

# Consensus Among Climate Models via Synchronized Chaos

Greg Duane  
U. Colorado

Ljupco Kocarev  
UCSD

Frank Selten  
KNMI

Supported by:  
DOE Grant# DE-SC0005238

# SUMMARY

- Problem: IPCC-class climate models give widely divergent predictions in regard to:

- a) magnitude of long-term climate change
- b) detailed regional predictions
- c) short-term climate change

Can we do better than averaging model outputs?

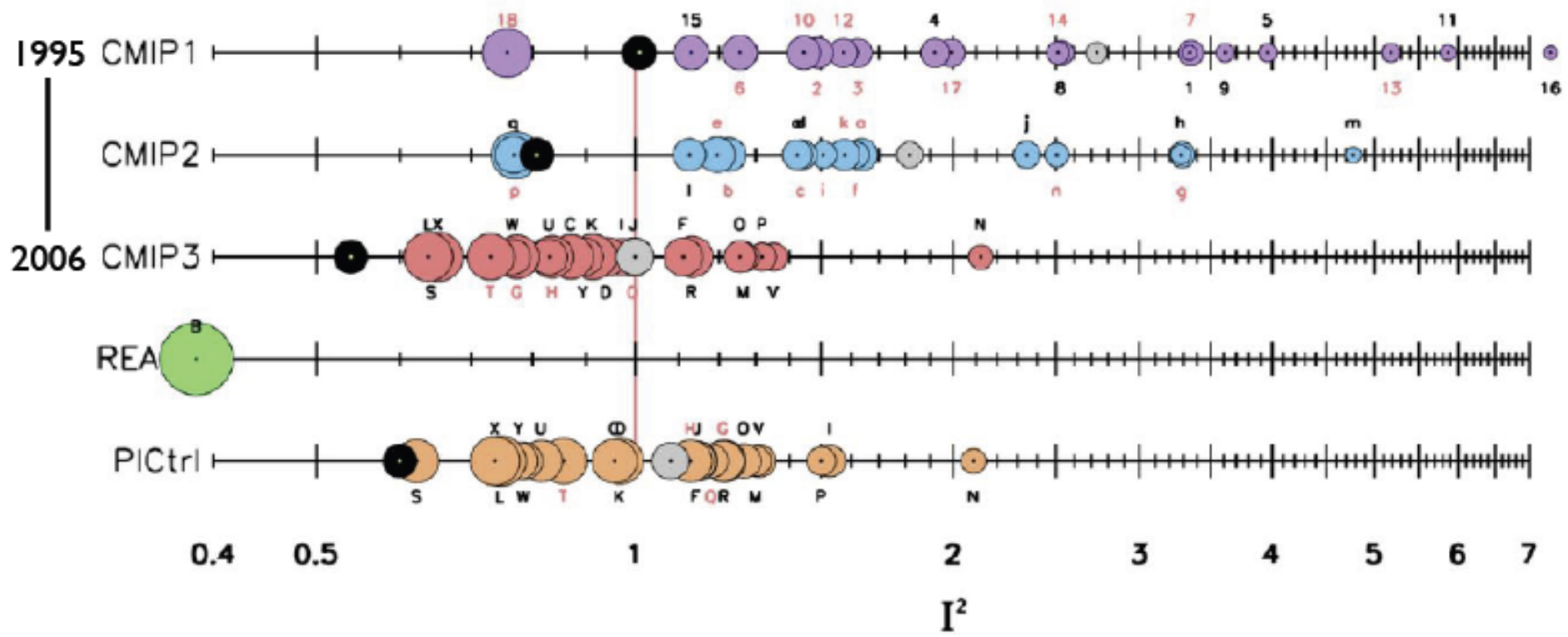
- Potential Solution: Take the synchronization view of data assimilation, and allow models to form a consensus (synchronize) by assimilating data from one another.
  - Sync extends the “nudging” approach to assimilation.
  - Parameters can be nudged as well as states *without ensembles*.
  - Choose the adaptable parameters to be connection coefficients linking corresponding variables in different models; adapt them using historical data.

# Coupled Model Intercomparison Project

Reichler, T., and J. Kim (2008): How Well do Coupled Models Simulate Today's Climate? *Bull. Amer. Meteor. Soc.*, **89**, 303-311.

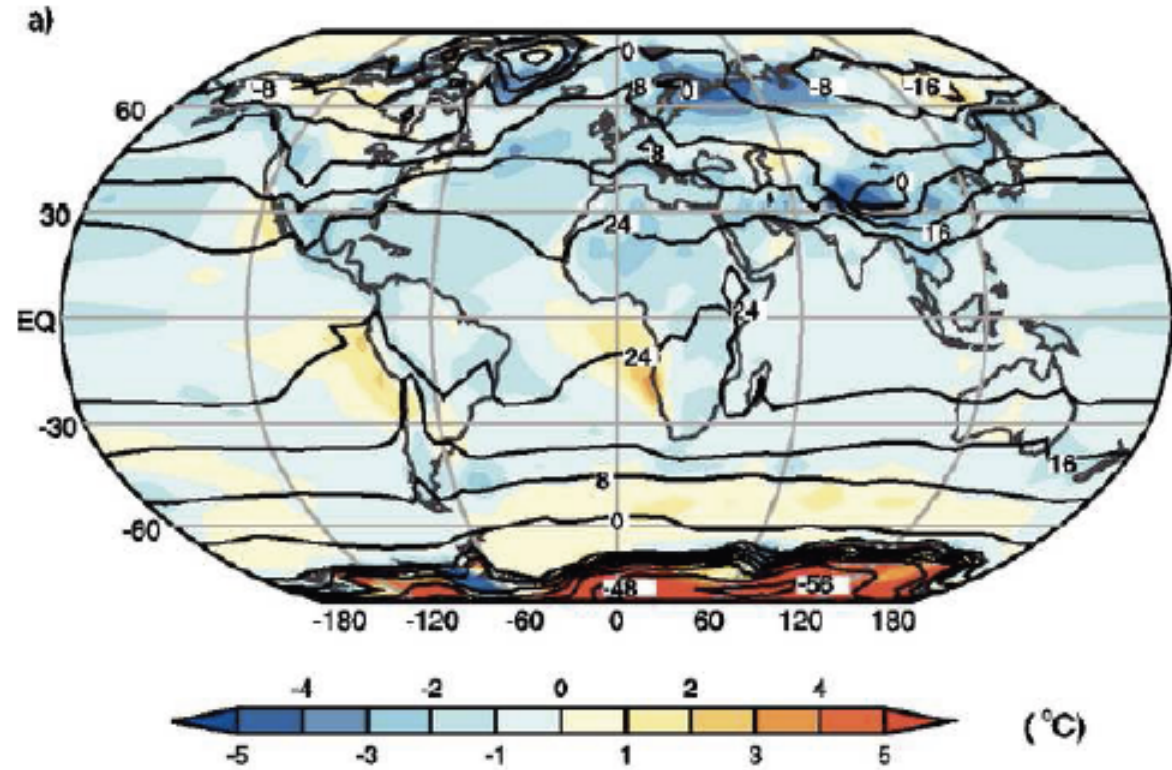
## Performance metric

Based on mean squared errors in time mean global temperatures, winds, precipitation, ....



● = index value based on multi model mean fields: outperforms individual models: why ?

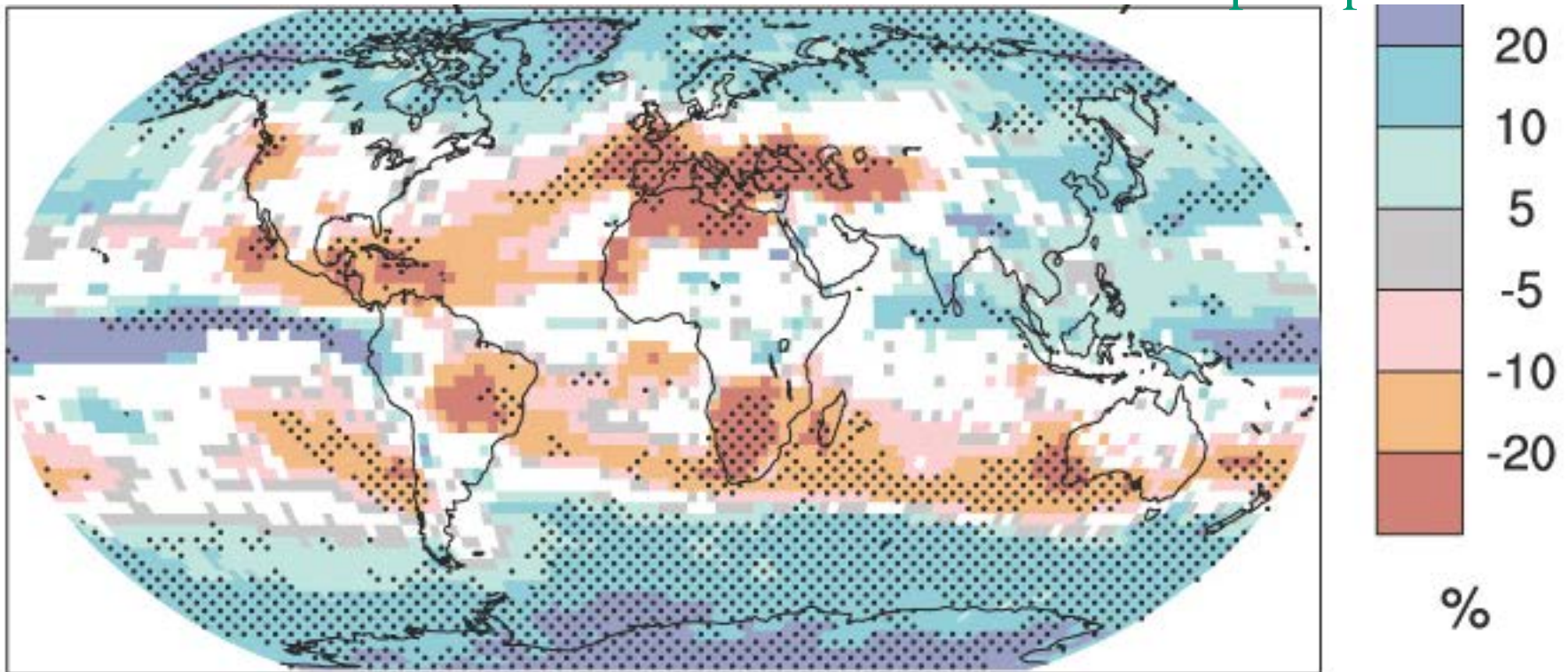
# Error in annual mean surface air temperatures multi model mean over all CMIP3 simulations



IPCC 2007

# EXAMPLE: DIVERGENT MODEL PROJECTIONS OF REGIONAL PRECIPITATION CHANGE

increased or decreased  
precipitation

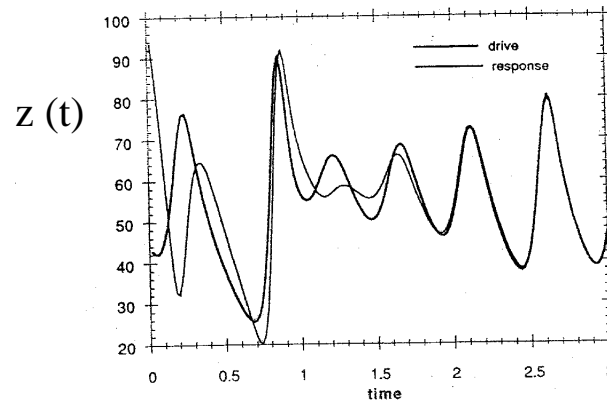
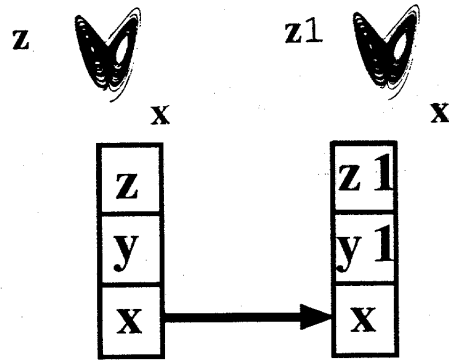


White areas: less than 2/3 of models agree on the sign of precipitation change

Stippled areas: more than 90% of models agree on the sign

# SUPPOSE THE WORLD IS A LORENZ SYSTEM AND ONLY $x$ IS OBSERVED

- two coupled chaotic systems can fall into synchronized motion along their strange attractors when linked through only one variable

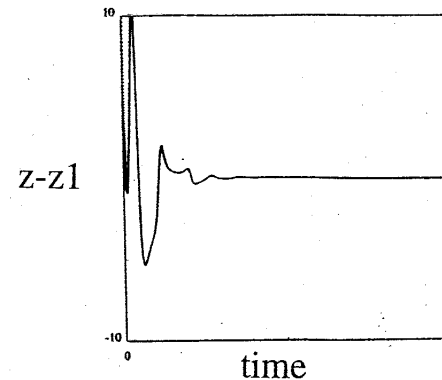


$$x' = \sigma(y-x)$$

$$y' = \rho x - y - xz \quad y_1' = \rho x - y_1 - x(z_1)$$

$$z' = -\beta z + xy \quad z_1' = -\beta(z_1) + x(y_1)$$

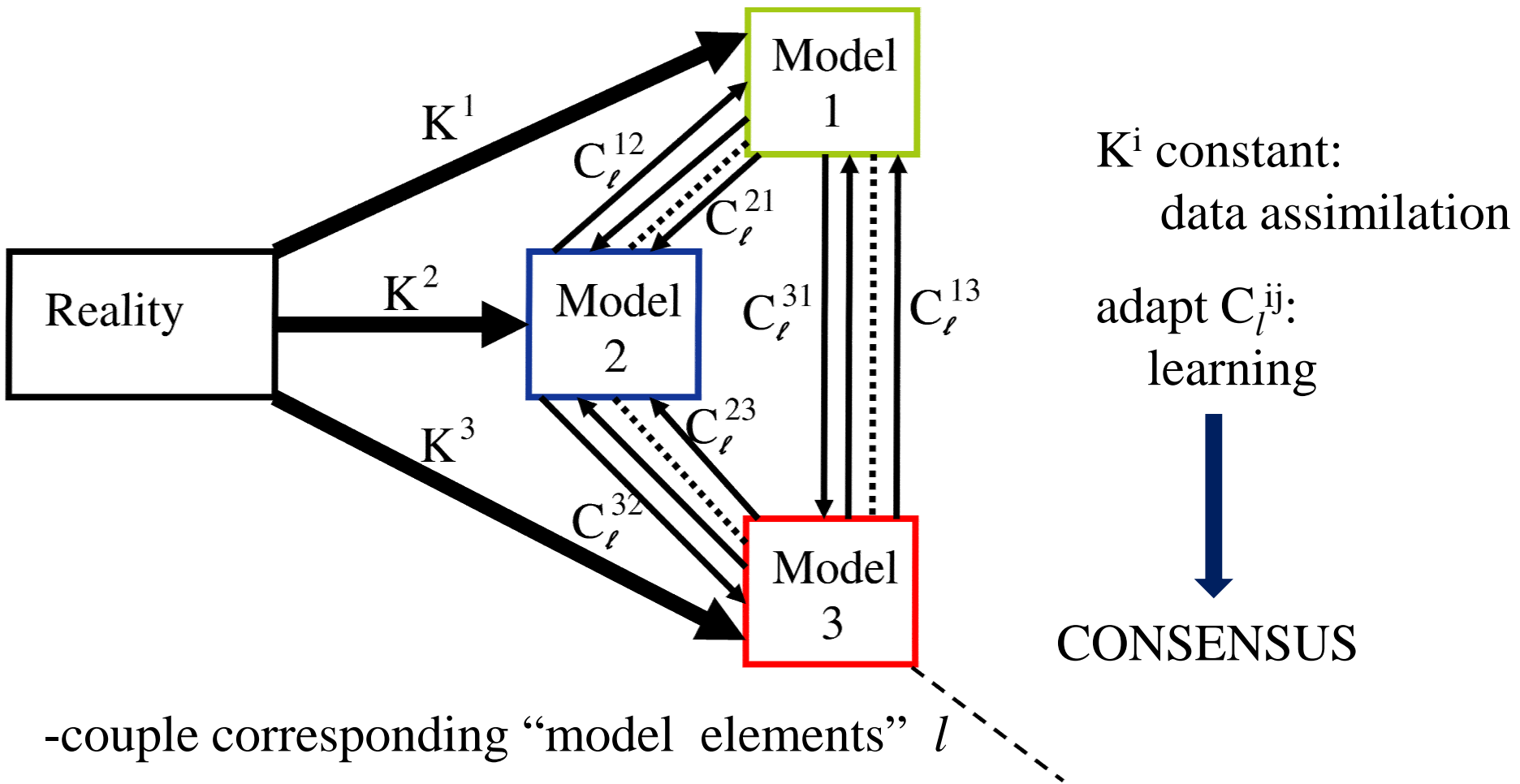
(also works for  $y$ -coupling, but not for  $z$ -coupling)



(Pecora and Carroll '90)

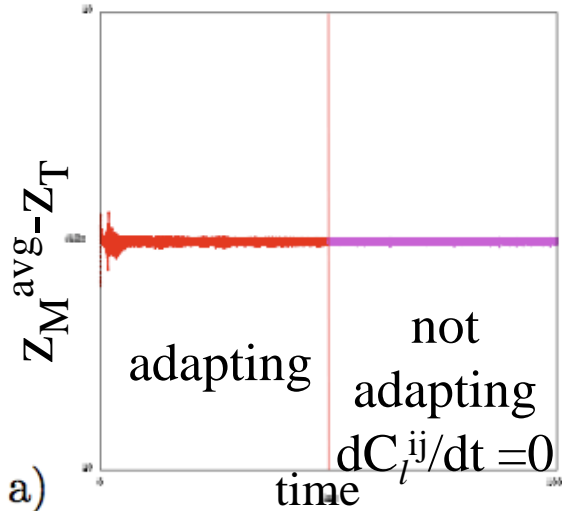
- SYNCHRONIZATION**  $\longrightarrow$  **DATA ASSIMILATION**

LET A COLLECTION OF MODELS  
ASSIMILATE DATA FROM  
(SYNCHRONIZE WITH) ONE ANOTHER;  
ADAPT THE COUPLING COEFFICIENTS

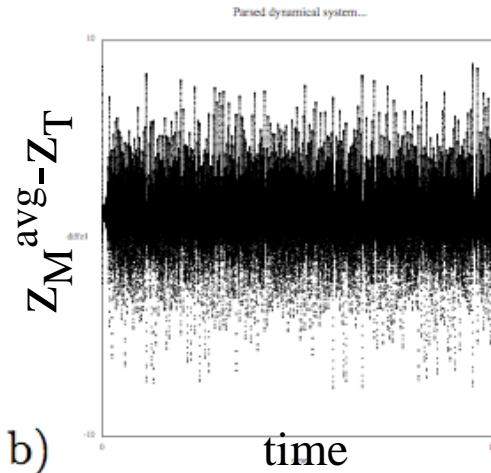


# Test Case: Fusing 3 Lorenz Systems With Different Parameters

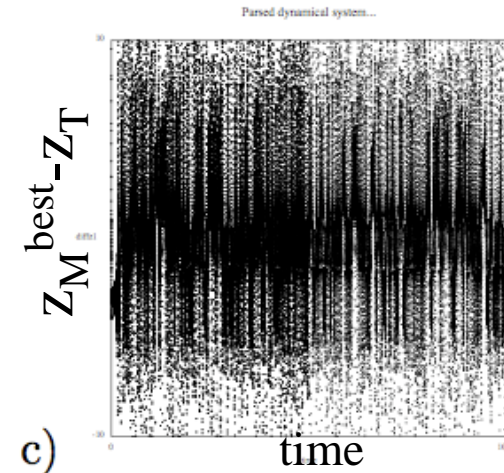
Fused Models



Average Output  
of Models (Unfused)



z from Model  
With Best z Eqn



$$\begin{aligned} \dot{x} &= \sigma(y - z) \\ \dot{y} &= \rho x - y - xz \\ \dot{z} &= -\beta z + xy \\ \dot{x}_i &= \sigma_i(y_i - z_i) + \sum_{j \neq i} C_{ij}^x(x_j - x_i) + K_x(x - x_i) \\ \dot{y}_i &= \rho x_i - y_i - x_i z_i + \mu_i + \sum_{j \neq i} C_{ij}^y(y_j - y_i) + K_y(y - y_i) \\ \dot{z}_i &= -\beta z_i + x_i y_i + \sum_{j \neq i} C_{ij}^z(z_j - z_i) + K_z(z - z_i) \end{aligned}$$

$$dC_x^{ij}/dt = a(x_j - x_i)(x - \frac{1}{3}\sum x_k)$$

$$i = 1, 2, 3$$

$$dC_y^{ij}/dt = \dots\dots\dots$$

$$dC_z^{ij}/dt = \dots\dots\dots$$

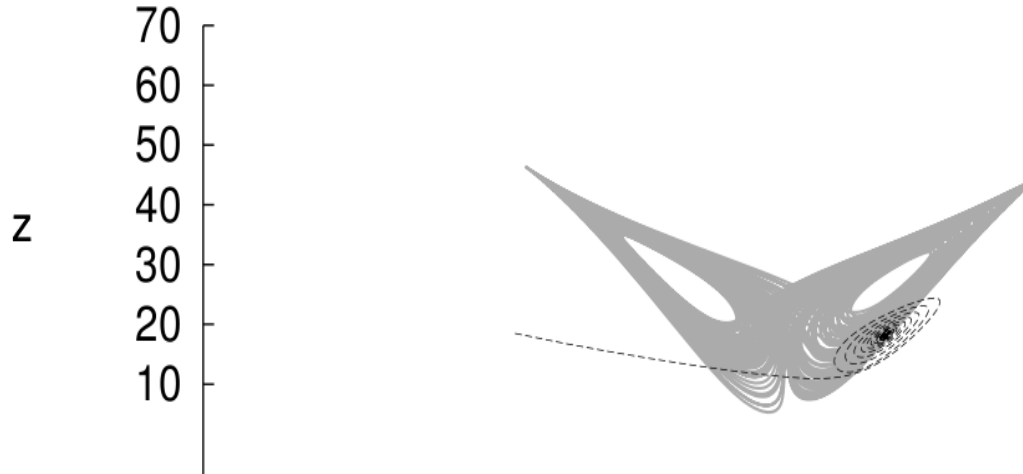
- Model fusion is superior to any weighted averaging of outputs



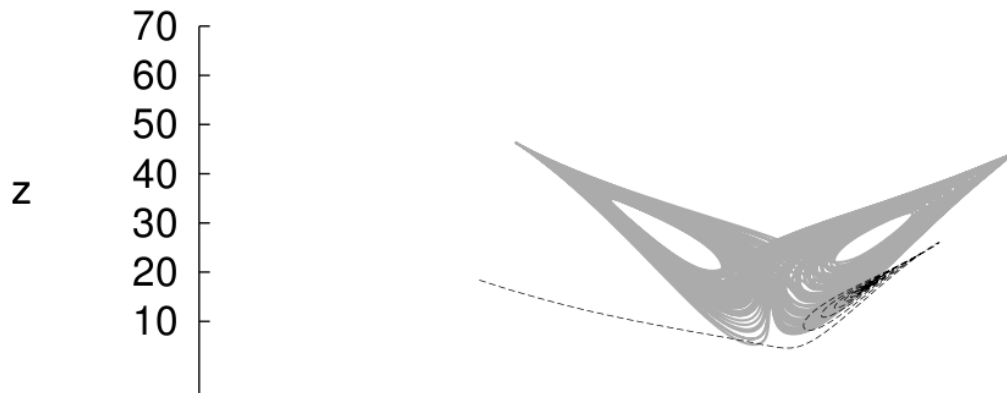
# .....OR CAN USE STANDARD MACHINE LEARNING METHODS TO ADAPT INTER-MODEL CONNECTIONS

*(Berge et al. 2010)*

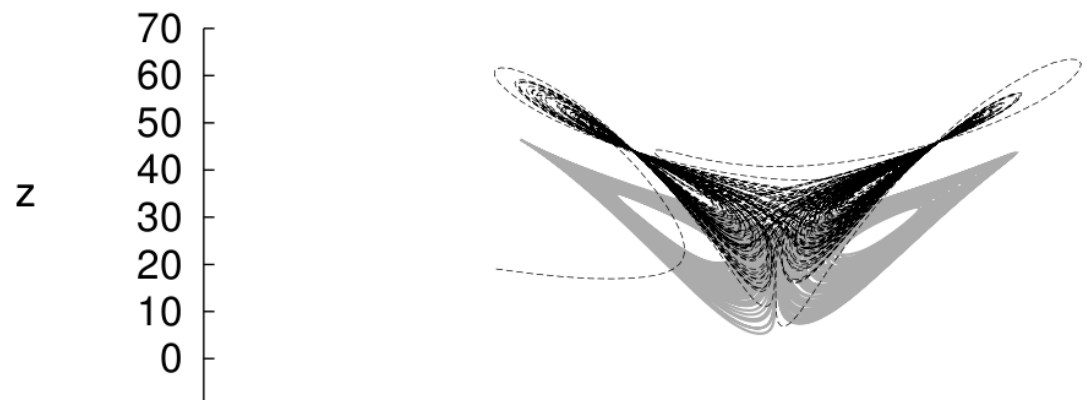
Truth —  
Model 1 - - -



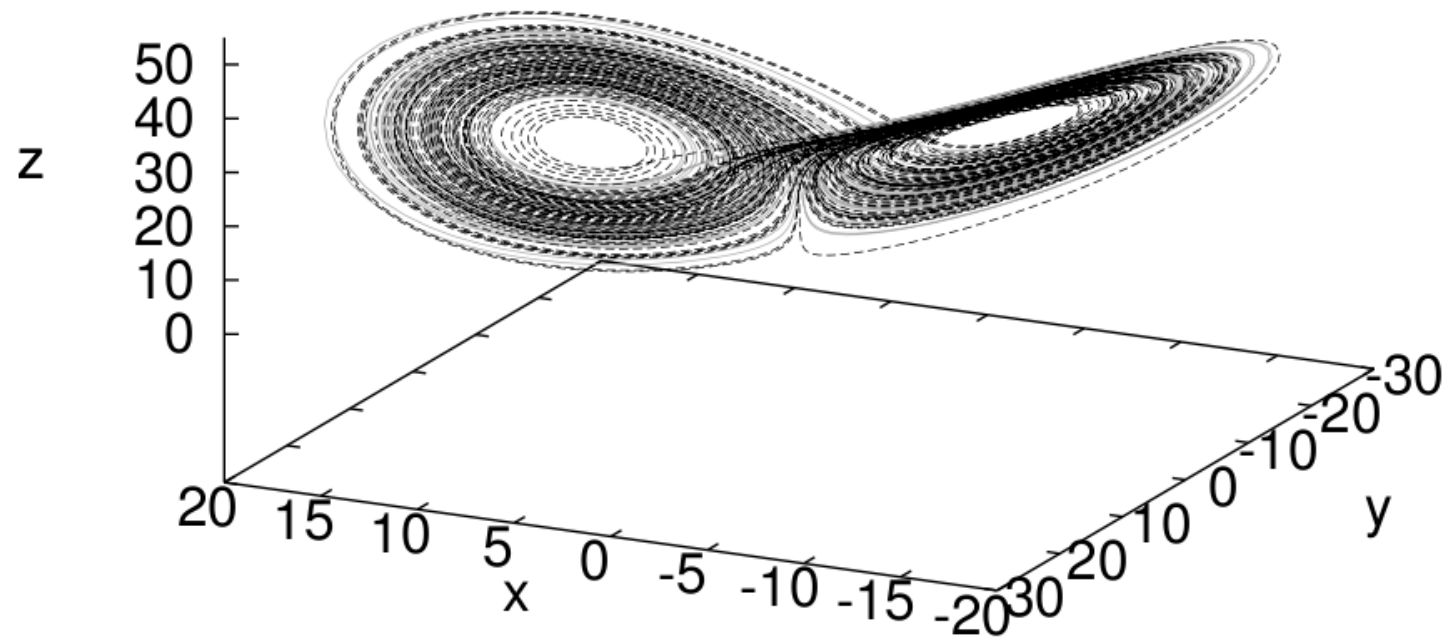
Truth —  
Model 2 - - -



Truth ———  
Model 3 - - - -



Truth ———  
Super-model - - - -



# PROJECT PLAN

- address theoretical issues using simple ODEs
  - negative connections if all models are biased in same direction
  - multiple time scales (ocean/atmosphere) in models
  - globally vs. locally optimal connection schemes
  - .....
- specialize to climate application using QG models
  - determine minimal spatial density of connections
  - choose variables to couple
  - test robustness of trained “supermodel” against increases in N-S temperature gradient
- apply to suite of 3 full climate models: 2 versions of CCSM and  
NOAA CFS

# Supermodeling Works With Multi-time-scale Models

Lorenz '84 coupled to ocean box model:

$$x' = -(y^2) - (z^2) - a x + a (F_0 + F_1 T) \quad f = \omega T - \xi S$$

$$y' = x y - b x z - y + G_0 + G_1 (T_{av} - T)$$

$$z' = b x y + x z - z$$

$$T' = k_a (\gamma x - T) - |f| T - k_w T$$

$$S' = \delta_0 + \delta_1 (y^2 + z^2) - |f| S - k_w S$$

$X_{\text{supermodel}} - X_{\text{truth}}$



$T_{\text{supermodel}} - T_{\text{truth}}$

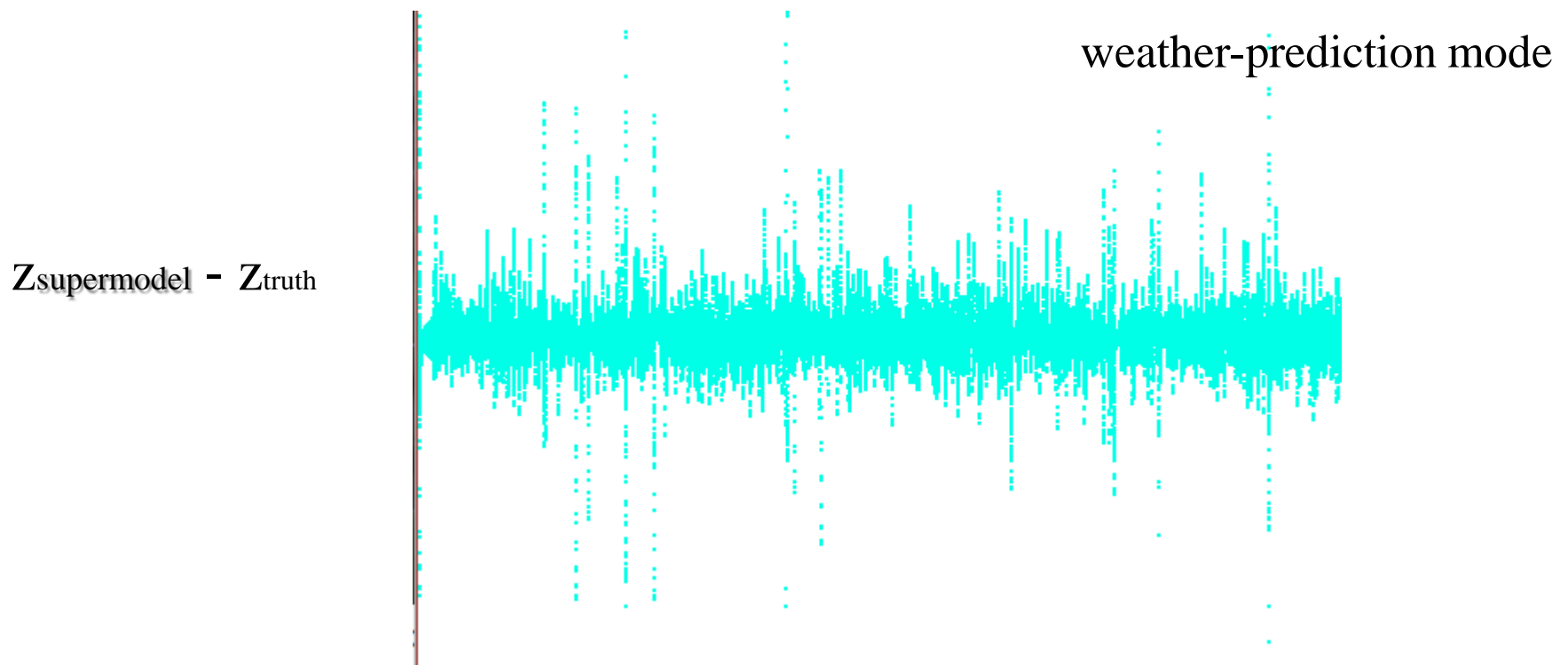


In “weather-prediction mode” ocean strongly nudged to truth so as to obtain an atmospheric supermodel. Ocean supermodel can be trained on longer time scales.

# What if all models are biased in same direction?

Lorenz supermodel with  $\sigma_{\text{truth}} < \sigma_1, \sigma_2, \sigma_3$

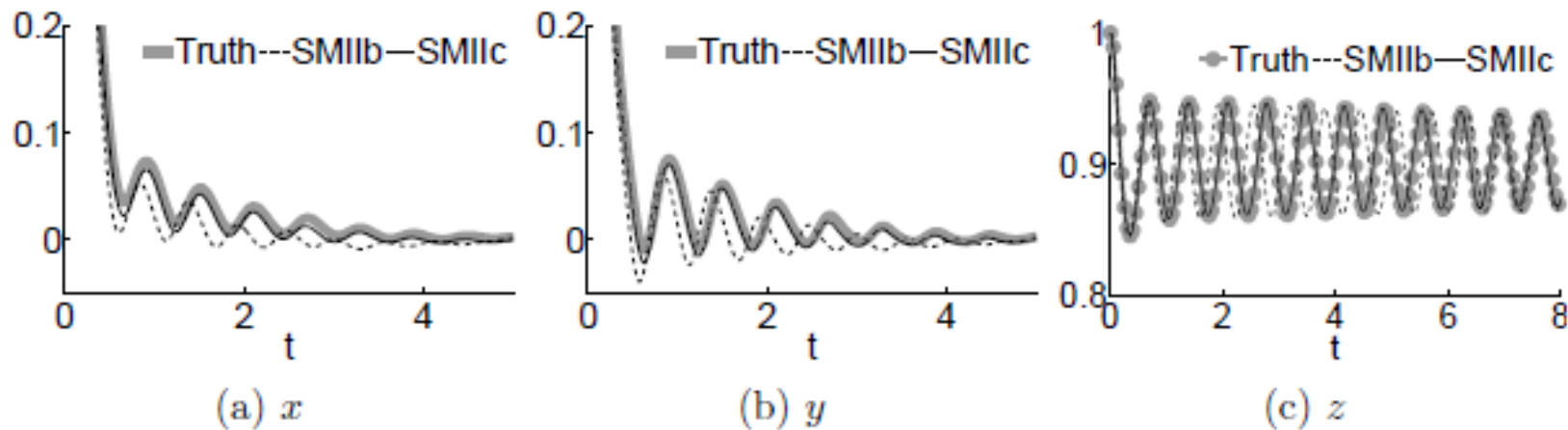
$\Rightarrow$  Some connections become negative



Not as effective as positive connections, but better than averaging.

# Stochastic Learning Methods Can Help Optimize Supermodel

## Autocorrelations for Truth and Two Supermodels



SMIlb is formed using a deterministic learning method

SMIlc is formed using a stochastic learning method

# Extension to PDE's: What is the required spatial density of inter-model coupling?

Synchronization of two 1D Kuramoto-Sivishinsky systems:

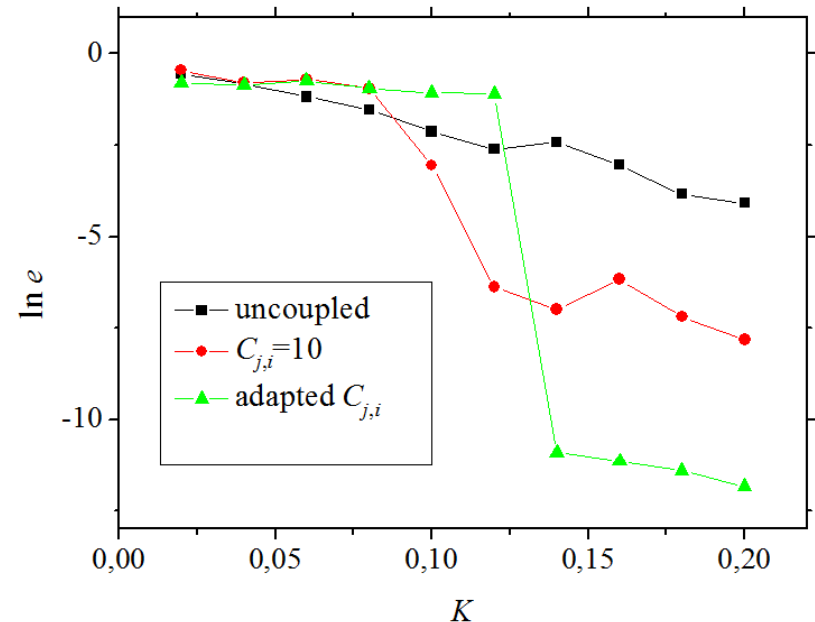
$$u_t = -u_{xxxx} - \alpha_u u_{xxx} - u_{xx} - 2uu_x$$

$$v_t = -v_{xxxx} - \alpha_v v_{xxx} - v_{xx} - 2vv_x + K[u(x) - v(x)]f(x)$$

$f(x)$  non-vanishing only at discrete points

Can form supermodel from 3 KS's:

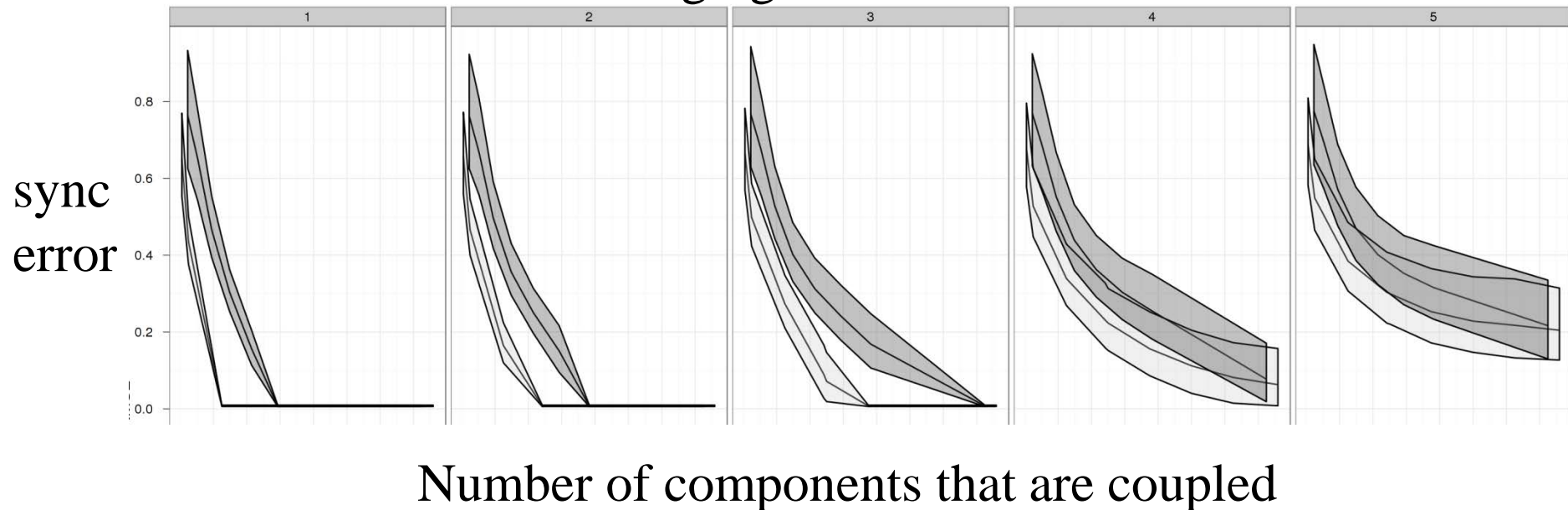
Maximum coupling distance is  
length scale of coherent structures:



# What variables should be coupled?

Consider 3-layer QG model on sphere with realistic topography and a forcing chosen to reproduce the observed winter mean state.

Compare coupling in a basis of spherical harmonics to a basis of EOF's:  
nudging time scale



dark grey: spherical harmonics

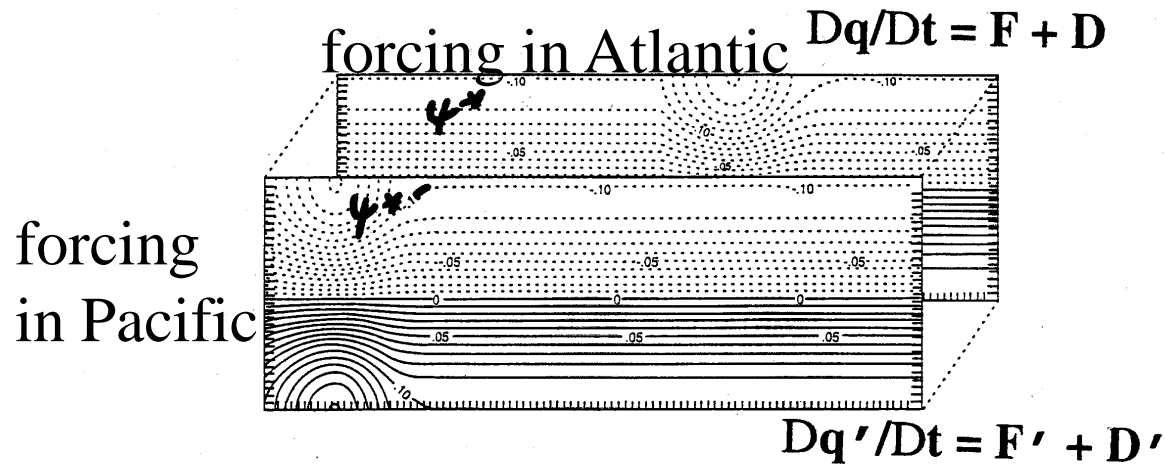
light grey: EOF's



# Immediate Plans

- Understand role of stochasticity in choosing among highly constrained connection schemes
- Study robustness of QG supermodel against changes in forcing
- Establish inter-model coupling within DART at NCAR

# Proposed Adaptive Fusion of Two QG Channel Models



$$F = f_0 (q - q^*) + c J(\psi, q - q')$$

$$F' = f_0 (q' - q^{*'}) + c J(\psi', q' - q)$$

$$c = 1/2 \quad \Downarrow$$

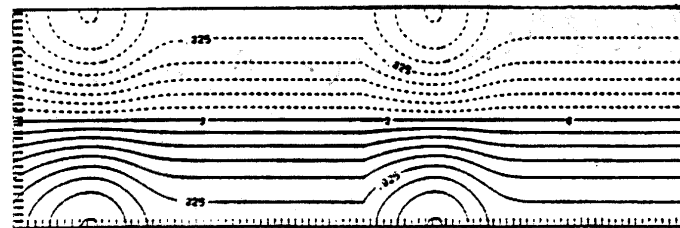
$$D/Dt (q + q')/2 = (F_0 + F_0')/2 + (D + D')/2$$

(k-dependence suppressed)

$$F_0 = f_0 (q - q^*)$$

$$F_0' = f_0 (q' - q^{*'})$$

- If the parallel channels synchronize, their common solution also solves the single-channel model with the average forcing



$$\frac{\Psi^* + \Psi^{*'}}{2}$$

To find  $c$  adaptively:

$$dc/dt = \int d^2x J(\psi, q' - q)(q - q_{obs}) + \int d^2x J(\psi', q - q')(q' - q_{obs})$$

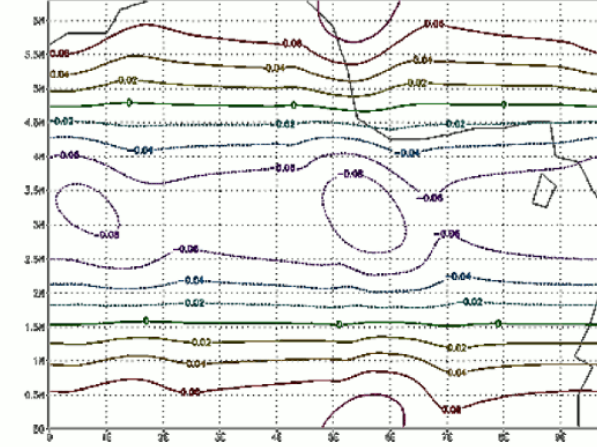
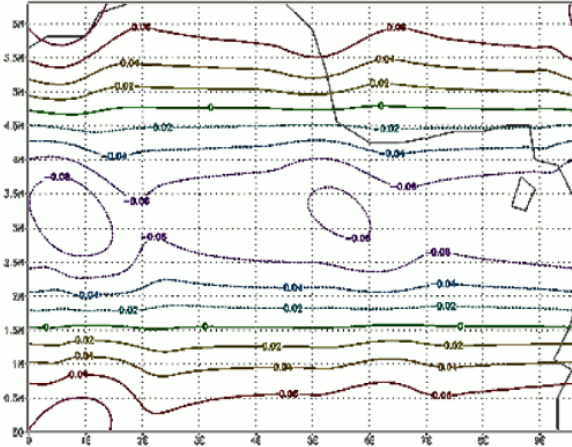
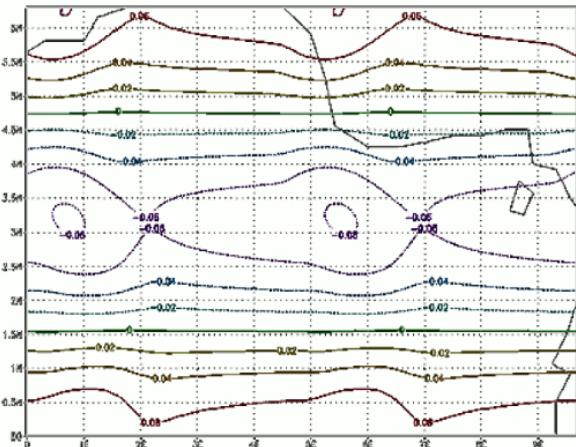
# Models Synchronize With Each Other and With “Truth”

“truth”

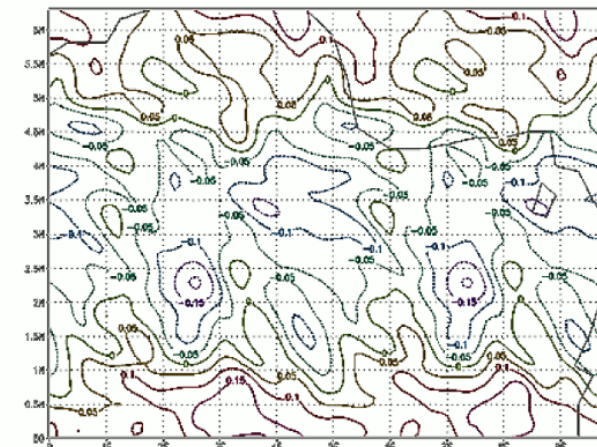
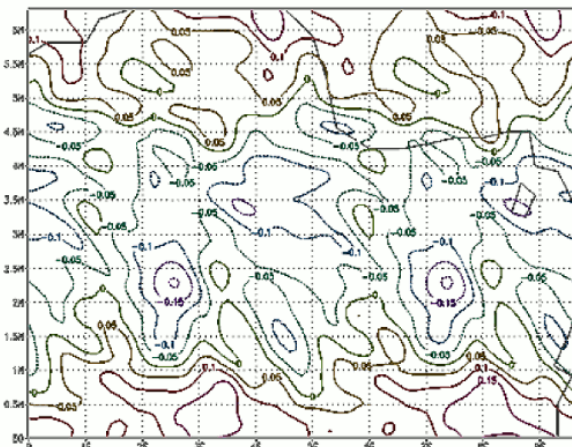
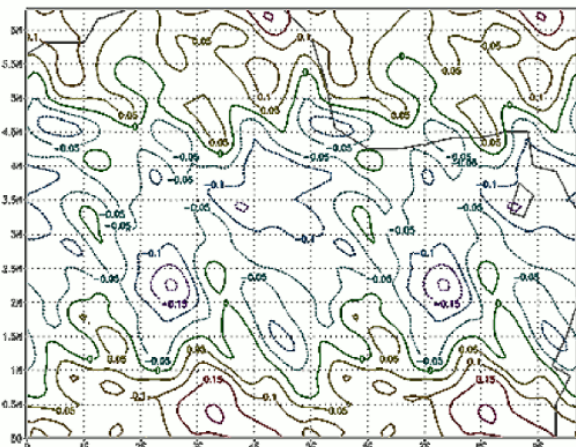
model with Atlantic forcing

model with Pacific forcing

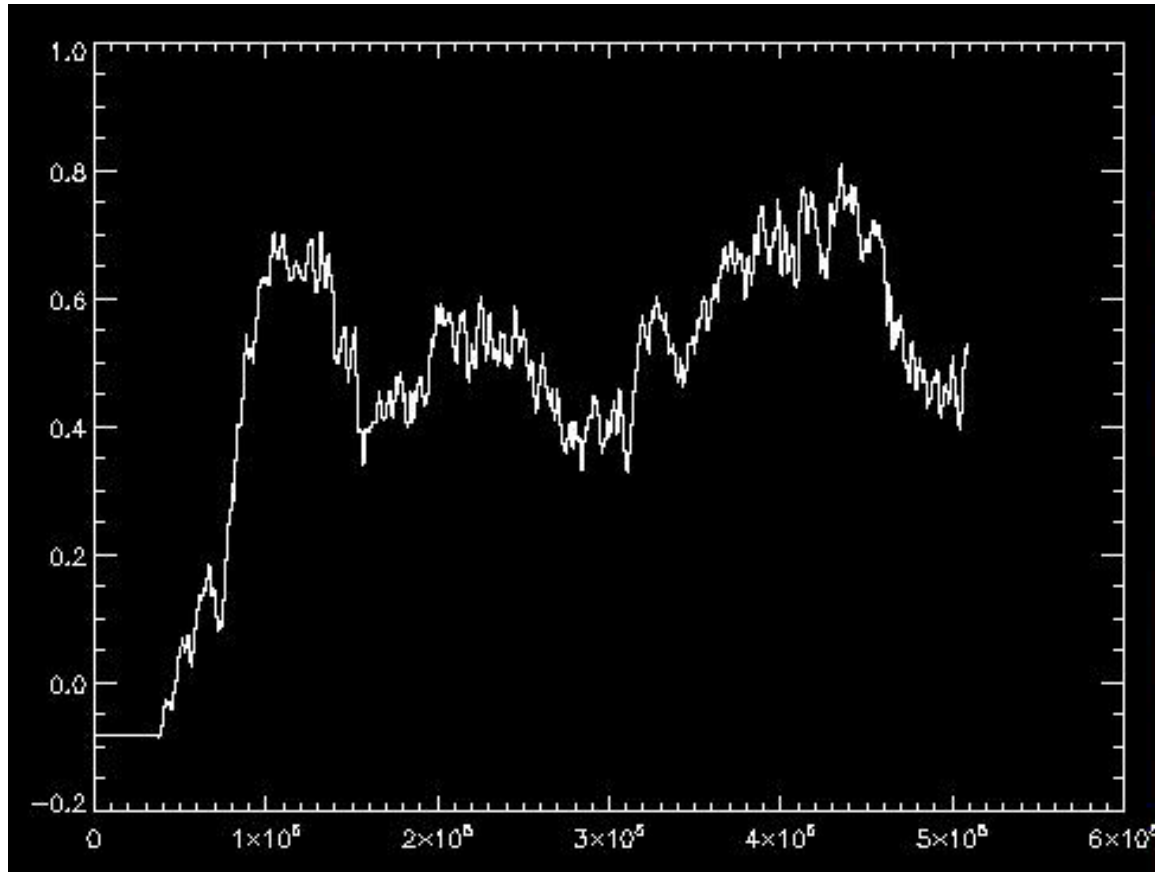
n = 1000:



n = 30000:

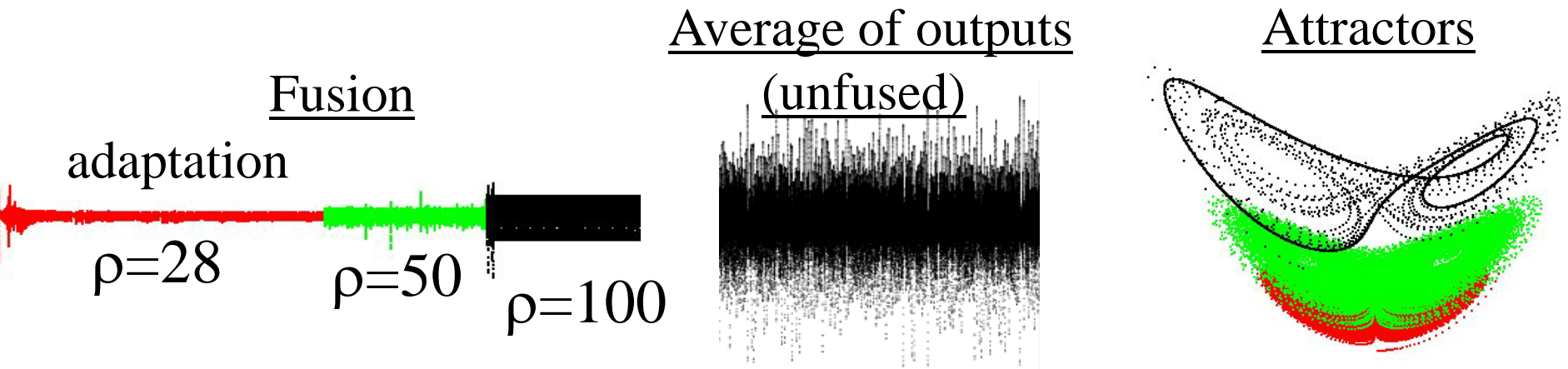


.....As the Adaptation Procedure Estimates the Intermodel Connection Coefficient  $c \rightarrow 1/2$



Possible Issue 1: What if the dynamical parameters change drastically in the 21<sup>st</sup> century as compared to the training period?

Train with Lorenz  $\rho=28$  and then reset  $\rho$  in “reality” and in 3 “models”



-fusion still better than averaging even when training and test systems differ by a large number of dynamical bifurcations

## Possible Issue 2: Do the results apply to climate projection (vs. weather prediction)?

-It is actually easier to achieve *non-isochronic* synchronization (a.k.a. *measure synchronization*), where the attractors of two coupled systems become the same, without any agreement between concurrent states.