Coherent Instability Induced By Electron Cooling

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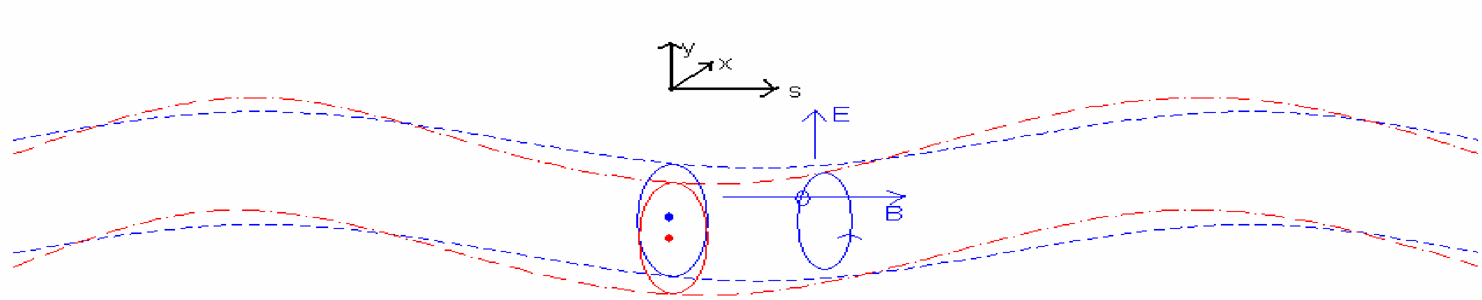
Outline

- Transverse Instability Induced by magnetized electron cooling beam:
  - Ion beam Transversal Instability Growth rate estimation.
  - Effects of the Ion clouds from residue gas on the Transversal Growth rate.
  - Possible Correction Methods.
- Transverse Instability Induced by non-magnetized electron cooling beam:
  - Ion beam Transversal Instability threshold estimation.
  - Coherent Damping effect due to the designed cooling electron beam.
- Longitudinal Instability due to electron beam for magnetized and non-magnetized cooling.

Two stream Instability Due to Magnetized Electron Beam



$$\frac{d}{dt}\overline{Z_e} + i\Lambda\overline{Z_e} = i\Lambda Z_i \longrightarrow \text{Drift Approximation}$$

$$\frac{d^2}{dt^2}Z_i + \omega_{ie}^2 Z_i + i\omega_{ci}\frac{d}{dt}Z_i = \omega_{ie}^2\overline{Z_e}$$

$Z_i \equiv X_i + iY_i$ $\quad Z_e \equiv X_e + iY_e$ $\quad \omega_{ce} = \frac{eB_{//}}{m_e} = 8.79\times10^{11}s^{-1}$ $\quad \omega_{ci} = \frac{Z_i eB_{//}}{M_i} = 1.93\times10^8 s^{-1}$

$\Lambda = \frac{\omega_{ei}^2}{\omega_{ce}} = 7.57\times10^5 s^{-1}$ $\quad \omega_{ei} = \sqrt{\frac{Z_i n_i e^2}{2m_e\varepsilon_0}} = 8.16\times10^8 s^{-1}$ $\quad \omega_{ie} = \sqrt{\frac{Z_i n_e e^2}{2M_i\varepsilon_0}} = 3.4\times10^7 s^{-1}$

Magnetized Beam Contin.1

cooling section transfer matrix

$$\begin{pmatrix} X_i(t) \\ X_i'(t) \\ Y_i(t) \\ Y_i'(t) \end{pmatrix} = T'_{cool} \begin{pmatrix} X_i(0) \\ X_i'(0) \\ Y_i(0) \\ Y_i'(0) \end{pmatrix}$$

$$T'_{cool} = \begin{pmatrix} A_{11} & A_{12}V_{//} & -B_{11} & -B_{12}V_{//} \\ \frac{A_{21}}{V_{//}} & A_{22} & \frac{-B_{21}}{V_{//}} & -B_{22} \\ B_{11} & B_{12}V_{//} & A_{11} & A_{12}V_{//} \\ \frac{B_{21}}{V_{//}} & B_{22} & \frac{A_{21}}{V_{//}} & A_{22} \end{pmatrix}$$

$$A_{11} = \frac{\omega_2(\cos(\omega_1 t) - T_1) - \omega_1(\cos(\omega_2 t) - T_2)}{\omega_2(1-T_1) - \omega_1(1-T_2)}$$

$$A_{12} = \frac{(T_2 - 1)\sin(\omega_1 t) - (T_1 - 1)\sin(\omega_2 t)}{\omega_2(1-T_1) - \omega_1(1-T_2)}$$

$$A_{21} = \frac{\omega_1\omega_2(\sin(\omega_2 t) - \sin(\omega_1 t))}{\omega_2(1-T_1) - \omega_1(1-T_2)}$$

$$A_{22} = \frac{\omega_2(1-T_1)\cos(\omega_2 t) - \omega_1(1-T_2)\cos(\omega_1 t)}{\omega_2(1-T_1) - \omega_1(1-T_2)}$$

$$T_\alpha = \left(1 - \frac{\omega_\alpha^2}{\omega_{ie}^2} + \frac{\omega_{ci}\omega_\alpha}{\omega_{ie}^2}\right)$$

$$B_{11} = -\frac{\omega_2(\sin(\omega_1 t) - T_1) - \omega_1(\sin(\omega_2 t) - T_2)}{\omega_2(1-T_1) - \omega_1(1-T_2)}$$

$$B_{12} = \frac{(T_2 - 1)\cos(\omega_1 t) - (T_1 - 1)\cos(\omega_2 t) + (T_1 - T_2)}{\omega_2(1-T_1) - \omega_1(1-T_2)}$$

$$B_{21} = \frac{\omega_1\omega_2(\cos(\omega_2 t) - \cos(\omega_1 t))}{\omega_2(1-T_1) - \omega_1(1-T_2)}$$

$$B_{22} = \frac{\omega_1(1-T_2)\sin(\omega_1 t) - \omega_2(1-T_1)\sin(\omega_2 t)}{\omega_2(1-T_1) - \omega_1(1-T_2)}$$

$$\omega_{1,2} = \frac{1}{2}\left[(\omega_{ci} + \Lambda) \pm \sqrt{(\omega_{ci} + \Lambda)^2 + 4(\omega_{ie}^2 - \Lambda\omega_{ci})}\right]$$

Magnetized Beam Contin.2

$$\begin{pmatrix} X_i(t) \\ X_i'(t) \\ Y_i(t) \\ Y_i'(t) \end{pmatrix}_{n+1} = T_{ring} \begin{pmatrix} X_i(0) \\ X_i'(0) \\ Y_i(0) \\ Y_i'(0) \end{pmatrix}_{n}$$

One turn transfer matrix

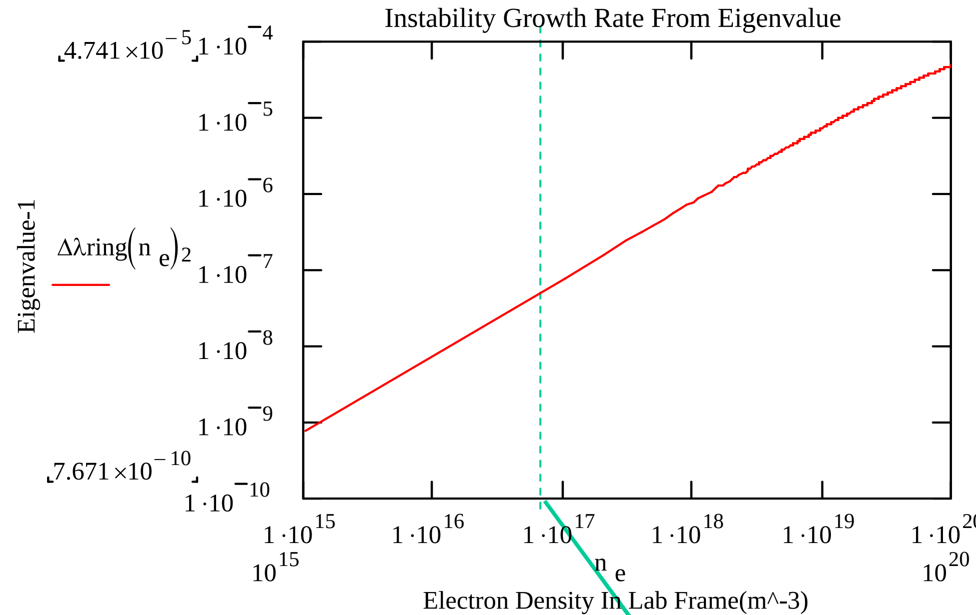
$$T_{ring} = L_{drift} T_{twiss} L_{drift} E_C^{-1} L_{lorentz}^{-1} T_{cool}^{'} L_{lorentz} E_C$$

$$\det(T_{ring}), |\lambda_{ring}| \begin{cases} \leq 1 & \text{Stable motion} \\ > 1 & \text{Unstable motion} \end{cases}$$

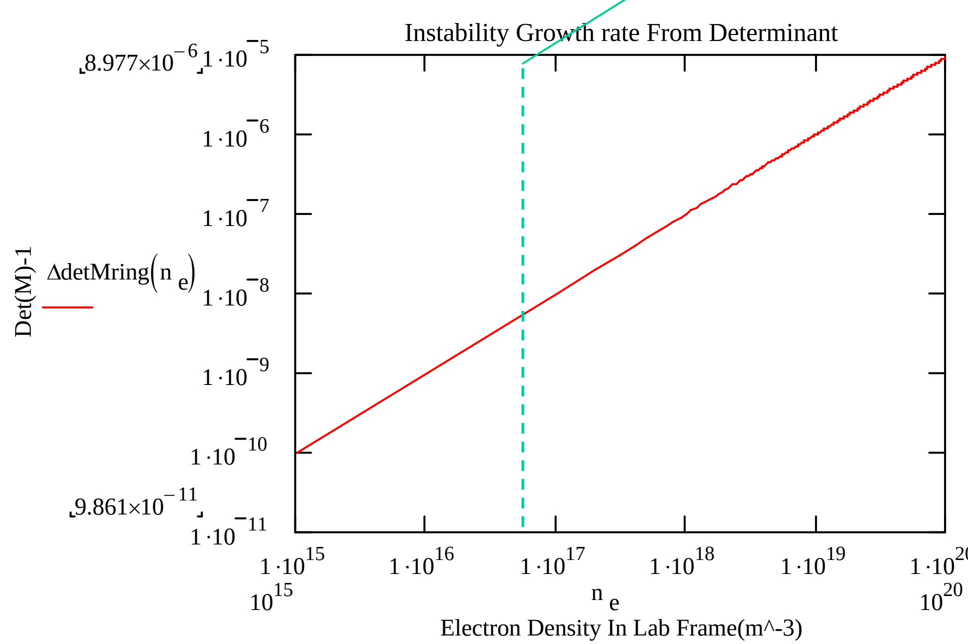
Rise time: 4minutes

$$|\lambda_{max}| - 1 = 5.5 \times 10^{-8}$$

$$\det(T_{ring}) - 1 = 7.0 \times 10^{-9}$$



For 5cm 20 nC beam



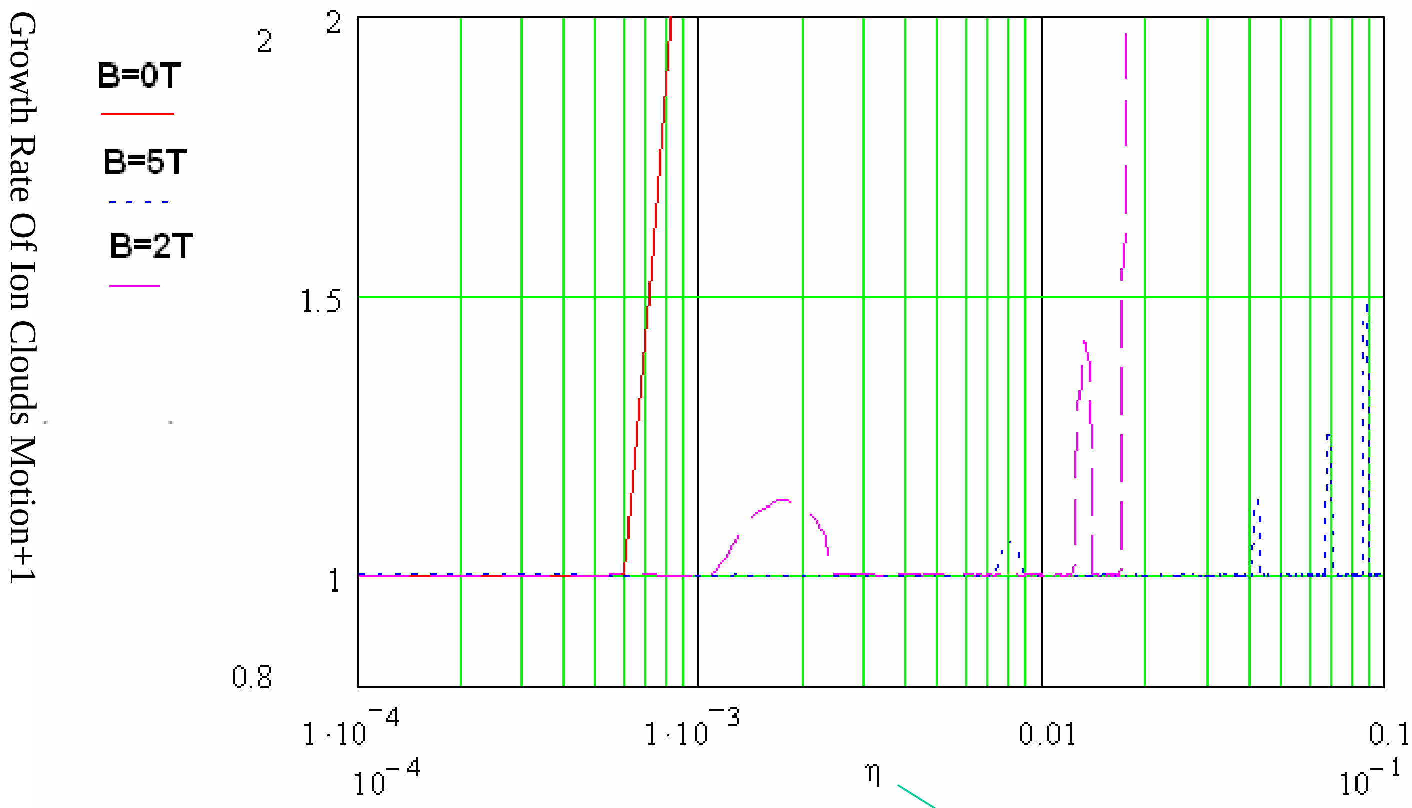
Magnetized E-cooling Designed parameters

Ion rms bunch length	0.37 m
Ion Density (Lab Frame)	$7.697 \times 10^{13} \ m^{-3}$
Solenoid field strength	5 T
Cooling section ion Beta function	60 m
Cooling section length	60 m
Electron rms beam size	1.225 mm
Electron rms bunch length	0.05 m
Electron beam charge	20 nC
Electron Density (Lab Frame)	$7.117 \times 10^{16} \ m^{-3}$

Magnetized Beam Contin.3 (ion cloud effects on growth rate)

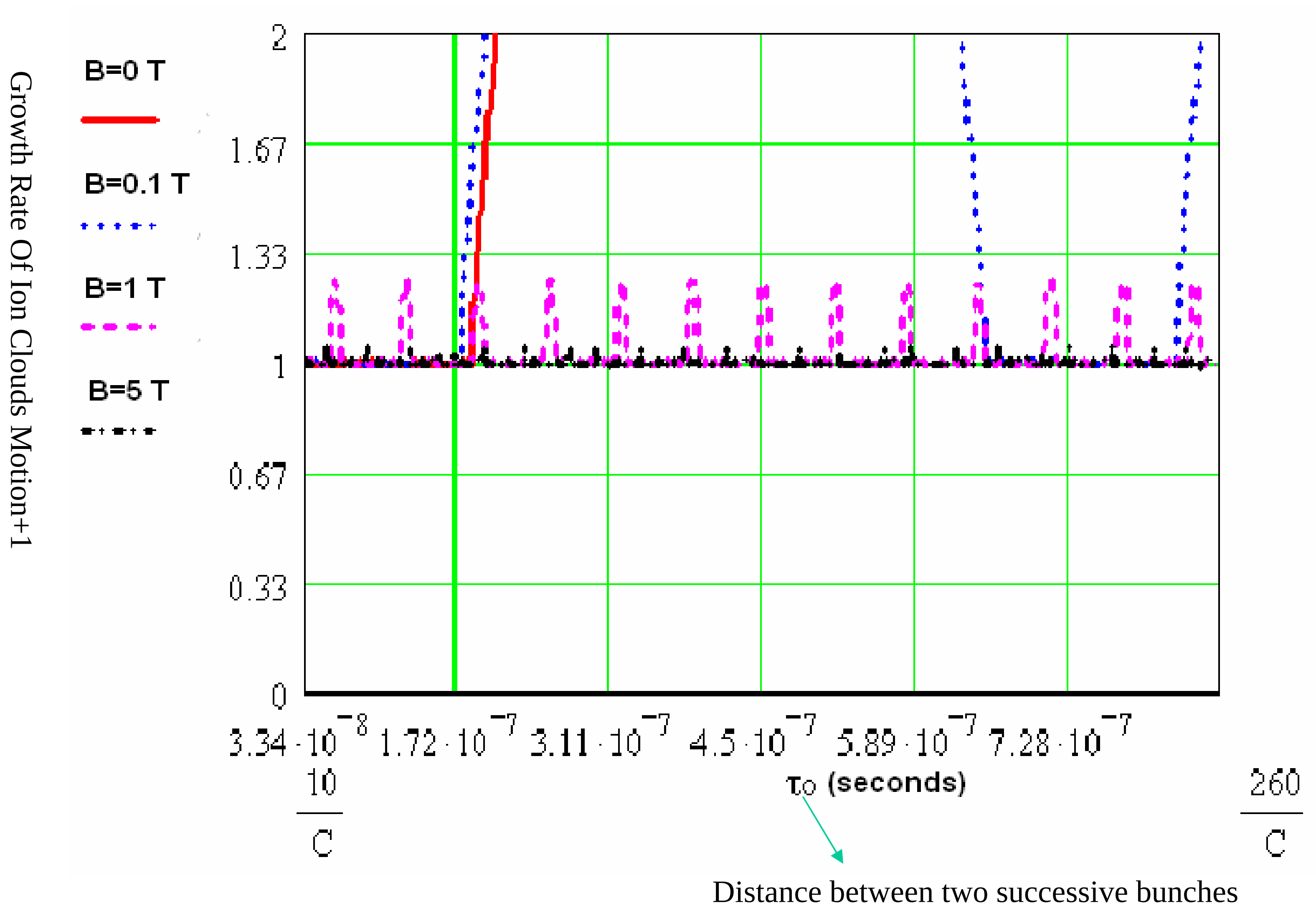
1.The longitudinal magnetic field helps for ion clouds to accumulate

Growth rate of Ion Clouds motion As a function of neutralization factor



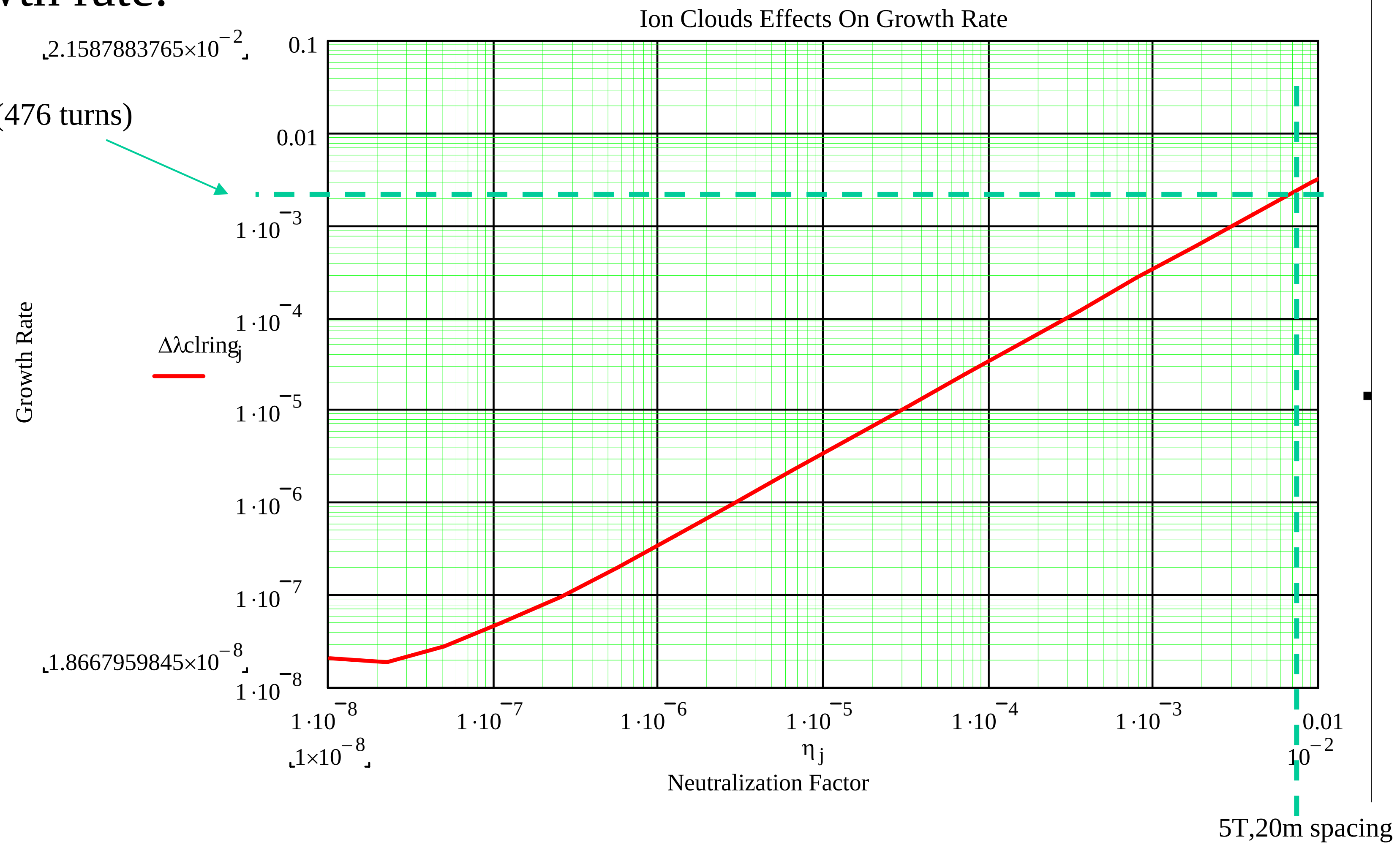
Neutralization Factor $\equiv \frac{n_{cloud}}{n_e}$

Growth rate of Ion Clouds motion As a function of bunch spacing



Magnetized Beam Contin.4 (Ion Cloud Effects On Growth Rate)

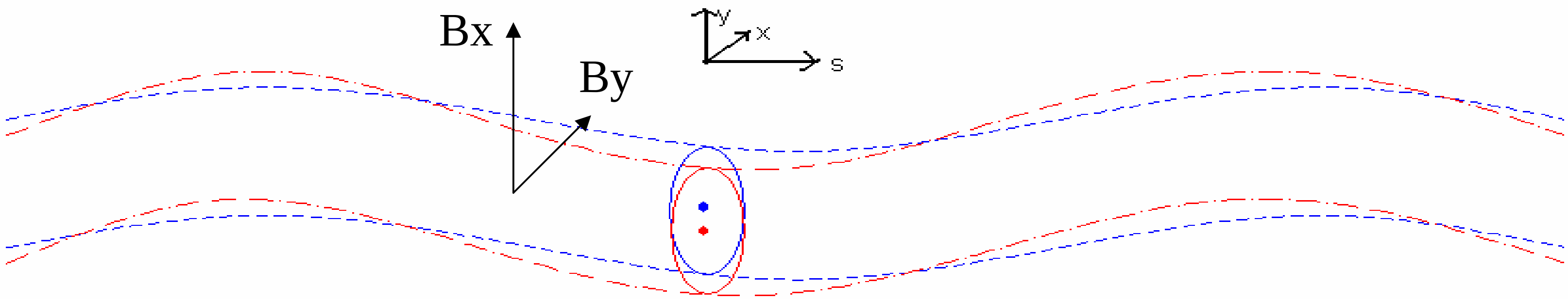
2.The accumulated ion clouds couple with the electron-ion beam oscillation and increase the ion beam instability growth rate.



Magnetized Beam Contin.5 (Possible Correction Methods)

1. Install clearing electrodes to remove the ion clouds from cooling section.
2. Increase the tune spread so that the landau damping is faster than the coherent growth rate.
3. Install Feedback system to damp the coherent oscillation.

Two stream Instability Due to Non-Magnetized Electron Beam



$$B_x = B_\perp \cos(\frac{2\pi s}{\lambda_w}) \qquad B_y = B_\perp \sin(\frac{2\pi s}{\lambda_w}) \qquad B_s = 0$$

$$\frac{d^2}{ds^2} X_i = -\Omega'^2_{ie} (X_i - X_e) - \frac{ZeB_\perp}{M_i \gamma c} \sin(\Omega'^2_w s)$$

$$\frac{d^2}{ds^2} X_e = -\Omega'^2_{ei} (X_e - X_i) + \frac{eB_\perp}{m_e \gamma c} \sin(\Omega'_w s)$$

Designed parameters

Ion number per bunch	10^9
Ion rms bunch length	0.37 m
Ion rms emittance	2.5 $\pi \mu m$
Wiggler field strength	0.001 T
Wiggler field wavelength	0.15 m
Cooling section ion Beta function	200 m
Cooling section length	60 m
Electron rms beam size	2.36 mm
Electron rms bunch length	0.009 m
Electron beam charge	5 nC
Ion Density (Lab Frame)	$2.31 \times 10^{13}\ m^{-3}$
Electron Density (Lab Frame)	$2.967 \times 10^{16}\ m^{-3}$

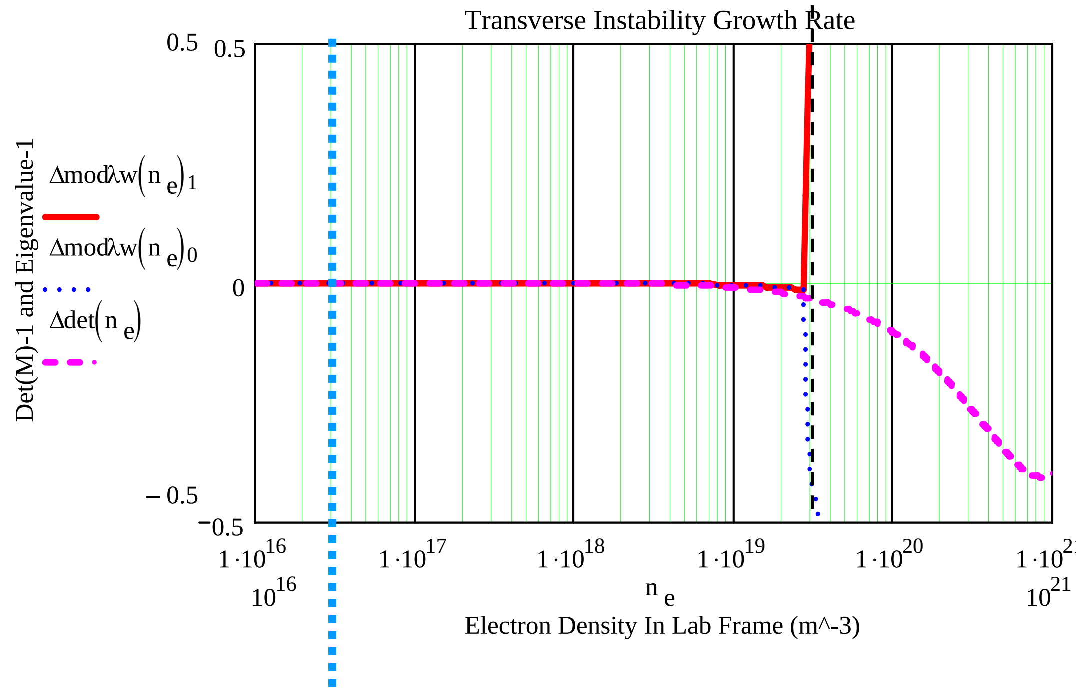
Non-magnetized Beam Contin.1

$$\begin{pmatrix} X_i \\ X'_i \\ 1 \end{pmatrix}_{n+1} = M_{ring} \begin{pmatrix} X_i \\ X'_i \\ 1 \end{pmatrix}_n$$

$$M_{ring} = R_x L_{drift}(-\frac{l_c}{2}) M_{cool} L_{drift}(-\frac{l_c}{2})$$

The instability threshold is three orders of magnitude bigger than the designed electron density.

$n_{eth} = 2.99 \times 10^{19} m^{-3}$



$n_e = 2.967 \times 10^{16} m^{-3}$ → Designed Electron Density within 1σ

$$M_{cool} = \begin{pmatrix} 1+\xi'(\cos(\Omega'_0 s)-1) & \frac{1}{\Omega'_0}[\Omega'_0 s(1-\xi') + \xi' \sin(\Omega'_0 s)] & a(s) \\ -\xi'\Omega'_0 \sin(\Omega'_0 s) & 1+\xi'(\cos(\Omega'_0 s)-1) & a'(s) \\ 0 & 0 & 1 \end{pmatrix}$$

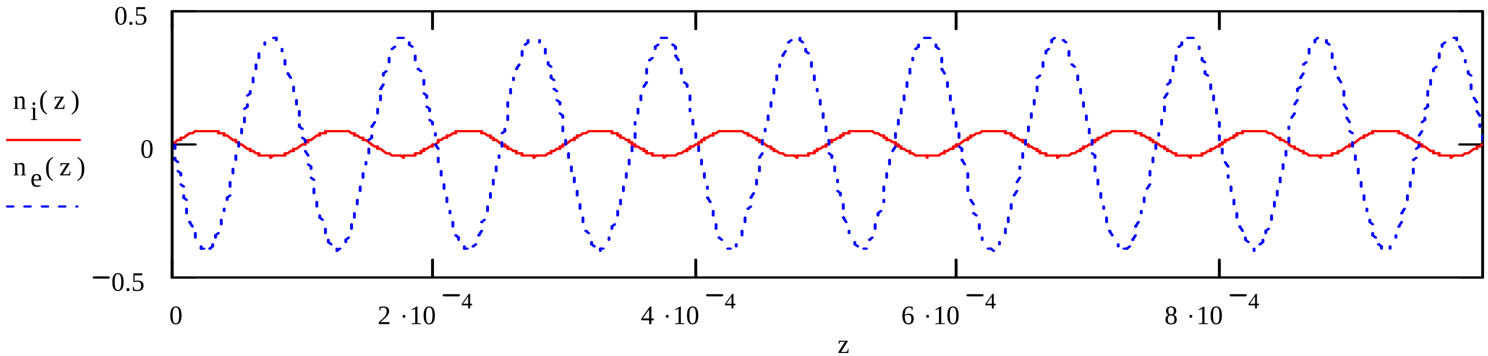
Non-magnetized Beam Contin.2

At the designed electron beam density, the two stream interaction will damp the ion beam coherent oscillation with the damping rate:

$$\left|\det_{transverse}\right| - 1 = -4.01 \times 10^{-6} \quad \text{Per turn}$$

Which corresponds to the damping time of 3.2 seconds.

Longitudinal Instability Induced by electron beam



$$\frac{d^2 s_i}{ds^2} + \frac{\omega_{pi}^2}{c^2} s_i = \frac{\omega_{ie}^2}{c^2} s_e$$

$$\frac{d^2 s_e}{ds^2} + \frac{\omega_{pe}^2}{c^2} s_e = \frac{\omega_{ei}^2}{c^2} s_i$$

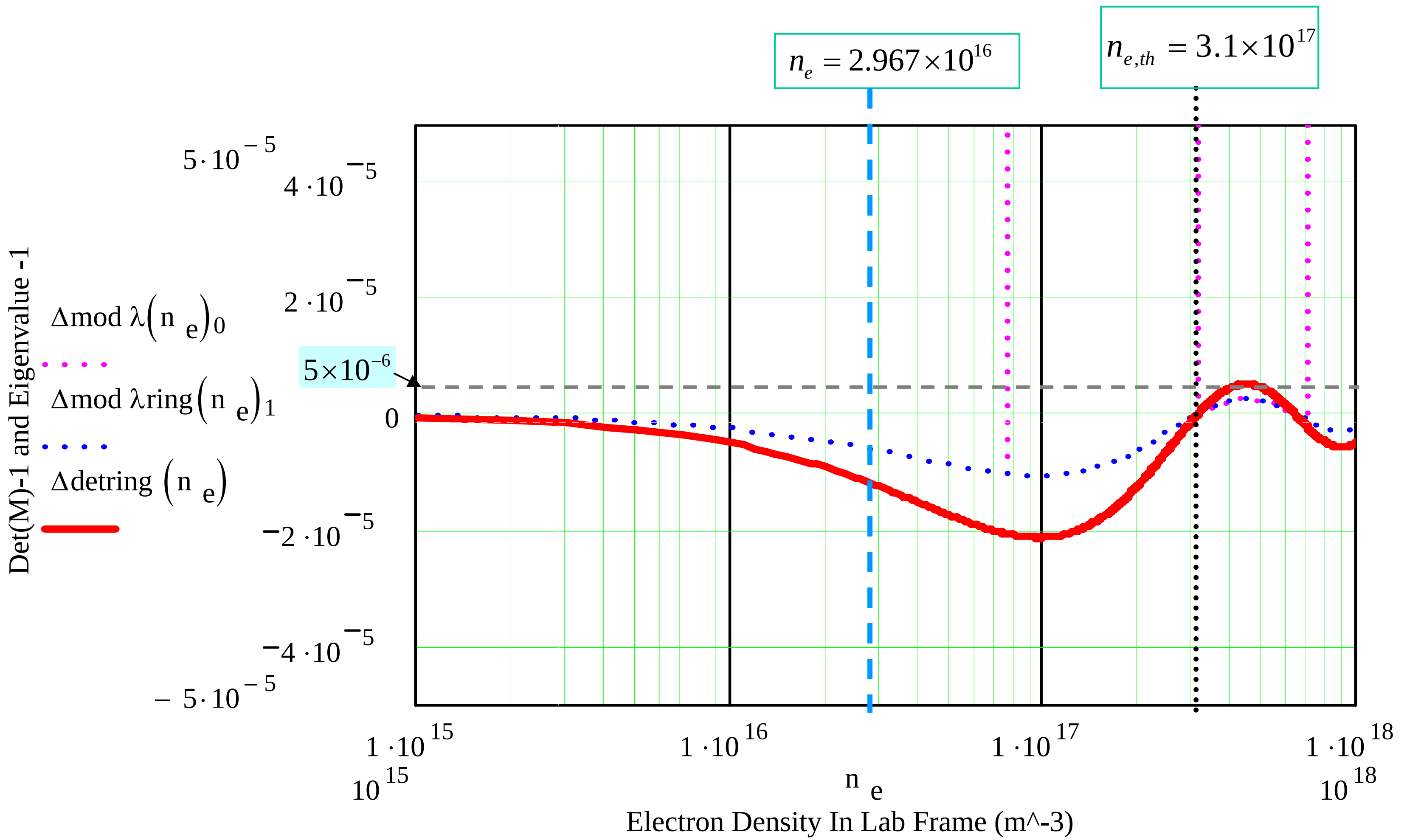
Inside Cooling section

$$\frac{d^2 s_i}{ds^2} + \frac{\omega_{pi}^2}{c^2} s_i = 0$$

→ Outside Cooling section

$$\begin{pmatrix} s_i \\ s_i' \end{pmatrix}_{n+1} = M_{ring} \begin{pmatrix} s_i \\ s_i' \end{pmatrix}_n \qquad M_{ring} = M_{cool} M_{out}$$

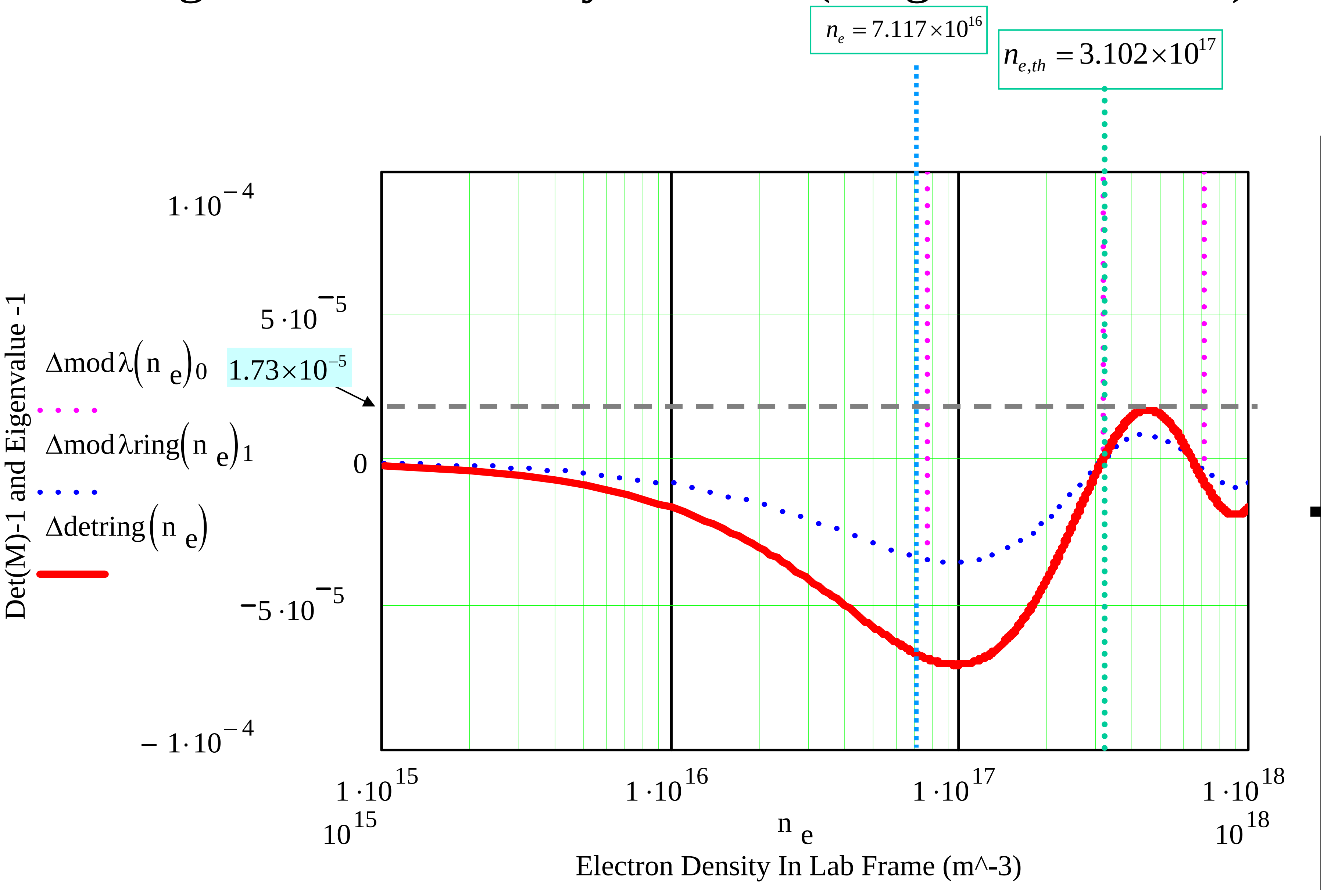
Longitudinal Instability Contin.1(Non-magnetized Beam)



$$M_{cool}=\begin{pmatrix} \xi\left(\cos\left(\frac{\omega_0}{c}l_{cool}\right)-1\right)+1 & \frac{1}{\omega_0}\left[\xi\sin\left(\frac{\omega_0}{c}l_{cool}\right)+(1-\xi)\frac{\omega_0}{c}l_{cool}\right] \\ -\xi\omega_0\sin\left(\frac{\omega_0}{c}l_{cool}\right) & \xi\left(\cos\left(\frac{\omega_0}{c}l_{cool}\right)-1\right)+1 \end{pmatrix}$$

$$M_{out}=\begin{pmatrix} \cos\left[\frac{\omega_0}{c}(Cir-l_{cool})\right] & \frac{\sin\left[\frac{\omega_0}{c}(Cir-l_{cool})\right]}{\omega_0} \\ -\omega_0\sin\left[\frac{\omega_0}{c}(Cir-l_{cool})\right] & \cos\left[\frac{\omega_0}{c}(Cir-l_{cool})\right] \end{pmatrix}$$

Longitudinal Instability Contin.2(Magnetized Beam)



Summary

1. Magnetized electron cooling beam would induce instability growth and the growth time is calculated to be 233 seconds.
2. Ion clouds generated from residue gas can increase the growth rate dramatically and cleaning electrodes would be necessary to keep the ion clouds density staying in a low level.
3. For the designed electron beam density, non-magnetized electron cooling would induce coherent damping instead of growth and the damping rate is calculated to be $0.31s^{-1}$. The electron density threshold for instability to take place is 3 orders of magnitude larger than the designed value.
4. The longitudinal plasma oscillation could grow if the electron rms bunch length smaller than 1.15 cm for magnetized cooling and 0.86 mm for non-magnetized cooling. However, the growth would be destroyed by synchrotron oscillation since the growth rate is much smaller than the synchrotron tune, 3.7×10^{-4} (Detailed study will be done in the future).