Global Sensitivity Analysis of the Dynamic Integrated Model of Climate and the Economy (DICE2007)

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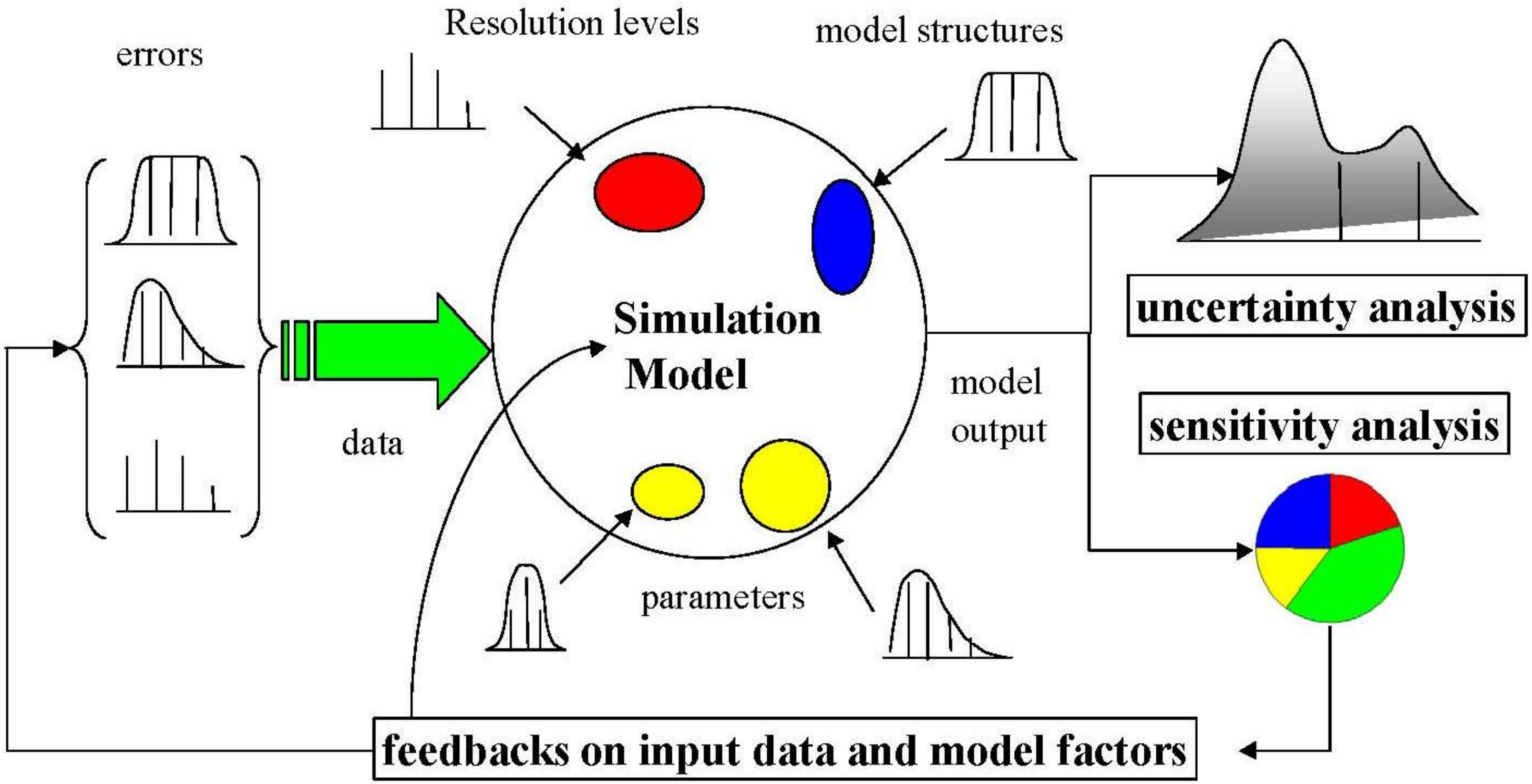
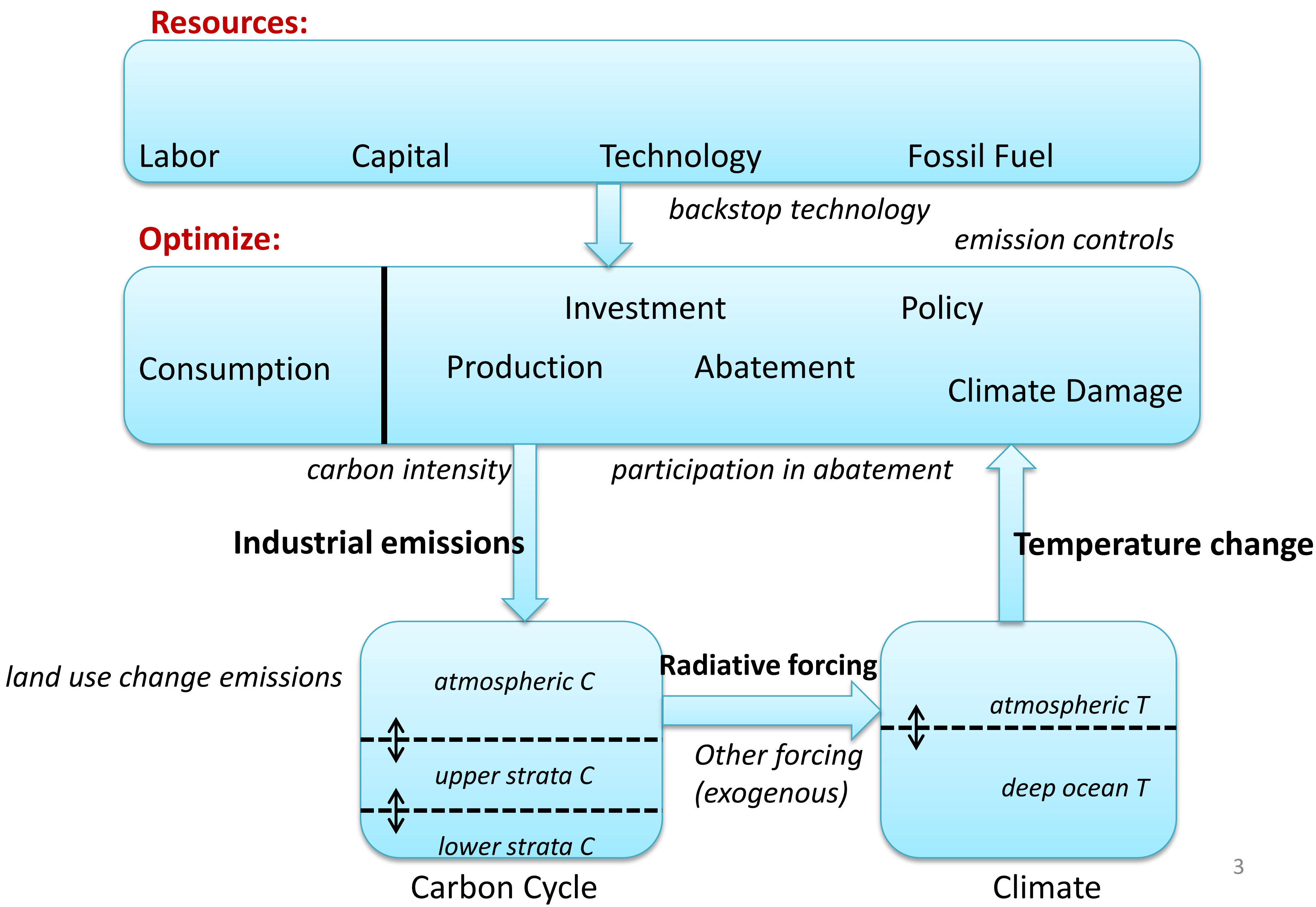
How do we gain confidence in the models we use?



[Andrea Saltelli (2009), http://sensitivity-analysis.jrc.ec.europa.eu]

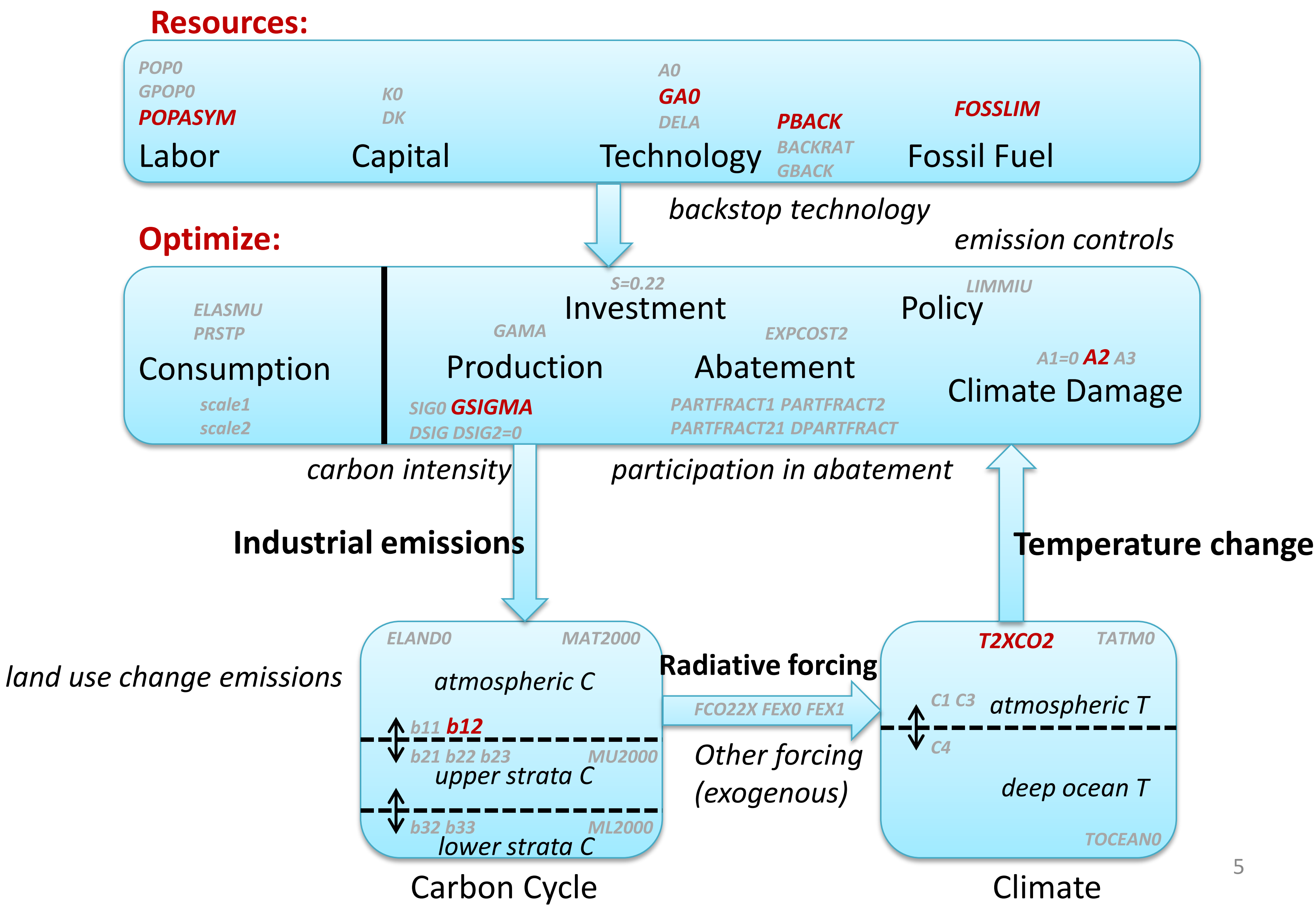
DICE2007 is a globally-aggregated Integrated Assessment Model



We analyze 8 DICE exogenous parameters based on suggestions by Nordhaus (2008, *A Question of Balance*, Table 7-1)

Factor	**Definition**	**Units**
GA0	Initial rate of growth of TFP	Per decade
GSIGMA	Rate of decarbonization	Per decade
T2XCO2	Equilibrium temperature sensitivity	°C per CO_2 doubling
A2	Damage coefficient (on quadratic term)	Fraction of GWP [/ΔT^2?]
PBACK	Price of backstop technology	Thousand $ per ton of carbon
POPASYM	Asymptotic global population	Millions
b12	Transfer coefficient in carbon cycle	Per decade
FOSSLIM	Total resources of fossil fuels	Gt Carbon

Which of these will be most important for temperature change?



The standard starting approach is to use one-at-a-time (OAT) testing, as shown in this dependence of the global temperature increase in 2105 on these eight exogenous parameters.

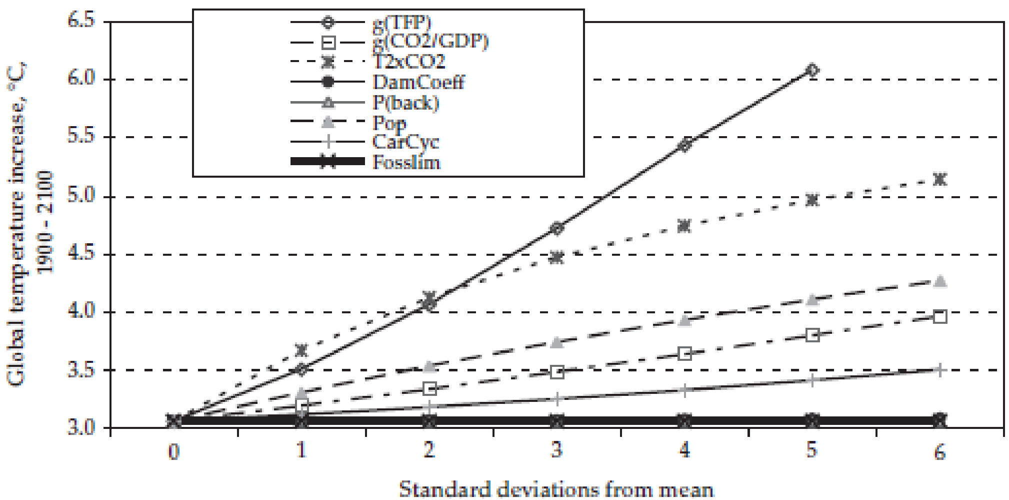


Figure 7-1 Nordhaus, *A Question of Balance* (2008)

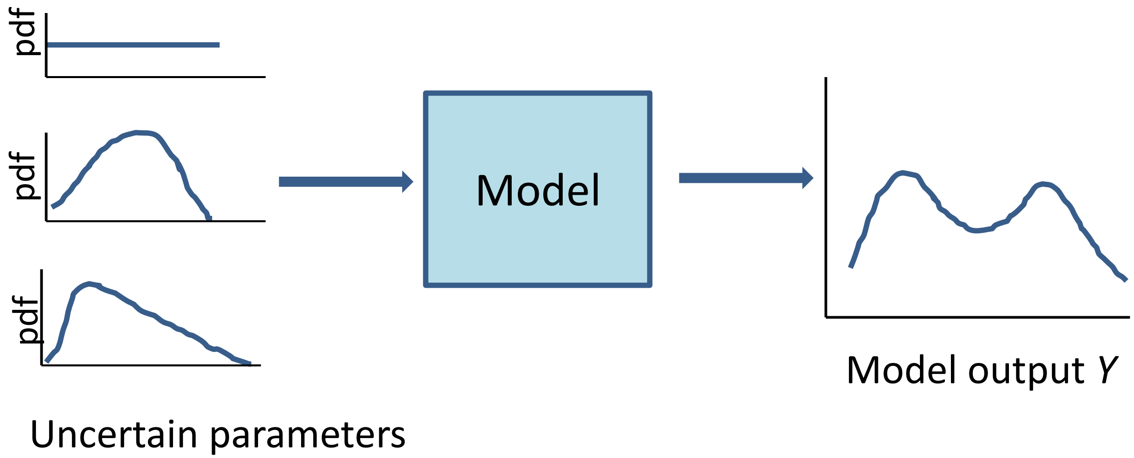
In order of importance:

1. Total factor productivity
2. Temperature sensitivity
3. Population limit
4. Decarbonization
5. Carbon cycle coefficient

We know this model is highly nonlinear.
We expect that there will be interactions between parameters.
We want to characterize the global effects of these parameters on key model outputs.
Often the next step is to examine different scenarios without understanding total effects of the exogenous parameters.

Sobol' Sensitivity Analysis is a global variance decomposition method

By sampling each of these parameters, we can attribute the uncertainty in the model output to the parameters and their interactions.



Total variance: $$V(Y) = V_1 + V_2 + V_3 + V_{12} + V_{23} + V_{13} + V_{123}$$

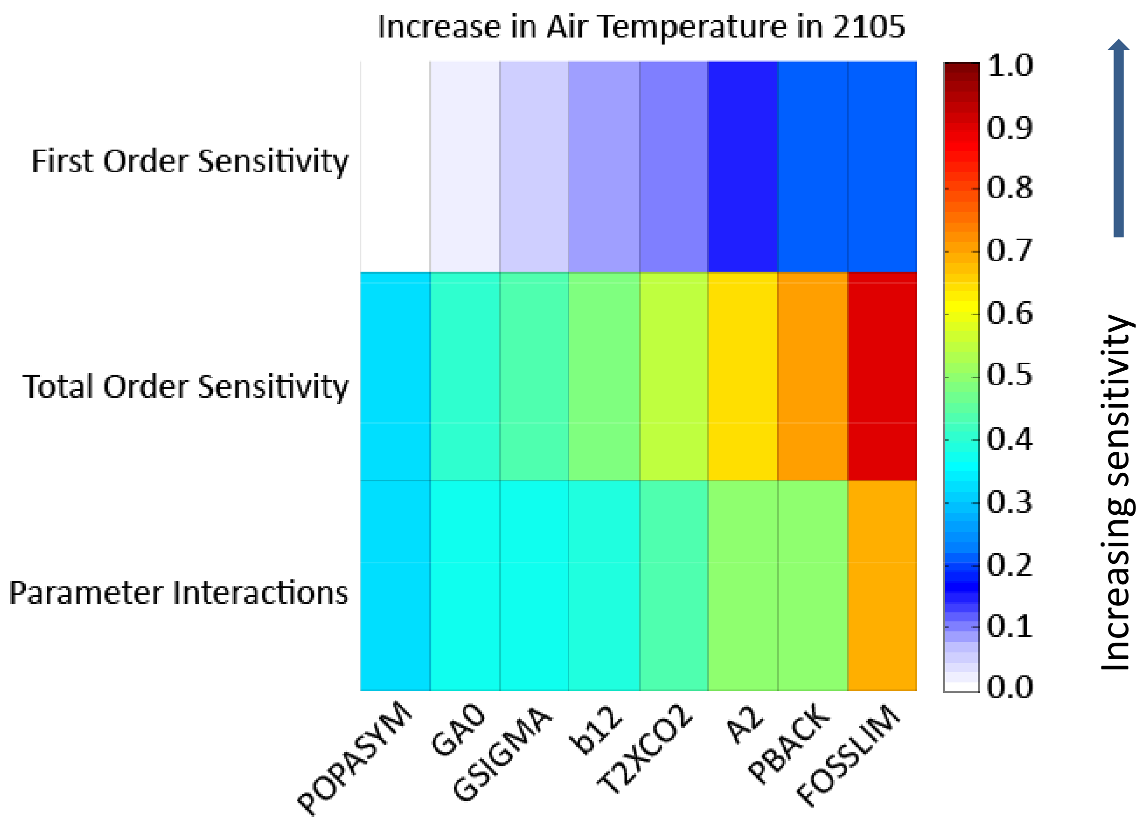
First order sensitivity index for Parameter 1: $$S_1 = \frac{V_1}{V}$$

Total order sensitivity index for Parameter 1: $$S_{T_1} = 1 - \frac{V_{\sim 1}}{V} = 1 - \frac{V_2 + V_3 + V_{23}}{V}$$

Sobol' Sensitivity Analysis setup for DICE2007

- We used the parameter bounds from the OAT experiment, where feasible
  - Reducing ranges on some parameters for logical and computational reasons
  - Extending the upper bound for Climate Sensitivity
- Quasi-random uniform sampling over the bounds for the 8 parameters => 135,000 samples
- DICE2007 configuration:
  - Optimal scenario, without fixing the savings rate
  - Climate damages, abatement costs, and emission control rates apply from the model start time
  - GAMS/CONOPT3
- Filtered the results, removing the ~1.5% of samples that failed to optimize

We compute the sensitivity indices of the global temperature increase to these 8 parameters using the Sobol' method

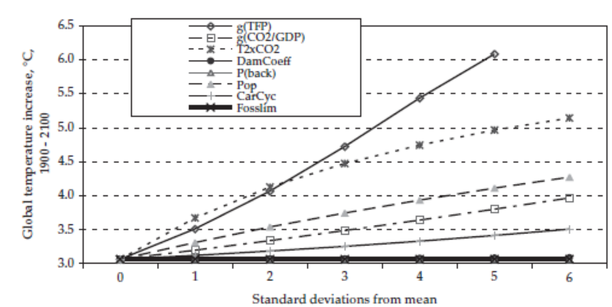


3 1 4 5 2

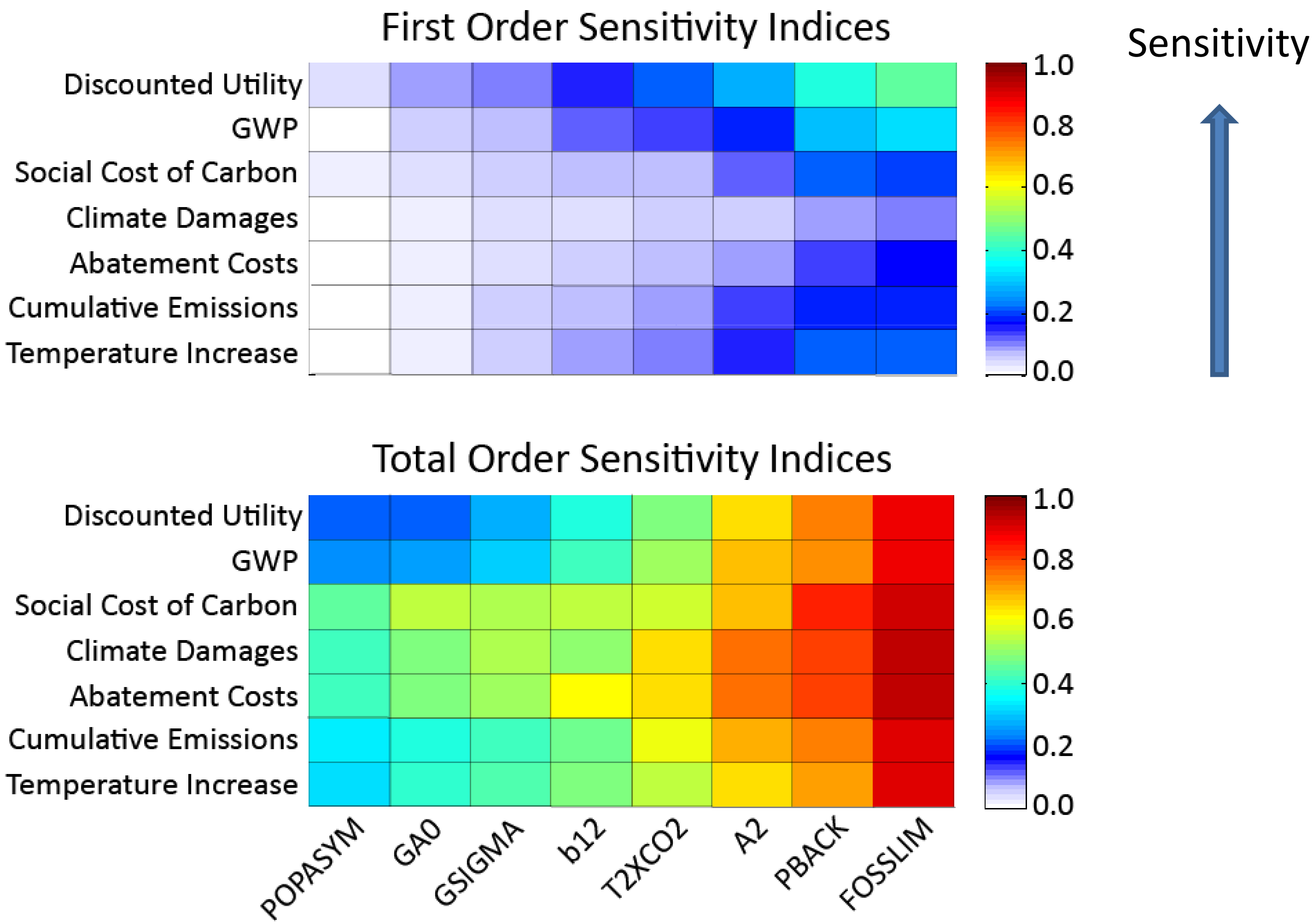
OAT order of importance

The Sobol' results suggest that parameter interactions are more important than the first order effects for this model output.

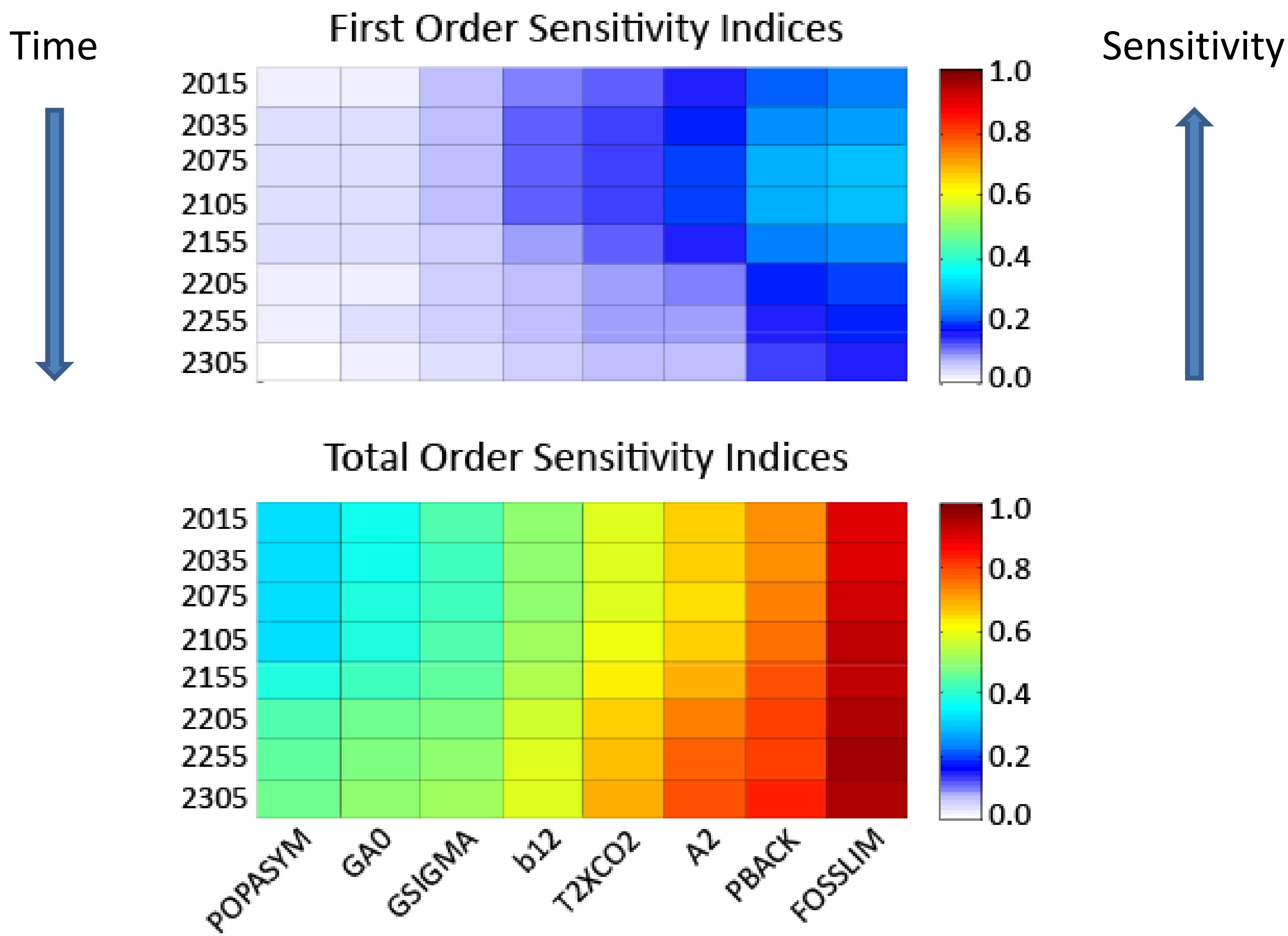
OAT order of importance:
1. Total factor productivity
2. Temperature sensitivity
3. Population limit
4. Decarbonization
5. Carbon cycle coefficient



Target model outputs are sensitive to different model parameters



Sensitivities to parameters may change over time: Emission Control Rates



Conclusions

- Sensitivity analysis is a complement to uncertainty analysis.
- Sensitivity analysis can attribute the output variance to specific parameters and parameter interactions.
  - **For DICE: Parameter interactions are very important.**
- One-at-a-time testing is not sufficient for non-linear models, especially those with extensive parameter interactions.
  - **For DICE: The fossil fuel limit is very influential in the sensitivity analysis, but not important at all in the OAT exercise.**
- Next steps:
  - Extension of the sensitivity analysis to more factors
  - Testing the model vs. testing the optimization

For More Information on Sensitivity Analysis

- Guides to good model practices
  - EPA, OMB, EC, IPCC
- Econometrics and Applied Statistics Unit, Joint Research Center, European Commission, http://sensitivity-analysis.jrc.ec.europa.eu
- Sobol' Sensitivity Analysis
  - Global Sensitivity Analysis: The Primer (A. Saltelli, et al., 2008) and references within.
  - Saltelli, A. (2002), Making best use of model evaluations to compute sensitivity indices, *Comp Phys Comm*, 145, 280-297.
  - Tang et al. (2007), Comparing sensitivity analysis methods to advance lumped watershed model identification, *HESS*, 11, 793-817.

How can we compute the sensitivity indices efficiently?

1. Make N(k+2) samples varying parameters $(X_1, \ldots, X_k)$ within given bounds using quasi-random sampling (Sobol' sequences) to minimize clumps and gaps in the samples.
2. Run the model for each parameter sample.
3. Calculate the total variance: $V(Y)$
4. Make Monte Carlo estimates of the conditional variances : $V[E(Y|X_i)]$ and $V[E(Y|X_{\sim i})]$
5. Calculate the sensitivity indices.
6. Use bootstrapping to calculate uncertainty estimates for the sensitivity indices.

For the example of [N=3; k=3], we need 15 parameter sets to calculate the conditional variances

Matrix A

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$$

Matrix B

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$$

Sample A and B;
From A and B,
construct a C matrix
for each parameter.

Matrix C_1

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$$

Matrix C_2

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$$

Matrix C_3

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$$

$$f_0 = \frac{1}{n}\sum_{s=1}^{n} Y_s^A \quad V(Y) = \frac{1}{n}\sum_{s=1}^{n}(Y_s^A)^2 - f_0^2$$

Mean & variance of model output Y

$$V[E(Y|x_1)] = \frac{1}{n}\sum_{s=1}^{n} Y_s^A\, Y_s^{C1} - f_0^2$$

Examples of conditional variances for parameter x_1

$$V[E(Y|{\sim}x_1)] = \frac{1}{n}\sum_{s=1}^{n} Y_s^B\, Y_s^{C1} - f_0^2$$

Conditional variances are scalar products.

Bounds for the 8 Parameters Tested

Factors	Definition	Units	Nominal	Standard Deviation	Range Low	Range High
GA0	Initial rate of growth of TFP	Per decade	0.092	0.040	~~-0.148~~ 0.001	~~0.332~~ 0.212
GSIGMA	Rate of decarbonization	Per decade	-0.073	0.020	-0.193	0.047
T2XCO2	Equilibrium temperature sensitivity	°C per CO_2 doubling	3.00	1.11	~~-3.66~~ 1	~~9.66~~ 15
AA2	Damage coefficient (on quadratic term)	Fraction of GWP [/ΔT^2?]	0.0028	0.0013	~~-0.0050~~ -0.002361	0.010639
PBACK	Price of backstop technology	Thousand $ per ton of carbon	1.170	0.468	~~-1.638~~ 0.001	3.978
POPASYM	Asymptotic global population	Millions	8600	1892	~~-2752~~ 750	19952
b12	Transfer coefficient in carbon cycle	Per decade	0.189288	0.017	0.087288	0.291288
FOSSLIM	Total resources of fossil fuels	Gt Carbon	6000	1200	~~-1200~~ 1200	13200

The last two columns show the bounds used in the Sobol' sensitivity analysis (bounds adjusted for GAMS optimization feasibility)