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by

David F. Findley, Brian C. Monsell,
Holly B. Shulman and Marian G. Pugh
Statistical Research Division
Bureau of the Census
Washington, DC 20233

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SLIDING SPANS DIAGNOSTICS FOR SEASONAL AND RELATED ADJUSTMENTS

David F. Findley, Brian C. Monsell, Holly B. Shulman, and Marian G. Pugh.[1]

ABSTRACT

When are the results of a seasonal adjustment procedure, or other smoothing procedure, likely to be of little value? The diagnostic approach we present offers an answer to this question and to other questions concerned with the comparison of competing adjustments. It is based on a straightforward idea. A minimal requirement of the output of any smoothing or adjustment procedure is stability: appending or deleting a small number of series values should not substantially change the smoothed values— otherwise, what reliable interpretation can they have? An important related principle is that, for a given series, if only one of several plausible signal extraction procedures has a stable output, then this procedure should be the preferred one for the series. To implement these principles successfully, the definition of stability must be made precise in an appropriate way. Our implementation is focused on multiplicative adjustments produced by the widely–used X–11 seasonal adjustment procedure, but it will be clear that the basic ideas are applicable more widely. The discussion addresses decisions about direct and indirect seasonal adjustment, trading day adjustment, trends, forecast extension prior to adjustment and other common adjustment issues.

KEYWORDS
Adjustment reliability, Indirect adjustments, Trading day adjustments, X–11 and X–11–ARIMA.

1. INTRODUCTION

The sliding spans technique involves the comparison of the correlated seasonal adjustments of a given month's datum obtained by applying the adjustment procedure to a sequence of three or four overlapping spans of data, all of which contain the month. Seasonally adjusted month–to–month and year–to–year change estimates and other related quantities are also examined. Excessive variability among such estimates indicates unreliability. Conversely, if there is no evidence of residual seasonality in the adjusted series, then one can interpret stability, meaning only moderate variability, as an indication that the estimates are reliable. This term does not mean "accurate" in an objective sense: "accuracy" is not an applicable concept here, because it seems impossible to give a completely objective definition of seasonality, see Bell and Hillmer (1984). Therefore, two adjustments of a series can be reliable, in this sense, and yet different, leaving room for additional criteria to be used to make a final choice. In this paper, we will give examples

[1]David F. Findley is Principal Researcher, Statistical Research Division, U.S. Bureau of the Census, Washington, DC 20233. Brian C. Monsell is Mathematical Statistician, Statistical Research Division, U.S. Bureau of the Census, Washington, DC 20233. Holly B. Shulman is currently Mathematical Statistician, Centers for Disease Control, Atlanta, GA 30333. Marian G. Pugh is currently a doctoral student at Harvard School of Public Health, Boston, MA 02115. The views expressed in this report are attributable to the authors and are not necessarily those of the Census Bureau.

demonstrating that the sliding spans technique provides insights not available from the traditional diagnostics about seasonal and trading day adjustments, seasonal filter length selection, forecast extension prior to adjustment and such questions as whether aggregated series should be adjusted indirectly or directly. Our analysis is limited to multiplicative seasonal adjustments.

2. SLIDING SPANS DIAGNOSTICS

To obtain sliding spans for a given series, an initial span is selected whose length depends on the seasonal adjustment filters being used. A second span is obtained from this one by deleting the earliest year of data and appending the year of data following the last year in the span. A third span is obtained from the second in this manner, and a fourth from the third, data permitting. This is done in such a way that the last span contains the most recent data. Figure 2.1 illustrates four eight–year spans of a series which includes January, 1974 and ends in December, 1984. There are four estimates $S_{1,81}(1),\ldots,S_{1,81}(4)$ for the seasonal factor of $X_{1,81}$, the observation occurring in January of 1981.

Each span is seasonally adjusted as though it were a complete series. Each month common to more than one span is examined to see if its seasonal adjustments or some related quantities vary more than a specified amount across the spans. Usually, there is a strong interest in having the analysis be based on data which are as close to contemporary as possible, which is an incentive to limit the number of spans. The investigations and suggested interpretations described in the remainder of the paper are based on the use of four spans. The use of more spans would increase the range of the calculated factors and related adjustments and therefore make it necessary to modify the interpretation of the sliding spans statistics given below for multiplicative adjustments.

We use the following notation.

$S_t(k)$ = the seasonal factor estimated from span k for month t;

$A_t(k)$ = the seasonally (and sometimes trading day) adjusted value from span k for month t;

$MM_t(k)$ = the month–to–month percentage change in the adjusted value from span k for month t (formula below);

$YY_t(k)$ = the year–to–year percentage change in the adjusted value from span k for month t (formula below);

$N_t = \{k : \text{month } t \text{ is in the } k\text{–th span}\}$;

$N1_t = \{k : \text{months } t \text{ and } t-1 \text{ are in the } k\text{–th span}\}$.

The Census Bureau's X–11.2 seasonal adjustment program (Monsell 1989) identifies or "flags" the (time series value associated with) month t as having an unreliable seasonal factor if

$$S_t^{max} = \frac{\max_{k\epsilon N_t} S_t(k) - \min_{k\epsilon N_t} S_t(k)}{\min_{k\epsilon N_t} S_t(k)} > 0.03. \qquad (2.1)$$

Many users adjust a series chiefly to get a "seasonally adjusted" value of the month–to–month percentage change. The estimate of this quantity,

$$MM_t(k) = \frac{A_t(k) - A_{t-1}(k)}{A_{t-1}(k)},$$

will be considered unreliable if

$$MM_t^{max} = \max_{k\epsilon N1_t} MM_t(k) - \min_{k\epsilon N1_t} MM_t(k) > 0.03. \qquad (2.2)$$

Relation (2.1) tests whether the maximum percentage difference in the seasonal factors for month t, S_t^{max}, is greater than 3 percent. Relation (2.2) tests whether the largest difference in the month–to–month percentage change in the adjusted data for a month t, MM_t^{max}, exceeds the same threshold. An unstable estimate of a month's seasonal factor can give rise to unstable estimates of the two associated month–to–month changes. So, with most series, more months are flagged for unreliable month–to–month changes than for unreliable seasonal factors.

It is useful to know whether months with unreliable adjustments cluster in certain years or calendar months and whether their sliding spans statistics barely or substantially exceed the threshold. For example, problems confined to early years can be a sign that seasonal adjustments should be calculated from a shorter series which excludes these years. In the sliding spans output of the X–11.2 program, summary tables are given in which the months flagged are grouped by year, by month, and by magnitude of each sliding spans statistic.

In the examples discussed in the sections 3–8, the adjustments which are analyzed and compared are all plausible in the weak sense that the seasonally adjusted series exhibit no residual seasonality, according to both the F–statistic of table D11 of X–11.2 (and X–11–ARIMA) and X–11.2's calculated spectrum of the differenced adjusted series. Only after such tests for residual seasonality have been performed is it appropriate to be concerned with the stability properties measured by the sliding spans statistics.

We also analyze the stability of such year–to–year changes, in order to obtain an indication of the stability of comparisons over spans of length greater then a few months. (Many consumers of seasonally adjusted data have a more direct interest in year–to–year changes. They compare a month's adjusted value with the adjusted value for the same calendar month a year earlier to estimate the direction of the trend. Unfortunately, such comparisons can be misleading when there is a trend turning point in the intervening months.) Setting

$$N12_t = \{k : \text{months } t \text{ and } t-12 \text{ are in span } k\},$$

we consider the estimate of seasonally adjusted year–to–year percent change,

$$YY_t(k) = \frac{A_t(k) - A_{t-12}(k)}{A_{t-12}(k)},$$

to be unreliable if

$$YY_t^{max} = \max_{k \epsilon N12_t} YY_t(k) - \min_{k \epsilon N12_t} YY_t(k) > 0.03. \qquad (2.3)$$

As we will show later by example, difficulties with trading day effect estimation can result in unreliable estimates of year–to–year percent change. Our principal use for (2.3) is to help detect such difficulties. Another useful indicator of an unstable trading day adjustment is a number of unstable month–to–month changes which is quite high relative to the number of months with unstable seasonal factors, see section 4. We have not found a useful way to directly assess the instability of trading day factor estimates, because of their small range. The indirect approach just described appears to be effective.

In our investigations, the earliest of which are summarized in Findley and Monsell (1986), we found that a series which seems to have good seasonal adjustments according to a variety of criteria, including those of Lothian and Morry (1978), Bureau of Labor Statistics (1977) and the opinions of subject matter experts, usually has fewer than fifteen percent of its months flagged for unstable seasonal factor estimates. Adjustments with more than twenty–five percent of the months flagged almost never seemed acceptable. Thus there is an interval of uncertainty between 15 and 25 percent. We recommend that seasonal adjustment not be performed if more than forty percent of the estimates of month–to–month change are flagged.

The threshold value of 0.03 in (2.1) – (2.3) will be too large if all the seasonal factors are close to 100, but a comparison of the factors and the month–to–month changes themselves across the different spans will probably be informative in such a case. Sometimes 0.03 is too small a threshold for series with very large seasonal movements. The histogram output of X–11.2 makes it possible to see the effect on the statistics of changing this value to 0.04 or 0.05. The adjuster could change the threshold according to his or her own sense of how much variability is acceptable. In fact, we obtained the value 0.03 by asking an expert from an econonomic statistics division at the Census Bureau how much uncertainty would be tolerable. It is important, however, to remember that the true variability is likely to be greater than the statistics suggest, because the different seasonal factors for a given month are produced from closely related filters applied to substantially overlapping spans of correlated data, and are therefore highly correlated. We caution against raising the threshold value or upper percent limits mentioned above without careful study both of the type of series being adjusted and of the uses intended for the adjusted data. The recommended values have been satisfactory for the great majority of the more than five hundred series to which they have been applied at the Census Bureau.

We do not have a recommended upper limit for the acceptable percentage of unstable year–to–year changes. Values around 2% are common with good series; 10% is quite high.

Table 2.1 shows the January, 1974 – February, 1977 section of a month–by–month sliding spans analysis of the estimates of seasonal factors produced by the X–11.2 program. In addition to the seasonal factors, the maximum percent differences S_t^{max} between factors are given for months common to more than one span, along with symbols flagging months for which S_t^{max} exceeds the 3% threshold. The specific symbols (%, %%, %%%) correspond to three levels of excess. They identify the members of histogram cells included in the program's sliding spans output.

In our subsequent discussion, we will refer to the following summary statistics:

S(%) = percentage of months with unreliable seasonal factor estimates ($S_t^{max} > 0.03$);

MM(%) = percentage of months with unreliable month–to–month percent change estimates ($MM_t^{max} > 0.03$);

YY(%) = percentage of months with unreliable year–to–year percentage change estimates ($YY_t^{max} > 0.03$).

The term "unreliable" is used in the sense discussed above, and "percentage" in each case is relative to the number of <u>candidate</u> <u>months</u>: this is the number of values in $\{t : N_t$ is nonempty$\}$ for S(%), $\{t : N1_t$ is nonempty$\}$ for MM(%), etc. With this notation, our recommendations are summarized in Table 2.2.

We close this section by describing how the choice of seasonal filter determines the span length. A 3x5 seasonal moving average is a simple centered average of three neighboring simple centered averages of five consecutive values of the same calendar month. Such filters are applied to a detrended version of the series (the SI ratios), see Shiskin et al. (1967) or Dagum (1983b). Throughout this paper, when 3x5 (respectively, 3x3 or 3x9) seasonal filters are used, then four eight–year spans (respectively, six– or eleven–year spans) are used to produce the sliding spans analysis. These span lengths are close to the smallest which can be used to adjust with the associated seasonal filters (the non–zero weights of a 3x5 filter span seven years, etc.), see Dagum (1982).

Table 2.1 Portion of Sliding Spans Analysis of Seasonal Factors for Crude Corn Oil Production

	SPAN					
MONTH	1/74 – 8/82	1/75 – 8/83	1/76 – 8/84	1/77 – 8/85	S_t^{max}	
1–74	93.3	–	–	–	–	
2–74	91.2	–	–	–	–	
3–74	103.6	–	–	–	–	
4–74	104.6	–	–	–	–	
5–74	108.3	–	–	–	–	
6–74	103.0	–	–	–	–	
7–74	101.3	–	–	–	–	
8–74	104.8	–	–	–	–	
9–74	100.8	–	–	–	–	
10–74	101.0	–	–	–	–	
11–74	95.0	–	–	–	–	
12–74	93.1	–	–	–	–	
1–75	93.2	93.6	–	–	0.45	
2–75	91.1	93.5	–	–	2.54	
3–75	103.8	104.3	–	–	0.51	
4–75	104.0	103.2	–	–	0.80	
5–75	108.6	109.4	–	–	0.76	
6–75	103.5	104.6	–	–	1.10	
7–75	101.3	103.8	–	–	2.47	
8–75	104.4	99.2	–	–	5.22	%%%
9–75	100.8	100.5	–	–	0.24	
10–75	101.5	102.7	–	–	1.25	
11–75	95.5	95.3	–	–	0.22	
12–75	92.6	89.8	–	–	3.16	%
1–76	92.9	93.8	91.1	–	2.97	
2–76	90.8	93.0	89.4	–	4.08	%%
3–76	104.3	104.6	104.9	–	0.54	
4–76	103.4	103.1	101.1	–	2.33	
5–76	109.0	109.7	109.2	–	0.72	
6–76	103.5	104.3	104.0	–	0.75	
7–76	101.4	103.3	101.1	–	2.25	
8–76	104.0	99.5	102.7	–	4.51	%%
9–76	100.5	100.5	100.8	–	0.32	
10–76	102.2	102.7	105.0	–	2.80	
11–76	96.3	95.7	99.7	–	4.16	%%
12–76	92.1	90.3	90.6	–	2.03	
1–77	93.0	93.8	91.4	93.7	2.54	
2–77	90.6	92.5	89.5	90.5	3.35	%
3–77	104.9	105.2	105.4	107.2	2.20	
4–77	102.6	102.7	101.1	100.8	1.87	

Table 2.2 Summary of Adjustment Recommendations for Series Whose Maximum and Minimum Seasonal Factors Differ by at Least 10.

S(%) and MM(%)	ADJUSTABLE?
S(%) ≤ 15.0 ; MM(%) ≤ 40.0	likely
15.0 < S(%) ≤ 25.0 ; MM(%) ≤ 40.0	less likely
S(%) > 25.0 or MM(%) > 40.0	unlikely

3. SEASONAL ADJUSTABILITY: SLIDING SPANS AND Q

One of the strengths of the X–11–ARIMA and X–11.2 enhancements of X–11 is the large number of diagnostics they provide to determine if the adjustments they calculate for a series are reliable. Among these diagnostics is a summary statistic, Q, which is a weighted average of eleven other diagnostic statistics (M1–M11). Despite remarks warning against relying exclusively on this summary measure in the X–11–ARIMA manual (Dagum 1983b) and in Lothian and Morry (1978), it is our experience that some users decide to adjust or not adjust an individual series solely according to whether or not the value of Q is less than the threshold value 1.0 emphasized by X–11–ARIMA. The two examples of this section show concretely that Q is not discrimminating enough to be used in this way.

We find Q to be a useful screening statistic in the sense that values of Q higher than 1.20 usually indicate that almost all diagnostics, ours as well as others, will recommend against adjustment. In this and later sections, a number of example series will demonstrate that lower values of Q do not adequately discriminate between adjustable series and series which cannot be reliably adjusted. Other information is needed, and the sliding spans diagnostics are especially informative, because of their direct connection with the quantities of interest.

We now consider two series for which the Q statistic and the sliding spans diagnostics contradict each other, Shipments of Building Papers (SBP) and Corn Oil Production (COP), see Table 3.1.

Table 3.1 Diagnostic Statistics for Two Series

Series	Dates	Q	S(%)	MM(%)	YY(%)
SBP	1/74–12/84	0.85	38.9	52.3	5.2
COP	1/71–8/85	1.14	1.3	11.3	1.4

SBP – Value of Shipments of Building Papers
COP – Crude Corn Oil Producion

The first series, SBP, has a low Q–value of 0.68, which might suggest good adjustability. However, the sliding spans diagnostics show that the adjustment is not stable. For this series, much evidence points to problems in the recent data. The graph in figure 3.1 shows that the character of the series changes in later years. The M10 and M11 statistics of X–11–ARIMA have failing scores (> 1.0) of 1.6 indicating that the seasonal factors over the last few years of data are evolving too rapidly. The sliding spans diagnostics reveal large numbers of months with unstable seasonal factors, month–to–month and year–to–year change estimates in the last three years, see Table 3.2. A later analysis in section 9 shows that the instability in the last two years is probably even understated by the sliding span statistics. The lack of stability in the seasonal factors for recent years does not cause a failing Q statistic but does cause failing sliding spans diagnostics. Should we adjust or not? Recent data are more important to most data users. Therefore we would choose not to adjust.

For the second series, COP, a high Q–value of 1.14 might be interpreted to mean that the series is not adjustable, but the sliding spans diagnostics suggest that the adjustment is stable. The high value of Q for COP is caused mainly by the M1 and M2 statistics, which measure the relative contribution of the irregular component. Both have values which are close to 3.0 (the maximum value for these statistics). It is somewhat unusual to encounter a series in which a large irregular component does not compromise the stability of the adjustments. Seeking additional evidence, we also calculated the

Table 3.2 SBP: Number of Months Flagged, by Year and by Sliding Spans Statistic

Year	S(%)	MM(%)	YY(%)
1975	2	3	0
1976	3	7	0
1977	4	5	0
1978	1	2	0
1979	2	3	0
1980	6	9	0
1981	10	12	1
1982	8	10	3
1983	6	5	1

statistics CPREV and CONRAT of Findley and Monsell (1986). CPREV measures the total absolute revision experienced by each month's successive seasonal adjustments, from the initial adjustment through to the final seasonal adjustment. CONRAT measures the rate of convergence to the final adjustment. This final adjustment is essentially obtained when enough data are available to apply a nearly symmetric seasonal filter. The values of CPREV and CONRAT were well within the range associated with acceptable seasonal adjustments in the cited paper. Therefore, in the COP example, we accept the conclusion suggested by the sliding spans statistics that this series is adjustable.

4. SLIDING SPANS AND X–11's TRADING DAY REGRESSION F–STATISTIC

Among X–11 and X–11–ARIMA users, the most commonly applied criterion for deciding whether or not to adjust a series for trading day variation is the F–statistic from the program's trading day regression. The "irregular" values to which this regression is applied are the output of a filtering operation, a situation which causes some problems, see Cleveland and Devlin (1980). One problem is that the irregular series is correlated, so the same is true of the "error" terms in the regression. As a consequence of this, the distribution of the regression F–statistic will differ from the F–distribution, and the program's use of critical values from an F–distribution will sometimes result in misleading conclusions concerning the statistical significance of a trading day component.

Table 4.1 starts with the trading day regression F–statistic and the sliding spans statistics for seven foreign trade import series. Assuming the F–distribution with the indicated degrees of freedom, the values given for the F–statistics (denoted F–TD) would all be highly significant, a strong indication for trading day adjustment. However, the sliding spans statistics show that application of X–11's trading day adjustment leads to many more unstable estimates of seasonally adjusted month–to–month and year–to–year change. This suggests the series should not be trading day adjusted. There is additional evidence favoring non–adjustment. Cleveland and Devlin (1980) and Cleveland (1983) describe the use of spectrum diagnostics applied to the irregular series to determine the presence of trading day variation. If no peaks are found at trading day alias frequencies, then it is unlikely there is statistically significant trading day variation in the series. The upper graph in Figure 4.1 is a spectrum estimate of the irregular modified for extremes of the Imports from European Common Market Countries series (IOECD). There are no peaks at the designated frequencies. The same was true for all the import series given in Table 4.1.

After this analysis was completed, it was determined that a much higher than expected percentage of Customs forms were arriving at the Census Bureau one or more months late and were being incorrectly assigned to the month of their arrival. These errors would mask any trading day effects in the Imports series, and they provide an explanation for the inadequacies identified in X–11's trading day adjustment of these series.

Our analysis of these and other series shows that the X–11 trading day F–statistic is inadequate, as a sole diagnostic, for deciding in favor of trading day adjustment (except for when its values are quite large, say, F–TD > 15.0). Sliding spans and spectral diagnostics provide useful additional information.

Table 4.1 Sliding Spans Analysis: Trading Day Adjustment/No Adjustment

Series	S(%)	MM(%)	YY(%)	F–TD
IANEC	22.9/10.4	45.3/29.5	19.0/0.0	4.7
IASIA	9.4/11.5	37.9/29.5	21.4/0.0	5.0
IOECD	12.5/7.3	42.1/24.2	31.0/0.0	6.0
IOEEC	13.5/9.4	36.8/25.3	17.9/0.0	3.8
IWEUR	12.5/9.4	35.8/24.2	26.2/0.0	5.4
IWGER	6.3/0.0	29.5/12.6	25.0/0.0	6.2
IWH	6.3/2.1	40.0/9.5	38.1/0.0	3.4
XOECD	8.3/2.8	28.0/10.5	2.5/0.0	5.9

IANEC – Imports from Certain Asian Countries
IASIA – Imports from Asia
IOECD – Imports from OECD Countries
IOEEC – Imports from European Common Market Countries except the United Kingdom and West Germany
IWEUR – Imports from Western Europe
IWGER – Imports from West Germany
IWH – Imports from the Western Hemisphere
XOECD – Exports from OECD Countries.

NOTE: F–TD is the trading day regression F–statistic from X–11.2 Table C15. The 1% critical values of the F–distribution with either (6,124) degrees of freedom (for the Import series) or (6,152) degrees of freedom (for XOECD) is approximately 2.96.

For the final series in Table 4.1, Exports to European Common Market Countries (XOECD), we have a different situation. There are broad peaks in the spectrum which encompass the trading day frequencies although they are not centered there, see the lower graph in Figure 4.1. This suggests a somewhat erratic trading day component, the lower graph and the sliding spans analysis show that there are some undesirable consequences from adjusting this series for trading day effects. However, both seasonal adjustments, with and without trading day adjustment, are acceptable according to the standards given in Table 2.1. Therefore, the decision whether to trading day adjust can based on what is more important to the analyst, eliminating signs of trading day variation from the seasonally adjusted series or generating more stable seasonally adjusted estimates.

5. DIRECT VERSUS INDIRECT ADJUSTMENT: SLIDING SPANS AND SMOOTHNESS

Suppose the series to be seasonally adjusted, X_t, is the sum of component series which are also seasonally adjusted. Then, in addition to the series obtained by direct adjustment of the X_t, a second seasonally adjusted version of the series can be obtained by summing the adjusted component series. This second approach is called indirect adjustment. It will yield a different series from the one obtained by direct adjustment, either because the individual components have differing seasonal patterns, or because of non– linearities in the adjustment procedure arising from outlier adjustment and from multiplicative seasonal adjustment calculations. Indirect adjustment is also an option to be considered when the series to be adjusted is a more complicated function of other series, see subsection 5.2 below.

How does one choose between adjustments of a series obtained from different plausible adjustment procedures? In statistical agencies, the choice is often based on characteristics of the adjusted series which many data users find desirable. For comparing direct and indirect adjustments, the property most often employed has been "smoothness", as measured by one or more "smoothness measures." The smoothness measures R1 and R2 calculated by X–11–ARIMA to facilitate the comparison of direct and indirect adjustments are defined as follows:

$$R1 = (N-1)^{-1}\Sigma_{t=2}^{N} (A_t - A_{t-1})^2 \text{, and}$$

$$R2 = N^{-1}\, \Sigma_{t=1}^{N} (A_t - H_t)^2 ,$$

where A_t $(1 \leq t \leq N)$ is the seasonally adjusted series (direct or indirect), H_t $(1 \leq t \leq N)$ is the associated trend obtained by smoothing the adjusted series with the Henderson trend weights, and N is the length of the series, see Dagum (1983b). Analogous quantities are also calculated for just the last three years of adjustments.

There are no theoretical models of seasonality whose ideal seasonal adjustment minimizes a quantity estimated by R1 or R2. For this reason the use of such measures to compare adjustments ("smoother" is "better") is somewhat unsatisfactory. It seems more prudent to examine the reliability of the competing adjustments, as measured by sliding spans statistics such as S(%) (calculated from implied seasonal factors $S_t^{ind} = X_t/A_t^{ind}$ in the case of indirect adjustment) and the corresponding MM(%) and YY(%). In fact, the smoothness and reliability criteria often agree, as the examples of subsection 5.2 below suggest, but we will begin in subsection 5.1 with an example where indirect adjustments are more stable, but less "smooth" in the sense of R1 and R2. Rather than give individual R–values, we will follow X–11–ARIMA's practice of giving percentage difference values,

$$\Delta = 100\cdot(R^{direct} - R^{indirect})/R^{direct} .$$

Therefore, according to the traditional use of these statistics, negative values of Δ1 or Δ2 favor direct adjustment.

5.1 An Aggregate Series

The series HS1FTL of total U.S. single family house construction starts is the sum of four regional series associated with totals from the northeastern, midwestern, southern and western states. Each regional series is seasonally adjusted so that an indirect adjustment of HS1FTL is available. The estimated seasonal patterns differ substantially among the regions, as would be expected from the differences in climate. This suggests that seasonality is better removed at the regional level; that is, an indirect adjustment

Table 5.1.1 X–11–ARIMA Smoothness Statistics for HS1FTL

Full Series		Last 3 years	
Δ1	Δ2	Δ1	Δ2
–43.9	–32.4	–55.1	–68.4

Table 5.1.2 Sliding Spans Statistics for HS1FTL

Adjustment	S(%)	MM(%)	YY(%)	Q
direct	24.5	41.8	0.0	0.27
indirect	13.6	26.0	0.0	0.37

should be more satisfactory, which is the conclusion suggested by the sliding spans diagnostics. However, the smoothness statistics favor direct adjustment rather strongly. Values of the smoothness and sliding spans statistics for the direct and indirect adjustments are given in Tables 5.1.1 and 5.1.2 below, along with the Q summary statistics.

The sliding spans statistics show, in fact, that the direct adjustment is not reliable, and a separate analysis by the statistics CONRAT and CPREV of Findley and Monsell (1986), discussed in section 3, confirmed this. Thus this example reveals the inadequacy of both the R and the Q statistics for determining the choice between direct and indirect adjustment. Lothian and Morry (1978) also warn against using the Q statistic to choose between direct and indirect seasonal adjustments.

5.2 Derived Series

The various New Orders series published by the Manufacturer's Shipments, Inventories, and Orders (M3) Branch of the Census Bureau's Industry Division are not measured directly but are obtained as the sum of the reported Value–of–Shipments series and the monthly change in the reported Unfilled Orders,

$$NO_t = VS_t + (UO_t - UO_{t-1}) \ ,$$

in a self–explanatory notation. The observed series VS_t and UO_t are seasonally adjusted, so both direct and indirect adjustment of NO_t can be considered.

We present in Tables 5.2.1 and 5.2.2 the smoothness, Q and sliding spans statistics for three New Orders series. The indirect adjustment of OLP seems unacceptable and caution seems called for with its direct adjustment. For the other series, too, direct adjustment seems preferable.

Table 5.2.1 X–11–ARIMA Smoothness Statistics for New Orders Series

Series	Full Series Δ1	Full Series Δ2	Last 3 yrs. Δ1	Last 3 yrs. Δ2
NFOPM	–10.1	– 1.1	– 1.4	– 3.5
BFOT	– 5.5	+ 0.4	– 1.5	– 2.1
OLP	–13.3	–19.1	–16.8	–21.0

Table 5.2.2 Sliding Spans, Q Statistics for New Orders Series

Series	Adjustment	S(%)	MM(%)	YY(%)	Q
NFOPM	direct	14.2	32.4	0.0	0.93
	indirect	17.0	36.2	1.1	1.00
BFOT	direct	10.4	22.9	0.0	0.73
	indirect	11.3	29.5	0.0	0.86
OLP	direct	23.6	33.3	0.0	0.64
	indirect	27.4	44.8	3.2	0.81

NFOPM – Nonferrous and Other Primary Metals
BFOT – Broadwoven Fabrics and Other Textiles
OLP – Other Leather Products

5.3 Raking

Sometimes, for reasons of consistency, the seasonally adjusted component series are modified to force them to have the same annual totals as the direct adjustment or some other adjustment of the aggregate. This is usually done by proportionally redistributing the difference between the indirect and the other adjustments, a procedure known as raking, see Ireland and Kullback (1968) and Fagan and Greenberg (1988), for example. Although we will not give an illustrative example here, it is worth mentioning that we have found sliding spans analysis to be a useful means of assessing the effect (usually benign, it seems) of such modifications on the quality of the seasonal adjustments of the components.

6. THE USE OF SLIDING SPANS DIAGNOSTICS TO ASSIST IN THE SELECTION OF SEASONAL FILTERS

How sensitive is the stability of an X–11 seasonal adjustment to the choice of seasonal filter? We will provide examples to show that different choices of filters can lead to dramatically different values of the sliding spans statistics. Table 6.1 presents the results for two series from the M3 survey and for the Total U.S. Imports series. Different span lengths were used with the different filter lengths, as described in section 3.

Table 6.1
Sliding Spans Statistics for Varying Seasonal Filter Lengths

Series	Seasonal Filter	S(%)	MM(%)	YY(%)
EDMISC	3x5	35.3	42.6	0.0
	3x9	15.1	29.8	0.0
UOOG	3x3	41.7	32.5	8.3
	3x5	37.0	28.1	4.2
	3x9	10.4	9.1	0.0
TOTIMP	3x3	8.5	28.6	10.6
	3x5	11.0	41.9	21.7

EDMISC – Total Consumption of Non–categorized Edible Products,
UOOG – Unfilled Orders of Opthalmic Goods, Watches and Watch Cases.
TOTIMP – Total U.S. Imports, including Freight and Insurance.

The results given in Table 6.1 strongly suggest the use of 3x9 seasonal filters for EDMISC and UOOG. The most carefully analyzed seasonal filter length selection criterion for X–11 with which we are familiar is the Global Moving Seasonality Ratio (GMSR, also called the $\bar{I}/\bar{S}$ ratio) analyzed by Lothian (1984), which is printed out in Table D9.A of X–11.2 and Table F2.H of X–11–ARIMA. For any series of the length we are considering (less than 15 years), Lothian recommends using (a) a 3x3 moving average if GMSR is between 2.3 and 4.1; (b) a 3x5 moving average if GMSR is between 4.1 and 5.2; (c) a 3x9 moving average if GMSR is between 5.2 and 6.5; and "stable" filters (described in section 9) otherwise.

For EDMISC, the value of GMSR is 6.47, favoring the same filter as the sliding spans statistics. For UOOG, GMSR's value is 3.75, favoring a 3x3 seasonal filter, whereas only the 3x9 filter's adjustment has acceptable sliding spans statistics. The graphical analysis procedure of Lothian (1984) was applied to the SI ratios of EDMISC but did not clearly favor either filter. It seems prudent to prefer the adjustment with 3x9 seasonal filters because of its much greater stability.

For TOTIMP, the value of GMSR is 5.1, suggesting the 3x5 filter, but the MM(%) value of 41.9 for this filter is not satisfactory, whereas the sliding spans statistics associated with the 3x3 filter seem acceptable. Results for 3x9 filters could not be calculated, because only 13 years of data were available for this series, so only three 11 year spans could be obtained for adjustments with 3x9 filters. Sliding spans statistics from only three spans tend to be smaller than those obtained with four spans, see section 9, making comparisons difficult. However, for this series, the F–statistics for the presence of stable seasonality from X–11.2 (and X–11–ARIMA) given below in Table 6.2, which are printed for each span in the sliding spans output of X–11.2, offer evidence that shorter than 3x9 filters should be used, as well as a shortened series. Because the SI ratios in Table D8 are

Table 6.2 "F"–Statistics for Stable Seasonality for TOTIMP

		Span 1	2	3	4
Filter	3x3	4.1	5.8	6.8	7.2
	3x5	4.3	4.8	6.9	7.0

correlated, being filtered (detrended) values of the observed series, these F–statistics do not follow an F–distribution precisely. The tradition at the Census Bureau and at Statistics Canada has been to interpret an F–statistic value less than 7.0 as indicating a seasonal pattern which is too weak to permit adjustment. By this criterion, only the last 6–8 years of TOTIMP are adjustable.

Our recommendations for filter length selection are the following: if the seasonal filter length suggested by Lothian's GMSR statistic does not yield an acceptably stable adjustment, then the sliding spans statistics can sometimes reveal a more suitable filter length. Usually (see section 9), but not always, as we just saw, the sliding spans statistics S(%), MM(%), etc. are smaller for longer filters. Finally, it may sometimes be appropriate to select different seasonal filters for different calendar months. Sliding span diagnostics can help to determine the effects of a set of such selections.

7. SEASONAL ADJUSTMENTS VERSUS TRENDS (X–11)

"Trends" receive much attention. However, the concept of trend is use–dependent rather than fixed. For example, analysts seeking long–term trends expect less fluctuating trends than investigators of short–term trends. There are statistical long–term forecasting procedures whose forecasted trend curves differ from those of corresponding short–term forecasting procedures in just this way, see Gersch and Kitagawa (1983).

The X–11 trends are obtained by applying 9, 13, or 23 term Henderson filters (moving averages) to the seasonally adjusted data. The resulting trends are short–term, and would be expected to have more stable month–to–month changes than the seasonally adjusted data itself. It is less clear what to expect for longer term comparisons such as year–to–year changes. In fact, in every case that we have observed, year–to–year changes are less stable. The examples in Table 7.1 are typical. Similar results were observed with trends estimated by BAYSEA, an empirical Bayesian seasonal adjustment program developed at the Institute of Statistical Mathematics, see Akaike (1980).

Some theoretical calculations for a special case would predict these phenomena. William Bell, in unpublished work, analyzed the linear filters which approximate the seasonal adjustment and trend estimation procedures of additive X–11. He calculated the coefficients of the filters which produce the revisions in the month–to–month and year–to–year changes as new data are added. The filter coefficients associated with month–to–month changes are much larger for seasonal adjustments than for trends.

Table 7.1 **Seasonal Adjustment/Trend Sliding Spans Statistics for the Foreign Trade Series of Table 4.1**

Series	S(%) / T(%)	MM(%)	YY(%)
IANEC	10.4 / 6.2	29.5 / 6.3	0.0 / 11.9
IASIA	11.5 / 6.2	29.5 / 1.1	0.0 / 11.9
IOECD	7.3 / 4.2	24.2 / 0.0	0.0 / 6.0
IOEEC	9.4 / 3.1	25.3 / 1.1	0.0 / 4.8
IWEUR	9.4 / 2.1	24.2 / 0.0	0.0 / 0.0
IWGER	0.0 / 0.0	12.6 / 0.0	0.0 / 0.0
IWH	2.1 / 5.2	9.5 / 0.0	0.0 / 9.5
IOECD	2.8 / 3.5	10.5 / 0.0	0.0 / 8.3

NOTE: T(%) is the analog of S(%) with the trend values used instead of seasonal factors

For year–to–year changes the situation is reversed. Large coefficients usually magnify the effects of differences in the input data to the filter. Consequently, with additive adjustments, one would expect the kinds of instabilities we observed.

- It appears that data users who choose to use X–11 trends instead of seasonal adjustments must be prepared to accept less stable estimates of year–to–year and longer term change.

8. ARIMA EXTENSION PRIOR TO SEASONAL ADJUSTMENT

The central seasonal filters of additive X–11 have better frequency response function characteristics than the asymmetric filters which are used near the ends of the series, see Dagum (1982). In general, suppose that a_k, $-m_0 \leq k \leq m_1$, are the coefficients ("weights") of a linear seasonal adjustment filter with desirable properties, and that we wish to calculate adjustments

$$A_t = \sum_{k=-m_0}^{m_1} a_k X_{t-k} \tag{8.1}$$

for $t = 1,\ldots, N$, but only have data $X_1,\ldots,X_N$. In this case, an approximation to A_t must be used if $t < m_1$ or $t > N-m_0$. The best linear approximation, in the least mean square sense, linear as a function of the data, is given by

$$A_t^* = \sum_{k=-m_0}^{m_1} a_k X_{t-k}^*,$$

with $X_s^* = X_s$ if $1 \leq s \leq N$ and, if $s < 0$ or $s > N$, with X_s^* equal to the best linear approximation to X_s from $X_1,\ldots,X_N$. Estimates of X_s^*, $s < 0$ or $s > N$, can be obtained from ARIMA models fitted to the data.

The argument given above captures the central theoretical motivation for the forecast and backcast extension procedure implemented in X–11–ARIMA, see Dagum (1983a) and Cleveland (1983), which permit the use of closer–to–symmetric seasonal filters near the ends of a series. However, when multiplicative adjustments are used, which is the most common situation, the adjusted values A_t are not obtained linearly as in (8.1). What evidence favors the use of extended series in the multiplicative situation? There are empirical results, see Otto (1985) for example, showing that revisions to past seasonal adjustments resulting from the addition of more recent data to the series are smaller, on average, if the original adjustment was obtained from a forecasted and backcasted series. With sliding spans, the oldest data are deleted when later data are added, so sliding spans analyses can provide a new kind of evidence. Such analyses show, in fact, that adjustments from series extended by ARIMA model forecasts and backcasts are usually more stable.

The results for six series presented in Table 8.1 are typical of what we have seen with the many series we have analyzed. The series were extended by 36 or 60 months in each direction, depending on whether 3x5 or 3x9 seasonal filters were used. The series extensions were produced from carefully fitted seasonal ARIMA models. Regression mean functions were included when needed to ensure that the ARIMA coefficient estimates were not affected by large additive outliers or by level shifts. The precise robustness strategy used is described in Findley et al. (1988).

For each series in Table 8.1, the spans used to obtain the statistics were the same for the case of no extension and the case of extension. For the latter case, the type of model fit to the entire series was used for each span, but two different extensions were

Table 8.1
Effects of Forecast and Backcast on the Sliding Spans Statistics of Six Series

No Extension/Extension/Extension without Reestimation

Series	S(%)	MM(%)	YY(%)
RSAPPL	2.7/7.3/5.5	5.5/14.7/7.3	0.0/0.0/0.0
RSAUTO	8.1/7.2/0.9	16.4/11.8/0.0	5.1/0.0/0.0
WSFURN	7.3/0.0/0.0	17.4/5.5/4.6	2.0/3.1/0.0
HSMW1F	42.2/15.6/8.2	50.7/26.0/9.6	3.7/0.0/0.0
HSSOTL	17.7/4.1/0.0	34.9/13.0/2.7	0.0/0.0/0.0
HSWETL	15.6/11.6/2.8	22.6/27.7/6.2	0.0/0.0/0.0

RSAPPL – Retail Sales of Appliance (1–67 to 2–89)
RSAUTO – Retail Sales of Automobile (1–67 to 2–89)
WSFRN – Wholesale Sales of Furniture (1–67 to 2–89)
HSMW1F – Midwest Single Family Housing Starts (1–64 to 3–89)
HSSOTL – South Total Housing Starts (1–64 to 3–89)
HSWETL – West Total Housing Starts (1–64 to 3–89)

obtained, one using coefficients estimated from data belonging only to the span, and the other using the coefficients estimated from the entire data set. The latter procedure is referred to as "extension without reestimation" in the table. For RSAPPL, the additional instability introduced by reestimating the coefficients for each span causes the adjustment of the extended series to look (slightly) less stable than adjustment without extension. A graphical analysis not presented shows that the forecasts beyond December, 1986 are somewhat unstable and inaccurate. The other series are better adjusted after extension. For HSMW1F, the adjustment obtained without extension is unacceptable. Figure 8.1 shows the forecasts (broken line) produced from the earliest span and the actual observations through March of 1989. The forecasts are good and appear to counterbalance the earlier volatile behavior.

In summary, the sliding spans diagnostics provide information about whether a model for a series produces forecasts and backcasts which increase the stability of its X–11 seasonal adjustments. The effect of different forecast horizons on stability can also be examined: for the series in Table 8.1, using only twelve months of forecasts led either to equivalent or to slightly less stable results.

9. BIAS IN THE SLIDING SPANS PROCEDURE

In a sliding spans analysis with four spans, some months are common to only two spans (we will call such months "2"–months), some only to three spans ("3"–months), and some occur in all four spans ("4"–months"). Are months for which more comparisons are available more likely to be flagged, as some basic results for range statistics of correlated data would suggest see chapter 4 and 5, especially Exercise 5.5.2, of David (1969)? Or, on the contrary, are "4"–months the least likely to have unstable adjustments, because their adjustments are produced by closer–to–symmetric filters (see Dagum (1982, 1983a), Wallis (1982), and Pierce and McKenzie (1987)), as some experts suggested to us?

To address these questions, we calculated the percentages of months with unstable seasonal factors ($S_t^{max} > 0.03$) or unstable month–to–month changes in the seasonally adjusted series ($MM_t^{max} > 0.03$) among the adjustments of 61 series. We used spans of length 7, 8, 11 and 13 years according to whether 3x3, 3x5, 3x9 or "stable" filters were applied. (The "stable" seasonal filter calculates a single seasonal factor for each calendar month. This factor is the simple average of the SI ratios of data from the calendar month.) The results are presented in Tables 9.1 and 9.2. They show that, on average, "2"–months are flagged less often than "3"–months, and "3"–months less often than "4"–months. The tables show that this conclusion applies regardless of whether or not the analysis is restricted to series which are adjustable with the chosen seasonal filter according to the criteria of Table 2.1.

We have not yet attempted to refine the threshold values to remove this bias, which could, perhaps, be done. There are various practical considerations which a refinement must take into account. For example, "2"–months, which are less numerous and are least likely to be flagged, are either the most recent (and therefore most interesting) or the oldest (and usually least interesting) of the months for which comparisons are available.

Table 9.1

Percentages of Months with Unstable Adjustments ($S_t^{max} > 0.03$), Classified by Position–In–Span and Filter Length Used.

Seasonally Adjustable Series

	"2" Months	"3" Months	"4" Months	All Months
3x3 Filter	9/ 672 1%	32/ 672 5%	42/ 1008 4%	83/ 2532 3%
3x5 Filter	11/ 768 1%	34/ 768 4%	78/ 1920 4%	123/ 3456 4%
3x9 Filter	13/ 936 1%	26/ 936 3%	101/ 3744 3%	140/ 5616 2%
Stable Filter	3/ 984 0%	17/ 984 2%	171/ 4428 4%	191/ 6396 3%
All Filters	36/3360 1%	109/3360 3%	392/11100 4%	537/17820 3%

All Series

	"2" Months	"3" Months	"4" Months	All Months
3x3 Filter	273/1464 19%	420/1464 29%	710/ 2196 32%	1403/ 5124 27%
3x5 Filter	201/1464 14%	357/1464 24%	879/ 3660 24%	1437/ 6588 22%
3x9 Filter	118/1464 8%	235/1464 16%	899/ 5856 15%	1252/ 8784 14%
Stable Filter	76/1464 5%	187/1464 13%	1296/ 6588 20%	1559/ 9516 16%
All Filters	668/5856 11%	1199/5856 20%	3784/18300 21%	5651/30012 19%

Table 9.2

Percentages of Months with Unstable Adjustments ($MM_t^{max} > 0.03$), Classified by Position–In–Span and Filter Length Used.

	"2" Months	"3" Months	"4" Months	All Months
Seasonally Adjustable Series				
3x3 Filter	68/ 851 8%	136/ 888 15%	226/ 1332 17%	430/ 3071 14%
3x5 Filter	63/ 897 7%	126/ 936 13%	308/ 2340 13%	497/ 4173 12%
3x9 Filter	57/1081 5%	128/1128 11%	503/ 4512 11%	688/ 6721 10%
Stable Filter	16/1035 2%	67/1080 6%	497/ 4860 10%	580/ 6975 8%
All Filters	204/3864 5%	457/4032 11%	1534/13044 12%	2195/20940 10%
All Series				
3x3 Filter	324/1403 23%	511/ 1464 35%	876/ 2196 40%	1771/ 5063 35%
3x5 Filter	281/1403 20%	454/ 1464 31%	1167/ 3660 32%	1902/ 6527 29%
3x9 Filter	187/1403 13%	334/ 1464 23%	1347/ 5856 23%	1868/ 8723 21%
Stable Filter	133/1403 9%	270/1464 18%	1731/ 6588 26%	2134/ 9455 23%
All Filters	925/5612 16%	1569/5856 27%	5121/18300 28%	7675/29768 26%

10. CONCLUDING SUMMARY

Findley and Monsell (1986) introduced several new diagnostics for detecting problematic seasonal adjustments from X–11. Among these, the sliding spans diagnostics seemed particularly appealing because of their simplicity, interpretability, and broad applicability. This paper has described the results of an expanded investigation of the sliding spans diagnostics which has demonstrated their great versatility.

Our analysis concerned multiplicative adjustments of monthly series. It applies to quarterly data as well. For additive adjustments, it is more difficult to find appropriate scale–free measures of stability. This is a topic for further research.

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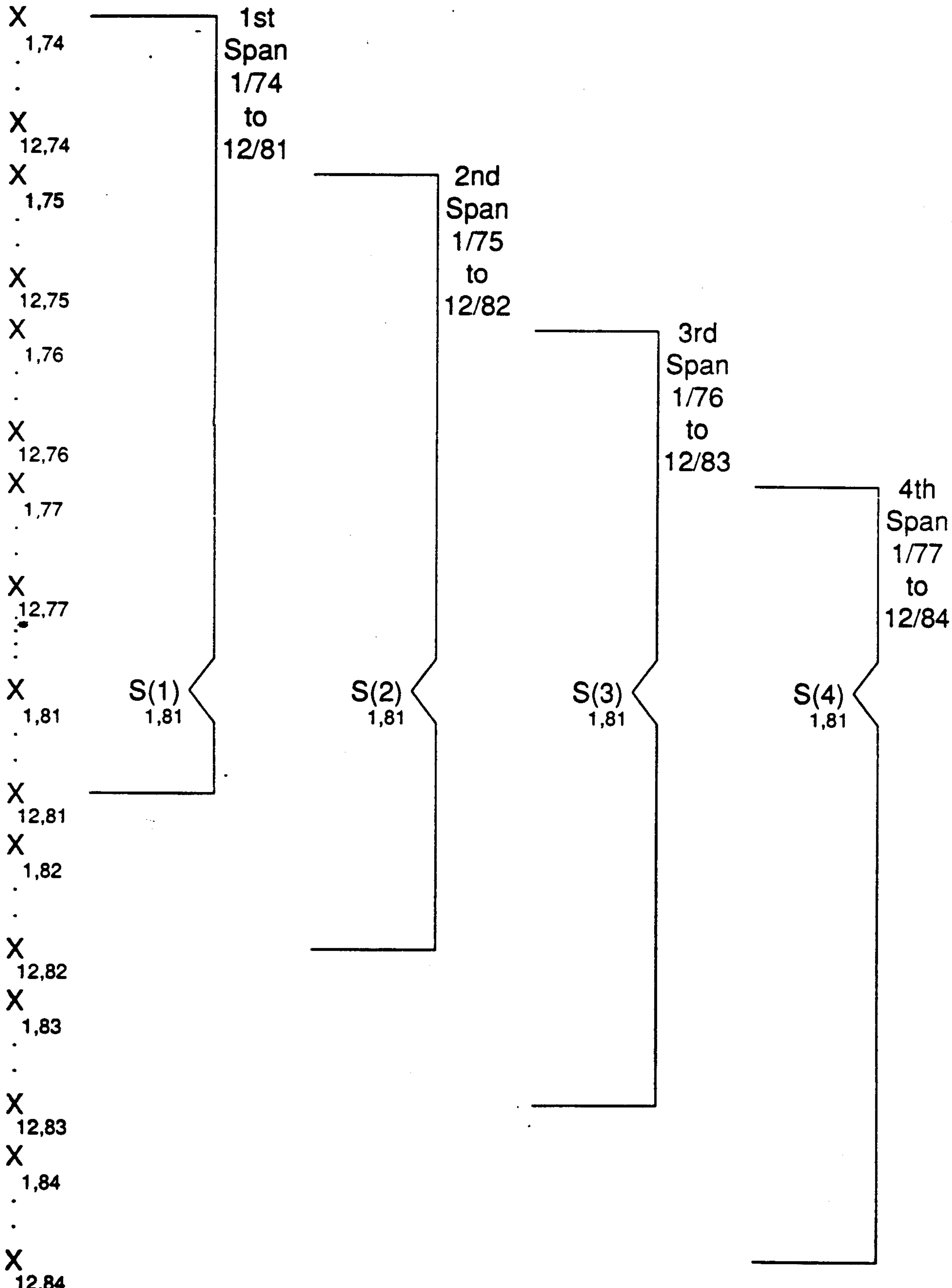


Figure 2.1 Illustration of Four Eight Year Span

Figure 3.1 Graph of SBP

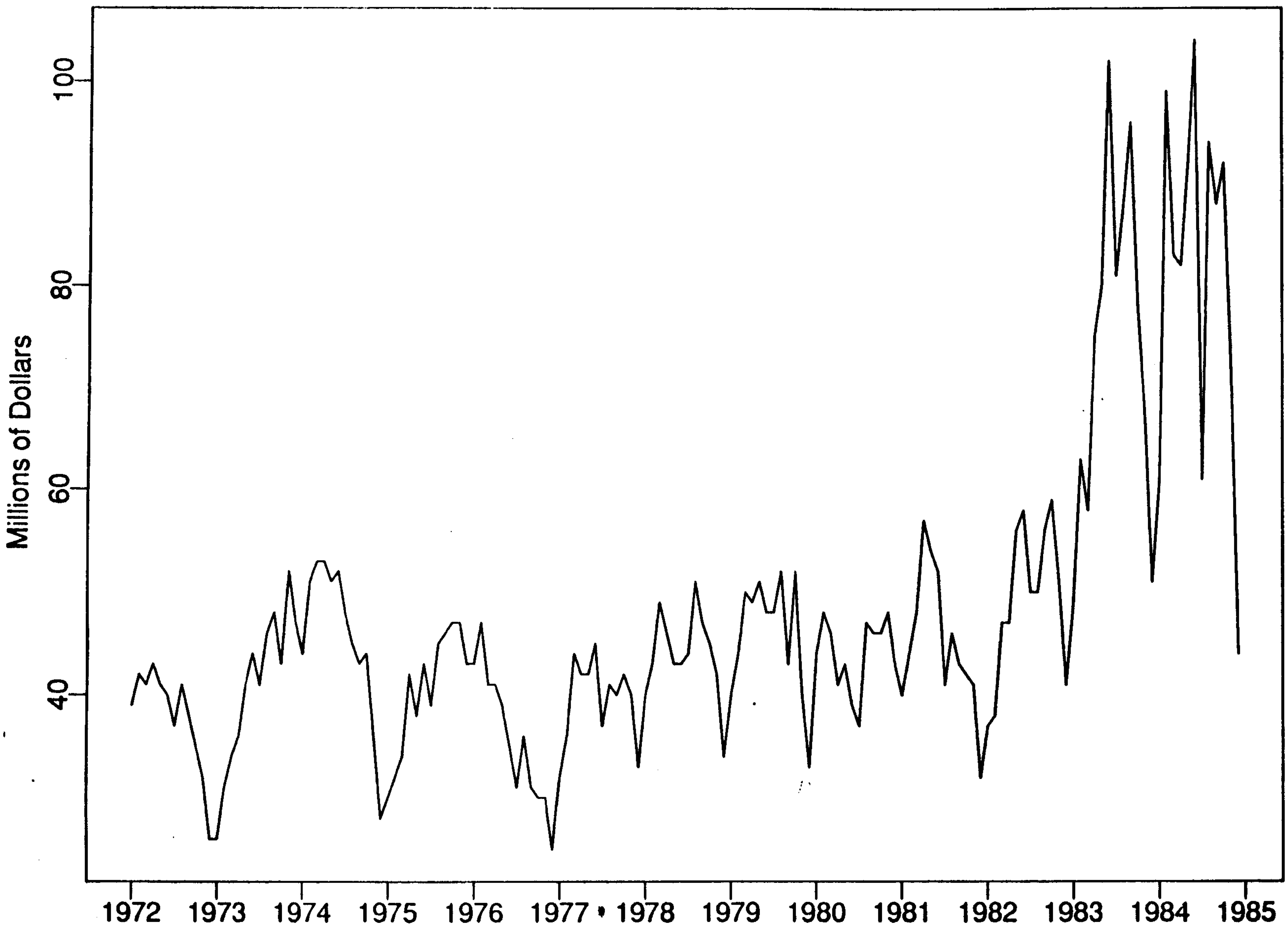


Figure 4.1 Smoothed Periodogram of the Irregular from IOECD and XOECD

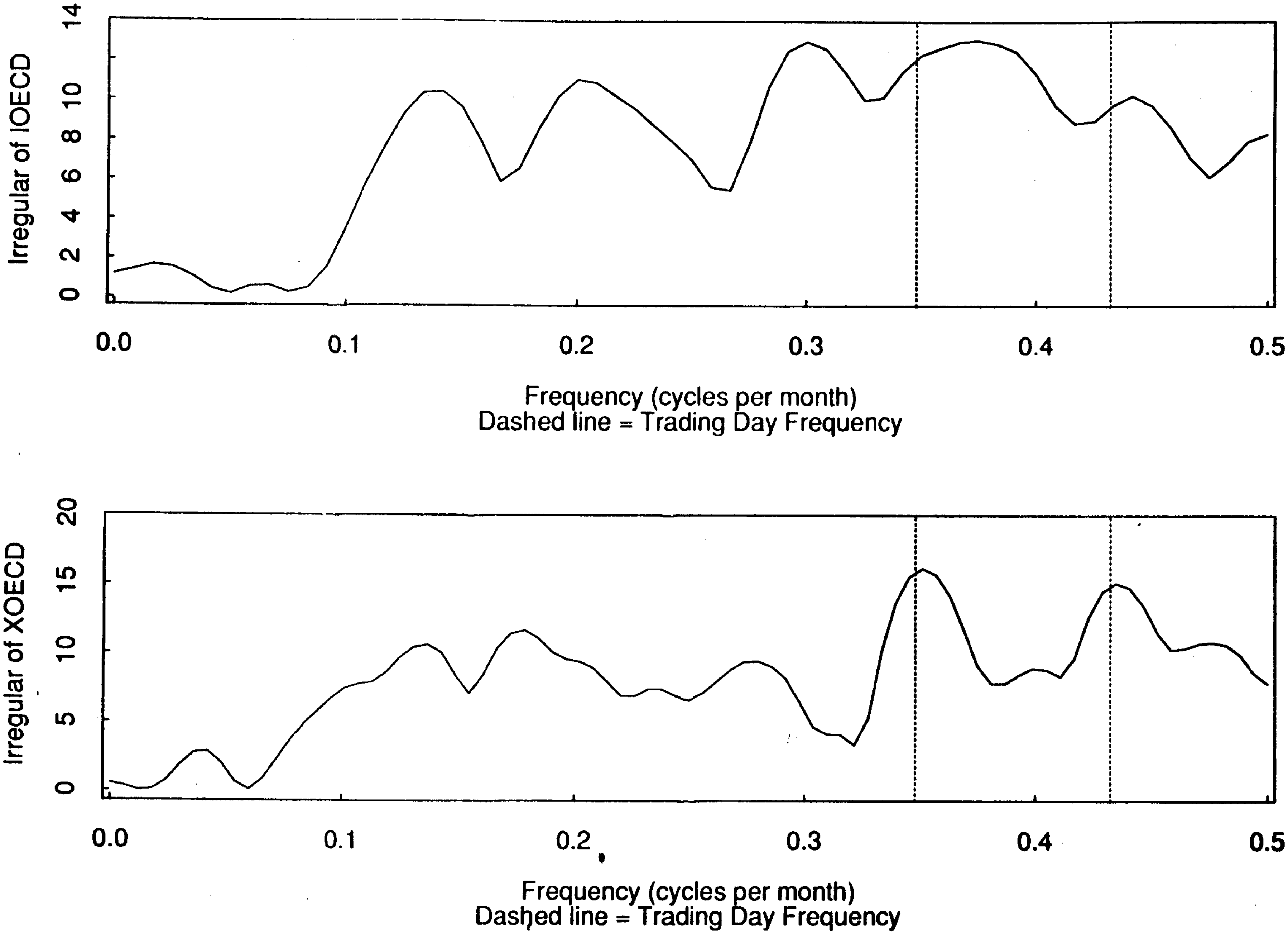
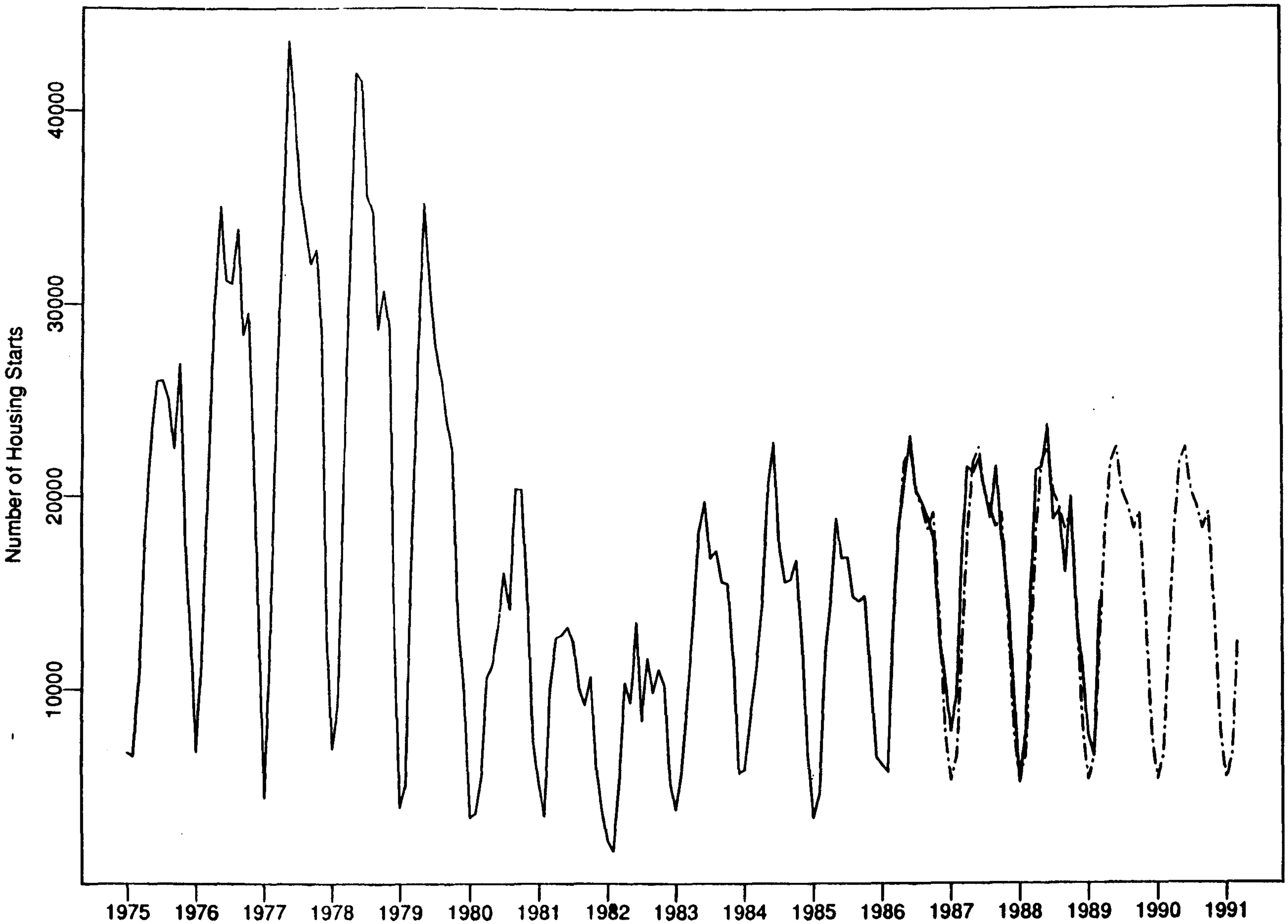


Figure 8.1 Graph of MW1FHS with ARIMA Forecasts



Solid line = MW1FHS

Dashed line = ARIMA Forecasts for MW1FHS Produced From the Span Ending in March 1985