A Gravimetric Geoid Model for the United States: The Development and Evaluation of USGG2009

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Introduction

A new gravimetric geoid model, USGG2009, is developed for the United States territory including the Continental US (CONUS), Hawaii, Guam, North Mariana Island, American Samoa, Puerto Rico and the US Virgin Island. USGG2009 is based on a 1′x1′ gravity grid derived from the NGS surface gravity database and the DNSC08 altimetry-derived anomalies, the SRTM-DTED1 3" Digital Elevation Model (DEM) for its topographic reductions, and the global geopotential model EGM08 (Pavlis et al., 2008) as a reference model.

The National Geodetic Survey (NGS) has been developing geoid models for the United States for nearly two decades starting with GEOID90 (Milbert, 1991; Smith and Milbert, 1999; Smith and Roman, 2001; Roman et al 2004). Past NGS geoid models were computed by combining the Faye anomaly (terrain-corrected surface gravity anomaly) with global geopotential models in a remove-restore fashion. The highest degree of past geopotential models used by NGS never exceeded 360. The satellite-derived long-wavelength content of those models was far from perfect. The satellite mission GRACE (Tapley et al, 2005) changed all this, giving rise to modern geopotential models such as EGM08 and the European EIGEN-series. These models were developed by combining very accurate low-degree GRACE-based geopotential models with improved surface and altimetry-derived gravity anomalies, and high-resolution DEMs. EGM08 is based on 5' and 15′ gravity block means and extends to degree 2160, while the European models extend to degree 360. Though these modern global geopotential models are still limited by their commission and omission errors, they are far more superior to previous models such as EGM96 (Lemoine et al., 1998) in accuracy and resolution.

Because of the high accuracy and resolution of EGM08, USGG2009 had to be computed slightly differently than previous US gravimetric geoid models. These differences can be briefly summarized as: 1) the USGG2009 long wavelength content is based on a contribution from the GRACE mission rather than the long wavelength content of the surface and altimetry-derived gravity data. This is achieved by using the method of kernel truncation. 2) We used the harmonic continuation method for geoid computation, since it is much easier to compute, and 3) all topographic reductions were done using 3" DEMs.

This document provides an overview of the USGG2009 gravimetric geoid model. It briefly describes the computation methods, data used and some comparisons and tests, with emphasis on the differences from previous NGS gravimetric geoid models.

Development in local gravimetric geoid computations

Theoretically, a precise geoid can be computed once global gravity and elevation data are available. In practice, however, gravity data are usually available only within the boundaries of the country or a limited region around it. Therefore, global geopotential models have been used to account for the necessary contribution of the rest of the globe, where no gravity data is available to us.

Global geopotential models take advantage of the accurate long wavelength portion of the gravity field provided by satellite technology. In the 1990s, the satellite-derived long wavelengths of the gravity field were estimated from analyses of observations of dozens of satellites. Although this satellite-derived portion of the gravity field was fairly accurate, it suffered from inconsistencies and imperfections that are inevitable when many different systems of hardware, models, and software are combined. Consequently, these models were usually used to substitute for the contribution of the parts of the globe where gravity data was not available, but were not allowed to play any major role in areas where gravity data was available.

This situation changed with the launch of the satellite mission GRACE, which by all accounts captures the long wavelength portion of the gravity field almost perfectly (Tapley et al., 2005). Therefore, in the computation of USGG2009 we allowed the GRACE contribution, represented by the low degrees of the geopotential model EGM08, to overwrite the long-wavelength portion of the surface and altimetry-derived gravity data in and around the US.

USGG2009 is the first NGS geoid not based on the theory of Helmert's 2nd condensation, which condenses the topography onto the geoid (resulting in the Helmert gravity anomaly), and downward continues it from the surface to the geoid. The Stokes integral is then evaluated, and finally, the indirect effect (potential change due to the condensation) is added. The Faye gravity anomaly was used in the past as a simplification of the Helmert anomaly, on the geoid. The Faye anomaly is easier to compute but involves several approximations and assumptions that may not meet today's geoid accuracy requirement.

Geopotential models, including EGM08, are developed using the theory of Harmonic Continuation. Unlike Helmert's 2nd condensation, this method does not condense the topography onto the geoid. Rather, it downward continues the surface gravity data, through the topographic masses, to the ellipsoid (e.g., Lemoine et al., 1998). Thus, before the Faye gravity anomalies can be combined with a geopotential model for computing a gravimetric geoid, one must insure that both are consistent in handling the topography.

The effect of the "topographic reduction inconsistency" was ignored in past geoid models, since other errors, e.g., long wavelength error in the geoid, were much larger. This situation changed dramatically after the release of EGM08. The accuracy of this model has been improved to such a level that the error due to the topographic reduction inconsistency becomes significant, and has to be corrected. There are two ways to do that: (a) by modifying the geopotential model to become compatible with Helmert's 2nd condensation. This can be done by a harmonic analysis of the topography, and using the resulting harmonic coefficients to compute and add the coefficients of the indirect topographic effect to the coefficients of the geopotential model; (b) by abandoning Helmert’s 2nd condensation all together, and adopting the harmonic continuation method for geoid computation instead (Wang 1990; Sjöberg, 2001, 2003).

Mathematically, the two methods are equivalent for a band-limited gravity field. The harmonic continuation method has its advantages in the simplicity and computational efficiency. It does not involve any topographic correction to the gravity data, which is very time consuming for ultra high resolution DEMs. Since the residual gravity anomalies with respect to the full EGM08 are of the order of a few mGals at most, the effect of the gravity downward continuation on the residual geoid is a fraction of a centimeter. Thus the downward continuation can be safely done with approximate methods. Since the downward continued gravity data are treated in the same way the global geopotential models are developed, these models can be used in a simple remove-restore fashion without having to modify them. For these reasons, the harmonic continuation method becomes a natural choice for geoid computations. We tested both methods in the process of computing USGG2009 and obtained very similar results, with the harmonic continuation method being very slightly better.

Data used

There are over 2 million terrestrial and ship-borne gravity data points in the NGS database that are a result of decades of data collection efforts by various sources. Another 88,000 points in CONUS were obtained recently from the NGA to add to this total. The old Canadian data, used in USGG2003, were replaced by a new version obtained from NRCAN (Veronneau, 2007).

Satellite altimetry-derived gravity anomalies have been improved over the years, culminating in the recent DSNC08 gravity anomaly model over the oceans (Anderson, 2008). The most noticeable improvement is the altimeter re-tracking for data near shore, which directly impacts the geoid in the states bordering the oceans. The altimetry-derived gravity data fill in the ocean areas surrounding the terrestrial data and provide continuity across the sea-land boundary. Altimetry-derived gravity data is very important for geoid models over islands, such as Puerto Rico, the Virgin Islands, American Samoa, Guam and the Northern Mariana Islands. Although altimetry re-tracking near shore was improved, it turned out that truncating altimetry-derived gravity in a 50 km strip near-shore gave a slightly better result.

Two other altimetry-derived models, GSFC00.1 (Wang 2001) and SIO/NOAA 2008 (Sandwell/Smith), were also tested and their performance compared to DNSC08. The DSNC08 and SIO/NOAA data performed very similarly. Both were significantly better than GSFC00.1 in the North East and very slightly better in the Gulf of Mexico. GSFC00.1 performed significantly better in the South East to the Mid Atlantic. All models performed similarly on the West Coast.

The Shuttle Radar Topography Mission (SRTM) DTED1 elevations were obtained from NGA over the window {10°≤lat≤60° ; 190°≤long≤308°}, in the form of several thousands of 1°x1° grids. The cell sizes of these grids are 3″x3″ below latitude 50° and 3″x6″ above it. Once these tiles were assembled (in two large tiles, one above and the other below latitude 50°), it turned out that they contain more than 12 million voids, several of which were larger than the size of a large county in the US (we would like to thank NGA for not having us start our SRTM analysis from the Radar waveforms).

To fill the SRTM voids and verify its quality (in the wake of finding millions of gaps), we had to assemble another continental-scale DEM using the National Elevation Database (NED) in the US, the National Canadian and the National Mexican DEMs. These DEMs come in thousands of tiles of different sizes and cell sizes, some in grid and others in raster (cell center) formats. All tiles were re-sampled on 3″x3″ cell grids and "mosaiced" to form the "National DEM", independent of SRTM. Voids above latitude 50° in SRTM-DTED1 were then filled and tiles re-sampled in 3″x3″ cells in the window {50°≤lat≤60° ; 190°≤long≤ 308°}. Voids below latitude 50° were filled and the resulting {10°≤lat≤50° ; 190°≤long≤308°} window "mosaiced" with the previous one to create a North/Central American 3″ SRTM DEM. Once the quality of this DEM was verified by comparison to the National DEM, it was used to compute all topographic reductions necessary for creating USGG2009. These include the terrain correction, which goes into the Helmert 2nd condensation geoid, and the RTM (Forsberg, 1984), which goes into the harmonic continuation geoid (see Table 1).

EGM08 contains wavelengths of the gravity field up to a resolution of 5' and it is used for data editing. The reference gravity to degree 2160 is computed at the location of the observation on the Earth's surface (latitude, longitude and height), and subtracted from the observed surface gravity anomalies to produce residuals with respect to EGM08. The RMS value of these residuals is 5.3 mGal for all data and 16.3 mGal for land-only data, with a range of ±300 mGal. The Beta USGG2009 model was based on a 3-sigma (16 mGal) outlier filter, which resulted in the rejection of about hundred thousand point-gravity values. The RMS value of the accepted residuals was 2.9 mGal. Once the final editing was completed, the EGM08 model was used in a remove-compute-restore manner adopting the spherical harmonic values through degree and order 360. Subsequent analysis was completed to determine the optimal combination of filtering and level of acceptance for the EGM08 model. The rejection threshold was then reduced to 6 mGal resulting in the rejection of several hundred thousand data points, while the degree of the adopted long wavelength portion from EGM08 was reduced to degree 120. The net effect was to reject more points that disagreed with EGM2008 but to retain more of the signal in the unrejected points.

The resolution of EGM2008 gravity anomalies is 5' at best. Thus, before point gravity data can be accurately compared to EGM08, all frequencies higher than 2160 (or 5') must be modeled and removed. This is done by computing the 3″-to-5' RTM gravity effects and removing them from the land gravity residuals (with respect to EGM08). The RMS difference of the resulting (land-only) residuals decreased to 5.1 mGal, from 16.3 mGal before applying the RTM reduction. Now, an outlier filter of 3-sigma kicked out only 1400 points.

In future geoid computations, new airborne data, collected within the project GRAV-D, will replace EGM08 in playing the "truth" role in the data editing. In addition, GRAV-D, when complete, will provide the long wavelength content of future gravimetric geoid models. The use of EGM08 is complicated by the fact that it is based on the same data we evaluated here. While this is circuitous, there is a lack of options for otherwise filtering the gravity data.

For Alaska, the original scheme of a 3-sigma filter and use of EGM08 through degree and order 360 was retained. No improvement resulted from variations on the original scheme. This is likely due to the very poor gravity and topographic data quality and coverage. In particular, great efforts were made to develop a single digital elevation model for the state. This required melding several different sources. The SRTM only reached to about 60 degrees North latitude. Hence, the majority of Alaska was not covered by it and the resulting geoid model suffered for that lack. There is an effort underway to provide a consistent DEM over the state but that is still in the nascent stages. For now, Alaska's USGG2009 model is more dependent on the underlying EGM2008 model for lack of additional information.

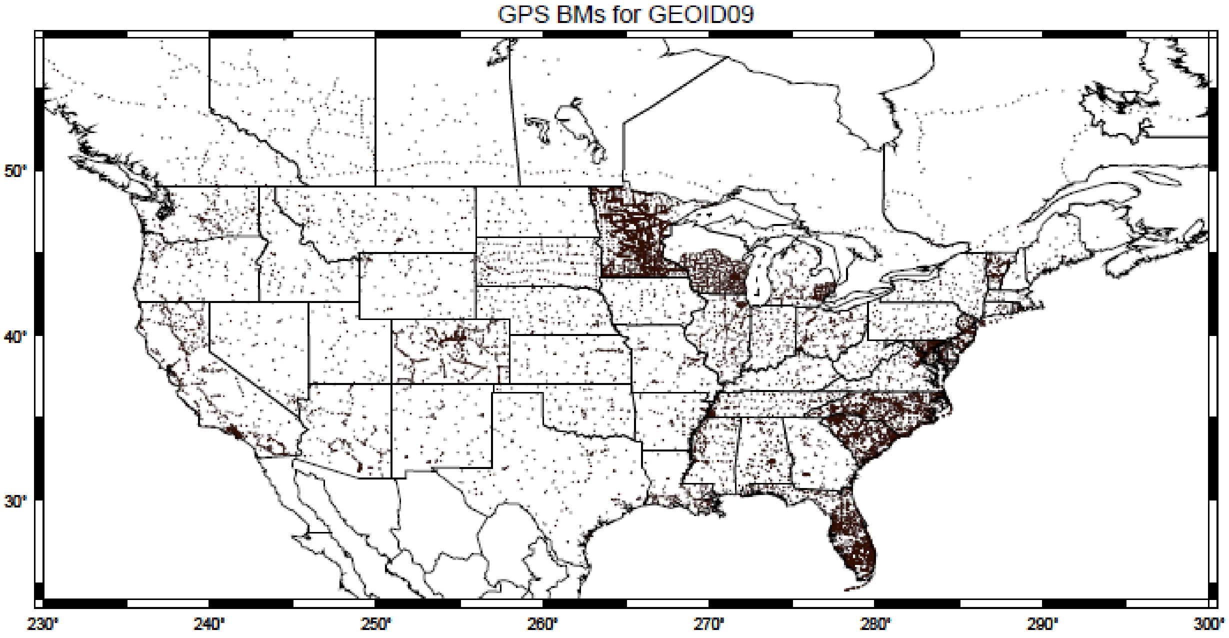

Figure 1: The 20446 GPS Bench Marks 2009 (GPSBMs09)

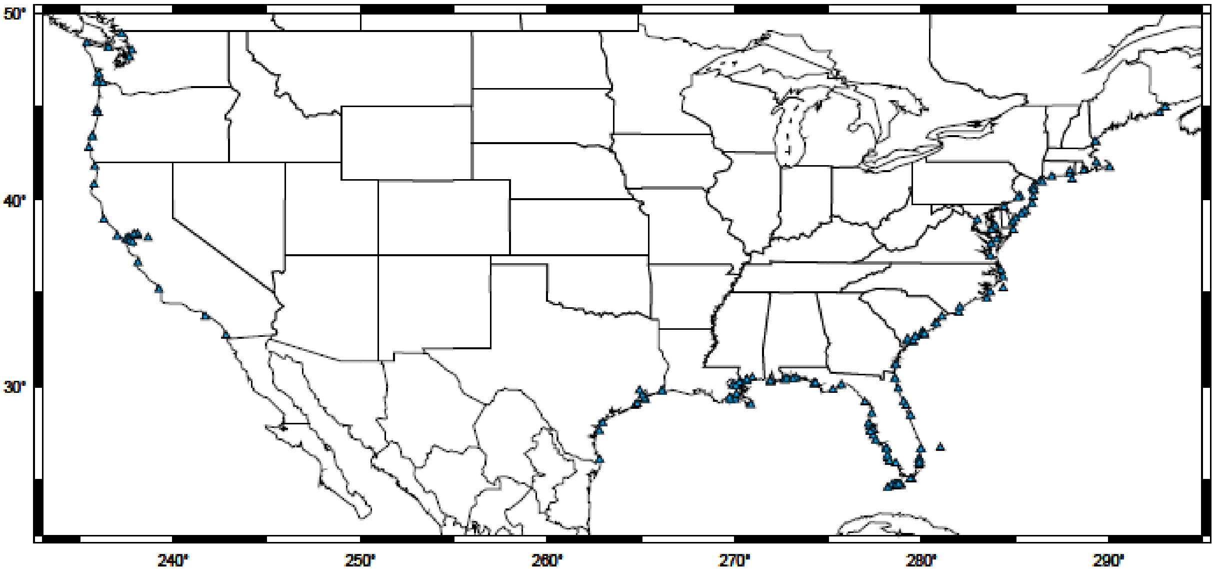

Figure 2: GPS-occupied Tide gages along the US coastlines

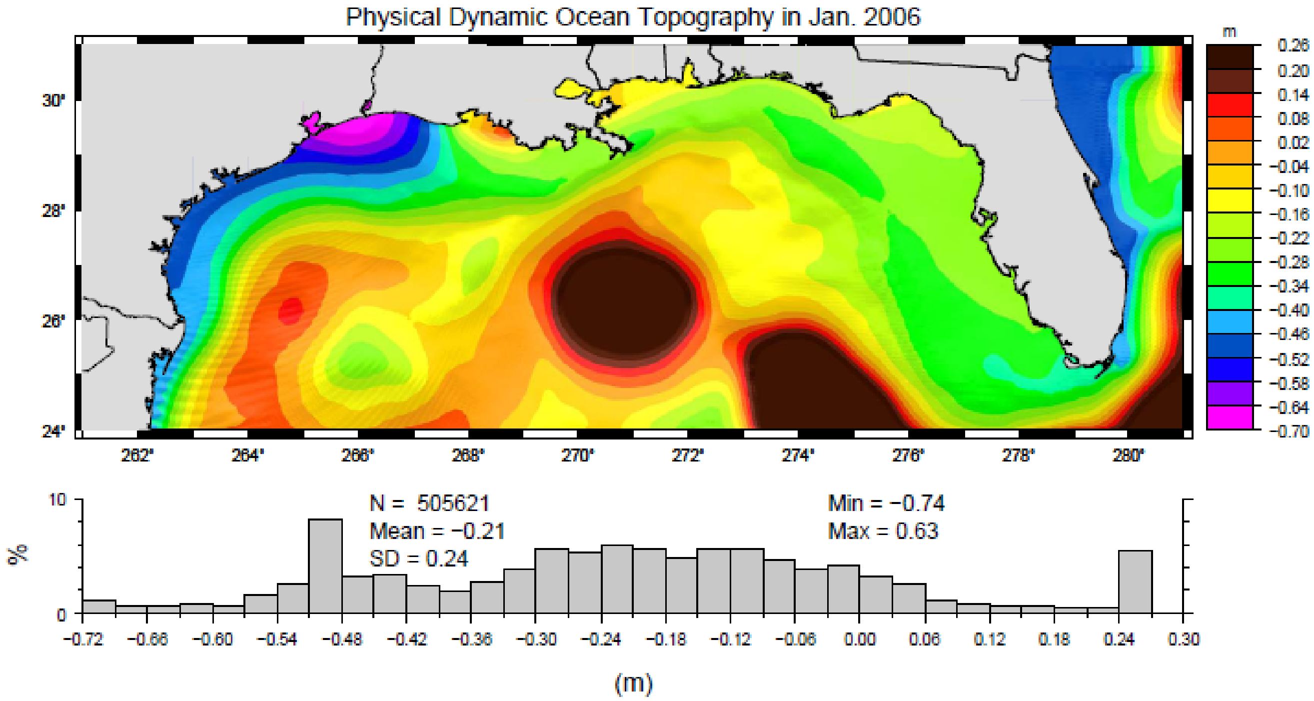

Figure 3: Dynamic Ocean Topography (DOT) in the Gulf of Mexico

Three other data sets served for testing the geoid models: 1) GPS Bench Marks (GPSBMs09), which include 20,446 GPS-occupied Bench Marks in the US and Canada (Figure 1); 2) More than 200 GPS-occupied tide gages along the US coast (Figure 2); 3) A dynamic ocean topography model for the Gulf of Mexico. This was computed, at NOAA, based on physical oceanographic principles and independent of any geoid (Figure 3) (Patchen, 2006).

Tests and Comparisons

Comparison with USGG2003

The changes from USGG2003 to USGG2009 are significant (Figure 4). The long wavelength changes are due to the differences between the EGM96 and EGM08 (Figure 5) and improved altimetry-derived gravity anomalies. In addition, some differences are due to the erroneous long wavelength content of the surface and possibly altimetry-derived gravity data, which was allowed to influence USGG2003 model but not USGG2009.

Over the oceans, there are clear features that are associated with the mean dynamic ocean topography of the Gulf Stream and other ocean currents. Geoid improvement in ocean areas is mostly due to better satellite gravity model determined from the GRACE mission and incorporated into EGM08. Along the shorelines, altimetry data re-tracking (Anderson, 2008) improves the gravity recovery and indirectly improves the geoid for the shoreline statistics. In previous models, altimetry data near the shorelines were edited out because of their poor quality. Typical distances offshore for exclusion were about 20-100 km corresponding to the 500-fathom curve where ocean models become more rigorous. Now, gravity data derived from re-tracked altimetry can approach the shoreline. Editing out altimetry-derived gravity near shore does improve the geoid statistics, but only very slightly. Over land areas, the long wavelength changes are due to the same thing plus the inconsistency caused between the methods of Helmert 2nd condensation and harmonic continuation. In addition, some differences are caused by the fact that altimetry-derived gravity was edited out near shore in USGG2003 but not in USGG2009. The short wavelength changes are the contributions of the edited surface gravity data, the improved topographic data and differences between residual gravity gridding methods.

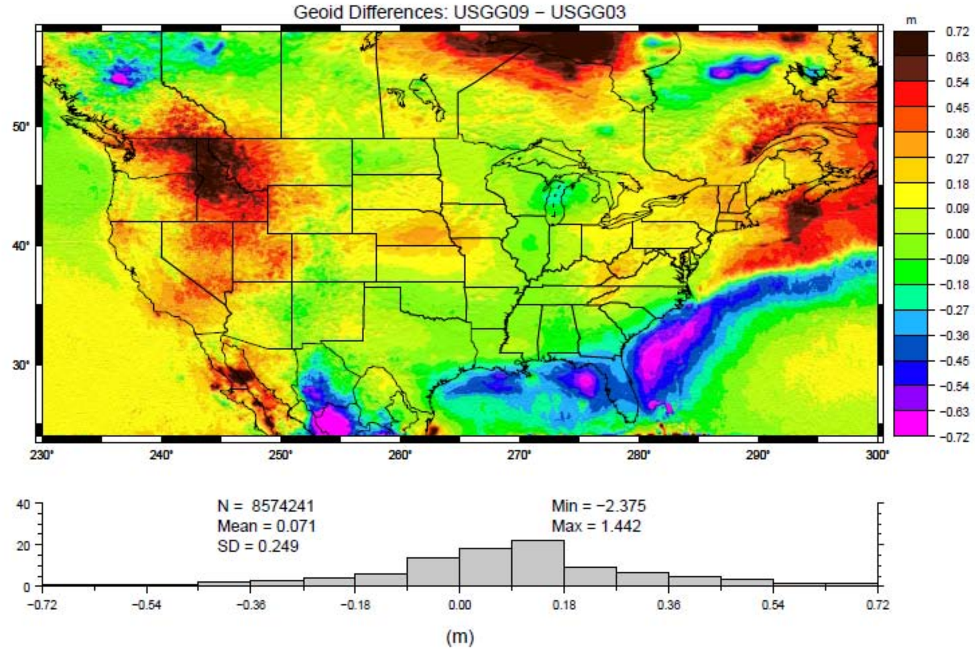

FIGURE 4: Difference between USGG2009 and USGG2003

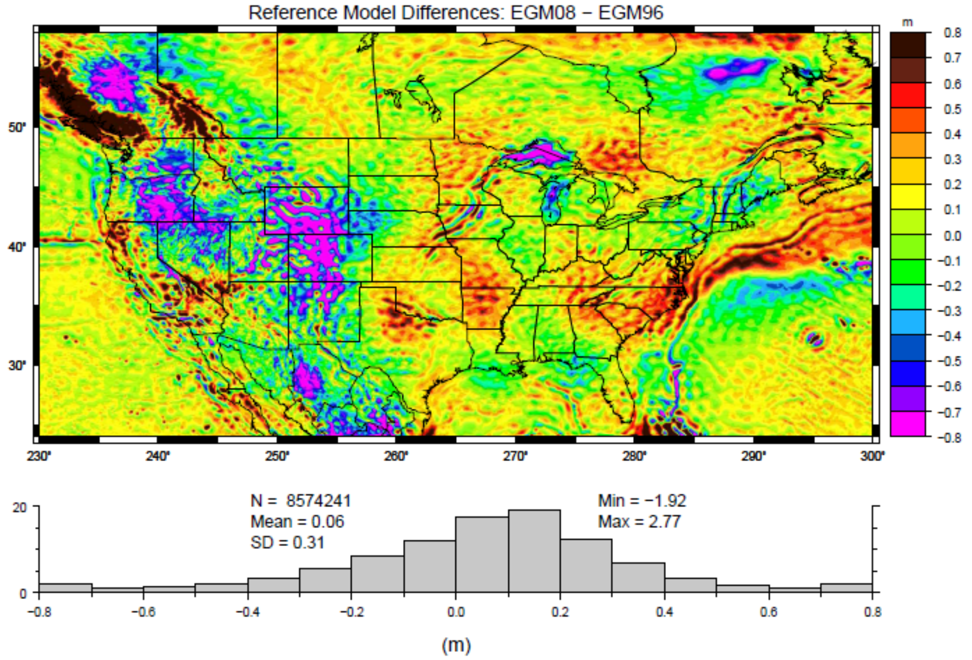

FIGURE 5: Difference between EGM08 and EGM96 geoid

Comparison of USGG2009 and USGG2003 with GPS/Bench Mark-derived geoid heights

USGG2009 and USGG2003 are compared with 18,398 US GPS Bench Marks (GPSBMs), after the NSRS2007 adjustment (Table 1). The comparison is done for each state separately, to minimize the effect of the long wavelength errors of NAVD88. The overall improvement, estimated by ($\sqrt{SD_{2003}^2 - SD_{2009}^2}\Big/SD_{2003}$), in the geoid precision is 66%, as can be seen in the last row of Table 1. USGG2003 performs as well as or slightly better in many areas with flat topography and good gravity data, including in the Mid-Atlantic, several small North Eastern and some Gulf states. The drastic improvements of USGG2009 over USGG2003 occur in the North West (WA, OR, MT, ID) and some Appalachian states (KY, WV, TN), where the improvement exceeds 80%. Large improvements of about 80% also occur in flat states of the upper Midwest and Western planes (MI, WI, IA, NE, ND). The improvement in California and some southern Rocky states (AZ, NM, NV) range from 50 to 75%. Texas improved by 67%. Even South East and North East states (FL, SC, NC, NY, MA, NH, VT and ME) improved significantly.

The long wavelength portion of USGG2009 is based on the gravity field as seen by the modern satellite mission GRACE, while that of USGG2003 is based on the dubious long wavelength content of the surface and a previous generation altimetry-derived gravity, and on the old-generation satellite-derived long wavelength content of EGM96 (Figure 5). Consequently, the bias of USGG2009 (see the “bias” column in Table 1) implies a South-East/North-West slope, much more regular than that of USGG2003 (Figures 6 and 7). The USGG2003 long wavelength differences in Figure 7 have several local maxima and minima over Washington and Oregon, Idaho, Southern California, South Western Texas, Minnesota and North Dakota, New England and the Appalachian Mountains. None of these local phenomena can be seen in Figure 6, which most likely implies that they are long wavelength errors of USGG2003.

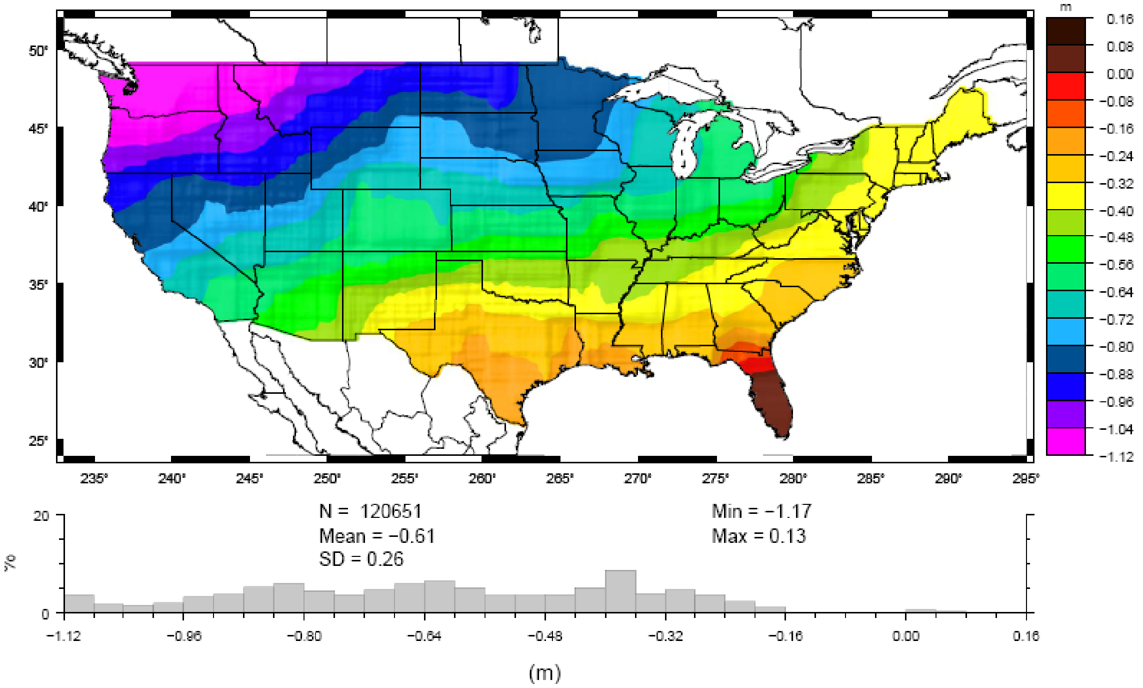

Figure 6: Long wavelength (5°) differences between USGG2009 and the GPSBMs09

Table 1: State by state statistics of the differences: Gravimetric geoid – GPSBM09-derived geoid

		USGG2009		USGG2003		USGG2009, No kernel truncation		USGG2009, 2nd Helmert Cond.		EGM08	
State	N	Bias	SD	Bias	SD	Bias	SD	Bias	SD	Bias	SD
AL	283	-0.2064	0.0501	-0.0499	0.0386	0.1255	0.0513	-0.2035	0.0489	-0.2082	0.0433
AZ	227	0.0152	0.0874	-0.0626	0.0900	-0.3867	0.1018	0.0259	0.0891	0.0154	0.0919
AR	133	-0.1162	0.0341	-0.0613	0.0329	0.1715	0.0567	-0.1146	0.0336	-0.1192	0.0372
CA	738	0.2339	0.1324	0.0178	0.1601	0.0142	0.1792	0.2417	0.1320	0.2330	0.1338
CO	562	0.1060	0.0834	0.0307	0.0747	-0.1774	0.1071	0.1315	0.0817	0.1073	0.0870
CT	20	-0.1422	0.0347	-0.3681	0.0262	-0.4726	0.0344	-0.1479	0.0351	-0.1314	0.0334
DE	35	-0.1787	0.0464	-0.2583	0.0235	-0.2734	0.0242	-0.1835	0.0458	-0.1690	0.0397
DC	16	-0.1175	0.0209	-0.2296	0.0195	-0.2095	0.0197	-0.1208	0.0206	-0.1208	0.0199
FL	2181	-0.5411	0.0834	-0.1586	0.0971	-0.2676	0.1228	-0.5306	0.0824	-0.5405	0.0837
GA	137	-0.2651	0.0640	-0.0618	0.0650	0.0627	0.0721	-0.2587	0.0645	-0.2629	0.0630
ID	97	0.4688	0.0785	-0.1522	0.0910	0.1764	0.1178	0.5037	0.0894	0.4721	0.0791
IL	334	0.1061	0.0909	0.1667	0.0892	0.3632	0.0542	0.1000	0.0881	0.1090	0.0921
IN	119	0.0263	0.0565	0.0025	0.0647	0.1594	0.0660	0.0221	0.0556	0.0272	0.0551
IA	100	0.1893	0.0604	0.0683	0.0857	0.2113	0.0728	0.1821	0.0589	0.1905	0.0596
KS	105	0.0704	0.0577	-0.0490	0.0374	0.0854	0.0496	0.0705	0.0568	0.0724	0.0558
KY	123	-0.0859	0.0377	-0.1432	0.0861	0.0147	0.1540	-0.0868	0.0375	-0.0870	0.0348
LA	217	-0.3550	0.1063	-0.1283	0.0741	-0.0958	0.0994	-0.3470	0.1059	-0.3646	0.1151
ME	65	-0.1442	0.0427	-0.3846	0.0578	-0.6421	0.0834	-0.1549	0.0421	-0.1548	0.0452
MD	511	-0.1258	0.0369	-0.2411	0.0252	-0.2491	0.0442	-0.1303	0.0359	-0.1344	0.0265
MA	35	-0.1626	0.0406	-0.4558	0.0565	-0.5312	0.0462	-0.1707	0.0405	-0.1556	0.0401
MI	410	0.0869	0.0425	0.0500	0.0761	0.0374	0.1046	0.0777	0.0423	0.0889	0.0439
MN	4089	0.3090	0.0378	0.2403	0.0412	0.2625	0.0670	0.2967	0.0371	0.3103	0.0390
MS	243	-0.1508	0.0477	-0.0654	0.0460	0.1607	0.0644	-0.1473	0.0467	-0.1507	0.0465
MO	138	0.0077	0.0738	-0.0697	0.0395	0.2055	0.0638	0.0057	0.0731	0.0066	0.0751
MT	151	0.4694	0.0904	0.1051	0.1773	0.2281	0.1157	0.4972	0.1241	0.4720	0.0893
NE	145	0.1771	0.0467	-0.0407	0.0958	0.0012	0.0691	0.1724	0.0491	0.1764	0.0459
NV	70	0.2471	0.0887	-0.0535	0.1348	-0.1355	0.1164	0.2631	0.1027	0.2521	0.0908
NH	14	-0.1413	0.0181	-0.4453	0.0272	-0.6621	0.0797	-0.1494	0.0175	-0.1394	0.0137
NJ	326	-0.1444	0.0276	-0.2604	0.0279	-0.3272	0.0611	-0.1490	0.0277	-0.1404	0.0242
NM	107	-0.1025	0.0913	-0.0806	0.1192	-0.4232	0.1222	-0.0933	0.0948	-0.1038	0.0943
NY	185	-0.1036	0.0644	-0.3047	0.0975	-0.4903	0.1575	-0.1113	0.0647	-0.1035	0.0630
NC	1676	-0.2261	0.0462	-0.1456	0.1113	-0.0960	0.0559	-0.2240	0.0470	-0.2255	0.0468
ND	47	0.4115	0.0326	0.3813	0.0537	0.3736	0.0442	0.3994	0.0344	0.4185	0.0374
OH	297	0.0219	0.0466	-0.0810	0.0475	-0.0212	0.0847	0.0162	0.0472	0.0232	0.0471
OK	79	-0.0887	0.0574	-0.0146	0.0460	0.0848	0.0594	-0.0844	0.0569	-0.0917	0.0541
OR	202	0.5227	0.0806	0.3285	0.1830	0.2946	0.1078	0.5247	0.0809	0.5249	0.0804
PA	96	-0.0804	0.0450	-0.2199	0.0474	-0.3167	0.0817	-0.0829	0.0459	-0.0768	0.0442
RI	29	-0.1472	0.0232	-0.4165	0.0258	-0.4580	0.0266	-0.1531	0.0238	-0.1248	0.0242
SC	1315	-0.2212	0.0572	-0.0732	0.0855	0.0213	0.0373	-0.2184	0.0548	-0.2256	0.0563
SD	242	0.2847	0.0624	0.1643	0.0682	0.2096	0.1087	0.2777	0.0611	0.2866	0.0600
TN	302	-0.1056	0.0313	-0.0757	0.0773	0.1330	0.1394	-0.1058	0.0318	-0.1015	0.0333
TX	218	-0.2573	0.0847	-0.1532	0.1141	-0.2648	0.2314	-0.2474	0.0842	-0.2563	0.0876
UT	55	0.2229	0.0903	-0.0891	0.0919	-0.1738	0.0815	0.2497	0.0931	0.2269	0.0865
VT	317	-0.1407	0.0296	-0.4560	0.0488	-0.7666	0.0450	-0.1406	0.0266	-0.1412	0.0268
VA	434	-0.1412	0.0398	-0.2401	0.0344	-0.2055	0.0505	0.1441	0.0390	-0.1456	0.0376
WA	259	0.6095	0.0833	0.1973	0.1582	0.3766	0.0769	0.6135	0.0906	0.6110	0.0771
WV	55	-0.0594	0.0451	-0.2562	0.0700	-0.2629	0.0449	-0.0620	0.0434	-0.0689	0.0428
WI	758	0.1719	0.0357	0.2161	0.0487	0.2458	0.0417	0.1624	0.0350	0.1726	0.0381
WY	101	0.2700	0.0888	0.0357	0.0897	-0.0659	0.0801	0.2901	0.1029	0.2759	0.0950
Mean	18398	-0.0099	0.0632	-0.0054	0.0838	0.0027	0.0908	-0.0101	0.0635	-0.0098	0.0636

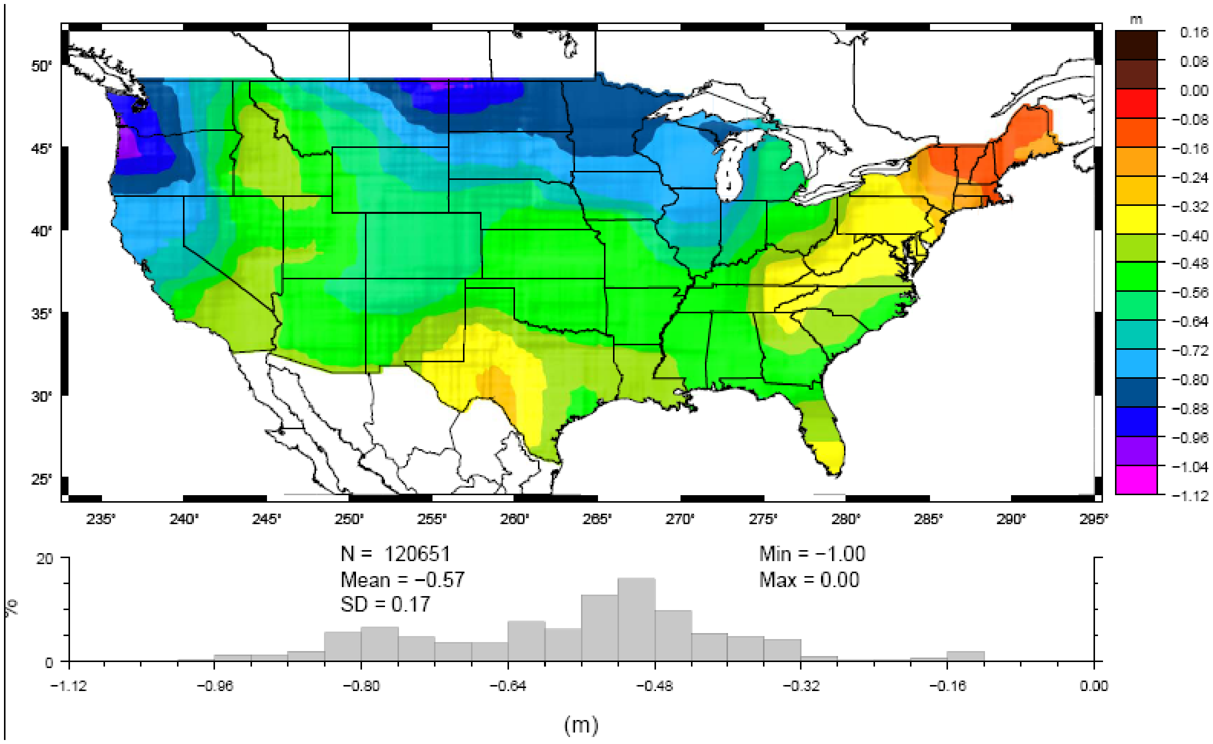

Figure 7: Long wavelength (5°) differences between USGG2003 and the GPSBMs09

Comparison with tide gage-derived geoid heights in the Gulf of Mexico

It has been argued that the GPSBMs may not be an entirely appropriate dataset for geoid testing, since they may be contaminated by motions of the BMs with time and leveling errors. Therefore, we introduce a second test, independent of differential leveling, which utilizes the GPS-occupied tide gages and a Dynamic Ocean Topography (DOT) model in the Gulf of Mexico. The DOT model was computed at NOAA, based on physical oceanographic data (Patchen, 2006). We derived the ellipsoidal heights of Mean Sea Level (MSL) at more than 70 tide gages on the Gulf coast, from the Florida Keys to the Mexican border (Figure 2). This was done by subtracting the vertical distance between the BM and the MSL of the tide gage from the BM's ellipsoidal height. We then computed the geoid at those tide gages by subtracting the DOT from the ellipsoidal height of the MSL of each tide gage. The resulting tide gage-derived geoid heights are independent of leveling. These geoid heights were compared to USGG2009. After examination of the differences, we excluded tide gages in the Florida Keys to avoid effects of the strong ocean currents there, and in Louisiana to avoid effects of the subsidence of the Louisiana coast. We also excluded all river tide gages, since their geoids exhibited clear biases, probably due to erroneous DOT near rivers. Finally, all Texas tide gages were excluded because they exhibited a large jump in their geoid. This is a consequence of the DOT model (see the very negative DOT along the Texas coast in Figure 3), which was originally developed for the FL, AL, MS and LA part of the Gulf. In fact, if the DOT is not applied to the ellipsoidal heights of MSL at the tide gages, the Texas tide gages do not exhibit an abhorrent behavior.

The remaining 40 tide gage geoid heights (Figure 8) were used for the comparison with USGG2009, USGG2003 and several other geoid models discussed below (Table 2).

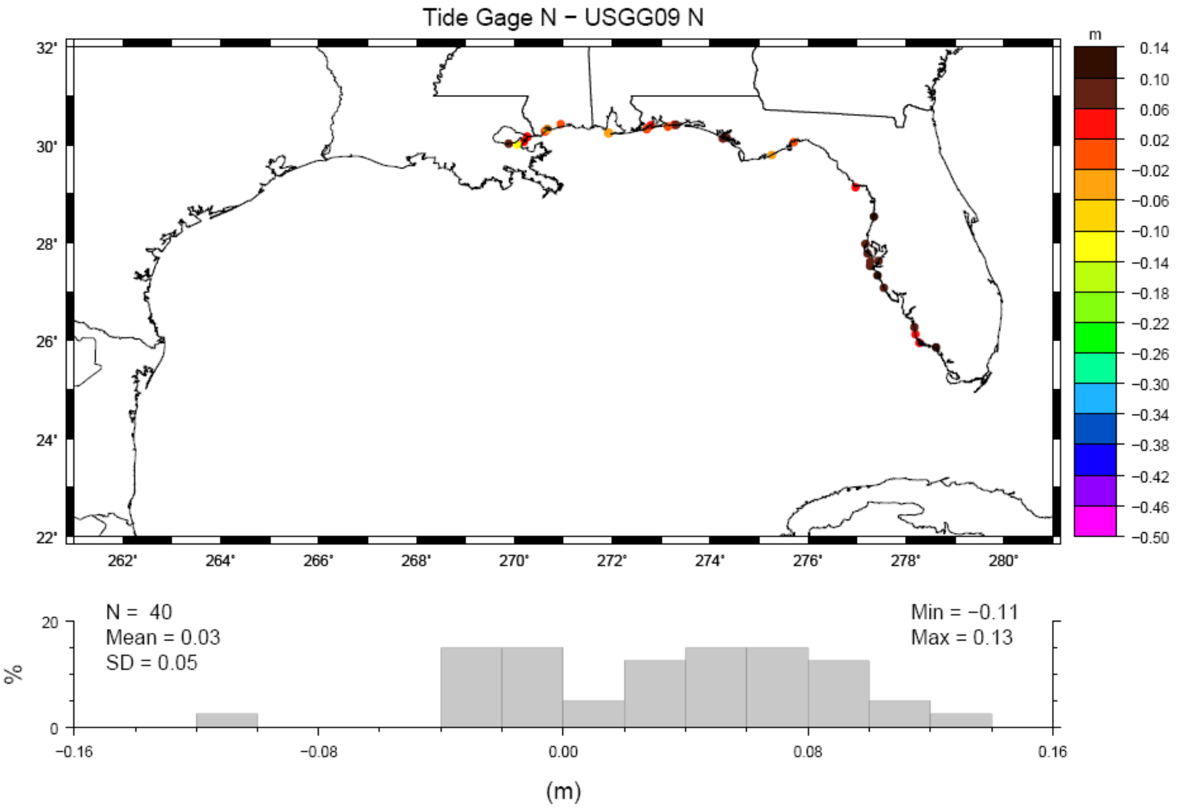

Figure 8: The difference between USGG2009 and tide-gage-derived geoid heights

Table 2: Statistics (in meters) of the differences between 40 Gulf-coast tide-gage-derived geoid heights and several gravimetric geoid models (see Figure 8).

Geoid\Statistic	Mean	SD	Min	Max
USGG2009	+0.033	0.050	-0.110	+0.128
USGG2003	-0.357	0.085	-0.566	-0.210
USGG2009, No Kernel Truncation	-0.527	0.079	-0.673	-0.307
USGG2009, Helmert 2nd Condensation	0.025	0.049	-0.118	0.117
EGM08	0.034	0.047	-0.099	0.123

The bias of USGG2003 relative to the Gulf's tide gages is –36 cm while that of USGG2009 is 3 cm. The much smaller bias of USGG2009 is due to the accurate contribution of GRACE. It implies that any future changes to the value of W_0 should be minor at most. The scatter of the results of USGG2009 is 5 cm, more than 80% improvement over the 8.5 cm of USGG2003.

Tests of the long wavelength content of USGG2009

To test the GRACE-derived long wavelength portion of the field and whether it is superior to the long wavelength content of the surface and altimetry-derived gravity data, we computed a gravimetric geoid based on the same gravity data used for USGG2009, but without any Stokes' kernel truncation. This "traditional-kernel-geoid" is purely based on the long wavelengths as implied by the surface and altimetry-derived gravity in and around North America, and to a much smaller extent on GRACE (through EGM08). The resulting geoid was compared to the same GPSBMs09 and tide gage-derived geoid heights mentioned above. Statistics of the differences from the GPSBMs are presented in Table 1. Figure 9 shows the long wavelength differences between this geoid and the GPSBMs09, and Table 2 presents the statistics of the differences between this geoid and the one computed using the Gulf tide gages and DOT.

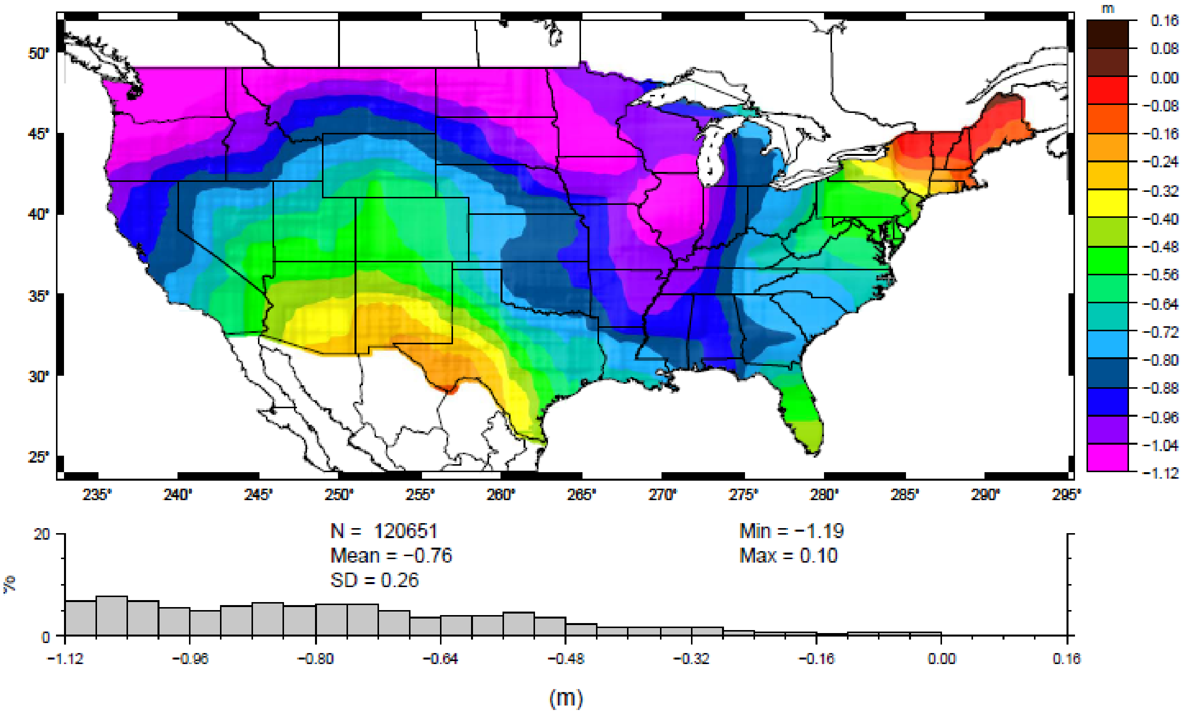

Figure 9: Long wavelength (5°) difference: the traditional-kernel-USGG2009 minus GPSBM09

The results do not leave any margin for doubt. USGG2009 unequivocally fits the GPSBMs09 and the tide-gage-derived geoid heights much better than the traditional-kernel-geoid. The improvement of USGG2009 over the traditional-kernel-geoid is 6.52 cm in the SD fit to GPSBMs09 (see Table 1) and 3.32 cm in the SD fit to the tide gages (compare the SDs of the first and third rows in Table 2). These differences are orders of magnitude larger than: (1) the effect of ellipsoidal corrections to the residual geoid, (2) the effect of the downward continuation of the gravity residuals (with respect to the full EGM08) to the geoid, and (3) the effect of any possible discrepancy between permanent tidal systems or reference fields.

Figure 6 shows the long wavelength (~ 5° or 550 km) difference between NAVD88 (through the Orthometric heights of the GPSBMs09) and a GRACE-derived geoid. The errors of the latter and of the ellipsoidal heights of the GPSBMs09 are almost two orders of magnitude smaller than the long wavelength leveling errors, coast to coast. Thus, one can think of Figure 6 as a representation of the long wavelength errors of NAVD88. Figure 9 represents NAVD88 errors plus the long wavelength errors of the surface and altimetry-derived gravity data. Differencing the two figures produces the long wavelength geoid errors due to the long wavelength errors of the NGS gravity database (Figure 10). These errors cannot be ignored anymore. They are hard to remove by correcting the point gravity data. Rather, one can avoid the effect of these errors on the geoid in two ways: (1) by simple truncation of Stokes’ kernel, or more accurately (2) by adopting the long wavelength content of the GRAV-D airborne data, once complete, to replace the long wavelength content of the surface data.

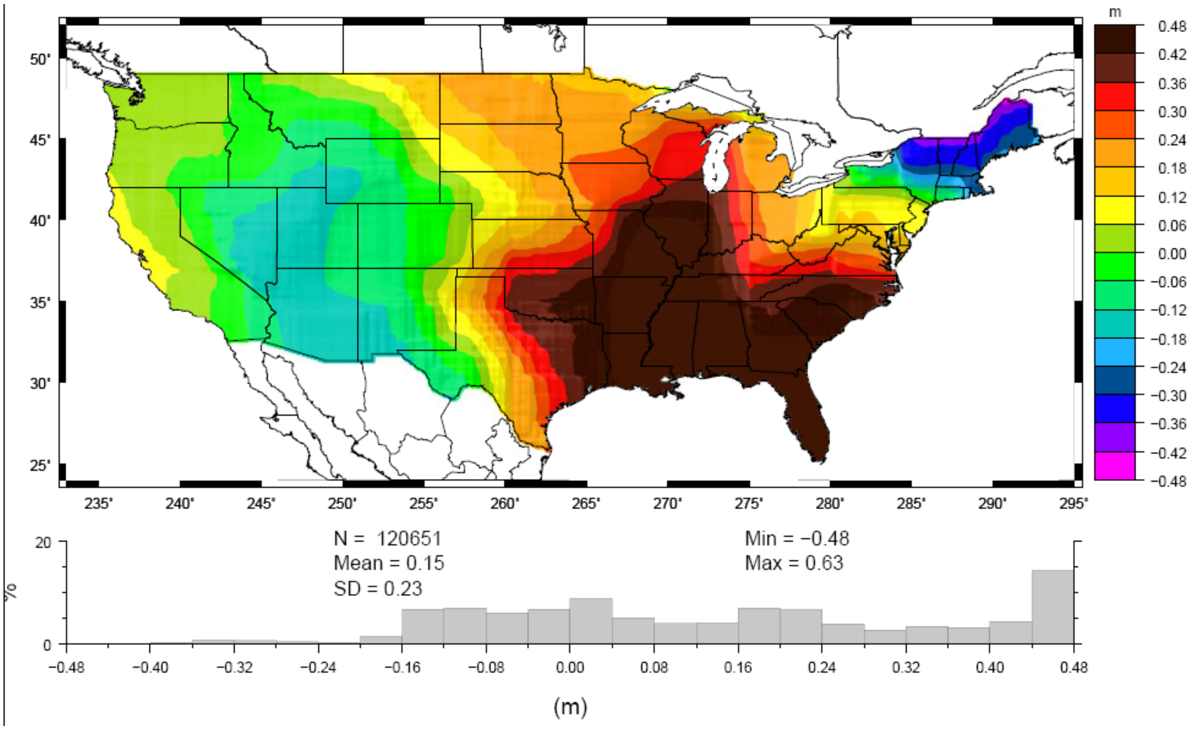

Figure 10: Estimated long wavelength (5°) geoid errors due to long wavelength errors in the NGS surface and altimetry-derived gravity data

Comparison between the harmonic continuation and the Helmert 2nd condensation geoids

The geoid models of these two methods were compared to the GPSBMs09 (Table 1). The harmonic continuation geoid is USGG2009 in Table 1. The state-by-state biases are almost identical in both models. Although it appears from the last row of Table 1 that USGG2009 is slightly better, a comparison with the Gulf tide gages (see Table 2), points in favor of the Helmert 2nd condensation method. However, it seems from Table 1 that the harmonic continuation method performs systematically significantly better in the Rocky Mountains (WA, ID, MT, WY, UT, NV, AZ, NM). Notice also that the Gulf coast region, where the tide gages of Table 2 are located, has a flat topography and a benign gravity signal.

The apparent advantage of the harmonic continuation method could be related to the fact that the terrain correction, which is a part of the Helmert 2nd condensation method, involves several approximations. It is possible that once the Helmert 2nd condensation method is computed more rigorously, replacing the Faye anomaly by the Helmert anomaly, this method could produce identical results to those of the harmonic continuation method. In that case, there remains only one disadvantage for the Helmert 2nd condensation method: it is much harder to compute.

Comparisons with EGM08

No matter how good a global geopotential model becomes, it is always possible to design and compute a local geoid that contains more high frequency information. It is futile, therefore, to compare global with local geoid models for any purpose other than the testing of global models. Nevertheless, we present in Table 3 the statistics of the differences between USGG2009 and EGM08, and GPSBMs09 over the US and its territories.

Table 3: Standard deviation (cm) of the differences: gravimetric geoid – GPSBMs09 geoid

Territory		N	USGG2009	EGM08
CONUS		18398	6.32	6.36
Alaska		198	27.5	27.7
Hawaii	Maui	5	2.8	3.9
	Honolulu	17	6.0	6.1
	Kauai	6	13.8	13.4
Guam		16	4.5	6.8
North Mariana Island	Saipan	10	2.6	3.3
	Tinian	35	2.0	1.7
	Rota	9	2.4	2.6
American Samoa		22	5.3	11.2
Puerto Rico and the US Virgin Islands		29	1.7	3.0

In the few instances where EGM08 outperforms USGG2009, the data editing process at NGS was most likely not careful enough. These few instances will be revisited and new geoid models computed. In the overwhelming majority of the US and its territories, however, USGG2009 fits the GPSBMs09 better, sometimes significantly. EGM08 fit to the tide gages in Table 2 is a testimony to the high quality of its long and short wavelength contents. We repeat, however, that the Eastern Gulf coast is a region of flat topography and benign gravity signal. Over most of the Rocky Mountains (NM, AZ, NV, CO, ID, WY), California and Texas, USGG2009 fits the state-by-state GPSBMs significantly better. EGM08 fits better in UT and WA, and several other states in the East, indicating that the NGS data editing in these states should be revisited.

Table 4: Mean and Standard Deviation (") of the differences between gravimetric and 3415 observed Astro-geodetic deflections of the vertical on the Earth surface in CONUS

Deflection component	USGG2009	EGM08
ξ	Mean = 0.02529 SD = 0.87338	Mean = -0.09113 SD = 0.97803
η	Mean = 0.16115 SD = 0.94117	Mean = 0.18889 SD = 1.03344

Table 4 presents the statistics of the differences between gravimetric deflections of the vertical and 3415 Astro-geodetic deflections in CONUS. Strictly speaking, before the comparison can be made, corrections should be applied to unify the coordinate systems of both sets of data and insure that all quantities refer to the same location. These corrections, however, have a very long wavelength character and mainly affect the mean values of the differences in Table 4. They change the standard deviations by a few thousandths of a second at most (Jekeli, 1999). Since deflections are very high frequency quantities, we need only concern ourselves with the standard deviations for the sake of comparing the two models. Therefore, we did not apply any coordinate system corrections to the data of Table 4. USGG2009 deflections outperform EGM08 deflections so much that corrections do not really matter or change the picture in any way.

Table 5: State by state accuracy (meters) for USGG2009, USGG2003 and EGM08

		USGG2009		USGG2003		USGG2009, No kernel truncation		USGG2009, 2^{nd} Helmert Cond.		EGM08	
State	N	Bias	SD	Bias	SD	Bias	SD	Bias	SD	Bias	SD
AL	283	0.0134	0.0378	0.1698	0.0503	0.3453	0.0690	0.0163	0.0373	0.0115	0.0302
AZ	227	0.0004	0.0478	-0.0774	0.1003	-0.4020	0.0590	0.0111	0.0517	0.0005	0.0520
AR	133	-0.0044	0.0304	0.0505	0.0355	0.2834	0.0545	-0.0027	0.0300	-0.0074	0.0332
CA	738	-0.0025	0.0576	-0.2186	0.0871	-0.2224	0.0987	0.0053	0.0588	-0.0034	0.0584
CO	562	0.0015	0.0757	-0.0737	0.0812	-0.2832	0.1032	0.0271	0.0751	0.0028	0.0803
CT	20	0.0041	0.0349	-0.2218	0.0259	-0.3263	0.0353	-0.0017	0.0352	0.0149	0.0337
DE	35	-0.0412	0.0436	-0.1209	0.0217	-0.1359	0.0234	-0.0461	0.0431	-0.0316	0.0371
DC	16	0.0189	0.0208	-0.0932	0.0194	-0.0731	0.0196	0.0156	0.0204	0.0156	0.0197
FL	2181	0.0003	0.0491	0.3827	0.0741	0.2738	0.0774	0.0108	0.0486	0.0009	0.0498
GA	137	0.0005	0.0397	0.2038	0.1208	0.3284	0.0833	0.0068	0.0399	0.0027	0.0417
ID	97	0.0077	0.0566	-0.6133	0.1274	-0.2853	0.0751	0.0426	0.0632	0.0111	0.0505
IL	334	0.0174	0.0474	0.0781	0.0415	0.2748	0.0407	0.0114	0.0454	0.0204	0.0474
IN	119	-0.0191	0.0345	-0.0429	0.0485	0.1141	0.0705	-0.0233	0.0337	-0.0182	0.0373
IA	100	-0.0315	0.0387	-0.1525	0.0729	-0.0095	0.0834	-0.0387	0.0405	-0.0302	0.0382
KS	105	-0.0024	0.0367	-0.1218	0.0592	0.0127	0.0686	-0.0023	0.0361	-0.0004	0.0350
KY	123	-0.0134	0.0369	-0.0708	0.0766	0.0871	0.1439	-0.0144	0.0380	-0.0145	0.0363
LA	217	-0.0533	0.0834	0.1734	0.0874	0.2060	0.0774	-0.0453	0.0832	-0.0628	0.0919
ME	65	0.0033	0.0436	-0.2372	0.0576	-0.4947	0.0845	-0.0074	0.0429	-0.0073	0.0462
MD	511	0.0087	0.0340	-0.1066	0.0280	-0.1146	0.0508	0.0042	0.0331	0.0001	0.0258
MA	35	-0.0182	0.0411	-0.3114	0.0572	-0.3869	0.0467	-0.0263	0.0411	-0.0112	0.0409
MI	410	0.0062	0.0429	-0.0307	0.0652	-0.0432	0.1019	-0.0029	0.0434	0.0083	0.0455
MN	4089	-0.0004	0.0270	-0.0691	0.0340	-0.0467	0.0666	-0.0127	0.0262	0.0009	0.0282
MS	243	0.0115	0.0468	0.0969	0.0500	0.3231	0.0351	0.0150	0.0474	0.0116	0.0426
MO	138	-0.0019	0.0337	-0.0793	0.0892	0.1961	0.1179	-0.0039	0.0337	-0.0029	0.0309
MT	151	0.0255	0.0458	-0.3389	0.2265	-0.2164	0.1025	0.0533	0.0725	0.0281	0.0439
NE	145	-0.0096	0.0379	-0.2274	0.0821	-0.1862	0.0512	-0.0143	0.0421	-0.0103	0.0384
NV	70	0.0041	0.0540	-0.2964	0.1058	-0.3797	0.0652	0.0202	0.0601	0.0092	0.0533
NH	14	0.0015	0.0188	-0.3025	0.0275	-0.5194	0.0823	-0.0066	0.0185	0.0034	0.0135
NJ	326	-0.0076	0.0269	-0.1237	0.0285	-0.1905	0.0617	-0.0123	0.0270	-0.0036	0.0237
NM	107	-0.0215	0.0477	0.0004	0.0645	-0.3429	0.0970	-0.0122	0.0486	-0.0227	0.0489
NY	185	0.0097	0.0406	-0.1914	0.0700	-0.3771	0.1410	0.0020	0.0414	0.0098	0.0371
NC	1676	-0.0014	0.0383	0.0791	0.1268	0.1287	0.0656	0.0007	0.0382	-0.0008	0.0388
ND	47	0.0502	0.0309	0.0201	0.0595	0.0124	0.0367	0.0382	0.0334	0.0573	0.0368
OH	297	-0.0022	0.0403	-0.1051	0.0469	-0.0452	0.0815	-0.0078	0.0423	-0.0009	0.0418
OK	79	-0.0085	0.0392	0.0657	0.0514	0.1655	0.0698	-0.0041	0.0392	-0.0114	0.0369
OR	202	-0.0038	0.0597	-0.1979	0.1683	-0.2320	0.0784	-0.0018	0.0668	-0.0015	0.0595
PA	96	0.0264	0.0399	-0.1131	0.0404	-0.2101	0.0627	0.0239	0.0392	0.0300	0.0349
RI	29	0.0014	0.0231	-0.2679	0.0261	-0.3093	0.0271	-0.0045	0.0237	0.0239	0.0241
SC	1315	0.0032	0.0482	0.1512	0.1008	0.2459	0.0343	0.0060	0.0460	-0.0012	0.0456
SD	242	0.0030	0.0396	-0.1174	0.0556	-0.0723	0.0801	-0.0041	0.0395	0.0048	0.0370
TN	302	0.0130	0.0358	0.0429	0.0653	0.2516	0.1229	0.0128	0.0357	0.0171	0.0365
TX	218	-0.0076	0.0628	0.0964	0.1284	-0.0151	0.2205	0.0022	0.0626	-0.0067	0.0618
UT	55	0.0146	0.0787	-0.2974	0.1178	-0.3835	0.0870	0.0414	0.0764	0.0186	0.0757
VT	317	-0.0026	0.0295	-0.3178	0.0476	-0.6288	0.0466	-0.0025	0.0264	-0.0030	0.0262
VA	434	0.0077	0.0331	-0.0912	0.0401	-0.0565	0.0636	0.0049	0.0331	0.0033	0.0329
WA	259	0.0053	0.0766	-0.4069	0.1531	-0.2276	0.0708	0.0092	0.0845	0.0067	0.0702
WV	55	0.0516	0.0444	-0.1452	0.0483	-0.1520	0.0648	0.0490	0.0403	0.0420	0.0323
WI	758	-0.0137	0.0254	0.0305	0.0660	0.0604	0.0590	-0.0232	0.0264	-0.0130	0.0244
WY	101	-0.0079	0.0642	-0.2422	0.1547	-0.3460	0.0885	0.0123	0.0664	-0.0020	0.0690
Mean	18398	0.0001	0.0433	0.0045	0.0805	0.0126	0.0768	-0.0001	0.0437	0.0002	0.0433

How accurate is USGG2009?

Figure 6 presents the long wavelength errors of NAVD88, assuming that GRACE is free of long wavelength errors, in particular continental scale tilts such as the one seen in Figure 6. There are still some sharp curves in the contours of Figure 6, due to some residual high frequency content. So before we could use Figure 6 as a sort of transformation from erroneous to NAVD88-error-free GPSBMs09, we had to smooth it a bit more. The resulting signal was then removed from the GPSBM09-derived geoid heights, giving GPSBM geoid heights that are almost free of NAVD88 long wavelength errors. These were then compared to USGG2009 and all other gravimetric geoid models (see Table 5).

We realize that these statistics are still contaminated by subsidence and tectonic movements affecting the GPSBMs in CA, LA, TX and possibly a few other western states. In the rest of the country, the standard deviations in Table 5 are due to errors in USGG2009, but also reflect short wavelength errors of NAVD88 and ellipsoidal height errors. Thus, it can be stated based on Table 5 that the accuracy of the slope of USGG2009 is about 3-4 cm, except in the Rocky Mountains, where it is of the order of 5-6 cm. The bias of USGG2009 is a few centimeters at most.

Concluding remarks

USGG2009 is a new 1'x1' gravimetric geoid for the US and its territories. It is based on the NGS gravity database, DNSC08 altimetry-derived gravity over the oceans, SRTM-DTED1 3" elevations for its terrain reductions and EGM08 as a reference geopotential model.

USGG2009 differs from its predecessors in two major ways: (1) its long wavelength (> ~300 km) content is based on GRACE rather than implied by the surface and altimetry derived gravity data, and (2) it is computed using the harmonic continuation method rather than the classical Faye anomaly.

Tests and comparisons presented in this document show unequivocally that this geoid outperforms USGG2003 significantly. These tests indicate that USGG2009: (1) fits the GPS/tide gage-derived geoid heights to better 5 cm, (2) fits the GPSBMs09 to about 3-4 cm except in the Rocky Mountains, where it fits to 5-6 cm, provided that the GPSBMs09 are corrected to remove the NAVD88 long wavelength errors, and (3) LA and TX are exceptions due to the subsidence of the GPSBMs there. The statistics (Table 5) of CA, WA, OR and possibly other western states could be inflated due to tectonics.

The iterative process of computing successive gravimetric geoid models seems to have reached an accuracy level of about 5 cm. The contribution of the satellite GRACE was monumental in improving the very long wavelength of the geoid. The satellite GOCE, which has been launched recently, will add some more accuracy to the medium wavelengths of the field up to a spatial resolution of 100 km.

The journey to arrive at the 1 cm level will depend on improving the gravity data under the Gravity for the Re-definition of the American vertical Datum (GRAV-D) project. USGG2009, however, does not include any of the recently acquired GRAV-D airborne gravity data. This model is intended as a baseline for determining the impact of GRAV-D; see http://www.ngs.noaa.gov/GRAV-D/ for more details. As GRAV-D moves forward, updated gravimetric geoid models will reflect the improvements. As more data and improved techniques are applied, this series of models will eventually lead to the selection of the optimal gravimetric geoid for use as the defining surface for the vertical datum, which will replace the North American Vertical Datum of 1988.

The USGG2009 model was used to develop the GEOID09 hybrid geoid model (http://www.ngs.noaa.gov/GEOID/GEOID09) in conjunction with the GPSBM2009 (http://www.ngs.noaa.gov/GEOID/GPSonBM09). These models were also used to create the USDOV2009 gravimetric vertical deflection model and the DEFLEC09 hybrid vertical deflection model (http://www.ngs.noaa.gov/GEOID/USDOV2009).

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