational speed against the rate of cycle accumulation predicted by the counting method.

3. Comparison of RMS and Maximum Stress Level

Stress data from the DOE/ALCOA 17m Low Cost VAWT at Rocky Flats is available in the form of either RMS stress level or maximum vibratory stress. These data make it possible to check the accuracy of using the RMS stress level to predict the distribution of vibratory stresses. If a Rayleigh distribution of vibratory stresses occurs during wind turbine operation, it will be reflected in the relationship between RMS level and maximum vibratory stress. Although both RMS and maximum vibratory stress change with wind speed, the relationship between them at all wind speeds is fixed by the Rayleigh distribution. These two measurements are compared below using the Rayleigh distribution assumption.

The Rayleigh probability density function:

$$P(S) = \frac{S}{\sigma^2} \exp\left(-\frac{S^2}{2\sigma^2}\right) \quad (2)$$

where:

S = vibratory stress level

σ = RMS of the stress vs. time signal

describes the probability of occurrence of any vibratory stress level.

The probability that vibratory stresses exceed a value S_0 is:

$$E(S_0) = \int_{S_0}^{\infty} P(S) ds \quad (3)$$

or:

$$E(S_0) = \exp\left(-\frac{S_0^2}{2\sigma^2}\right) \quad (4)$$

Each 'quick look' vibratory stress plotted in Figure 1 is the largest amplitude cycle observed in five seconds. With a dominant frequency of 2.5 Hz, each data point in Figure 1 represents a 1 in 12.5 occurrence, $E(S_0) = 0.08$. The RMS of the stress signal from some 30-40 second records are plotted vs. wind speed as x's in Figure 7; the dashed line is a rough best fit to this RMS data. Using equation (4) with $E(S_0) = 0.08$, leads to an average expected maximum over five seconds indicated by the solid line in Figure 7. The 'quick look' stresses obtained experimentally are also plotted for reference. The excellent fit of the data to the predicted vibratory stress level further supports the assumption that the distribution of vibratory stress levels can be characterized by the RMS of the stress vs. time signal.

4. Applying the Cumulative Damage Rule

If the vibratory stresses are characterized by a probability density function such as the Rayleigh distribution in equation (2), the number of cycles with magnitudes within $\pm \Delta$ of the i^{th} stress level S_i will be:

$$n_i = \int_{S_i - \Delta}^{S_i + \Delta} P(S) n_t dS \quad (5)$$

where:

n_i = number of cycles at the i^{th} stress level

n_t = total number of applied cycles

The number of cycles to failure under constant amplitude loading can be written as a function of S based upon the S-N curve. For most materials this relationship takes on the form:

$$N(S) = K \exp (bS) \quad (6)$$

or,

$$N(S) = K(S)^b$$

where:

K and b are constants and

$N(S)$ may be defined piecewise over S .

Equations (5) and (6) can be incorporated into Miner's Rule [equation (1)] to produce

$$D = \sum_{i=1}^k \frac{\int_{S_i - \Delta}^{S_i + \Delta} P(S) n_t dS}{N(S_i)} \quad (7)$$

In the limiting case where $\Delta \rightarrow 0$ and $k \rightarrow \infty$, equation [7] becomes:

$$D = \int_0^{\infty} \frac{P(S) n_t}{N(S)} dS \quad (8)$$

When n_t equals the number of cycles to failure (n_f), $D = 1$ and the above expression reduces to:

$$\frac{1}{n_f} = \int_0^{\infty} \frac{P(S)}{N(S)} dS \quad (9)$$

Thus, given appropriate S-N data and the probability density function of the vibratory stresses, the number of cycles to failure can be computed. This probability density function is based upon a single stress RMS value and is, therefore, only applicable to a single windspeed. The prediction of the number of cycles to failure by equation (9) would apply to steady operation of a wind turbine at a constant wind speed. The total blade life will depend on the relative length of time the wind turbine is operating at each particular wind speed.

To account for changing wind speeds, Miner's Rule can be employed a second time:

$$D = \sum_{i=1}^k \frac{n_{wi}}{N_{wi}} \quad (10)$$

where:

- n_{wi} = the number of cycles at the i^{th} wind speed
- N_{wi} = the number of cycles to failure at the i^{th} wind speed

The wind speed distribution may also be described by a probability density function, $Q(V)$, where V is the wind speed. Assuming a constant rate of cycle accumulation over all wind speeds, the expression for n_{wi} within $\pm\Delta$ of the wind speed V is:

$$n_{wi} = \int_{V - \Delta}^{V + \Delta} Q(V) n_t dV \quad (11)$$

The RMS stress level may also be written as a function of wind speed. This will make the probability density function for vibratory stress a function of both vibratory stress level and wind speed, $P(S,V)$. Since N_{wi} in equation (10) is n_f in equation (9), this expression for n_f can be substituted into equation (10). Equation (11) is also substituted into equation (10) while the limiting case of $\Delta \rightarrow 0$ and $k \rightarrow \infty$ is evaluated as before to produce:

$$D = \int_0^{\infty} Q(V) n_t \int_0^{\infty} \frac{P(S,V)}{N(S)} dS dV \quad (12)$$

where:

$Q(V)$ = the probability density function of wind speed
As before, when $n_t = n_f$, $D = 1$.

$$\frac{1}{n_f} = \int_0^{\infty} Q(V) \int_0^{\infty} \frac{P(S,V)}{N(S)} dS dV \quad (13)$$

Here, n_f is the total number of cycles to failure under all wind speeds encountered at the wind turbine site. Equation (13) includes the stresses induced by all wind speeds in a single expression. This would be appropriate for a turbine operating at all times, and in all wind speeds.

However, the wind turbine will be operating only in wind speeds between cut-in and cut-out, and will be parked in winds below cut-in and above cut-out. The difference in stress

levels between parked and operating turbines is significant. A second probability density function for stresses while parked, $P_p(S, V)$, must be introduced. The integral must be split into three wind regimes:

$$\begin{aligned}
 \frac{1}{n_f} &= \int_0^{V_{IN}} Q(V) \int_0^{\infty} \frac{P_p(S, V)}{N(S)} dS dV \\
 &+ \int_{V_{IN}}^{V_{OUT}} Q(V) \int_0^{\infty} \frac{P(S, V)}{N(S)} dS dV \\
 &+ \int_{V_{OUT}}^{\infty} Q(V) \int_0^{\infty} \frac{P_p(S, V)}{N(S)} dS dV
 \end{aligned} \tag{14}$$

where:

- V_{IN} = the turbine cut-in wind speed
- V_{OUT} = the turbine cut-out wind speed
- $P_p(S, V)$ = the probability density function of vibratory stresses while the turbine is parked.

Since the stress levels are substantially lower when the turbine is not operating the first term in equation (14), which accounts for the fatigue damage done while the turbine is parked in winds below cut-in, will almost always be negligible. The same may often be true of the damage while parked in high winds, as expressed in the third term in equation (14). If damage is only accumulated while the turbine is operating, both sides of equation (14) can be multiplied by the dominant frequency of blade vibration during operation to obtain an expression for the elapsed time (clock time) to failure:

$$\frac{1}{T_{f_0}} = f_0 \int_{V_{IN}}^{V_{OUT}} Q(V) \int_0^{\infty} \frac{P(S, V)}{N(S)} dS dV \tag{15}$$

where:

- T_{f_o} = elapsed time to failure if the turbine is operating in wind speeds between V_{IN} and V_{OUT} and no damage is accumulated while parked
- f_o = the dominant frequency of blade vibration during operation (cycle rate)

To evaluate the life when damage is accumulated both while operating and while parked in high winds, the hypothetical (but unrealistic) case of no damage occurring except while parked is considered. The life is then expressed as:

$$\frac{1}{T_{f_p}} = f_p \int_{V_{OUT}}^{\infty} Q(V) \int_0^{\infty} \frac{P_P(S,V)}{N(S)} dS dV \quad (16)$$

where:

- T_{f_p} = elapsed time to failure if damage is only accumulated while the turbine is parked in winds above V_{OUT}
- f_p = the dominant frequency of blade vibration while parked

The left sides of equations (15) and (16) are expressions of the amount of damage done per unit time. Making use of the linearity of Miner's cumulative damage rule to combine the parked and operating conditions yields:

$$\frac{1}{T_f} = \frac{1}{T_{f_o}} + \frac{1}{T_{f_p}} \quad (17)$$

where:

- T_f = elapsed time to failure if damage occurs while operating and while parked.

This is equivalent to multiplying each term in equation (14) by the appropriate frequency of vibration.

$$\begin{aligned} \frac{1}{T_f} = & f_o \int_{V_{IN}}^{V_{OUT}} Q(V) \int_0^{\infty} \frac{P(S,V)}{N(S)} dSdV \\ & + f_p \int_{V_{OUT}}^{\infty} Q(V) \int_0^{\infty} \frac{P_P(S,V)}{N(S)} dSdV \end{aligned} \quad (18)$$

Selecting the frequency of vibration can be done by examining the blade stress spectral density. If one frequency clearly dominates, it can be used in equation (18). If there is no one dominant frequency of vibration, the zero crossing (or mean crossing) rate can be used as an average frequency. Variations in frequency have a linear effect on the estimate of time to failure. This effect is less pronounced than variations in the S-N data or RMS stress level which will have exponential effects on the life estimate.

The frequency content of the stress vs. time signal while parked will not have a lower bound as is the case when the turbine is operating. More care will have to be taken in assessing the relative contribution of these low frequency cycles to the fatigue damage. Also, fluctuations in the mean stress level may have to be accounted for when dealing with the parked condition.

This problem of estimating the fatigue life in an environment where there is no one obviously dominant frequency of vibration has been examined by Broch [2]. Tests were conducted with both one and two degree-of-freedom systems by varying RMS values of stress and "average" zero-crossing frequency. The conclusion was that for Gaussian random excitation -

"For a given RMS Stress level and a given 'average' frequency (number of zero crossings per second) the most dangerous loading case seems to be that which produces a Rayleigh distribution of stress-response reversals (single resonance response)."

This implies that if the RMS and zero crossing "average" frequency are known, the conservative approach would be to assume that there is a Rayleigh distribution of vibratory stresses characterized by the RMS stress level.

It should be noted that when using equations (15) or (18), the wind turbine is assumed to be operating whenever the winds are between cut-in and cut-out wind speeds, and not operating at all other times. There is likely to be a grey zone of at least 2.5 m/s (5 mph) around the cut-out wind speed where the turbine will be operating at times and parked at times. Since the stress levels go up drastically with wind speed, operation in this wind speed zone around cut-out is likely to account for a significant amount of fatigue life. The algorithm used to control high wind shut-down and start-up will decide the relative amount of time operating near cut-out, and is therefore an important factor in the fatigue life of wind turbine blades. Use of equation (15) or (18) does provide a method of comparison between different wind sites and the relative effects of cut-in and cut-out wind speeds at a particular wind site.

Summary

Examination of the real time stress signal from VAWT blades during operation, as well as a plot of 'quick look' data shown in Figure 1, demonstrate that a single vibratory stress level at each wind speed does not characterize the state of stress of an operating turbine's blades. The stochastic effects of a constantly varying wind causes the stresses in the blades to be stochastic as well. The use of a Rayleigh distribution to describe the different vibratory stress levels present during operation at each wind speed seems to be appropriate for VAWT blades. The only parameter of the real time stress signal required to characterize the Rayleigh distribution is the RMS stress level. These RMS levels can then be expressed as a function of wind speed as shown in Figure 7.

With all the vibratory stress levels described, a cumulative damage rule must be employed to account for the relative amounts of damage done by these stresses. Miner's Rule is the damage rule used most often mainly because of its simplicity and ease of implementation. There is, however, general disagreement on the accuracy of Miner's Rule. The fact that it does not account for the sequence of load applications appears to be less important in random loading than in block loading.[14] Since VAWT loading is random, Miner's Rule is used as the cumulative damage rule, and the emphasis is placed on carefully defining the stress history of the blades.

Combining the Rayleigh distribution with the S-N data using Miner's Rule results in an expression for the number of operating cycles to failure at each wind speed. Implementing the cumulative damage rule again to account for the wind speed distribution provides a method of predicting the total wind turbine life for a given wind site and given cut-in and cut-out wind speeds. This method assumes operation whenever the winds are between cut-in and cut-out and no operation when the winds are not. In reality, the high wind cut-out algorithm will have a large effect on the amount of operating time that a turbine will see at wind speeds near cut-out, and therefore will have a significant effect on blade life.

Another assumption in this approach is that the stress cycle rate remains constant during operation over the entire range of wind speeds. For a constant RPM machine this will usually be true although it is quite possible that the modes of vibration will be excited at different levels in different wind speeds. A variable RPM machine will definitely not have a constant stress cycle rate. In that case, a solution for the time to failure [as in equation (15)] will not simply multiply the damage integral by the dominant frequency, but will require an expression for the frequency as a function of wind speed included inside the integral.

A great deal of emphasis has been placed on the use of the Rayleigh distribution of vibratory stresses. Because the

Rayleigh distribution of peaks is present in Gaussian narrow band processes, it is a very common distribution. It is easy to work with because it is a single parameter distribution (RMS) and therefore simple to describe. The Rayleigh distribution has been emphasized here mostly because it describes the vibratory stresses for the DOE/ALCOA Low Cost 17m very well. However, the expressions presented here for elapsed time to failure of wind turbine blades do not require that the probability density function of vibratory stress be Rayleigh. Any probability density function for stress, $P(S)$, as well as for wind speed, $Q(V)$, can be used.

Future Work

To implement this method of fatigue analysis, the RMS stress level must be determined as a function of wind speed. Figure 7 has data from two collection methods which can be used to determine this relationship, but neither of these methods is very direct. Work is being done to create a data collection scheme that characterizes the distribution of vibratory stress levels in a simple, direct way. The Method of Bins approach [1] looks most promising for producing a representative RMS stress level vs. wind speed curve.

Work is also continuing in statistical testing of time series records of both operating and parked (in high wind) turbines. The DOE/ALCOA 17m Low Cost turbines have provided, and continue to provide, a good data base for such testing. Proposed structural modifications of the Low Cost which change the resonant frequencies will also allow comparative studies of turbines that have different characteristic power spectral densities of blade vibration.

Comparative studies of cut-out algorithms using computer simulations of a wind turbine automatic controller in an actual wind environment are continuing. Because both the power output and the rate of fatigue damage are highest at the cut-out wind speed, the control algorithm will have a significant effect on the cost effectiveness of the VAWT.

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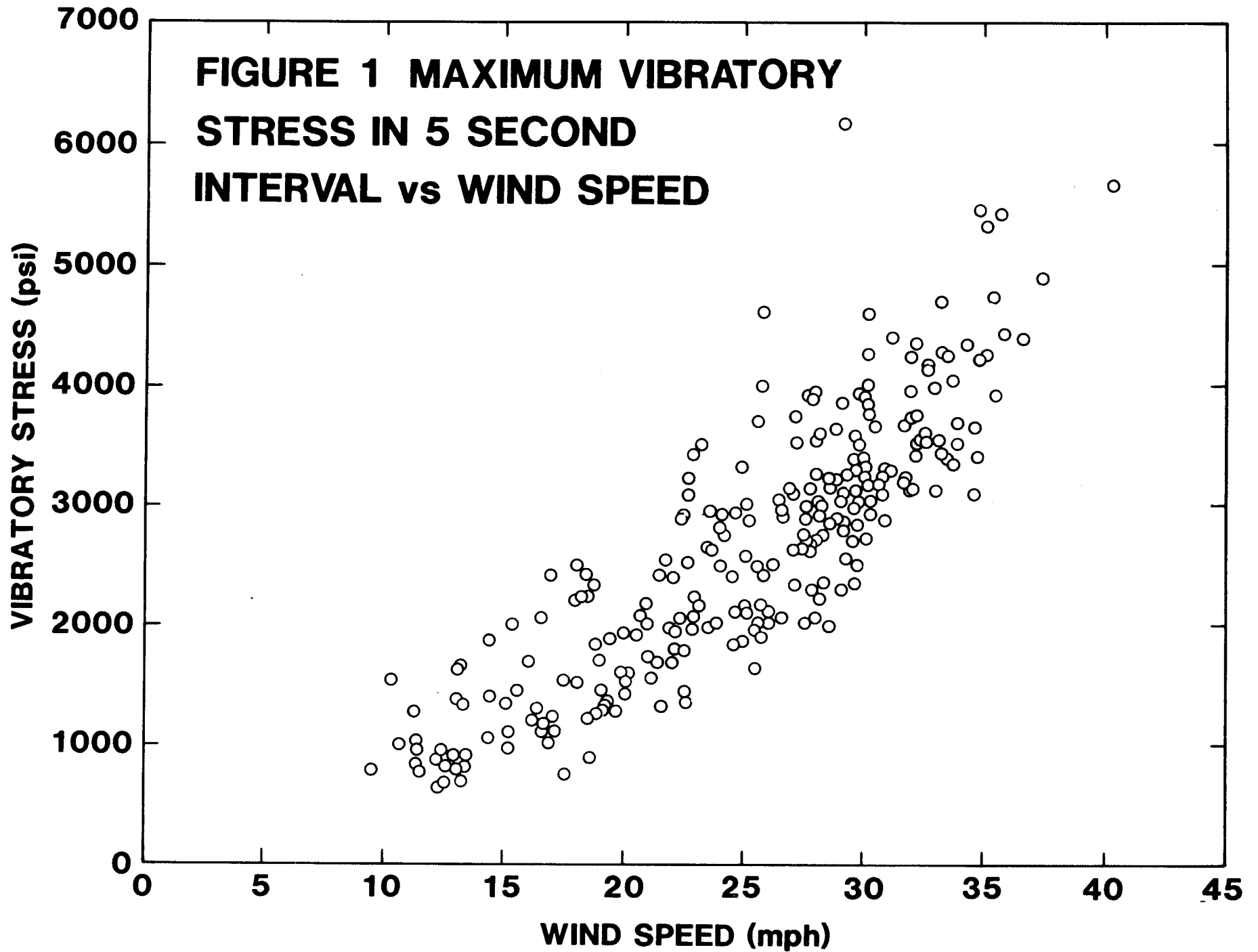
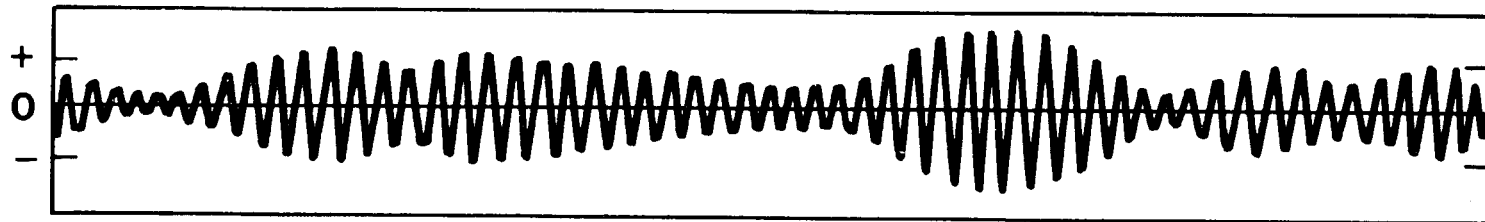
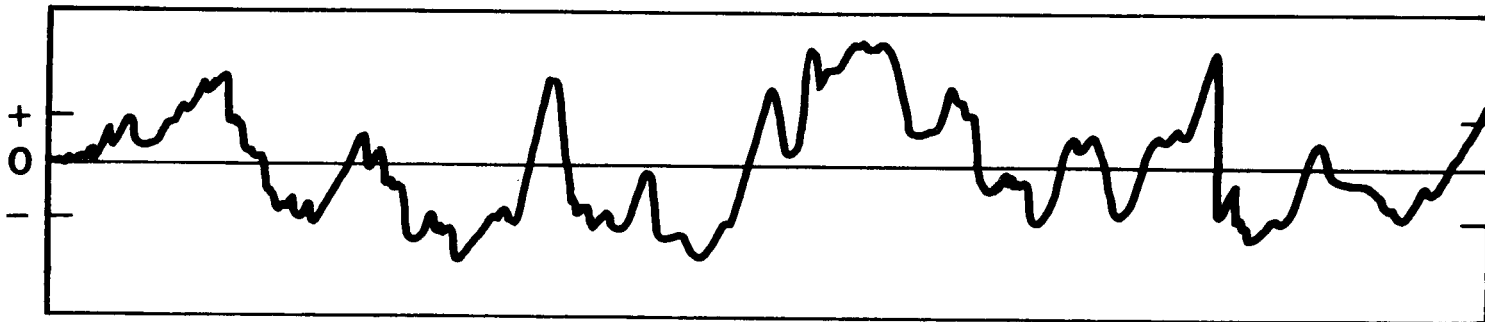


FIGURE 2

GAUSSIAN SIGNALS

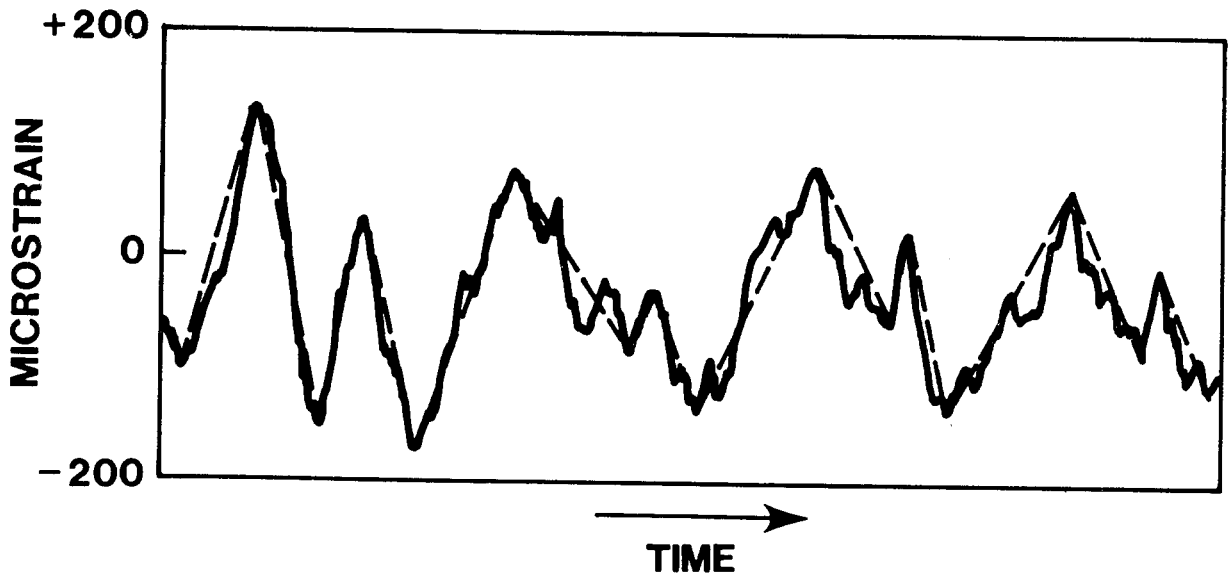


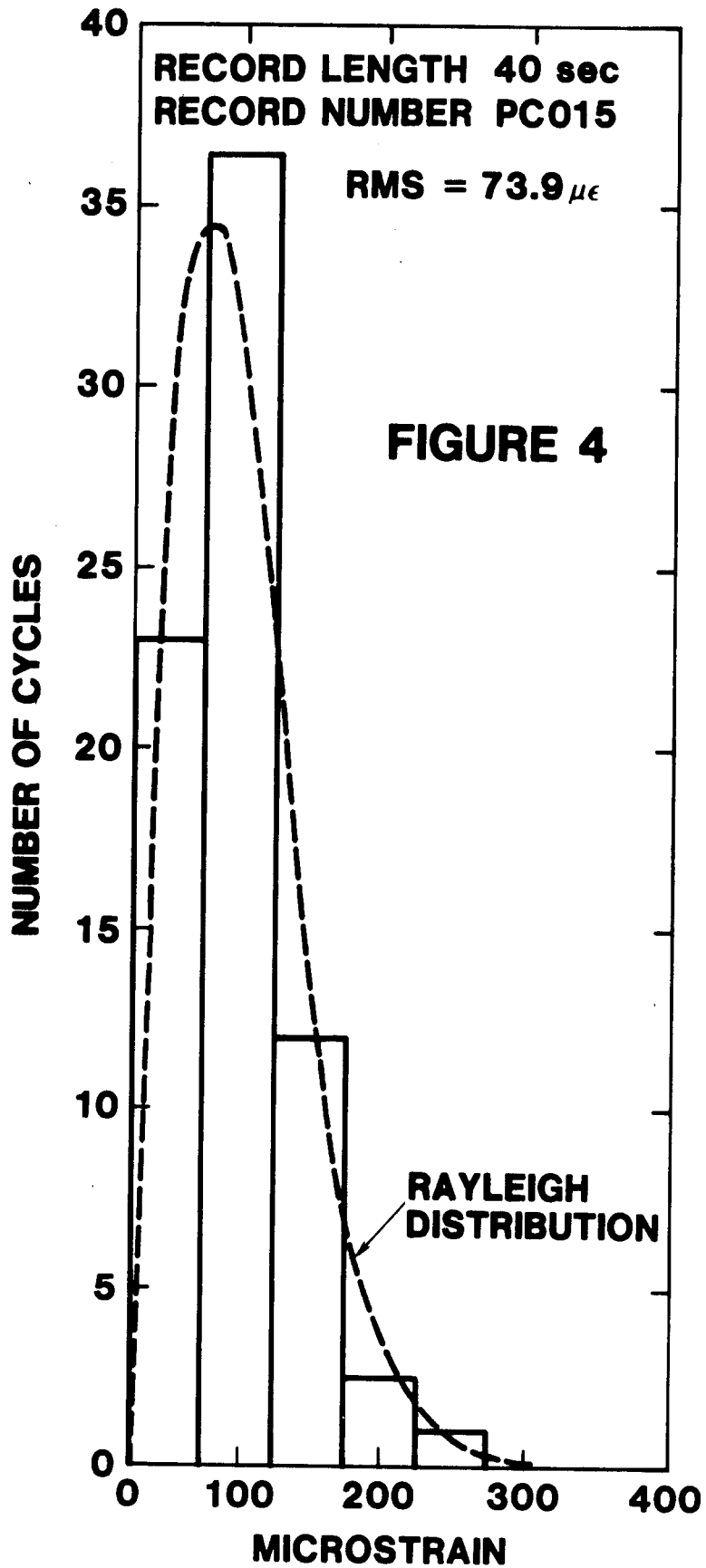
NARROW BAND

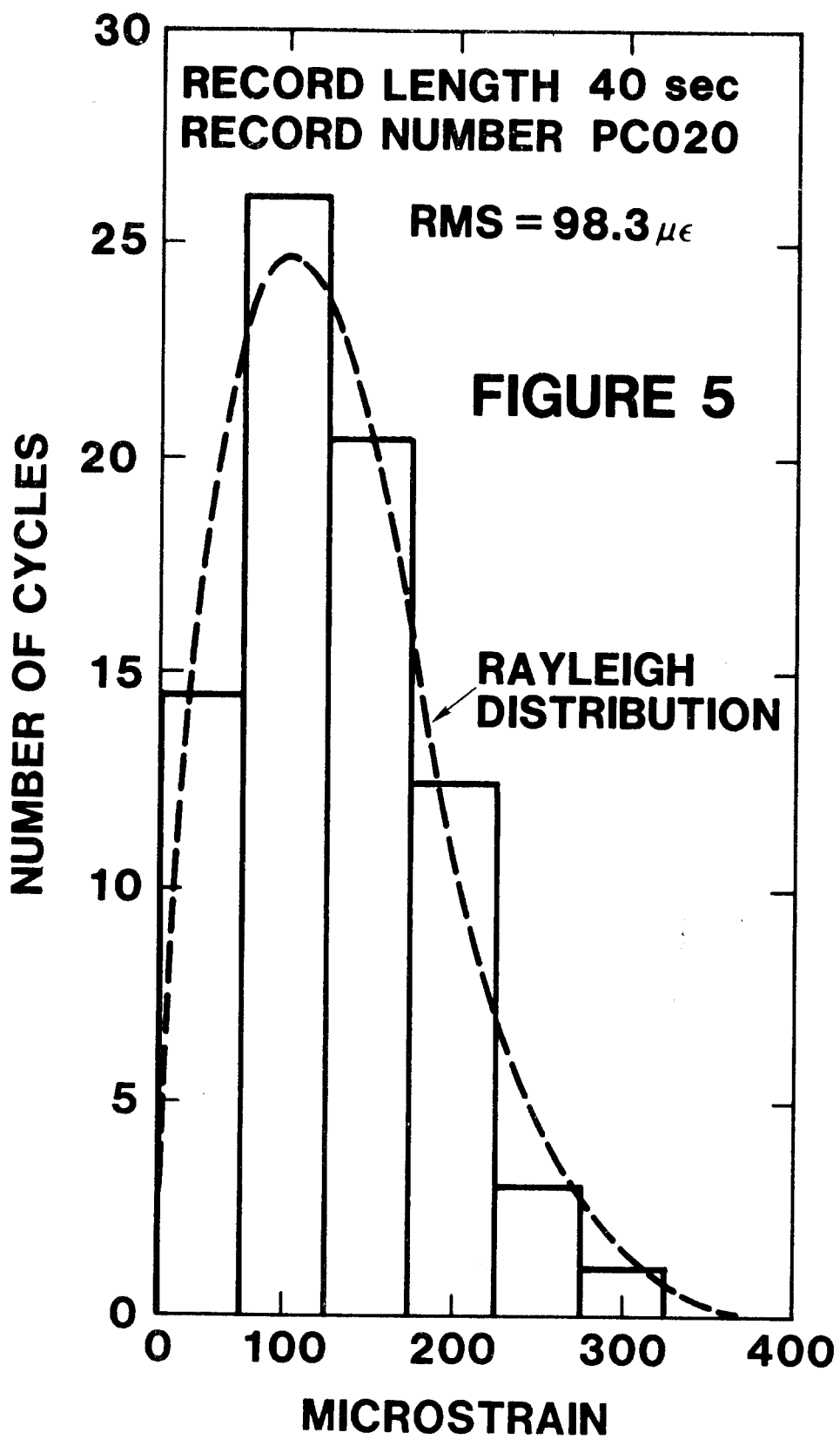


WIDE BAND

FIGURE 3
TIME SERIES STRAIN SIGNAL
WITH APPROXIMATION USED TO COUNT CYCLES







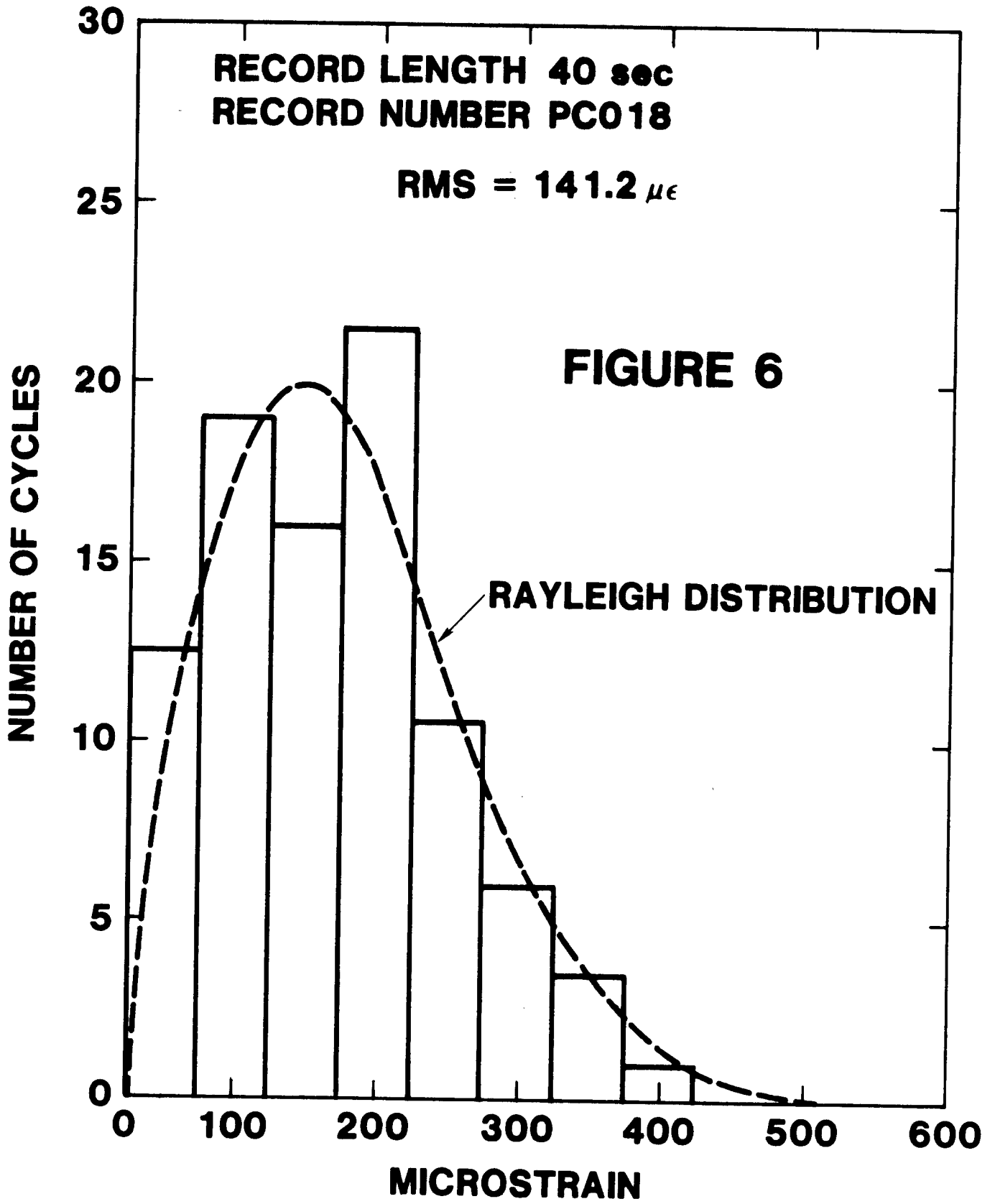
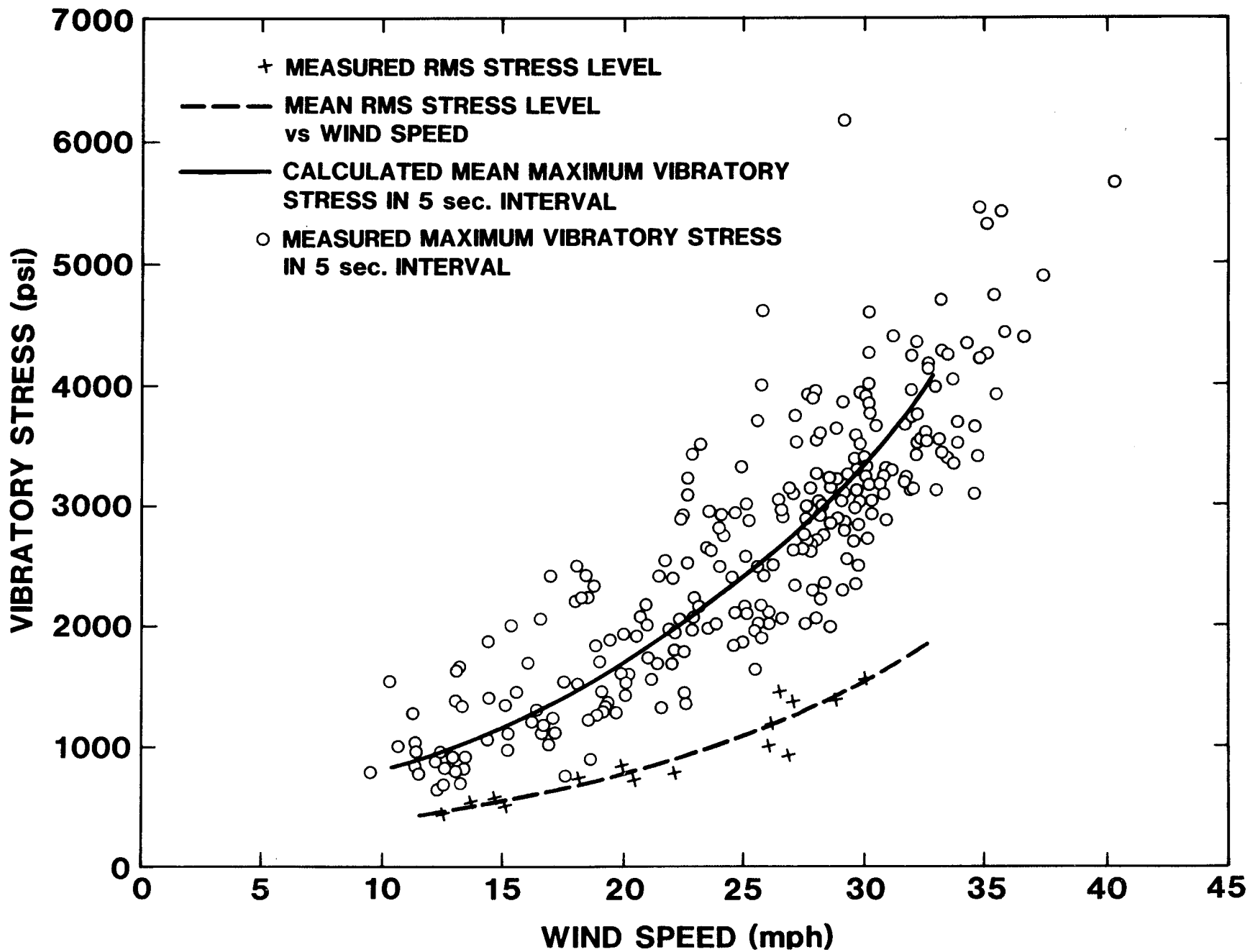


FIGURE 7



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