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Wind Characteristics at the Vawt Test Facility

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WIND CHARACTERISTICS AT THE VAWT TEST FACILITY

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ABSTRACT

A limited program of field measurements was undertaken in order to define the wind characteristics of the DOE/Sandia vertical axis wind turbine test facility. Because micrometeorological conditions under which a particular wind turbine is tested may have an effect on the performance of that turbine, it is important that these conditions be properly documented. The mean velocity profile, longitudinal and lateral turbulence intensities, longitudinal velocity spectra, and cross-correlations between the velocities at select locations were measured for a representative sample of incident winds. These measurements are summarized in a form which should facilitate comparison with other sites.

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I. Introduction

Although the effects of such wind characteristics as the mean velocity profile, the incident turbulence intensity, or the scales of the turbulence on the performance of wind energy conversion systems (WECS) are not well understood, it is important that the wind characteristics of a particular test facility be documented for use in interpretation of test results obtained at that facility. A limited program of measurements was undertaken at the DOE/Sandia vertical axis wind turbine (VAWT) test facility in order to document these properties of the lower regions of the atmospheric boundary layer at the test facility. This program was not intended to be a state-of-the-art micrometeorological investigation, but was instead planned to use existing instrumentation with the minimum effort devoted which was necessary to obtain satisfactory results. Measurements were made so as not to interfere with operations at the facility. The period of observations used in compiling the wind characteristics at the facility spans a nine month period from March 1977 to December 1977. Some additional measurements will be continued in calendar year 1978 to verify the values reported in this document.

This report was written to provide a brief summary which should be easily understood by those individuals interested in the test results obtained at the VAWT test facility. In order to accomplish this goal, some explanations of the atmospheric boundary layer have been simplified. Adequate background is provided to enable a reader unfamiliar with the concepts associated with wind characteristics to understand the measurements without the necessity of reading additional references.

The following sections provide a physical description of the test facility, a brief summary of the background necessary to understand the measurements discussed, an explanation of the equipment and techniques used, and a summary of the wind characteristics at the VAWT test facility.

II. Test Site Layout/Physical Description

The VAWT test facility is located on Kirtland Air Force Base, Albuquerque, New Mexico. The facility is located at an elevation of 1658 m MSL on a mesa which rises gradually eastward to an elevation of 1830 m at the base of the Sandia and Manzano Mountain Ranges. The test facility is about 8 km west of the nearest point along these mountains. About 5 km west of the test facility, the land falls off rapidly to the floor of the Rio Grande Valley. The river is about 7 km west of the test facility. Tijeras Arroyo, which is up to 60 m deep and varies from 1 to 3 km in width, runs west-south-westward about 1 km southeast of the test facility. This arroyo begins at the juncture of the Sandia and Manzano Mountains at Tijeras Pass about 12 km east of the test facility and enters the Rio Grande Valley about 5 km southwest of the test facility. The Sandia and Manzano Mountains form a north-south chain that extends for a distance of approximately 100 km. The highest elevation in the Sandias is 3260 m MSL about 24 km northeast of the test facility. The highest point of the Manzano Range is 3080 m MSL about 48 km south-southeast of the test facility. The lowest point in Tijeras Pass, located 12 km to the east, is at an elevation of 2100 m MSL. West of the river, the terrain rises gradually to an elevation of about 2400 m MSL at the Continental Divide, a distance of 150 to 160 km. The only important interruption in this gradual rise to the Divide is Mount Taylor, approximately 3050 m MSL, located 80 km to the west. The Rio Puerco Valley, located 32 km westward and oriented north-south, is almost as broad and as deep as the Rio Grande Valley. The Jemez Mountains rise to a maximum elevation of 3640 m MSL, about 100 km north of the test facility. Figure 1 is a map which shows these features.

On a smaller scale, the layout of the VAWT test facility as of February 1978 is shown in Fig. 2. Items of interest are the relative locations of the test pads for the 17 M, 5 M, and 2 M turbines, as well as the location of the 30 m

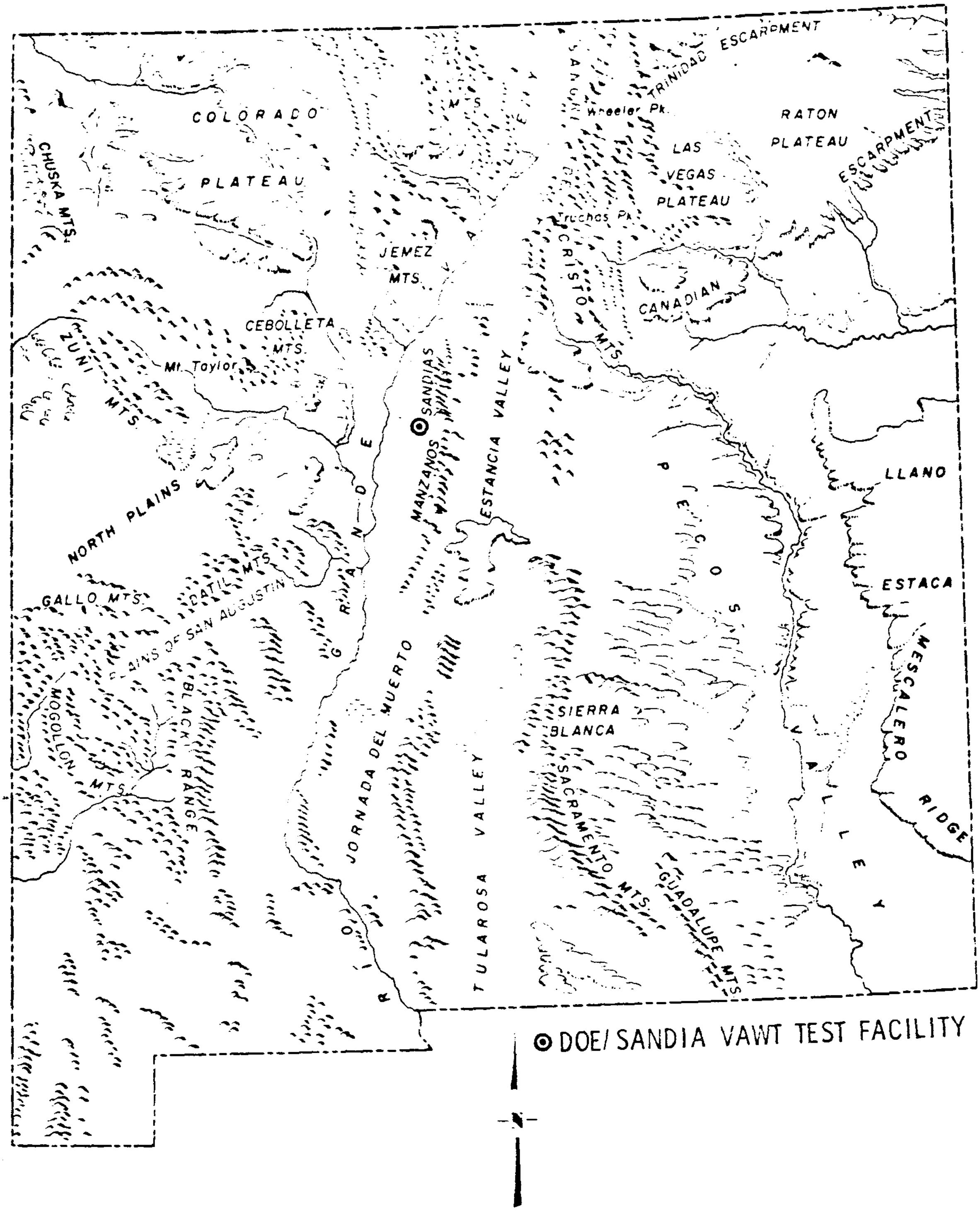

FIGURE 1
Map of Albuquerque Region

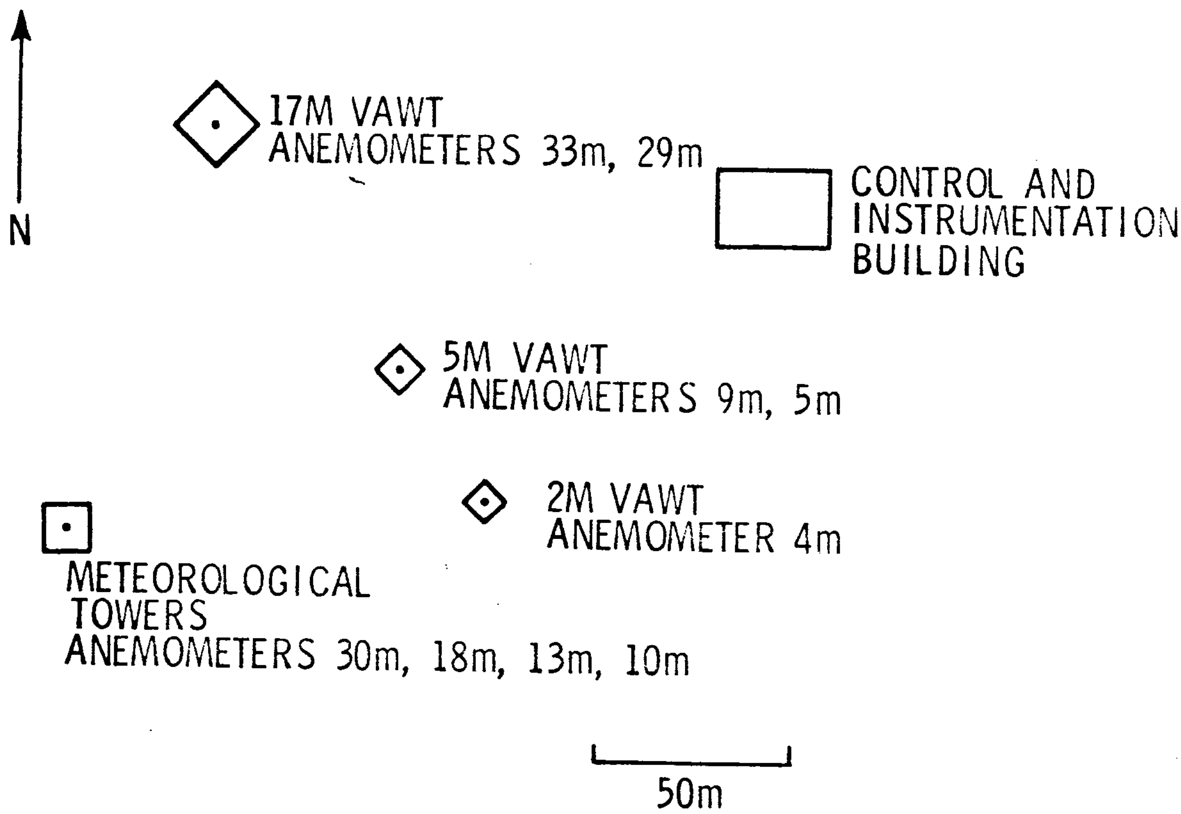

FIGURE 2
Test Facility Layout Including Anemometer Locations

meteorological tower. Locations and heights of reference anemometers are also indicated on Fig. 2. Figure 3 is a photograph of the test facility. This picture provides a physical feel for the terrain and vegetation in the immediate vicinity of the test facility. No major buildings or trees are within 1 km of the test facility. The predominate vegetation is that characteristic of the high desert of the southwest, sparse vegetation less than 0.5 m high.

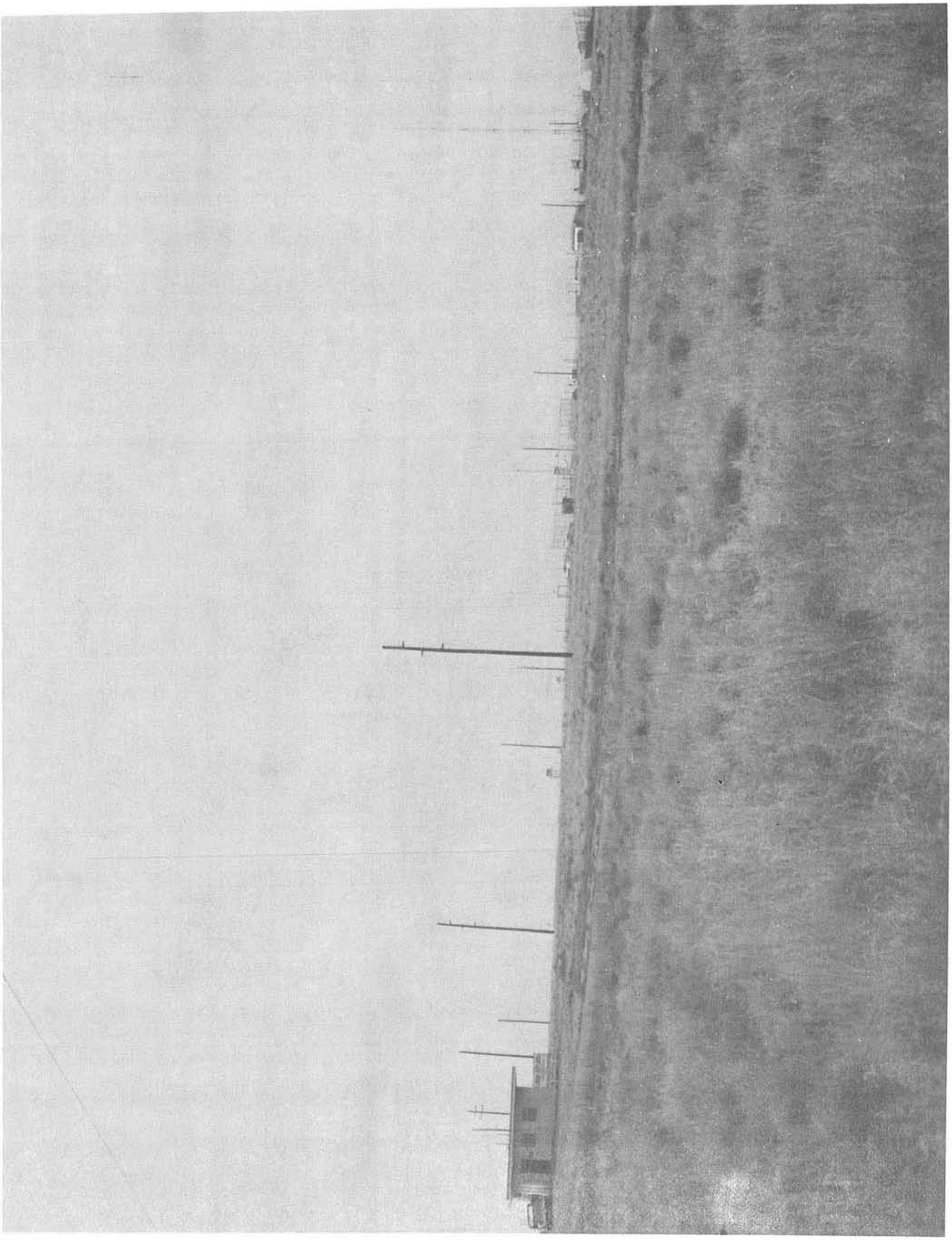

FIGURE 3
Photograph of DOE/Sandia VAWT Test Facility

III. Background

The wind is a highly variable random quantity. Anyone who has observed the motion of leaves on a windy day has some idea of the complexity of atmospheric motions. There are many methods of describing the motion of the lower 500 m of the atmosphere, involving various levels of complexity. This section will explain what is felt to be the minimum measurements necessary to adequately describe the relevant wind characteristics required for the interpretation of performance of a wind turbine.

The most elementary concept associated with atmospheric flow in the lowest 500 m is that of the atmospheric or planetary boundary layer. Well above the surface of the earth, the wind is caused by large-scale pressure differences. At a sufficient height, the winds are independent of the conditions on the surface of the earth. At the surface of the earth, the velocity of the wind is always zero. The region between the zero velocity at the surface and the geostrophic wind is termed the atmospheric boundary layer. The atmospheric boundary layer can be considered the region in which the surface of the earth has an effect on the flow of air over the surface. The geostrophic wind occurs at a height where surface effects cease. In this layer, the wind speed increases with height in most strong wind (> 5 m/s) situations. Wind speeds in weather forecasts are usually predicted for a level 10 m above the surface. The winds at a height less than 10 m will be less than those predicted at 10 m while the winds above 10 m will be greater than those predicted for the 10 m level. This variation of wind speed with height is termed wind shear. The manner in which wind speed increases with height is in general a function of the features of the upwind fetch, the vertical variation of temperature, and the wind speed. Conditions of importance to wind power studies are characterized by strong winds (> 5 m/s). In such cases, the effects of temperature variation with height are not important. This condition is termed a neutrally stable atmosphere or an adiabatic boundary layer. The remainder of this section will deal with the neutral or adiabatic boundary layer. A number of extensive reviews of the properties of the atmospheric boundary layer exist. Counihan (1) has condensed most available data reported prior to 1972 and provided a useful summary. More analytical reviews have been published by Plate (2) and Panofsky (3). These references should be consulted for further information on topics discussed in this section.

Two expressions are frequently used to describe the vertical variation of horizontal wind speed in the atmospheric boundary layer. These are the logarithmic or log formulation and the power-law formulation. There is an analytical basis for the logarithmic expression. The power-law expression is an empirical relationship. Figure 4 is a plot of the vertical variation of mean wind speed in the atmospheric boundary layer.

The log profile can be derived in many different ways based on an assumption of constant shear stress in the lower portion of the atmospheric boundary layer. The expression for the log profile in a neutral atmosphere is:

$$V(z) = \frac{u^*}{k} \ln \frac{z + z_o}{z_o} \qquad (1)$$

where $V(z)$ is the mean wind speed at height, z

$u^* \equiv \sqrt{\frac{\tau_{surface}}{\rho}}$, the friction or shear velocity

k is the von Karman constant

z_o is the surface roughness length

$\tau_{surface}$ is the surface shear stress

ρ is the density of air

In order to apply this expression if a reference wind speed at a fixed height, z_{ref}, is known, the equation can be solved for u^* if z_o is known or conversely for z_o if u^* is known. In most applications related to wind power, sufficient

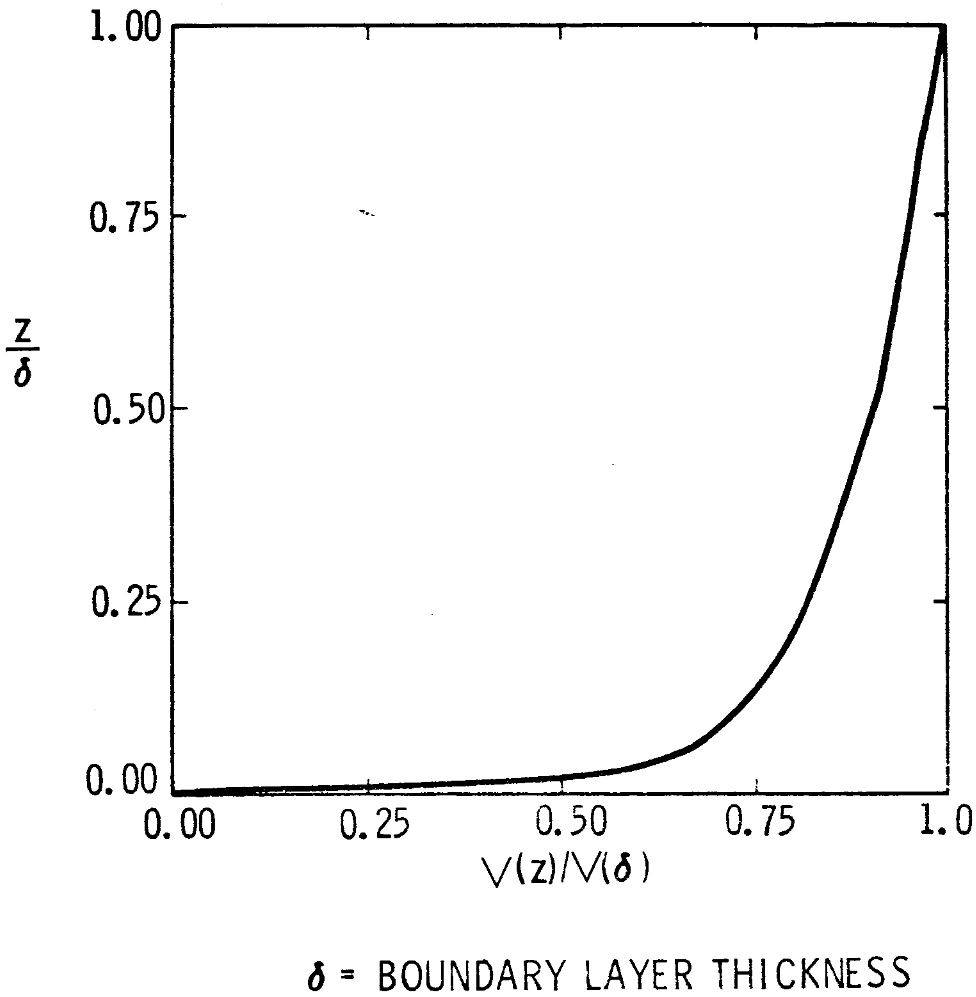

FIGURE 4
Typical Mean Velocity Profile

measurements to determine u* will not be available. A number of sources are available which relate the physical description of a location to the value of the surface roughness length, z_o [(1), (4)]. The value of z_o is independent of the mean wind speed at a particular reference level and is considered a unique property of a particular location. Measured values of z_o for terrain which might be considered as potential wind power locations range from 10^{-5} m to 1.5 m. Such a range could easily result in an order of magnitude error in an estimate of z_o based on a physical description of a location. A direct measurement of z_o requires a simultaneous measurement of the mean wind speed at a minimum of two elevations and preferably at several. In spite of these potential problems associated with the evaluation of z_o at a particular location, the best summary of available data (1) reports properties of the atmospheric boundary layer as a function of z_o.

The power-law method of describing the wind shear has evolved primarily to fill a need for a simple expression to describe the vertical variation of wind speed. It is used in the specification of wind loading of structures and other similar applications. The power-law relationship is given by:

$$V(z) = V(z_{ref}) \left(\frac{z}{z_{ref}}\right)^{\alpha} , \qquad (2)$$

where $V(z)$ is the mean wind speed at height, z

$V(z_{ref})$ is the mean wind speed at height, z_{ref}

α is the exponent of the power-law

Application of this formula requires a $V(z_{ref})$ and an exponent, α. Estimates of α as a function of surface roughness, z_o, or based upon a physical description of a location are available [(1), (4)]. Typical values of α range from 0.10 for very smooth terrain to 0.35 or 0.40 for large urban areas. If a value of wind speed at a reference height is known for a particular location, the power-law provides a simple method for extrapolating this value to another elevation. It should be emphasized that the power-law is an empirical formulation. Recent work (5) has suggested some dependence of the exponent on mean wind speed, but for strong wind situations, such a dependence is not yet well established. The log- or power-law provides a means of describing the mean wind speed profile over a period of the order of 10 minutes to an hour. At each elevation, z, a mean wind speed, $\overline{V}(z)$, can be measured. The variations of wind speed about the mean are important both for the design of wind turbines and for the analysis of performance data. Figure 5 is a plot of wind speed as a function of time showing random fluctuations about the mean. These fluctuations may be expressed as

$$V(t) = \overline{V} + v(t)$$

where V(t) is the wind speed made up of the sum of a mean and a fluctuating component (the fluctuating component can be positive or negative). Although much theoretical and experimental work has been undertaken to describe and predict the properties of atmospheric turbulence, for the purposes of this report, only the most elementary properties of atmospheric turbulence will be used.

The most readily understood measure of turbulence is a root-mean-square (rms) value defined by

$$v' = \left[\frac{1}{T}\int_{0}^{T} v^2(t)dt\right]^{\frac{1}{2}}$$

This quantity may be calculated for alongwind, acrosswind, and vertical velocity fluctuations. A cup anemometer is only capable of measuring the alongwind fluctuations (or more accurately the speed fluctuations). A cup anemometer and a direction vane allow measurement of both the alongwind and acrosswind fluctuations in

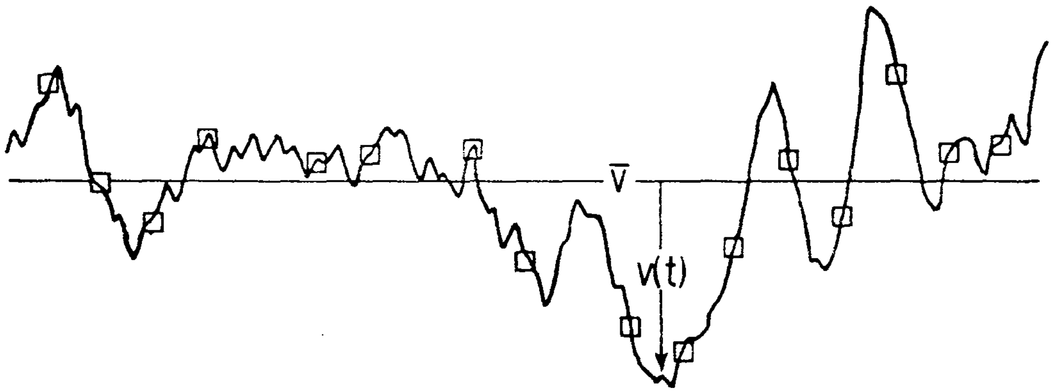

FIGURE 5
Mean and Fluctuating Velocity

velocity. More sophisticated instrumentation is necessary to measure the vertical fluctuations. A common method of reporting the rms values of velocity is in terms of a turbulence intensity defined as the ratio of the rms wind speed to the mean wind speed. In general, the alongwind turbulence intensity is greater than the acrosswind turbulence intensity which in turn is greater than the vertical turbulence intensity (1).

The turbulent fluctuations do not occur at a single discrete frequency, but consist of contributions from a wide range of frequencies. The contribution to the variance of the wind speed as a function of frequency is termed the spectral density. The most readily measured spectral density is that of the alongwind turbulence. In this particular application, the spectral density of the wind speed fluctuations can be measured using the output of a cup anemometer. The spectral density of the longitudinal turbulence can be used to identify ranges for which appreciable energy exists in the velocity fluctuations. Such information is of value in evaluating both the structural response and the performance of a wind turbine.

The average size of a turbulent eddy is described by an integral length scale. A length scale exists for the alongwind or longitudinal component, the acrosswind component and the vertical component. The primary importance of an integral scale is in comparison to the size of the wind turbine or significant components of the turbine. If the integral scale is larger than the diameter of a WECS, the energy containing gusts will envelop the entire structure. If the integral scale is smaller than the diameter of a WECS, most gusts will only affect a portion of the structure.

One additional property of turbulence which is of interest in performance evaluation is the cross-correlation between the alongwind velocity fluctuations at two separate locations. The cross-correlations between the wind speed measured by an anemometer above a turbine and the wind speed at the centerline of the turbine or between the wind speed measured at a meteorological tower and the wind speed measured at a turbine location are of particular interest. In order to obtain accurate measurements of performance, this cross-correlation should be greater than zero. It is not possible to assign an arbitrary value which the cross-correlation coefficient of wind speeds at two locations must assume to insure a valid measure of performance. In addition, the time lag for which the cross-correlation of wind speeds at two locations is a maximum is an indication of any time delay which should be applied to a measure of WECS output measured at one location when compared with a wind speed measured at the second location.

IV. Experimental Techniques

The physical layout of the test facility is shown in Fig. 2. The locations of anemometers and wind direction indicators are of particular interest in the interpretation of results presented in the following section. Instruments are located at four levels on the meteorological tower (30 m, 18 m, 13 m, and 10 m), at two levels on the 17 M turbine (33 m and 29 m) and at two levels near the 5 M turbine (9 m and 5 m). The anemometer at the 2 M test pad was not in place during the period of these measurements. The signals from the instruments are fed through an underground cable system to the central building. All data collection and reduction was done on a Hewlett-Packard HP21MX mini-computer system (6).

The anemometers used are Teledyne Geotech Model 1564B with Model 170-41 cups. This instrument has a distance constant of 1.5 m and a threshold speed of 0.25 m/s. The direction indicators used were Teledyne Geotech Model 1565B with a Model 53.4 vane. This instrument has a distance constant of 1.1 m and a threshold speed of 0.30 m/s.

Data were collected at a sample rate of 10 samples per second for periods of from 10 to 60 minutes. Mean velocities and turbulence intensities were computed for 5 minute periods using the techniques listed in Appendix A. Spectral density functions and cross-correlations were computed using digital data reduction techniques (7), (8).

V. Results

This section will provide a summary of the measurements conducted at the test facility. An item of general interest is the wind speed probability density function for the Albuquerque area. The test facility was not selected because of high wind power potential, yet the environment that a turbine located at the facility will experience is of interest. Figure 6 shows the wind speed probability density function for Albuquerque at a height of 15 m above the ground. The data used to generate this figure were 20 years of hourly readings from 1950 to 1970 obtained by combining data from a number of sources.

The remainder of the measurements were taken for periods of from 10 to 60 minutes between March and December 1977. Figure 7 shows a diagram of the resultant direction and mean speed at 30 m for the cases where data were collected. The wind directions have been rounded to the nearest 10°.

The exponent, α, of the power-law expression was determined by fitting the 5 minute averages using a least-squares technique. The mean value for all cases was 0.10 with a standard deviation of 0.03. This value of α corresponds to smooth terrain and is smaller than might be predicted for the test facility. The value of z_o, the surface roughness length, was determined by using a least-squares approximation to Eq. (1). The average value of z_o was found to be 1 cm with a standard deviation of 0.7 cm. This value of z_o can be used to predict α using an expression proposed by Counihan (1). The predicted value of α for z_o of 1 cm is 0.11. This value is consistent with the measured value of 0.10.

The measured values of alongwind and acrosswind turbulence intensity are shown in Table 1. The turbulence intensities are based upon the mean wind speed at the height of the measurement. Some directional dependence may be seen in these values. Winds from the south and east approach over a smoother surface

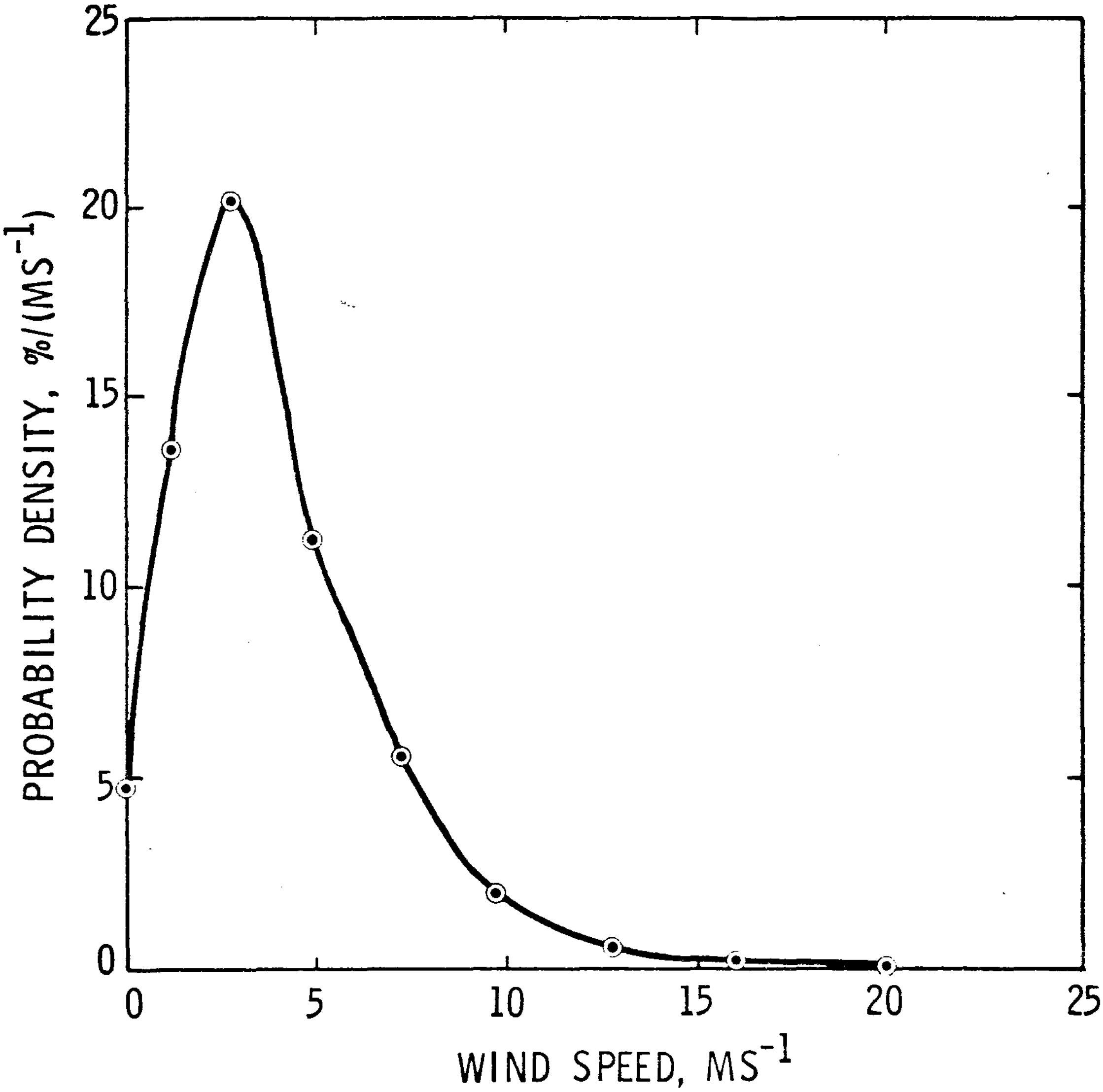

FIGURE 6
Probability Density Function of Wind Speed,
Albuquerque, NM, 1951-1970

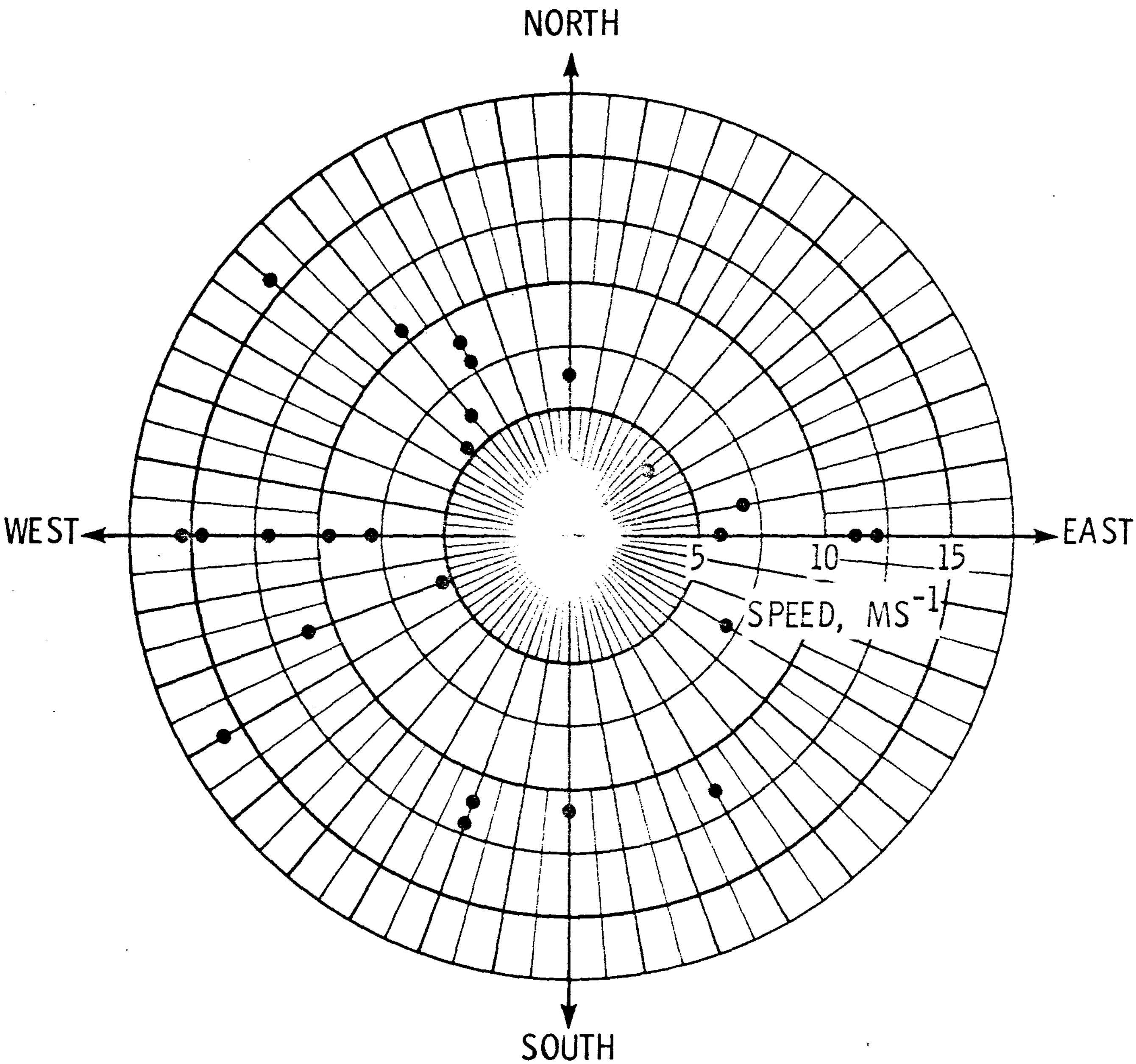

FIGURE 7
Schematic of Measurement Conditions Used in Data Reduction

NORTH (315° - 045°)

Height	3m	10m	13m	18m	30m
Alongwind	28%	25%	23%	24%	23%
Acrosswind	-	-	25%	23%	21%

EAST (045° - 135°)

	3m	10m	13m	18m	30m
Alongwind	17%	14%	-	-	10%
Acrosswind	-	-	-	-	-

SOUTH (135° - 225°)

	3m	10m	13m	18m	30m
Alongwind	21%	19%	16%	17%	14%
Acrosswind	-	17%	18%	16%	14%

WEST (225° - 315°)

	3m	10m	13m	18m	30m
Alongwind	21%	18%	22%	20%	17%
Acrosswind	-	-	25%	23%	21%

TABLE 1
Alongwind and Acrosswind Turbulence Intensities

than those from the north and west. The turbulence intensities are lower for southerly and easterly winds. The values at 13 m correspond to the centerline of the 17 M turbine.

A typical longitudinal wind speed spectral density is shown in Fig. 8. While considerable scatter exists in the lower frequency range, the region from 0.1 to 5 hz is well defined. This spectral density was obtained from a fifteen minute record of wind speed sampled at 10 samples per second. The format used in the plot of Fig. 8 is termed a reduced normalized spectral density. The ordinate is $nS(n)/\sigma^2$ where n is the frequency, S(n) is the spectral density and σ^2 is the variance of the wind speed fluctuations. The abscissa is the logarithm of the frequency. In this format, the area under the curve is equal to 1.0 and the ordinate at a particular frequency is proportional to the contribution of that frequency to the variance of the wind speed fluctuations. The spectral density shown in Fig. 8 was measured at a height of 13 m corresponding to the centerline of the 17 M turbine. The mean wind speed for the record was 13 m/s. Also included on this figure are curves corresponding to two empirical expressions for the spectral density of longitudinal turbulence (9), (10). The expression proposed by Davenport (9) is widely used in predicting wind loading of structures. The expression suggested by Frost (10) is a modification of a form proposed by Kamial, et. al. (11) and has been recommended for use in the design of wind turbines. It is felt that either of these expressions adequately describes the data from the VAWT test facility. Full-scale measurements of the structure of the wind are subject to considerable scatter and exact agreement should not be expected.

The longitudinal spectral density will shift along the frequency axis as a function of mean wind speed. Higher mean wind speeds will have contributions to the variance of the wind speed at higher frequencies than those shown in Fig. 8 for a mean wind speed of 13 m/s.

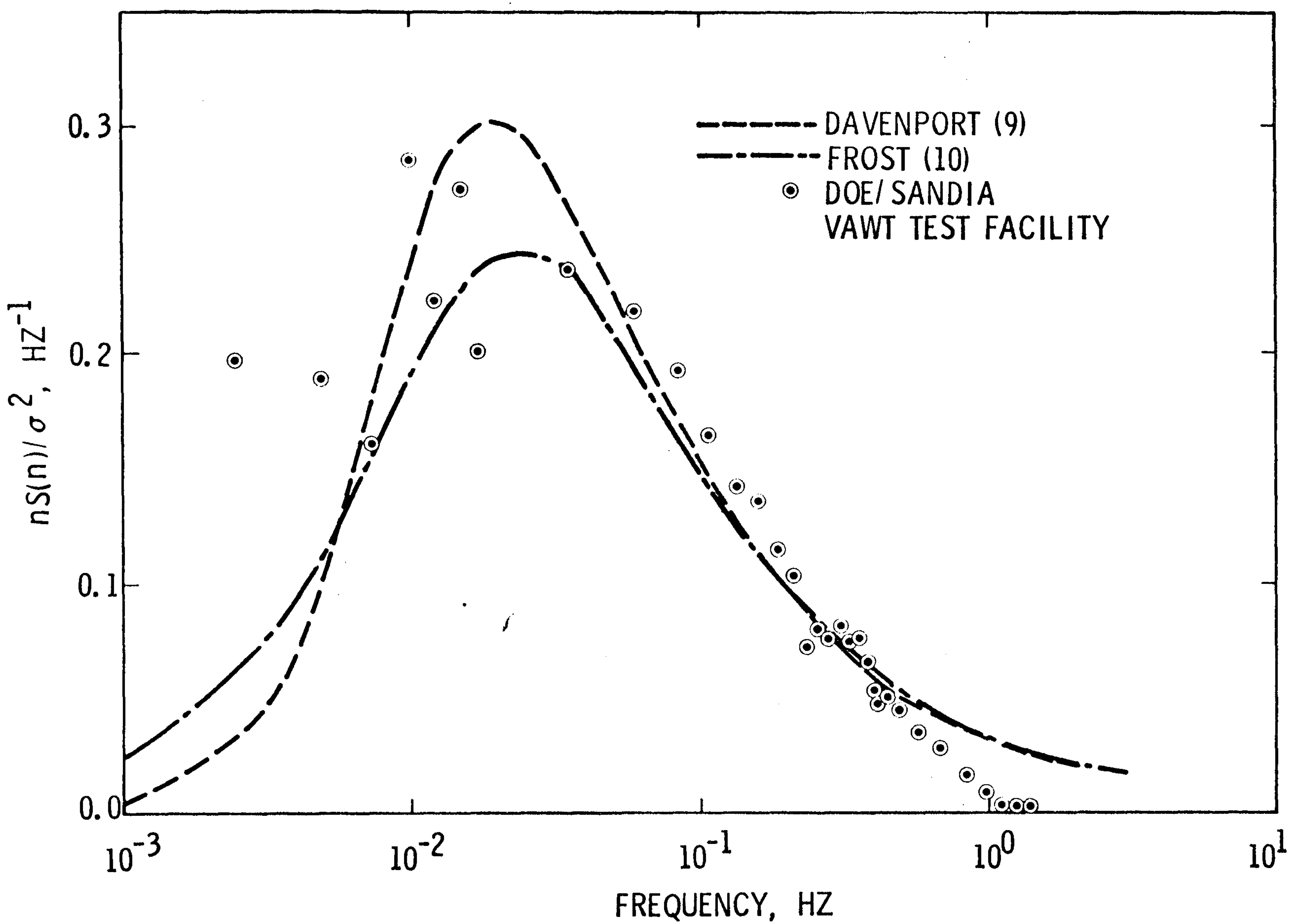

FIGURE 8
Longitudinal Wind Speed Spectral Density

Measurements of the longitudinal integral scale at a height 30 m above the ground ranged from 60 m to 120 m. In all cases, the longitudinal integral scale was larger than the diameter of the 17 M turbine. These values of integral scale were obtained both from the zero intercept of the spectral density and from the integral of the auto-correlation function to the first zero crossing. A detailed explanation of these techniques can be found in Hinze (12).

Two cross-correlations were selected as being representative of situations of particular interest. The first case is the cross-correlation between the wind speed above a turbine and the wind speed at the centerline of the turbine. The dimensions of the 17 M turbine were chosen to define this vertical separation distance. Anemometers located at 30 m and 13 m on the meteorological tower were used to measure the cross-correlation between wind speed at these two heights. In the performance evaluation of the 17 M turbine, a reference wind speed is measured at a height of either 33 m or 29 m. This value is corrected to a height of 13 m, the centerline of the turbine. This correction is accomplished using the power-law expression [Eq. (2)] assuming a cross-correlation coefficient of 1.0 and a zero time delay. The actual cross-correlation measured is shown in Fig. 9. The data shown in this figure are an average of five measurements. Two important aspects of this cross-correlation are the fact that its value is less than 1.0 at a lag time of 0.0 and that the peak cross-correlation occurs at a time lag of 0.0. These properties of the cross-correlation indicate that an instantaneous shear correction is appropriate but not exact.

The cross-correlation between the meteorological tower and the 17 M test location at a height of 30 m is shown in Fig. 10. These data are an average of three cases for northwest wind approximately perpendicular to a line connecting the two measurement locations. The data show very little correlation between the wind speed fluctuations at the two locations.

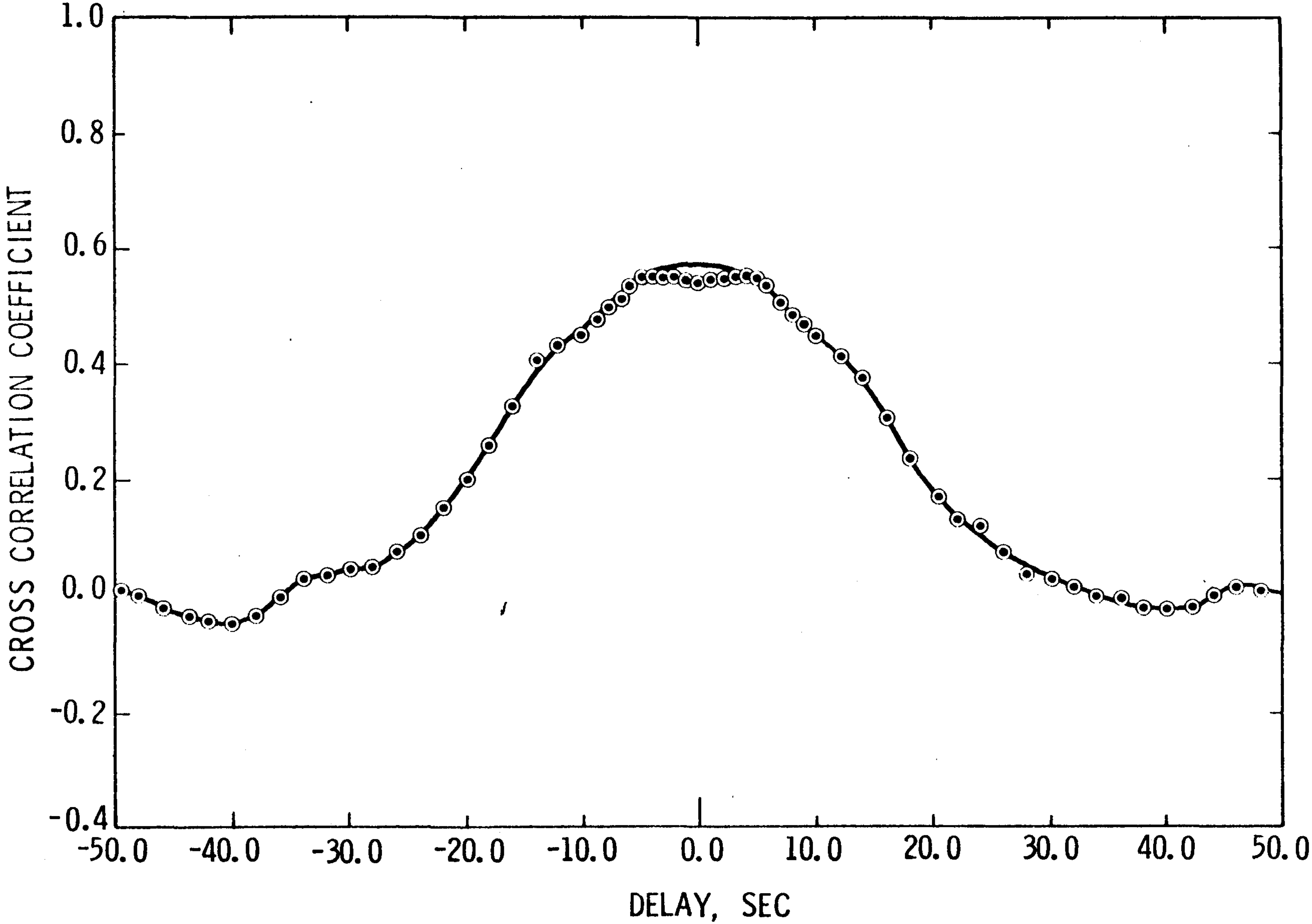

FIGURE 9
Cross-Correlation 30 m and 13 m

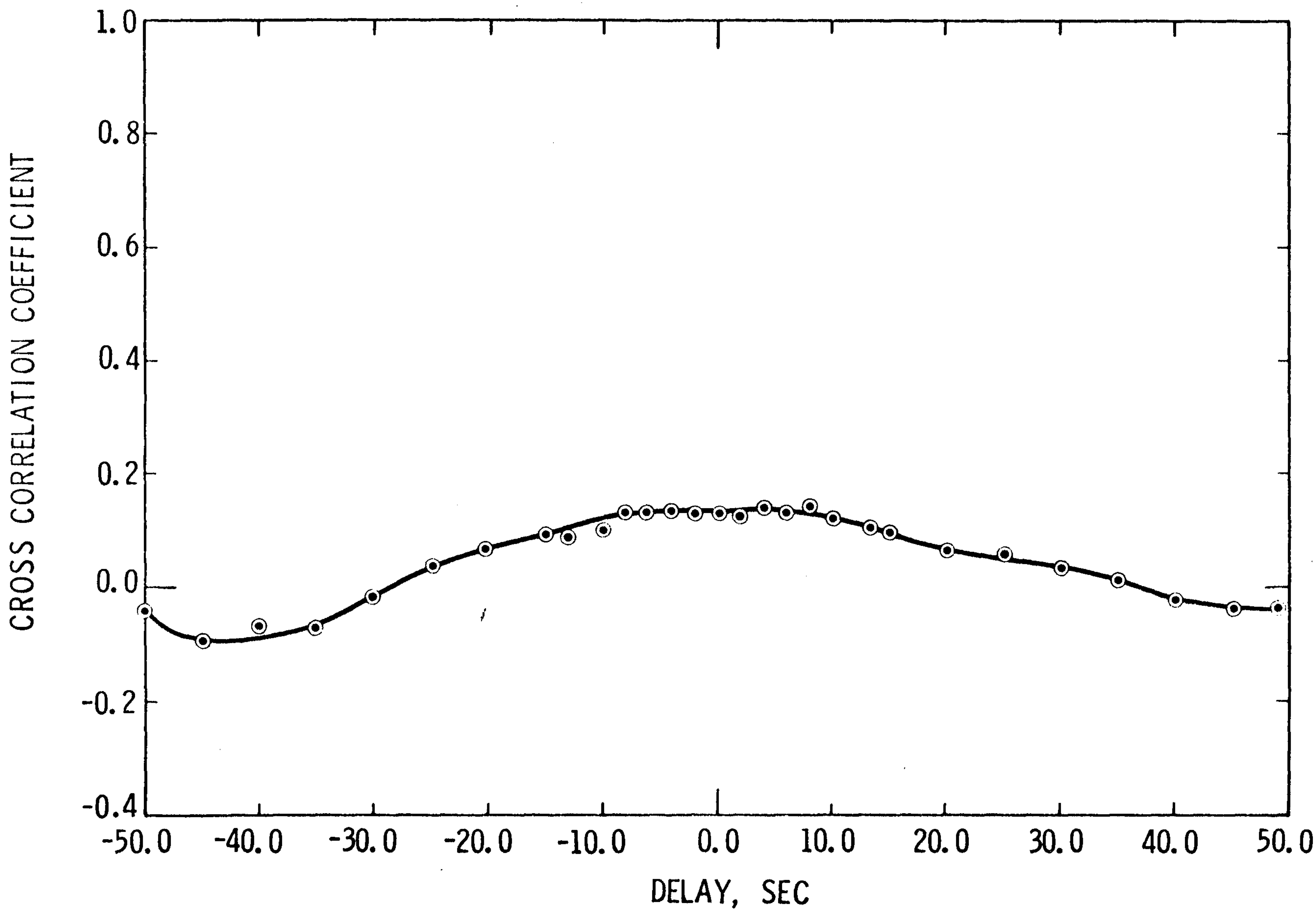

FIGURE 10
Cross-Correlation 83 m Horizontal Separation

VI. Summary

A limited program to determine the micrometeorological characteristics of the DOE/Sandia vertical axis wind turbine test facility allows the following statements to be made.

1. The power-law exponent in the lower 30 m at the test facility is 0.10.
2. Incident turbulence intensity at an elevation of 13 m (centerline of the 17 M turbine) ranges from 14 to 25%.
3. The longitudinal integral scale of the wind speed fluctuations at an elevation of 30 m ranges from 60 m to 120 m.
4. The average surface roughness length is 1 cm.
5. The cross-correlation of wind speed for a vertical separation of 17 m is less than 1.0 at a zero time delay. The maximum value of cross-correlation for a vertical separation of up to 17 m occurs at a zero time delay.

Appendix A

Averaging Techniques for Wind Velocity

The instrumentation at the test facility provides an instantaneous measure of wind speed and wind direction. In order to properly average these quantities over a period of time, it is necessary to convert these values into two vector components and average these components. At the end of a period, the time-averaged components may be combined to provide a resultant wind speed and direction as well as the alongwind and acrosswind turbulence intensities.

The coordinate system used in these calculations is shown in Fig. A1. The instrument output will provide the magnitude of the vector, W, and a direction, D, from which the wind blows, measured clockwise from north. This vector can be resolved into a north-south component, v, and an east-west component, u, according to the following relationships:

$$u = W\cos\theta \tag{A1}$$

$$v = W\sin\theta \tag{A2}$$

but

$$\begin{aligned} D &= 90 - \theta + 180 \\ &= 270 - \theta \end{aligned} \tag{A3}$$

$$\begin{aligned} \sin D &= \sin(270 - \theta) \\ &= \sin(270)\cos\theta - \cos(270)\sin\theta \\ &= (-1)\cos\theta - (0)\sin\theta \\ &= -\cos\theta \quad . \end{aligned} \tag{A4}$$

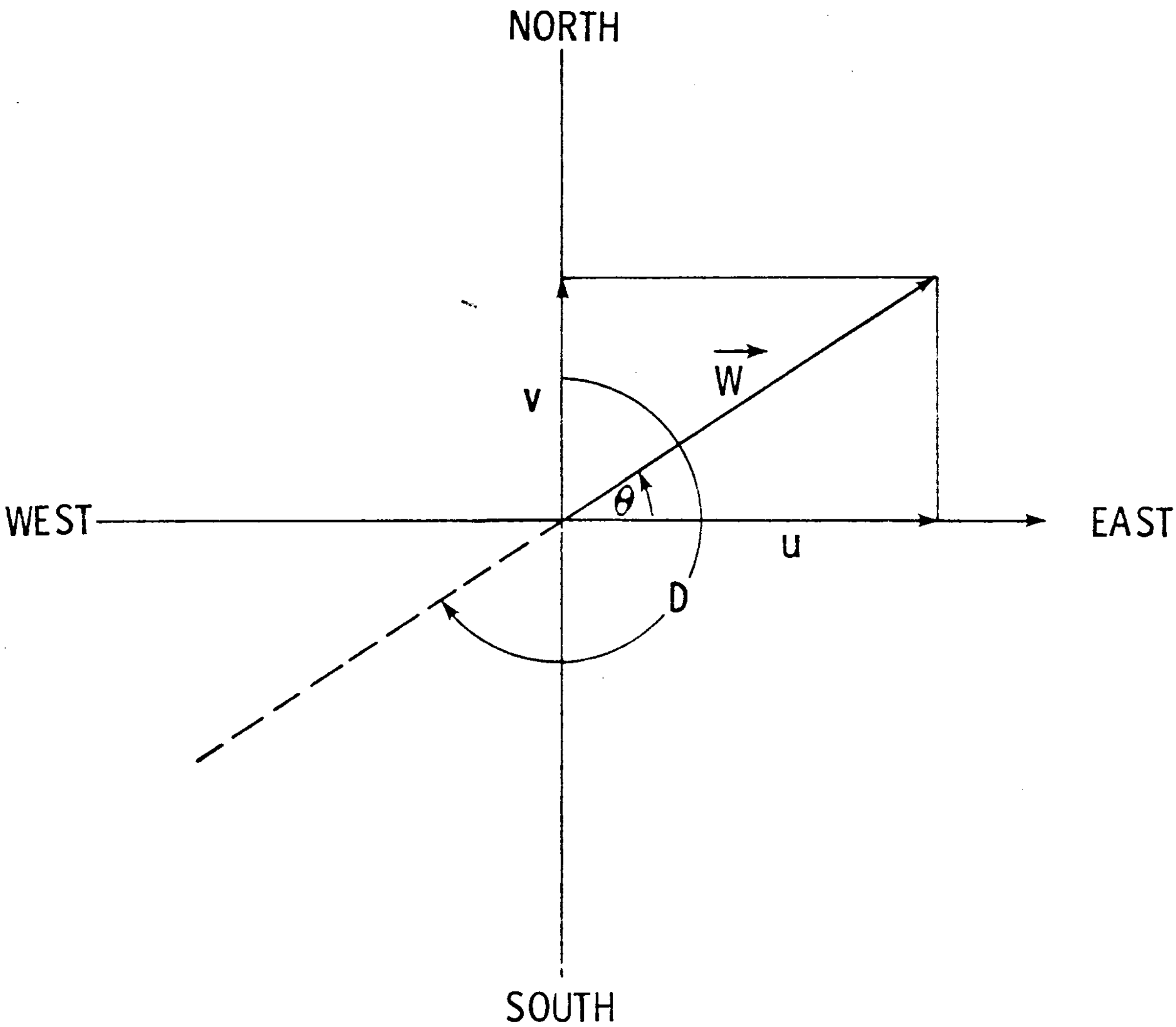

FIGURE A1
Coordinate System Wind Velocity Averaging

$$\begin{aligned} \cos D &= \cos(270 - \theta) \\ &= \cos(270)\cos\theta + \sin(270)\sin\theta \\ &= (0)\cos\theta + (-1)\sin\theta \\ &= -\sin\theta \quad . \end{aligned} \tag{A5}$$

Therefore, in terms of the measured wind direction, D,

$$u = -W \sin D \tag{A6}$$

$$v = -W \cos D \quad . \tag{A7}$$

These components consist of a mean and a fluctuating portion,

$$u = \overline{u} + u' \tag{A8}$$

$$v = \overline{v} + v' \tag{A9}$$

where the overbar indicates time averaging. Note that $\overline{u'}$ and $\overline{v'} = 0$ by definition. After a period of data collection, five quantities can be computed:

$$\overline{u},\ \overline{u^2},\ \overline{v},\ \overline{v^2},\ \overline{uv} \quad .$$

From these values, the variance of u and v can be computed.

$$\overline{u'^2} = \overline{u^2} - \overline{u}^2 \tag{A10}$$

$$\overline{v'^2} = \overline{v^2} - \overline{v}^2 \quad . \tag{A11}$$

Similar expressions exist for the covariance

$$\overline{u'v'} = \overline{uv} - (\overline{u})\ (\overline{v}) \tag{A12}$$

and the cross-correlation coefficient

$$\rho_{uv} = \overline{u'v'} \Big/ \left[\overline{u'^2} \; \overline{v'^2} \right]^{\frac{1}{2}} \quad . \qquad (A13)$$

With these quantities known, the resultant wind speed and direction may be computed

$$W_R = (\overline{u}^{\,2} + \overline{v}^{\,2})^{\frac{1}{2}} \qquad (A14)$$

$$D_R = \tan^{-1} \frac{\overline{u}}{\overline{v}} \quad . \qquad (A15)$$

In order to calculate the alongwind variance (w') and the acrosswind variance (c'), the following equations are required;

$$w' = u'\cos\theta + v'\sin\theta \qquad (A16)$$

$$c' = u'\sin\theta - v'\cos\theta \quad . \qquad (A17)$$

From Eqs. (A4), (A5)

$$w' = -u'\sin D_R - v'\cos D_R$$

$$c' = -u'\cos D_R + v'\sin D_R$$

$$w'^2 = (u'\sin D_R)^2 + 2u'v'\sin D_R \cos D_R + (v'\cos D_R)^2$$

$$c'^2 = (u'\cos D_R)^2 - 2u'v'\sin D_R \cos D_R + (v'\sin D_R)^2 \quad .$$

Taking the time average of these two equations

$$\overline{w'^2} = \overline{u'^2}\, \sin^2 D_R + 2\, \overline{u'v'} \sin D_R \cos D_R + \overline{v'^2}\, \cos^2 D_R \tag{A18}$$

$$\overline{c'^2} = \overline{u'^2}\, \cos^2 D_R - 2\, \overline{u'v'} \sin D_R \cos D_R + \overline{v'^2}\, \sin^2 D_R \quad . \tag{A19}$$

From Fig. A1

$$\cos D_R = \overline{-v}/W_R$$

$$\sin D_R = \overline{-u}/W_R$$

therefore,

$$\overline{w'^2} = \frac{\overline{u'^2}\,\bar{u}^2}{W_R^2} + \frac{\overline{v'^2}\,\bar{v}^2}{W_R^2} + \frac{2\left(\overline{u'v'}\right)(\bar{u})(\bar{v})}{W_R^2} \tag{A20}$$

and

$$\overline{c'^2} = \frac{\overline{u'^2}\,\bar{v}^2}{W_R^2} + \frac{\overline{v'^2}\,\bar{u}^2}{W_R^2} - \frac{2\,\overline{u'v'}\,\bar{u}\,\bar{v}}{W_R^2} \tag{A21}$$

The alongwind turbulence intensity defined by

$$I_w = \frac{\left(\overline{w'^2}\right)^{\frac{1}{2}}}{W_R}$$

is equal to

$$I_w = \frac{\left[\left(\overline{u'^2}\right)\left(\bar{u}\right)^2 + \left(\overline{v'^2}\right)\left(\bar{v}\right)^2 + 2\left(\overline{u'v'}\right)\left(\bar{u}\right)\left(\bar{v}\right)\right]^{\frac{1}{2}}}{W_R^2} \quad . \tag{A22}$$

The acrosswind turbulence intensity defined by

$$I_c = \frac{\left(\overline{c'^2}\right)^{\frac{1}{2}}}{W_R} \quad . \tag{A23}$$

is equal to

$$I_c = \frac{\left[\left(\overline{u'^2}\right)\left(\bar{v}\right)^2 + \left(\overline{v'^2}\right)\left(\bar{u}\right)^2 - 2\left(\overline{u'v'}\right)\left(\bar{u}\right)\left(\bar{v}\right)\right]^{\frac{1}{2}}}{W_R^2} \quad . \tag{A23}$$

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