APPENDIX F

ESTIMATION OF DREDGED MATERIAL CONSOLIDATION BY FINITE STRAIN TECHNIQUE

F-1. General. In this appendix, the technique for estimating consolidation by finite strain techniques is described. Also, the practical problem of a single dredged fill layer deposited on a compressible foundation will be solved for settlement as a function of time by both small strain and linear finite strain theories. The solutions will involve only hand calculations and the appropriate percent consolidation curves given previously in this report.

F-2. Estimation of Consolidation Using Finite Strain Technique.

a. Laboratory Test Data. Consolidation of dredged material due to self weight must be estimated using results from appropriate laboratory tests. The following procedure for hand computation uses standard consolidation (oedometer) laboratory test data. Procedures for these tests are described in Chapter 3 and Appendix D. The laboratory tests yield a relationship between void ratio and effective stress as shown in Figure F-1. An exponential form for the relationship should be determined by curve fitting techniques. The fit should be of the form:

$$e = (e_{oo} - e_{\infty}) \exp(-\lambda\sigma') + e_{\infty} \qquad \text{(F-1)}$$

where e_{oo} is void ratio at zero effective stress and e_{∞} is the void ratio at infinite effective stress. Such a curve is also shown in Figure F-1 where λ, e_{oo}, and e_{∞} were chosen to give the best apparent fit to the test data.

b. Determination of Layer Thicknesses. The void ratio at the end of the sedimentation phase as well as initial thickness of the deposited layer will be determined from column settling tests as described in Chapter 3. The layer thickness in reduced coordinates for each deposited layer should be calculated as follows:

$$\ell = \frac{h}{1 + e_{oo}} \qquad \text{(F-2)}$$

where h is the layer thickness as deposited and e_{oo} is the initial void ratio since the effective stress is assumed initially zero throughout the layer. In a normally consolidated layer or layer having any other than uniform void ratio distribution, ℓ can be calculated to sufficient accuracy by dividing the layer into a number, m, of sublayers and using

$$\ell = \sum_{i=1}^{m} \ell_i = \sum_{i=1}^{m} \frac{h_i}{1 + e_i} \qquad \text{(F-3)}$$

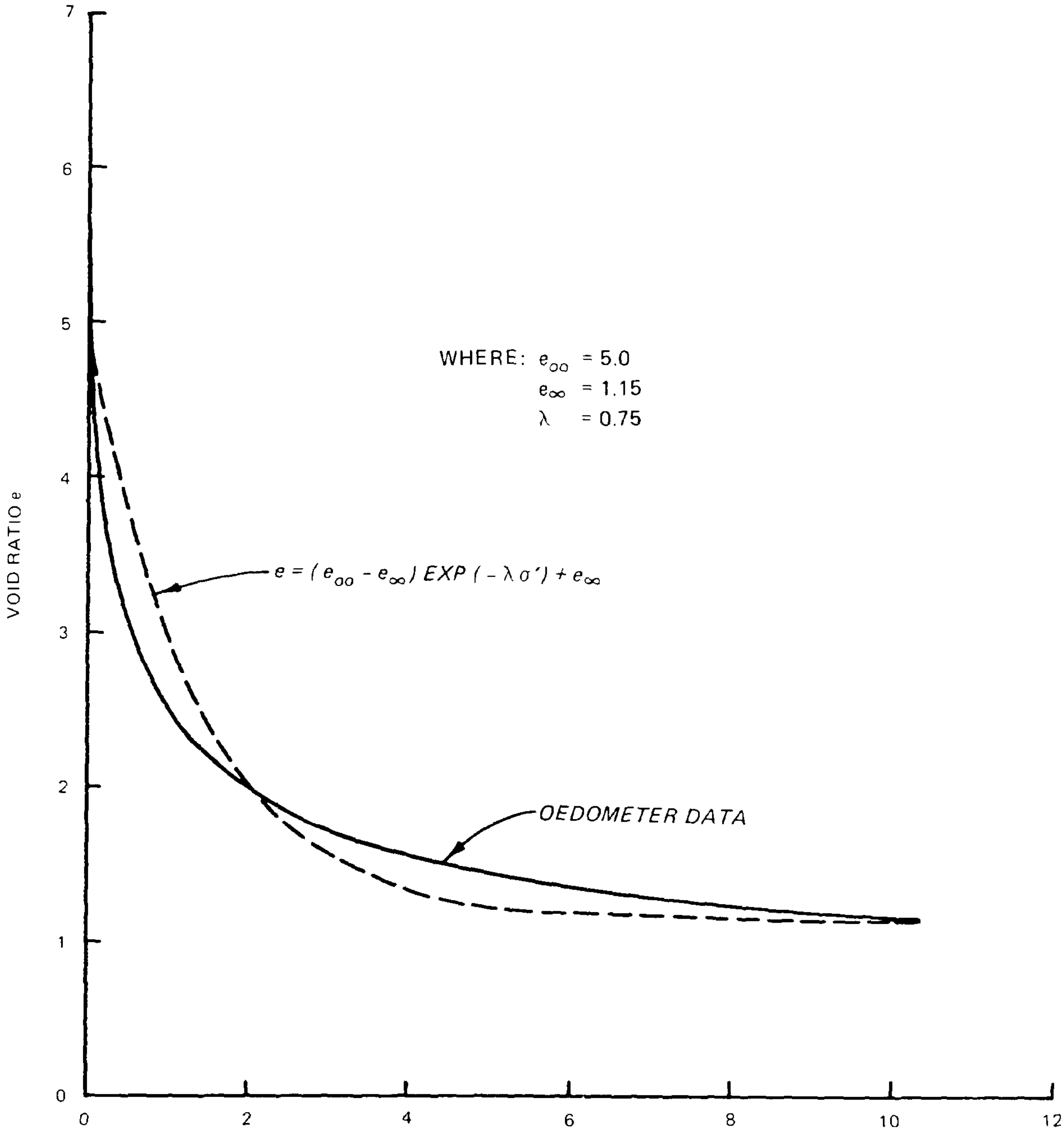


Figure F-1. Exponential void ratio-effective stress relationship compared to oedometer data, 0-12.0 tsf where h_i is the sublayer height and e_i is the average void ratio in the sublayer. The sublayer void ratio is obtained from the $e - \log \sigma'$ curve for the material by considering the effective weight of all material and surcharge above the center of the sublayer or by direct measurement of the saturated water content of the sublayer.

c. Calculation of Ultimate Settlement.

(1) The ultimate settlement of a consolidating fine-grained layer is defined as that which has occurred after all excess pore pressures have dissipated. Within the layer, the material assumes a void ratio distribution due to the buoyant weight of material above plus any surcharge, and this void ratio is related to the effective stress by the material's $e - \log \sigma'$ curve as determined by laboratory testing. Therefore, initial and final void ratio distributions are known or can be calculated.

(2) Ultimate settlement is calculated by dividing the total layer into a number, m , of sublayers such that

$$\delta(\infty) = \sum_{i=1}^{m} \delta_{i,\infty} = \sum_{i=1}^{m} (e_{i,o} - e_{i,\infty})\, \ell_i \qquad \text{(F-4)}$$

where δ is the settlement, ℓ_1 is defined in Equation F-3, and $e_{i,o}$ and $e_{i,\infty}$ are the average initial and final void ratios of each sublayer, respectively. The ultimate average effective stress is then calculated for each sublayer by

$$\sigma'_i = \frac{1}{2} \ell_i (\gamma_s - \gamma_w) + \begin{pmatrix}\text{effective weight}\\ \text{of all sublayers}\\ \text{above it}\end{pmatrix} + (\text{surcharge}) \qquad \text{(F-5)}$$

where the effective weight of each sublayer is $\ell_i (\gamma_s - \gamma_w)$. Then, using this average effective stress, an average void ratio is picked from the oedometer test data and substituted into Equation F-4.

d. Calculation of Settlement Versus Time.

(1) The coefficient of consolidation for finite strain, g , should be determined from a plot such as shown in Figure F-2 for the void ratio corresponding to an average effective stress during the consolidation process if the coefficient is relatively constant over the range of expected void ratios. If there is substantial variation in the coefficient of consolidation over the expected range of void ratios, the coefficient can be periodically updated during the calculation to conform to the average void ratio in the layer at the time consolidation is calculated.

(2) The nondimensional time factor for the real time in question is calculated as follows:

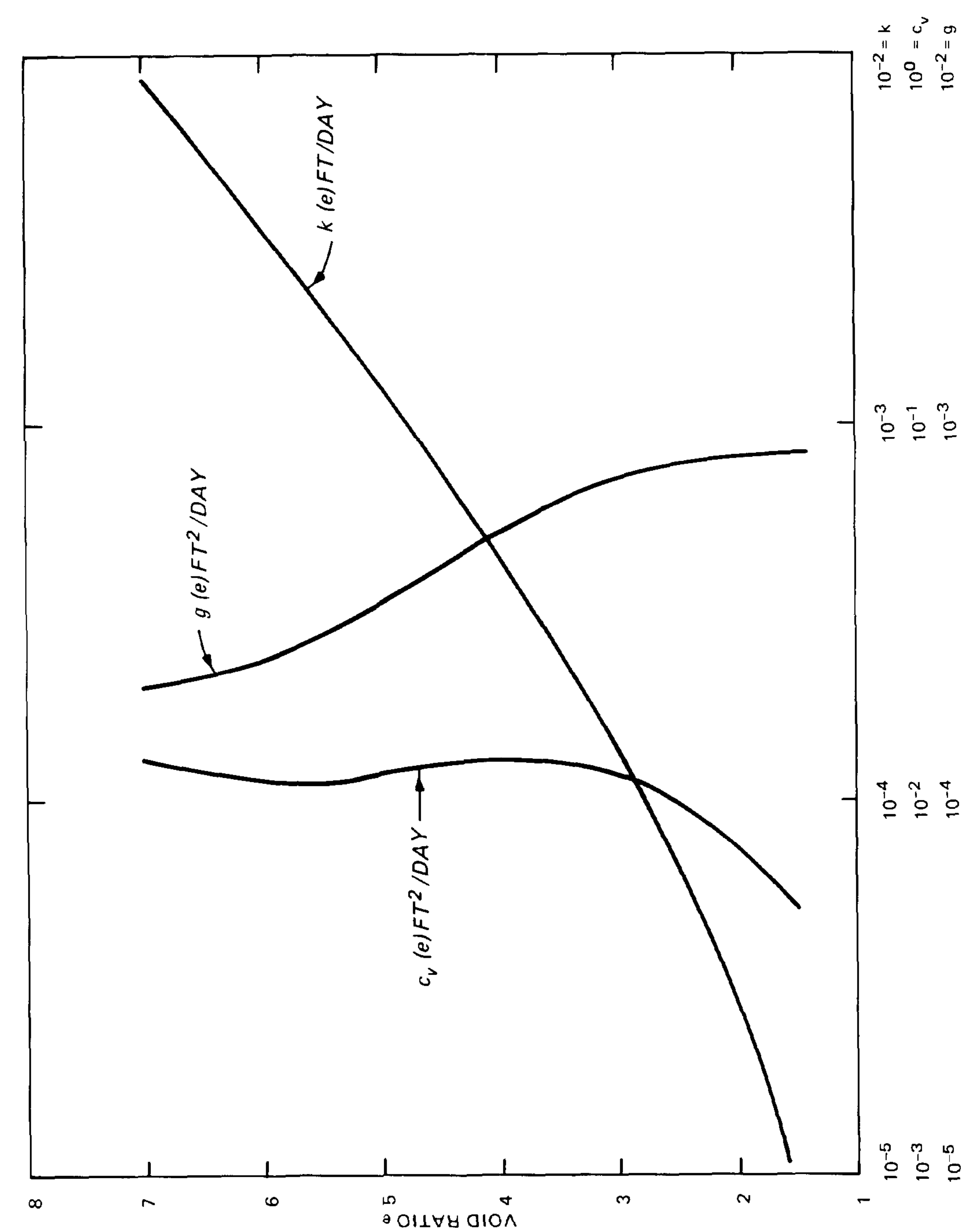


Figure F-2. Typical permeability and coefficients of consolidation as a function of void ratio

$$T_{fs} = \frac{gt}{\ell^2} \qquad \text{(F-6)}$$

(3) Calculate the dimensionless parameter N as follows:

$$N = \lambda \, \ell \, (\gamma_s - \gamma_w) \qquad \text{(F-7)}$$

(4) The percent consolidation, U , is then read from Figure F-3 thru F-6, depending on the value of N and initial conditions and boundary conditions for the calculated time factor.

(5) With the percent consolidation known, settlement is then

$$\delta(T_{fs}) = \delta_\infty \cdot U(T_{fs}) \qquad \text{(F-8)}$$

at the real time t chosen in calculating T_{fs} .

(6) An example of this procedure for a single dredged fill layer deposited on a compressible foundation is solved in F-4 by both a small strain and linear finite strain formulation. In the example, an updated coefficient of consolidation and layer height are used in calculating the dimensionless time factor.

F-3. Empirical Estimate of Settlement Due to Desiccation.

a. Determination of Void Ratio at Saturation and Desiccation Limits.

(1) The void ratio at the saturation limit, e_{SL} , can be determined empirically as follows:

$$e_{SL} = \frac{1.8LL\ G_s}{100} \qquad \text{(F-9)}$$

where

e_{SL} = void ratio at saturation limit
LL = Atterberg liquid limit of dredged material in percent
G_s = Specific gravity at the dredged material

(2) The void ratio at the desiccation limit can be determined empirically as:

$$e_{DL} = \frac{1.2\ PL\ G_s}{100} \qquad \text{(F-10)}$$

where

e_{DL} = void ratio of desiccation limit

PL = Atterberg plastic limit of dredged material in percent

b. Calculation of Desiccation Depths.

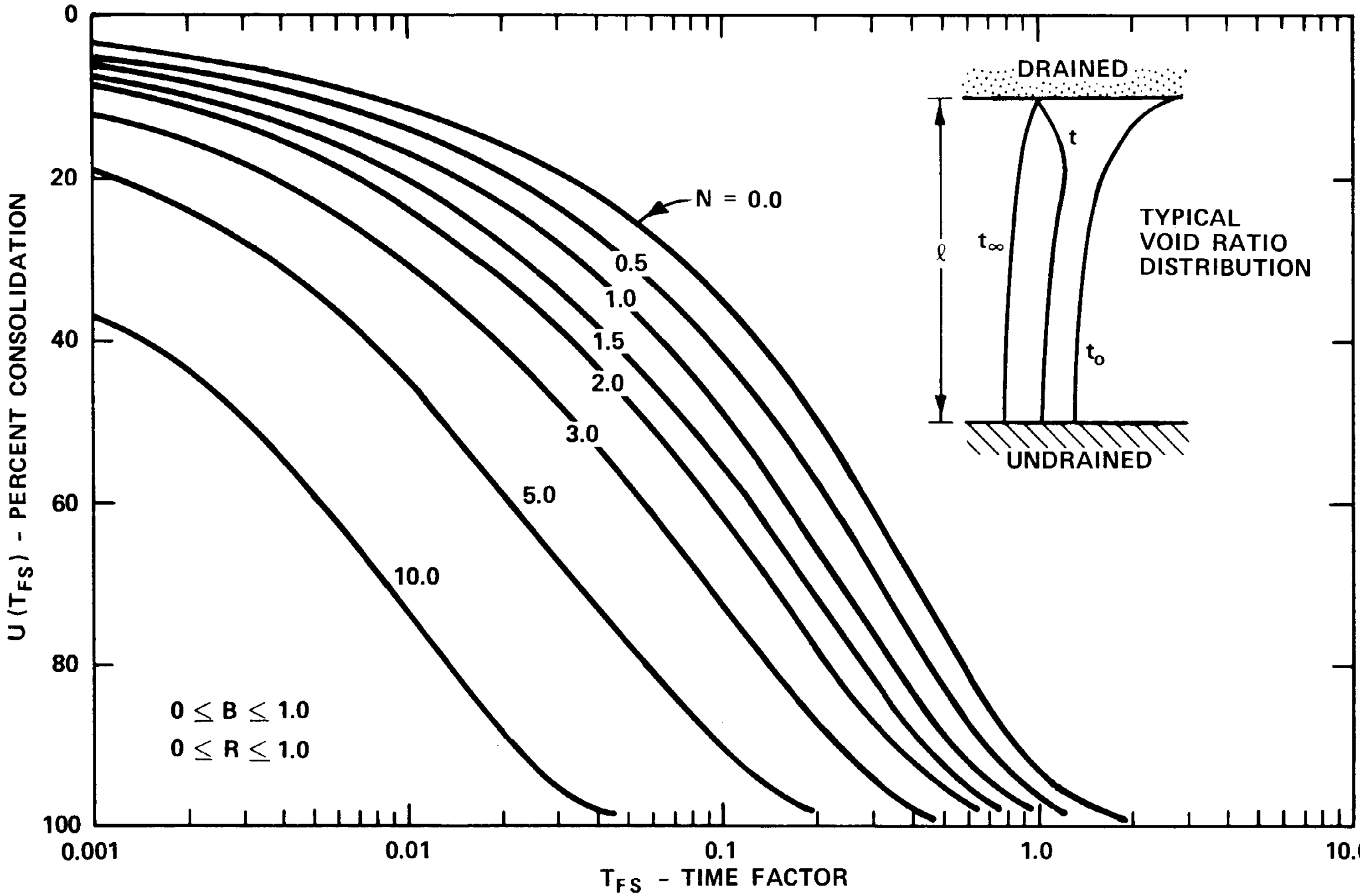


Figure F-3. Degree of consolidation as a function of the time factor for normally consolidated, singly drained layers by linear finite strain theory

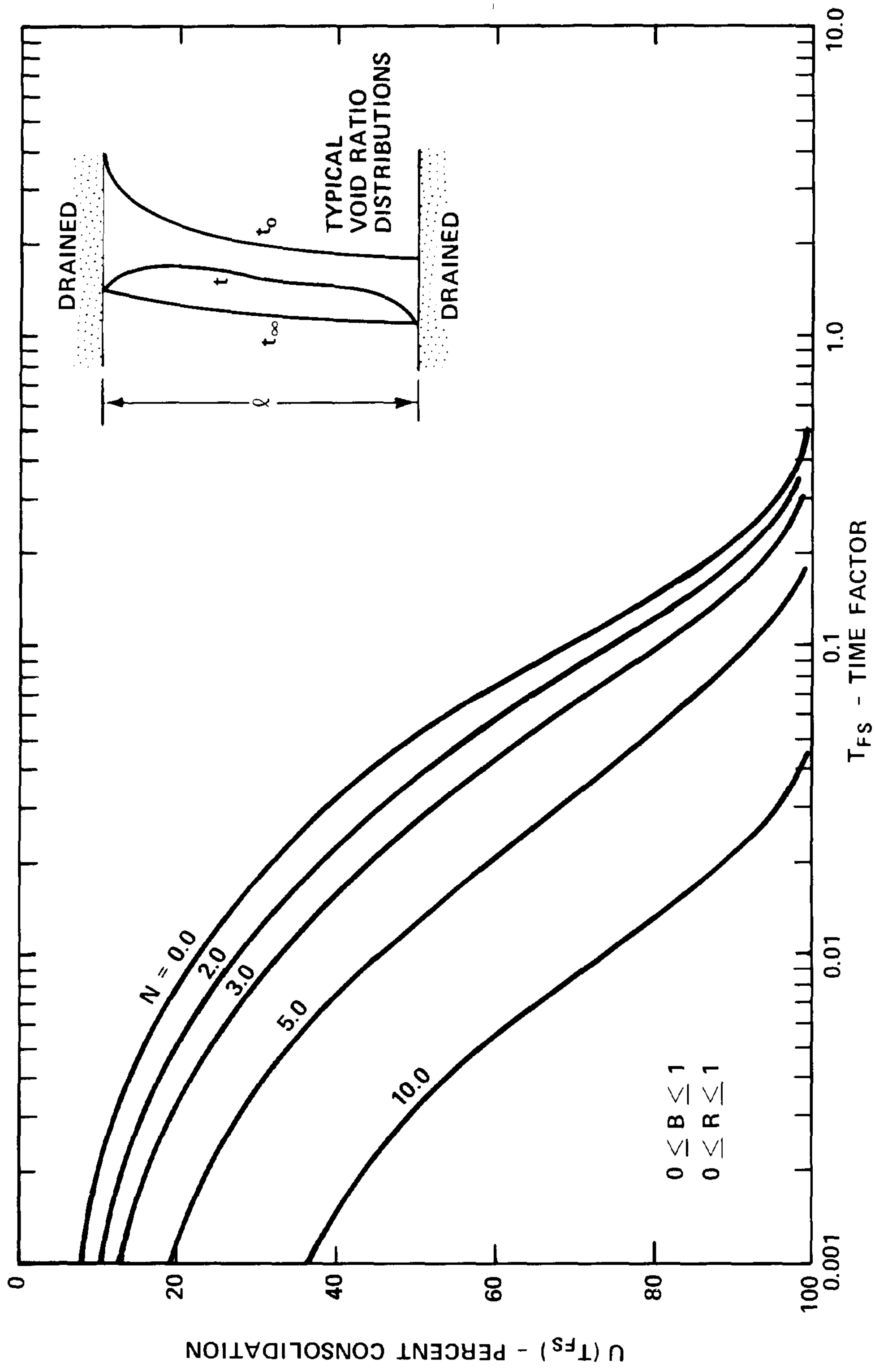


Figure F-4. Degree of consolidation as a function of the time factor for normally consolidated, doubly drained layers by linear finite strain theory

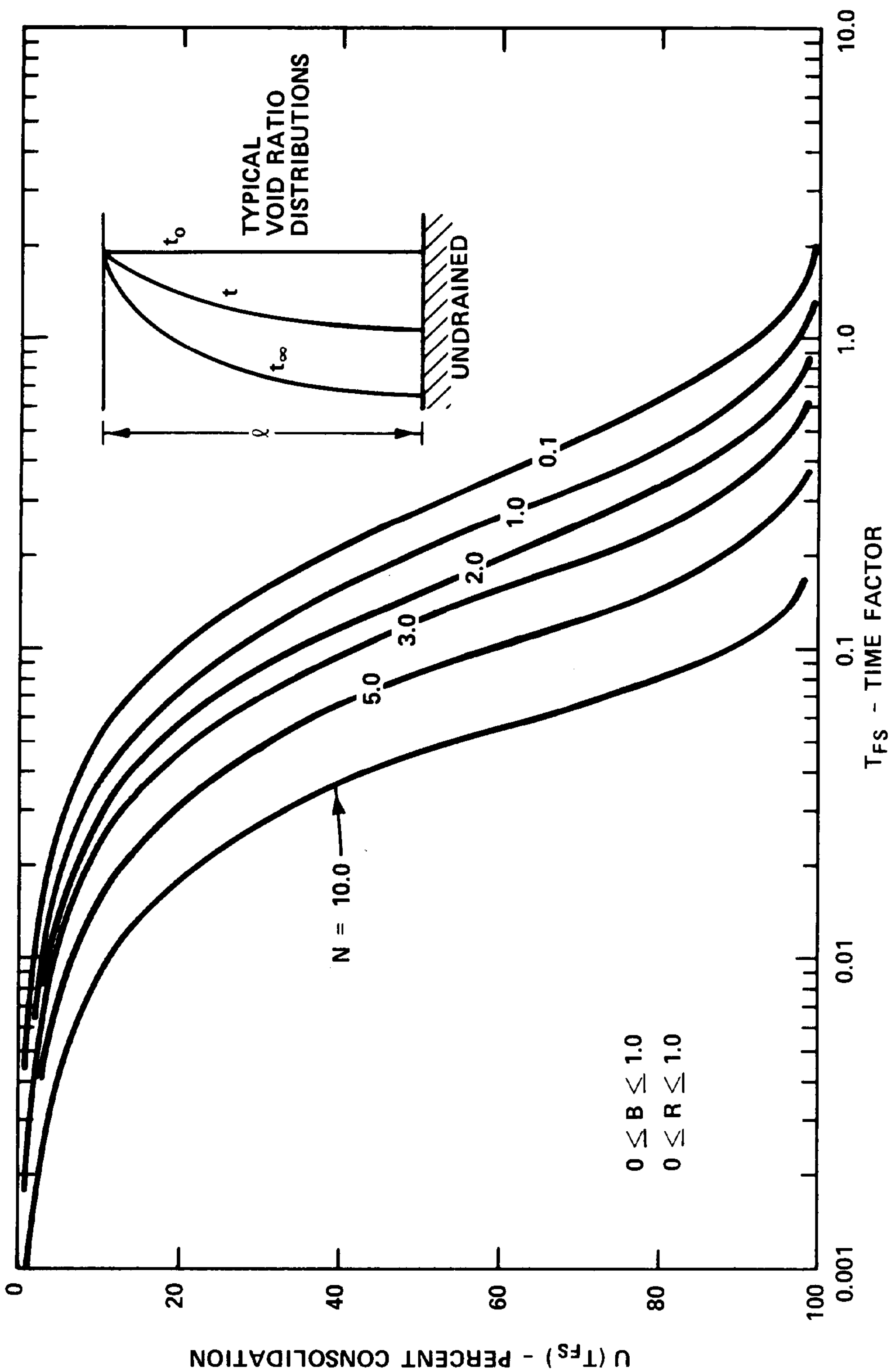


Figure F-5. Degree of consolidation as a function of the time factor for dredged fill, singly drained layers by linear finite strain theory

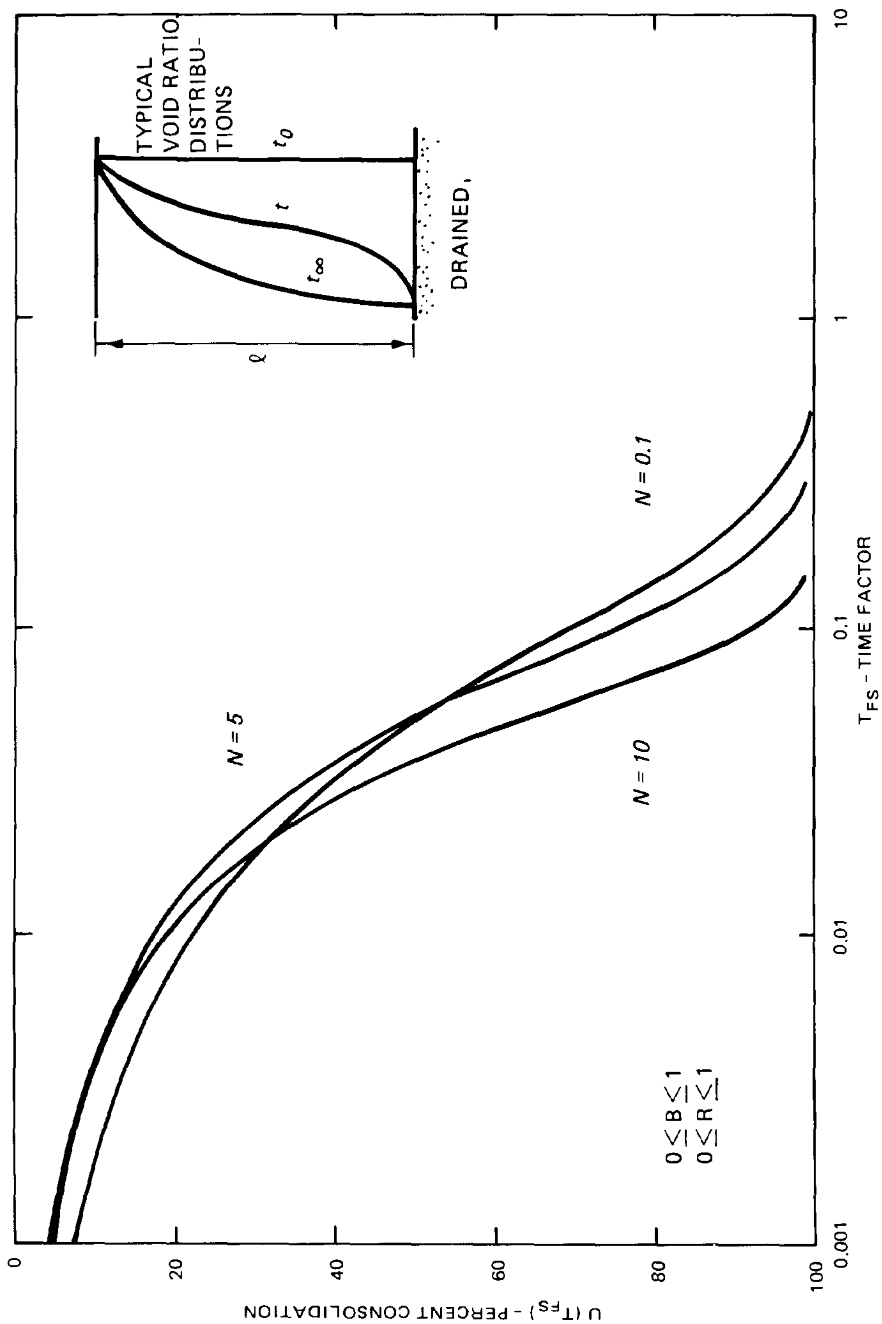


Figure F-6. Degree of consolidation as a function of the time factor for dredged fill, doubly drained layers by linear finite strain theory

(1) As long as the material remains saturated and the free water table is at the surface, the effects of evaporative drying cannot extend deeper than the intersection of the ordinate denoting e_{SL} and the ultimate void ratio distribution curve (See Figure F-7). Thus, the maximum depth to which first-stage drying can occur is

$$h_{1st} = (\ell - z_{SL})(1 + e_{SL}) \qquad \text{(F-11)}$$

where

h_{1st} = maximum depth of first-stage drying

z_{SL} = material coordinate at intersection of e_{SL} and ultimate void ratio distribution curve

While void ratios lower than e_{SL} may exist in the dredged material below z_{SL}, they are due to self-weight consolidation and not surface desiccation during first-stage drying.

(2) The absolute maximum depth to which second-stage drying can occur is the water table depth (which sometimes can be measured in the field) or the intersection of the ordinate denoting e_{DL} with the ultimate void ratio

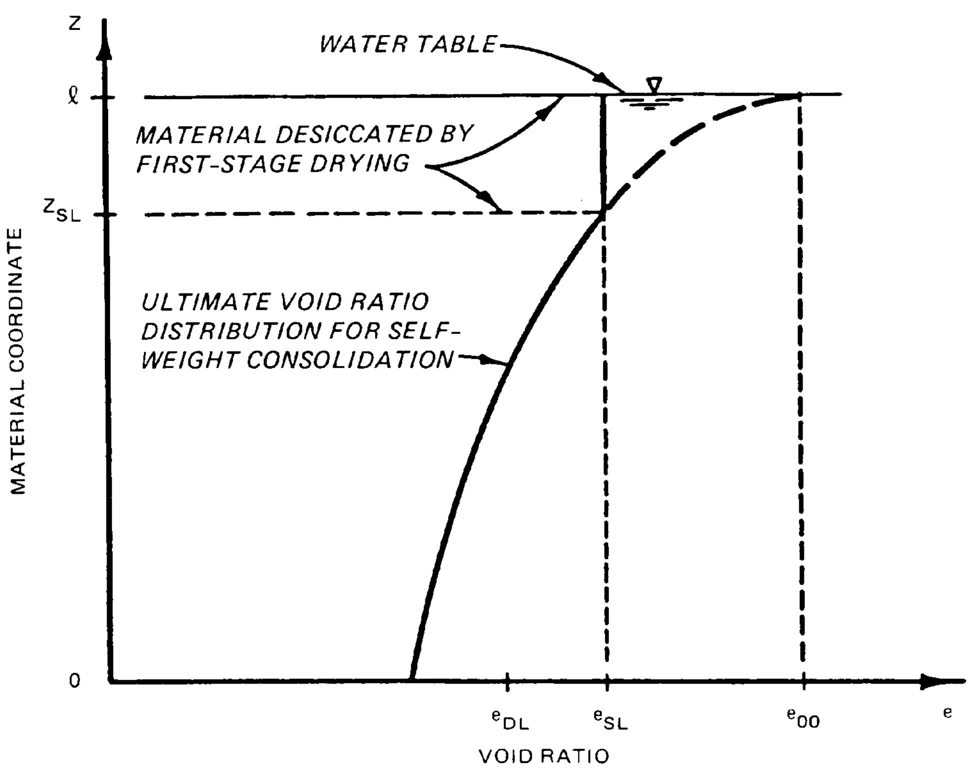


Figure F-7. Maximum depth of material desiccated by first-stage drying

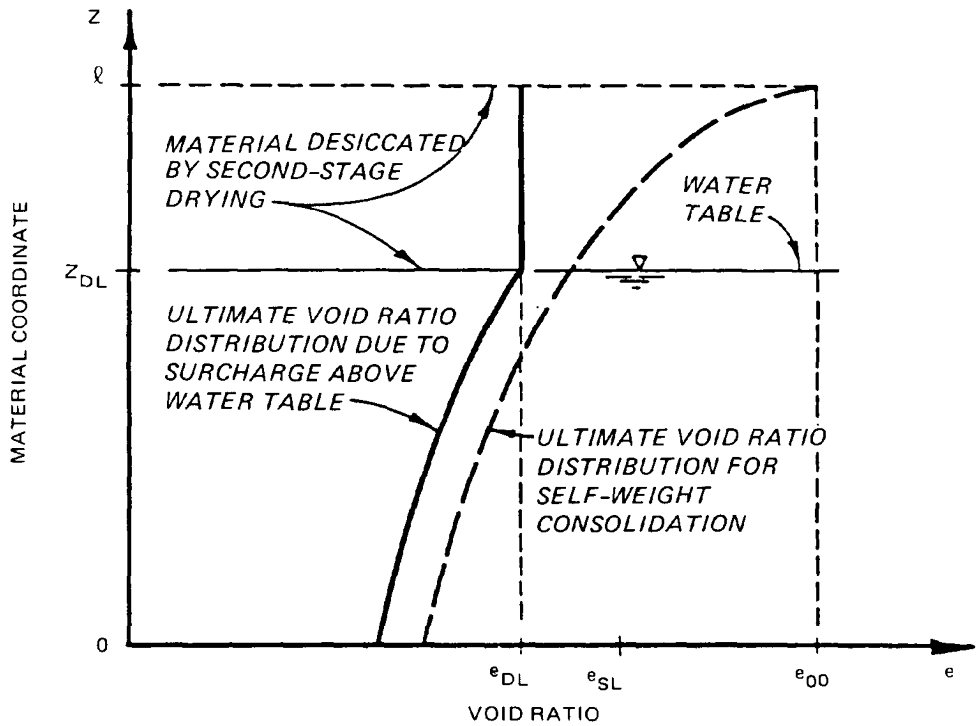


Figure F-8. Maximum depth of material desiccated by second-stage drying

distribution curve that is based on the surcharge induced (See Figure F-8). In equation form

$$h_{2nd} = (\ell - z_{DL})\ (1 + e_{DL}) \qquad \text{(F-12)}$$

where

h_{2nd} = maximum depth of second-stage drying

z_{DL} = material coordinate at intersection of e_{DL} and ultimate void ratio distribution curve

Again it can be seen that void ratios lower than e_{DL} may exist below z_{DL} due to consolidation effects. It is also important to note that h_{1st} can be larger than h_{2nd} due to the low void ratio of a completely desiccated dredged material. A field indicator of the depth to which second-stage drying can be effective is the depth of cracks in the dredged material. Of course, cracks subjected to periodic rainfall are probably shallower than they would be under constant evaporative conditions.

(3) The preceding two equations form a rational basis for estimating the depths of crust formation in dredged material under first- and second-stage drying. They should be applicable whenever sufficient dredged material is

present to provide an intersection between the ultimate void ratio distribution and the appropriate limiting void ratio, and there is no external influence limiting the water table depth. If insufficient material is present, the entire dredged fill layer may be subjected to first- and second-stage drying processes in turn. If the water table depth is limited, the second-stage drying depth will be similarly limited. Again, the practical maximum depth of second-stage drying is best estimated from the maximum depth of desiccation cracks.

(4) The maximum depth of first-stage drying as expressed in Equation 5-11 should be a realistic measure for most fine-grained soils whose e_{SL} intersects the consolidated void ratio curve above the material coordinate defining the soil's maximum field crust thickness. For those soils with an e_{SL} so low that z_{SL} is greater than z_{DL} when based on the preceding considerations, the z_{SL} should be limited to no greater than z_{DL} .

c. Evaporation Efficiencies. The expression for defining the drying rate during second-stage evaporation will be simply a linear function of the water table depth:

$$C_E = C'_E \; 1 - \frac{h_{wt}}{h_{2nd}} \quad \text{for} \quad h_{wt} \leqq h_{2nd} \qquad \text{(F-13)}$$

where

C_E = evaporation efficiency

h_{wt} = depth of water table below surface

h_{2nd} = maximum depth of second-stage drying

C'_E = maximum evaporation efficiency for soil type

d. Water Loss and Desiccation Settlement.

(1) The water lost from a dredged material layer during first-stage drying can be written

$$\Delta W' = CS - C'_E \cdot EP + (1 - C_D)\ RF \qquad \text{(F-14)}$$

where

$\Delta W'$ = water lost during first-stage drying
CS = water supplied from lower consolidating material
EP = pan evaporation rate
CD = drainage efficiency
RF = rainfall

Even though some minor cracks may appear in the surface during this stage, the material will remain saturated, and vertical settlement is expected to correspond with water loss or

$$\delta'_D = -\Delta W' \qquad \text{(F-15)}$$

where δ'_D = settlement due to second-stage drying.

(2) Water lost during second-stage drying can be written

$$\Delta W'' = CS - C'_E \; 1 - \frac{h_{wt}}{h_{2nd}} \cdot EP + (1 - C_D)RF \qquad \text{(F-16)}$$

where

$\Delta W''$ = water lost during second-stage drying.

(3) Two things prevent an exact correspondence between water loss and settlement during second-stage drying. First is appearance of an extensive network of cracks that may encompass up to 20 percent of the volume of the dried layer. Second is the probable loss of saturation within the dried material itself. Combining these two occurrences into one factor enables the vertical settlement to be written

$$\delta''_D = -\Delta W'' - 1 - \frac{PS}{100} h_{wt} \qquad \text{(F-17)}$$

where

δ''_D = settlement due to second-stage drying

PS = gross percent saturation of dried crust that includes cracks

In determining the second-stage drying settlement, there are three unknowns and only two equations. Therefore, hand calculation involves an iterative procedure.

(4) The empirical approach as outlined and interaction of consolidation and desiccation are incorporated in the computer solutions described in 5-2.d. Use of the computer solutions is recommended for evaluation of the long-term storage capacity of confined disposal areas.

F-4. Example Consolidation Calculations.

a. Problem Statement. It is required to determine the time rate of surface settlement of a 10.0-foot-thick, fine-grained dredged fill material having a uniform initial void ratio after sedimentation of 7.0. The layer will be deposited on a normally consolidated compressible foundation 10.0 feet thick that overlies an impermeable bedrock. Laboratory oedometer testing of the dredged material resulted in the σ'-e relationship shown in Figure F-l and k-e , c_v-e , and g-e relationships as shown in Chapter 5. Laboratory oedometer testing of the foundation material resulted in the relationships shown in Figures F-9 and F-10. Laboratory testing also revealed specific gravity of solids G_s = 2.75 in the dredged material and G_s = 2.65 in the foundation material.

b. Void Ratio Distributions.

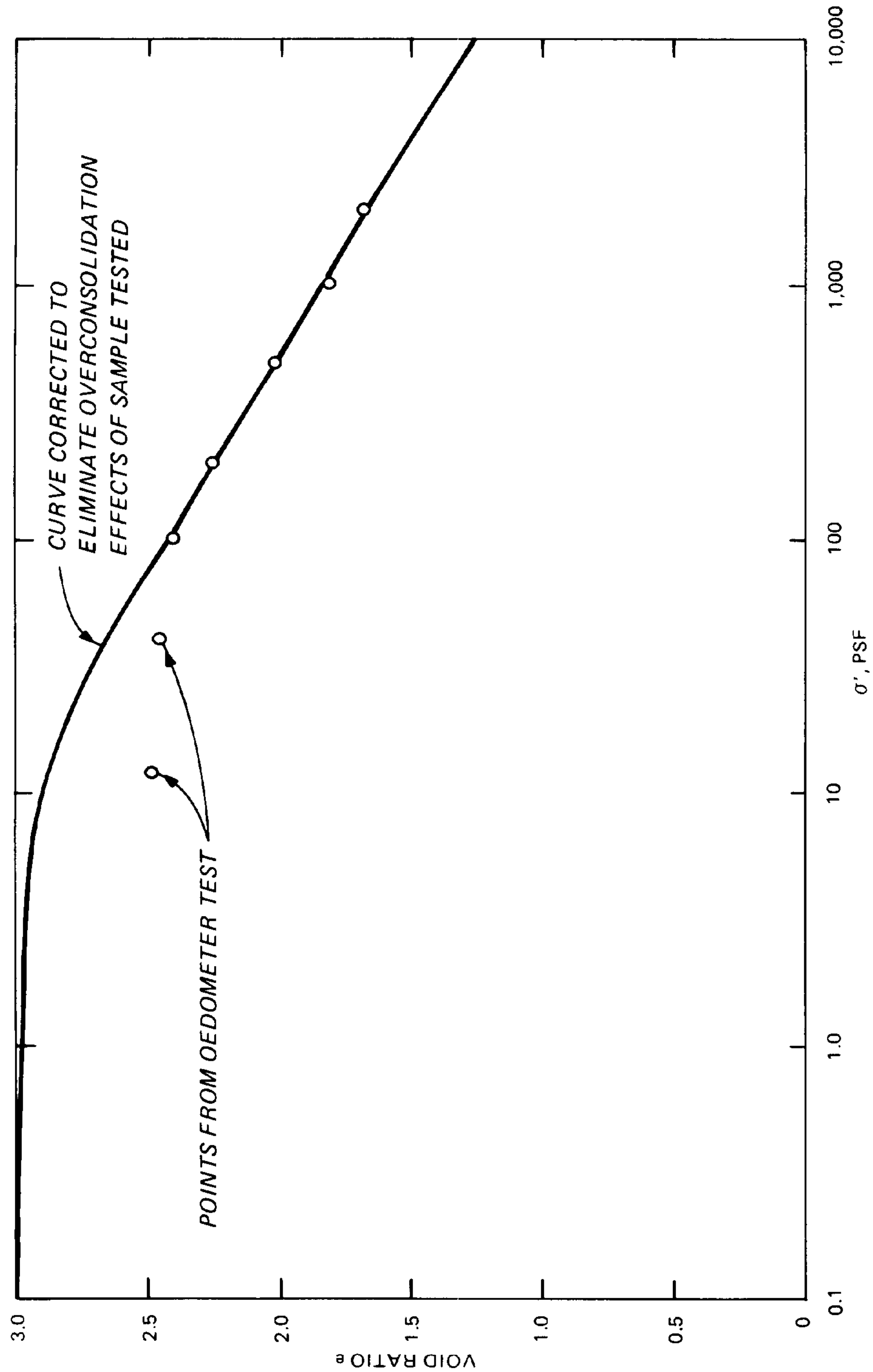


Figure F-9. Relationship between void ratio and effective stress, e-log σ' curve, for compressible foundation for oedometer testing

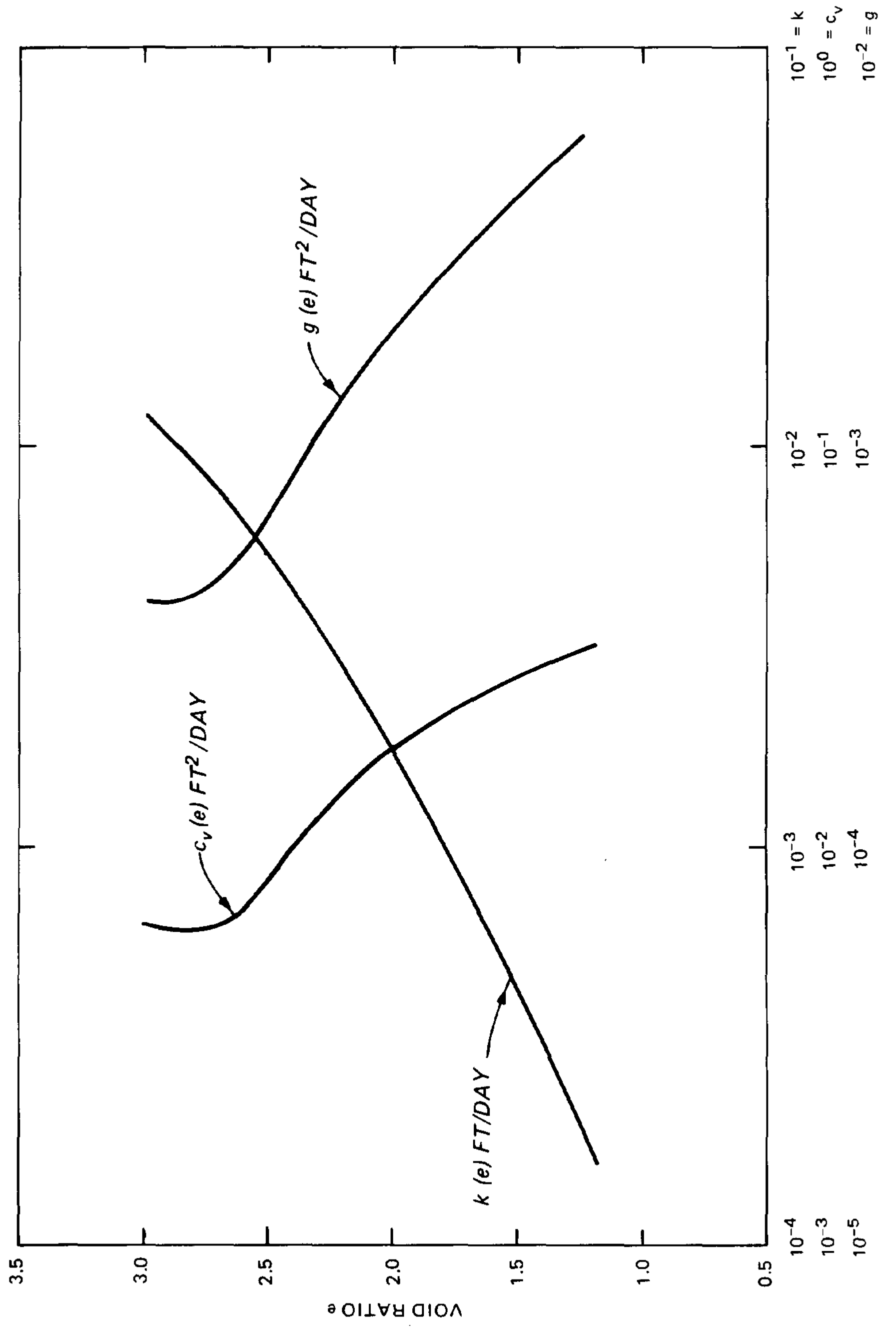


Figure F-10. Permeability and coefficients of consolidation as a function of void ratio for a compressible foundation

Table F-1
Void Ratio Distribution and Ultimate Settlement Calculations*

i	$h_{i,o}$ ft	ℓ_i ft	$\sigma'_{i,o}$ psf	$e_{i,o}$	$\sigma'_{i,\infty}$ psf	$e_{i,\infty}$	$h_{i,\infty}$ ft	$\delta_{i,\infty}$ ft
1	1.0	0.125	0.0	7.00	6.8	6.52	0.94	0.06
2	1.0	0.125	0.0	7.00	20.5	5.93	0.87	0.13
3	1.0	0.125	0.0	7.00	34.1	5.57	0.82	0.18
4	1.0	0.125	0.0	7.00	47.8	5.34	0.79	0.21
5	1.0	0.125	0.0	7.00	61.4	5.14	0.77	0.23
6	1.0	0.125	0.0	7.00	75.1	4.98	0.75	0.25
8	1.0	0.125	0.0	7.00	102.4	4.75	0.72	0.28
9	1.0	0.125	0.0	7.00	116.0	4.65	0.71	0.29
10	1.0	0.125	0.0	7.00	129.7	4.57	0.70	0.30
	Σ = 10.0	Σ = 1.250					Σ = 7.80	Σ = 2.20

Foundation

i	$h_{i,o}$ ft	ℓ_i ft	$\sigma'_{i,o}$ psf	$e_{i,o}$	$\sigma'_{i,\infty}$ psf	$e_{i,\infty}$	$h_{i,\infty}$ ft	$\delta_{i,\infty}$ ft
1	1.0	0.259	13.3	2.86	149.8	2.31	0.86	0.14
2	1.0	0.275	40.9	2.64	177.4	2.26	0.90	0.10
3	1.0	0.286	69.7	2.50	206.2	2.23	0.92	0.08
4	1.0	0.293	99.5	2.41	236.0	2.20	0.94	0.06
5	1.0	0.299	130.0	2.35	266.5	2.17	0.95	0.05
6	1.0	0.305	161.1	2.28	297.6	2.14	0.95	0.04
7	1.0	0.308	192.6	2.25	329.1	2.11	0.95	0.04
8	1.0	0.312	224.6	2.21	361.1	2.09	0.96	0.04
9	1.0	0.314	256.8	2.18	393.3	2.07	0.96	0.03
10	1.0	0.317	289.2	2.15	425.7	2.05	0.90	0.03
	Σ = 10.0	Σ = 2.968					Σ = 9.38	Σ = 0.61

* Symbols are defined in the main text.

(1) For the most accurate calculations, it is necessary to know the distribution of void ratios throughout the consolidating layers both before consolidation begins and after it ends. As an aid in this and later calculations, Table F-I is set up where the layers are subdivided into ten increments each. Entries in the table correspond to average conditions at the center of each sublayer.

(2) Completion of the table is a straightforward exercise for the dredged fill layer. The column for $e_{i,o}$ is given in the problem statement and the initial effective stress $\sigma'_{i,o}$ will always be zero by definition. The sublayer depth in reduced coordinates is calculated directly from Equation F-2.

$$\ell_i = \frac{h_{i,o}}{1 + e_{i,o}} = \frac{1.0}{1 + 7.0} = 0.125 \text{ ft}$$

The ultimate effective stress $\sigma'_{i,\infty}$ column is computed from Equation F-5.

Thus

$$\sigma'_{1,\infty} = \frac{1}{2}\ell_1 \ (\gamma_s - \gamma_w) = \frac{0.125}{2} \ (2.75 - 1.0)62.4 \ = 6.8 \text{ psf}$$

and

$$\sigma'_{2,\infty} = \frac{1}{2}\ell_2 \ (\gamma_s - \gamma_w) + \ell_1(\gamma_s - \gamma_w) = 20.5 \text{ psf}$$

The final void ratio $e_{i,\infty}$ is read from the laboratory oedometer test curve. The usual $e\text{-}\log\ \sigma'$ curve is more accurate for this purpose than the curve given in Figure F-3. The final sublayer height $h_{i,\infty}$ is also calculated by substitution into Equation 5-2:

$$h_{1,\infty} = \ell_1(1 + e_{1,\infty}) = 0.125(1 + 6.52) = 0.94 \text{ ft}$$

(3) Completion of the table for the compressible foundation layer is not quite as simple since the initial void ratio is not usually known. However, it can be calculated given its $e\text{-}\log\ \sigma'$ curve in the normally consolidated state as shown in Figure F-9. An iterative process is required. First assume an initial void ratio for the first layer, $e_{1,o}$. Based on this void ratio, calculate ℓ from Equation F-2. Thus, assuming $e_{1,o} = 3.0$

$$\ell_i = \frac{h_1}{1 + e_{1,o}} = \frac{1.0}{1 + 3.0} = 0.250 \text{ ft}$$

Using this value of ℓ_1, $\sigma'_{1,o}$ is calculated from Equation F-5 as

$$\sigma'_{1,o} = \frac{1}{2}\ell_1(\gamma_s - \gamma_w) = \frac{0.250}{2}(2.65 - 1.0)62.4 = 12.9 \text{ psf}$$

Based on this value of $\sigma'_{1,o}$, a new estimate of $e_{1,0}$ is made from Figure F-9 and the process repeated until no further iterations are required. (Usually three iterations are required for an accuracy ±0.01 in the void ratio.) Using the total effective weight of the first layer, a first estimate of the void ratio in the second layer is made from Figure F-1 and its true average void ratio determined as was done with the first sublayer. The first estimate of each following sublayer is based on the effective weight of those above it.

(4) Once the initial void ratios and effective stresses have been determined throughout the compressible foundation, the final void ratios and effective stresses are easily found. The final effective stress $\sigma'_{1,\infty}$ is its initial value plus the effective weight of the dredged fill layer. Thus if

$$\sigma'_{\text{dredged fill}} = \ell_{d.f.}(\gamma_s - \gamma_w) = 136.5 \text{ psf}$$

then

$$\sigma'_{i,\infty} = \sigma'_{i,o} + 136.5$$

for the foundation. The final sublayer void ratio can then be read from the e-log σ' curve, and the final sublayer height $h_{i,\infty}$ can be calculated from Equation F-3.

c. Ultimate Settlement. Ultimate settlements for the compressible layers are calculated directly from Equation F-4 or from the difference in the sum of the sublayer heights initially and finally. As shown in Table F-l, for the dredged fill, δ_∞ = 2.20 feet, and for the foundation, δ_∞ = 0.61 feet. The fact that ultimate settlement plus total sublayer final heights in the foundation does not equal the initial total sublayer heights is due to round-off errors in the calculations.

d. Settlement as a Function of Time.

(1) A prerequisite to determining settlement as a function of time is the selection of an appropriate coefficient of consolidation during the course of consolidation, and in the case of linear finite strain theory, appropriate values for λ and N .

(2) For the dredged fill layer, a look at Table F-1 shows the void ratio will vary between the extremes 7.00 to 4.57. Figure F-2 is used to determine the appropriate coefficient of consolidation for the average void ratio during consolidation. For the foundation, where the void ratio extremes are 2.86 to 2.05, Figure F-10 is used.

(3) The value of λ must be determined by approximating the laboratory-determined curve with one of the form of Equation F-1. Figure F-12a shows that an appropriate value for the dredged fill is

$$\lambda = 0.026 \text{ cubic foot per pound}$$

and Figure F-12b shows that an appropriate value for the foundation is:

$$\lambda = 0.009 \text{ cubic foot per pound}$$

These curves were fitted in the range of expected void ratios only and should not be used in computations outside those ranges.

(4) All that remains is to calculate the dimensionless time factor from Equation F-6 where H = 10.0 feet initially for both layers with appropriate constants. By small strain theory, Figure F-11 is used to determine percent consolidation. Curve type I is used for the foundation and type III for the dredged fill. By linear finite strain theory, Figure F-3 is used for the foundation and Figure F-5 for the dredged fill. The calculations are organized in Table F-2 and results plotted in Figures F-13 and F-14.

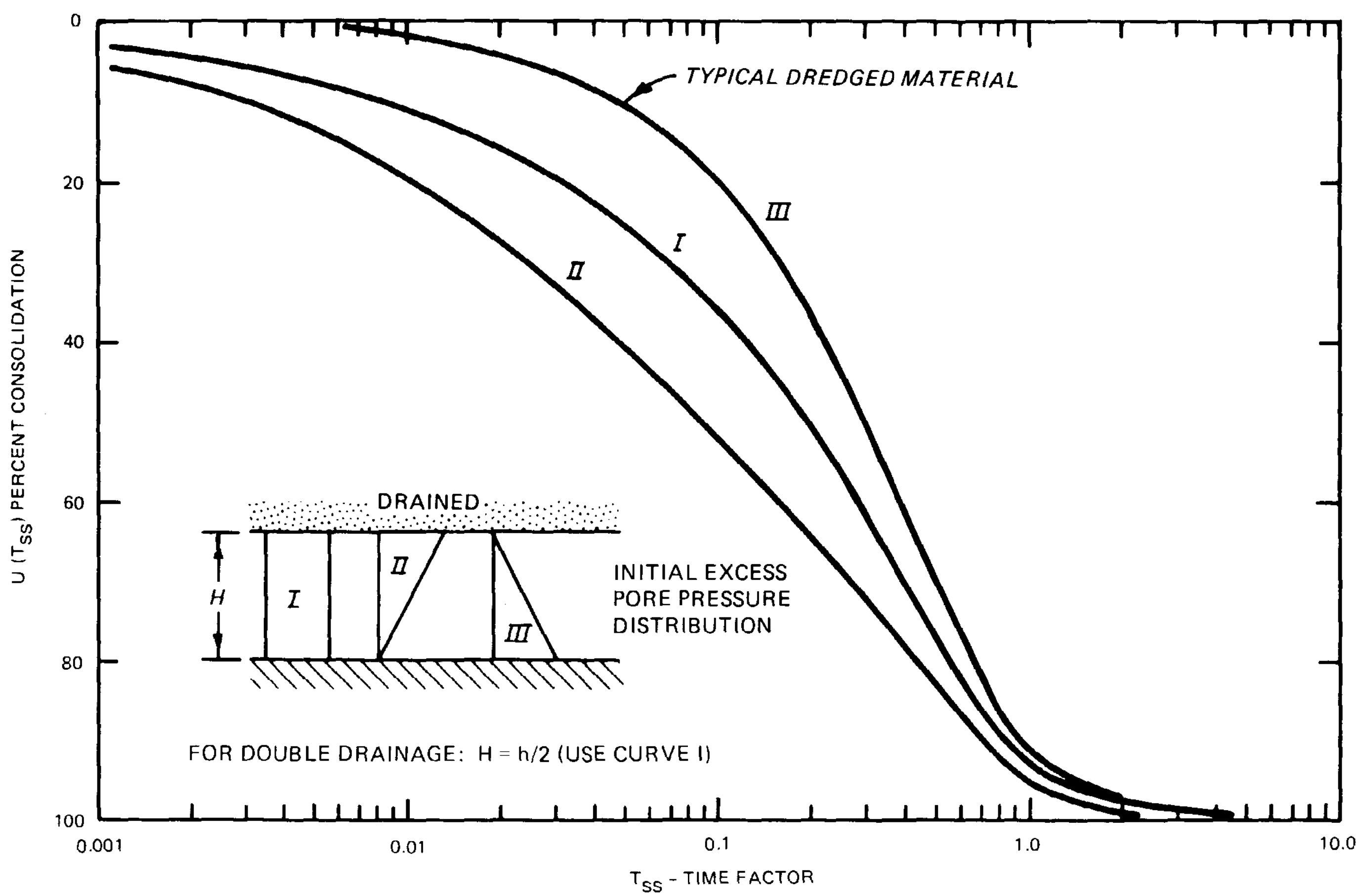
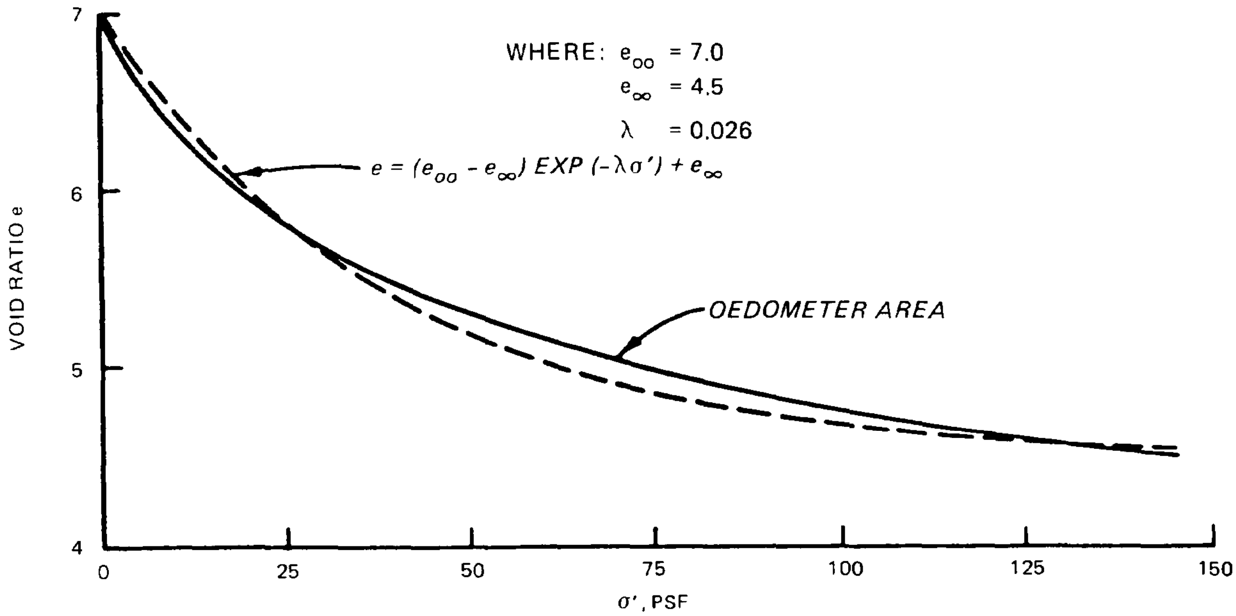
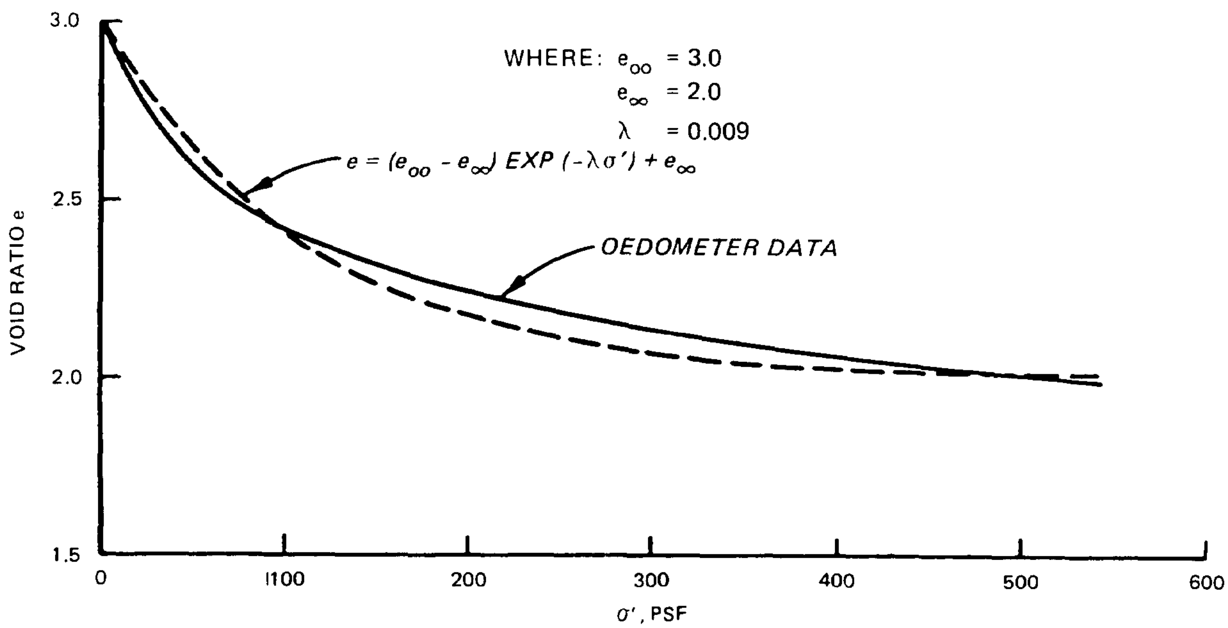


Figure F-11. Degree of consolidation as a function of the time factor for various initial conditions by small strain theory



a. DREDGED FILL



b. FOUNDATION

Figure F-12. Exponential void ratio-effective stress relationship fitted to oedometer data

Table F-2
Percent Consolidation and Settlement Calculations

				Small Strain Theory						Linear Finite Strain Theory		
t days	$\bar{e}$	$\bar{H}$ ft	c_v ft^2/day	T	U %	δ ft	$\bar{e}$	$\bar{H}$ ft	g ft^2/day	T	U %	δ ft
				Dredged Fill								
500	6.8	9.75	1.25×10^{-2}	0.066	14	0.31	6.4	9.25	2.16×10^{-4}	0.069	33	0.73
1,000	6.5	9.38	1.20×10^{-2}	0.136	26	0.57	5.9	8.63	2.410×10^{-4}	0.154	64	1.41
1,500	6.3	9.13	1.17×10^{-2}	0.211	39	0.86	5.5	8.13	2.73×10^{-4}	0.262	85	1.87
2,000	6.1	8.88	1.15×10^{-2}	0.292	50	1.10	5.3	7.87	2.96×10^{-4}	0.379	94	2.07
2,500	5.9	8.63	1.14×10^{-2}	0.383	60	1.32	5.3	7.87	2.96×10^{-4}	0.474	97	2.13
3,000	5.8	8.50	1.13×10^{-2}	0.469	68	1.50	5.3	7.87	2.96×10^{-4}	0.57	99	2.18
3,500	5.7	8.38	1.13×10^{-2}	0.56	74	1.63	5.3	7.87	2.96×10^{-4}	0.66	100	2.20
4,000	5.6	8.25	1.13×10^{-2}	0.66	80	1.76					100	2.20
4,500	5.5	8.13	1.13×10^{-2}	0.77	85	1.87					100	2.20
5,000	5.4	8.00	1.14×10^{-2}	0.89	89	1.96					100	2.20
				Foundation								
500	2.30	9.79	1.15×10^{-2}	0.060	28	0.17	2.25	9.65	1.19×10^{-3}	0.068	62	0.38
1,000	2.30	9.79	1.15×10^{-2}	0.120	40	0.24	2.20	9.50	1.30×10^{-3}	0.148	78	0.48
1,500	2.25	9.65	1.24×10^{-2}	0.200	51	0.31	2.20	9.50	1.30×10^{-3}	0.221	87	0.53
2,000	2.25	9.65	1.24×10^{-2}	0.266	58	0.35	2.15	9.35	1.45×10^{-3}	0.329	93	0.57
2,500	2.25	0.65	1.24×10^{-2}	0.333	65	0.40	2.15	9.35	1.45×10^{-3}	0.412	96	0.59
3,000	2.20	9.50	1.32×10^{-2}	0.439	73	0.45	2.15	9.35	1.45×10^{-3}	0.494	98	0.60
3,500	2.20	9.50	1.32×10^{-2}	0.51	77	0.47	2.15	9.35	1.45×10^{-3}	0.58	99	0.60
4,000	2.20	9.50	1.32×10^{-2}	0.59	81	0.49	2.15	9.35	1.45×10^{-3}	0.66	100	0.61
4,500	2.20	9.50	1.32×10^{-2}	0.66	84	0.51					100	0.61
5,000	2.20	9.50	1.32×10^{-2}	0.73	87	0.53					100	0.61

Dredged material: $\delta_\infty = 2.20$ ft ; N = 3.55
Foundation: $\delta_\infty = 0.61$ ft ; N = 2.75

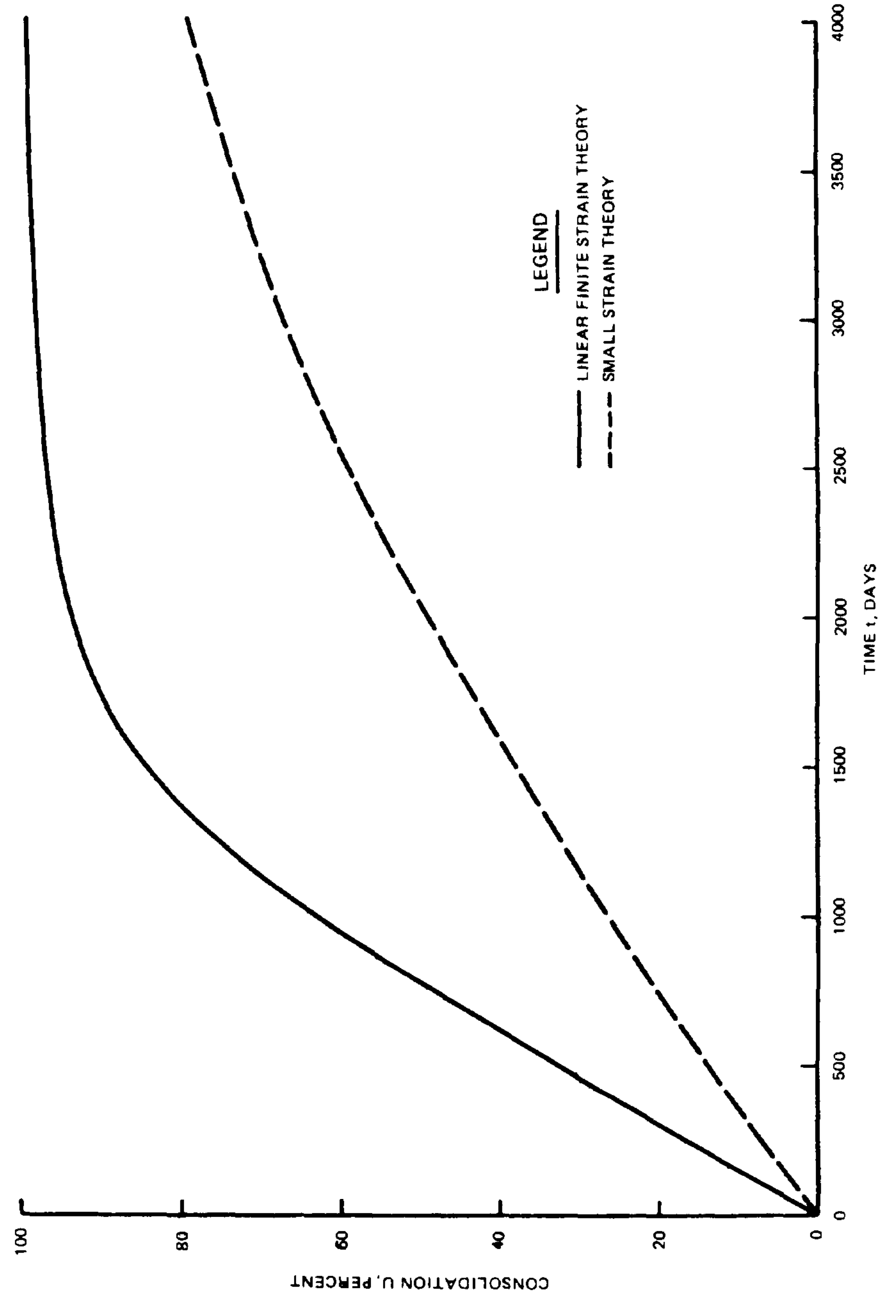


Figure F-13. Comparison of consolidation percentages in the dredged fill layer as a function of time

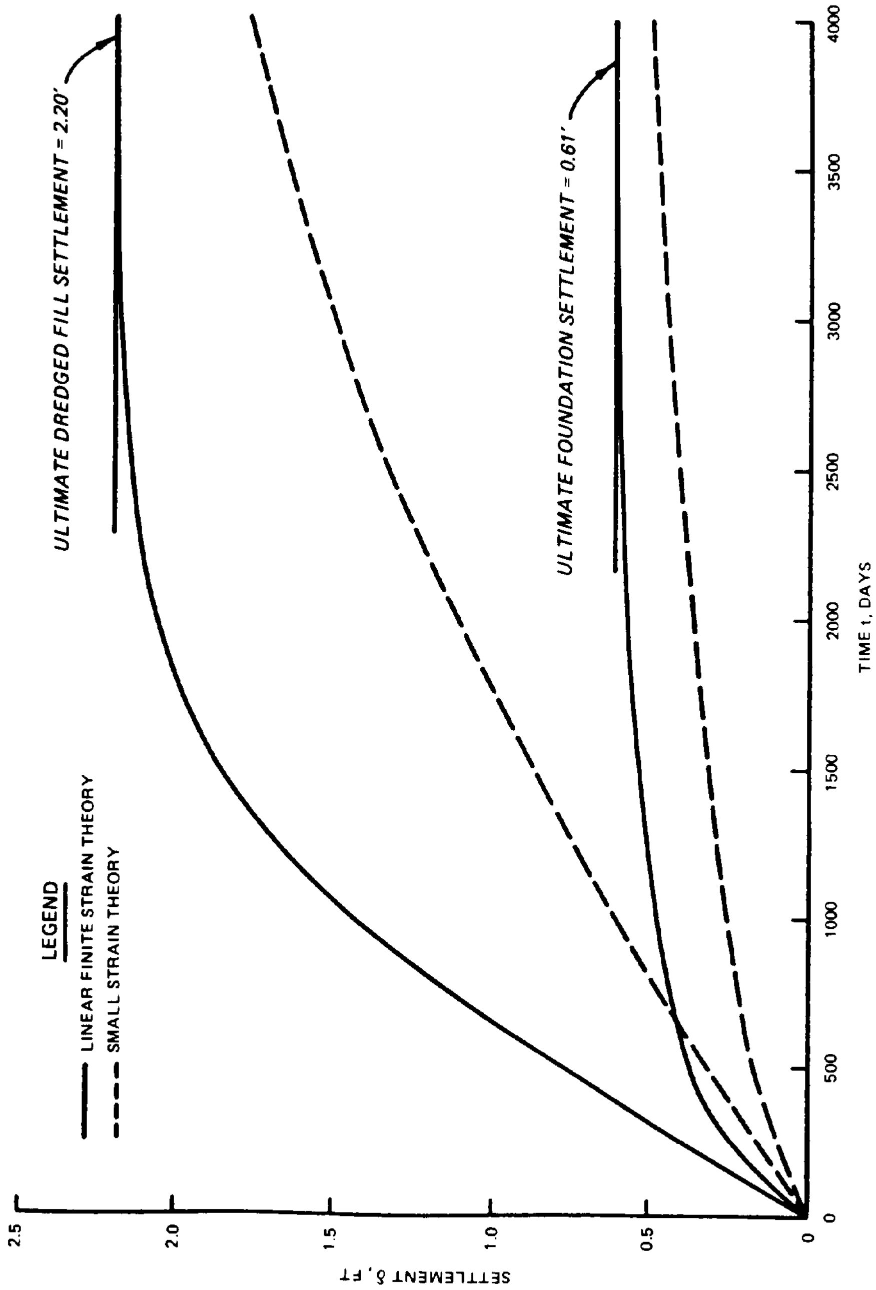


Figure F-14. Comparison of settlement predictions by small strain and linear finite strain theories