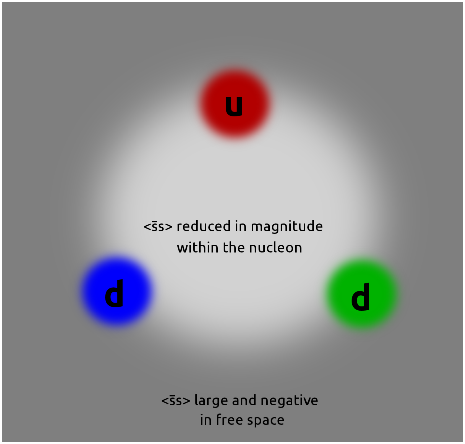
The strangeness and charm of the nucleon: from QCD to dark matter

Walter Freeman

The George Washington University
The MILC Collaboration

December 17, 2012

The nucleon strangeness: interpretation



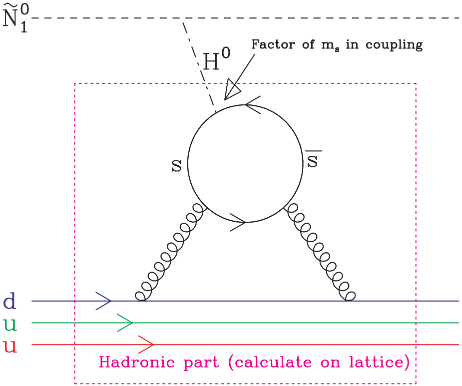
- Strong dynamics cause spontaneous chiral symmetry breaking
- $\bar{s}s$ (etc.) obtain a vacuum expectation value (quark condensate)
- Nucleon strangeness actually the partial suppression of the vacuum condensate
- Specifically, the nucleon strangeness is the matrix element $\langle N | \int d^3x\, \bar{s}s | N \rangle - \langle 0 | \int d^3x\, \bar{s}s | 0 \rangle$
- Does not imply that there are a great many virtual $\bar{s}s$ pairs in the nucleon
- Nucleon carves out a region of different quark-sea properties from the vacuum

The nucleon strangeness: overview

- $\langle N|\bar{s}s|N\rangle - \langle 0|\bar{s}s|0\rangle$ equal to $\frac{\partial M_N}{\partial m_s}$ by Feynman-Hellman theorem
  - Partition function contains $e^{-m_s \bar{s}s}$ – differentiating partition function with respect to m_s brings down $\bar{s}s$, and gives correct vacuum subtraction
  - Only true if action contains "simple" mass term; not true for Wilson quarks!
- Important for our overall understanding of nucleon structure
- Particularly important for dark matter detection experiments

- Not really accessible to either pure theory or the laboratory: $\rightarrow$ lattice QCD
- We developed a new method to do this calculation, and applied it to large MILC configuration library
- First result published in 2009; first modern calculation with full dynamical strange quarks and well-controlled systematics
- We have continued to refine the technique and applied it to related matrix elements for the light and charm quarks
- Latest result: arXiv:1204.3866

The nucleon strangeness: dark matter detection



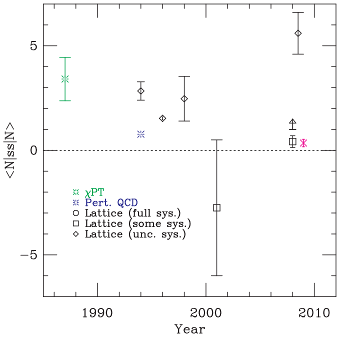
- WIMP interacts with sea heavy quark via Higgs (or Higgs-like) exchange: factor of $|m_q \langle N|\bar{q}q|N\rangle|^2$
- Heavy quark loops may be quite important to overall cross section!
- Early results for $m_s \langle N|\bar{s}s|N\rangle$ said to be 300 MeV or even more!
- ... does this suggest a large enhancement to the cross section?

The nucleon strangeness: dark matter detection, history

- Interpretation of dark matter detection experiments requires knowledge of nucleon-neutralino scattering cross-section
- Ellis *et al.* (2008) point out that one of the largest uncertainties is $\langle N|\bar{q}q|N\rangle$, particularly for the strange quark
- Historical estimates for $\langle N|\bar{s}s|N\rangle$ all over the map
- Problem made worse by various calculations and discussions of quantities like the y-parameter that mix strange and light matrix elements

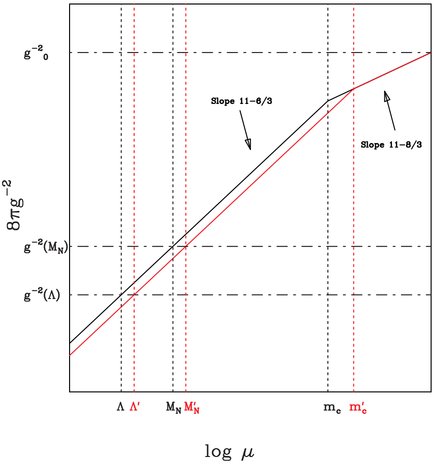
"By far the largest single uncertainty is that in spin-independent scattering induced by our ignorance of the $\langle N|\bar{q}q|N\rangle$ matrix elements linked to the π-nucleon σ term, which affects the ratio of cross sections on proton and neutron targets as well as their absolute values. This uncertainty is already impacting the interpretations of experimental searches for cold dark matter. *We plead for an experimental campaign to determine better the π-nucleon σ term.*"
–John Ellis, 2008 (italics original)

The nucleon strangeness: status as of mid 2009



- Theoretical estimates disagree wildly
- Early χPT result suggested value substantially above natural scale
- This would be a big deal for DM searches
- Quantity is fundamentally nonperturbative: need lattice QCD
- Early lattice results have uncontrolled systematics, huge error bars, etc.
- Need to sort this out!

The nucleon strangeness: perturbative estimate



- Nucleon sea quark content equal to $\frac{\partial M_N}{\partial m_q}$ by Feynman-Hellman theorem
- Low energy dynamics of QCD set mostly by running of gauge coupling
- Changing the running of g changes all low-energy scales, including M_N
- Running of gauge coupling depends on number of light quark flavors: $\frac{\partial g^{-2}}{\partial \log \mu} = \frac{1}{8\pi^2}(11 - \frac{2n_f}{3})$
- Changing a heavy quark mass affects the scale where it freezes out, shifting the running of g and thus shifting M_N
- For all heavy quarks, $m_q \langle N|\bar{q}q|N\rangle \approx 75$ MeV
    - This implies they all contribute about the same amount to DM scattering
- No reason this perturbative result should apply to strange quark, but it does set a natural scale

Introduction to lattice QCD: motivation

- Need a fundamentally nonperturbative way to model QCD
- Use quantum Monte Carlo to do the Feynman path integral
- Discretize spacetime on a 4D lattice to make theory amenable to the computer
  - Define a lattice action that reduces to continuum QCD as $a \to 0$
  - Absorb Euclidean-time weight factor $e^{-\mathcal{S}}$ in the path integral into Monte Carlo configuration weight
- Can't just naïvely replace derivatives in action with finite differences
  - This leads to an unwanted doubling of the ground state: each quark flavor splits into 2^4 copies!
- Have to address this "fermion doubling problem"
- Can do any perverse thing you want to the action, so long as it has the right continuum limit

Lattice actions

Three broad classes of actions:

Wilson quarks:

- Add a second-derivative term to the action to kill doublers
- Earliest fermion action used
- Fairly fast to simulate
- No simple mass term in action
- Explicit chiral symmetry breaking
- Additive mass renormalization

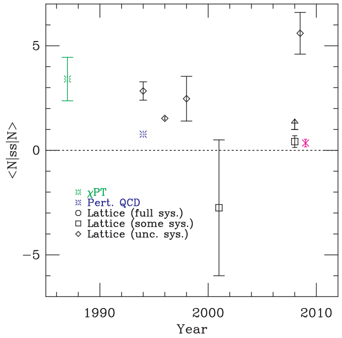
Staggered quarks (us):

- Use fields with only one component per site
- Use far corners of Brillouin zone to carry Dirac structure
- Very fast to simulate
- Remnant chiral symmetry protects from additive mass renormalization
- Doublers reduce to four "tastes" which decouple as $a \to 0$
- Remnant couplings between tastes at finite a are largest lattice artifact
- "Taste splittings" can be handled in analysis

Chiral quarks:

- Preserve (very nearly) exact chiral symmetry (unlike Wilson)
- No doublers or remnant doublers (unlike staggered)
- Very, very slow to simulate
- Performance problems limit them to small volumes, coarse lattices, or Herculean efforts

The nucleon strangeness: status as of mid 2009



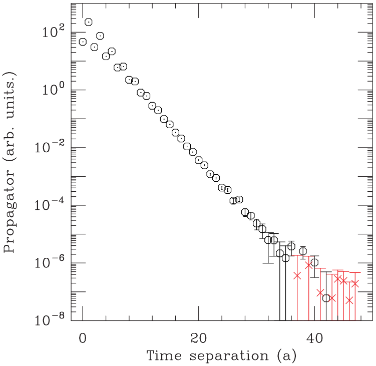
- Turns out the problem in these early results is mostly the additive mass renormalization in Wilson quarks
- Michael (2001) realized this problem and attempted to correct for it, and got the giant error bar shown here

The MILC simulation program

- MILC has undertaken a large-scale program of lattice QCD simulations over the last decade or so using improved staggered quarks
  - Too expensive (mostly) to simulate high statistics at physical m_l, so use a variety of light quark masses and extrapolate
  - Simulate at a variety of lattice spacings and extrapolate to $a = 0$
  - Goal: generate a general-purpose library of gauge configurations usable for a variety of studies
  - Configurations available to anyone, code released under GPL
- Old simulations (using "Asqtad" action) complete: 26,000 configurations
  - Include dynamical light and strange quarks: "2+1 flavor"
- New simulations (using "HISQ" action) ongoing: 11,000 configurations and counting
  - Also include dynamical charm: "2+1+1 flavor"

Measuring the nucleon mass

- Going to need nucleon mass to apply Feynman-Hellman theorem
- Lattice QCD lets us measure correlation functions of operators
- The one we need here is the nucleon two-point function $P(T) = \langle N^\dagger(0) N(T) \rangle$ ("propagator" or "correlator")
- In Euclidean time, this should go as $P(T) \propto e^{-M_N T}$ – like an energy in QM
- Do a fit, extract M_N? Not quite:
- Our operator overlaps with all sorts of stuff ("excited states") other than just the nucleon
- Result at long distances too noisy
- Result at short distances polluted by excited states
- Choose some fit region in the middle: explicitly consider some excited states, wait for rest to decay away



Methods for the nucleon strangeness: spectrum method

- Directly apply Feynman-Hellman theorem: find $\frac{\partial M_N}{\partial m_s} = \langle N|\bar{s}s|N\rangle$ by looking at how M_N changes for different values of m_s
- In principle, very straightforward
- Problem: Only a few of the MILC ensembles have m_s at nonphysical values
- Worse, ensembles have different coupling constants: we need $\frac{\partial M_N}{\partial m_s}$ with the coupling held fixed
  - The Feynman-Hellman theorem requires that we take the derivative with all lattice parameters held fixed, *not* physical things like the lattice spacing

Methods for the nucleon strangeness: direct method

- The spectrum method needs special lattice parameters which we don't have; do we have another approach?
- MILC has computed $\int d^4x \, \bar{s}s$ on all of the configurations
- Could we just evaluate $\langle N|\bar{s}s|N\rangle - \langle 0|\bar{s}s|0\rangle$ directly?
- Get result for individual ensembles $\Rightarrow$ potentially better chiral extrapolation
- Problem: don't have measurements of $\bar{s}s$ on particular timeslices, or of nucleon correlator for individual sources
- We could use some computer time to get them, but this would be expensive on $\sim$ 40k configurations

Methods for the nucleon strangeness: hybrid method

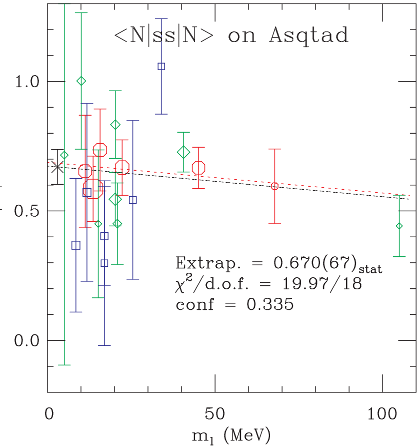
- We can combine aspects of these methods to get $\frac{\partial M_N}{\partial m_s}$ on a single ensemble with no extra computer time
- M_N obtained from a fit to propagator at various times T_i, and is just some complicated function of $P(T_i)$
- Write $\frac{\partial M_N}{\partial m_s} = \sum_i \frac{\partial M_N}{\partial P(T_i)} \frac{\partial P(T_i)}{\partial m_s}$ using the chain rule
  - The first of these can be evaluated numerically
  - The second of these can be gotten with the Feynman-Hellman theorem in reverse: $\frac{\partial P(T_i)}{\partial m_s} = \langle P(T_i)\bar{s}s\rangle - \langle P(T_i)\rangle \langle \bar{s}s\rangle$
- This method has multiple advantages:
  - It explicitly considers excited states (through the M_N fit form)
  - It can be applied to a single ensemble with any lattice parameters
  - Multiple ensembles can be used to extrapolate to the physical point and to improve statistics

Hybrid method: Analysis

- Need to determine minimum distance T_{min} for mass fits that provides good balance between statistical and excited-state systematic error
- Values at different lattice spacings: need to convert everything to consistent RG scheme [we use $\overline{\mathrm{MS}}(2\,\mathrm{GeV})$] first
- Values not at quite the right strange quark mass: need to account for that
- Need to extrapolate to physical m_l:
  - Examine χPT form for M_N and differentiate with respect to m_s: no terms leading to large curvature
  - Since there are no dangerous terms, just do a simple chiral fit linear in m_l
- Need to extrapolate to $a = 0$:
  - The lowest-order discretization errors in the Asqtad action are $\mathcal{O}(\alpha_s a^2)$, so add a term $\propto a^2$ to the fit (α_s relatively constant)
- A fairly simple chiral and continuum fit suffices:

$$\langle N|\bar{s}s|N\rangle = A + Bm_l + Ca^2$$

Hybrid method on the Asqtad lattices: result and error budget



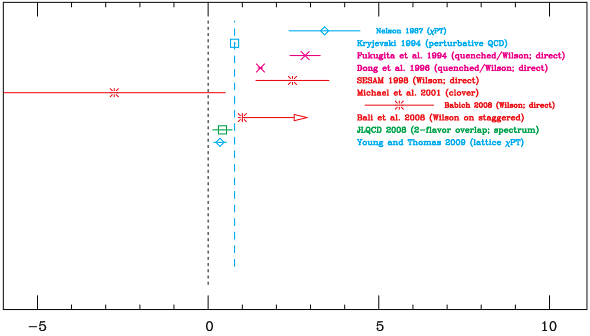
- This extrapolation looks worse than it is: a few ensembles with low errors control the fit

Error budget:

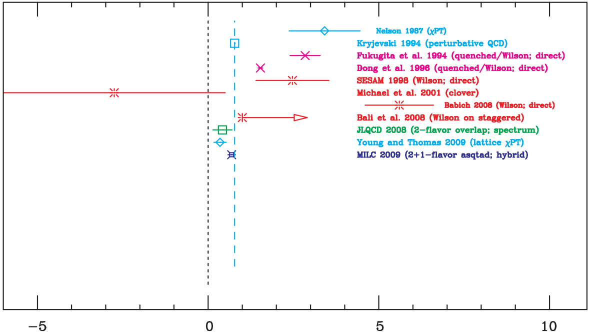
Source	Error
Statistical	0.07
Higher order χPT	0.05
Excited states	0.03
Finite volume	0.02
Renormalization	0.03

At the physical point, we have
$\langle N|\bar{s}s|N\rangle = 0.670(67)_{\rm stat}(70)_{\rm sys}$

How does this result compare?



How does this result compare?



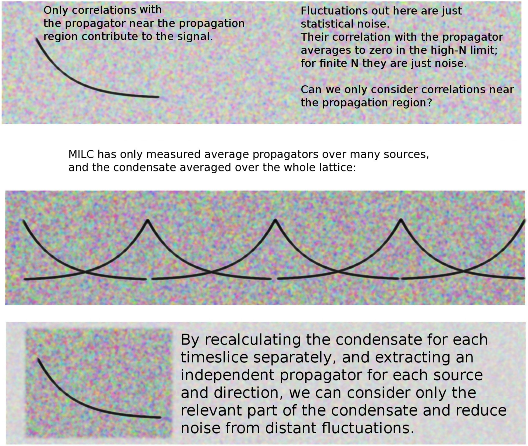
- Our result has fairly small overall error
- We also have multiple lattice spacings, chiral extrapolation, large volumes, etc.
- See arXiv:0905.2432v2 (PRL) and arXiv:0912.1144 (PoS Latt '09)

Improved hybrid method

- Large source of statistical error: fluctuations of $\bar{s}s$ far from nucleon propagation region
- Each propagator covers only a small region of the temporal extent of the lattice
- We use the entire lattice by averaging many propagators with different source times
- No physical reason for $\bar{s}s$ to be correlated with $P(t)$ far from propagation region
- In the limit of high statistics, correlations far from this region will average to zero...
- ... but for finite N they contribute statistical noise
- Can we only consider correlations between the propagator and the condensate in regions that contribute real signal?

Improved hybrid method

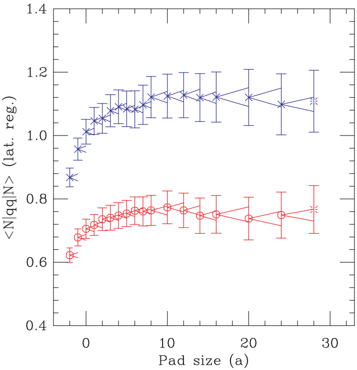
- Only consider the condensate at times between source and sink operators of the propagator, plus variable "padding" of a few time units at each end



- This requires new computer work – cheap on $a = 0.12$ fm ensembles, though

Improved hybrid method: choice of pad size

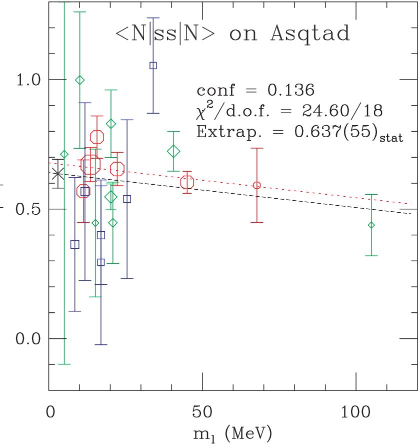
- The previous result suggests that (for the $a \approx 0.12$ fm ensembles), a padding size of around 4a will include all physically-meaningful correlations
- Have done the needed extra supercomputer work for all the $a \approx 0.12$ fm ensembles, and one of the $a \approx 0.09$ fm ones
- Plot the result vs. pad size (averaged over all these ensembles) to see what pad size to use
- Conservative choice: pad size of $6a$
- Method does successfully reduce statistical errors
- See [arXiv:1204.3866] for details



Red points are strangeness, blue points are sea light quark content.

Carats indicate errors on difference between adjacent points, as the error bars are correlated.

Improved hybrid method: result

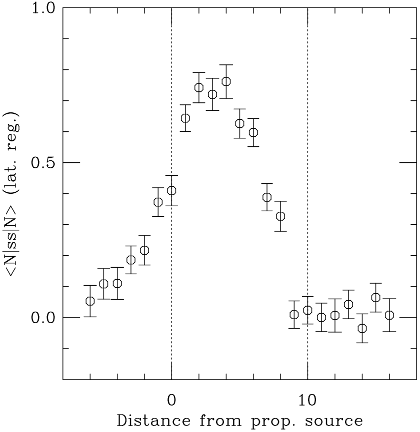


- Despite significant reduction in error bars on coarse ensembles, overall error not decreased that much
- Half of the remaining statistical error comes from the continuum extrapolation: need improved data on (more) finer ensembles!
- Systematic errors unchanged, with the exception of the addition of a 1% error (conservative) from the improved method
- At the physical point, we have $\langle N|\bar{s}s|N\rangle = 0.637(055)_{\rm stat}(074)_{\rm sys}$

Direct method: approach and insertion point

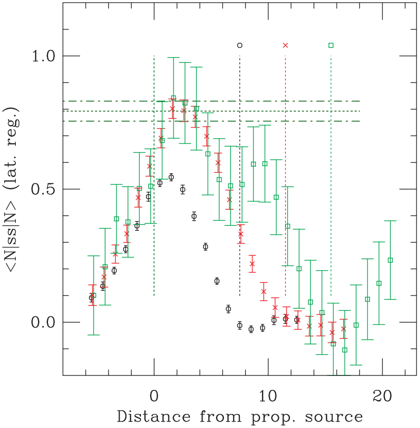
- We rejected this method before because we didn't have the needed measurements
- ... but the measurements required to apply this method are the same as those for the improved hybrid method
- In principle, and using the logic behind the improved hybrid method, we could just calculate $\frac{\langle N(T)\bar{s}s(t_1)N^\dagger(0)\rangle}{\langle N(T)N^\dagger(0)\rangle} - \langle \bar{s}s \rangle$ for any time $0 < t_1 < T$
- In practice, since our operators are imperfect, we must choose an intermediate time t_1 sufficiently far from source and sink to avoid excited state pollution
- Need longer propagators to do this, but they are noisy...
- This method favored by other modern calculations: applying it to our data would provide a nice cross-check with them!

Direct method: finding a plateau



- Appearance of a plateau for t_1 around $3a$, but not broad enough to say anything definitive
- Asymmetry due to different nucleon interpolating operators at source and sink
    - One disadvantage of staggered fermions: clever smeared nucleon operators are harder
- Try different propagator lengths: need to find a plateau both in T and t_1 to be convincing

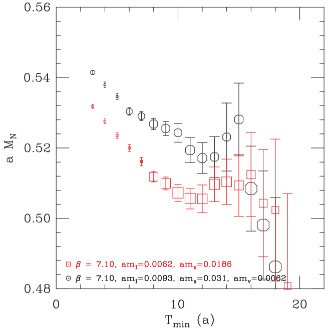
Direct method: combining results



- Why use just one propagator and the condensate on just one timeslice?
- Average two adjacent propagator lengths and four adjacent condensate timeslices to beat down noise
- Plot result as a function of both propagator length and $\bar{s}s$ insertion point
- This result lends stronger support to that from the improved hybrid method: results are consistent
- Better results can be extracted from this if you trust the "slope method" of Liu and Gong

Spectrum method: another check

- Need runs identical in all ways except for sea masses
- Difficult on MILC Asqtad because of changes in β to keep a fixed
- Two ensembles accidentally run at the same value of β (and u_0); can we use this "mistake"?
  - A: $\beta = 7.10, am_l = 0.0093, am_s = 0.031$
  - B: $\beta = 7.10, am_l = 0.0062, am_s = 0.186$
- Use hybrid method to compute $\frac{\partial M_N}{\partial m_s}$ and $\frac{\partial M_N}{\partial m_{l,\mathrm{sea}}}$ on both
- Compute M_N on both ensembles, setting $m_{\mathrm{val}} = 0.0062$ on ensemble A
- Results from hybrid method should predict difference in M_N



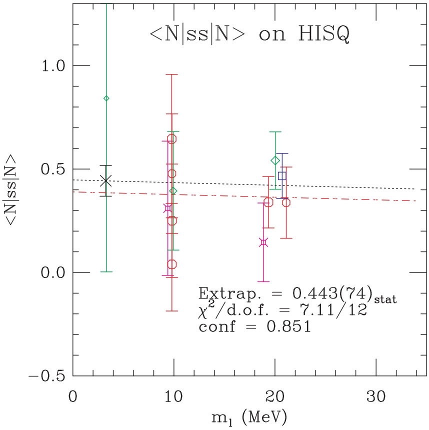
- Choose $t_{\mathrm{min}} = 11$ for correlator fits
- Direct measurement gives $a\Delta M_N = 0.013(5)$
- Hybrid-method results on these two ensembles predict $a\Delta M_N = 0.011(4)$

$\rightarrow$ The two methods agree

Hybrid method on the HISQ ensembles

- We may apply the same analysis to the newer HISQ ensembles, with a few corrections
- No ensembles with heavier light quarks, so slope of chiral extrapolation not that well controlled
  - Apply a Bayesian prior from the Asqtad result for the slope and its uncertainty
  - In principle we could do a lot with partial quenching, but that requires more computer time
- Generally lower statistics and fewer ensembles (generation project still in progress)

Hybrid method for HISQ ensembles: result

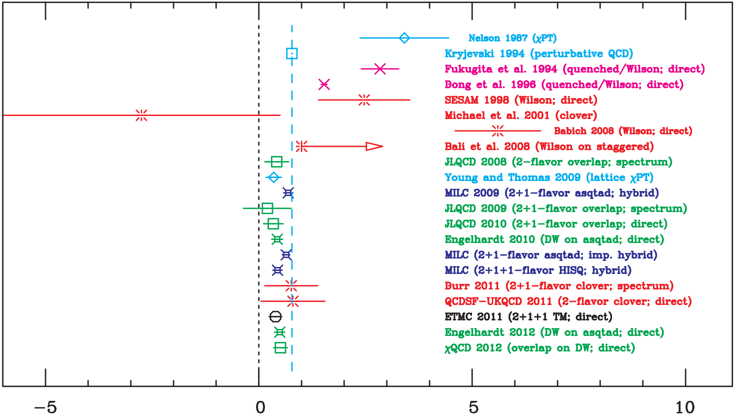


Error budget:

Source	Error
Statistical	0.08
Higher order χPT	0.03
Excited states	0.02
Finite volume	0.01
Renormalization	0.06

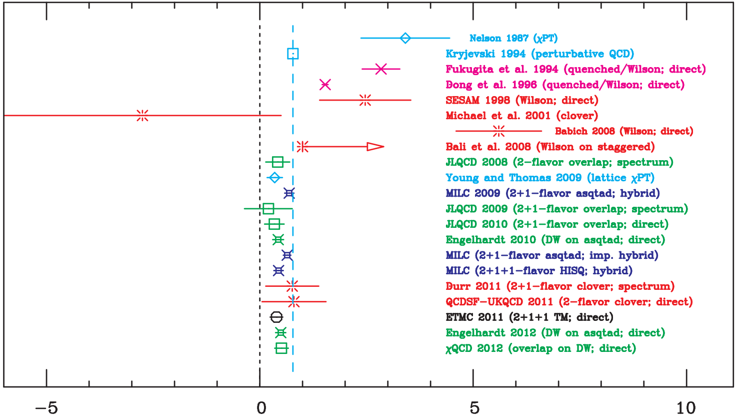
Evaluated at the physical point, we get $\langle N|\bar{s}s|N\rangle = 0.443(74)_{\mathrm{stat}}(71)_{\mathrm{sys}}$ This is somewhat lower than, but not wildly inconsistent with, the Asqtad result.

Conclusions and consequences: nucleon strangeness



- (Improved) hybrid method on the MILC library of gauge configurations allows determination of $\langle N|\bar{s}s|N\rangle$ and $\sigma_{\pi N,\mathrm{disc}}$ with low statistical errors and good control of systematics
- A variety of methods give consistent results for $\langle N|\bar{s}s|N\rangle$
- Good control of systematic errors (we think)

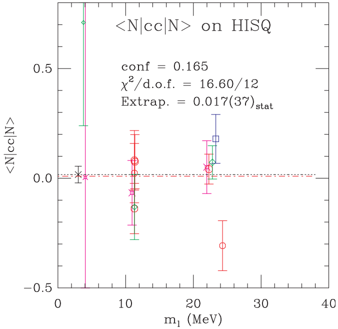
Conclusions and consequences: nucleon strangeness

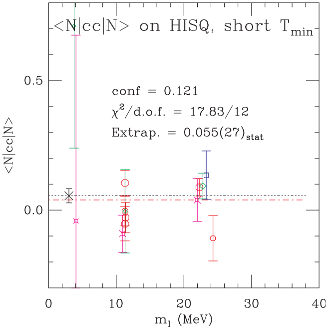


- Modern calculations all pretty much consistent (although some have large errors)
- Minor differences remain, but the biggest question has been answered:
- “Is the nucleon strangeness substantially enhanced above its natural scale?” No!

Intrinsic charm: hybrid method on HISQ

- Can calculate the intrinsic charm using the hybrid method on the HISQ ensembles
- Charm is heavy enough that perturbative result should be valid; provides a nice check
- No chiral extrapolation possible; too much noise (and no heavy m_l)
  - χQCD used partially quenched valence quarks and found an increase as $m_{\rm val} \to 0$
- Not so worried about excited state pollution with 50-100% statistical errors (and it should be lower anyway); optionally use $T_{\rm min}$ smaller by a factor of 2/3





- Using the shorter $T_{\rm min}$ values, can resolve difference from zero by 2σ, in agreement with perturbative prediction!

Conclusions and consequences: dark matter searches

- The relevant quantity for dark matter scattering is $\sum m_q \langle N|\bar{q}q|N\rangle$: "nucleon sigma term"
- Early results (thanks mostly to Wilson operator mixing) suggested that the strange sigma term might be ¿300 MeV
- Modern results are in the 35-60 MeV range (with us at the high end)
- We (and independently χQCD) get a value consistent with the perturbative result for $m_c \langle N|\bar{c}c|N\rangle$ of around 75 MeV
- Presumably the bottom and top quarks contribute about the same amount
- For the light quark, there are both sea and valence (disconnected and connected) contributions to $\sigma_{\pi N}$
  - The overall value is usually held to be around 65 MeV, although there is some debate
- So up+down (together) and each of the heavy flavors all contribute about the same, with the strangeness being the lowest!

Future directions: how could we do this better?

How could we improve these results with a little computer time to invest?

- Use improved method on all ensembles (costs nothing if you do it from the beginning)
- Improve the nucleon two-point function: more sources, better sources, "all-to-all" techniques
- Improving the stochastic estimator for $\langle \bar{s}s \rangle$ is not necessary for the strangeness, but it is for the charm
- Partial quenching will give a much better handle on the chiral extrapolation
- Staggered quarks excellent for this sort of work
  - No additive mass renormalization
  - Taste breaking artifacts not significant for nucleon, can select "correct" taste for $\bar{s}s$
  - Staggered construction does make it harder to use a "good" nucleon operator (ours overlaps with the Δ)
  - Very fast: can get needed statistics and afford large volumes

Future directions: other quantities

- Calculating various related nucleon properties that involve disconnected diagrams is a growing industry:
  - Strange quark spin fraction (M. Engelhardt has a nice result here)
  - Strange quark magnetic moment...
  - ... and more
- Results for these quantities are generally done with the direct method on fewer configurations
  - All these quantities tend to be very noisy, involve delicate vacuum subtractions, etc.
  - Surprisingly large improvements can be had by simply doing the two-point function better
  - "Bury it with statistics": this work is based on 37,000 gauge configurations generated over almost a decade. Could other such nucleon matrix elements be computed on the MILC library fairly cheaply?
  - Any configuration generation project that emphasized high statistics would be beneficial not just for nucleon structure, but for anything involving disconnected diagrams: η' mass, some mesonic decays, etc.

Conclusions

- Contrary to early suggestions from χPT and a few lattice calculations that $\langle N|\bar{s}s|N\rangle \approx 3-4$, substantially enhanced above its natural scale, we measure $\langle N|\bar{s}s|N\rangle = 0.64(6)_{\rm stat}(7)_{\rm sys}$
- Other modern calculations using a variety of actions and methods get similar values
- Also able to measure $\sigma_{\pi N,\rm disc}$ and $\langle N|\bar{c}c|N\rangle$
- Strange quark loops in the nucleon contribute about the same as charm, bottom, top to DM scattering cross section
- While there may be some remaining discrepancies in the lattice values, Ellis' plea has been answered
- These matrix elements are connected to the one of the prevailing mysteries of QCD: how do strong dynamics give rise to the quark condensate?
- Potentially understanding how and why the quark condensate is suppressed in a nucleon will lead to greater understanding of how it arises in the first place

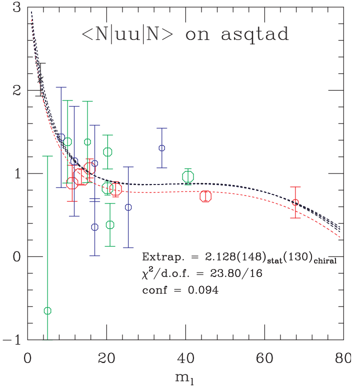
Spare slides follow

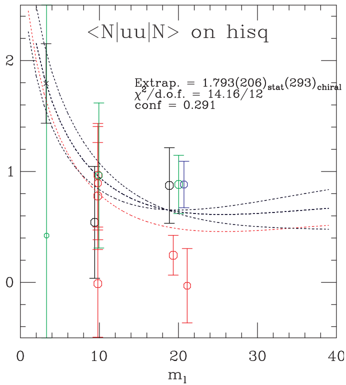
Light quark sea content on Asqtad

- Can also calculate the light quark matrix element $\langle N|\bar{u}u|N\rangle_{\mathrm{disc}}$
- (There is also a connected contribution from valence quarks; together they comprise the "pion-nucleon sigma term")
- Going to need a more sophisticated chiral extrapolation, but everything else is the same

Light quark sea content: chiral extrapolation

- The nonanalytic parts of the PQχPT form for M_N have no unknown parameters. Differentiate that form numerically, and perform a chiral/continuum fit with a quadratic analytic term:
- $\langle N|\bar{u}u|N\rangle_{\rm disc} = A + Bm_l + Cm_l^2 Da^2 + \left.\frac{\partial M_N^{(3/2)}}{\partial m_l}\right|_{m_{\rm val}}$
- On HISQ, need to constrain both linear and quadratic terms with priors taken from Asqtad fit





Light quark sea content: error budgets and result

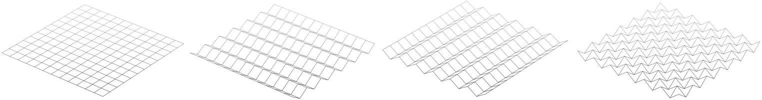
Error budgets:

Source	Error on Asqtad	Error on HISQ
Statistical	0.15	0.21
Uncertainties in χPT	0.13	0.13
Higher order χPT	0.15	0.13
Excited states	0.11	0.09
Finite volume	0.06	0.05
Renormalization	0.08	0.14
Result for $\langle N\|\bar{u}u\|N\rangle_{\mathrm{disc}}$	2.13(15)(25)	1.79(21)(22)

- These results are broadly consistent with each other, but the errors are somewhat larger than for the strangeness
- Substantial contribution to error budget from uncertainty in m_l: we use 3.25(17) from MILC's χPT analysis
- Result often quoted as the disconnected part of the pion-nucleon sigma term: $\sigma_{\pi N} \equiv \left\langle N|\bar{u}u + \bar{d}d|N\right\rangle$
- Disconnected part of $\sigma_{\pi N}$: $\sigma_{\pi N,\mathrm{disc}} = 13.8(1.0)_{\mathrm{stat}}(1.6)_{\mathrm{sys}}$ MeV from the Asqtad result
- Result should be treated as somewhat preliminary; quite sensitive to chiral fits and parameters, and requires some more thought

Fermion actions and the doubling problem

- Putting the Dirac action onto a lattice is harder than it looks
- Can't just replace derivatives with finite differences: leads to doubling of ground state
  - One doubler per dimension: gives 16 copies!



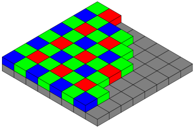
  - Have to modify the action, often in bizarre ways, to eliminate these doublers
    - Modifications okay if they reduce to continuum QCD as $a \to 0$
- All known actions have tradeoffs between accuracy at finite a, analytical ease, and computational effort required
- In particular, the most common actions ("Wilson quarks") explicitly break chiral symmetry at finite a
- The chiral condensate is the order parameter of chiral symmetry breaking: this causes problems trying to study it with Wilson quarks

Staggered quarks

- Doublers come from distant corners of the Brillouin zone
- Reduce the extent of the BZ – or, rather repurpose the far corners
  - Divide the lattice into 2^4 hypercubes
  - Use only one-component fermion fields per site, giving 16 degrees of freedom per hypercube
  - Enough for four Dirac four-vectors: different linear combinations of these 16 sites represent different Dirac components
  - Dirac-matrix structure becomes factors of -1 in hopping from site to site
  - **Distant corners of BZ now partially used to carry Dirac information**
- Reduces doublers by a factor of 4: now only have a four-fold degeneracy rather than 16-fold
- Also reduces number of degrees of freedom by a factor of four: this makes staggered quarks very fast

Staggered quarks: analogy to the Bayer filter

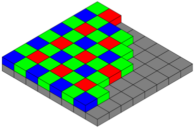
- This seems weird: using the high-frequency part of the BZ to carry Dirac information
- Not without precedent: digital color cameras work this way too!
  - CCD's only see black and white: put a primary color filter on top



  - Image processing algorithms interpret high spatial frequencies on the CCD as chrominance information (phase specifies which color)
  - Ambiguity exists between high-frequency luminance detail and low-frequency color information: detail at spatial frequencies near edge of BZ gets interpreted as color information, leading to moiré
  - Solved in photography by smearing (blurring) the image before it gets to the color filter

Staggered quarks: analogy to the Bayer filter

- This seems weird: using the high-frequency part of the BZ to carry Dirac information
- Not without precedent: digital color cameras work this way too!
  - CCD's only see black and white: put a primary color filter on top

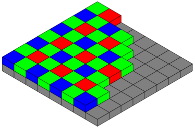


  - Image processing algorithms interpret high spatial frequencies on the CCD as chrominance information (phase specifies which color)

  - Ambiguity exists between high-frequency luminance detail and low-frequency color information: detail at spatial frequencies near edge of BZ gets interpreted as color information, leading to moiré
  - Solved in photography by smearing (blurring) the image before it gets to the color filter

Staggered quarks: analogy to the Bayer filter

- This seems weird: using the high-frequency part of the BZ to carry Dirac information
- Not without precedent: digital color cameras work this way too!
  - CCD's only see black and white: put a primary color filter on top



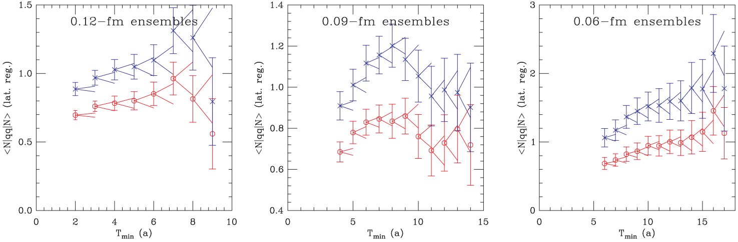
  - Image processing algorithms interpret high spatial frequencies on the CCD as chrominance information (phase specifies which color)
  - Ambiguity exists between high-frequency luminance detail and low-frequency color information: detail at spatial frequencies near edge of BZ gets interpreted as color information, leading to moiré
  - Solved in photography by smearing (blurring) the image before it gets to the color filter

Staggered quarks: problems and solutions

- As $a \to 0$ these "tastes" decouple; in that limit we can just divide the quark action by 4 to get a single taste
- At finite a, taste coupling can happen via highly virtual gluons
  - Quark absorbs a gluon near the edge of the BZ and scatters to a far corner: taste mixing
- Suppress coupling to these high momentum gluons by smearing the gauge field that the quarks see
  - "Asqtad" action: older form of this smearing
  - "HISQ" action: newer, more sophisticated form
  - Just like the optical blurring to fix moiré
- Some residual taste mixing remains, but it is not too bad, and goes away quickly as $a \to 0$

Hybrid method: minimum fit distance

- As always there is a tradeoff in picking $T_{\min}$ for nucleon fits
  - Shorter distances mean systematic error from excited state pollution
  - Longer distances mean higher statistical error from declining propagator SNR
  - These matrix elements, like anything with disconnected diagrams, are very noisy – use smaller $T_{\min}$ than we use to determine M_N



- Want $T_{\min}$ relatively consistent in physical units across lattice spacings: choose $T_{\min} \approx 0.6$ fm, giving $5a$, $7a$, $10a$ on the three spacings

Hybrid method: renormalization

- In lattice QCD, the lattice itself provides the cutoff: "Asqtad at $a \approx 0.12\,\mathrm{fm}$" is as valid a renormalization scheme as "$\overline{\mathrm{MS}}$ (2 GeV)
- $\langle N|\bar{s}s|N\rangle$ is a RG-dependent quantity
- The community would prefer us to quote values in $\overline{\mathrm{MS}}$ (2 GeV)
- Either way, we're going to have to combine values at different lattice spacings $\rightarrow$ different cutoffs
- As a first step in the analysis, convert to $\overline{\mathrm{MS}}$ using Z-factors computed by HPQCD

Hybrid on Asqtad: adjustment for m_s

- The correct lattice value of m_s is only known *a posteriori*
  - The lattice spacing can only be determined after the lattices are analyzed
  - The *physical* value of m_s, in fact, has been determined most accurately from lattice QCD
- The original simulation values of m_s were off, sometimes by as much as 20%
- Need to extrapolate values to the physical value of m_s (89 MeV, per MILC analysis)
- Determine $\frac{\partial \langle N|\bar{s}s|N\rangle}{\partial m_s}$ with a trick
  - Hybrid method also works to determine $\langle N|\bar{u}u|N\rangle$; should be similar in character to $\langle N|\bar{s}s|N\rangle$ for ensembles with m_l far from chiral limit
  - Compute $\frac{\langle N|\bar{s}s|N\rangle - \langle N|\bar{u}u|N\rangle}{m_s - m_l}$ on these "heavy light-quark ensembles" and fit to a constant

β	a (nominal, fm)	am_l	am_s	$\frac{\partial}{\partial m_s}\langle N|\bar{s}s|N\rangle$ (MeV^{-1})
6.81	0.12	0.30	0.50	-0.0033(21)
6.79	0.12	0.20	0.50	-0.0021(6)
6.79	0.12	0.20	0.50	-0.0030(3) (improved)
7.10	0.09	0.093	0.31	-0.0067(19)
7.11	0.09	0.124	0.31	-0.0046(8)
7.48	0.06	0.72	0.18	-0.0046(24)
Average				-0.00331(28)

- Use these values to extrapolate results to the physical m_s

Light quark sea content: chiral extrapolation

- Use partially-quenched chiral perturbation theory: find $\left.\frac{\partial M_N}{\partial m_l}\right|_{m_{\mathrm{val}}}$
- The non-analytic parts are:

$$M_N^{(3/2)} = \frac{1}{8\pi f^2}\left[4D(F-\frac{1}{3}D)m_\pi^3 + \frac{1}{3}(5D^2+6DF+9F^2)(2m_{ju}^3+m_{ru}^3) \right.$$
$$\left. +(D-3F)^2 G_{\pi,\pi} - \frac{8D^2}{3\pi}\left(F_{ju}+F_\pi+\frac{1}{2}F_{ru}\right)\right]$$

where F_H (H is some hadron with mass m) is

$$F_H = (m^2-\Delta^2)\left[\sqrt{\Delta^2-m^2}\log\left(\frac{\Delta-\sqrt{\Delta^2-m^2+i\epsilon}}{\Delta+\sqrt{\Delta^2-m^2+i\epsilon}}-\Delta\log\left(\frac{m^2}{\mu^2}\right)\right)\right]$$
$$-\frac{1}{2}\Delta m^2\log\left(\frac{m^2}{\mu^2}\right)$$

and

$$G_{\pi,\pi} = -\frac{1}{3}\left[\frac{(m_X^2-m_{jj}^2)(m_X^2-m_{rr}^2)}{m_X^2-m_\pi^2}\right]m_X^3$$

where $m_X^2 = \frac{1}{3}(m_{jj}^2+2m_{rr}^2)$.