Need for Caution in Interpreting Extreme Weather Statistics

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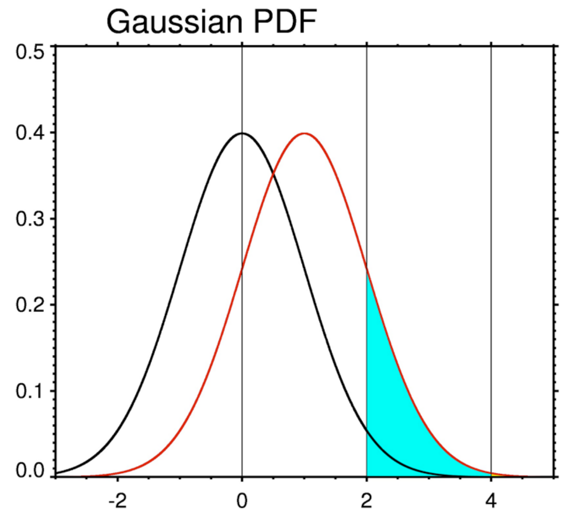
Main Point

The PDFs of daily atmospheric anomalies are not Gaussian. They are generally skewed and heavy tailed. This has enormous implications for the statistics of extreme weather.

Non-Gaussianity has enormous implications for the probabilities of extreme values, and for our ability to estimate their changes using limited samples

*Consider Gaussian vs non-Gaussian PDFs, **both p(0,1)**, and shifted by 1 sigma*

Gaussian PDFs

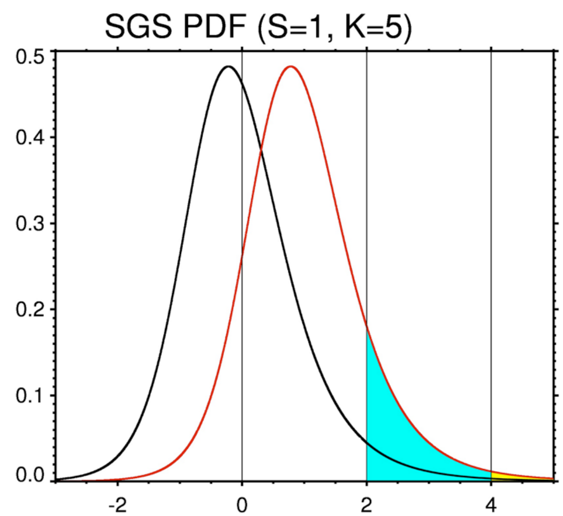


$P(x \geq 2) = 2.3\%$
and increases by
a factor of 7

$P(x \geq 4) = 0.003\%$
and increases by
a factor of 43

Non-Gaussian PDFs

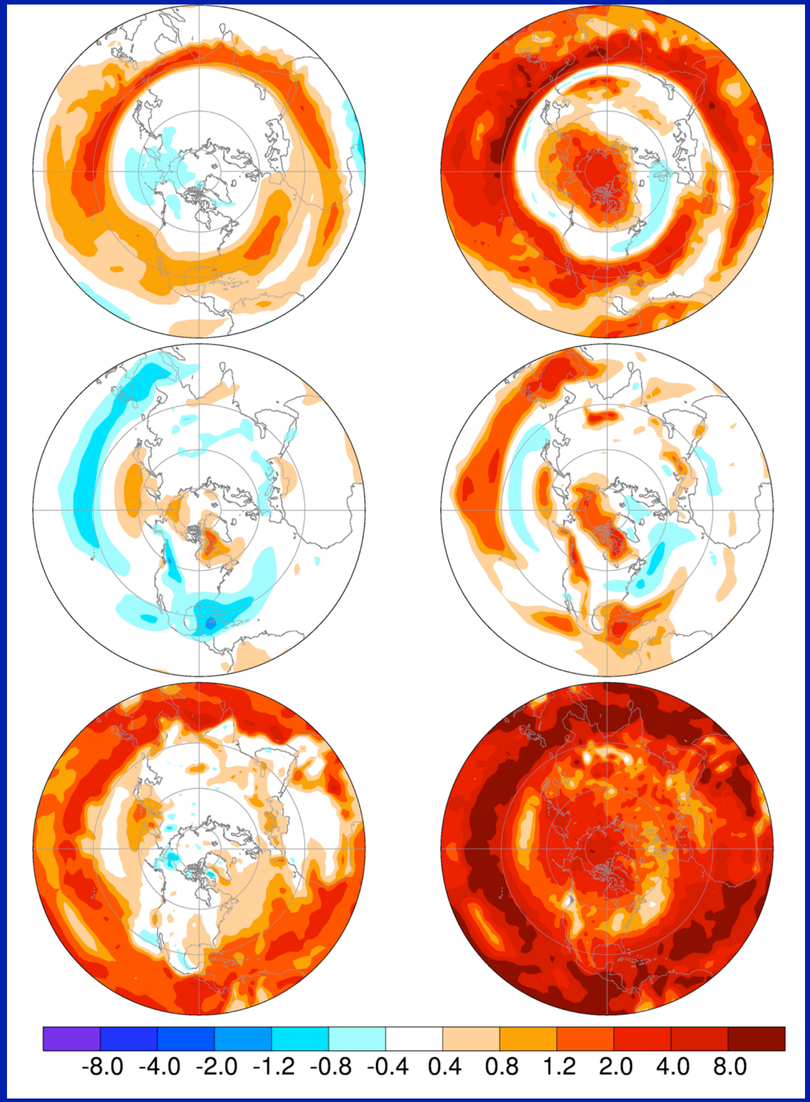
skewed and heavy-tailed with
Skewness $S = 1$
Kurtosis $K = 5$



$P(x \geq 2) = 3.4\%$
and increases by
only a factor of 4

$P(x \geq 4) = 0.34\%$
and increases by
only factor of 3

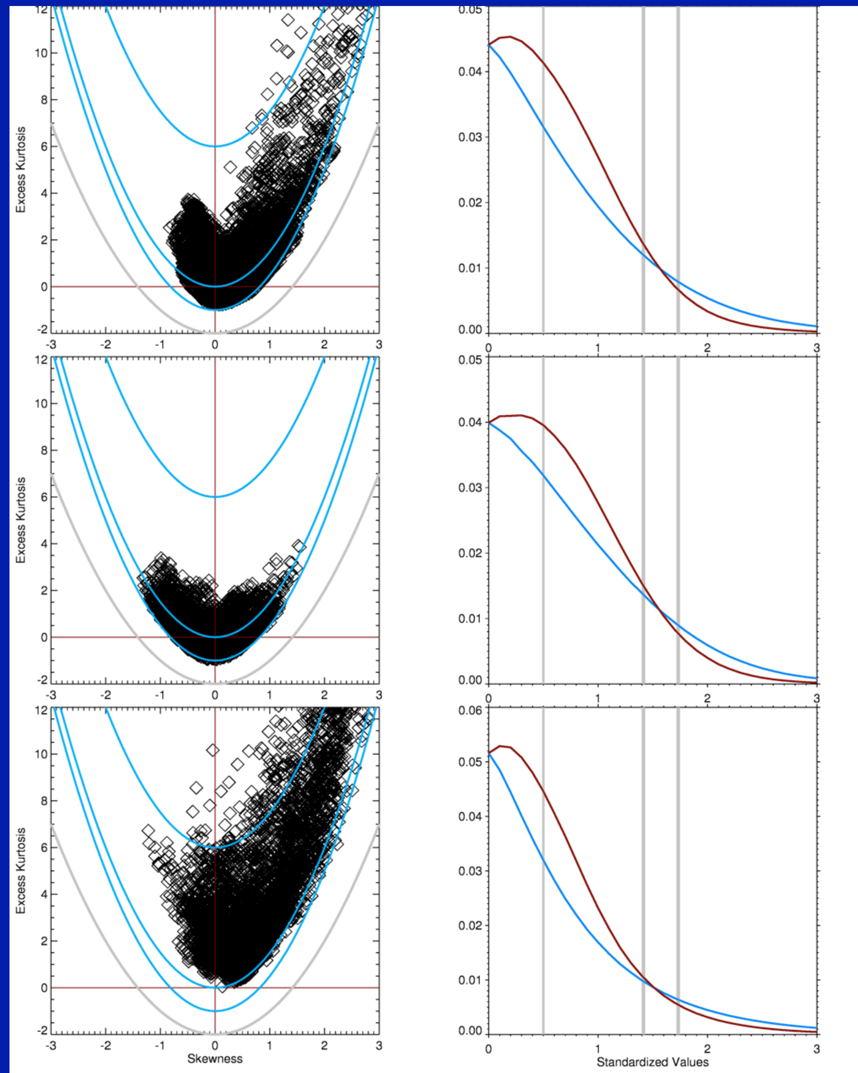
Skewness $S = \langle x^3 \rangle / \sigma^3$ **and Kurtosis** $K = \langle x^4 \rangle / \sigma^4 - 3$ **of daily anomalies in winter computed over 137 winters (1871-2007) in the 20CR dataset (Compo et al 2011)**



Some distinctive features of the non-Gaussianity of standardized daily anomalies at all N.H. grid points

computed using 137 winters (1871-2007) of 20CR data

K vs S **Average Histograms**



250 mb Vorticity

850 mb Air temperature

500 mb Vertical Velocity

Note the parabolic inequality $K \geq 3/2\ S^2$

Note that the crossover point where $p(x) = p(-x)$ lies between 1.4σ and 1.7σ

A generic "Stochastically Generated Skewed" (SGS) probability density function (PDF) suitable for describing non-Gaussian climate variability *(Sardeshmukh and Sura J. Clim 2009)*

$$p(x) = \frac{1}{\mathcal{N}}\left[(Ex+g)^2+b^2\right]^{-\left(1+\frac{\lambda}{E^2}\right)} \exp\left[-\frac{2\lambda g}{E^2 b}\ \arctan\left(\frac{Ex+g}{b}\right)\right]$$

If $E \rightarrow 0$, then $p(x) \rightarrow$ a Gaussian PDF

$\lambda > 0 \qquad b \geq 0$

$g \geq 0$ or $g < 0$

$E \geq 0$

Such a PDF has power-law tails, its moments satisfy $K \geq (3/2)\, S^2$, and $p(x) = p(-x)$ at $\hat{x} \approx \sqrt{3}\ \sigma$

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This PDF arises naturally as the PDF of the simplest 1-D damped linear Markov process that is perturbed by Correlated Additive and Multiplicative white noise ("CAM noise")

$$\frac{dx}{dt} = -\left(\lambda + \frac{1}{2}E^2\right)x + b\,\eta_1 + (Ex+g)\,\eta_2 - \frac{1}{2}Eg$$

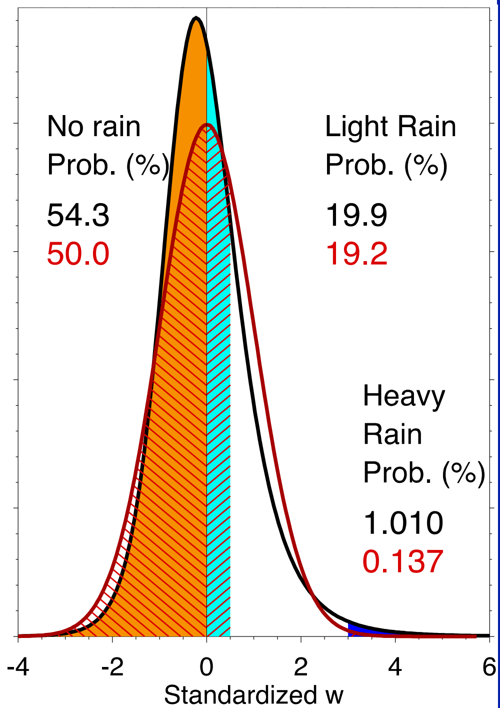
If $E \to 0$, this is just the evolution equation for Gaussian "red noise"

η_1 and η_2 are Gaussian white noises of unit amplitude.

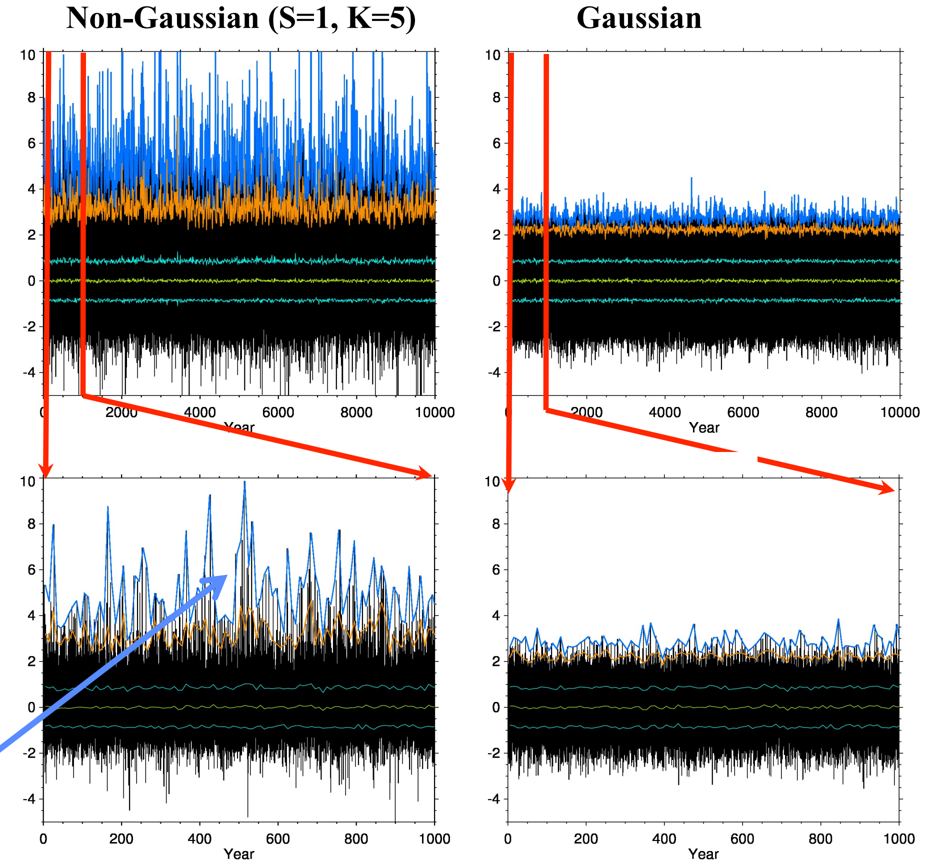
The parameters of this model (and of the PDF) can be estimated using the first four moments of x and its correlation scale. **The model can then be run to generate Monte Carlo estimates of extreme statistics**

Sharply contrasting behavior of extreme *w* anomalies (and by implication, of extreme precipitation anomalies) obtained in 10^8-day runs (equivalent to 10^6 100-day winters) of the Gaussian and non-Gaussian Markov models

Gaussian (red) and non-Gaussian (black, *S*=1, *K*=5) PDFs with same mean and variance



Note how one can obtain spurious 100-yr trends of decadal extremes in the non-Gaussian case *even in this statistically stationary world*.

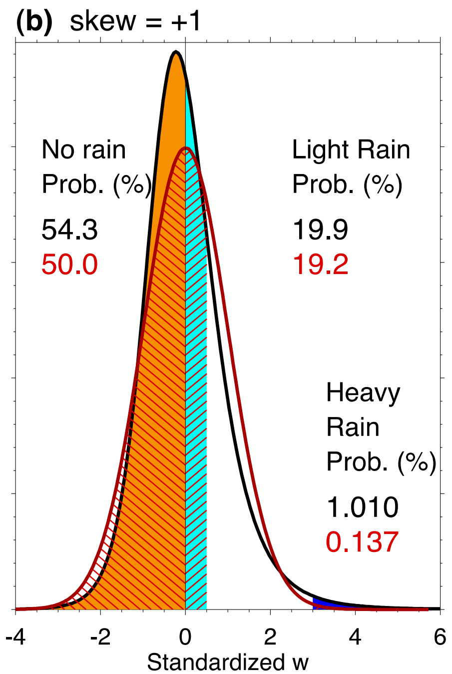


Blue curves: Time series of decadal maxima (i.e the largest daily anomaly in each decade = 1000 days = 10 100-day winters)

Orange curves: Time series of 99.5th decadal percentile (i.e. the 5th largest daily anomaly in each decade)

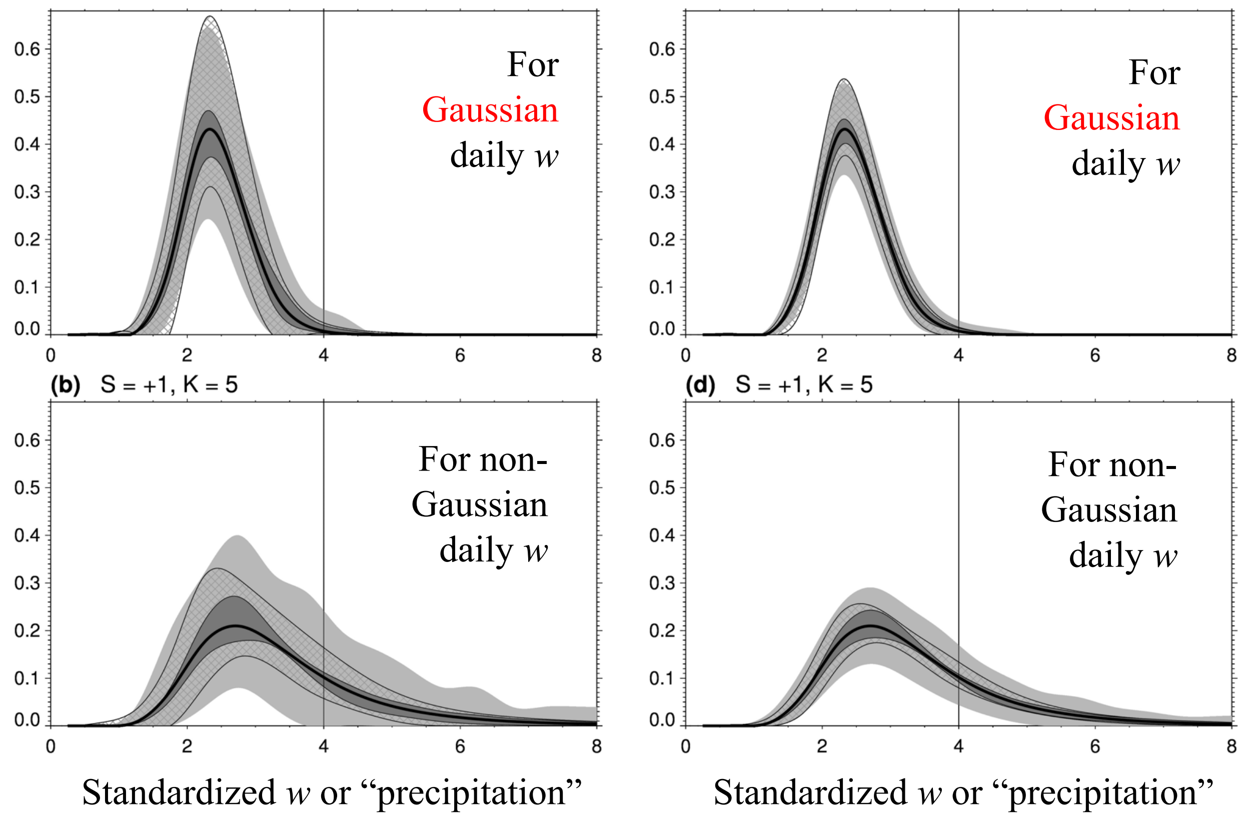
The PDFs of winter maxima are VERY DIFFERENT if the PDFs of the daily values are Gaussian or “SGS”. They are also more accurately estimated by fitting SGS distributions to *all* daily values than by fitting GEV distributions to just maximum values

PDFs of daily *w* :
Gaussian
and non-Gaussian
(S=1, K=5)



PDFs of daily winter *maxima* (Extreme Value PDFs) estimated using

25-winter records **100-winter records**



Black Curves: Extreme Value PDFs of winter *w* (or “precipitation”) maxima estimated from 10^6 model winters, when the PDF of daily *w* is Gaussian or non-Gaussian

Shaded bands: 95% intervals of Extreme Value PDFs estimated using 25- or 100- winter records

Outer bands: 95% intervals of raw histogram-based estimates using the 25 or 100 winter maxima

Inner grey bands: 95% intervals of GEV PDFs fitted to the 25 or 100 winter maxima

Darkest grey bands: 95% intervals of Extreme Value PDFs derived from SGS distributions fitted to all daily values in the 25 or 100 winters

Models differ in their representation of observed non-Gaussian behavior, and do not "converge" simply by increasing resolution.

Below are some results from ECMWF (Wedi et al, 2010)

Higher resolution influence (250hPa vorticity)

Wedi et al 2010

Variance:

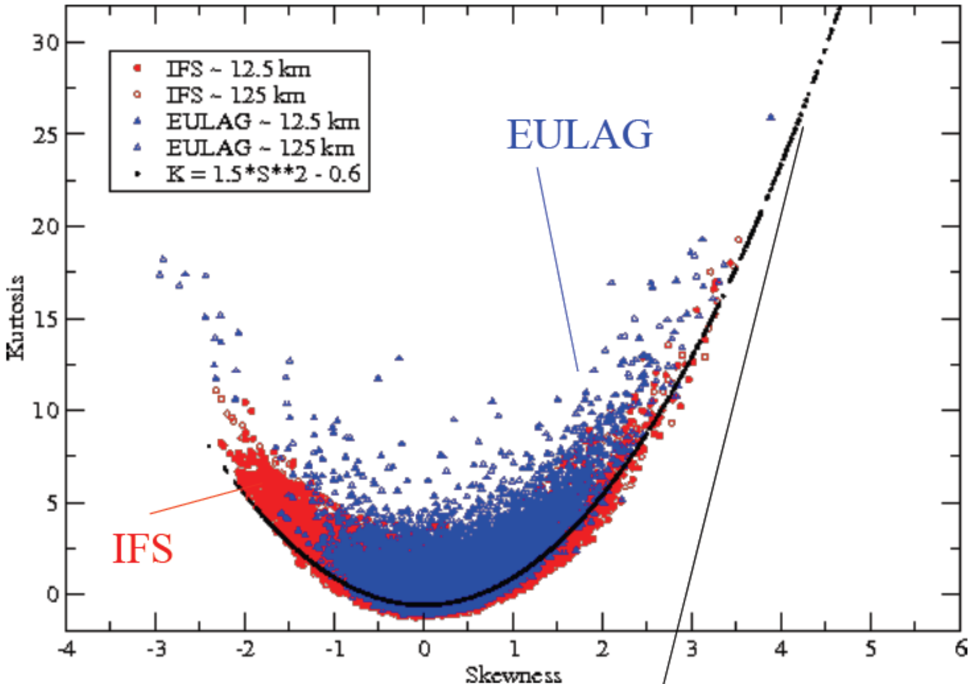
$$\sigma^2 = \overline{(x-\overline{x})^2}$$

Skewness:

$$S = \frac{\overline{(x-\overline{x})^3}}{\sigma^3}$$

Kurtosis:

$$K = \frac{\overline{(x-\overline{x})^4}}{\sigma^4} - 3$$

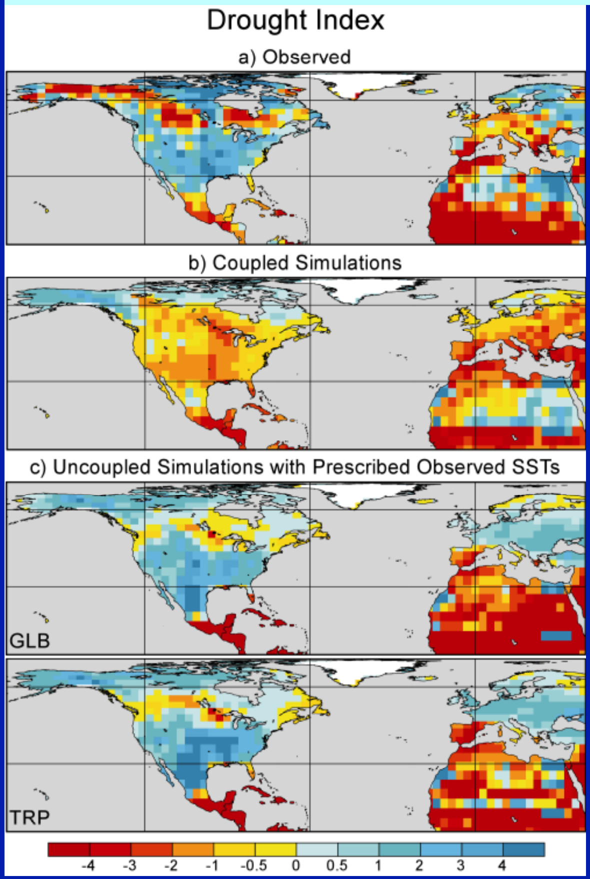


Predicted from linear stochastic models forced with Correlated Additive and Multiplicative ("CAM") noise (Sardeshmukh and Sura, 2009)

<u>Principal Conclusions from the Shin and Sardeshmukh study (Climate Dynamics 2011)</u> :

1. **In order to get regional-scale climate changes right, it is important to get the tropical SSTs right, even in a radiatively warming world.**
2. **Cimate models are not getting the tropical SSTs right.**

Trend of annual Palmer Drought Severity Index (PDSI) over 1951-1999



Observed drought trends (blue shading indicates a 50-yr trend toward *reduced* drought, red shading a tendency toward increased drought)

Simulated trends in **COUPLED** atmosphere-ocean IPCC climate models with prescribed observed radiative forcing changes

Simulated trends in **UNCOUPLED** atmospheric models with *observed* ocean temperature changes prescribed globally, *but with no explicitly specified radiative forcing changes*

Simulated trends in **UNCOUPLED** atmospheric models with observed ocean temperature changes prescribed <u>only in the Tropics </u>(30N-30S), *but with no explicitly specified radiative forcing changes*

Summary

1. **The PDFs of daily anomalies are significantly skewed and heavy-tailed. This fact has enormous implications both for the probabilities of extremes and for estimating changes in those probabilities.** Direct estimations from raw histograms or GEV distributions become even more prone to sampling errors than in the Gaussian case.

2. We have demonstrated the relevance of "stochastically generated skewed" (SGS) distributions for describing daily atmospheric variability, that arise from simple extensions of a linear Markov "red noise" process.

3. We have shown that extreme-value distributions can be estimated more accurately from limited-length records using such a Markov model than through direct GEV approaches.

4. To accurately represent extreme weather statistics and their changes, it is necessary for climate models to accurately represent the first four moments of daily variability. The good news is that for many purposes this may also be sufficient. The bad news is that currently they do not adequately capture the changes of even the first moment (the mean), primarily because of misrepresenting tropical SSTs.

5. **Increasing model resolution may not be a panacea, unless it also improves the representation of tropical SSTs and the first four moments of daily atmospheric variability.**

Trend of annual-mean Tropical Ocean Temperatures over 1951-1999

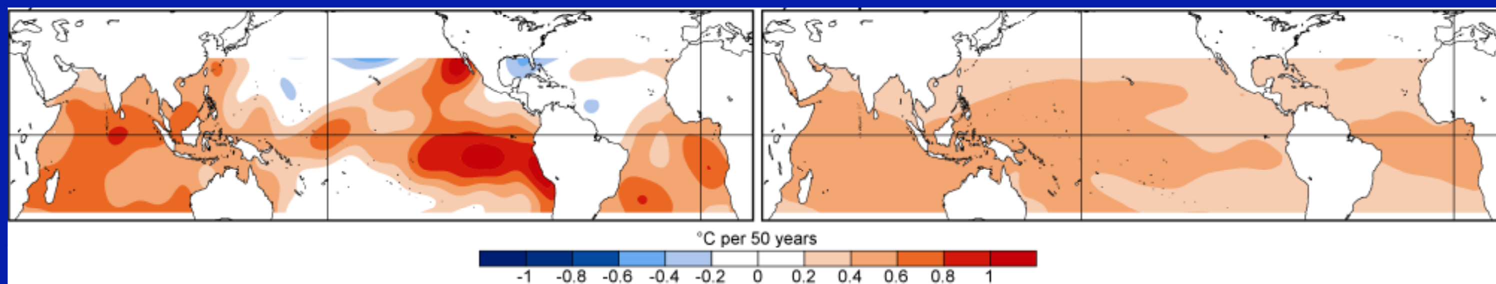
The lower right panel shows that IPCC/AR4 models did not capture the spatial variation of the observed trend field

OBSERVED TREND

(average of 3 datasets)

SIMULATED TREND

(average of 76 coupled IPCC/AR4 simulations)



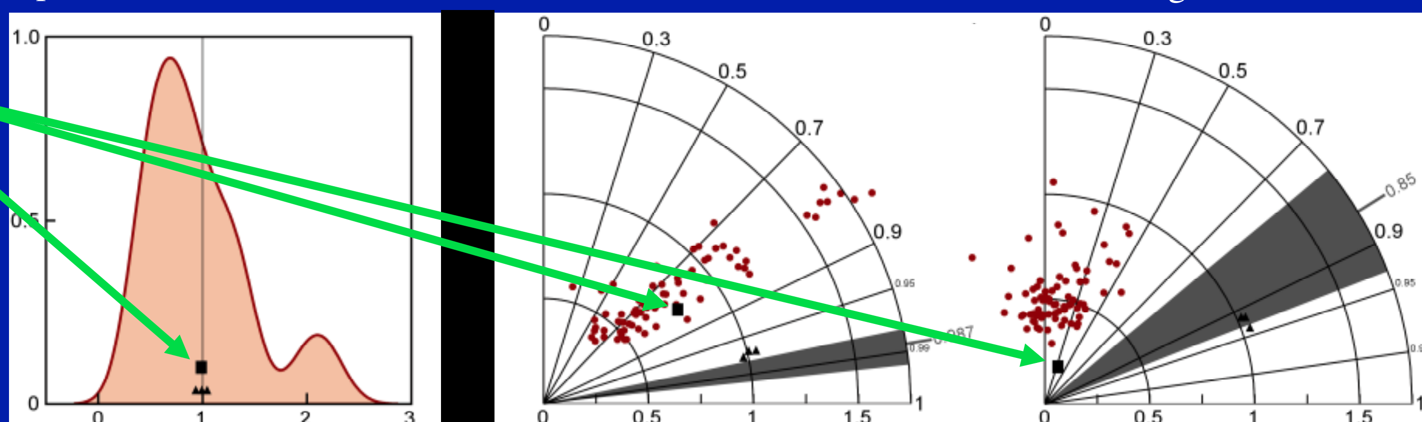
Fidelity of all 76 simulated ocean temperature trend fields

p.d.f. of area-mean trends

Including area-mean trends

Excluding area-mean trends

Multi-model Ensemble Mean trend



A 50-yr record is not long enough to distinguish, using GEV methods, a change in precipitation extremes (red curve) associated with a 15% change in mean precipitation

Winter extremes of Gaussian daily *w*

Winter extremes of SGS daily *w*