Intermittent Purchases and
Welfare-Based Price Deflators for Durable Goods

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WP2007-02
February 2007

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A B S T R A C T

The consensus among inflation watchers is that biases in official price indexes like the consumer price index overstate true inflation. We identify a new source of bias that works in the other direction. In particular, we use a simple dynamic model for durable goods—one type of good typically purchased intermittently—to derive equivalent variations and a true COL index. We show that, all else held equal, price indexes that rely on market prices will tend to understate the true COL index. With intermittent purchases, the true COL index is based on reservation prices and those prices lie below the market price in periods when consumers opted not to make a purchase. Hence, an equivalent variation that compensates consumers for observed changes in prices will be too generous, thus causing a downward bias in price indexes that rely on market prices.

* Bureau of Economic Analysis, 1441 L Street, NW, Washington, DC 20005. This paper benefited from comments from Roger Betancourt and an anonymous referee. Jordan Boslego provided research assistance. The views expressed here are those of the authors and do not represent views of staff members at the Bureau of Economic Analysis.

1. Introduction

The consensus among inflation watchers is that biases in official price indexes like the consumer price index overstate true inflation (see Lebow and Rudd (2003) for a recent synopsis). Substitution bias arising from the use of fixed weights and the inability to adequately control for quality improvements in new goods are the two main sources of bias (National Research Council (2002)).

We identify a new source of bias that works in the other direction. The problem arises when goods are purchased intermittently. Durable goods—like new vehicles, computers and household appliances—are typically purchased only every few years. Nondurable goods may also be purchased infrequently—Christmas decorations—and many services are also purchased only on occasion—legal advice, visits to physicians, vacations, visits to movies or restaurants.

In contrast, standard price measures implicitly assume that consumers purchase some amount of all available goods in every period. For example, traditional cost of living (COL) theory assumes that a representative consumer purchases some amount of all the existing goods in every period. Similarly, the model typically used to justify hedonic analysis (Rosen (1974)) allows heterogeneous consumers to purchase different quantities of goods each period, but nonetheless assumes that consumers purchase at least some amount of all available goods every period.

We develop an alternative COL index that is based on a simple demand model for durable goods—a class of goods that have important macroeconomic implications. Two important features of these goods leads us to a discrete, dynamic choice model, where a fixed cost incurred at the time of purchase generates intermittent purchases by heterogeneous households. Although some studies in the measurement literature have extended traditional paradigms to consider the construction of cost of living indexes in the context of dynamic problems, and others to allow for heterogeneous consumers making discrete choices, we are not aware of any that consider both issues at the same time.[1]

[1]The theory underlying cost of living indexes uses the concept of utility to provide a solid foundation for the construction of indexes that track changes in the cost of living (COL) for nondurable goods that are consumed and purchased in the same period (Diewert (1993)). Another strand of the literature uses production possibilities boundaries to justify the construction of price deflators. However, both of these approaches rely on static models that do not explicitly take into account the intertemporal nature of durable goods purchases.

We use the value functions implied by our dynamic model to define equivalent variations associated with price change (Bajari, Benkard and Krainer (2005)) and translate those equivalent variations into a true COL index (Trajtenberg (1990)). Following Griliches and Cockburn (1994) and Fisher and Griliches (1995), we use reservation prices to deal with intermittent purchases and the arrival of new goods. In our context, we show that the equivalent variations implied by our model may be restated in terms of market and reservation prices. We derive upper bounds to these reservation prices using observed market prices for continuing goods and hedonic-based bounds for new goods (Pakes (2003)). Using those bounds in place of the unobserved reservation prices in our COL index provides a way to calculate a lower bound to the true COL index.

The true COL index implied by our model is a Paasche index that compares market prices at time *t* to reservation prices in some base period. This has two implications for price indexes for durable goods. First, it implies that superlative indexes like the Fisher cannot be viewed as approximations to a COL index; in our model, the Fisher index is undefined and does not have a welfare interpretation. With regard to prices, our bounds for the reservation price imply that, all else held equal, indexes that use market prices will show *slower* price growth than the true COL index.

The paper is organized as follows. We begin by writing down a simple model and deriving algebraic expressions for the equivalent variations and the COL price deflator implied by our model. In section 3, we translate the price index in terms of reservation prices and find bounds that allow us to define an observable lower bound for the COL deflator. We illustrate these points using results and data from Copeland, Dunn and Hall (2005) for the motor vehicle industry in section 4. A final section concludes.

2. A Simple Model

The traditional treatment for durable goods relies on user-cost or rental-price concepts to track the period-to-period price change associated with using an existing asset (Hall, Jorgenson, Hulten and Wykoff). While useful for purposes of tracking changes in the cost of living, for example, these approaches do not readily provide a price deflator for newly-produced assets except under strong assumptions.

Feenstra and Shapiro (2000) were the first to develop price indexes for goods that have an intertemporal dimension without appealing to user-cost/ rental-cost concepts, but they do not explicitly consider the complications that arise when pricing is designed to exploit heterogeneous consumers. Fisher and Griliches (1995) and Griliches and Cockburn (1994) do take heterogeneity explicitly into account in a static setting but do not consider the intertemporal nature of durable goods purchases.

This section uses a simple discrete, dynamic choice model to derive equivalent variations and a COL index for durable goods.[2]

Consider a model where a consumer faces $J+1$ mutually-exclusive options in each of T periods; the consumer either purchases one of the $j=\{1, 2,\ldots J\}$ goods or no good at all ($j=0$). The utility associated with each period is a function of the service flows from a durable good and from the consumption of a bundle of nondurable goods, specified as a fixed fraction of current income, λY^{t}. The service flows from the durable good are a function of a vector of the good's characteristics, X_j^{τ}, where $\tau \in \{1,2, \ldots, t\}$ denotes the period in which the durable good was purchased. The time t utility for a consumer who receives a flow of services from a durable good j bought at time τ is: $U(X_j^{\tau}, \lambda Y^{t})$.

Each period, the consumer, taking into account his expectations about future prices and the availability of future goods and their characteristics, plans which models to purchase and in which periods. In the recursive version of the model, the state variables in this problem are the asset the consumer currently holds, say X_g^{τ}, the consumer's wealth, W_t, and the set of information available to consumers, Ω^{t}. This information set includes prices and characteristics for all current and expected products. Because we do not allow borrowing, all purchases must be financed through wealth. For a period t, we can write the alternative-specific value function associated with purchasing alternative $j=\{1,2,\ldots,J\}$ as

$$V_j^{\,t}(X_g^{\,\tau}, W^{t};\Omega^{t}) = U\,(X_j^{\,t}, \lambda Y^{t}) + \delta E\,[\; V^{t+1}(X_j^{\,t}, W^{t+1}; \Omega^{t+1})\,],$$

where δ is the consumer's discount rate and expectations are taken over next period's information set (e.g. prices and characteristics of products next period). The evolution of wealth is given by $W^{t+1} = W^{t} + (1\text{-}\lambda)\, Y^{t} - P_j^{t} - K + P_g^{t,\tau}$, where P_j^{t} is the t period price of the new durable, $P_g^{t,\tau}$ is the t period trade-in (or scrap) value of a durable $(t\text{-}\tau)$ periods old, and $K>0$ is a transaction cost. The utility associated with choosing good j has two parts. There is the current period flow of services, the first term in the equation above, and the expected discount flow of services in the future, the second term. Naturally, the current flow of services is provided by the newly-bought asset, j. The second term—the

[2] See Keane and Wolpin (1994) for a good review of these methods.

expected utility from making the optimal choice in the next period—depends on the state variables: the characteristics of the asset he holds, his end-of-period wealth, and on expectations about future goods and prices.

The utility associated with not purchasing a new good ($j=0$) is given by:

$$V_0{}^t(X_g{}^\tau, W^t;\Omega^t) = U\,(X_g{}^\tau, \lambda Y^t) + \delta\, E\ [\ V^{t+1}(X_g{}^\tau, W^{t+1}; \Omega^{t+1})\ ].$$

In this case, the consumer decides not to buy a new asset, but rather continues to consume the old durable good he currently possesses. Since doing so does not require an outlay, the evolution of wealth is simply $W^{t+1} = W^t + (1\text{-}\lambda)\, Y^t$.

Using this notation, the solution to the consumer's problem may be stated as:

$$V^t(X_g{}^\tau, W^t;\Omega^t) = max_{\,(j=0,1,2,\ldots J)}\, V_j{}^t(X_g{}^\tau, W^t;\Omega^t)$$

Following Bajari, Benkard and Krainer (2005), we use value functions to compare the expected lifetime utility for an observed outcome in some time period, t, with that of a hypothetical counterfactual. This equivalent variation (*EV*) is the dollar amount one must give to (or take from) the consumer in the counterfactual to make him indifferent to the two scenarios. In our context, the two scenarios are (1) the observed time t choice, which is assumed optimal given time t tastes, state variables, and choice set, and (2) the optimal choice when the consumer, with time t tastes and state variables, chooses from the choice set in some base period, 0.[3]

Formally, let alternative n be the optimal choice from the time t choice set, $C^t=\{X^t, P^t\}$, with time t tastes and state variables, where the vectors $\{X^t, P^t\}$ denote the characteristics and prices of all vehicles sold in period t:

$$\boldsymbol{V}_n{}^t(X_g{}^\tau, W^t, \Omega^t \mid C^t) = V^t(X_g{}^\tau, W^t,\ \Omega^t \mid C^t).$$

[3] Traditional COL theory assumes that goods are homogenous over time, in which case, changing the "choice set" boils down to changing prices. Our model allows for the possibility that characteristics change over time and, so, the relevant counterfactual is one where both prices and the associated characteristics change. As we show in section 3, finding bounds for reservation prices will require us to place restrictions on the movement of unobserved characteristics over time. But, those restrictions are not required to obtain an analytical expression for the equivalent variations or indexes in this section.

Define alternative c as the optimal choice when choosing from the base period choice set, $C^0 = [\, X^{\,0}, P^{\,0}\,]$ with time t tastes and state variables:

$$\boldsymbol{V_c}^{\,t}(X_g^{\,\tau}, W^t, \Omega^t \mid C^0) = V^{\,t}(X_g^{\,\tau}, W^{\,t}, \Omega^{\,t} \mid \mathrm{C}^0)$$

Note that both the actual and hypothetical choices involve the same tastes, state variables $(X_g^{\,\tau}, W^{\,t})$ and information set $(\Omega^{\,t})$—only the choice set is different.

The equivalent variation, EV, is defined as the change in the consumer's wealth required to equate the expected lifetime utilities from these two alternatives and may be implicitly obtained from:

$$\text{(2)} \qquad \boldsymbol{V_n}^{\,t}(X_g^{\,\tau}, W^t, \Omega^t \mid C^{\,t}) = \boldsymbol{V_c}^{\,t}(X_g^{\,\tau}, W^{\,t}+EV\,, \Omega^{\,t} \mid C^{\,0})$$

This defines an equivalent variation for a consumer that purchased good n at time t. It is possible to define these equivalent variations for all consumers (buyers and non-buyers) which forces the question of whose equivalent variation to include in the price index. Because we are interested in a deflator for time t sales, we define our index to include EVs for all time t buyers since they are the only ones that actually paid the time t price.[4]

To translate these EVs into price indexes, we explicitly write the EV in (2) as $EV_{i,j}^{\,t}$, the equivalent variation for individual i that purchased good j at time t. Denote $I(j,t)$ as the set consumers that bought good j at time t, and denote its cardinality by I_j^t. Then the average equivalent variation for those that purchased good j is $\overline{EV}_j^{\,t} = \Sigma_{\,i\in I(j,t)} EV_{i,j}^{\,t} \;/\; I_j^t$. Further averaging over all goods, and noting that the number of consumers buying each good (I_j^t) is equal to the number of units sold (Q_j^t), the average equivalent variation over all buyers is:

$$\overline{EV}^{\,t} = \Sigma_{\,j=1,\,J}\,(\,\overline{EV}_j^{\,t}\; Q_j^t) \;/\; (\,\Sigma_{\,j=1,\,J}\, Q_j^t\,).$$

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[4] Certainly, if one were constructing a price index for the cost of holding and using a durable asset, one would want to consider a broader range of consumers (i.e., all those that hold a durable). But, for an index that is to be used as a deflator, it makes sense to include only those that actually purchased durables in the reference period.

Following Trajtenberg (1990), we use this average equivalent variation to define the price index:[5]

$$(3) \qquad COL_{0,t} = \overline{P}^{\,t} / (\overline{P}^{\,t} + \overline{EV}^{\,t})$$

where $\overline{P}^{\,t}$ is the average price: $\overline{P}^{\,t} = (\Sigma_{j=1,\,J} P_j^{\,t} Q_j^t) / (\Sigma_{j=1,\,J} Q_j^t)$. This index measures constant-utility price change from some base period, *0*, to time *t*. It compares the average price paid at time *t* to what the average price would have to be under the counterfactual to equate the buyers' welfare across the two scenarios. With increasing prices and no change in the choice set, the equivalent variation is negative (we must take income away from the consumer in the counterfactual) and the price index exceeds one; the reverse is true with falling prices.

Given estimates for the parameters of the value functions, one could, in principle, obtain the *EV* as follows: (i) calculate the value function for the observed time *t* choice, (ii) simulate the model to find the optimal choice associated with the counterfactual, and (iii) use the associated value functions in (2) to calculate the average *EV* and the attendant price index. This type of exercise has been done in a static setting by Pakes, Berry and Levinsohn (1993) and Nevo (2003), among others. The advantage to this approach is that, given the econometric assumptions, the resulting *EV* and price index are very general: they account for the arrival of new goods, substitution possibilities, and unobserved characteristics. The down side is in the potentially restrictive econometric assumptions used in the structural approach.

Below, we consider nonparametric alternatives to estimating the dynamic demand system.

3. Reservation Prices

Following Fisher and Griliches (1995) and Griliches and Cockburn (1994), we use the concept of reservation prices to define our cost of living index.[6] In our context,

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[5] While traditional COL theory uses expenditure functions to derive the COL index, we use the dual. The two yield theoretically equivalent *EV*s.

[6] In their application, the patent for a branded drug expires and a generic drug enters the market. But, the

we restate the equivalent variation in terms of prices—market prices for the time *t* choice and reservation prices for the counterfactual. We, later, find bounds for the reservation prices and those bounds ultimately allow us to place bounds on the price index.

Consumers implicitly have a reservation price for every good in the choice set. Under the counterfactual, we define the reservation price for each good *j* as the price such that the (expected lifetime) utility obtained from that good exactly equals that of the optimal choice (good *c*).

Formally, the reservation price for good *n* under the counterfactual is the R_n that satisfies:

$$(4) \qquad \boldsymbol{V_n}^{t}(X_g^{\tau}, W^t, \Omega^t | C^0(R_n)) = \boldsymbol{V_c}^{t}(X_g^{\tau}, W^t, \Omega^t | C^0),$$

where C^0 is the the actual base period choice set from which the buyer chooses the optimal good, and $C^0(R_n)$ is the hypothetical choice set that we use to define the reservation price. If good *n* is a continuing good, present in both the current and base periods, we define $C^0(R_n) = \{(X_{-n}^0, P_{-n}^0); (X_n^0, R_n)\}$, where *-n* designates all goods except good *n*. Hence, when facing this choice set, a consumer faces all the goods available in period *0*. The prices for all goods but *n* are period *0* market prices, while the price for good *n* is the buyer's reservation price. Equation (4) defines the reservation price as the price such that the buyer is just indifferent between buying good *n* and buying the optimal good *c*.

If good *n* is a new good, we augment the base period choice set to include the characteristics of the new good, X_n^t, and the buyer's reservation price: $C^0(R_n) = \{(X_{-n}^0, P_n^0); (X_n^t, R_n)\}$. The reservation price answers the following question: If good *n* had existed in the base period, what price would have made the consumer just indifferent between buying that good vs. buying the optimal good *c*? This is the conceptual solution to the "new goods" problem.

To find this buyer's *EV,* we assume that increases in wealth have the same effect

slow diffusion of knowledge about the generic drug generates staggered switching to the new good: not everyone switches to the lower-cost generic drug in the period of introduction even if the two drugs are chemically identical because it takes time for consumers to realize that the generic is a safe alternative to the higher-priced branded drug. They handle this heterogeneity across consumers by defining an index in terms of reservation prices. Their index is not applicable in our case because it is derived from a static model where choices are continuous whereas our problem calls for a dynamic model with discrete choices.

on expected lifetime utility regardless of which good the consumer purchases:

$$\boldsymbol{V_n}^{\,t}(X_g^{\,\tau}, W^t + EV, \Omega^t \,|\, C^0(R_n)) = \boldsymbol{V_c}^{\,t}(X_g^{\,\tau}, W^t + EV, \Omega^t \,|\, C^0),$$

Using this expression, we can restate the equivalent variation in (2) solely in terms of good *n*:

$$\text{(2')} \qquad \boldsymbol{V_n}^{\,t}(X_g^{\,\tau}, W^t + EV, \Omega^t \,|\, C^0(R_n)) = \boldsymbol{V_n}^{\,t}(X_g^{\,\tau}, W^t, \Omega^t \,|\, C^t)$$

Because wealth is linear in prices, the increase in wealth may be measured instead as a reduction in price: that is, $\boldsymbol{V_n}^{\,t}(X_g^{\,\tau}, W^t + EV, \Omega^t \,|\, C^0(R_n))$ may be restated as $\boldsymbol{V_n}^{\,t}(X_g^{\,\tau}, W^t, \Omega^t \,|\, C^0(R_n - EV))$ and we rewrite the expression for the *EV* as:

$$\text{(2'')} \qquad \boldsymbol{V_n}^{\,t}(X_g^{\,\tau}, W^t, \Omega^t \,|\, C^0(R_n - EV)) = \boldsymbol{V_n}^{\,t}(X_g^{\,\tau}, W^t, \Omega^t \,|\, C^t)$$

We suppose that product innovation occurs through the introduction of new goods rather than the incremental improvement of existing goods. Note, then, that if goods' characteristics are the same in both scenarios (i.e., $X^t = X^0$ for continuing goods—they are the same for new goods by construction), then the only difference in the two value functions is in the prices and equating the value functions only requires equating the prices: $P_n^{\,t} = R_n - \boldsymbol{EV}^t$, which implies an equivalent variation of $\boldsymbol{EV}^t = R_n - P_n^{\,t}$.[7]

Using this average $\overline{EV}^{\,t}$ in (3) to restate the COL index in terms of prices, we obtain a Paasche index, where reservation prices take the place of the usual base-period market price: [8]

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[7] Our model can accommodate the presence of changes in unobserved characteristics, so long as the average across goods is constant over time, one assumption often made in this literature in order to construct price indexes. Let $X_k^{\,t} = X_k + \xi_k^{\,t}$ and suppose that these changes in characteristics over time, once averaged across all goods, are constant: $\overline{\xi}^{\,t} = \overline{\xi}^{\,t-1}$. Then, one can show that the previous result holds on average: $\overline{EV} = \overline{R} - \overline{P}^{\,t}$, where $\overline{R} = (\Sigma_{j=1,..J}\, \overline{R}_j^{\,t} Q_j^t) / (\Sigma_{j=1,J} Q_j^t)$ is the average reservation price and $\overline{R}_j = (\Sigma_{i=1,..I(j,t)} R_{i,j}^{\,t} / I_j^t)$ is the average reservation price for consumers that purchased good *j* at time *t*. This assumption is necessary because if goods' unobserved attributes are trending up over time: If $X^t > X^0$, then $EV > R_n - P_n^t$ and market prices do not provide a bound to the EVs.

[8] An alternative way to restate the index is directly in terms of reservation prices. In particular, the index

$$(5) \qquad COL_{t,o} = \left(\Sigma_{j=1,J} \ P_j^t Q_j^t \right) / \left(\Sigma_{j=1,J} \overline{R}_j \ Q_j^t \right)$$

This index is very similar to the COL index that Fisher and Griliches (1995) and Griliches and Cockburn (1995) developed using a cost function approach in a static setting; their index is also a Paasche that relies on reservation prices. Our model shows that their approach may be applied in a dynamic setting to obtain a similar index.

The use of a Paasche index is counterintuitive in that the usual practice is to treat the Laspeyres as an upper bound to price change and a Paasche as a lower bound and the average of the two, the Fisher index, as the best guess for the true underlying COL index. Note, however, that because our index is only for time *t* buyers, a Laspeyres for those buyers is undefined—since their time *t-1* expenditure weights are zero—and, hence, so is the Fisher. So, these indexes do not have a welfare interpretation under the assumptions of our model.

Bounding Reservation Prices and the COL Index

Of course, reservation prices are not observed. Our approach is to find upper bounds for these reservation prices and use them to obtain bounds for the true COL index. For continuing goods, we show that the average base period market price is an upper bound to the average reservation price. For new goods, we discuss and apply the two assumptions typically made in the measurement literature. To the extent that these corrective measures adequately account for welfare gains generated by new goods, a Paasche index provides a lower bound to the true COL index.

The case of continuing goods is one where a consumer purchases a good (good *n*) that also existed in the base period. For goods that exist in both periods, our definition of the reservation price in (2’’) implies that buying good *n* at the reservation price brings at least as much utility as all other options available under the counterfactual, including

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in (5) may be rewritten as $COL_{t,o} = P^t / \overline{R}$. This says the index implicitly compares the average price at time *t* to the average reservation price in the counterfactual. This makes clear the similarities in the problem we consider and the standard “new goods” problem. In the latter, the problem is that a price for a new good is not observed in the base period and the solution is to use a reservation price. Here, we do not have a base period price for consumers because they did not make a purchase in that period and the solution is, again, to use a reservation price.

good n at its actual price: $\boldsymbol{V_n}^t(X_g^{\tau}, W^t, \Omega^t | C^0(\boldsymbol{R_n}) \geq \boldsymbol{V}_{\mathrm{n}}^t(X_g^{\tau}, W^t, \Omega | C^0(P_n^0))$. Note that the only difference between the two value functions is in the prices: consumer pays the reservation price in one and the market price in the other. Hence, this inequality can only hold if the reservation price is no greater than the market price for good n in the base period, so that the market price for good n in the base period provides an upper bound to the consumer's reservation price for that good in the counterfactual: $B_j^0 \equiv P_n^0 \geq \boldsymbol{R_n}$. We are able to define a bound because a price for the good exists in the base period.

The case of new goods is one where good n did not exist in the base period. There are two ways typically used to incorporate such goods in price indexes: the matched-model and hedonic assumptions. Under the matched-model assumption, the relationship between market and reservation prices for continuing goods is assumed to equal that between market and reservation prices for new goods: $(\overline{P}^t / \overline{R}) = (\overline{P}_n^t / \overline{R}_n)$. Only then will the Paasche for continuing goods equal that which includes new goods. This implicitly assumes the following bound for an average reservation price for the new good, n: $B_{mm}^0 = \overline{P}_n^t(\overline{P}_j^0 / \overline{P}_j^t) \geq \overline{R}_n$.

An alternative is to deal with the arrival of new goods explicitly.[9] Pakes (2003) showed that a predicted price from a hedonic regression can, under certain assumptions, provide an upper bound for the compensating variation in a Laspeyres index. That logic is directly applicable in our case, except the predicted value provides an (hedonic) upper bound to time t buyers' average reservation price: $B_h^0 \equiv h^0(X_n^t) \geq \overline{R}_n$.

An upper bound on the reservation price implies the following lower bound to the true COL index:[10]

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[9] The new goods problem is extremely difficult. Traditionally, predicted prices from a hedonic regression have been used (in the so-called imputation method) to for a price relative for new goods. Pakes (2003) provided a justification for that practice in the context of a static model. We are not aware of a parallel justification for durable goods and use the standard imputation method to deal with new goods in our dynamic model.

[10] For completeness, we can also define lower bounds to the reservation price and the implied upper bound on the COL index, but those bounds are of limited usefulness. When prices are falling, the utility from purchasing good n at base period prices is less than what it would be if the consumer could pay the (lower) time t price. To see this, note that, by construction, the value function for the purchase of good c under the counterfactual equals that of good n purchased at the reservation price. If all prices fall over time, then the optimum under the counterfactual brings less utility than the optimum when facing the lower time t prices. Thus, $\boldsymbol{V_c}^t(X_g^{\tau}, W^t, \Omega^t | C^0) = \boldsymbol{V_n}^t(X_g^{\tau}, W^t, \Omega^t | C^0(R_n)) \leq \boldsymbol{V_n}^t(X_g^{\tau}, W^t, \Omega^t | C^t)$. For this to be true, it must be the case that the reservation price exceeds the time t market price for good n: $\boldsymbol{R_n} \geq P_n^t$. This bound on

$$(6) \qquad COL_{t,o} \geq \left(\Sigma_{j=1,J}\ P^t_j\, Q_j^t \right) / \left(\Sigma_{j=1,J}\, B_j^0\, Q_j^t \right)$$

where

$B_j^0 = P_j^0$,   for continuing goods

$= h^0(X_j^t)$, for new goods.

The arithmetic can be counterintuitive. With falling prices, the standard Paasche would be a number like .8 and the true COL index would be greater than that: a number like .9. Therefore, a standard Paasche overstates price declines when prices are falling. With rising prices, the standard Paasche is a number like 1.2 and the true COL index is greater than that: a number like 1.3 and the standard Paasche understates price growth.

To sum up, our model implies the following:

- the true COL index implied by our model is a Paasche index that compares market prices at time *t* to reservation prices in the base period.
- A Paasche index that uses our bounds for reservation prices gives a lower bound for the true EV-based COL index.
- In our model, the Laspeyres and Fisher indexes are undefined and do not have a welfare interpretation.

4. Illustration

Our theoretical model shows that a standard Paasche index is a lower bound to the true COL index while, assuming prices decline over time, zero price change is an upper bound. In most applications, these bounds can be quite far apart and may provide little guidance on the true COL index. In this section we use an empirical model of consumer demand to explore under what circumstances the standard Paasche is close to the true COL index. We also consider the practical usefulness of our bound in interpreting existing measures.

The model, parameter estimates, and data are from Copeland, Dunn and Hall (2005), hereafter known as CDH, who estimate a discrete-choice model of consumer

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the reservation price implies that the EV is no less than 1—increases in utility—something we already know when prices are falling. Alternatively, when prices are rising, the EV is no greater than 1—utility is automobile demand for Ford, GM and Chrysler (a.k.a. Big Three) products. CDH use a discrete-choice model that fits into the framework developed by Berry, Levinsohn and Pakes (1995); these types of models are well-known for their ability to provide accurate estimates of consumers' substitution patterns. Further, there is a substantial literature that uses discrete choice models of consumer demand to compute consumer surplus or willingness-to-pay (see, for example, Trajtenberg, (1990), Petrin (2002) and McFadden 1997). Unlike the theoretical model presented earlier, our empirical model is static, and so does not directly capture consumers' inter-temporal substitution. This force is captured indirectly, however, as the empirical model allows a consumer to not purchase a Big Three vehicle, but instead purchase an outside good. Choosing the outside product incorporates, among other things, waiting to buy a Big Three vehicle at a later date. The indirect utilities estimated in CDH can be interpreted as the present discounted value of purchasing a new automobile, or the alternative-specific value functions in our theoretical model.[11]

The empirical model we use specifies that consumer *i*'s utility from purchasing a product *j* depends on the interaction between a consumer's characteristics and a product's characteristics. Consumers are heterogeneous in income and in their tastes for certain product characteristics. Indirect utility is given by

$$V_{ij}^{t} = \delta_{ij} + \xi_{j}^{t} - \alpha_i P_j^t + \varepsilon_{ij}^{t},$$

where δ_{ij} (a product of data and estimated parameters) represent consumer *i*'s valuation of model-specific characteristics, which are constant over the life of the good. Because δ_{ij} includes fixed effects for each model, ξ_j^t represents the change in a product's unobserved characteristic over time. As detailed in Nevo (2003), this change can be considered as a change in tastes, or a change in the utility that consumers obtain from the product. The price of the product is P_j^t and α_i is a parameter measuring consumer *i*'s distaste for price. Finally, ε_{ij}^t is independently and identically distributed extreme value.

decreasing—and the COL index is no less than 1, something we already know from the standard Paasche.

[11] There is a small, growing industrial organization literature that estimates dynamic demand models (see Gowrisankaran and Rysman (2005) for example). Once these models become more established, their explicit modeling of dynamics will likely provide more accurate COL indexes than the current, static approach.

In the original CDH specification, there is a quadratic time trend in the indirect utility. Having a time trend in the valuation of the inside goods or the outside good does not significantly affect the results reported below. Our preference is to include the time trend in the outside good, which lends itself to the interpretation that the outside good is getting better over time.[12] As discussed earlier, consumers' equivalent valuations are computed by comparing the price of the product they bought today, against their reservation price for that product in a base period. To calculate the COL index, we need to compute consumers' average reservation price when they use today's tastes to choose from the base period choice set. Differences in the consumer's problem arise both from the entry or exit of goods and changes in price. Note that in our empirical examples, we interpret ξ_j^t as a change in tastes. This provides a clean comparison between our COL index and matched-model indexes; under the matched model approach, there cannot be an unobserved characteristic changing over time. With regard to the $\varepsilon_{ij}{}^t$, we follow the literature and interpret these i.i.d. variables as taste shocks.

4.1. When do the Paasche bound and the COL index diverge?

Unlike the true COL index, the Paasche bound "forces" buyers to purchase the same good in both periods. The key driver of differences between our bound and the true COL index occur when changes in price or in the choice of available goods under the counterfactual result in the consumer buying a different good. If this switching generates nontrivial utility gains, then there will be significant a wedge between the standard Paasche and true COL indexes.

Naturally, the likelihood of gaining nontrivial utility from switching under the counterfactual differs across markets. When consumers face many similar products, large changes in relative prices will cause lots of switching. Further, consumers will garner significant utility gains from switching because they will choose a similar product with a lower relative price. When goods are relatively far apart in characteristics space (e.g. there is substantial product differentiation), one is less likely to observe switching. Durable goods, however, allow consumers to optimally time their purchase. Hence, even in markets where goods are not close substitutes, the durability of these goods allows consumers to inter-temporally substitute, increasing both the likelihood of switching and

[12] The time trend is both the quadratic time trend, as well as the mean value of ξ_j^t for continuing goods.

the resulting utility gains.

To illustrate the importance of accounting for consumers not only switching to other goods under the counterfactual, but also delaying their purchase decisions, we simulate our demand model and track consumers' purchasing decisions. For every period in the model, we consider an "alternative" last period where consumers face the same set of goods in the current period, but face a price vector where every product's price is 10 percent higher, $P_j^{t-1} = P_j^t * (1/0.9)$.[13] We choose to fix the choice set and impose a uniform percentage price increase so that every consumer is strictly worse off under the counterfactual. In the counterfactual, consumers can choose to purchase the same vehicle they bought when facing the current period's prices, purchase a different new vehicle, or choose the outside option.

Using consumers' optimal purchase decisions, we compute their reservation prices. To aggregate across consumers and quarters, we normalize the reservation price so that it falls between 0 and 1, by letting $\tilde{R}_{i,j} = (R_{i,j} - P_j^t) / (P_j^{t-1} - P_j^t)$, were P represents market prices and R reservation prices. Hence $\tilde{R}_{i,j}$ is equal to 1 whenever $R_{i,j}$ is equal to P_j^{t-1}, or the consumer purchases the same good when facing both price vectors.

Figure 1 shows the cumulative distribution function of the normalized reservation values. There is a large mass at 1, signifying that most consumers by far did not change their purchase decisions under this simulation. This behavior reflects both the highly differentiated nature of the automobile market as well as the fact that, by construction, relative prices among inside goods do not change much under the counterfactual.

For those consumers that did switch goods, the distribution of reservation utilities is convex.[14] This convexity reflects that consumers, more often than not, only slightly improved their utility when they switched their purchase decision to another new vehicle. This is most clearly seen by separating consumers into groups depending on whether they switched to an inside good or to the outside good. The cumulative distribution functions of each group's reservation utilities, which includes only the mass of consumers who switched, are graphed in figure 2. The solid linear line represents those consumers who switched to the outside good; the dotted convex line represents those consumers who

[13] We also add a constant positive utility value to all inside goods to generate a higher fraction of consumers purchasing an inside good.

[14] As a point of comparison to the distribution of reservation values, Fisher and Griliches (1995) and Griliches and Cockburn (1994) assumied a uniform distribution for $R_{i,j}$.

switched to an inside good. Given that *all* prices are declining 10 percent, switching to an inside good, more often than not, does not make consumers much better off. This drives the convex shape of the cumulative distribution function of these consumers' normalized reservation values.

As shown in figure 2, consumers who switched to the outside good had reservation prices that were evenly distributed between P_j^t and P_j^{t-1}. Recall that in our demand model consumers differ along several dimensions: their income, their tastes of certain product characteristics, and through i.i.d. taste shocks. This heterogeneity creates a smooth distribution in the value of the outside option across consumers. We interpret consumers' switch to the outside good as a delay in their purchase decision. Given that a significant number of consumers garnered large utility gains from switching to the outside good, this simulation reveals that consumers' ability to time purchases is an important force in the automobile industry and will drive a wedge between the true COL and a standard Paasche indexes. Indeed, in this simulation, the average true COL monthly index was 0.906, 0.06 above the standard Paasche.

These results highlight that markets with highly substitutable goods are not the only cases where the standard Paasche and true COL indexes might differ. Durable goods markets are also prime candidates, because of consumers' ability to time their purchases.

4.2. A Comparison of Price Indexes

To provide a concrete example of the degree to which a standard Paasche index overstates constant-quality price change, we consider the automobile market. Using the empirical model, we compute the COL index through model simulations. For every period, we simulate the purchase decisions of 5,000 individuals a 1,000 times. For each simulation, we compute individuals' optimal choices in the current and base periods and calculate their resulting indirect utilities. For those individuals that purchased a good, we then determine which product they would buy if they, instead, faced the base period choice set and price vector. We consider the case where the previous period is the base period, and hence determine which product time t consumers would purchase given the $t-1$ choice set and price vector. Using equation (4), we can then back out these individuals' reservation prices, which, once averaged, give the COL index (see equation (5)). For each simulation, we use this algorithm to compute the true COL and standard Paasche indexes. For each index, we then take the mean across all simulations to obtain average true COL and standard Paasche indexes.[15]

Our focus is to compare the standard Paasche index—i.e., our bound to the COL index—with the true COL index implied by the model. We start by considering two cases. In the first case, we only use consumers who bought continuing goods in the current period to construct our indexes (the case illustrated by Pakes, Berry and Levinsohn (1993)). Continuing goods are those goods which were offered both in period *t* and *t-1*. We compute the COL index holding the goods in the choice set constant. Hence, we determine what period *t* buyers of continuing goods would choose when facing the same set of continuing goods but with period *t-1* prices. Constructing the Paasche is straightforward in this case because we have data on prices and quantity sold in both periods.

In the second case, we consider the same group of consumers, but for the COL index we now allow consumers to buy *any* good available in period *t-1*. Hence, in this second case, we allow the *t-1* choice set to expand, by including goods that exited between period *t-1* and *t*. Our prior is that this provides more possibilities for switching to a good that yields higher utility and, thus, is more likely to generate a wedge between the Paasche and COL indexes.

Table 1 presents the COL and standard Paasche indexes for these two cases.[16] The quarters refer to the automotive model year, where the first quarter is composed of August, September, and October. The data starts in May 1999 and ends in January of 2004. Columns 3 and 4 of the table list the COL index for cases 1 and 2, while column 5 lists the standard Paasche index. The Paasche is always weakly smaller than the COL index by construction and the difference between these two indexes is minimal for case 1. Indeed, the mean quarterly values of the two indexes are only different by 0.0016. Recall, the COL and Paasche index are the same whenever consumers do *not* change

[15] Alternatively, we could have performed one simulation with many more potential consumers. However, given that an average of about 10 percent of all households purchase a new motor vehicle in our sample, we would need a prohibitively high number of potential consumers in order to minimize any simulation error.

[16] Among the issues we plan to explore in future work, the most important is an exploration of the practice of chaining price indexes over time to measure price change over many periods. In the context of a representative consumer framework, it makes sense to cumulate period-to-period welfare gains through chaining. However, it's not clear that this practice makes sense in markets where purchases are intermittent so that those that bought a good in one period are not the same consumers that bought a good in the next.

their purchase decisions in the base period relative to the current period. In case 1, because the choice set is held constant, consumers only change their purchase decisions if relative prices change or if prices are sufficiently high to move them to choose the outside good. The demand estimates show that, under these circumstances, motor vehicle buyers will not tend to change their purchase decisions, in which case, the standard Paasche is numerically close to the COL index.

Case 2, however, shows that the gap between the standard Paasche and COL index can be large when exiting goods are taken into account. As seen in the last row, the Paasche falls, on average, 2.5 percent per quarter, almost a full percentage point above the average 1.6 percent quarterly decline in the COL index. The COL and Paasche indexes differ in this case because consumers are given a larger set of goods over which to re-optimize, thus providing more opportunities to find a good in the counterfactual that is preferred to good *j*.

In these cases, when consumers purchase continuing goods, the Paasche is a lower bound to the COL index. The U.S. automotive industry, however, frequently introduces new products into the marketplace. In the first quarter of the automotive model year, automakers typically introduce a new vintage of their product line. This results in a substantial number of new goods every four quarters.

For our third and final case, we expand case 2 to include *all* consumers that bought inside goods in period *t*—including those that purchased new goods—and, when constructing the COL index, allow them to purchase any available good in period *t-1*. This last case completely takes into account changes in the choice set between periods. While the COL index is straightforward to compute for this case, we face the standard new-goods problem in trying to construct the standard Paasche, namely that the Paasche requires an unobserved period *t-1* price for all new, period *t* goods.

We apply the two standard solutions to the new goods problem discussed in the theory section. The Paasche index in table 1 uses prices for only continuing goods and, so, gives the bound constructed under the matched-model assumption. We also consider Pakes (2003) hedonic solution. We estimate a hedonic regression for every quarter, using vehicles' observed characteristics as the independent variables to explain the log of price and use it to estimate the price of new goods in the period before their arrival. Details about the hedonic regressions appear in the Appendix, but, in summary, the hedonic regressions appear to predict prices quite well; the R-squared of these regressions range from the 0.75 to 0.86. [17] Yet, the demands on the hedonic approach are admittedly heavy in this application because of the simultaneous introduction of so many new goods in the same period. Columns 3 and 4 of table 2 list the ratio of the number of continuing to total goods and the revenue share of continuing goods, respectively. In the first quarters of the automotive model year, the massive introduction of new goods results in continuing goods making up only 50 to 60 percent of all vehicles in the market, and accounting for roughly 70 to 80 percent of revenue.

For new goods, the theoretically-correct bound would be an upper bound to the average reservation price. As discussed in Pakes (2003), the hedonic corrects for a selection problem—it measures price change for all goods, not just goods sold in both periods. But, Pakes warns that the hedonic will not necessarily provide a bound on reservation prices for new good buyers when the reservation prices exceed the highest observed price for a good. For autos, the highest observed price is typically the introduction price and so the hedonic will not capture the infra-marginal rents that accrue to those that buy the new good in the introduction period. Note, however, that our method properly accounts for those that buy the new models in subsequent periods.

To the extent that infra-marginal rents are important, the hedonic Paasche index will not provide a lower bound to the true COL index. Indeed, this appears to be the case in our data. The last two columns of table 2 compare the COL index with the hedonic Paasche index. In quarters where introductions are heavy—the shaded rows—the COL index is, on average, about 8 percentage points below the Paasche index. The hedonic Paasche does provide a lower bound on the COL index in other periods; the gap between the average values of both indexes is about 1 percentage point.

Our demand model, thus, suggests that infra-marginal rents are high in this industry. However, there is reason to somewhat discount these estimates. It has been noted that demand models that include a logit term—as in the CDH model—will tend to overstate welfare gains from new goods (Petrin, 2002). A back-of-the-envelope calculation suggests that this is the case here. For the first quarter of the 2001 model

[17] A concern is that the hedonically-imputed prices are subject to prediction error. In particular, Pakes warns that the hedonic estimates can be imprecise when the characteristics of new goods are not observed a lot in the base period. The characteristics of new goods, for all but 21 observations in our sample are within the range of continuing goods' characteristics—a simple check that new and continuing goods are "close"

year, for example, the 13.5 percent decline in the COL index implies over a 40% decline in the index for buyers of new goods. This implies that the arrival of the new car was worth, average, about ½ the price of the car to new buyers, a number that seems implausibly large.[18] In contrast, the hedonic Paasche implies that buyers that purchased the new good in the introduction period would have been willing to pay, on average, 16% more for the vehicle. Although this still seems a bit high, we take a literal view of the Paasche hedonic as our upper bound. Under that interpretation, we conclude that the true COL index declines slower than the 2.8% declines seen in the Paasche bound.

Despite the difficulties in handling the new-goods problem, these results demonstrate that the standard Paasche does overstate the constant-quality price decline in the automobile industry. The results from case 2 and 3 (ignoring the first quarter of the model year) indicate that the true COL index falls 1 percent slower than the Paasche, a substantial difference. Importantly, this wedge between the indexes exists in spite of the highly differentiated nature of the automobile market. In other durable goods markets with less product differentiation, we expect the difference between the standard Paasche and true COL indexes to become even larger.

4.3. Practical Relevance

It is worth noting that a Fisher index—undefined in our model but widely viewed as the proper way to construct matched-model indexes—declines 2.7% per quarter in these data, about the same as the hedonic-Paasche bound. Given that the Paasche bound can deviate substantially from the true COL index, a Fisher index that is numerically close to the lower bound may better be viewed as a bound, rather than our best guess at true price change.

An important industry where we think existing indexes may underestimate the true COL index is the IT sector, where there exist close substitutes and consumers appear to be sensitive to prices (see, for example, Song (2007)). Moreover, as is well known, the average price level for individual models of PCs and other electronics fall quite rapidly over time, opening opportunities for potential buyers to defer their purchases into the future. Price indexes for these goods typically show rapid declines, reflecting steady

to one another.

[18] In Petrin's study for minivans, he found that the welfare gains were overstated by about 1/3 of the price of the vehicle, substantially less than what we find. Therefore, it is unlikely that the logit-related bias can

declines over the life of each good (Berndt and Rappaport (2001)). While it is undoubtedly true that the rapid rates of product innovation in these markets has generated welfare gains to consumers that would tend to pull down price indexes, our concern is that, all else held equal, the numerical measure of those welfare gains may be overstated.

For example, we can use estimates for PCs recently reported by Pakes (2003) to make this point. He reports two indexes that are relevant for us: a hedonic Laspeyres that falls between 15-17% per quarter and a hedonic Paasche that falls about 18 percent. Our model suggests that the hedonic Paasche provides a lower bound on the true COL index; the true COL falls no faster than 18 percent and our empirical illustration suggests that it could fall substantially slower than this. That interpretation is very different from the standard view of the 15-17 percent declines in the Laspeyres index as an *upper* bound.

There are other cases where calculated Paasche indexes show slower declines than the usual Fisher index—software is one such case (see Prud'homme and Yu (2005) and Abel, Berndt and White (2003)). In those cases, our interpretation of the Paasche as a lower bound calls to question the use of indexes that show faster declines (like the Fisher).

5. Conclusions

This paper uses a simple dynamic model to obtain an expression for the true COL index for durable goods. Although the index is based on unobserved reservation prices, we show that a Paasche index that uses market prices provides a lower bound for the COL index implied by our model. An empirical illustration shows that the bound can be far below the true COL index.

We believe that pinning down this issue will require a fully-developed model that captures all three of the important features of durable goods purchases: *heterogeneous* consumers making *discrete* choices in a *dynamic* setting. We view models such as the one proposed by Gowrisankaran and Rysman (2005) and the studies they cite as promising approaches for assessing the potential importance of the problem we have identified. In the meantime, we feel the calculation of our Paasche bound can often provide perspective on indexes currently in use.

While our work points to a new source of bias, several other sources of biases in

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explain the entire gap.

standard price indexes have been identified in the literature (see National Research Council (2002) for a review of the traditional issues). Some impart an upward bias—as in recent work by Pakes (2003), Bils and Klenow (2001) and Bils (2004)—and some impart a downward bias—see Hobijn (2002), Harper (2003), Feenstra and Knittel (2004), and Gordon (2004). While we identify a new source of downward bias, we do not resolve how all these potentially offsetting effects might net out.

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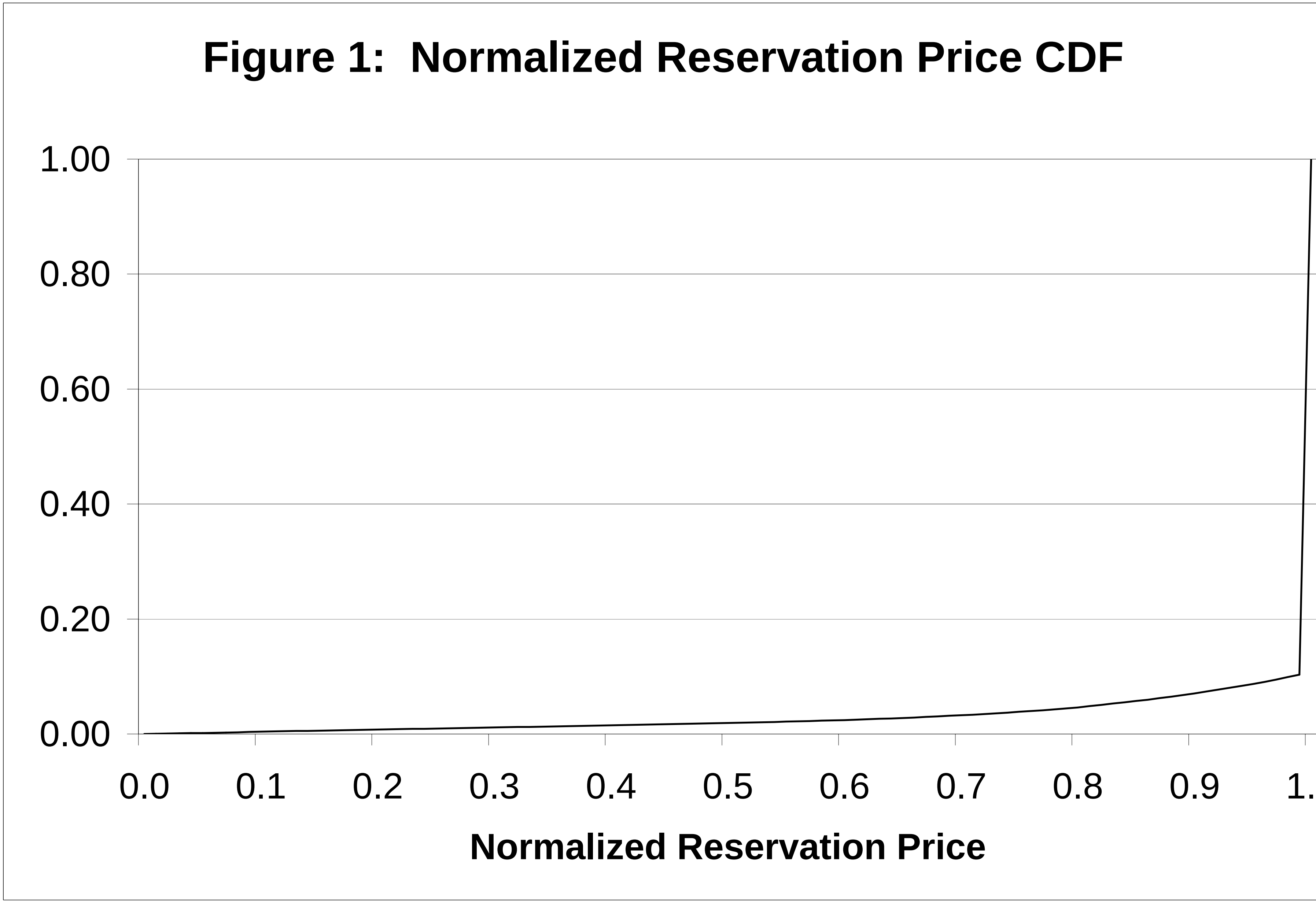
Figure 1: Normalized Reservation Price CDF
1.00
0.80
0.60
0.40
0.20
0.00
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Normalized Reservation Price

Figure 2: Conditional Normalized Reservation Price CDF

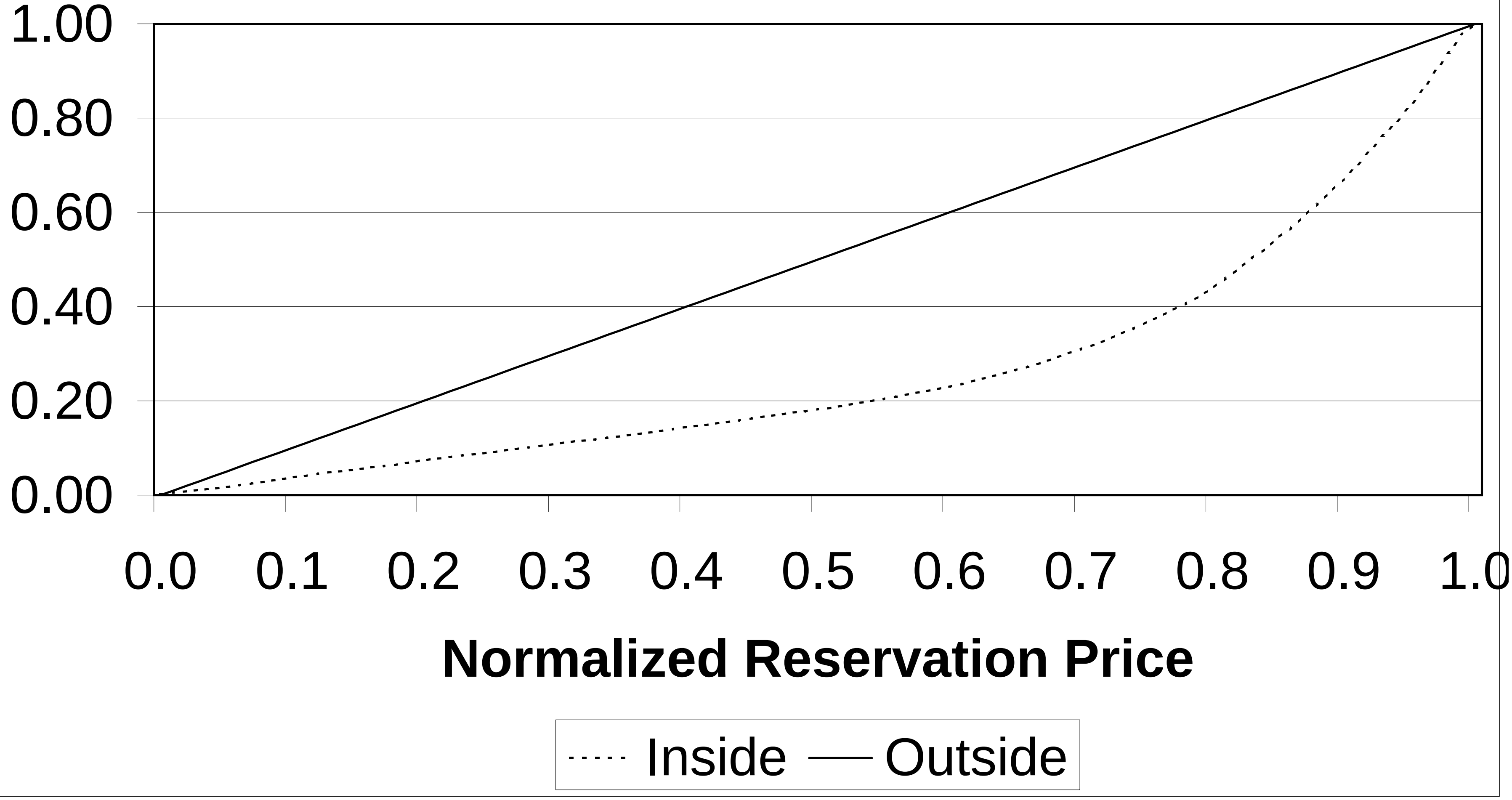

Table 1: Prices Indexes for Case 1 and Case 2

Model Year	Quarters	True COL Case 1	True COL Case 2	Paasche
1999	4	0.9849	0.9869	0.9842
		(0.9849,0.9850)	(0.9867, 0.9870)	(0.9842, 0.9843)
2000	1	0.9714	0.9714	0.9700
		(0.9713, 0.9714)	(0.9713, 0.9715)	(0.9699, 0.9701)
2000	2	0.9803	0.9904	0.9786
		(0.9803, 0.9804)	(0.9902, 0.9907)	(0.9786, 0.9787)
2000	3	0.9842	0.9918	0.9836
		(0.9842, 0.9843)	(0.9916, 0.9920)	(0.9835, 0.9836)
2000	4	0.9861	0.9881	0.9854
		(0.9861, 0.9862)	(0.9880, 0.9883)	(0.9854, 0.9855)
2001	1	0.9769	0.9769	0.9753
		(0.9768, 0.9770)	(0.9769, 0.9770)	(0.9752, 0.9753)
2001	2	0.9839	0.9954	0.9821
		(0.9839, 0.9840)	(0.9951, 0.9957)	(0.9820, 0.9821)
2001	3	0.9789	0.9988	0.9777
		(0.9789,0.9789)	(0.9984, 0.9991)	(0.9777, 0.9778)
2001	4	0.9769	0.9797	0.9760
		(0.9769, 0.9770)	(0.9796, 0.9798)	(0.9760, 0.9761)
2002	1	0.9932	0.9992	0.9924
		(0.9931, 0.9932)	(0.9990, 0.9993)	(0.9923, 0.9924)
2002	2	0.9950	1.0123	0.9942
		(0.9950, 0.9951)	(1.0120, 1.0126)	(0.9941, 0.9942)
2002	3	0.9513	0.9678	0.9488
		(0.9513, 0.9513)	(0.9675, 0.9681)	(0.9488, 0.9489)
2002	4	0.9834	0.9845	0.9828
		(0.9834, 0.9835)	(0.9844, 0.9846)	(0.9827, 0.9828)
2003	1	0.9886	0.9915	0.9860
		(0.9885, 0.9887)	(0.9913, 0.9917)	(0.9859, 0.9861)
2003	2	0.9519	0.9695	0.9490
		(0.9519, 0.9520)	(0.9692, 0.9698)	(0.9489, 0.9490)
2003	3	0.9579	0.9702	0.9556
		(0.9579, 0.9580)	(0.9700, 0.9705)	(0.9555, 0.9556)
2003	4	0.9757	0.9795	0.9734
		(0.9756, 0.9757)	(0.9793, 0.9796)	(0.9733, 0.9734)
2004	1	0.9656	0.9686	0.9641
		(0.9656, 0.9657)	(0.9685, 0.9688)	(0.9640, 0.9641)
2004	2	0.9688	0.9778	0.9667
		(0.9688, 0.9689)	(0.9777, 0.9780)	(0.9667, 0.9668)
Mean		0.9766	0.9842	0.9750

Note: Numbers in parenthesis are the bounds of the 95 percent confidence interval. `Quarters' designates an automotive model-year quarter. August, September and October compose the first quarter of the automotive year.

Table 2: Price Indexes when incorporating new goods

Model Year	Quarters	Continuing Goods (% of all goods)	Continuing Goods (revenue share)	True COL	Paasche
1999	4	0.93	0.99	0.9825	0.9819
				(0.9818, 0.9819)	(0.9822, 0.9827)
2000	1	0.57	0.70	0.8727	0.9319
				(0.8720, 0.8734)	(0.9317, 0.9322)
2000	2	0.96	0.97	0.9846	0.9730
				(0.9843, 0.9850)	(0.9728, 0.9731)
2000	3	0.98	0.99	0.9891	0.9809
				(0.9889, 0.9893)	(0.9809, 9810)
2000	4	0.94	0.98	0.9824	0.9844
				(0.9821, 0.9826)	(0.9844, 0.9845)
2001	1	0.59	0.78	0.9087	0.9713
				(0.9081, 0.9093)	(0.9711, 0.9714)
2001	2	0.98	1.00	0.9947	0.9817
				(0.9944, 0.9950)	(0.9817, 0.9818)
2001	3	0.94	0.99	0.9950	0.9795
				(0.9947, 0.9954)	(0.9794, 0.9795)
2001	4	0.90	0.98	0.9743	0.9711
				(0.9741, 0.9745)	(0.9710, 0.9712)
2002	1	0.59	0.73	0.9135	1.0401
				(0.9128, 0.9141)	(1.0398, 1.0404)
2002	2	0.94	1.00	1.0117	0.9940
				(1.0114, 1.0120)	(0.9939, 0.9940)
2002	3	1.00	1.00	0.9677	0.9488
				(0.9674, 0.9680)	(0.9488, 0.9489)
2002	4	0.91	0.98	0.9802	0.9801
				(0.9800, 0.9804)	(0.9800, 0.9802)
2003	1	0.59	0.72	0.9072	0.9873
				(0.9066, 0.9078)	(0.9872, 0.9875)
2003	2	0.95	1.00	0.9688	0.9487
				(0.9684, 0.9691)	(0.9487,0.9488)
2003	3	0.93	0.98	0.9654	0.9525
				(0.9652, 0.9657)	(0.9524, 0.9525)
2003	4	0.97	1.00	0.9786	0.9734
				(0.9785, 0.9787)	(0.9734, 0.9735)
2004	1	0.57	0.78	0.9107	0.9275
				(0.9102, 0.9112)	(0.9273, 0.9277)
2004	2	0.96	0.98	0.9737	0.9686
				(0.9735, 0.9739)	(0.9685, 0.9686)
Geomean	(all quarters)			0.9604	0.9722
	(all but 1st quarters)			0.9820	0.9727
	(only 1st quarters)			0.9024	0.9708

Note: Numbers in parenthesis are the bounds of the 95 percent confidence interval. `Quarters' designates an automotive model-year quarter. August, September and October compose the first quarter of the automotive year.

Appendix:

Price Hedonic Results

To construct a standard Paasche for case 3 described in section 4 of the paper, we need to estimate reservation prices of consumers who bought new goods in the current period. We followed Pakes (2003) in using a hedonic price equation to get predicted reservation prices. The basic idea is to use the estimated coefficients from a hedonic estimated on period *t-1* data to get a predicted price of a new good from period *t*.

The characteristics we use to estimate a new vehicle's price closely hew to those used in the Industrial Organization literature to understand consumers' motor vehicle purchasing behavior. The characteristics are horsepower over weight (a measure of acceleration), height, size (length multiplied by width), miles per dollar (miles per gallon divided by the price of a gallon of gas), and safety (a dummy variable equal to 1 if the vehicle offers driver, passenger, and side airbags). To estimate the relationship between prices and characteristics, we regressed the log of price on the characteristics and model-year dummies for each quarter in the data. We tried alternative specifications, including regressing the price level on characteristics and model-year dummies, and the log of price on the log of characteristics and model-year dummies, but found little-to-no improvement in the adjusted r-squareds.

Following the advice of Pakes (2003), we checked to make sure that the new goods are "close" to goods sold in the previous period. We accomplished this by making sure that every new good's characteristics were within the range of *t-1* goods' characteristics. This condition was violated in only 21 instances; these observations were not used to construct the standard Paasche index.

Table A reports the estimated coefficients on the characteristics from each hedonic regression, including the adjusted r-squared and the number of observations. The adjusted r-squareds are all above 0.79 and the estimated coefficients are stable across time, indicating that the hedonics do a strong job in approximating the relationship between prices and characteristics.

We do not report the estimated coefficients for the model-year dummies. We used the oldest observed model-year dummy variable as the reference point. In all cases the estimated coefficients on the model-year dummies were positive, but in most cases these estimates were statistically insignificant. In the few quarters where vehicles from three different model years were being sold, the point-estimates on the model-year dummies increased from oldest to newest, as expected.

Table A: Price Hedonic Coefficients by Quarter

	1999:Q4	2000:Q1	2000:Q2	2000:Q3	2000:Q4	2001:Q1	2001:Q2	2001:Q3	2001:Q4
Constant	3.80	3.87	3.88	3.24	3.20	3.02	3.36	2.64	2.78
Hp/wgt	0.98	1.02	1.02	1.16	1.17	1.14	1.05	1.20	1.20
Height	-0.23	-0.34	-0.42	0.06	0.10	0.06	-0.25	0.14	0.12
Size	0.16	0.14	0.13	0.24	0.25	0.29	0.32	0.40	0.41
Miles per Dollar	-9.09	-9.49	-10.54	-9.59	-9.25	-8.80	-8.93	-7.42	-6.62
Safety	0.19	0.17	0.15	0.16	0.15	0.15	0.16	0.14	0.16
Adjusted R-squared	0.81	0.83	0.85	0.84	0.79	0.79	0.80	0.80	0.78
Obs	100	170	156	118	103	170	160	118	106

	2002:Q1	2002:Q2	2002:Q3	2002:Q4	2003:Q1	2003:Q2	2003:Q3	2003:Q4	2004:Q1
Constant	2.67	2.88	2.79	2.79	2.84	2.82	2.44	2.84	2.92
Hp/wgt	1.15	1.19	1.27	1.26	1.10	1.17	1.28	1.09	1.05
Height	0.07	-0.03	0.26	0.28	0.14	0.25	0.72	0.25	0.03
Size	0.50	0.38	0.23	0.33	0.38	0.36	0.41	0.42	0.52
Miles per Dollar	-5.43	-5.78	-6.46	-6.63	-6.75	-8.28	-6.92	-8.40	-8.32
Safety	0.16	0.19	0.19	0.20	0.23	0.26	0.26	0.26	0.27
Adjusted R-squared	0.81	0.81	0.82	0.79	0.81	0.80	0.81	0.81	0.82
Obs	167	157	107	105	170	155	120	106	168

Note: The above quarters are over the automobile model year. August, September and October make up the first quarter of the automotive model year.