Additional Results: Adding Dynamics to the Model

In the model presented in the main paper, we assumed that the common and idiosyncratic components of measured GDP and GDI were serially uncorrelated. These simplifying assumptions allowed us to make the main points of the paper in as clear and uncluttered a framework as possible, but as table 6 shows, some autocorrelation is present in the data.[1] In this additional appendix we relax the simplifying assumptions. We consider first the pure noise model, then the pure news model, and finally the general mixed news and noise model. We derive intuitive generalizations to equations (4), (5) and (3) in the main paper when dynamics are confined to the common component of the estimates. We have been unable to derive similarly intuitive expressions when dynamics govern the idiosyncratic components of the estimates as well as the common component, but the appendices show how to employ the Kalman filter to produce estimates of "true" unobserved GDP growth under these circumstances.

We first express the static version of the pure noise model in a state space framework, which is popular for modelling unobserved variables, and provides a convenient point of departure for including dynamics, as in Howrey (2003). The static version of the pure noise model, with the assumption that $E\left(\Delta y_t^{\star}|\mathcal{F}_t^k\right) = \Delta y_t^{\star}$, posits that:

$$\begin{aligned}\Delta y_t^1 &= \Delta y_t^{\star} + \varepsilon_t^1, \quad \text{and:} \\ \Delta y_t^2 &= \Delta y_t^{\star} + \varepsilon_t^2.\end{aligned}$$

Our other distributional assumptions can be summarized as:

$$\begin{bmatrix} \Delta y_t^\star - \mu \\ \varepsilon_t^1 \\ \varepsilon_t^2 \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \tau_1^2 & 0 \\ 0 & 0 & \tau_2^2 \end{bmatrix} \right).$$

Then one state space representation of this model is as follows:

$$\xi_t = (\Delta y_t^\star - \mu) \qquad \text{(state equation)}$$

$$\begin{pmatrix} \Delta y_t^1 \\ \Delta y_t^2 \end{pmatrix} = \begin{pmatrix} \mu \\ \mu \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} \xi_t + \begin{pmatrix} \varepsilon_t^1 \\ \varepsilon_t^2 \end{pmatrix} \qquad \text{(observation equations).}$$

The estimate of the state variable at time t conditional on the observed time t estimates, $\widehat{\xi}_{t|t}$, can be computed from standard Kalman filter formulas - see, for example, Hamilton (1994), and $\widehat{\xi}_{t|t} = E(\Delta y_t^\star - \mu | \mathcal{H}_t) = \widehat{\Delta y_t^\star} - \mu$ yields the same result as equation (4) in the main paper. Here we define $\mathcal{H}_t = \{1, \Delta y_t^1, \Delta y_{t-1}^1, \ldots, \Delta y_t^2, \Delta y_{t-1}^2, \ldots\}$, simply the history of the two observed estimates plus a constant. The next proposition shows how equation (4) changes when $\Delta y_t^\star - \mu$ follows any covariance-stationary process:

Proposition 1 *Let $(\Delta y_t^\star - \mu)$ follow any ARMA(p,q) process, so:*

$$\begin{aligned} \Delta y_t^\star - \mu &= \phi_1 \left(\Delta y_{t-1}^\star - \mu\right) + \phi_2 \left(\Delta y_{t-2}^\star - \mu\right) + \ldots + \phi_p \left(\Delta y_{t-p}^\star - \mu\right) \\ &\quad + \nu_t + \theta_1 \nu_{t-1} + \ldots + \theta_q \nu_{t-q}, \end{aligned}$$

with the ν_{t-q} white noise innovations, and let the noise model assumptions govern

$[\Delta y_t^1 \quad \Delta y_t^2]$, *with* ε_t^1 *and* ε_t^2 *uncorrelated with* ν_t *at all leads and lags. Then:*

$$E\left(\Delta y_t^{\star}|\mathcal{H}_t\right) = \frac{\tau_2^2 \Delta y_t^1 + \tau_1^2 \Delta y_t^2 + \frac{\tau_1^2 \tau_2^2}{\text{var}(\Delta y_t^{\star}|\mathcal{H}_{t-1})} E\left(\Delta y_t^{\star}|\mathcal{H}_{t-1}\right)}{\tau_1^2 + \tau_2^2 + \frac{\tau_1^2 \tau_2^2}{\text{var}(\Delta y_t^{\star}|\mathcal{H}_{t-1})}}.$$

Proof: See Appendix A1.

This equation is identical to the static formula for $\widehat{\Delta y_t^{\star}}$ in the main paper (inclusive of the mean μ), the only differences being the time-varying mean $E\left(\Delta y_t^{\star}|\mathcal{H}_{t-1}\right)$ replaces μ and $\text{var}\left(\Delta y_t^{\star}|\mathcal{H}_{t-1}\right)$ replaces σ^2. This new variance will converge to a steady state value,[2] so the weights in this formula are time invariant in the steady state.

Appendix A1 gives the state-space representation of models where dynamics govern ε_t^1 and ε_t^2 as well as $\Delta y_t^{\star}$, and the Kalman filter algorithms for computing $\widehat{\xi}_{t|t}$ and $E\left(\Delta y_t^{\star}|\mathcal{H}_t\right)$. In our data we find it necessary to fit dynamics to the idiosyncratic components, and we employ such a model.

We next consider the pure news model. One way to motivate the static model in the paper is to assume that only contemporaneous, time t information is useful in estimating $\Delta y_t^{\star}$. However even in this case where lagged variables are useful for estimating $\Delta y_t^{\star}$, adding dynamics to the news model will be appropriate only if the conditional expectations ignore the information content in the lagged variables. To solidify concepts, assume that $\Delta y_t^{\star}$ is correlated with variables in the lagged information sets $\mathcal{F}_{t-1}^1, \mathcal{F}_{t-1}^2, \mathcal{F}_{t-2}^1, \mathcal{F}_{t-2}^2, \ldots$, as well as variables in the contemporaneous $\mathcal{F}_t^1$ and $\mathcal{F}_t^2$. Now, if:

$$\begin{aligned} \Delta y_t^1 &= E\left(\Delta y_t^{\star}|\mathcal{F}_t^1, \mathcal{F}_{t-1}^1, \mathcal{F}_{t-1}^2, \mathcal{F}_{t-2}^1, \mathcal{F}_{t-2}^2, \ldots\right), \quad \text{and:} \\ \Delta y_t^2 &= E\left(\Delta y_t^{\star}|\mathcal{F}_t^2, \mathcal{F}_{t-1}^1, \mathcal{F}_{t-1}^2, \mathcal{F}_{t-2}^1, \mathcal{F}_{t-2}^2, \ldots\right), \end{aligned}$$

the lagged information is already in the estimates, appearing as part the information common to the two estimates.[3] Adding dynamics to the news model would be redundant and inappropriate if the conditional expectations have been formed in this way. On the other hand, if no lagged information is employed in the construction of the conditional expectations, even though it is useful in predicting $\Delta y_t^\star$, so:

$$\begin{aligned} \Delta y_t^1 &= E\left(\Delta y_t^\star | \mathcal{F}_t^1\right), \quad \text{and:} \\ \Delta y_t^2 &= E\left(\Delta y_t^\star | \mathcal{F}_t^2\right), \end{aligned}$$

then adding a dynamic component to the estimates may improve the accuracy of $\widehat{\Delta y_t^\star}$. It is not clear which set of assumptions is closer to the truth, but for hueristic purposes, we study this second case, where no lagged information is employed in the construction of the conditional expectations even through it is relevant for predicting $\Delta y_t^\star$.

Before adding dynamic components to the pure news model, we first rewrite its static version, decomposing the two efficient estimates in the following way:

$$\begin{aligned} \Delta y_t^1 &= \eta_t + \eta_t^1, \quad \text{and:} \\ \Delta y_t^2 &= \eta_t + \eta_t^2, \quad \text{with:} \end{aligned}$$

$$\begin{bmatrix} \eta_t - \mu \\ \eta_t^1 \\ \eta_t^2 \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \tau_1^2 & 0 \\ 0 & 0 & \tau_2^2 \end{bmatrix} \right).$$

Each estimate is the sum of a common component (the overlap in the information sets, essentially), and an idiosyncratic component orthogonal to the common component η_t and the other idiosyncratic component. As noted in the text of the main paper, we assume the variance of each individual estimate is larger than the covariance between the two, but if this condition holds, this decomposition is not restrictive: the model estimates three variance parameters (σ^2, τ_1^2, and τ_2^2) from a variance-covariance matrix consisting of three moments.

Using this decomposition, appendix A2 shows how to write the static pure news model in state space form and compute $\widehat{\Delta y_t^\star}$ from the estimated state variables; this $\widehat{\Delta y_t^\star}$ coincides with equation (5) in the main paper. Using these results as a jumping off point, the appendix then derives dynamic analogs to this static estimator. If the common component η_t follows an arbitrary ARMA process, with the idiosyncratic η_t^1 and η_t^2 remaining white noise, this analogous dynamic estimate is:

$$E\left(\Delta y_t^\star|\mathcal{H}_t\right) = \frac{\left(\tau_1^2+\frac{\tau_1^2\tau_2^2}{\text{var}(\eta_t|\mathcal{H}_{t-1})}\right)\Delta y_t^1+\left(\tau_2^2+\frac{\tau_1^2\tau_2^2}{\text{var}(\eta_t|\mathcal{H}_{t-1})}\right)\Delta y_t^2-\left(\frac{\tau_1^2\tau_2^2}{\text{var}(\eta_t|\mathcal{H}_{t-1})}\right)E(\eta_t|\mathcal{H}_{t-1})}{\tau_1^2+\tau_2^2+\frac{\tau_1^2\tau_2^2}{\text{var}(\eta_t|\mathcal{H}_{t-1})}}.$$

With $E\left(\eta_t|\mathcal{H}_{t-1}\right)$ replacing μ and the var $\left(\eta_t|\mathcal{H}_{t-1}\right)$ replacing σ^2, this formula is identical to the $\widehat{\Delta y_t^\star}$ estimate in the main text given by (5). Appendix A2 also shows how to estimate $\Delta y_t^\star$ when dynamics govern η_t^1 and η_t^2 as well as η_t; the state space representation of this model is identical to the representation of the pure noise model with dynamics in the idiosyncratic and common components.

Finally consider the mixed news and noise model. Decomposing the efficient estimates as before, we have:

$$\begin{aligned}\Delta y_t^1 &= \eta_t + \eta_t^1 + \varepsilon_t^1, \quad \text{and} \\ \Delta y_t^2 &= \eta_t + \eta_t^2 + \varepsilon_t^2.\end{aligned}$$

For the distributions of the relevant variables, we have:

$$\begin{bmatrix} \eta_t - \mu \\ \eta_t^1 \\ \eta_t^2 \\ \varepsilon_t^1 \\ \varepsilon_t^2 \end{bmatrix} \sim N \left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & 0 & 0 & 0 & 0 \\ 0 & \chi_1 \tau_1^2 & 0 & 0 & 0 \\ 0 & 0 & \chi_2 \tau_2^2 & 0 & 0 \\ 0 & 0 & 0 & (1-\chi_1)\tau_1^2 & 0 \\ 0 & 0 & 0 & 0 & (1-\chi_2)\tau_2^2 \end{bmatrix} \right).$$

Modelling dynamics as in the news model (reiterating caveats about whether it is appropriate to do so), we again find an intuitive generalization of the appropriate static formula, equation (3) from the main paper in the case, when we take η_t to be an arbitrary ARMA process with the idiosyncratic components remaining white noise. Appendix A3 derives this formula:

$$\begin{aligned} E\left(\Delta y_t^\star | \mathcal{H}_t\right) &= \frac{\left(\chi_1 \tau_1^2 + (1-\chi_2)\tau_2^2 + \chi_1 \frac{\tau_1^2 \tau_2^2}{\mathrm{var}(\eta_t|\mathcal{H}_{t-1})}\right) \Delta y_t^1}{\tau_1^2 + \tau_2^2 + \frac{\tau_1^2 \tau_2^2}{\mathrm{var}(\eta_t|\mathcal{H}_{t-1})}} \\ &+ \frac{\left(\chi_2 \tau_2^2 + (1-\chi_1)\tau_1^2 + \chi_2 \frac{\tau_1^2 \tau_2^2}{\mathrm{var}(\eta_t|\mathcal{H}_{t-1})}\right) \Delta y_t^2}{\tau_1^2 + \tau_2^2 + \frac{\tau_1^2 \tau_2^2}{\mathrm{var}(\eta_t|\mathcal{H}_{t-1})}} \\ &+ \frac{(1-\chi_1-\chi_2)\left(\frac{\tau_1^2 \tau_2^2}{\mathrm{var}(\eta_t|\mathcal{H}_{t-1})}\right) E\left(\eta_t|\mathcal{H}_{t-1}\right)}{\tau_1^2 + \tau_2^2 + \frac{\tau_1^2 \tau_2^2}{\mathrm{var}(\eta_t|\mathcal{H}_{t-1})}}. \end{aligned}$$

Further dynamics could be added to the idiosyncratic components in the mixture model, as before in the pure news and pure noise models.

Dynamic Estimation Results

Table 7 reports estimates of dynamic versions of the pure news and pure noise models estimated using latest available data. To fit the first few autocorrelations of GDP growth and GDI growth, we found it necessary to allow dynamics in both the component common to the two series and their idiosyncratic components; specifically, the idiosyncratic component of GDP exhibited some negative serial correlation. Fitting AR1 processes to these three components, a particular case of (A9) in Appendix A, produced a good fit; in addition we allowed for the 1984Q3 break in μ and σ^2. The botton panel of table 7 reports autocorrelations and partial autocorrelations to innovations to the common component (labelled Δy_t^c) and the two idiosyncratic components (Δy_t^{i1} and Δy_t^{i2}), computed as the appropriate elements of $\widehat{\xi_{t|t}} - \widehat{\xi_{t|t-1}}$. We see little residual autocorrelation. The top panel reports estimated parameters, where we see the estimated positive autocorrelation of the common component and negative autocorrelation of the idiosyncratic component for GDP, and the middle panel reports the variance of the predicted values for "true" GDP growth and weights on current and lagged GDP and GDI for each model and sub-period.[4] The weights on current GDP and GDI are not so dissimilar to those reported in table 5 of the main paper. The weights on the lags for the news model are evidently equal to minus the weights on the lags for the noise model, a property that carries over from more restrictive models where dynamics are confined to the common component; in these more restrictive models this property can be seen quite clearly in the formulas.

Appendix A1: Proof of the Proposition, and Further Results on the Dynamic Pure Noise Model

Hamilton (1994) shows how to write an ARMA(p,q) process in a state-space representation. Defining $r = \max(p, q{+}1)$ and using those results in Hamilton, the state-space representation of the noise model described in the Proposition is:

$$\text{(A1)} \qquad \xi_t = \begin{bmatrix} \phi_1 & \phi_2 & \dots & \phi_{r-1} & \phi_r \\ 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & 1 & 0 & 0 \\ 0 & \dots & 0 & 1 & 0 \end{bmatrix} \xi_{t-1} + \begin{pmatrix} \nu_t \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \Delta y_t^1 \\ \Delta y_t^2 \end{pmatrix} = \begin{pmatrix} \mu \\ \mu \end{pmatrix} + \begin{bmatrix} 1 & \theta_1 & \theta_2 & \dots & \theta_{r-1} \\ 1 & \theta_1 & \theta_2 & \dots & \theta_{r-1} \end{bmatrix} \xi_t + \begin{pmatrix} \varepsilon_t^1 \\ \varepsilon_t^2 \end{pmatrix}.$$

Following the notation in Hamilton (1994), define:

$$R = \begin{bmatrix} \tau_1^2 & 0 \\ 0 & \tau_2^2 \end{bmatrix} \quad \text{and:} \quad H' = \begin{bmatrix} 1 & \theta_1 & \theta_2 & \dots & \theta_{r-1} \\ 1 & \theta_1 & \theta_2 & \dots & \theta_{r-1} \end{bmatrix}$$

$$\text{(A2)} \qquad = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & \theta_1 & \theta_2 & \dots & \theta_{r-1} \end{bmatrix}.$$

We have:

$$\text{(A3)} \qquad E\left(\Delta y_t^\star | \mathcal{H}_{t-1}\right) = \begin{bmatrix} 1 & \theta_1 & \theta_2 & \dots & \theta_{r-1} \end{bmatrix} \widehat{\xi_{t|t-1}},$$

and with:

$$P_{t|t-1} = E\left[\left(\xi_t - \widehat{\xi_{t|t-1}}\right)\left(\xi_t - \widehat{\xi_{t|t-1}}\right)'\right], \tag{A4}$$

we have:

$$\operatorname{var}(\Delta y_t^\star|\mathcal{H}_{t-1}) = \begin{bmatrix} 1 & \theta_1 & \theta_2 & \dots & \theta_{r-1} \end{bmatrix} P_{t|t-1} \begin{bmatrix} 1 \\ \theta_1 \\ \theta_2 \\ \dots \\ \theta_{r-1} \end{bmatrix}. \tag{A5}$$

The formula for updating $E(\Delta y_t^\star|\mathcal{H}_{t-1})$ with respect to time t information is (again see, for example, Hamilton (1994)):

$$\begin{aligned} E(\Delta y_t^\star|\mathcal{H}_t) &= E(\Delta y_t^\star|\mathcal{H}_{t-1}) \\ &+ \begin{bmatrix} 1 & \theta_1 & \theta_2 & \dots & \theta_{r-1} \end{bmatrix} P_{t|t-1} H \left(H' P_{t|t-1} H + R\right)^{-1} \begin{pmatrix} \Delta y_t^1 - E(\Delta y_t^\star|\mathcal{H}_{t-1}) \\ \Delta y_t^2 - E(\Delta y_t^\star|\mathcal{H}_{t-1}) \end{pmatrix}. \end{aligned} \tag{A6}$$

Now, given (A2) and (A5), we have:

$$\begin{aligned} H' P_{t|t-1} H &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \operatorname{var}(\Delta y_t^\star|\mathcal{H}_{t-1}) \begin{bmatrix} 1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \operatorname{var}(\Delta y_t^\star|\mathcal{H}_{t-1}) & \operatorname{var}(\Delta y_t^\star|\mathcal{H}_{t-1}) \\ \operatorname{var}(\Delta y_t^\star|\mathcal{H}_{t-1}) & \operatorname{var}(\Delta y_t^\star|\mathcal{H}_{t-1}) \end{bmatrix}, \end{aligned}$$

so:

$$\left(H' P_{t|t-1} H + R\right)^{-1} = \frac{1}{\operatorname{var}(\Delta y_t^\star|\mathcal{H}_{t-1})(\tau_1^2 + \tau_2^2) + \tau_1^2 \tau_2^2} \begin{bmatrix} \operatorname{var}(\Delta y_t^\star|\mathcal{H}_{t-1}) + \tau_2^2 & -\operatorname{var}(\Delta y_t^\star|\mathcal{H}_{t-1}) \\ -\operatorname{var}(\Delta y_t^\star|\mathcal{H}_{t-1}) & \operatorname{var}(\Delta y_t^\star|\mathcal{H}_{t-1}) + \tau_1^2 \end{bmatrix}.$$

Similarly, we have:

$$\begin{aligned} \left[\begin{array}{ccccc} 1 & \theta_1 & \theta_2 & \ldots & \theta_{r-1} \end{array} \right] P_{t|t-1} H &= \text{var}\left(\Delta y_t^\star | \mathcal{H}_{t-1}\right) \left[\begin{array}{cc} 1 & 1 \end{array} \right] \\ &= \left[\begin{array}{cc} \text{var}\left(\Delta y_t^\star | \mathcal{H}_{t-1}\right) & \text{var}\left(\Delta y_t^\star | \mathcal{H}_{t-1}\right) \end{array} \right]. \end{aligned}$$

Substituting these expressions into (A6), we have:

$$E\left(\Delta y_t^\star | \mathcal{H}_t\right) = E\left(\Delta y_t^\star | \mathcal{H}_{t-1}\right) + \frac{\text{var}\left(\Delta y_t^\star | \mathcal{H}_{t-1}\right)}{\text{var}\left(\Delta y_t^\star | \mathcal{H}_{t-1}\right)\left(\tau_1^2 + \tau_2^2\right) + \tau_1^2 \tau_2^2} \left[\begin{array}{cc} \tau_2^2 & \tau_1^2 \end{array} \right] \left(\begin{array}{c} \Delta y_t^1 - E\left(\Delta y_t^\star | \mathcal{H}_{t-1}\right) \\ \Delta y_t^2 - E\left(\Delta y_t^\star | \mathcal{H}_{t-1}\right) \end{array} \right)$$

Rearranging produces the result reported.

When adding dynamics to $[\varepsilon_t^1 \quad \varepsilon_t^2]$, we continue to assume that the innovations to these variables are mutually orthogonal at all leads and lags, and also orthogonal to the innovations to $\Delta y_t^\star - \mu$ at all leads and lags. With $\Delta y_t^\star - \mu$ as before, let:

$$\text{(A7)} \qquad \begin{aligned} \varepsilon_t^1 &= \phi_1^1 \varepsilon_{t-1}^1 + \phi_2^1 \varepsilon_{t-1}^1 + \ldots + \phi_{p^1}^1 \varepsilon_{t-p^1}^1 \\ &\quad + \nu_t^1 + \theta_1^1 \nu_{t-1}^1 + \ldots + \theta_{q^1}^1 \nu_{t-q^1}^1, \end{aligned}$$

and:

$$\text{(A8)} \qquad \begin{aligned} \varepsilon_t^2 &= \phi_1^2 \varepsilon_{t-1}^2 + \phi_2^2 \varepsilon_{t-1}^2 + \ldots + \phi_{p^2}^2 \varepsilon_{t-p^2}^2 \\ &\quad + \nu_t^2 + \theta_1^2 \nu_{t-1}^2 + \ldots + \theta_{q^2}^2 \nu_{t-q^2}^2. \end{aligned}$$

Define $r^1 = \max(p^1, q^1 + 1)$ and $r^2 = \max(p^2, q^2 + 1)$. The state space representation of this model is:

$$
\text{(A9)} \qquad \xi_t = \begin{bmatrix} \phi_1 & \dots & \phi_r & 0 & \dots & 0 & 0 & \dots & 0 \\ 1 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \dots & 1 & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & \dots & 0 & \phi_1^1 & \dots & \phi_{r^1}^1 & 0 & \dots & 0 \\ 0 & \dots & 0 & 1 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & 0 & \dots & 1 & 0 & 0 & \dots & 0 \\ 0 & \dots & 0 & 0 & \dots & 0 & \phi_1^2 & \dots & \phi_{r^2}^2 \\ 0 & \dots & 0 & 0 & \dots & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & 0 & 0 & \dots & 0 & \dots & 1 & 0 \end{bmatrix} \xi_{t-1} + \begin{pmatrix} \nu_t \\ 0 \\ \vdots \\ 0 \\ \nu_t^1 \\ 0 \\ \vdots \\ 0 \\ \nu_t^2 \\ 0 \\ \vdots \\ 0 \end{pmatrix}
$$

$$
\begin{pmatrix} \Delta y_t^1 \\ \Delta y_t^2 \end{pmatrix} = \begin{pmatrix} \mu \\ \mu \end{pmatrix} + \begin{bmatrix} 1 & \theta_1 & \dots & \theta_{r-1} & 1 & \theta_1^1 & \dots & \theta_{r^1-1}^1 & 0 & 0 & \dots & 0 \\ 1 & \theta_1 & \dots & \theta_{r-1} & 0 & 0 & \dots & 0 & 1 & \theta_1^2 & \dots & \theta_{r^2-1}^2 \end{bmatrix} \xi_t .
$$

The expected value of $\Delta y_t^\star$ is computed in the same way as before, however, setting the additional elements of ξ_t to zero:

$$
\text{(A10)} \qquad E\left(\Delta y_t^\star | \mathcal{H}_t\right) = \begin{bmatrix} 1 & \theta_1 & \dots & \theta_{r-1} & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \end{bmatrix} \widehat{\xi_{t|t}} .
$$

Appendix A2: Results on the Dynamic Pure News Model

To get an idea of how to proceed with the dynamics, we start by considering again the static model. With estimates decomposed as described, the static news model can be written in the same state-space form as the static noise model, with a different interpretation for ξ_t and the errors in the observation equations:

$$\xi_t = \eta_t - \mu \qquad \text{(state equation)}$$

$$\begin{pmatrix} \Delta y_t^1 \\ \Delta y_t^2 \end{pmatrix} = \begin{pmatrix} \mu \\ \mu \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} \xi_t + \begin{pmatrix} \eta_t^1 \\ \eta_t^2 \end{pmatrix} \qquad \text{(observation equations).}$$

But how do we compute $\widehat{\Delta y_t^\star}$ in this model? To justify the decompositon, we think of decomposing $\mathcal{F}_t^1$ into a set of variables representing its intersection with $\mathcal{F}_t^2$, $\mathcal{F}_t^C = \mathcal{F}_t^1 \cap \mathcal{F}_t^2$ where C stands for common, and a set of variables representing the remainder of the information, $\mathcal{F}_t^{1-C}$, where these variables are orthogonal to $\mathcal{F}_t^C$. Then we can write:

$$\begin{aligned} \Delta y_t^1 &= E\left(\Delta y_t^\star | \mathcal{F}_t^1\right) \\ &= E\left(\Delta y_t^\star | \mathcal{F}_t^C, \mathcal{F}_t^{1-C}\right) \\ &= E\left(\Delta y_t^\star | \mathcal{F}_t^C\right) + E\left(\Delta y_t^\star | \mathcal{F}_t^{1-C}\right) \\ &= \eta_t + \eta_t^1, \end{aligned}$$

using the orthogonality of the variables in each part of the information set and properties of linear conditional expectations, and defining η_t and η_t^1 as the conditional expectations in the second to last line. If we similarly decompose $\mathcal{F}_t^2$ into $\mathcal{F}_t^C$ and $\mathcal{F}_t^{2-C}$, we have:

$$\begin{aligned} \Delta y_t^2 &= E\left(\Delta y_t^\star | \mathcal{F}_t^C\right) + E\left(\Delta y_t^\star | \mathcal{F}_t^{2-C}\right) \\ &= \eta_t + \eta_t^2. \end{aligned}$$

The variables in $\mathcal{F}_t^{2-C}$ will be orthogonal to the variables in $\mathcal{F}_t^{1-C}$, since all the common information resides in $\mathcal{F}_t^C$. Then:

$$E\left(\Delta y_t^\star | \mathcal{F}_t^C, \mathcal{F}_t^{1-C}, \mathcal{F}_t^{2-C}\right) = \eta_t + \eta_t^1 + \eta_t^2.$$

Hence we conjecture that the best estimate of $\Delta y_t^\star$ is $E\left(\eta_t + \eta_t^1 + \eta_t^2 | \mathcal{H}_t\right) = \widehat{\eta_{t|t}} + \widehat{\eta_{t|t}^1} + \widehat{\eta_{t|t}^2}$, and this conjecture turns out to be correct. With the estimated $\widehat{\xi_{t|t}} = \widehat{\eta_{t|t}} - \mu$ given by the right hand side of equation (4) in the main paper, and the estimated idiosyncratic errors given by the difference between each estimate and the state variable (so $\widehat{\eta_{t|t}^k} = \Delta y_t^k - \mu - \widehat{\eta_{t|t}}$), we have:

$$\begin{aligned}
\widehat{\eta_{t|t}} + \widehat{\eta_{t|t}^1} + \widehat{\eta_{t|t}^2} &= \left(\Delta y_t^1 - \mu\right) + \left(\Delta y_t^2 - \mu\right) - \left(\widehat{\eta_{t|t}} - \mu\right) \\
&= \frac{\left(\tau_1^2 + \frac{\tau_1^2 \tau_2^2}{\sigma^2}\right)\left(\Delta y_t^1 - \mu\right) + \left(\tau_2^2 + \frac{\tau_1^2 \tau_2^2}{\sigma^2}\right)\left(\Delta y_t^2 - \mu\right)}{\tau_1^2 + \tau_2^2 + \frac{\tau_1^2 \tau_2^2}{\sigma^2}} \\
&= \widehat{\Delta y_t^\star} - \mu \quad \text{from equation (5) in the main paper.}
\end{aligned} \tag{A11}$$

With this result in mind, we proceed with the dynamics, first considering the case where the common factor $\eta_t - \mu$ follows an arbitrary ARMA process:

$$\begin{aligned}
\eta_t - \mu &= \phi_1\left(\eta_{t-1} - \mu\right) + \phi_2\left(\eta_{t-2} - \mu\right) + \ldots + \phi_p\left(\eta_{t-p} - \mu\right) \\
&\quad + \nu_t + \theta_1 \nu_{t-1} + \ldots + \theta_q \nu_{t-q},
\end{aligned}$$

If η_t^1 and η_t^2 remain white noise uncorrelated with each other and with ν_t at all leads and lags, it is clear from the Proposition that:

$$E\left(\eta_t | \mathcal{H}_t\right) = \frac{\tau_2^2 \Delta y_t^1 + \tau_1^2 \Delta y_t^2 + \frac{\tau_1^2 \tau_2^2}{\operatorname{var}(\eta_t | \mathcal{H}_{t-1})} E\left(\eta_t | \mathcal{H}_{t-1}\right)}{\tau_1^2 + \tau_2^2 + \frac{\tau_1^2 \tau_2^2}{\operatorname{var}(\eta_t | \mathcal{H}_{t-1})}}.$$

Given this formula, we perform the same manipulations as in (A11), computing $\widehat{\eta_{t|t}} + \widehat{\eta_{t|t}^1} + \widehat{\eta_{t|t}^2}$ to arrive at the reported result.

In the case where η_t, η_t^1, and η_t^2 each follow arbitrary ARMA processes, the state space form of the model is the same as in (A9). However $E\left(\Delta y_t^\star | \mathcal{H}_t\right) = \widehat{\eta_{t|t}} + \widehat{\eta_{t|t}^1} + \widehat{\eta_{t|t}^2}$, so:

$$\text{(A12)} \quad E\left(\Delta y_t^{\star}|\mathcal{H}_t\right) = \left[\begin{array}{ccccccccccccc} 1 & \theta_1 & \ldots & \theta_{r-1} & 1 & \theta_1^1 & \ldots & \theta_{r^1-1}^1 & 1 & \theta_1^2 & \ldots & \theta_{r^2-1}^2 \end{array}\right] \widehat{\xi_{t|t}}.$$

Appendix A3: Results on the Mixed News and Noise Model with Dynamics

The static version of this model can be written in state space form as:

$$\xi_t = \begin{pmatrix} \eta_t - \mu \\ \eta_t^1 \\ \eta_t^2 \end{pmatrix} \qquad \text{(state equations)}$$

$$\begin{pmatrix} \Delta y_t^1 \\ \Delta y_t^2 \end{pmatrix} = \begin{pmatrix} \mu \\ \mu \end{pmatrix} + \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \xi_t + \begin{pmatrix} \varepsilon_t^1 \\ \varepsilon_t^2 \end{pmatrix} \qquad \text{(observation equations).}$$

As before in the pure news model (see Appendix A2), the estimate of the true unobserved state of the economy is $\widehat{\eta_{t|t}} + \widehat{\eta_{t|t}^1} + \widehat{\eta_{t|t}^2} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \widehat{\xi}_{t|t}$, which produces the same $\widehat{\Delta y_t^{\star}}$ as equation (3) in the main paper. Taking η_t to be an arbitrary ARMA process with the idiosyncratic components remaining white noise, the mixed news and noise model with dynamics in η_t can be written in state space form as:

(A13)
$$\xi_t = \begin{bmatrix} \phi_1 & \phi_2 & \dots & \phi_{r-1} & \phi_r & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ 0 & \dots & 1 & 0 & 0 & 0 & 0 \\ 0 & \dots & 0 & 1 & 0 & 0 & 0 \\ 0 & \dots & 0 & 0 & 0 & 0 & 0 \\ 0 & \dots & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xi_{t-1} + \begin{pmatrix} \nu_t \\ 0 \\ \vdots \\ 0 \\ \eta_t^1 \\ \eta_t^2 \end{pmatrix}$$

$$\begin{pmatrix} \Delta y_t^1 \\ \Delta y_t^2 \end{pmatrix} = \begin{pmatrix} \mu \\ \mu \end{pmatrix} + \begin{bmatrix} 1 & \theta_1 & \theta_2 & \dots & \theta_{r-1} & 1 & 0 \\ 1 & \theta_1 & \theta_2 & \dots & \theta_{r-1} & 0 & 1 \end{bmatrix} \xi_t + \begin{pmatrix} \varepsilon_t^1 \\ \varepsilon_t^2 \end{pmatrix}.$$

The variance-covariance matrix of $[\eta_t^1 \quad \eta_t^2 \quad \varepsilon_t^1 \quad \varepsilon_t^2]$ is given by our distributional assumptions. As before in the pure noise model, define:

$$R = \begin{bmatrix} (1-\chi_1)\,\tau_1^2 & 0 \\ 0 & (1-\chi_2)\,\tau_2^2 \end{bmatrix} \quad \text{and:} \quad H' = \begin{bmatrix} 1 & \theta_1 & \theta_2 & \dots & \theta_{r-1} & 1 & 0 \\ 1 & \theta_1 & \theta_2 & \dots & \theta_{r-1} & 0 & 1 \end{bmatrix}.$$

The dynamic analog to the static estimator in the mixed model is:

(A14)
$$E\left(\Delta y_t^\star | \mathcal{H}_{t-1}\right) = \begin{bmatrix} 1 & \theta_1 & \theta_2 & \dots & \theta_{r-1} & 1 & 1 \end{bmatrix} \widehat{\xi_{t|t-1}},$$

and we have:

(A15)
$$\operatorname{var}\left(\eta_t | \mathcal{H}_{t-1}\right) = \begin{bmatrix} 1 & \theta_1 & \theta_2 & \dots & \theta_{r-1} \end{bmatrix} P_{t|t-1} \begin{bmatrix} 1 \\ \theta_1 \\ \theta_2 \\ \dots \\ \theta_{r-1} \end{bmatrix}.$$

We again employ the standard Kalman filter formula for updating $E\left(\Delta y_t^\star|\mathcal{H}_{t-1}\right)$ with respect to time t information:

$$\begin{aligned} E\left(\Delta y_t^\star|\mathcal{H}_t\right) &= E\left(\Delta y_t^\star|\mathcal{H}_{t-1}\right) \\ \text{(A16)} \qquad &+ \left[\begin{array}{ccccccc} 1 & \theta_1 & \theta_2 & \ldots & \theta_{r-1} & 1 & 1 \end{array}\right] P_{t|t-1} H \left(H' P_{t|t-1} H + R\right)^{-1} \left(\begin{array}{c} \Delta y_t^1 - E\left(\eta_t|\mathcal{H}_{t-1}\right) \\ \Delta y_t^2 - E\left(\eta_t|\mathcal{H}_{t-1}\right) \end{array}\right). \end{aligned}$$

Using the same type of manipulations as in the dynamic noise model, we have:

$$\left(H' P_{t|t-1} H + R\right)^{-1} = \frac{1}{\operatorname{var}\left(\eta_t|\mathcal{H}_{t-1}\right)\left(\tau_1^2+\tau_2^2\right)+\tau_1^2\tau_2^2} \left[\begin{array}{cc} \operatorname{var}\left(\eta_t|\mathcal{H}_{t-1}\right)+\tau_2^2 & -\operatorname{var}\left(\eta_t|\mathcal{H}_{t-1}\right) \\ -\operatorname{var}\left(\eta_t|\mathcal{H}_{t-1}\right) & \operatorname{var}\left(\eta_t|\mathcal{H}_{t-1}\right)+\tau_1^2 \end{array}\right],$$

and:

$$\left[\begin{array}{ccccccc} 1 & \theta_1 & \theta_2 & \ldots & \theta_{r-1} & 1 & 1 \end{array}\right] P_{t|t-1} H = \left[\begin{array}{cc} \operatorname{var}\left(\eta_t|\mathcal{H}_{t-1}\right)+\chi_1\tau_1^2 & \operatorname{var}\left(\eta_t|\mathcal{H}_{t-1}\right)+\chi_2\tau_2^2 \end{array}\right].$$

Substituting these expressions into (A16) gives the reported result, after some manipulations. To allow dynamics in $[\eta_t^1 \quad \eta_t^2 \quad \varepsilon_t^1 \quad \varepsilon_t^2]$, (A13) could be suitably generalized along the lines of (A9).

Table 6: Autocorrelations and Partial Autocorrelations, GDP and GDI
Latest Available Vintage

Panel A: 1978-2002

		1	2	3	4	5	6	7	8	9	10	11	12
GDP	Autocorrelations	0.42	0.31	0.12	0.03	-0.07	0.06	0.05	-0.04	0.21	0.18	0.31	0.04
	Partial Autocorrelations	0.42	0.16	-0.03	-0.05	-0.12	0.13	0.01	-0.11	0.24	0.01	0.18	-0.25
GDI	Autocorrelations	0.48	0.33	0.22	0.09	-0.10	0.02	-0.02	-0.06	0.03	0.17	0.20	0.16
	Partial Autocorrelations	0.48	0.09	0.12	-0.12	-0.21	0.16	-0.08	0.01	0.04	0.12	0.05	-0.00

Panel B: 1984Q3-2002

		1	2	3	4	5	6	7	8	9	10	11	12
GDP	Autocorrelations	0.30	0.34	0.07	0.19	0.13	-0.06	-0.03	-0.17	0.15	-0.01	0.04	-0.21
	Partial Autocorrelations	0.30	0.28	-0.09	0.12	0.09	-0.20	-0.01	-0.10	0.21	0.03	-0.09	-0.19
GDI	Autocorrelations	0.44	0.31	0.20	0.25	0.12	0.02	-0.04	0.02	-0.03	-0.16	-0.22	-0.10
	Partial Autocorrelations	0.44	0.14	0.04	0.15	-0.04	-0.06	-0.03	0.06	-0.01	-0.18	-0.09	0.04

Table 7: Dynamic Estimates of True Unobserved GDP Growth, AR1 for All Components 1984Q3 Break in μ and σ^2, Latest Available Data

μ(pre84Q3)	μ(post84Q3)	ϕ	ϕ^1	ϕ^2	σ^2(pre84Q3)	σ^2(post84Q3)	τ_1^2	τ_2^2
9.25	**5.51**	**0.55**	**-0.59**	**0.06**	**29.77**	**1.96**	**0.91**	**2.39**
(1.90)	(0.37)	(0.10)	(0.16)	(0.13)	(8.39)	(0.49)	(0.42)	(0.49)

Variances of Estimated $\Delta y^\star$ and Weights on Contemporaneous and Lagged GDP and GDI

1978Q1-1984Q2

		Weights									
	var $\widehat{\Delta y^\star}$	GDP_t	GDI_t	GDP_{t-1}	GDI_{t-1}	GDP_{t-2}	GDI_{t-2}	GDP_{t-3}	GDI_{t-3}	GDP_{t-4}	GDI_{t-4}
Noise Model	30.18	0.65	0.32	0.16	-0.13	-0.05	0.05	0.02	-0.02	-0.01	0.01
News Model	34.36	0.35	0.68	-0.16	0.13	0.05	-0.05	-0.02	0.02	0.01	-0.01

1984Q3-2002Q4

		Weights									
	var $\widehat{\Delta y^\star}$	GDP_t	GDI_t	GDP_{t-1}	GDI_{t-1}	GDP_{t-2}	GDI_{t-2}	GDP_{t-3}	GDI_{t-3}	GDP_{t-4}	GDI_{t-4}
Noise Model	2.57	0.51	0.21	0.23	-0.04	-0.03	0.01	0.00	-0.00	-0.00	0.00
News Model	6.46	0.49	0.79	-0.23	0.04	0.03	-0.01	-0.00	0.00	0.00	-0.00

Autocorrelations of Innovations to Components of GDP and GDI

		1	2	3	4	5	6	7	8	9	10	11	12
Δy_t^{i1}	Autocorrelations	-0.03	-0.00	-0.02	-0.09	-0.09	-0.00	-0.09	-0.19	0.26	0.10	0.13	-0.13
	Partial Autocorrelations	-0.03	-0.00	-0.02	-0.09	-0.10	-0.01	-0.10	-0.21	0.24	0.09	0.11	-0.18
Δy_t^{i2}	Autocorrelations	-0.04	0.03	0.07	0.02	-0.18	0.03	-0.13	-0.13	0.09	0.01	-0.03	0.06
	Partial Autocorrelations	-0.04	0.03	0.07	0.02	-0.18	0.01	-0.13	-0.11	0.08	-0.01	-0.00	-0.00
Δy_t^{c}	Autocorrelations	-0.01	-0.06	0.10	-0.03	-0.28	0.11	-0.02	-0.18	0.05	0.06	0.11	-0.21
	Partial Autocorrelations	-0.01	-0.06	0.11	-0.05	-0.26	0.11	-0.10	-0.14	0.00	-0.03	0.15	-0.25

Notes

[1]We have 100 observations in Panel A of table 6, and 74 observations in Panel B, so approximate two standard error bands for the sample partial autocorrelations are ± 0.20 and ± 0.23, respectively.

[2]This value is given by an algebraic Riccati equation (see Harvey, 1989)

[3]Here we assume that current estimates of GDP employ the information in both lagged GDP and lagged GDI, as do the estimates of current GDI; a more realistic model may be one where current estimates of GDP employ the information in lagged GDP only, and current estimates of GDI employ the information in lagged GDI only, so:

$$\begin{aligned} \Delta y_t^1 &= E\left(\Delta y_t^\star | \mathcal{F}_t^1, \mathcal{F}_{t-1}^1, \mathcal{F}_{t-2}^1, \dots\right), \quad \text{and:} \\ \Delta y_t^2 &= E\left(\Delta y_t^\star | \mathcal{F}_t^2, \mathcal{F}_{t-1}^2, \mathcal{F}_{t-2}^2, \dots\right), \end{aligned}$$

We leave study of this more complicated model for future research.

[4]These weights were estimated by ordinary least squares regression. Since we know the predicted values for "true" GDP growth are linear combinations of current and lagged GDP and GDI, the only choice here is where to cut off the number of lags included in the regression; for this particular choice of cutoff, standard errors were all less than 0.001.