5. Assessment of the Deepwater Flatfish Stock Complex in the Gulf of Alaska (Executive Summary)

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5.1 Introduction

In 2006, the deepwater flatfish complex (consisting of Dover sole, Greenland turbot and deepsea sole) was moved to a biennial stock assessment schedule to coincide with new survey data. A discussion at the September 2006 Groundfish Plan Team meetings concluded the following two important points for updating information in off-year assessments for species in Tier 3 or higher:

1) Anytime the assessment model is re-run and presented in the SAFE Report, a full assessment document **must** be produced.
2) The single-species projection model **may** be re-run using new catch data without re-running the assessment model.

Thus for stocks in Tier 3 or higher, on alternate (even) years, parameter values from the previous year's assessment model and total catch information for the current and previous year can, at the author's discretion, be used to make projections via the single species projection model for the following two years and to recommend ABC levels for those years. Projection models are not available to update stocks falling into Tiers 4-6; consequently, the harvest recommendations for these stocks remain the same as those presented in the previous year.

Since 2004, Dover sole has been assessed using an age-structured model and Tier 3 status determination. However, last year (2011) that model was considered unsatisfactory and Dover sole was assessed using Tier 5 criteria. A new Tier 3 model is currently being developed for Dover sole (see Appendix to this chapter for a description), but final results were not ready for this assessment cycle. Consequently, it is recommended that Dover sole remain in Tier 5 for 2012. Thus, 2012-2013 ABCs and OFLs developed last year for Dover sole carry over unchanged for 2013-2014.

Greenland turbot and deepsea sole fall under Tier 6. Because species-level ABC's and OFL's for Tier 6 species are based on historical catch levels, these quantities cannot be updated. Consequently, as in previous years (Stockhausen et al. 2011[1]), the species-level ABC is 179 t for Greenland turbot while the species-level OFL is 238 t for both 2013 and 2014. For deepsea sole, the ABC is 4 t and OFL is 6 t. The ABC for the complex is the sum of the single species ABC's and the OFL for the complex is the sum of the single species OFL's.

5.2 Updated catch and projection

Total catch of deepwater flatfish in 2011 was 465 t, of which 461 t was Dover sole. As of Sept. 22, the total catch of deepwater flatfish in 2012 was 238 t, all of which was Dover sole. Because Dover sole is currently in Tier 5 and Greenland turbot and deepsea sole are in Tier 6, ABCs and OFLs do not depend on updated catch. The OFL's and recommended complex-level ABC's for 2013 and 2014 are the same as in 2012: 6,834 t for OFL and 5,126 t for ABC. The principal reference values are presented in the following table, with the recommended values in bold:

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[1]Stockhausen, W., M. Wilkins and M. Martin. 2011. 5. Assessment of the Deepwater Flatfish Stock in the Gulf of Alaska. In: Stock assessment and fishery evaluation report for the groundfish resources of the Gulf of Alaska. North Pacific Fishery Management Council, PO Box 103136, Anchorage, AK. http://www.afsc.noaa.gov/REFM/docs/2011/GOAdeepflat.pdf.

Species	Quantity	As estimated or specified last year (2011)		As estimated or specified this year (2012)	
		2012	2013	2013	2014
Dover sole	M (natural mortality)	0.085	0.085	**0.085**	**0.085**
	Specified/recommended tier	5	5	**5**	**5**
	Tier 5 biomass (Survey biomass,t)	77,531	77,531	**77,531**	77,531
	F_{OFL}	0.085	0.085	**0.085**	0.085
	max F_{ABC}	0.064	0.064	**0.064**	0.064
	recommended F_{ABC}	0.064	0.064	**0.064**	0.064
	OFL (t)	6,590	6,590	**6,590**	6,590
	max ABC (t)	4,943	4,943	**4,943**	4,943
	ABC (t)	4,943	4,943	**4,943**	4,943
Greemnland Turbot	Specified/recommended tier	6	6	**6**	**6**
	OFL (t)	238	238	**238**	238
	max ABC (t)	179	179	**179**	179
	ABC (t)	179	179	**179**	179
Deepsea sole	Specified/recommended tier	6	6	**6**	**6**
	OFL (t)	6	6	**6**	6
	max ABC (t)	4	4	**4**	4
	ABC (t)	4	4	**4**	4
Deepwater flatfish complex	OFL (t)	6,834	6,834	**6,834**	**6,834**
	max ABC (t)	5,126	5,126	**5,126**	**5,126**
	ABC (t)	5,126	5,126	**5,126**	**5,126**
	Status	As determined last year (2011) for:		As determined this year (2012) for:	
		2010	2011	**2011**	**2012**
	Overfishing	no	n/a	**no**	n/a

5.3 Area Apportionment

In September 2012 the Plan Teams recommended that assessment authors retain status quo assessment approaches for the November 2012 SAFE report but also apply the Kalman filter or random effects survey averaging methods for Tier 5 stocks and summarize the analytical results for comparison purposes only. Because this is an executive summary, the status quo assessment approach was retained. Next year the full assessment will evaluate and apply the recommended survey averaging approach for apportionment.

Area apportionment for ABC is currently based on the relative abundance (biomass) of each species in the stock complex found within each management area in the last GOA groundfish survey. The recommended ABC area apportionment percentages are identical to last year because the last GOA groundfish survey was conducted in 2011. Thus, the recommended ABC area apportionments for 2013-14 are:

Quantity	Species	Western	Central	West Yakutat	Southeast Outside	Total
Area Apportionment	Dover sole	1.1%	45.8%	31.8%	21.3%	100.0%
	Greenland turbot	68.2%	22.3%	5.0%	4.5%	100.0%
	Deepsea sole	0.0%	100.0%	0.0%	0.0%	100.0%
2013 ABC (t)	Dover sole	54	2,264	1,572	1,053	4,943
	Greenland turbot	122	40	9	8	179
	Deepsea sole	0	4	0	0	4
	Deepwater flatfish	176	2,308	1,581	1,061	5,126
2014 ABC (t)	Dover sole	54	2,264	1,572	1,053	4,943
	Greenland turbot	122	40	9	8	179
	Deepsea sole	0	4	0	0	4
	Deepwater flatfish	176	2,308	1,581	1,061	5,126

5.4 Research Priorities

As noted above, a new assessment model is under development that incorporates length-based approaches to fishery and survey selectivity, size-based natural mortality, environmental predictors of recruitment or catchability (e.g., temperature), density-dependent stock-recruit functions, multiple fisheries and surveys, ageing error, and temporally-varying parameter values. The new model will also allow the incorporation of additional data sources into the model, including ADFG surveys (as suggested by the SSC) and fishery age compositions. See the Appendix to this chapter for a more detailed description.

5.5 Summaries for the Plan Team

Species	Year	Biomass[1]	OFL[2,3]	ABC[2,3]	TAC[2,3]	Catch[4]
Deepwater flatfish	2011	77,531	7,823	6,305	6,305	465
	2012	--	6,834	5,126	5,126	238
	2013	--	6,834	5,126		
	2014	--	6,834	5,126		

[1] Dover sole survey biomass (2011).
[2] http://www.fakr.noaa.gov/sustainablefisheries/specs11_12/goa_table1.pdf
[3] http://www.fakr.noaa.gov/sustainablefisheries/specs12_13/goa_table1.pdf
[4] As of Sept. 22, 2012.

Stock/ Assemblage	Area	2012				2013		2014	
		OFL[1]	ABC[1]	TAC[1]	Catch[2]	OFL[3]	ABC[3]	OFL[3]	ABC[3]
Deepwater flatfish	W	--	176	176	5	--	176	--	176
	C	--	2,308	2,308	227	--	2,308	--	2,308
	WYAK	--	1,581	1,581	4	--	1,581	--	1,581
	SEO	--	1,061	1061	2	--	1,061	--	1,061
	Total	6,834	5,126	5,126	238	6,834	5,126	6,834	5,126

[1] http://www.fakr.noaa.gov/sustainablefisheries/specs12_13/goa_table1.pdf
[2] As of Sept. 22, 2012.
[3] Based on the Tier 5 calculations for Dover sole and Tier 6 calculations for Greenland turbot and deepsea sole.

Appendix: Description of a New Stock Assessment Model Framework for Several Flatfish Stocks

Introduction

Progress is underway to implement a new age/size-structured assessment model framework for the following stocks: BSAI flathead sole, GOA flathead sole, GOA rex sole, and GOA Dover sole (as part of the GOA Deepwater Flatfish stock complex). Currently, these stocks are assessed using several different models, each with differing model configuration options, model processes, potential input data sources and formats, and output formats. In addition, the reliability of the current age-structured assessment model for Dover sole has been called into question (Stockhausen et al., 2011). As a consequence, it was considered desirable to construct an "integrated" assessment model framework (IAMF) for these flatfish stocks that would provide a common set of model configuration options, model processes, input data sources and formats, and output formats suitable for processing by an integrated set of analysis routines. Although the model framework is still being developed, this appendix is intended to provide an overview of its main features and how it differs from the current set of models used to assess the BSAI flathead sole, GOA flathead sole, GOA rex sole, and GOA Dover sole stocks.

Features

A brief summary of key features in the IAMF is presented in Table 1 for six general categories. New functionality, relative to the current set of assessment models, is highlighted in more detail below.

In the current set of assessment models, it is not possible to specify prior probability density functions (pdfs) for most model parameters. As a consequence, the current models cannot be used to conduct fully Bayesian analyses, thereby complicating model comparison and selection. In the IAMF, prior density functions (or none) can be specified for any model parameter. The prior types and hyper-parameter values are specified in theby input files, so testing alternative priors does not require re-coding the model. Consequently, fFully Bayesian analyses can consequently be conducted by specifying priors on all parameters.

Initial values for model parameters can only be specified in input files in the current set of assessment models. This complicates making multiple model runs from different sets of initial parameter values to evaluate model convergence. In the IAMF, initial values can be randomly generated within the model for parameters with prior density functions.

In the current set of assessment models, model processes such as natural mortality or fishery selectivity are generally fixed over the entire assessment period. Consequently, potential changes in, for example, fishery selectivity over time associated with changes in management regulations cannot be addressed using these models. To address this deficiency, multiple "time stanzas" can be defined for each model process in the IAMF. Model parameter values associated with a given model process are constant across each time stanza defined for the process, but may differ among stanzas.

The IAMF incorporates several sex-specific selectivity functions (e.g., logistic, double normal), each of which can be either age or size dependent. In addition, multiple time stanzas can be defined such that different selectivity functions are defined for different time stanzas. In the current GOA models, selectivity functions are strictly age-dependent while they are strictly size-dependent in the BSAI model (and only logistic).

The current set of assessment models can only accommodateOnly a single fishery (with constant selectivity). can be accommodated in the current set of assessment models. In addition, the current GOA assessment models do not incorporate fishery age composition data. In the IAMF, multiple fisheries (fleets) can be accommodated, with fits to annual catch, size compositions, and/or age composition data.

It is not possible to incorporate data from different surveys (gear types or areas) into the current BSAI model, although it is possible in the GOA models. Conversely, The the current GOA models cannot incorporate environmental factors (such as temperature) as correlates in any model processes; while the current BSAI model can incorporates environmental effects (temperature) on survey catchability. TLike the BSAI, the IAMF has been expanded to incorporate data from different surveys and a suite of environmental factors as correlates in model processes.
can incorporate environmental effects on survey catchability.

Finally, the IAMF allows estimation of ageing error, weight-at-size and size-at-age both inside and outside the model. These could only be estimated outside the model in the current assessment models.

Model processes

Model processes incorporated in the IAMF consist of: 1) natural mortality, 2) maturity, 3) recruitment, 4) sex ratio at recruitment, 5) weight-at-size, 6) size-at-age, 7) fully-selected fishing mortality, 8) fully-selected survey catchability, 9) selectivity and 10) ageing error. Parameterizations and function options are listed in Tables 2-11 and discussed in more detail below. For each model process, multiple time stanzas can be defined such that parameter values (and function options) are the same for each year within the stanza but may be different in different stanzas. Current models do not incorporate multiple time stanzas—model parameters have no time-varying component (other than annual fishing mortality and recruitment deviations).

Natural mortality can be specified as either a sex-specific constants across age and size (as in current models) or as size-dependent functions using Lorenzen's equation (Table 2). Only the first option is available in the current models.

In the IAMF, female maturity can be incorporated as either an age-specific or a size-specific logistic relationship (Table 3). Maturity parameters must be estimated outside the model. In the

current models, maturity is generally based on a size-specific logistic function estimated outside the model.

Recruitment can be modeled using one of three stock-recruit relationships: mean recruitment (independent of stock size), a Ricker function or a Beverton-Holt function (Table 4). Currently, only the first is available in the GOA assessment models while all three are available in the BSAI flathead sole model.

The sex ratio at recruitment is assumed to be 1:1 in the current models. In the IAMF, the fraction of recruits that are female is described using a logit function with (potentially-estimable) annual deviations from a logit-scale mean value (Table 5).

Weight-at-size is expressed in the IAMF as a sex-specific power-law function of size (Table 6). This is the same functional form used in the current models, but the associated parameters in the current models must be fit outside the model whereas the IAMF includes the option to estimate weight-at-size inside the model (according to multiple time stanzas).

Mean size-at-age is expressed in the IAMF using the Complete Expected Value Parameterization for the von Bertalanffy growth curve (Table 7); variance in size-at-age can be modeled using several simple parameterizations (e.g., cv constant, cv linear with age, cv linear with size). The probability function of size-at-age is represented as a lognormal function of mean size-at-age and variance. Both mean size-at-age and variance can (potentially) be estimated inside the model in the IAMF, whereas these quantities must be estimated outside the model in the current assessment models.

Fully-selected fishing mortality is parameterized using the median fully-selected fishing mortality on females, an optional log-scale offset for males, and a time series of annual log-scale deviations (Table 8). While this is similar to current models, these latter can only incorporate one type of fishery (fleet plus gear type) whereas multiple fisheries can be accommodated in the IAMF.

Fully-selected survey catchability is parameterized using the median fully-selected catchability for females, an optional log-scale offset for males, and a covariate for an environmental time series (Table 9). The current GOA models incorporate an optional log-scale offset for males, while the current BSAI flathead sole model incorporates a temperature-dependent environmental covariate. Like the IAMF, the GOA models can also incorporate multiple surveys whereas the BSAI flathead sole model only accommodates a single survey.

Fishery and survey selectivity functions in the IAMF can be age- or size-dependent (Table 10), whereas they are strictly age-dependent in the current GOA models and strictly size-dependent in the current BSAI flathead sole model. Up to six parameters per selectivity function are available to fit logistic equations (2 parameters) or double normal equations (6 parameters). Both these functions are available in the current GOA models, but only the logistic equation is available in the BSAI flathead sole model.

Finally, the IAMF includes functions to estimate reader-specific ageing error matrices from reader/tester ageing data (Table 11). The estimation model is based on Punt et al. (2008).

Population dynamics

The equations governing population dynamics in the IAMF are given in Table 12. One notable feature of the IAMF is that the dynamics are truly age and size dependent, whereas the current models are either, strictly age-specific (GOA models) or use approximations to true age/size dynamics (the BSAI flathead sole model). An approximation to true age/size dynamics is also made in the IAMF, however, in the IAMF: the size structure of the population during each year is consistent with the size-at-age distribution function for the corresponding time stanza. That is, within-year growth is not followed (allowing for in-year growth would be prohibitively expensive from a computational viewpoint). Instead, an "effective" size-at-age is calculated for the entire year based on a fixed age offset ($0 \leq \delta a \leq 1$). As a consequence, only sex-specific numbers-at-age ($N_{y,x,a}$, y = year, x = sex, a = age) need to be specified at the beginning of each year (or carried over from the end of the previous year; Table 3).

Total recruitment to age 1 at the beginning of each year is based on log-scale deviations from the stock-recruit function for the corresponding time stanza. The fraction of these recruits that are female (x=1) in each year is based on a logit-scale mean parameter fraction and annual deviations. Numbers-at-age at the beginning of each year are then expanded to numbers at age and size ($N_{y,x,a,z}$) using the sex-specific size-at-age conversion matrix for the corresponding time stanza. Age, size, and sex-specific rates for natural mortality and fishing mortality are then computed and applied to $N_{y,x,a,z}$ to compute the numbers at age and size at the end of the year, as well as at spawning time and the time of each survey.

Annual numbers caught in each fishery and survey are fully sex-, age- and size-specific (Tables 13 and 14). Numbers caught at age and numbers caught at size are calculated as simple sums of the full numbers at age and size over the appropriate dimension. Age and size compositions are calculated as proportions at age and size over both males and females; numbers at age are passed through an appropriate ageing error matrix prior to calculating the age composition. Annual fishery catches (biomass) and survey biomasses are sums of numbers caught at age and size weighted by weight-at-size.

Likelihoods

Model parameters are estimated by minimizing the objective function

$$\mathcal{O} = -2 \cdot \sum_i ln\mathcal{L}_i - 2 \cdot \sum_j ln\mathcal{Pr}_j + \sum_k \mathcal{P}^k$$

where the $ln\mathcal{L}_i$ are the log-likelihood components outlined in Tables 15-18, the $ln\mathcal{Pr}_j$ are prior log-likelihoods defined for each model parameter, and the $\mathcal{P}^j$ are additional penalties imposed to improve model convergence. Each likelihood component is based on an assumed process or observation error distribution (lognormal or multinomial). On the whole, the functions used to calculate these components are quite similar to those used for the corresponding components in the current models. The likelihood components for weight-at-size, size-at-age, and ageing error data are, of course, new in the IAMF because the current models cannot fit these data sources. Similarly, the likelihood component associated with annual deviations in the initial sex ratio is new to the IAMF, as are the prior likelihoods for the model parameters.

One innovation (relative to the current models) the IAMF introduces is in the handling of size composition data. While age samples are always sexed from both fishery and survey samples, this is not true of size samples—particularly for smaller-sized fish sampled by fishery observers. In the current assessment models, the sex ratio of unsexed individuals within a size bin is assumed to be the same as that of the sexed individuals within the same size bin. In the IAMF, this assumption is no longer made. Sex-specific model size compositions can be collapsed to unsexed compositions when unsexed individuals account for a large fraction of the observations in the corresponding data size bin. As a consequence, the size-composition-related equations given in Tables 15 and 16 are somewhat misleading because they suggest the likelihoods are based strictly on sexed individuals. This is not the case, but the actual equations used in the IAMF are rather cumbersome and are not presented here in the interest of simplification.

Tables

Category	Features
Parameters	• priors specified by input files • initial values can be randomly-generated
Time stanzas	• possible to define multiple time stanzas • different stanzas for different processes
Fisheries	• multiple fisheries • "historic" catch level • catch (yield) • size compositions • age compositions
Surveys	• multiple surveys • biomass • size compositions • age compositions • environmental factors (e.g., temp)
Selectivity	• multiple functions • sex/age-specific • sex/size-specific
Ageing error	• can estimate within the model • estimation based on Punt et al (2008)
Weight-at-size	• can estimate within the model
Size-at-age	• can estimate within the model

Table 1. A summary of key model features.

Category	Details
Parameters	M_{0t} female mortality ΔM_t log-scale male-specific offset
Functions	$M_{x,y,a,z} = \begin{cases} M_{0t} \cdot \exp(\Delta M_t \cdot \delta_x) \\ M_{0t} \cdot \exp(\Delta M_t \cdot \delta_x) \cdot \dfrac{z_0}{z} \end{cases}$

Table 2. Parameterization and function options for natural mortality.

Category	Details
Parameters	$x50_t$ age or size at 50% female maturity b_t slope at 50% female maturity
Functions	$\mu_{y,a,z} = \begin{cases} [1 + \exp(-b_t \cdot [a - x50_t])]^{-1} \\ [1 + \exp(-b_t \cdot [z - x50_t])]^{-1} \end{cases}$

Table 3. Parameterization and function options for maturity. Maturity parameters are estimated outside the model.

Category	Details
Parameters	R_{0t} equilibrium recruitment h_t slope-at-origin τ_y log-scale deviations
Functions	$SR_t = \begin{cases} R_{0t} \\ \text{BevHolt}(R_{0t}, h_t) \\ \text{Ricker}(R_{0t}, h_t) \end{cases}$ $R_y = SR_t(SB_{y-1}) \cdot \exp(\tau_y)$

Table 4. Parameterization and function options for total recruitment.

Category	Details
Parameters	$\widehat{\vartheta}_t$ mean logit-scale fraction female ϵ_y logit-scale deviations
Functions	$\vartheta_y = logit^{-1}(\widehat{\vartheta}_t + \epsilon_y)$

Table 5. Parameterization and function options for the fraction female at recruitment.

Category	Details
Parameters	a_t female log-scale intercept b_t female log-scale size coefficient Δa_t male offset to log-scale intercept Δb_t male offset to log-scale size coefficient
Functions	$w_{x,y,a,z} = \exp(a_t + b_t \cdot \ln(z) + \delta_x \cdot [\delta a_t + \delta b_t \cdot \ln(z)])$

Table 6. Parameterization and function options for weight-at-size.

Category	Details	
Parameters	$Z1_{tx}$	size-at-age "1"
	$Z2_{tx}$	size-at-age "2"
	ZM_{tx}	size at midpoint of ages "1" and "2"
	$VarA_{tx}$	variance coefficient A
	$VarB_{tx}$	variance coefficient B
	$VarC_{tx}$	variance coefficient C
Functions	$\tilde{Z}_{x,y,a} = \text{CEVP}(a; Z1_{tx}, Z2_{tx}, ZM_{tx})$	
	$\sigma^2_{x,y,a} = f(a; VarA_{tx}, VarB_{tx}, VarC_{tx})$	
	$\Phi_{x,y,a,z} = Lognormal(z|\tilde{Z}_{x,y,a}, \sigma^2_{x,y,a})$	

Table 7. Parameterization and function options for size-at-age.

Category	Details	
Parameters	$F_{0f,t}$	median FS female mortality
	$\Delta F_{f,t}$	log-scale male-specific offset
	$\varphi_{f,y}$	log-scale annual deviations
Functions	$F_{f,x,y} = F_{0f,t} \cdot \exp(\Delta F_{f,t} \cdot \delta_x + \varphi_{f,y})$	

Table 8. Parameterization and function options for fully-selected fishing mortality.

Category	Details	
Parameters	$Q_{0s,t}$	median female catchability
	$\Delta Q_{s,t}$	log-scale male-specific offset
	$QT_{s,t}$	temperature coefficient
Functions	$Q_{x,y} = Q_{0s,t} \cdot \exp(\Delta Q_{s,t} \cdot \delta_x + QT_{s,t} \cdot \delta T_y)$	

Table 9. Parameterization and function options for fully-selected survey catchability.

Category	Details	
Parameters	p_{1t}	selectivity parameter
	p_{2t}	selectivity parameter
	p_{3t}	selectivity parameter
	p_{4t}	selectivity parameter
	p_{5t}	selectivity parameter
	p_{6t}	selectivity parameter
Functions	• $S_{x,y,a,z} = f_{x,t}(a, z, \{p_{it}\})$	
	• logistic (5%, 95% locations parameterization)	
	• double normal	

Table 10. Parameterization and function options for fishery and survey selectivity functions.

Category	Details (based on Punt et al, 2008)
Parameters	3-parameters for reader bias 3-parameters for reader imprecision
Functions	3-parameter function for reader bias 3-parameter function for reader imprecision $\Theta_{r,\acute{a},a}$

Table 11. Parameterization and functions for ageing error.

Term	Description
x, y, t, a, z	indices for sex (1=females, 2=males), year, time stanza corresponding to year, age, size
$N_{x,y,a}$	sex-specific numbers-at-age start of year
$R_y = SR_t(SB_{y-1}) \cdot \exp(\tau_y)$	total recruitment (age 1). SR=stock/recruit function, SB = female spawning biomass, τ_y = log-scale deviation from SR.
$\vartheta_y = logit^{-1}(\hat{\vartheta}_t + \epsilon_y)$	fraction of recruitment that is female
$N_{x=1,y,1} = \vartheta_y \cdot R_y$ $N_{x=2,y,1} = (1 - \vartheta_y) \cdot R_y$	sex-specific recruitment
$\tilde{N}_{x,y,a,z} = \Phi_{x,t,a+\delta a,z} \cdot N_{x,y,a}$	expansion from numbers-at-age to numbers-at-age/size ($0 \le \delta a \le 1$ to account for growth during year)
$M_{x,y,a,z} = \begin{cases} \hat{M}_{0t} \cdot \exp(\hat{\Delta M}_t \cdot \delta_x) \\ \hat{M}_{0t} \cdot \exp(\hat{\Delta M}_t \cdot \delta_x) \cdot \dfrac{z_0}{z} \end{cases}$	natural mortality
$F_{f,x,y} = \hat{F}_{0\,f,t} \cdot \exp(\hat{\Delta F}_{f,t} \cdot \delta_x + \varepsilon_{f,y})$	fully-selected fishing mortality (in fishery f)
$F_{f,x,y,a,z} = s^F_{f,x,t,a,z} \cdot F_{f,x,y}$	fishing mortality (in fishery f)
$Z_{x,y,a,z} = \sum_f F_{f,x,y,a,z} + M_{x,y,a,z}$	total mortality
$\acute{N}_{x,y,a,z} = \tilde{N}_{x,y,a,z} \cdot e^{-Z_{x,y,a,z}}$ $N_{x,y+1,a+1} = \sum_z \acute{N}_{x,y,a,z}$	• survivors at year end • numbers advanced to next year
$\mu_{t,a,z} = \begin{cases} [1 + \exp(-b_t \cdot [a - x50_t])]^{-1} \\ [1 + \exp(-b_t \cdot [z - x50_t])]^{-1} \end{cases}$	female maturity (either age- or size-dependent)
$SB_y = \sum_a \sum_z w_{x=1,t,z} \cdot \mu_{t,a,z} \cdot \tilde{N}_{x=1,y,a,z} \cdot e^{-\delta t_S \cdot Z_{x,y,a,z}}$	female spawning biomass ($0 \le \delta t_S \le 1$ to account for in-year mortality prior to spawning)

Table 12. Model population dynamics.

Term	Description
$C_{f,x,y,a,z} = \dfrac{F_{f,x,y,a,z}}{Z_{x,y,a,z}} \cdot (1 - e^{-Z_{x,y,a,z}}) \cdot \tilde{N}_{x,y,a,z}$	numbers caught at sex x, age a, size z in fishery f
$C_{f,x,y,a} = \sum_z C_{f,x,y,a,z}$	numbers caught at age by sex in fishery f
$p^{F,A}_{f,x,y,\acute{a}} = \dfrac{\sum_a \Theta_{t,\acute{a},a} \cdot C_{f,x,y,a}}{\sum_x \sum_a C_{f,x,y,a}}$	proportions caught at age (age compositions)
$C_{f,x,y,z} = \sum_a C_{f,x,y,a,z}$	numbers caught at size by sex in fishery f
$p^{F,Z}_{f,x,y,z} = \dfrac{C_{f,x,y,z}}{\sum_x \sum_a C_{f,x,y,z}}$	proportions caught at size (size compositions)
$C_{f,y} = \sum_x \sum_a \sum_z w_{x,t,z} \cdot C_{f,x,y,a,z}$	total catch (biomass) in fishery f

Table 13. Fishery dynamics.

Term	Description
$N^S_{s,x,y,a,z} = \tilde{N}_{x,y,a,z} \cdot e^{-Z_{x,y,a,z} \cdot \delta t_s}$	stock numbers at time of survey s
$Q_{s,x,y} = Q_{0s,t} \cdot exp(\Delta Q_{s,t} \cdot \delta_x + QT_{s,t} \cdot \delta T_y)$	survey catchability
$C^S_{s,x,y,a,z} = \alpha_s \cdot Q_{s,x,y} \cdot S^S_{s,x,t,a,z} \cdot N^S_{s,x,y,a,z}$	numbers caught at sex x, age a, size z in survey s α_s = availability; $S^S_{s,x,t,a,z}$ = selectivity
$C^S_{s,x,y,a} = \sum_z C^S_{s,x,y,a,z}$	numbers caught at age by sex in survey s
$p^{S,A}_{s,x,y,a} = \dfrac{\sum_a \Theta_{t,\acute{a},a} \cdot C^S_{s,x,y,a}}{\sum_x \sum_a C^S_{s,x,y,a}}$	proportions caught at age (age compositions)
$C^S_{s,x,y,z} = \sum_a C^S_{s,x,y,a,z}$	numbers caught at size by sex in survey s
$p^{S,Z}_{s,x,y,z} = C^S_{s,x,y,z} / \sum_x \sum_a C^S_{s,x,y,z}$	proportions caught at size (size compositions)
$B^S_{s,y} = \sum_x \sum_a \sum_z w_{x,t,z} \cdot C^S_{s,x,y,a,z}$	total survey biomass in survey s

Table 14. Survey dynamics.

Component	Description
$ln\mathcal{L}_{HF} = -\sum\limits_{f}\left[\lambda_f^{HF}\cdot\left\{\ln(\tilde{\sigma}_f^{HF})+\left[\dfrac{\ln(\tilde{C}_f^{HF}+\eta)-\ln(C_f^{HF}+\eta)}{\sqrt{2}\cdot\tilde{\sigma}_f^{HF}}\right]^2\right\}\right]$	• "historical" fishery catch (yield) data • assumes observation errors are lognormally-distributed • η is a small constant ($<10^{-5}$) • The λ_f^{HF} represent user-specified likelihood weights for each fishery
$ln\mathcal{L}_{FC} = -\sum\limits_{f}\left[\lambda_f^{F}\cdot\sum\limits_{y\in\{y^F\}}\left\{\ln(\tilde{\sigma}_{f,y}^{F})+\left[\dfrac{\ln(\tilde{C}_{f,y}^{F}+\eta)-\ln(C_{f,y}^{F}+\eta)}{\sqrt{2}\cdot\tilde{\sigma}_{f,y}^{F}}\right]^2\right\}\right]$	• fishery catch (yield) data • assumes observation errors are lognormally-distributed
$ln\mathcal{L}_{FA} = -\sum\limits_{f}\left[\lambda_f^{F,A}\cdot\sum\limits_{y\in\{y^{F,A}\}}\sum\limits_{x}\sum\limits_{a}\tilde{n}_{f,y}^{F,A}\cdot\left\{\tilde{p}_{f,x,y,a}^{F,A}\cdot\ln(p_{f,x,y,a}^{F,A}+\eta)-\tilde{p}_{f,x,y,a}^{F,A}\cdot\ln(\tilde{p}_{f,x,y,a}^{F,A}+\eta)\right\}\right]$	• fishery age composition data • assumes sampling distribution is multinomial • $n_{f,y}^{F,A}$ is the observed sample size in fishery f for year y
$ln\mathcal{L}_{FZ} = -\sum\limits_{f}\left[\lambda_f^{F,Z}\cdot\sum\limits_{y\in\{y^{F,Z}\}}\sum\limits_{x}\sum\limits_{a}\tilde{n}_{f,y}^{F,Z}\cdot\left\{\tilde{p}_{f,x,y,a}^{F,Z}\cdot\ln(p_{f,x,y,a}^{F,Z}+\eta)-\tilde{p}_{f,x,y,a}^{F,Z}\cdot\ln(\tilde{p}_{f,x,y,a}^{F,Z}+\eta)\right\}\right]$	• fishery size composition data • assumes sampling distribution is multinomial • $\tilde{n}_{f,y}^{F,Z}$ is the observed sample size in fishery f for year y

Table 15. Likelihood components related to fishery data.

Component	Description
$ln\mathcal{L}_{SB} = -\sum\limits_{s}\left[\lambda_s^{S}\cdot\sum\limits_{r\in\{t^S\}}\left\{\ln(\tilde{\sigma}_{s,y}^{S})+\left[\dfrac{\ln(\tilde{B}_{s,y}^{S}+\eta)-\ln(B_{s,y}^{S}+\eta)}{\sqrt{2}\cdot\tilde{\sigma}_{s,y}^{S}}\right]^2\right\}\right]$	Survey biomass data; assumes lognormal distributions
$ln\mathcal{L}_{SA} = -\sum\limits_{s}\left[\lambda_s^{S,A}\cdot\sum\limits_{y\in\{y^{S,A}\}}\sum\limits_{x}\sum\limits_{a}\tilde{n}_{s,y}^{S,A}\cdot\left\{\tilde{p}_{s,x,y,a}^{S,A}\cdot\ln(p_{s,x,y,a}^{S,A}+\eta)-\tilde{p}_{s,x,y,a}^{S,A}\cdot\ln(\tilde{p}_{s,x,y,a}^{S,A}+\eta)\right\}\right]$	• survey age composition data • assumes sampling distribution is multinomial. • $\tilde{n}_{s,y}^{S,A}$ is the observed sample size in survey s for year y.
$ln\mathcal{L}_{SZ} = -\sum\limits_{s}\left[\lambda_s^{S,Z}\cdot\sum\limits_{y\in\{y^{S,Z}\}}\sum\limits_{x}\sum\limits_{z}\tilde{n}_{s,y}^{S,Z}\cdot\left\{\tilde{p}_{s,x,y,z}^{S,Z}\cdot\ln(p_{s,x,y,z}^{S,Z}+\eta)-\tilde{p}_{s,x,y,z}^{S,Z}\cdot\ln(\tilde{p}_{s,x,y,z}^{S,Z}+\eta)\right\}\right]$	• survey size composition data • assumes sampling distribution is multinomial • $\tilde{n}_{t}^{S,L}$ is the observed sample size in survey s for year y

Table 16. Likelihood components related to survey data.

Component	Description
$ln\mathcal{L}_{WatZ} = -\lambda^{WatZ} \cdot \sum_i \left\{ ln(\sigma^W_{x,y,z}) + \left[\dfrac{ln(\tilde{W}_i + \eta) - ln(W_{x,y,z} + \eta)}{\sqrt{2} \cdot \sigma^W_{x,y,z}} \right]^2 \right\}$	• individual weight-at-size data • i represents individual weight samples • x, y, z correspond to values for i
$ln\mathcal{L}_{ZatA} = -\lambda^{ZatA} \cdot \sum_i \left\{ n_i \cdot ln(\sigma^Z_{x,y,a}) + \dfrac{n_i - 1}{2} \left(\dfrac{\tilde{\sigma}^Z_i}{\sigma^Z_{x,y,a}} \right)^2 + \left[\dfrac{ln(\tilde{Z}_i + \eta) - ln(Z_{x,y,a} + \eta)}{\sqrt{2} \cdot \sigma^Z_{x,y,a}} \right]^2 \right\}$	• mean size at age data. • i represents mean size-at-age • n_i represents the number of samples in the mean • $\tilde{\sigma}^Z_i$ represents the standard deviation in the estimate • x, y, a correspond to values for i
Ageing error	See Punt et al. (2008).

Table 17. Likelihood components related to weight-at-size, size-at-age and ageing error data.

Component	Description
$ln\mathcal{L}_{Rec} = -\sum_t \left[\lambda^{Rec}_t \cdot \sum_{y \in \{y_t\}} \left\{ ln(\sigma^{Rec}_t) + \left[\dfrac{\tau_y}{\sqrt{2} \cdot \sigma^{Rec}_t} \right]^2 \right\} \right]$	• recruitment process error • t represents individual time stanzas • σ^{Rec}_t is the assumed log-scale standard deviation for stanza t
$ln\mathcal{L}_{SR} = -\sum_t \left[\lambda^{SR}_t \cdot \sum_{y \in \{y_t\}} \left\{ ln(\sigma^{SR}_t) + \left[\dfrac{\varepsilon_y}{\sqrt{2} \cdot \sigma^{SR}_t} \right]^2 \right\} \right]$	• initial sex ratio process error • t represents individual time stanzas • σ^{SR}_t is the assumed logit-scale standard deviation for stanza t

Table 18. Likelihood components related to process error: recruitment deviations and initial sex ratio deviations.

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