

# News, Noise, and Estimates of the “True” Unobserved State of the Economy

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## Abstract

Which provides a better estimates of the growth rate of “true” U.S. output, gross domestic product (GDP) or gross domestic income (GDI)? Past work has assumed the idiosyncratic variation in each estimate is pure *noise*, taking greater variability to imply lower reliability. We develop models that relax this assumption, allowing the idiosyncratic variation in the estimates to be partly or pure *news*; then greater variability may imply higher information content and greater reliability. Based on evidence from revisions, we reject the pure noise assumption for GDI growth, and our results favor placing sizable weight on GDI growth because of its relatively large idiosyncratic variability. This calls into question the suitability of the pure noise assumption in other contexts, including dynamic factor models.

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## 1 Introduction

For analysts of economic fluctuations, estimating the true state of the economy from imperfectly measured official statistics is an ever-present problem. Since no one statistic is a perfect gauge of the state of the economy, taking some type of weighted average of multiple imperfectly measured statistics seems sensible. Examples include composite indexes of coincident indicators,<sup>1</sup> and averages of the different measures of aggregate output. For the case of U.S. output growth, the NBER Business Cycle Dating Committee's announcement of a peak in December 2007 noted marked differences between two different estimates, GDP and GDI, and essentially decided to give each estimate some implicit weight: "in examining the behavior of domestic production, we consider not only the conventional product-side GDP estimates, but also the conceptually equivalent income-side GDI estimates. The differences between these two sets of estimates were particularly evident in 2007 and 2008." In this paper, we take the state of the economy to mean the growth rate of output as traditionally defined in the U.S. National Income and Product Accounts (NIPAs), and work out methodologies for reconciling differences between these two estimates.<sup>2</sup>

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<sup>1</sup>See Stock and Watson (1989) and the subsequent literature on dynamic factor models referenced below.

<sup>2</sup>National Income accountants face two fundamental problems. First, they must define an interesting and useful measure of aggregate economic activity, and second, they must design methods for estimating the value of that measure, taking the definition as fixed. Our concern in this paper is with the second issue, using the definition of economic activity (output) traditionally employed by National Income accountants. It is a value-added measure with the private sector component restricted to marketed economic activity for the most part - i.e. non-market activities such home production and changes

The main point of our paper is as follows. To our knowledge, all prior attempts to produce such a weighted average of imperfectly measured statistics have made a strong implicit assumption that drives their weighting: that the idiosyncratic variation in each measured statistic is pure *noise*, or completely uncorrelated with information about the true state of the economy. Under this assumption, a statistic with greater idiosyncratic variance is given a smaller weight because it is assumed to contain more noise. We consider the implications of relaxing this assumption, allowing the idiosyncratic variation in each measured statistic to contain *news*, or information about the true state of the economy.<sup>3</sup> If the idiosyncratic variation is mostly news, the implied weighting is diametrically opposite that of the noise assumption: a statistic with greater idiosyncratic variance should be given a larger weight because it contains more information about the true state of the economy. The implicit noise assumption relied upon in numerous prior papers is arbitrary, and more information must be brought to bear on this issue.

Focusing on GDP and GDI allows us to make this basic point in a simple bivariate context. These two measures of the size of the U.S. economy would equal one another if all the transactions in the economy were observed, but measurement difficulties lead to the statistical discrepancy between the two; their quarterly growth rates often diverge significantly. Weale (1992) and others<sup>4</sup> have estimated the growth rate of “true” unobserved output as a combination of measured GDP growth and GDI growth, generally concluding that GDI growth should be given more weight than measured GDP growth.

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in natural resources are excluded. For more discussion and references, see Sir Richard Stone’s Nobel Memorial lecture, Stone (1984).

<sup>3</sup>Our terminology follows Mankiw and Shapiro (1986) and Mankiw, Runkle and Shapiro (1984), who coined the news and noise terminology describing revisions. Subsequent work on revisions includes Dynan and Elmendorf (2001), Faust, Rogers and Wright (2005), and Fixler and Grimm (2006). Sargent (1989) contrasts noisy and optimally filtered estimates of income, consumption, and investment in the context of an accelerator model of investment demand.

<sup>4</sup>See Howrey (2003) and the related work of Weale (1985) and Smith, Weale, and Satchell (1998).

Is GDI really the more accurate measure? We argue for caution, as the results are driven entirely by the noise assumption: the models implicitly assume that since GDP growth has higher variance than GDI growth over their sample period, it must be noisier, and so should receive a smaller weight. However GDP may have higher variance because it contains more information about “true” unobserved output (this is the essence of the news assumption); then measured GDP should receive the higher weight.

In the general version of our model that allows the idiosyncratic component of each measured statistic to be a mixture of news and noise, virtually any set of weights can be rationalized by making untestable assumptions about the mixtures. More information must be brought to bear on the problem; otherwise the choice of weights will be arbitrary. While this fundamental indeterminacy is somewhat disturbing, in the case of combining GDP and GDI we bring more information to bear on the problem to help pin down the weights. Contrary to the sample employed in Weale (1992), GDI growth has more idiosyncratic variation than GDP growth in our sample, which starts in the mid 1980s after the marked reduction in the variance of the measured estimates - see McConnell and Perez-Quiros (2000). However the initial GDI growth estimates have negligible idiosyncratic variance (i.e. its variance is close to its covariance with GDP growth); it is only through revisions that the variance of GDI growth becomes relatively large. If the revisions add news, and not noise - an assumption that is consistent with our knowledge of the revisions process and that follows previous research such as Mankiw, Runkle and Shapiro (1984) and Mankiw and Shapiro (1986) - then there must be a strong presumption that the relatively large variance of GDI growth represents news, news derived from the revisions.

In this paper we develop new techniques for decomposing revisions into news and noise, and show how to place bounds on the shares of the idiosyncratic variation in GDP and GDI that are news. Based on these bounds we test the assumptions of the

pure noise model, rejecting them at conventional significance levels. Due to its relatively large idiosyncratic variation, GDI growth should be weighted more heavily, not less, in estimating “true” unobserved output growth. Measured GDP growth then understates the true variability of the economy’s growth rate, a finding with implications for real business cycle, asset pricing, and other models.

Weighting GDI growth more heavily leads to some interesting modifications to the time series of output growth. For example, both before and after the 1990-1991 recession, economic growth is weaker than indicated by measured GDP growth, and the 2001 recession was substantially deeper than indicated by measured GDP growth. For the 2007-2008 episode discussed by the NBER business cycle dating committee, we reserve judgment until the data have passed through more revisions, which may eliminate the large discrepancies between the growth rates of GDP and GDI. But if they do not, our results provide a rigorous methodology for reconciling these discrepancies, which could prove quite useful for historical analysis of business cycles.

The rest of the paper is organized as follows. Section 2 discusses the news vs noise assumptions in the bivariate context of GDP and GDI, drawing out their implications for constructing weighted averages. Section 3 describes the GDP and GDI data and discusses the information content of revisions. Section 4 shows how to decompose revisions into news and noise, and place bounds on the fractions of GDP and GDI that are news or noise. Section 5 constructs estimates of “true” unobserved output growth as weighted averages of GDP and GDI, and tests the assumption that the idiosyncratic variation in GDI is noise. Section 6 draws conclusions.

## 2 Theory: The Competing News and Noise Models

### 2.1 Review of News and Noise

Let  $\Delta y_t^*$  be the true growth rate of the economy, let  $\Delta y_t^i$  be one of its measured estimates, and let  $\varepsilon_t^i$  be the difference between the two, so:

$$\Delta y_t^i = \Delta y_t^* + \varepsilon_t^i.$$

The noise model makes the classical measurement error assumption that  $\text{cov}(\Delta y_t^*, \varepsilon_t^i) = 0$ ; this is the precise meaning of the statement that  $\varepsilon_t^i$  is *noise*. One implication of a noisy estimate  $\Delta y_t^i$  is that its variance is greater than the variance of the true growth rate of the economy, or  $\text{var}(\Delta y_t^i) > \text{var}(\Delta y_t^*)$ .

In contrast, if an estimate  $\Delta y_t^i$  were constructed efficiently with respect to a set of information about  $\Delta y_t^*$  (call it  $\mathcal{F}_t^i$ ), then  $\Delta y_t^i$  would be the conditional expectation of  $\Delta y_t^*$  given that information set:

$$\Delta y_t^i = E(\Delta y_t^* | \mathcal{F}_t^i).$$

Writing:

$$\Delta y_t^* = \Delta y_t^i + \zeta_t^i,$$

the term  $\zeta_t^i$  represents the information about  $\Delta y_t^*$  that is unavailable in the construction of  $\Delta y_t^i$ . Then  $\Delta y_t^i$  and  $\zeta_t^i$  represent mutually orthogonal pieces of *news* about  $\Delta y_t^*$ , employing the terminology in Mankiw and Shapiro (1986), and  $\text{cov}(\Delta y_t^i, \zeta_t^i) = 0$ . This leads us to an implication of the news model that we employ later, namely that  $\text{cov}(\Delta y_t^i, \Delta y_t^*) = \text{var}(\Delta y_t^i)$ . We also have  $\text{var}(\Delta y_t^*) > \text{var}(\Delta y_t^i)$ , an implication opposite

to that of the noise model.

These two models are clearly extremes; the next section considers a general model that allows differing degrees of news and noise in the estimates.

## 2.2 The Mixed News and Noise Model

We consider a model with two estimates of “true” unobserved output growth, each an efficient estimate plus noise:

$$\begin{aligned}\Delta y_t^1 &= E(\Delta y_t^* | \mathcal{F}_t^1) + \varepsilon_t^1, \quad \text{and} \\ \Delta y_t^2 &= E(\Delta y_t^* | \mathcal{F}_t^2) + \varepsilon_t^2.\end{aligned}$$

The noise components  $\varepsilon_t^1$  and  $\varepsilon_t^2$  are mutually uncorrelated and, naturally, uncorrelated with true unobserved GDP. Taking  $\Delta y_t^1$  to be GDP and  $\Delta y_t^2$  to be GDI, the information in  $\mathcal{F}_t^1$  likely would consist of personal consumption expenditures, investment, net exports, and the other components that sum to GDP, while the information in  $\mathcal{F}_t^2$  likely would consist of wage and salary income, corporate profits, proprietors’ income, and the other components that sum to GDI.<sup>5</sup> We assume each information set includes a constant, so both  $\Delta y_t^1$  and  $\Delta y_t^2$  consistently estimate the mean  $\mu$  of  $\Delta y_t^*$ , and there may be a substantial amount of additional overlap between the two information sets. Consumption growth may be highly correlated with the growth rate of wages and salaries, for example. However a key feature of our model is that it recognizes that the two

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<sup>5</sup>We should note that our efficiency assumption is weaker than some others that have been tested in the literature, such as those in Dynan and Elmendorf (2001) and Fixler and Grimm (2006). We only assume that the estimates are efficient with respect to the internal information used to compute them, not with respect to the entire universe of available information - we do not consider efficiency with respect to the slope of the yield curve, stock prices, and so on.

information sets are not necessarily identical.<sup>6</sup>

To clearly illustrate the main points of the paper, we focus on the simple case where all variables are jointly normally distributed, and where measured GDP and GDI are serially uncorrelated.<sup>7</sup> With normality, the conditional expectation of the true growth rate of the economy is a weighted average of GDP and GDI; netting out means yields:

$$(1) \quad \begin{aligned} E(\Delta y_t^* - \mu | \Delta y_t^1, \Delta y_t^2, \mu) &= \\ \widehat{\Delta y_t^*} - \mu &= \omega_1 (\Delta y_t^1 - \mu) + \omega_2 (\Delta y_t^2 - \mu), \end{aligned}$$

calling the conditional expectation  $\widehat{\Delta y_t^*}$ . The weights  $\omega_i$  can be derived using standard formulas for the population version of ordinary least squares:

$$(2) \quad \begin{aligned} \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} &= \begin{pmatrix} \text{var}(\Delta y_t^1) & \text{cov}(\Delta y_t^1, \Delta y_t^2) \\ \text{cov}(\Delta y_t^1, \Delta y_t^2) & \text{var}(\Delta y_t^2) \end{pmatrix}^{-1} \begin{pmatrix} \text{cov}(\Delta y_t^1, \Delta y_t^*) \\ \text{cov}(\Delta y_t^2, \Delta y_t^*) \end{pmatrix} \\ &= \begin{pmatrix} \text{var}(\Delta y_t^1) & \text{cov}(\Delta y_t^1, \Delta y_t^2) \\ \text{cov}(\Delta y_t^1, \Delta y_t^2) & \text{var}(\Delta y_t^2) \end{pmatrix}^{-1} \begin{pmatrix} \text{var}(E(\Delta y_t^* | \mathcal{F}_t^1)) \\ \text{var}(E(\Delta y_t^* | \mathcal{F}_t^2)) \end{pmatrix}, \end{aligned}$$

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<sup>6</sup>It is natural to ask whether it is possible to compute an efficient estimate of  $\Delta y_t^*$  given that it is unobserved. A couple of things should be kept in mind. First, though  $\Delta y_t^*$  itself is unobserved, it is defined quite precisely - see footnote 2. Second, the BEA and statisticians in general draw on a large stock of knowledge about the data they employ, and it's reliability. More reliable data sources are generally given greater weight, and less reliable data sources less weight; through such procedures it may be possible to produce estimates that are close to efficient even though  $\Delta y_t^*$  is never observed. To illustrate, suppose that the source data used to compute a component of GDP is contaminated with sampling error, and the variance of the sampling error is known (as is often the case); then procedures may be employed to downweight the estimate in proportion to the variance of the sampling error, producing an efficient estimate for that component even though it's true value is never observed. See Sargent (1989).

<sup>7</sup>In a set of additional results available from the authors, the model is extended to allow for serial correlation of arbitrary linear form in GDP and GDI. The main points of the paper carry through in this setting, and the empirical estimates with dynamics are similar to the empirical estimates of the static models presented here.



using  $\text{cov}(\Delta y_t^*, \varepsilon_t^i) = 0$  and the property of efficient estimates that their covariance with the variable they estimate is simply their variance.

It is useful to introduce some additional notation. Call the covariance between the two estimates  $\sigma^2$ ; this arises from the overlap between the information sets used to compute the efficient estimates, and correlation between the measurement errors  $\varepsilon_t^1$  and  $\varepsilon_t^2$ . The model imposes the condition that the variance of each estimate is at least as large as their covariance; let  $\sigma^2 + \tau_1^2$  and  $\sigma^2 + \tau_2^2$  be the variances of the  $\Delta y_t^1$  and  $\Delta y_t^2$ , respectively. The idiosyncratic variance in each estimate, the  $\tau_i^2$  for  $i = 1, 2$ , arises from two potential sources. The first source is the idiosyncratic news in each estimate - the information in each efficient estimate missing from the other, and the second source is noise.

Let the share of the covariance between the two estimates that is news, or common information, be  $\chi$ . Similarly, let the news share of the idiosyncratic variance in the  $i$ th estimate be  $\chi_i$ , so  $(1 - \chi_i)$  is the fraction of the idiosyncratic variance that is noise. Then equation (2) becomes:

$$\begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} = \begin{pmatrix} \sigma^2 + \tau_1^2 & \sigma^2 \\ \sigma^2 & \sigma^2 + \tau_2^2 \end{pmatrix}^{-1} \begin{pmatrix} \chi\sigma^2 + \chi_1\tau_1^2 \\ \chi\sigma^2 + \chi_2\tau_2^2 \end{pmatrix}.$$

Solving and substituting into (2) gives:

$$(3) \quad \widehat{\Delta y_t^*} - \mu = \frac{\left(\chi_1\tau_1^2 + (\chi - \chi_2)\tau_2^2 + \chi_1\frac{\tau_1^2\tau_2^2}{\sigma^2}\right)(\Delta y_t^1 - \mu)}{\tau_1^2 + \tau_2^2 + \frac{\tau_1^2\tau_2^2}{\sigma^2}} + \frac{\left(\chi_2\tau_2^2 + (\chi - \chi_1)\tau_1^2 + \chi_2\frac{\tau_1^2\tau_2^2}{\sigma^2}\right)(\Delta y_t^2 - \mu)}{\tau_1^2 + \tau_2^2 + \frac{\tau_1^2\tau_2^2}{\sigma^2}}.$$

Before examining (3) in greater depth, note that the weights on the two component

variables here do not necessarily sum to one; the weights on the two components variables and the mean  $\mu$  sum to one. But in some situations the econometrician may have little confidence in the estimated mean  $\mu$ , so it may be inadvisable to use it as the third component in the weighted average. One way around this problem is to force the weights on  $\Delta y_t^1$  and  $\Delta y_t^2$  to sum to one, with  $\omega_2 = 1 - \omega_1$ ; substituting into (1) and rearranging yields:

$$(1') \quad E(\Delta y_t^* - \Delta y_t^2 | \Delta y_t^1, \Delta y_t^2) = \omega_1 (\Delta y_t^1 - \Delta y_t^2).$$

Adding back in  $\Delta y_t^2$  to equation (1') yields  $\widehat{\Delta y_t^*}$ . The solution to the general model then becomes:

$$(3') \quad \widehat{\Delta y_t^*} = \frac{(\chi_1 \tau_1^2 + (1 - \chi_2) \tau_2^2) \Delta y_t^1 + (\chi_2 \tau_2^2 + (1 - \chi_1) \tau_1^2) \Delta y_t^2}{\tau_1^2 + \tau_2^2}.$$

With the assumptions of the pure noise model discussed below, this particular estimator is equivalent to the estimator proposed by Weale (1992) and Stone et al (1942). Appendix A clarifies the relation between these earlier estimators and those derived here.

It is clear that not all of the parameters of the unconstrained model are identified: we observe three moments from the variance-covariance matrix of  $[\Delta y_t^1 \quad \Delta y_t^2]$ , which is not enough to pin down the six parameters  $\sigma^2$ ,  $\tau_1^2$ ,  $\tau_2^2$ ,  $\chi$ ,  $\chi_1$ , and  $\chi_2$ . Imposing values for  $\chi$ ,  $\chi_1$ , and  $\chi_2$  allows identification of the remaining parameters. Some illuminating special cases are examined next, which show how assumptions about the idiosyncratic news shares  $\chi_1$  and  $\chi_2$  are critical for determining the relative weights on the two component variables.

### 2.2.1 The Pure Noise Model

Previous attempts to estimate models of this kind have focused on one particular assumption for the idiosyncratic news shares:  $\chi_1 = \chi_2 = 0$ . The implication is that the two information sets must coincide, at least in the universe of information that is relevant for predicting  $\Delta y_t^*$ , so  $E(\Delta y_t^* | \mathcal{F}_t^1) = E(\Delta y_t^* | \mathcal{F}_t^2)$ . We call this the *pure noise model*; equation (3) is then:<sup>8</sup>

$$(4) \quad \widehat{\Delta y_t^*} - \mu = \frac{\chi \tau_2^2 (\Delta y_t^1 - \mu) + \chi \tau_1^2 (\Delta y_t^2 - \mu)}{\tau_1^2 + \tau_2^2 + \frac{\tau_1^2 \tau_2^2}{\sigma^2}}.$$

In the pure noise model, the weight for one measure is proportional to the idiosyncratic variance of the other measure - since the idiosyncratic variance in each estimate is assumed to be noise, the “noisier” measure is downweighted. The weights on the (net of mean) estimates sum to less than one; as is typical in the classical measurement error model, coefficients on noisy explanatory variables are downweighted. In fact, as the common variance  $\chi \sigma^2$  approaches zero, the signal-to-noise ratio in the model approaches zero as well, and the formula instructs us to give up on the estimates of GDP and GDI for any given time period, using the overall sample mean as the best estimate for each and every period.

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<sup>8</sup>Previous work typically has imposed the additional assumption that  $E(\Delta y_t^* | \mathcal{F}_t^i) = \Delta y_t^*$ , for  $i = 1, 2$ , leading to the first case in subsection 2.1. Equation (4) holds with or without this additional assumption; the only difference lies in the interpretation of the parameters. With this assumption,  $\sigma^2$  identifies the variance of “true” GDP growth; without it,  $\sigma^2$  merely identifies  $\text{var}(E(\Delta y_t^* | \mathcal{F}_t^1)) = \text{var}(E(\Delta y_t^* | \mathcal{F}_t^2))$ , which must be less than the variance of “true” GDP growth.

### 2.2.2 The Pure News Model

The opposite case is what we call the *pure news model*, where  $\chi_1 = \chi_2 = 1$ . Equation (3) then becomes:

$$(5) \quad \widehat{\Delta y_t^*} - \mu = \frac{\left(\tau_1^2 + (\chi - 1)\tau_2^2 + \frac{\tau_1^2\tau_2^2}{\sigma^2}\right)(\Delta y_t^1 - \mu)}{\tau_1^2 + \tau_2^2 + \frac{\tau_1^2\tau_2^2}{\sigma^2}} + \frac{\left(\tau_2^2 + (\chi - 1)\tau_1^2 + \frac{\tau_1^2\tau_2^2}{\sigma^2}\right)(\Delta y_t^2 - \mu)}{\tau_1^2 + \tau_2^2 + \frac{\tau_1^2\tau_2^2}{\sigma^2}}.$$

The weight for each measure is now proportional to its own idiosyncratic variance - the estimate with greater variance contains more news and hence receives a larger weight. This result is diametrically opposed to that of the noise model.

Under some circumstances it may be reasonable to assume that the covariance between the estimates is pure news, in which case the  $(\chi - 1)$  terms vanish; then the weights (on the net of mean estimates) sum to a number greater than unity, again opposite the pure noise model. As  $\sigma^2 \rightarrow 0$  (i.e. as the variance common to the two estimates approaches zero), the weight for each estimate approaches unity. In this case, we are essentially adding together two independent pieces of information about GDP growth. To illustrate, suppose we receive news of a shock that moves  $\Delta y_t^*$  two percent above its mean, and then receive news of another, independent shock that moves  $\Delta y_t^*$  one percent below its mean. The logical estimate of  $\Delta y_t^*$  is then the mean plus one percent - i.e. the sum of the two shocks. In Appendix B we work through another example, of two estimates of GDP growth, each based on the growth rate of a different sector of the economy; if the growth rates of the sectors are uncorrelated, we simply add up the net-of-mean contributions to GDP growth of the two sectors, and then add back in the mean.

### 2.2.3 Arbitrary Weights

Finally, consider another case of interest: if  $\chi_i = \chi$  and  $\chi_j = 0$ , then  $\omega_i = \chi$  and  $\omega_j = 0$ , placing all the weight on variable  $i$ . If placing all the weight on either variable can be justified with such assumptions about the idiosyncratic news shares, perhaps any set of weights is possible. This turns out to be the case. Let the ratio of the weights  $\frac{\omega_1}{\omega_2} = r$ , so:

$$(6) \quad r(\chi, \chi_1, \chi_2) = \frac{\chi_1 \tau_1^2 + (\chi - \chi_2) \tau_2^2 + \chi_1 \frac{\tau_1^2 \tau_2^2}{\sigma^2}}{\chi_2 \tau_2^2 + (\chi - \chi_1) \tau_1^2 + \chi_2 \frac{\tau_1^2 \tau_2^2}{\sigma^2}},$$

where we've expressed  $r$  as a function of  $\chi$ ,  $\chi_1$  and  $\chi_2$ . The following proposition shows that, for any  $0 < \chi \leq 1$ , any set of weights can be rationalized by making untestable assumptions about the degree of news and noise in the idiosyncratic components of the two variables:

**Proposition 1** *Let  $r$  be any non-negative real number, and let  $r(\chi, \chi_1, \chi_2)$  be given by (6), where  $\tau_1^2$ ,  $\tau_2^2$ , and  $\sigma^2$  are each constant, positive real numbers, and  $0 < \chi \leq 1$ . Then there exists a pair  $(\chi_1^*, \chi_2^*)$ , with  $\chi_1^* \in [0, 1)$  and  $\chi_2^* \in (0, 1]$ , such that  $r(\chi_1^*, \chi_2^*) = r$ .*

**Proof:** Consider an example that meets the conditions of the proposition, where  $\chi_2 = \chi - \chi_1$ . Then  $r(\chi_1, \chi_2) = \frac{\chi_1}{\chi - \chi_1}$ . Since  $r(\chi_1, \chi_2)$  is a continuous function,  $r(0, \chi) = 0$ , and  $\lim_{\chi_1 \rightarrow \chi} r(\chi_1, \chi - \chi_1) = \infty$ , the result holds by theorem 4.23 of Rudin (1953). We have  $\chi_1 = \frac{\chi r}{1+r}$ , which produces the desired  $\chi_1^* \in [0, 1)$  and  $\chi_2^* \in (0, 1]$  for any non-negative real  $r$ .

One set of weights is as justifiable as any other; without further information about the estimates, the choice of weights will be arbitrary. In the empirical work below on GDP and GDI, we do bring further information to bear on the problem, and examine which news shares are likely closest to reality.

### 3 Data

The most widely-used statistic produced by the U.S. Bureau of Economic Analysis (BEA) is GDP, its expenditure-based estimate of the size of the economy; this statistic is the sum of personal consumption expenditures, investment, government expenditures, and net exports. However the BEA also produces an income-based estimate of the size of the economy, gross domestic income (GDI), from different information. National income is the sum of employee compensation, proprietors' income, rental income, corporate profits and net interest; adding consumption of fixed capital and a few other balancing items to national income produces GDI. Computing the value of GDP and GDI would be straightforward if it were possible to record the value of all the underlying transactions included in the NIPA definition of the size of the economy, in which case the two measures would coincide. However all the underlying transactions are not recorded: the BEA relies on various surveys, censuses and administrative records, each imperfect, to compute the estimates, and differences between the data sources used to produce GDP and GDI, as well as other measurement difficulties, lead to the statistical discrepancy between the two measures.

Likelihood-ratio tests show breaks in the means and variances of our GDP and GDI growth series in 1984Q3; in this version of the paper we restrict our attention to the post-1984Q3 period. Figure 1 plots the annualized quarterly growth rates of the “latest available” versions of nominal GDP (solid) and GDI (dashed) from 1984Q3 to 2005, pulled from the BEA web site in September 2009.<sup>9</sup> These “latest available” estimates have been revised numerous times by the BEA. The “final current quarterly” estimates, released for each quarter about three months after the quarter ends, is the first set of

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<sup>9</sup>We choose to focus on nominal data because the BEA does not produce a deflator for GDI. Our results using GDP and GDI deflated by the GDP deflator were broadly similar to those reported.

estimates with a complete time series of both GDP and GDI growth over our sample period. Because the “final current quarterly” nomenclature is somewhat confusing (indeed, the BEA changed it in 2009), we call this the “first” set of estimates, and the “latest” available vintage as of 2009 the “last” set of estimates. Historically, each “first” estimate has been revised three times at annual revisions, and then periodically every five years at benchmark revisions. We restrict our sample to end in 2005 so that all of our “last” vintage observations have passed through the three annual revisions; the time series we employ was last benchmarked in the summer of 2009, to the 2002 input-output tables.<sup>10</sup>

At each of the annual revisions and at a benchmark revision, the BEA incorporates more comprehensive and accurate source data. For the “first” estimates, most available source data is based on samples, which may contain some noise from sampling errors. Later vintages are based on more comprehensive samples, or sometimes universe counts, so incorporation of these data has the potential to reduce noise.

In addition to potentially noisy data, at the time of the current quarterly estimates the BEA has little hard data at all on some components of GDP and GDI, including much of services consumption.<sup>11</sup> For these components the BEA often resorts to “trend extrapolations,” assuming the growth rate for the current quarter some average of past growth rates, which can be thought of as approximating conditional expectations based on past history. In later vintages when the BEA receives and substitutes actual data for these extrapolated components, news is added to the estimates. For some missing components the BEA substitutes related data instead of “trend extrapolations”; for example the BEA borrows data from the income-side, using employment, hours and

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<sup>10</sup>One could argue that we should cut our sample off in 2002 instead of 2005, since 2002 is the last year to which the data have been benchmarked. Results were similar using this smaller sample.

<sup>11</sup>For a detailed description of the missing GDP data, see Grimm and Weadock (2006).

earnings as an extrapolator from some components of services consumption. These estimates may be thought of as approximating conditional expectations based on related labor market information, although if the labor market data contain noise, it is possible that these procedures may introduce common noise into the current quarterly estimates.

## 4 Identifying News vs. Noise from Revisions

Our model for “first” and “last” vintage GDP and GDI growth ( $i = 1, 2$ ) is:

$$\begin{aligned}\Delta y_t^{i,f} &= E\left(\Delta y_t^* | \mathcal{F}_t^{i,f}\right) + \varepsilon_t^{i,f} \\ \Delta y_t^{i,l} &= E\left(\Delta y_t^* | \mathcal{F}_t^{i,l}\right) + \varepsilon_t^{i,l}.\end{aligned}$$

Our working assumption is that the revision from “first” to “last” brings the estimates closer to the truth  $\Delta y_t^*$ , through some combination of increased news and decreased noise.<sup>12</sup> On the news side, we assume  $\mathcal{F}_t^{i,f}$  is strictly smaller than  $\mathcal{F}_t^{i,l}$ , so  $\mathcal{F}_t^{i,f} \subset \mathcal{F}_t^{i,l}$ .

Writing:

$$E\left(\Delta y_t^* | \mathcal{F}_t^{i,l}\right) = E\left(\Delta y_t^* | \mathcal{F}_t^{i,f}\right) + \zeta_t^{i,fl},$$

the term  $\zeta_t^{i,fl}$  is the increase in news embedded in the revision, and is uncorrelated with  $E\left(\Delta y_t^* | \mathcal{F}_t^{i,f}\right)$ . This increase in news increases the variance of the estimates. On the noise side,  $\text{var}\left(\varepsilon_t^{i,l}\right) < \text{var}\left(\varepsilon_t^{1,f}\right)$ , and we write this as:

$$\varepsilon_t^{i,f} = \varepsilon_t^{i,l} + \varepsilon_t^{i,fl},$$

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<sup>12</sup>This assumption need not hold for each individual annual and benchmark revision, only for the sum of all these revisions that we consider; we assume that if a revision adds some noise, that noise is revised away over subsequent revisions.



$\varepsilon_t^{i,fl}$  uncorrelated with  $\varepsilon_t^{i,l}$ . This reduction in noise decreases the variance of “last” relative to “first”. These noise terms are assumed uncorrelated with all relevant conditional expectations.

If the revision from “first” to “last” reflects increased news, the variance of “last” should exceed the variance of “first,” and if the revision reflects decreased noise, the opposite should hold, as discussed in Mankiw and Shapiro (1986). Here we show how to identify the fraction of revision variance that stems from increased news, and the fraction that stems from decreased noise. The variance of the revision is:

$$\begin{aligned}
\text{var}(\Delta y_t^{i,l} - \Delta y_t^{i,f}) &= \text{var}(\zeta_t^{i,fl} - \varepsilon_t^{i,fl}) \\
(7) \qquad \qquad \qquad &= \text{var}(\zeta_t^{i,fl}) + \text{var}(\varepsilon_t^{i,fl}),
\end{aligned}$$

since  $\zeta_t^{i,fl}$  and  $\varepsilon_t^{i,fl}$  are independent. Contrast this with the change in the variance of the estimate:

$$\begin{aligned}
\text{var}(\Delta y_t^{i,l}) - \text{var}(\Delta y_t^{i,f}) &= \text{var} \left( E \left( \Delta y_t^* | \mathcal{F}_t^{i,l} \right) + \varepsilon_t^{i,l} \right) - \text{var} \left( E \left( \Delta y_t^* | \mathcal{F}_t^{i,f} \right) + \varepsilon_t^{i,f} \right) \\
&= \text{var} \left( E \left( \Delta y_t^* | \mathcal{F}_t^{i,f} \right) + \zeta_t^{i,fl} + \varepsilon_t^{i,l} \right) \\
&\quad - \text{var} \left( E \left( \Delta y_t^* | \mathcal{F}_t^{i,f} \right) + \varepsilon_t^{i,fl} + \varepsilon_t^{i,l} \right) \\
&= \text{var} \left( E \left( \Delta y_t^* | \mathcal{F}_t^{i,f} \right) \right) + \text{var}(\zeta_t^{i,fl}) + \text{var}(\varepsilon_t^{i,l}) \\
&\quad - \text{var} \left( E \left( \Delta y_t^* | \mathcal{F}_t^{i,f} \right) \right) - \text{var}(\varepsilon_t^{i,fl}) - \text{var}(\varepsilon_t^{i,l}) \\
(8) \qquad \qquad \qquad &= \text{var}(\zeta_t^{i,fl}) - \text{var}(\varepsilon_t^{i,fl}),
\end{aligned}$$

again relying on the lack of covariance between various terms. Equations (7) and (8) pin down the news increase  $\text{var}(\zeta_t^{i,fl})$  and noise decrease  $\text{var}(\varepsilon_t^{i,fl})$ .

Pinning down the news increase and noise decrease from revision is interesting because it allows us to place bounds on the fraction of the variance in an estimate that is

news. For example, if the “first” estimate starts out with little variance relative to the “last” estimate, then most of the variance of the “last” estimate must be news, since it came from revision. Similarly, if the noise decrease from revision is close to the variance of the “first” estimate, then most of the variance of that “first” estimate must be noise. The revisions identify noise in the “first” estimate and news in the “last” estimate, placing an upper bound on the fraction of variance of “first” that is news and a lower bound on the fraction of variance of “last” that is news.

The first two rows of Table 1 show, for GDP growth and GDI growth, the variances of their “first” and “last” vintages, the change in variance (equation 8), the revision variance (equation 7), and the increase in news and decrease in noise implied by these statistics. For GDP growth, the variance of “last” is only slightly larger than the variance of “first”, implying the increase in variance from greater news is only slightly larger than the decrease in variance from less noise. The variance of the revision then tells us that a little less than a percentage point of the variance of the “first” estimate of GDP growth must be noise, and a little more than a percentage point of the variance of the “last” estimate must be news. For GDI growth, the variance of “last” is substantially larger than the variance of “first”, implying most of the revision variance stems from increased news. More than two percentage points of the variance of the “last” estimate of GDI growth must be news.

Note that “first” GDI growth has little idiosyncratic variance - its variance is about equal to its covariance with GDP growth - but after passing through revisions its idiosyncratic variance is substantial. Since the idiosyncratic variance of this “last” estimate stems from revisions, which add news variance but not noise variance, it is tempting to conclude that this idiosyncratic variance must be news. However, as the equations in Appendix C illustrate, the increase in news covariance and decrease in noise covariance are not identified, as would be necessary to pin down precisely the increase in idiosyncratic

news. Some of the intuition developed earlier for variances applies to the covariance as well: an increase in news can only increase the covariance, while a decrease in noise can only decrease the covariance. The third line of the table shows that the covariance falls after the revision, implying that the “first” estimates must contain some common noise eliminated through revision. One possibility is that this common noise is eliminated from GDP growth but not GDI growth, so it becomes idiosyncratic noise in GDI growth. This possibility prevents us from concluding definitively that the idiosyncratic variation in the “last” estimate of GDI growth is news.

For the “first” estimates of GDP and GDI growth, let  $\chi_{1,f}$ ,  $\chi_{2,f}$  and  $\chi_f$  be the shares of their idiosyncratic variances and covariances that are news, and for the “last” estimates, let these news shares be  $\chi_{1,l}$ ,  $\chi_{2,l}$  and  $\chi_l$ . Despite the complications described above, the revisions do place some bounds on these news shares; since the revisions must add news or subtract noise, and the news increases and noise decreases must appear somewhere, in either the covariance between the estimates or their idiosyncratic variances. The bounds are defined by the equations in Appendix C. We make one additional assumption, that the covariance between the estimates is news unless explicitly identified as noise by the revisions; formally, we assume equation (C.10) holds with equality. This is certainly in the spirit of popular dynamic factor models that assume that covariance is signal.

The first column on table 2 shows news shares resulting from minimization of the total idiosyncratic news in the “last” estimates, subject to the revision equations described in Appendix C. For the “last” estimates, the pure noise model cannot be squared with the revisions: the idiosyncratic news share for GDI growth of 0.67 is its lowest possible value. Under this set of assumptions, all of the 0.93 reduction in the noise variance of “first” GDP is assumed to come from a reduction in common noise, with none of that noise removed from GDI growth so all of it becomes idiosyncratic noise in the “last” GDI estimate. The remainder of this idiosyncratic variance, about two-thirds, must be

news. However, the true idiosyncratic news share is likely well above this lower bound, since two of the assumptions made in this case are unlikely: (i) that all of the decrease in GDP noise occurs in the common component, and (ii) that none of the common noise removed from GDP growth is removed from GDI growth. Regarding (i), some of the idiosyncratic variance of “first” GDP growth is likely noise, since it relies on noisy samples not employed in estimating GDI growth, and revisions likely eliminating some of that. And regarding (ii), the revisions to GDI growth likely reduce its noise from sampling errors as well, since they incorporate virtual census counts from administrative and tax records.

The second column on table 2 shows news shares resulting from maximizing the total idiosyncratic news in the “last” estimates. While the pure news model cannot be squared with the revisions evidence either, something close to the pure news model with  $(\chi_{1,t}, \chi_{2,t})$  equal to  $(1, 0.87)$  is admissible. Of the 0.93 reduction in noise variance in the “first” GDP growth estimate, part stems from a reduction in idiosyncratic GDP noise, part stems from a reduction in common noise also removed from GDI growth, and part stems from a reduction in common noise not also removed from GDI growth. This appears quite reasonable to us, and we take this case as our preferred set of assumptions.<sup>13</sup>

The last two columns show news shares resulting from minimization of the ratio of optimal weights on the two components, with determined by equation (3). Minimizing the relative weight on GDP growth amounts to minimizing the overall idiosyncratic news shares, and minimizing the relative weight on GDI growth yields results similar to maximizing the idiosyncratic news shares, except for the assumption on  $\chi_{2,f}$ .

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<sup>13</sup>The assumption that  $\chi_{2,f} = 1$  is unlikely in this scenario, but given the small size of that idiosyncratic component, this assumption makes very little difference to the optimal combination formulas.

## 5 Estimates of “True” Unobserved Output Growth

Table 3 reports maximum likelihood estimates of the means, covariances, and idiosyncratic variances of GDP and GDI growth, with standard errors beneath the the estimates. These are slightly different from the statistics reported in table 1, because the estimation here imposes equality of mean GDP growth and mean GDI growth for each vintage.<sup>14</sup> The statistics for the two vintages are estimated jointly, along with covariances between vintages. This allows us to decompose the revision variances into news and noise as in the previous section, and recompute the bounds on the news shares implied by the equations in Appendix C; these bounds were very similar to those reported in in table 2. The news shares corresponding to each of these bounds imply optimal weights for GDP and GDI growth via equation (3); these are reported in table 3 with standard errors.<sup>15</sup>

Consider first the weights for the “first” vintage estimates. Under all sets of assumptions considered, the weights on GDP and GDI growth sum to less than one: it is optimal to downweight the “first” estimates, shrinking them back towards their mean. This is a consequence of the common noise in the “first” estimates implied by the revisions evidence - the fact that revisions reduce the covariance between the estimates. The down-weighting filters some of this noise out of the data. Regarding the relative weight to be placed on the GDP vs GDI for the “first” estimates, the bounds do not rule out weighting schemes that place most of the relative weight on either GDP or GDI.<sup>16</sup>

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<sup>14</sup>Since “true” output growth  $\Delta y_t^*$  has only one unconditional mean, imposing this through the estimation seemed natural.

<sup>15</sup>As in the previous section, these weights assume equation (C.10) holds with equality; the weights for the “last” estimates assume  $\chi_l = 1$ , making the assumption typical of dynamic factor models that covariance is signal.

<sup>16</sup>However, Nalewaik (2007a) uses real-time data to show that GDI growth has tended to recognize cyclical turning points faster than GDP growth, suggesting that it is optimal to place at least some weight on the “first” estimates of GDI growth.

Consider next the weights for the “last” estimates. Two main points stand out. First, the weights on GDP and GDI growth exceed one, opposite the usual noise result that down-weighting is optimal; see section 2.2.2. Second, and probably more important, GDI receives a substantial weight, no matter what set of assumptions we make. In fact, the weights on GDI growth are remarkably uniform across these different sets of assumptions, ranging from 0.59 to 0.65. Even when we minimize the relative weight on GDI growth, its weight is 0.59 and about equal to the weight on GDP growth. In the other sets of assumptions, GDI growth receives a larger weight than GDP growth.<sup>17</sup>

In the first or third set of assumptions, the lower bound of  $\chi_{2,l}$  based on the revisions evidence is binding. This lower bound shows that the pure noise model for the “last” estimates is inconsistent with the assumption that revisions either add news or decrease noise, but the bound is a function of estimated parameters, so there is some uncertainty about whether this lower bound is really above zero. A statistical test of whether  $\chi_{2,l} > 0$  is equivalent to a test of whether the difference between  $\tau_{2,l}$  and the estimated reduction in noise in  $\Delta y_t^{1,f}$  is greater than zero (since that noise reduction in GDP may add idiosyncratic noise to GDI), where the noise reduction is computed as  $\sigma_f^2 + \tau_{1,f}^2 - \text{cov}(\Delta y_t^{1,l}, \Delta y_t^{1,f})$ . This difference is 1.63, with a standard error of 0.58; we reject the pure noise model at conventional significance levels based on evidence from revisions, even taking on board the unlikely assumption that all of the reduction in noise in “final” GDP growth stems from the common component, with none of that noise removed from GDI growth.

As discussed in the previous section, the second set of assumptions is the set we

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<sup>17</sup>One sensible way to proceed may be to choose the midpoint of this range of feasible relative weights, which would place a greater weight on GDI. Minimax estimation over the unidentified parameters of the model may lead one to choose such a midpoint of the feasible set of relative weights. Thanks to Mark Watson suggesting the Minimax approach; see Watson (1987) and Lehmann and Casella (1998) for an example and description of the Minimax approach.

consider most likely. In this case, the informativeness of GDI relative to GDP increases in the revision from “first” to “last”, as a greater amount of useful information is incorporated into GDI. This interpretation is consistent with the findings in Nalewaik (2007a, 2007b), who shows that although GDI appears to be more informative than GDP in recognizing recessions (or, more precisely, more informative in recognizing the state of the world in a two-state Markov switching model for the economy’s growth rate), much of that greater information content comes from the information in annual and benchmark revisions.

Placing a greater weight on GDI growth in analyzing the historical behavior of the economy leads to some interesting modifications of economic history, as illustrated in Figures 2 and 3. These figures show “last” GDP and GDI growth, and a weighted average of the two using the weights from our preferred set of news shares in table 3, with the three series deflated by the GDP deflator. Compared with GDP, the composite estimate shows weaker recoveries from the recessions of 1990-1991 and 2001. Average annualized output growth over the last three quarters of 1991 was about three-quarters of a percentage point less than recorded by measured GDP. The economy leading up to the 1990-1991 recession was also weaker, with growth over the four quarters of 1989 a full percentage point less than measured GDP. Finally, the 2001 recession was substantially more severe than GDP indicates, with output over the four quarters of 2001 contracting 0.6%; GDP shows an expansion of 0.4%. In fitting structural economic relationships, these results should be useful.

It is interesting to note that in the fourth quarter of 1999, the growth rate of the combined estimate exceeds the growth rate of both GDP and GDI, while in the third quarter of 2001, the combined growth is less than each estimate. These examples reflect weights on the component series that sum to more than one, a consequence of the assumption that the idiosyncratic variances of the component series are largely news.

$\chi_{1,l}\tau_{1,l}$  and  $\chi_{2,l}\tau_{2,l}$  are independent pieces of information about “true” output growth, independent of each other and the common information  $\sigma_l^2$ . Adding these three terms together gives an estimate of the variance of “true” GDP growth  $\Delta y_t^*$ , based on the information in GDP and GDI growth, and this represents a lower bound on the actual variance of  $\Delta y_t^*$  since there is likely additional information about  $\Delta y_t^*$  contained in neither estimate. This lower bound of 6.50 is greater than the variance of either GDP growth (4.30) or GDI growth (5.50), a fact with potentially important implications for a wide class of economic models that depend importantly on the variance of the growth rate of the economy, for example many real business cycle and asset pricing models.

## 6 Conclusions

The main contributions and insights of this paper are the following;

- The paper derives a simple decomposition of revisions into news and noise, which uses only the variance of the revision, and the variance of the pre- and post-revision estimates. This decomposition should help sharpen such studies in the future.
- Using the revision decomposition, we obtain interesting implications for GDP growth and GDI growth, two measures of output growth that differ due to differences in source data. The paper shows how to use revisions to place bounds on the share of the idiosyncratic variation in each measure that is news or signal about “true” output growth. The initial GDI growth estimates have little idiosyncratic variation, less than GDP growth, but after passing through revisions the idiosyncratic variance of GDI increases substantially. The fact that this idiosyncratic variation stems from revisions, combined with the assumption that revisions add



news but not noise,<sup>18</sup> leads to the strong presumption that this variation is news. Formally testing the hypothesis that the idiosyncratic variation in GDI growth is noise, we reject at conventional significance levels.

- The fact that some of the idiosyncratic variation in GDI growth is news or signal runs contrary to heretofore implicit assumptions employed in taking weighted averages of imperfectly measured statistics. Previous attempts to produce the best possible estimate of “true” output growth by combining measured GDP growth and GDI growth have made the strong implicit assumption that the idiosyncratic variation in each measured statistic is pure noise, or completely uncorrelated with “true” output. We develop new models that relax this assumption, allowing the idiosyncratic variation in each measured statistic to be partly or pure news - i.e. correlated with “true” output. This generalized model may weight more heavily the statistic with higher idiosyncratic variance, since it may contain a greater amount of information about “true” output, in contrast to previous models which weight less heavily the statistic with higher idiosyncratic variance, assuming it contains more noise. In fact, we show that absent evidence shedding some light whether variation is news or noise, the weights in any weighted average of imperfectly measured statistics are totally arbitrary.
- When combining the GDP and GDI growth estimates in the period after the mid-1980s, we show that placing a large weight on GDI is optimal, precisely because of its relatively large idiosyncratic variation. Doing so alters economic history in interesting ways. For example, the 2001 recession was more severe than indicated by measured GDP growth, and economic growth around the 1990-1991 recession

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<sup>18</sup>This assumption is that of Mankiw, Runkle, and Shapiro (1984), Mankiw and Shapiro (1986), and numerous other papers following their seminal work.

was weaker than measured GDP growth. Over other time periods, such as the mid- and late-1990s, GDP understated output growth. In sorting out the large discrepancies between GDP and GDI growth in 2007 and 2008, we await more evidence from revisions, but our results suggest the NBER Business Cycle Dating Committee was right to place at least some weight on the relatively weak GDI growth estimates.

- Our results indicate that the true variance of the growth rate of the economy is not equal to the variance of measured GDP growth, as is often assumed in real business cycle, asset pricing, and other models; the true variance is actually higher.

The news vs. noise considerations highlighted here are ubiquitous when attempting to estimate unobservable. Take the well known index of coincident indicators as constructed by Stock and Watson (1989), used by Diebold and Rudebusch (1996) and many other economists. Stock and Watson decompose each of four time series into a common factor plus an idiosyncratic component; a time series that covaries relatively less with the other three will receive less weight in the common factor and have higher idiosyncratic variance. Stock and Watson define the state of the economy as this common factor, so a series with greater (relative) idiosyncratic variance receives less weight in this construct. Is this best weighting? There may be good reasons to define the state of the economy as this common factor, following the venerable tradition of Burns and Mitchell (1946). However if we define the state of the economy as something other than this common factor, the answer to this question is unclear: if the idiosyncratic components of the time series are noise, the Stock and Watson approach is appropriate, but if the idiosyncratic components are news, then time series that contain much idiosyncratic variation are uniquely informative about the state of the economy, and should be weighted *more* heavily.

Unfortunately, before such issues can be sorted out, a clearer definition of what we are attempting to measure in a factor analysis must be forthcoming. Without a clear definition of the unobservable of interest, it is not even clear which variables to include in a factor analysis, let alone how we should weight them with factor loadings.

Similar issues obviously arise in the burgeoning literature on dynamic factor models using large datasets. Often the common factors are used for pure forecasting as in Stock and Watson (2002a,b), and our results have little relevance for those applications. But sometimes they are equated with unobservables of interest, assuming the idiosyncratic components of the variables in the dataset are uninteresting noise.<sup>19</sup> For example, Bernanke et al (2005) equate linear combinations of common factors with four unobserved variables: (1) the output gap, (2) a cost-push shock, (3) output, and (4) inflation. They take these last two as unobservable due to measurement difficulties, in the same spirit as our work here. However it is unlikely that the idiosyncratic components of all 120 time series they use to extract the common factors are uncorrelated with these four unobservables. For example, our results indicate that information from the income side of the national accounts probably contains useful information about the growth rate of output, above and beyond the information contained in expenditure-side variables. So it may be possible to improve the results in Bernanke et al (2005), for example by allowing correlation between unobservables (1) or (3) and the idiosyncratic components of their employment and income variables.

These examples illustrate that the noise assumption, treating idiosyncratic variance as a bad, is often implicit in models of imperfect measurement. We have identified

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<sup>19</sup>We have heard that some of the consistency results in this literature do not rely on the idiosyncratic terms being uncorrelated with the factors. Apparently, as  $N, T \rightarrow \infty$ , the factors estimated by principle components converge to the “true” underlying factors even if the idiosyncratic terms are correlated with the “true” factors. The issue we raise is different: whether the weightings on a fixed set of  $N$  time series are optimal, in the sense that they minimize the squared deviations of the estimated factors from the “true” factors.

circumstances where this assumption is inappropriate, where some idiosyncratic variation should be treated as a good rather than a bad. While realizing this leads to some fundamental indeterminacies, we have taken some initial steps here towards resolving them.

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**Appendix A: Relation to Earlier Work Based on  
Stone, Champernowne, and Meade (1942)**

Equation (3') with the pure noise assumptions yields  $\widehat{\Delta y_t^*} = \frac{\tau_2^2 \Delta y_t^1 + \tau_1^2 \Delta y_t^2}{\tau_1^2 + \tau_2^2}$ , essentially the estimator presented in Weale (1992).<sup>20</sup> This paper applied to the case of U.S. GDP and GDI the techniques developed in Stone, Champernowne, and Meade (1942) and Byron (1978); see also Weale (1985), and Smith, Satchell, and Weale (1998). In the general case, Stone et al (1942) considered a row vector of estimates  $x$  that should but do not satisfy the set of accounting constraints  $Ax = 0$ . They produce a new set of estimates  $\tilde{x}^*$  that satisfy the constraints by solving the constrained quadratic minimization problem:

$$(A.1) \quad \begin{array}{ll} \text{MIN} & \\ \tilde{x}^* & (\tilde{x}^* - x)' V^{-1} (\tilde{x}^* - x) \\ \text{S.T.} & A\tilde{x}^* = 0. \end{array}$$

The matrix  $V$  represents a variance-covariance matrix of  $x^* - x$ , where  $x^*$  is the vector of “true” values estimated by  $x$ , so  $V^{-1}$  is an estimate of “precision”. The case at hand maps to this framework with the minimization problem looking like:

$$\begin{array}{ll} \text{MIN} & \\ \widetilde{\Delta y_t^{1*}}, \widetilde{\Delta y_t^{2*}} & \left( \begin{array}{cc} \widetilde{\Delta y_t^{1*}} - \Delta y_t^1 & \widetilde{\Delta y_t^{2*}} - \Delta y_t^2 \end{array} \right) V^{-1} \left( \begin{array}{c} \widetilde{\Delta y_t^{1*}} - \Delta y_t^1 \\ \widetilde{\Delta y_t^{2*}} - \Delta y_t^2 \end{array} \right) \\ \text{S.T.} & \widetilde{\Delta y_t^{1*}} - \widetilde{\Delta y_t^{2*}} = 0. \end{array}$$

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<sup>20</sup>Weale (1992) allowed for covariance between the measurement errors  $\varepsilon_t^1$  and  $\varepsilon_t^2$ . This has no impact on the weights when they are constrained to sum to one.



Substituting the constraint into the objective function, we have:

$$(A.2) \quad \underset{\widetilde{\Delta y_t^*}}{\text{MIN}} \left( \begin{array}{cc} \widetilde{\Delta y_t^*} - \Delta y_t^1 & \widetilde{\Delta y_t^*} - \Delta y_t^2 \end{array} \right) V^{-1} \left( \begin{array}{c} \widetilde{\Delta y_t^*} - \Delta y_t^1 \\ \widetilde{\Delta y_t^*} - \Delta y_t^2 \end{array} \right),$$

with  $\widetilde{\Delta y_t^*} = \widetilde{\Delta y_t^{1*}} = \widetilde{\Delta y_t^{2*}}$ . The judgement in this approach involves the choice of  $V$ . Stone et al (1942) are not so specific in their recommendations, but it seems logical to use estimates of the variance of measurement errors, as defined in the noise model, to compute  $V$ , and this is the tack taken by much of the literature following Stone et al (1942). The main point of this paper is that it is also important to consider the relative information content of the different estimates: if one estimate contains much more news than the other estimate, we may want to adjust that estimate less than the other, even if it contains more noise as well. Weale (1992) assumes the idiosyncratic variances of GDP and GDI, the  $\tau_i^2$ , are measurement errors, as in the noise model above. Under these assumptions, we have:

$$V = \begin{pmatrix} \tau_1^2 & 0 \\ 0 & \tau_2^2 \end{pmatrix}.$$

Solving the quadratic minimization problem with this  $V$ , we have  $\widetilde{\Delta y_t^*} = \frac{\tau_2^2 \Delta y_t^1 + \tau_1^2 \Delta y_t^2}{\tau_1^2 + \tau_2^2}$ , the same result as the restricted pure noise model.

Problem (A.2) is a different minimization problem than the least squares minimization problems that we solve in this paper, where we solve for the weights in (1) or (1') and then compute the predicted values  $\widehat{\Delta y_t^*}$ ; problem (A.2) solves for  $\widetilde{\Delta y_t^*}$  directly, leaving the weights implicit. In solving for the weights in (1) or (1'), assumptions must be made about the covariances between  $\Delta y_t^*$  and the estimates  $\Delta y_t^i$ , whereas in (A.2) assumptions must be made about  $V$ ; as we have seen, when these assumptions are equivalent and

when some constraints are applied to (1), the two approaches can give the same result. Comparing the Stone, Champernowne, and Meade (1942) approach with the approach taken here, in a more general setting such as in (A.1), is beyond the scope of this paper, but is an interesting avenue for future research.

## Appendix B: A Simple Example of the Bivariate News Model

We will consider two efficient estimates of true GDP growth, one based on consumption growth, and the other based on the growth rate of investment. After constructing each efficient estimate, we will discuss how to produce the improved estimate of true GDP growth by combining them with equation (5).

Let  $\Delta C_t$ ,  $\Delta I_t$ ,  $\Delta G_t$ , and  $\Delta NX_t$  be the contributions to true GDP growth  $\Delta y_t^*$  of consumption, investment, government, and net exports, so:

$$\Delta y_t^* = \Delta C_t + \Delta I_t + \Delta G_t + \Delta NX_t.$$

Our first efficient estimate of  $y_t^*$ ,  $\Delta y_t^1$ , is based on  $\mathcal{F}_t^1 = [1, \Delta C_t]$ , a constant and consumption growth, and the second is based on  $\mathcal{F}_t^2 = [1, \Delta I_t]$ , a constant and investment growth; the constant in either information set reveals  $\mu$ , the mean of  $y_t^*$ , as well as the means of the component growth rates. Then our efficient estimates will take the form:

$$\begin{aligned} \Delta y_t^1 &= \mu + (\Delta C_t - \mu_C) + E(\Delta I_t - \mu_I | \mathcal{F}_t^1) + E(\Delta G_t + \Delta NX_t - \mu_G - \mu_{NX} | \mathcal{F}_t^1); \\ \Delta y_t^2 &= \mu + (\Delta I_t - \mu_I) + E(\Delta C_t - \mu_C | \mathcal{F}_t^2) + E(\Delta G_t + \Delta NX_t - \mu_G - \mu_{NX} | \mathcal{F}_t^2). \end{aligned}$$

For simplicity, we will examine the case where neither  $\mathcal{F}_t^1$  nor  $\mathcal{F}_t^2$  contains any useful information about  $\Delta G_t + \Delta NX_t - \mu_G - \mu_{NX}$ , so the last term in each of the above expressions is zero, and  $\Delta G_t + \Delta NX_t - \mu_G - \mu_{NX}$  represents the information about  $y_t^*$

contained in neither of our two estimates.

The relation between  $\Delta C_t$  and  $\Delta I_t$  determines the nature of the efficient estimates and weights on  $\Delta y_t^1$  and  $\Delta y_t^2$  in equation (5). Consider first the case where these variables are independent. Then:

$$\begin{aligned}\Delta y_t^1 &= \mu + (\Delta C_t - \mu_C) \quad \text{and:} \\ \Delta y_t^2 &= \mu + (\Delta I_t - \mu_I).\end{aligned}$$

There is no information common to  $\mathcal{F}_t^1$  and  $\mathcal{F}_t^2$ , no covariance between the estimates, so  $\sigma^2 = 0$ . Equation (5) instructs us to remove the mean from each estimate, and then simply add them. Adding back in the mean, we have the natural result:

$$\widehat{\Delta y_t^*} = \mu + (\Delta C_t - \mu_C) + (\Delta I_t - \mu_I).$$

The weight on each estimate (net of mean) is just one; as mentioned in the previous subsection, this is the case where we are essentially adding independent contributions to GDP growth.

Next consider the case where  $\Delta C_t$  and  $\Delta I_t$  are perfectly correlated, so:

$$(\Delta I_t - \mu_I) = a (\Delta C_t - \mu_C),$$

where  $a$  is some constant. Then:

$$\begin{aligned}\Delta y_t^1 &= \mu + (1 + a) (\Delta C_t - \mu_C) = \mu + (\Delta C_t - \mu_C) + (\Delta I_t - \mu_I) \quad \text{and:} \\ \Delta y_t^2 &= \mu + (1 + \frac{1}{a}) (\Delta I_t - \mu_I) = \mu + (\Delta C_t - \mu_C) + (\Delta I_t - \mu_I).\end{aligned}$$

Given that  $\Delta y_t^1 = \Delta y_t^2$ , taking a weighted average of the two produces the same estimate as long as the weights in the average sum to one. There is no idiosyncratic variance to either estimate, so  $\tau_1^2 = \tau_2^2 = 0$ , and equation (5) instructs us to use a weight of 0.5 for each estimate.<sup>21</sup>

Finally consider the general linear case. In this case:

$$\begin{aligned} E(\Delta I_t - \mu_I | \mathcal{F}_t^1) &= a(\Delta C_t - \mu_C) \quad \text{and:} \\ E(\Delta C_t - \mu_C | \mathcal{F}_t^2) &= b(\Delta I_t - \mu_I) \end{aligned}$$

Least squares projections tell us that  $a = \frac{\sigma_{ci}}{\sigma_c^2}$ , where  $\sigma_{ci}$  is the covariance between  $\Delta I_t$  and  $\Delta C_t$ , and  $\sigma_c^2$  is the variance of  $\Delta C_t$ . Similarly,  $b = \frac{\sigma_{ci}}{\sigma_i^2}$ , where  $\sigma_i^2$  is the variance of  $\Delta I_t$ , and the fraction of the variance of each variable explained by the other,  $R^2$ , is  $\frac{\sigma_{ci}^2}{\sigma_i^2 \sigma_c^2}$ . The efficient estimates of  $\Delta y_t^*$  are:

$$\begin{aligned} \Delta y_t^1 &= \mu + (1 + a)(\Delta C_t - \mu_C) \quad \text{and:} \\ \Delta y_t^2 &= \mu + (1 + b)(\Delta I_t - \mu_I). \end{aligned}$$

The variance parameters of the news model are identified from the following relations:

$$\begin{aligned} \sigma^2 &= \text{cov}(\Delta y_t^1, \Delta y_t^2) = (1 + a)(1 + b)\sigma_{ci}, \\ \tau_1^2 &= \text{var}(\Delta y_t^1) - \text{cov}(\Delta y_t^1, \Delta y_t^2) = (1 + a)^2\sigma_c^2 - (1 + a)(1 + b)\sigma_{ci} \quad \text{and:} \\ \tau_2^2 &= \text{var}(\Delta y_t^2) - \text{cov}(\Delta y_t^1, \Delta y_t^2) = (1 + b)^2\sigma_i^2 - (1 + a)(1 + b)\sigma_{ci}. \end{aligned}$$

Substituting  $a = \frac{\sigma_{ci}}{\sigma_c^2}$  and  $b = \frac{\sigma_{ci}}{\sigma_i^2}$ , we see that both  $\tau_1^2 > 0$  and  $\tau_2^2 > 0$  if  $\sigma_{ci}^2 < \sigma_i^2 \sigma_c^2$ , or if  $R^2 < 1$ . If  $R^2 = 1$ , we are back to the perfect correlation case with  $\tau_1^2 = 0$  and  $\tau_2^2 = 0$ ;

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<sup>21</sup>These weights can be derived through application of L'Hopital's rule.

if  $R^2 = 0$ , we are back to independence with  $\sigma^2 = 0$ . In all intermediate cases, the sum of the two weights (net of mean) will range between 1 and 2.

It should be pointed out that, when combining  $\Delta y_t^1$  and  $\Delta y_t^2$  in this particular example, using equation (5) is not the most natural way to proceed. An easier and more intuitive procedure would be to set  $a(\Delta C_t - \mu_C)$  to zero in  $\Delta y_t^1$ , set  $b(\Delta I_t - \mu_I)$  to zero in  $\Delta y_t^2$ , and then combine, producing:

$$\widehat{\Delta y_t^*} = \mu + (\Delta C_t - \mu_C) + (\Delta I_t - \mu_I).$$

This is the best possible estimate of  $\widehat{\Delta y_t^*}$  given the information in  $\mathcal{F}_t^1$  and  $\mathcal{F}_t^2$ , so any estimate based on (5) can only be worse. This result highlights one of the key assumptions of the model: it assumes that the econometrician does not have enough information to set to zero or re-weight individual components of either estimate  $\Delta y_t^i$ ; the econometrician must take each  $\Delta y_t^i$  in its totality. Considering different weights for different components of GDP and GDI is another interesting avenue for future research.

### Appendix C: Revision Equations Determining Bounds on $\chi$ Parameters

Consider first the covariance between the revision to GDP growth and the revision to GDI growth:

$$\begin{aligned} \text{cov}(\Delta y_t^{1,l} - \Delta y_t^{1,f}, \Delta y_t^{2,l} - \Delta y_t^{2,f}) &= \text{cov}(\zeta_t^{1,fl} - \varepsilon_t^{1,fl}, \zeta_t^{2,fl} - \varepsilon_t^{2,fl}) \\ \text{(C.1)} \qquad \qquad \qquad &= \text{cov}(\zeta_t^{1,fl}, \zeta_t^{2,fl}) + \text{cov}(\varepsilon_t^{1,fl}, \varepsilon_t^{2,fl}). \end{aligned}$$

The change in the covariance between GDP growth and GDI growth (pre- and post-revision) is a more complicated expression:

$$\begin{aligned}
\text{cov}(\Delta y_t^{1,l}, \Delta y_t^{2,l}) - \text{cov}(\Delta y_t^{1,f}, \Delta y_t^{2,f}) &= \text{cov}\left(E\left(\Delta y_t^* | \mathcal{F}_t^{1,l}\right) + \varepsilon_t^{1,l}, E\left(\Delta y_t^* | \mathcal{F}_t^{2,l}\right) + \varepsilon_t^{2,l}\right) \\
&\quad - \text{cov}\left(E\left(\Delta y_t^* | \mathcal{F}_t^{1,f}\right) + \varepsilon_t^{1,f}, E\left(\Delta y_t^* | \mathcal{F}_t^{2,f}\right) + \varepsilon_t^{2,f}\right). \\
\text{(C.2)} \qquad \qquad \qquad &= \text{cov}\left(E\left(\Delta y_t^* | \mathcal{F}_t^{1,l}\right), E\left(\Delta y_t^* | \mathcal{F}_t^{2,l}\right)\right) + \text{cov}(\varepsilon_t^{1,l}, \varepsilon_t^{2,l}) \\
&\quad - \text{cov}\left(E\left(\Delta y_t^* | \mathcal{F}_t^{1,f}\right), E\left(\Delta y_t^* | \mathcal{F}_t^{2,f}\right)\right) - \text{cov}(\varepsilon_t^{1,f}, \varepsilon_t^{2,f}),
\end{aligned}$$

using the independence of the noise terms from the conditional expectations.

Drilling down further, for the covariance between the conditional expectations in (C.2), we have  $\text{cov}\left(E\left(\Delta y_t^* | \mathcal{F}_t^{1,f}\right), E\left(\Delta y_t^* | \mathcal{F}_t^{2,f}\right)\right) = \chi_f \sigma_f^2$ , and:

$$\begin{aligned}
\text{cov}\left(E\left(\Delta y_t^* | \mathcal{F}_t^{1,l}\right), E\left(\Delta y_t^* | \mathcal{F}_t^{2,l}\right)\right) &= \text{cov}\left(E\left(\Delta y_t^* | \mathcal{F}_t^{1,f}\right) + \zeta_t^{1,fl}, E\left(\Delta y_t^* | \mathcal{F}_t^{2,f}\right) + \zeta_t^{2,fl}\right) \\
&= \chi_f \sigma_f^2 + \text{cov}\left(E\left(\Delta y_t^* | \mathcal{F}_t^{1,f}\right), \zeta_t^{2,fl}\right) \\
\text{(C.3)} \qquad \qquad \qquad &\quad + \text{cov}\left(\zeta_t^{1,fl}, E\left(\Delta y_t^* | \mathcal{F}_t^{2,f}\right)\right) + \text{cov}(\zeta_t^{1,fl}, \zeta_t^{2,fl}) \\
&= \chi_l \sigma_l^2.
\end{aligned}$$

The common news in the “last” estimates is equal to the common news in the “first” estimates plus terms stemming from the revisions. The last term  $\text{cov}(\zeta_t^{1,fl}, \zeta_t^{2,fl})$  is information revealed to both estimates that was reflected in neither “first” estimate. The two  $\text{cov}\left(E\left(\Delta y_t^* | \mathcal{F}_t^{j,f}\right), \zeta_t^{i,fl}\right)$  terms are information reflected in one “first” estimate but not the other, that is then revealed to the other estimate through revisions, thus making the information common to the “last” estimates. Put differently, this is information in  $\mathcal{F}_t^{j,f}$  (and thus  $\mathcal{F}_t^{j,l}$  since these information sets can only increase), not in  $\mathcal{F}_t^{i,f}$ , but in  $\mathcal{F}_t^{i,l}$ . Each of these terms is positive, so common news can only increase through revisions.

Similarly, for the noise terms in (C.2),  $\text{cov}(\varepsilon_t^{1,l}, \varepsilon_t^{2,l}) = (1 - \chi_l)\sigma_l^2$ , and:

$$\begin{aligned}
\text{cov}(\varepsilon_t^{1,f}, \varepsilon_t^{2,f}) &= \text{cov}(\varepsilon_t^{1,l} + \varepsilon_t^{1,fl}, \varepsilon_t^{2,l} + \varepsilon_t^{2,fl}) \\
&= (1 - \chi_l)\sigma_l^2 + \text{cov}(\varepsilon_t^{1,fl}, \varepsilon_t^{2,l}) + \text{cov}(\varepsilon_t^{1,l}, \varepsilon_t^{2,fl}) + \text{cov}(\varepsilon_t^{1,fl}, \varepsilon_t^{2,fl}) \\
&= (1 - \chi_f)\sigma_f^2, \quad \text{so:} \\
\text{(C.4)} \quad (1 - \chi_l)\sigma_l^2 &= (1 - \chi_f)\sigma_f^2 - \text{cov}(\varepsilon_t^{1,fl}, \varepsilon_t^{2,l}) - \text{cov}(\varepsilon_t^{1,l}, \varepsilon_t^{2,fl}) - \text{cov}(\varepsilon_t^{1,fl}, \varepsilon_t^{2,fl}).
\end{aligned}$$

The common noise in the “last” estimates equals the common noise in the “first” estimates minus three revision terms. The last term  $\text{cov}(\varepsilon_t^{1,fl}, \varepsilon_t^{2,fl})$  is the common noise in the “final” estimates removed from both estimates by revision. The two  $\text{cov}(\varepsilon_t^{i,l}, \varepsilon_t^{j,fl})$  terms are common noise in the “first” estimates removed from one estimate by revision, but not the other. These three terms all reduce the common noise in the “last” estimates, so common noise can only fall through revision.

Substituting (C.3) and (C.4) into (C.2) yields:

$$\begin{aligned}
\text{cov}(\Delta y_t^{1,l}, \Delta y_t^{2,l}) - \text{cov}(\Delta y_t^{1,f}, \Delta y_t^{2,f}) &= \text{cov}\left(E\left(\Delta y_t^* | \mathcal{F}_t^{1,f}\right), \zeta_t^{2,fl}\right) \\
&\quad + \text{cov}\left(\zeta_t^{1,fl}, E\left(\Delta y_t^* | \mathcal{F}_t^{2,f}\right)\right) + \text{cov}(\zeta_t^{1,fl}, \zeta_t^{2,fl}) \\
&\quad - \text{cov}(\varepsilon_t^{1,fl}, \varepsilon_t^{2,l}) - \text{cov}(\varepsilon_t^{1,l}, \varepsilon_t^{2,fl}) \\
\text{(C.5)} \quad &\quad - \text{cov}(\varepsilon_t^{1,fl}, \varepsilon_t^{2,fl}).
\end{aligned}$$

The relation between (C.1) and (C.5) is evidently a bit more complicated than the relation between (7) and (8). The covariances between the revisions and the initial estimates provides some additional information:

$$\begin{aligned}
\text{cov}(\Delta y_t^{1,f}, \Delta y_t^{2,l} - \Delta y_t^{2,l}) &= \text{cov}\left(E\left(\Delta y_t^* | \mathcal{F}_t^{1,f}\right) + \varepsilon_t^{1,f}, \zeta_t^{2,fl} - \varepsilon_t^{2,fl}\right) \\
&= \text{cov}\left(E\left(\Delta y_t^* | \mathcal{F}_t^{1,f}\right) + \varepsilon_t^{1,l} + \varepsilon_t^{1,fl}, \zeta_t^{2,fl} - \varepsilon_t^{2,fl}\right) \\
\text{(C.5')} \quad &= \text{cov}\left(E\left(\Delta y_t^* | \mathcal{F}_t^{1,f}\right), \zeta_t^{2,fl}\right) - \text{cov}(\varepsilon_t^{1,l}, \varepsilon_t^{2,fl}) - \text{cov}(\varepsilon_t^{1,fl}, \varepsilon_t^{2,fl}).
\end{aligned}$$

Similarly:

$$(C.5'') \quad \text{cov}(\Delta y_t^{2,f}, \Delta y_t^{1,l} - \Delta y_t^{1,f}) = \text{cov}\left(E\left(\Delta y_t^* | \mathcal{F}_t^{2,f}\right), \zeta_t^{1,fl}\right) - \text{cov}\left(\varepsilon_t^{2,l}, \varepsilon_t^{1,fl}\right) - \text{cov}\left(\varepsilon_t^{2,fl}, \varepsilon_t^{1,fl}\right).$$

However, (C.1), (C.5), (C.5') and (C.5'') are linearly dependent, and we have been unable to discover additional restrictions on the six unknowns appearing in these equations, leaving us with no unique solution.

Next consider the idiosyncratic news in the “last” estimate of  $i$ . This is equal to the idiosyncratic news in the “first” estimate of  $i$ , minus the part of this idiosyncratic news revealed to  $j$  by revision (and hence transforming it to common news), plus the idiosyncratic news added to  $i$  by revision,  $\gamma_{i,fl}\tau_{i,l}^2$ :

$$(C.6) \quad \chi_{i,l}\tau_{i,l}^2 = \chi_{i,f}\tau_{i,f}^2 - \text{cov}\left(E\left(\Delta y_t^* | \mathcal{F}_t^{i,f}\right), \zeta_t^{j,fl}\right) + \gamma_{i,fl}\tau_{i,l}^2,$$

The overall increase in news from revisions, computed from (7) and (8), is the sum of this change in idiosyncratic news (C.6) and the change in common news as computed from (C.3):

$$(C.7) \quad \begin{aligned} \text{var}(\zeta_t^{i,fl}) &= \text{var}(\zeta_t^{i,l}) - \text{var}(\zeta_t^{i,f}) \\ &= (\chi_l\sigma_l^2 - \chi_f\sigma_f^2) \\ &\quad + (\chi_{i,l}\tau_{i,l}^2 - \chi_{i,f}\tau_{i,f}^2) \\ &= \text{cov}\left(E\left(\Delta y_t^* | \mathcal{F}_t^{i,f}\right), \zeta_t^{j,fl}\right) + \text{cov}\left(\zeta_t^{i,fl}, E\left(\Delta y_t^* | \mathcal{F}_t^{j,f}\right)\right) + \text{cov}(\zeta_t^{i,fl}, \zeta_t^{j,fl}) \\ &\quad - \text{cov}\left(E\left(\Delta y_t^* | \mathcal{F}_t^{i,f}\right), \zeta_t^{j,fl}\right) + \gamma_{i,fl}\tau_{i,l}^2 \\ &= \text{cov}\left(\zeta_t^{i,fl}, E\left(\Delta y_t^* | \mathcal{F}_t^{j,f}\right)\right) + \text{cov}(\zeta_t^{i,fl}, \zeta_t^{j,fl}) + \gamma_{i,fl}\tau_{i,l}^2. \end{aligned}$$



Finally consider the idiosyncratic noise in each “last” estimate. Let  $(1 - \psi_{i,fl})\tau_{i,f}^2$  be the idiosyncratic noise in  $\Delta y_t^{i,f}$  eliminated by revision. Noise common to the two “first” estimates that is eliminated from  $j$  but not  $i$  now appears as idiosyncratic noise in the “last”  $i$  estimate, so:

$$(C.8) \quad (1 - \chi_{i,l})\tau_{i,l}^2 = (1 - \chi_{i,f})\tau_{i,f}^2 + \text{cov}(\varepsilon_t^{j,fl}, \varepsilon_t^{i,l}) - (1 - \psi_{i,fl})\tau_{i,f}^2.$$

The overall noise reduction from revisions, computed from (7) and (8), is the sum of this change in idiosyncratic noise (C.8) and the change in common noise as computed from (C.4):

$$\begin{aligned} \text{var}(\varepsilon_t^{i,fl}) &= \text{var}(\varepsilon_t^{i,f}) - \text{var}(\varepsilon_t^{i,l}) \\ &= ((1 - \chi_f)\sigma_f^2 - (1 - \chi_l)\sigma_l^2) \\ &\quad + ((1 - \chi_{i,f})\tau_{i,f}^2 - (1 - \chi_{i,l})\tau_{i,l}^2) \\ &= \text{cov}(\varepsilon_t^{j,fl}, \varepsilon_t^{i,l}) + \text{cov}(\varepsilon_t^{j,l}, \varepsilon_t^{i,fl}) + \text{cov}(\varepsilon_t^{j,fl}, \varepsilon_t^{i,fl}) \\ &\quad - \text{cov}(\varepsilon_t^{j,fl}, \varepsilon_t^{i,l}) + (1 - \psi_{i,fl})\tau_{i,f}^2 \\ (C.9) \quad &= \text{cov}(\varepsilon_t^{j,l}, \varepsilon_t^{i,fl}) + \text{cov}(\varepsilon_t^{j,fl}, \varepsilon_t^{i,fl}) + (1 - \psi_{i,fl})\tau_{i,f}^2. \end{aligned}$$

Equations (C.7) and (C.9) for  $i = 1, 2$ , (C.1), (C.5), (C.5') and (C.5'') are eight linearly dependent equations in ten unknowns ( $\psi_{i,fl}$  and  $\gamma_{i,fl}$  for  $i = 1, 2$  and the six terms on the right-hand side of (C.5)). These equations limit the admissible values for the ten unknowns, which in turn limit the range of admissible values for  $\chi_{i,l}$  and  $(1 - \chi_{i,l})$  as can be seen from (C.6) and (C.8). For the “first” estimates, the admissible values for  $(1 - \chi_{i,f})$  are constrained by  $(1 - \psi_{i,fl})\tau_{i,f}^2 \leq (1 - \chi_{i,f})\tau_{i,f}^2$ , while  $\chi_{i,f}$  is constrained

by  $\text{cov}\left(E\left(\Delta y_t^* | \mathcal{F}_t^{j,f}\right), \zeta_t^{i,fl}\right) \leq \chi_{i,f} \tau_{i,f}^2$ . We also have:

$$(C.10) \quad (1 - \chi_f) \sigma_f^2 \geq \text{cov}\left(\varepsilon_t^{1,fl}, \varepsilon_t^{2,l}\right) + \text{cov}\left(\varepsilon_t^{1,l}, \varepsilon_t^{2,fl}\right) + \text{cov}\left(\varepsilon_t^{1,fl}, \varepsilon_t^{2,fl}\right).$$

**Table 1: Summary Statistics: Variances and Covariances, Growth Rates of GDP and GDI, 1984Q3-2005**

	(1)	(2)	(3)	(4)	(5)	(6)
Variance Measure:	First	Last	(2)-(1)	Revision Variance:		
				Total	↑ News	↓ Noise
$\text{var}(GDP), i = 1$	4.06	4.30	0.24	2.10	1.17	0.93
$\text{var}(GDI), i = 2$	3.60	5.50	1.90	2.58	2.24	0.34
$\text{cov}(GDP, GDI)$	3.42	2.91	-0.51	0.39	?	?

**Notes:**  $\Delta y^{i,f}$  is the “first” available estimate of either GDP growth ( $i = 1$ ) or GDI growth ( $i = 2$ ).  $\Delta y^{i,l}$  is the “last” or “latest” available estimate of either GDP or GDI growth.

**Table 2:  
Lower and Upper Bounds on News Shares**

	$\min\left(\frac{\chi_{1,l}\tau_{1,l+}}{\chi_{2,l}\tau_{2,l}}\right)$	$\max\left(\frac{\chi_{1,l}\tau_{1,l+}}{\chi_{2,l}\tau_{2,l}}\right)$	$\min\left(\frac{w_{1,l}}{w_{2,l}}\right)$	$\min\left(\frac{w_{2,l}}{w_{1,l}}\right)$
$\chi_{1,l}$	0.43	1.00	0.43	1.00
$\chi_{2,l}$	0.67	0.87	0.67	0.80
$\chi_{1,f}$	0.17	0.58	0.17	0.63
$\chi_{2,f}$	1.00	1.00	1.00	0.14
$\chi_f$	0.65	0.81	0.65	0.80

**Notes:**  $\tau_{1,l}$  and  $\tau_{2,l}$  are the idiosyncratic variances of the “last” estimates of GDP and GDI growth, where idiosyncratic variance means the variance of the estimate minus its covariance with the other estimate.  $\chi_{1,l}$  and  $\chi_{2,l}$  are the shares of the idiosyncratic variances of “last” GDP and GDI that are news, or signal, rather than noise.  $w_{1,l}$  and  $w_{2,l}$  are the optimal

weights on “last” GDP and GDI growth, using equation (3).  $\chi_{1,f}$ ,  $\chi_{2,f}$ , and  $\chi_f$  are the shares of the idiosyncratic variances and covariance of the “first” estimates of GDP and GDI growth that are news.

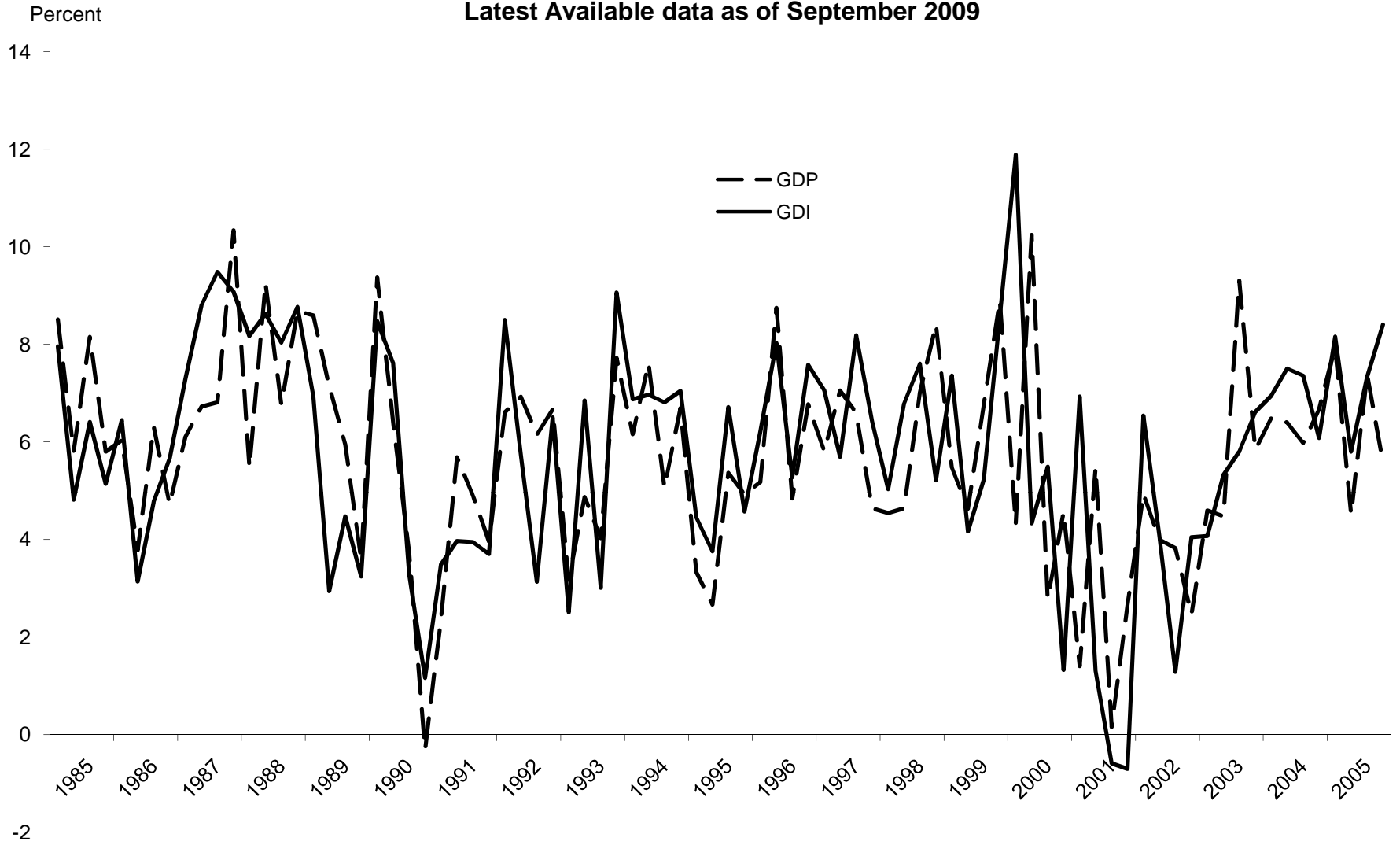
**Table 3: Estimates of True Unobserved GDP Growth**

Vintage	$\mu$	$\sigma$	$\tau_1^2$	$\tau_2^2$	$\min \begin{pmatrix} \chi_{1,l}\tau_{1,l+} \\ \chi_{2,l}\tau_{2,l} \end{pmatrix}$		$\max \begin{pmatrix} \chi_{1,l}\tau_{1,l+} \\ \chi_{2,l}\tau_{2,l} \end{pmatrix}$	
					$w_1$	$w_2$	$w_1$	$w_2$
First	<b>5.60</b>	<b>3.37</b>	<b>0.67</b>	<b>0.19</b>	<b>0.06</b>	<b>0.61</b>	<b>0.42</b>	<b>0.42</b>
	(0.21)	(0.55)	(0.22)	(0.19)	(0.11)	(0.12)	(0.16)	(0.19)
Last	<b>5.83</b>	<b>2.88</b>	<b>1.38</b>	<b>2.56</b>	<b>0.37</b>	<b>0.65</b>	<b>0.57</b>	<b>0.64</b>
	(0.21)	(0.60)	(0.47)	(0.57)	(0.01)	(0.01)	(0.09)	(0.06)

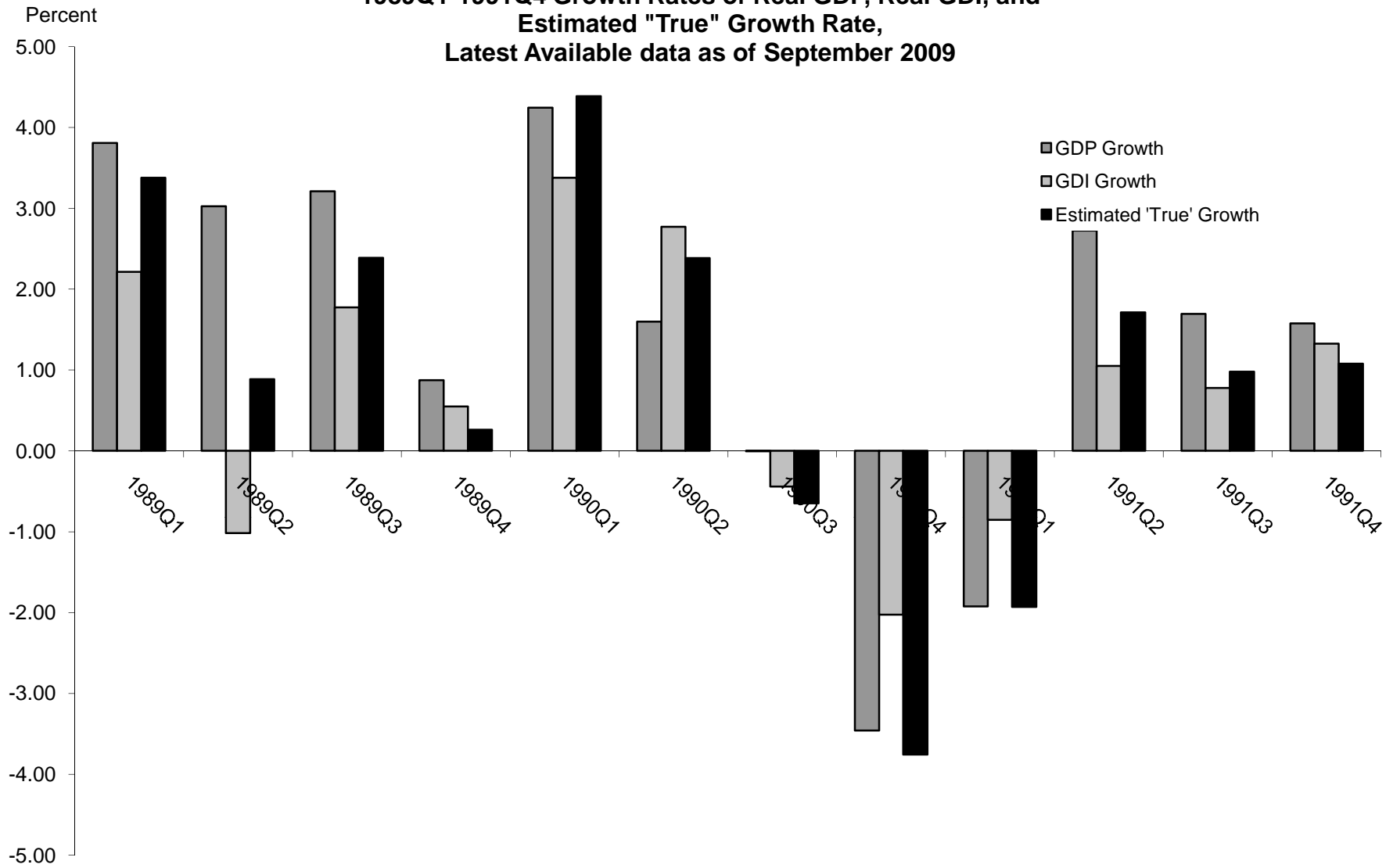
Vintage	$\min \begin{pmatrix} w_{1,l} \\ w_{2,l} \end{pmatrix}$		$\min \begin{pmatrix} w_{2,l} \\ w_{1,l} \end{pmatrix}$	
	$w_1$	$w_2$	$w_1$	$w_2$
First	<b>0.06</b>	<b>0.61</b>	<b>0.63</b>	<b>0.16</b>
	(0.11)	(0.12)	(0.01)	(0.01)
Last	<b>0.37</b>	<b>0.65</b>	<b>0.60</b>	<b>0.59</b>
	(0.01)	(0.01)	(0.09)	(0.06)

**Notes:** Vintage subscripts are suppressed in the “Vintage” row (the second subscript elsewhere, either  $f$  or  $l$ ).  $\mu$  is mean growth,  $\sigma$  is the covariance between the estimates,  $\tau_1$  and  $\tau_2$  are idiosyncratic variances of GDP and GDI growth, while  $w_1$  and  $w_2$  are the weights on GDP and GDI growth, respectively, using equation (3). See the notes to table 2 for further notation definitions.

**Figure 1:**  
**1985 to 2005 Growth Rates of Nominal GDP and GDI,**  
**Latest Available data as of September 2009**



**Figure 2:  
1989Q1-1991Q4 Growth Rates of Real GDP, Real GDI, and  
Estimated "True" Growth Rate,  
Latest Available data as of September 2009**



**Figure 3:  
1999Q4-2003Q2 Growth Rates of Real GDP, Real GDI, and  
Estimated "True" Growth Rate,  
Latest Available data as of September 2009**

