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6. Data-rich approaches

6.X. Probabilistic approaches to setting catch levels

K. W. Shertzer, M. H. Prager, E. H. Williams, SEFSC, Beaufort, NC

6.X.1. Introduction

6.X.1.1. Probability-based reference points

The use of probability theory to derive fishery reference points (typically targets and limits) has been described by several authors. Caddy and McGarvey (1996) described a procedure to set a target reference point, given a limit reference point F_{lim} specified as a point estimate, such that the realized fishing mortality rate in the next period F_t would exceed F_{lim} with only some specified probability P^*. The procedure assumes that F_t will be centered on the target but not necessarily equal it, because of imperfect implementation of management controls or imperfect estimation in the stock assessment.

In the Caddy and McGarvey (1996) framework, the probability of overfishing in the next year is computed from F_{lim} and the probability density function (PDF) ϕ_{F_t} of F_t:

$$\Pr(F_t > F_{\text{lim}}) = \int_{F_{\text{lim}}}^{\infty} \phi_{F_t}(F)\,dF \qquad (1)$$

The catch target can be set to position the distribution of F_t so that the allowable probability of overfishing is achieved; i.e., $\Pr(F_t > F_{\text{lim}}) = P^*$ (Figure 1). That fishing mortality rate is the target for the next period.

By "allowable probability," we mean a probability allowed in any single year, such that P^* is the annual probability of overfishing. In a series of n years, the probability that overfishing will occur in at least one year, assuming independence among years, increases to $p_n = 1 - (1 - P^*)^n$. For $n = 5$ and $P^* = 0.2$, $p_n = 0.672$.

Prager et al. (2003) extended the work of Caddy and McGarvey (1996) in several ways. The revised procedure, termed REPAST, allows for uncertainty both in estimating the limit reference point and in attaining the target; uses ratios to reduce possible covariance between quantities; and considers reference points in biomass as well as in fishing mortality rate. Also, the authors suggested that an adjustment (bias correction) be made when past catches have not been centered on their targets.

In more detail: Prager et al. (2003) pointed out that $F_{\lim}$ should be described when possible by its PDF ($\phi_{F_{\lim}}$) rather than by a point estimate, to account for scientific uncertainty in the estimate of the limit reference point (which in the context of U.S. fisheries is MFMT). Then, the probability of overfishing is computed

$$\Pr(F_t > F_{\lim}) = \int_0^{\infty} \left[\int_F^{\infty} \phi_{F_t}(\theta) d\theta \right] \phi_{F_{\lim}}(F) dF \tag{2}$$

where θ is a dummy integration variable. As before, a target is computed by positioning the corresponding distribution of F_t to achieve $\Pr(F_t > F_{\lim}) = P^*$.

An assumption of eq. 2 is that $F_{\lim}$ and F_t are independent. If the two are correlated, the probability of overfishing could be computed from the bivariate PDF $\phi_{F_{\lim},F_t}$,

$$\Pr(F_t > F_{\lim}) = \int_0^{\infty} \int_F^{\infty} \phi_{F_{\lim},F_t} d\theta dF \tag{3}$$

Although use of a bivariate PDF is more general, it may not be possible to estimate $\phi_{F_{\lim},F_t}$ from most data sets. Fortunately, simulation analysis has supported the assumption of independence, and thus the less general eq. 2 should in many cases be a suitable approximation (Shertzer et al. 2008).

6.X.1.2. New requirements

The reauthorizing legislation (MSRA 2006) established several new requirements for federal fishery management. The most notable in this context are that fishery management councils must set annual catch limits (ACLs), and that those limits may not exceed recommendations of the councils' scientific advisers.

The NS1 Guidelines (USOFR 2009) suggest that, as one precursor to establishment of an annual catch limit, the council's scientific advisors should determine acceptable biological catch (ABC) by reducing the overfishing level (OFL) to account for scientific uncertainty. In that context, the overfishing level is generally defined as the catch obtainable from current stock biomass at the limit reference point (or "threshold") in fishing mortality rate $F_{\lim}$ (in practice often set to F_{MSY}).

6.X.2. Simple method for computing ABC for one time step

It is not yet clear how the new statutory requirements and guidance will be implemented by fishery management councils, but it seems likely that some will take a serial approach, asking the scientific advisors to set ABC each year, and then setting some annual catch limit and annual catch target at lower levels in a separate step. In this section, we describe a simple procedure (Prager and Shertzer 2009) by which a council's scientific advisors could set the ABC below the overfishing level. The approach is intended for use in a single year for which estimates of current stock biomass B and the limit reference point $F_{\lim}$ are available. An estimate of OFL can be derived from those two estimates, and we demonstrate two ways to approximate the variance (or distribution) of OFL, as well.

Our one-step-ahead approximations are based on propagation of error, modeled either analytically or numerically. The analytical approximation can be used for fishery models using the Baranov catch equation, and we show that similar results can be obtained through numerical sampling (Monte Carlo simulation). We present a hypothetical example and illustrate its solution by each method.

Some stock assessments provide estimates of OFL *and* its distribution. In that case, the estimated distribution is used directly (instead of being approximated) to set ABC from probability points of OFL. The rest of the procedure is the same as in our examples (e.g., Fig. 2b).

6.X.2.1. Analytical approximation — Baranov equation

Most age-structured fishery models use the Baranov catch equation, which considers natural mortality M and fishing mortality F as continuous-time processes. Setting F to $F_{\lim}$ in the catch equation, and disregarding age-specific considerations, we obtain the overfishing level:

$$\mathrm{OFL} = \frac{F_{\lim} B (1 - e^{-M - F_{\lim}})}{F_{\lim} + M} \tag{4}$$

where M is the annual instantaneous rate of natural mortality.

Given the distribution of OFL, one could use a procedure symmetrical to that of Caddy and McGarvey (1996) to set ABC < OFL such that Pr(ABC > OFL) is less than some chosen value P^* (Fig. 2a). In the Caddy and McGarvey (1996) procedure (Fig. 1), the limit reference point $F_{\lim}$ is considered a fixed quantity, while the corresponding target is considered uncertain. Here, the situation is reversed: the limiting value OFL is assumed uncertain, while the ABC will be a point value. Still, the general concept of Caddy and McGarvey (1996) applies.

We assume here that OFL is centered at the value given by eq. 4 and that it is normally distributed (Fig. 2a). Thus, all that remains to describe its distribution is the variance. Most assessment provide either the variance of OFL explicitly or else components necessary to compute it. Specifically, when an assessment provides the variances of B and $F_{\lim}$, the variance of OFL can be approximated by the delta method (Seber 1973). The delta method approximates the variance of a function $G(\theta_1, \theta_2, \ldots, \theta_K)$ through a Taylor expansion. If one assumes—as we do for this example—that covariances among the θ_k are zero, the estimated variance of G is

$$\text{Var}[G(\theta)] \approx \sum_k \text{Var}[\theta_k](\partial G/\partial\theta_k)^2 \tag{5a}$$

A more complex form is used to account for nonzero covariances. Casting eq. 5a into a form more explicit for the current problem, we obtain

$$\text{Var}[\text{OFL}] \approx \text{Var}[F_{\text{lim}}]\left(\frac{\partial \text{OFL}}{\partial F_{\text{lim}}}\right)^2 + \text{Var}[B]\left(\frac{\partial \text{OFL}}{\partial B}\right)^2 \tag{5b}$$

Variances of F_{lim} and B should be available from a stock assessment, so that only the partial derivatives need be computed to apply eq. 5b. Differentiating eq. 4 with respect to F_{lim} and letting symbol $Z = F_{\text{lim}} + M$ gives

$$\frac{\partial \text{OFL}}{\partial F_{\text{lim}}} = \frac{Be^{-Z}(Me^{Z} + F_{\text{lim}}M - M + F_{\text{lim}}^2)}{Z^2}, \tag{6}$$

and differentiating eq. 4 with respect to B gives

$$\frac{\partial \text{OFL}}{\partial B} = \frac{F_{\text{lim}}(1 - e^{-Z})}{Z}. \tag{7}$$

We then can estimate the variance of OFL by substituting eqs. 6 and 7 into eq. 5b. Once that is done, the distribution of OFL is sufficiently described to proceed.

Let P^* be the chosen probability that ABC > OFL. We choose ABC from the distribution of OFL so that P^* is satisfied. This procedure is symmetrical to the one described by Caddy and McGarvey (1996), as shown in Figs. 1 and 2a.

A similar procedure can be used when the model under consideration has a different catch equation. For example, catch equations for the Schaefer surplus-production model are given by Prager (1994).

6.X.2.2. Numerical sampling — Baranov equation

The preceding development used an analytical approximation to arrive at propagation of error. A more modern way of accomplishing the same result is by numerical approximation, e.g., Monte Carlo sampling, in which many draws are made from the distributions of F_{lim} and B, and the corresponding distribution of OFL is computed. The ABC is taken as the percentile of the distribution of OFL corresponding to P^*. Monte Carlo simulation can be programmed directly by the analyst or implemented through a computer program such as WinBUGS (Lunn et al. 2000). Among the strengths of the numerical approach are that any distribution can be used, including empirical distributions, and that a complete distribution of OFL results, which need not be assumed normal.

6.X.2.3. Example — simple one-year-ahead method

In this example, we assume a stock assessment using the Baranov equation. The stock assessment gives the estimates B = 1000 t and F_{lim} = 0.3/yr. The assessment model also estimates that the coefficient of variation (CV) of B is 0.2 and the CV of F_{lim} is 0.25. We know from prior work that M = 0.2/yr.

By applying eq. 4, we estimate that OFL = 236.1 t. We then apply eqs. 5–7 and find that the variance of OFL is 4820 t^2. Assuming normality (Fig. 2a), the ABC can be computed for various levels of P^* (Fig. 3a). For example, if no more than a 30% chance that ACL > OFL is desired (P^* = 0.3), the ABC should be set at 200 t.

The same values were used to generate the distribution of OFL by a Monte Carlo simulation programmed in R (R Development Core Team 2008). The resulting distribution of OFL is given in Fig. 2b, and the ABCs in Fig. 3b. The corresponding example of ABC at P^* = 0.3 is 195 t.

6.X.2.4 Simple method with age or size structure

The example above did not consider a structured population model, however the same methods could be applied with age or size structure. The OFL of a structured population can be computed from eq. 4 summed over age or size classes, and its approximate variance from eq. 5. It is not obvious whether the assumption of independence among age- or size-specific terms, as in eq. 5, would bias the estimated variance of the OFL, and if so, in which direction.

6.X.3. Multi-year methods for ABC and ACT

In the next section, we describe two related, probabilistic approaches to setting catch levels for more than one year into the future. These approaches are modified slightly from Shertzer et al. (2008) for consistency with the 2008 NS1 Guidelines. Each approach calculates an acceptable biological catch (ABC) accounting for scientific uncertainty and an annual catch target (ACT) accounting for both scientific uncertainty and implementation uncertainty. One approach considers the two sources of uncertainty simultaneously, the other sequentially.

The 2008 NS1 Guidelines specify that the annual catch limit (ACL) must be less than or equal to the ABC. The approaches here do not refer to an ACL, but leave the ACL to be set anywhere below or equal to the ABC and sufficiently above the ACT so that accountability measures (AMs) will not be triggered too frequently.

We term both approaches PASCL, for Probability Approach to Setting Catch Levels. One approach is *Integrated PASCL,* the other, *Sequential PASCL.* The methods are based on probabilities of future events, probabilities whose allowable levels are assumed to have been set a priori, each less than 0.5:

- *P** is the allowable probability that the ABC will exceed the overfishing level (OFL). (*P** is used in both sequential and integrated PASCL.)

- *P*** is the allowable probability that catch from an ACT will exceed the ABC. (*P*** is used in sequential PASCL only.)

- *P**** is the allowable probability that catch from an ACT will exceed the OFL. (*P**** is used in integrated PASCL only. To ensure ACT ≤ ABC, P*** ≤ P*.)

In each variant of PASCL, ABC is set below OFL using a control rule such that, based on scientific uncertainty, the probability that ABC > OFL is *P**. In each variant, the corresponding ACT is set below the ABC, but the way that ACT is determined differs between approaches.

In Integrated PASCL, the ACT is set below the OFL in nearly the same way that the ABC is set below the OFL, but accounting for both scientific and implementation uncertainties. That is, integrated PASCL sets the ACT such that, considering scientific and implementation uncertainty, overfishing will occur with probability *P****.

In Sequential PASCL, the ABC is set first, then the ACT is set from the ABC, rather than directly from the OFL as in integrated PASCL. Because the ABC has been set accounting for scientific uncertainty, the ACT control rule here accounts only for implementation uncertainty: it sets the ACT so that future catch exceeds the ABC with annual probability *P***. The variants of PASCL are described below in more detail.

The difference between the two methods can be summarized—

1. Integrated PASCL sets the ACT by reference to the OFL, while Sequential PASCL sets the ACT by reference to the ABC (also the ACL if it is set equal to the ABC).

2. Integrated PASCL sets the ACT to control the probability of overfishing directly, while Sequential PASCL sets the ACT to avoid exceeding the ABC (also the ACL if it is set equal to the ABC).

6.X.3.1. Integrated PASCL method

In integrated PASCL (Fig. 4), the goals are to compute the ACT such that $\Pr(F_t > F_{\lim}) = P^{***}$ and the ABC such that $\Pr(F_t' > F_{\lim}) = P^{*}$. In this notation, the values of F_t' and F_t differ because they correspond to the different catch levels: F_t' are hypothetical fishing rates that would occur if the catch were set to achieve the ABC, and F_t are actual fishing rates predicted to occur when the catch is set to achieve the ACT. The goals can be accomplished through use of a projection model similar to the many such models already used by NMFS. When adapted to PASCL, such a model can be structured to describe any individual stock, and it can incorporate any source of uncertainty considered important. For example, scientific uncertainties can include uncertainty in assessment results (e.g., in estimating $F_{\lim}$ or initial abundance) and stochasticity in future stock conditions (e.g., recruitment or life-history characteristics). Implementation uncertainty is modeled as stochasticity in achieved catch from a target catch.

The probabilistic approach can be applied through a projection model with the following steps (modified from Shertzer et al. 2008):

1. ***Initialize N replicates of the stock,*** each different in abundance and age structure to reflect uncertainty in estimated current state of the stock.

2. ***Compute the ABC***
   a. Choose a trial value C' of the ABC without considering implementation uncertainty.

   b. Compute for each replicate the fishing mortality rate F_t' that yields C'. This produces *N* values of F_t' that define its empirical probability density (ϕ'_{F_t}).

   c. Given ϕ'_{F_t} and $\phi_{F_{\lim}}$, compute $P = \Pr(F_t' > F_{\lim})$ from eq. 2.

   d. Using a numerical optimization method, adjust C' until $P = P^*$. The adjusted C' is that year's ABC.

3. ***Compute the ACT***
   a. In the presence of implementation uncertainty (imprecision of management measures in controlling catch), each ACT will be the central tendency μ of a probability distribution ϕ_C. Choose a trial value of μ, and draw *N* values {C_1 … C_N} from the corresponding distribution. Catch C_1 is the catch taken from stock replicate N_1, C_2 from N_2, and so forth.

   b. To combine uncertainties in state of the stock and implementation, compute for each replicate the fishing mortality rate that yields C_n. This produces *N* values of F_t to define its empirical probability density (ϕ_{F_t}).

   c. Given ϕ_{F_t} and $\phi_{F_{\lim}}$, compute $P = \Pr(F_t > F_{\lim})$ from eq. 2.

   d. Using a numerical optimization method, adjust μ until $P=P^{***}$. The adjusted μ is that year's ACT.

4. ***Project each replicate one year forward,*** by applying recruitment and natural mortality and taking catch C_n.

5. ***Repeat steps 2–4 for T years**.* The duration *T* should be chosen to extend the projection at least until catch levels based on the next assessment can be implemented.

The preceding procedure gives an ABC and ACT for each year in the period, with the annual probability of overfishing kept at *P****. (This *P**** corresponds to *P** in the notation of Shertzer et al. (2008)). To ensure that ACT ≤ ABC, the constraint $P^{***} \leq P^*$ should be applied, although even if $P^{***} = P^*$, the ACT would be expected to fall below the ABC because it includes additional (implementation) uncertainty. In integrated PASCL, the order of computation of this year's ABC and ACT does not matter (step 2 could just as well be done after step 3).

6.X.3.2. Example—Integrated PASCL method

In this example, we apply a three year projection with integrated PASCL to compute values of ABCs, ACTs, and spawning biomass. The projection model extends from the 2008 assessment of vermilion snapper off the Southeastern U.S. (SEDAR 2008). It includes stochasticity in future recruitment and in the initial stock structure (i.e., number at age). For the example, we used *P** = 0.4 and *P**** = {0.1, 0.2, 0.3}, and assumed that the CV of management implementation was *CV* = {0.2, 0.4, 0.6}. As one might expect, lower values of *P**** corresponded with lower ACTs, but also higher spawning biomass and consequently higher ABCs (Fig. 5). For a given *P****, greater precision in management implementation (i.e., lower CV) allowed higher ACTs.

6.X.3.3 Sequential PASCL method

In the sequential method (Fig. 6), the ABC accounts only for scientific uncertainty, as before, and is computed as in the integrated method such that $\Pr(F_t' > F_{\mathrm{lim}}) = P^*$. Unlike the integrated method, however, the value of the ABC is used explicitly when computing the ACT, so that the buffer between ABC and ACT accounts for implementation uncertainty only, with probability *P***. Any of eqs. 1–3 could be used, but would be recast in terms of catch rather than fishing mortality rate. It is expected that the modified eq. 1 would most often apply:

$$\Pr(C_t > ABC) = \int_{ABC}^{\infty} \phi_{C_t}(C)\,dC \qquad (8)$$

where (ϕ_{C_t}) is the PDF of catch in year *t*, defined by the ACT and implementation uncertainty. As before, the ACT is adjusted to position the distribution of C_t so that the allowable probability of exceeding the ABC is achieved [i.e., $\Pr(C_t > ABC) = P^{**}$].

The sequential method could be applied in a procedure similar to that of the integrated method, but with the following modification to step 3:

3. Compute the ACT

a. Given implementation uncertainty in controlling catch, each ACT will be the central tendency μ of a probability distribution ϕ_C. Choose a trial value of μ, and draw N values $\{C_1 \dots C_N\}$ from the corresponding distribution. Catch C_1 is the catch taken from stock replicate N_1, C_2 from N_2, and so forth.

b. Given ϕ_C and the ABC, compute $P = \Pr(C_t > ABC)$ from eq. 8.

c. Using a numerical optimization method, adjust μ until $P = P^{**}$. The adjusted μ is that year's ACT.

In the sequential method, step 2 must come before step 3, because the ABC is used to derive the ACT. To ensure that ACT ≤ ABC, it is not necessary that $P^{**} \le P^{*}$.

6.X.4 Discussion

Because in fisheries analyses, statistical assumptions are rarely (if ever) met, it is unlikely that any of the methods described here will produce perfectly accurate results. We certainly do not claim perfect accuracy in determining probabilities. What makes these methods valuable is that they are objective, repeatable, and computable from standard assessment outputs (or clear assumptions about variance). What is more, they are explicit in use of a priori probabilities of exceeding reference points. Because it is impossible to avoid overfishing with full certainty (unless $F=0$), we think that it is critical to define the allowable probability of overfishing clearly and transparently.

6.X.4.1 Implementation imprecision and bias

When computing ACT, both variants of PASCL account for implementation uncertainty (Rosenberg and Brault 1993; Rice and Richards 1996), as well as scientific uncertainty. We have assumed so far that that implementation is unbiased, i.e., that actual catches are centered on the ACT. If, however, catches tend to be higher or lower than the ACT, that assumption can be avoided by including a bias-correction term in the computation (step 3). Prager et al. (2003) suggested that the correction could be based on a running average of observed bias in immediately preceding years. Semmens (2008) recommended "adjusting for quota overages by sector after considering all sources of uncertainty." That procedure is theoretically equivalent to, but at times may be more practical than, including a bias-correction in the overall computation.

Whether to account for imprecision or to correct for bias, an estimate of the form (distribution) of implementation uncertainty is needed. For some stocks, that will be estimable from data on fishery performance; if not, assumptions will be needed. With widespread application of ACTs, implementation uncertainty, including implementation bias, eventually should become estimable for many stocks.

6.X.4.2 Setting catch levels for multiple years

If catch levels are set just one year in advance, the ABC and ACT can be set under separate consideration, as in §6.x.2 of this chapter. If catch levels for a series of years are required, it seems better to set ACT and ABC under joint consideration, as in the two PASCL methods, because of the feedback loop between catch levels and stock abundance (Rice and Richards 1996). Each year's ACT influences the actual catch taken (C_t), which in turn influences the next year's stock size, and thus its OFL and ABC.

6.X.4.3. Choosing P*

What value should be chosen for P^*? That is at least in part a policy issue, so no definitive answer can be given here. However, we note that setting ABC < OFL is only one step of a multi-step process. A council is then charged with setting annual catch limits and annual catch targets such that ACL ≤ ABC and ACT < ACL. Because accountability measures (e.g., future catch reductions) are invoked when the actual catch exceeds the ACL, councils may want to set the ACT low enough to avoid that condition. Because the P^* described here is only one buffer of a multi-buffer system, it seems reasonable to set it higher than if it were the only buffer. A reasonable range might be $0.3 \le P^* < 0.5$.

6.X.4.4. Integrated or sequential method?

We have described both an integrated and a sequential version of PASCL. While we have described both for completeness, it seems likely that handling the uncertainties sequentially will result in more protection, and thus less catch, than might have been envisioned, because of the fundamental non-additivity of sequential probability events. That conclusion was supported in a study by Semmens (2008), who wrote: "Importantly, the results suggest that all sources of uncertainty and variability should be assessed together to determine the appropriate buffer, a contrast to the currently suggested separation of biological and management steps where the SSC handles the biological uncertainty buffer and Councils handle the management uncertainty buffer." For that reason, when setting ACT, methods such as integrated PASCL, which consider all forms of variability together, may be preferable to methods that consider them sequentially. We also note that, by setting P***, integrated PASCL directly controls the probability of overfishing, an important consideration.

References

Caddy, J. F. 1998. A short review of precautionary reference points and some proposals for their use in data-poor situations. FAO Fisheries Technical Paper 379. United Nations, Rome.

Caddy, J. F. and R. McGarvey. 1996. Targets or limits for management of fisheries? North American Journal of Fisheries Management 16:479–487.

Lunn, D. J., Thomas, A., Best, N., and Spiegelhalter, D. 2000. WinBUGS—a Bayesian modelling framework: concepts, structure, and extensibility. Statistics and Computing 10:325–337.

MSRA (Magnuson–Stevens Fishery Conservation and Management Reauthorization Act of 2006). 2006. U.S. Public Law 109-479, 120 Statute 3575.

USOFR (U.S. Office of the Federal Register). 2009. Magnuson–Stevens Act Provisions: National Standard 1—Optimum Yield. Code of Federal Regulations, Title 50, Part 600.310. Available: ecfr.gpoaccess.gov/. (March 2009).

Prager, M. H. 1994. A suite of extensions to a nonequilibrium surplus-production model. Fishery Bulletin 92: 374–389.

Prager, M. H., C. E. Porch, K. W. Shertzer, and J. F. Caddy. 2003. Targets and limits for management of fisheries: a simple probability-based approach. North American Journal of Fisheries Management 23:349–361.

Prager, M. H. and K. W. Shertzer. 2009. Deriving "Acceptable Biological Catch" for the next year through simple probability theory. Unpublished manuscript, available from the authors upon request.

R Development Core Team. 2008. R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. ISBN 3-900051-07-0, URL http://www.R-project.org. Last accessed October 2, 2008.

Rice, J. C. and L. J. Richards. 1996. A framework for reducing implementation uncertainty in fisheries management. N. Am. J. Fish. Manage. 16:488–494.

Rosenberg, A. A. and S. Brault. 1993. Choosing a management strategy for stock rebuilding when control is uncertain. *In* Risk evaluation and biological reference points for fisheries management (S. J. Smith, J. J. Hunt, and D. Rivard, eds.), p. 243–249. Can. Spec. Publ. Fish. Aquat. Sci. 120.

Seber, G. A. F. 1973. The estimation of animal abundance and related parameters. Hafner Press, New York. 506p.

SEDAR (SouthEast Data Assessment and Review). 2008. SEDAR 17 Document for Peer Review: South Atlantic Vermilion Snapper. South Atlantic Fishery Management Council, Charleston, SC.

Semmens, K. A. 2008. Buffering uncertainty: setting annual catch limits under the Magnuson–Stevens Fishery Conservation and Management Reauthorization Act of 2006. Master's thesis in Marine Policy, University of Delaware.

Shertzer, K. W., M. H. Prager, and E. H. Williams. 2008. A probability-based approach to setting annual catch levels. Fishery Bulletin 106:225–232.

Figure 1. Probability density of future fishing mortality rate (ϕ_{F_t}). Shaded area is the probability that future fishing mortality rate exceeds a point estimate of the limit fishing mortality rate.

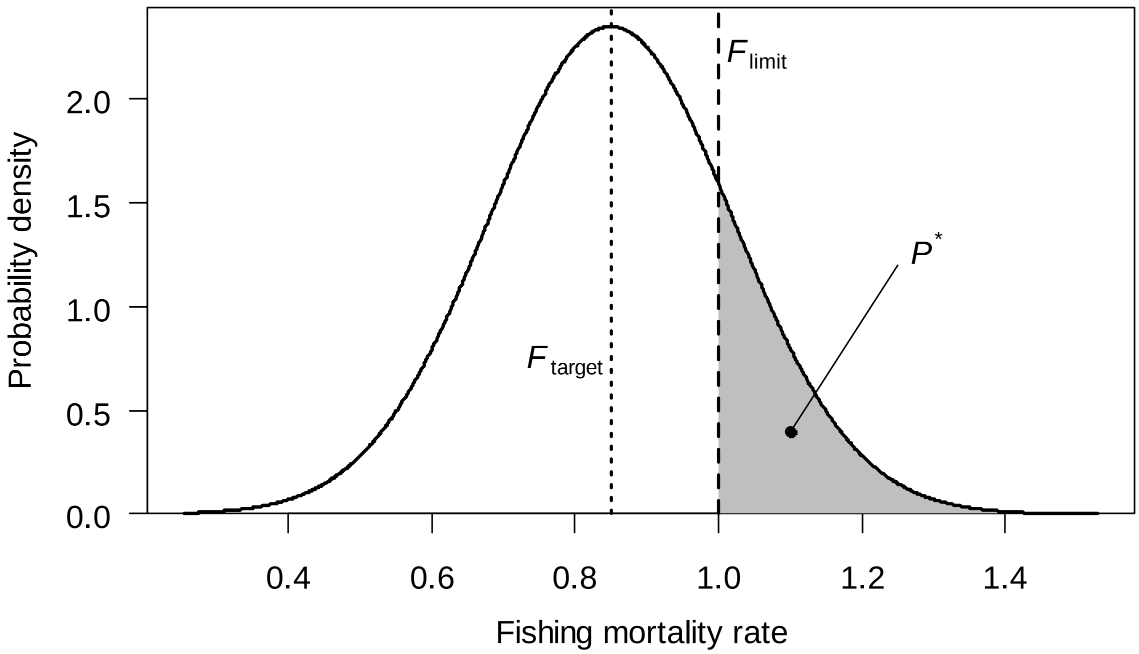

Figure 2. Probability density of an example OFL (a) assuming the normal distribution or (b) computed from numerical Monte Carlo sampling. The shaded area represents *P* = 0.3*. Dashed line in (a) represents a point estimate of the OFL.

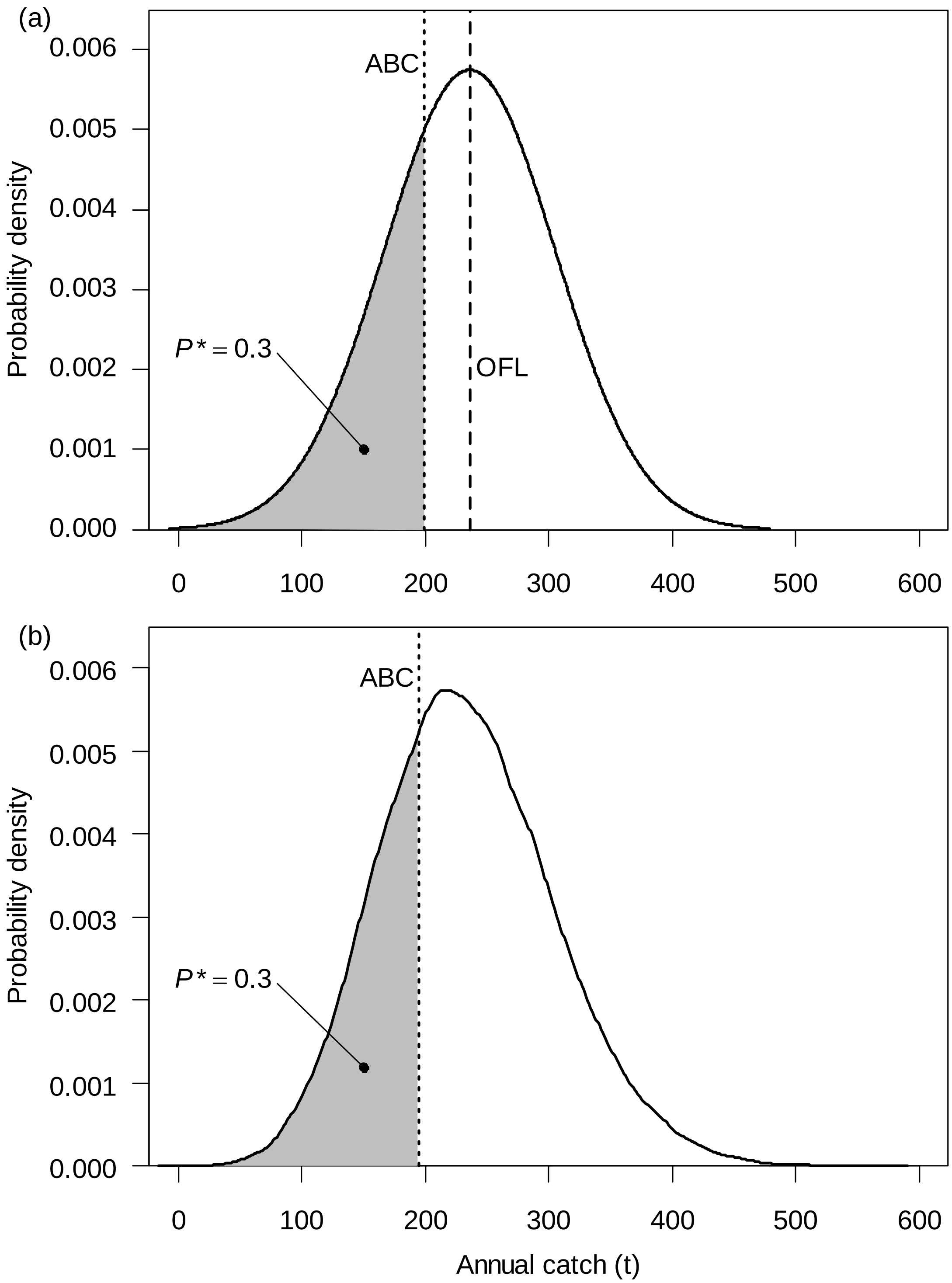

Figure 3. Example of ABCs as a function of P^*, where the distribution of OFL (a) is normal or (b) computed from numerical sampling.

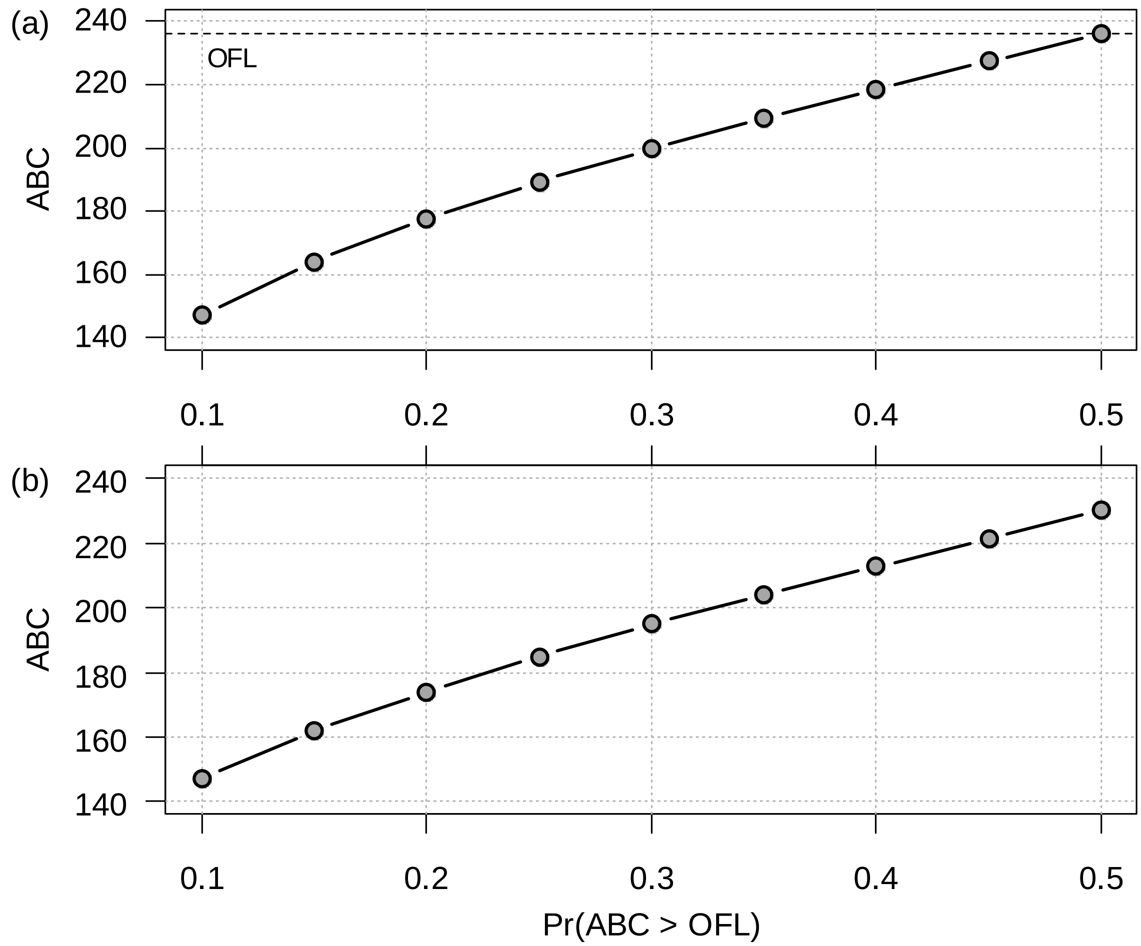

Figure 4. Flowchart of integrated PASCL.

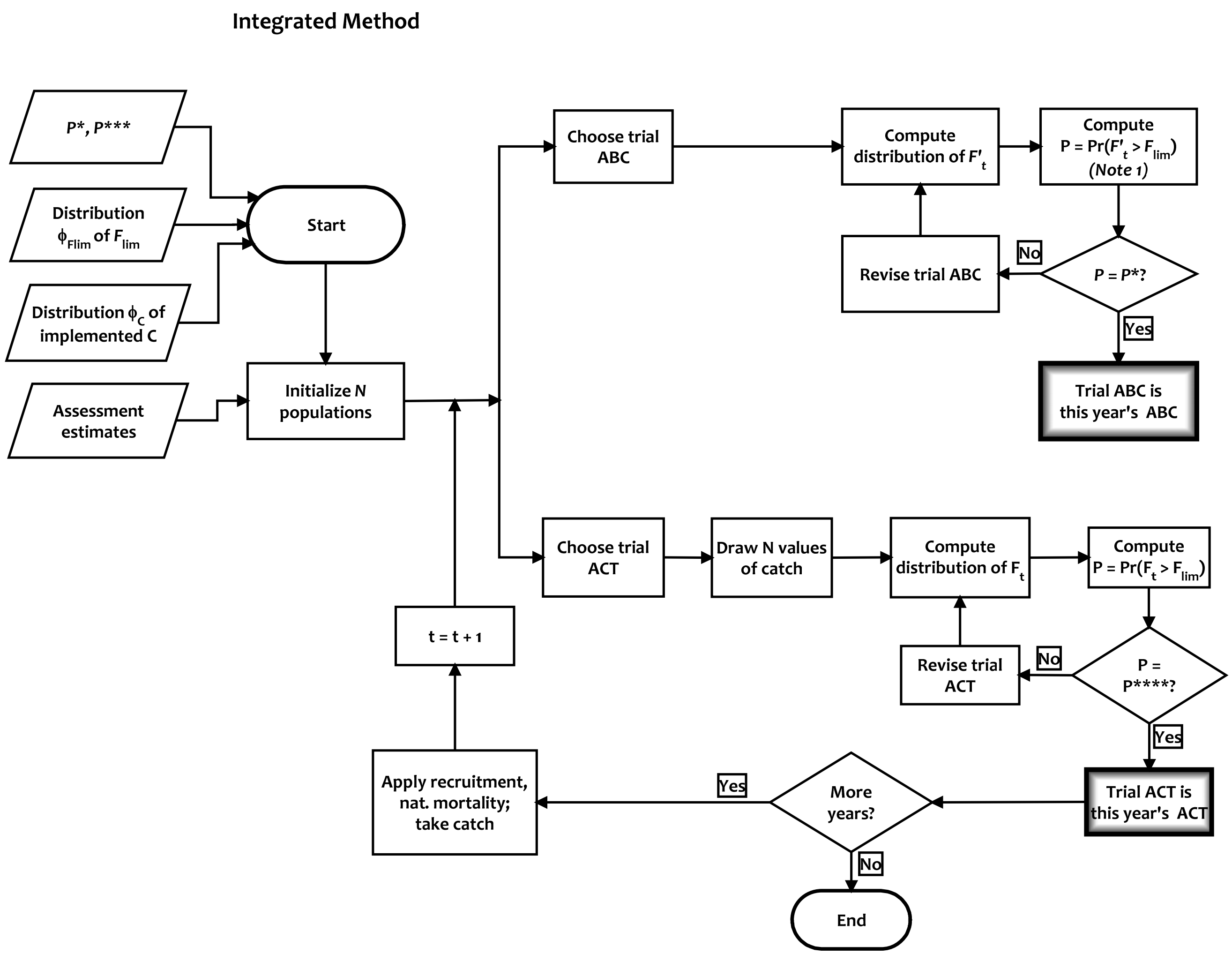

Figure 5. Example of integrated PASCL. Here, P^* = 0.4, and P^{***} = 0.1, 0.2, or 0.3, as indicated. The *CV* of management implementation was assumed to be 0.2, 0.4, or 0.6, as shown in column headings.

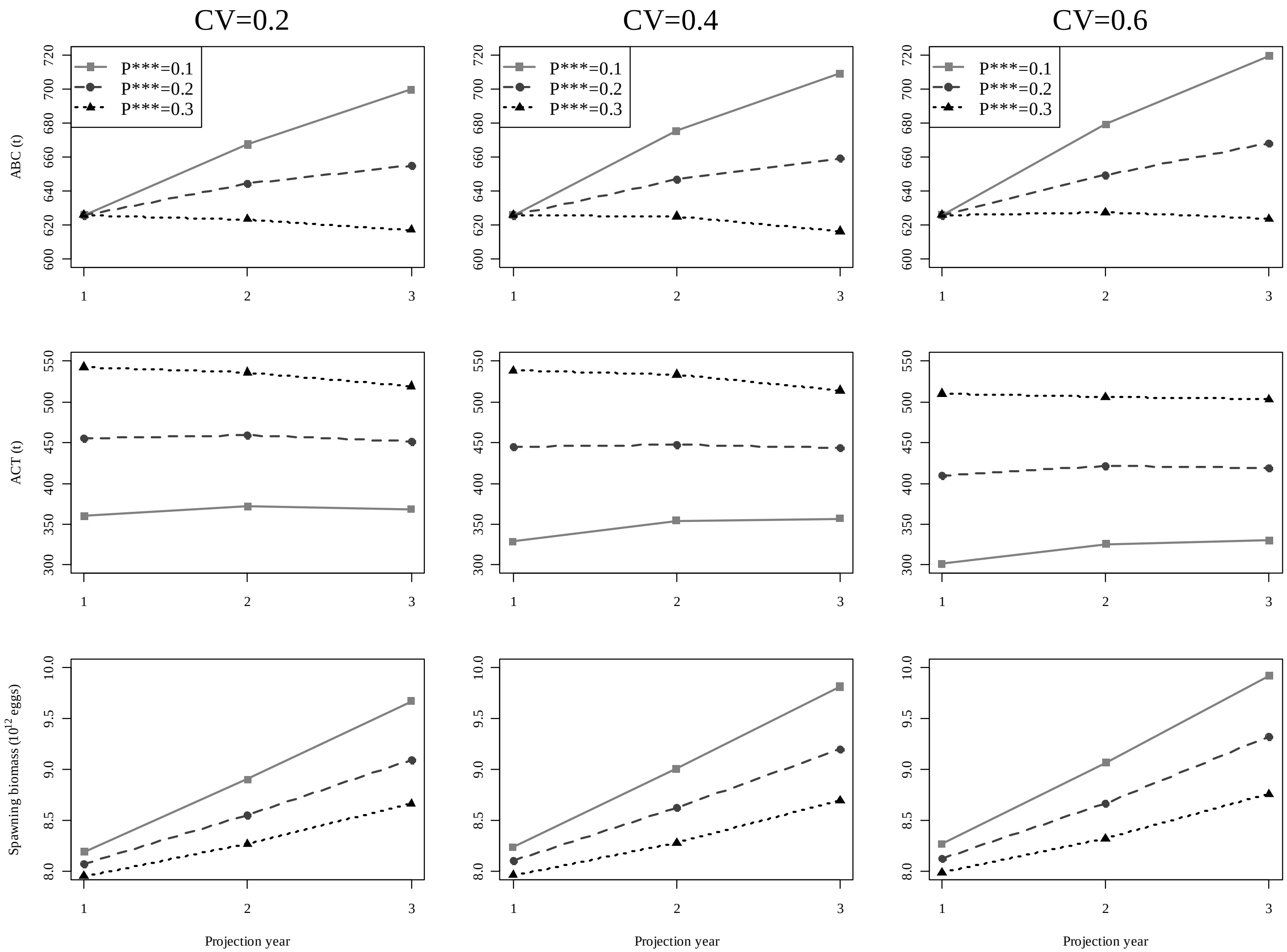

Figure 6. Flowchart of sequential PASCL.