An explicit SU(12) Family Unification Model with Natural Fermion Masses and Mixings

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in collaboration with
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Outline

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Review of the SU(5) model

Motivation for the SU(5) model

- Unification of Standard Model (SM) forces yielding only one coupling constant
- Rank four simple Lie group with SM gauge group as a maximal subgroup:
$$SU(5) \to SU(3)_C \times SU(2)_L \times U(1)_Y$$
- All SM fermions of one family fit into the totally antisymmetric irreducible representations (irreps) **10** and $\mathbf{\overline{5}}$.
- Right-handed neutrinos can be assigned to SU(5) singlets to accommodate for neutrino oscillation.
- The quantization of the electric charge and the observed values arise naturally.

Problems and Limitations

- Unifies only one family, does not solve the flavor problem
- Simplest version of the SU(5) model predicts proton decay at a rate incompatible with experimental bounds. But new calculations of the proton stability ameliorate the situation. (Martin, Stavenga [arXiv:1110.2188])

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Review of the SU(5) model

Standard Model Decomposition

Decomposition of the **10** and $\mathbf{\bar{5}}$ to the SM gauge group $\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y}$:

$$\mathbf{10} \rightarrow (\mathbf{\bar{3}},\mathbf{1})(\text{-}^4/_3) + (\mathbf{3},\mathbf{2})(^1/_3) + (\mathbf{1},\mathbf{1})(2)$$

$$\mathbf{\bar{5}} \rightarrow (\mathbf{\bar{3}},\mathbf{1})(^2/_3) + (\mathbf{1},\mathbf{2})(\text{-}1)$$

via an adjoint Higgs irrep $\mathbf{24}_H$ and electroweak (EW) breakdown by $\mathbf{5}_H$:

$$\mathrm{SU(5)} \xrightarrow{\mathbf{24}_H} \mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y}$$

$$\xrightarrow{\mathbf{5}_H} \mathrm{SU(3)_C \times U(1)_{em}}$$

Particle Identification

SM group	particles	Q	I_3	Y
$(\bar{3},1)(\text{-}^4/_3)$	$(u^c)_L$	$\text{-}^2/_3$	0	$\text{-}^4/_3$
$(3,2)(^1/_3)$	$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$^2/_3$ $\text{-}^1/_3$	$^1/_2$ $\text{-}^1/_2$	$^1/_3$
$(1,1)(2)$	e^+_L	1	0	2
$(\bar{3},1)(^2/_3)$	$(d^c)_L$	$^1/_3$	0	$^2/_3$
$(1,2)(\text{-}1)$	$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$	0 -1	$^1/_2$ $\text{-}^1/_2$	-1

Matrix Notation of Fermion Assignment

$$(\psi^{\bar{5}})_L = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ \nu_e \end{pmatrix}_L, \qquad (\psi^{10})_L = \begin{pmatrix} 0 & u_3^c & -u_2^c & -u_1 & -d_1 \\ -u_3^c & 0 & u_1^c & -u_2 & -d_2 \\ u_2^c & -u_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^+ \\ d_1 & d_2 & d_3 & e^+ & 0 \end{pmatrix}_L$$

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Three family unification and reproduction of mass and mixing hierarchy

Assignments

- Larger SU(N)'s have more complex *basic irreps* than SU(5) for an exotic-fermion free assignment:
  SU(5): $\mathbf{5}$, $\mathbf{10}$, $\overline{\mathbf{10}}$, $\overline{\mathbf{5}}$
  SU(12): $\mathbf{12}$, $\mathbf{66}$, $\mathbf{220}$, $\mathbf{495}$, $\mathbf{792}$, $\overline{\mathbf{792}}$, $\overline{\mathbf{495}}$, $\overline{\mathbf{220}}$, $\overline{\mathbf{66}}$, $\overline{\mathbf{12}}$
- These irreps have more anomaly free subsets than SU(5):
  SU(5): $\mathbf{5} + \overline{\mathbf{10}}$
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Natural Mass and Mixing Hierarchy

- Family triplication in SU(5) as $3(\mathbf{10} + \overline{\mathbf{5}})$.
  In higher SU(N)'s families can be assigned to different irreps resulting in different Yukawa interactions.
- Assuming underlying naturalness of Yukawa couplings, the measured values can be understood in an effective field theory scenario.

Supersymmetric Model

- We set up a supersymmetric model broken at a scale high enough to be inaccessible by current collider experiments $\rightsquigarrow$ we do not consider the superparterns of Standard Model particles.
- In supersymmetric models loops are suppressed $\rightsquigarrow$ higher dimensional operators must stem from tree-level diagrams. (Froggatt-Nielson-type diagrams)

Effective Theory Setup

Setup of the Effective Operators

- Describe top-quark mass term as four-dimensional, renormalizable Yukawa coupling and all others as effective couplings.
- Introduce vectorlike heavy fermions with masses at the SU(12) unification scale $M_{\mathrm{SU(12)}}$.
- Introduce SU(12) Higgs fields with an SU(5) singlet vacuum expectation value $\langle 1 \rangle_{\mathrm{SU(5)}}$ about 50 times smaller than the SU(12) unification scale M_{GUT}:

$$\epsilon = \frac{\langle 1 \rangle_{\mathrm{SU(5)}}}{M_{\mathrm{GUT}}} \sim \frac{1}{50} \tag{1}$$

- Yukawa interactions of dimension $4 + n$ have matrix elements of the form:

$$h_{ij} \epsilon^n v \, u_{iL}^T u_{jL}^c , \tag{2}$$

  with the Yukawa couplings h_{ij} and the electroweak VEV v=174 GeV.
- The dimensionless quantity ϵ parametrizes the mass and mixing hierarchy in our model.

Symmetry Breaking

Symmetry Breaking SU(12) $\to$ SU(5) $\to$ SM

- We use SU(5) as an intermediate step with the same embedding of one SM family as the Georgi-Glashow model plus additional assignments to right-handed neutrinos.
- Breaking SU(12) to SU(5) with a single SU(12) adjoint ($\mathbf{143}_H$) as
$$SU(12) \to SU(5) \otimes SU(7) \otimes U(1) \tag{3}$$
preserves supersymmetry.
- Four SU(7) adjoint (stemming from more SU(12)) adjoints break SU(7) to U(1)'s:
$$SU(5) \otimes SU(7) \otimes U(1) \to SU(5) \otimes U(1)^7. \tag{4}$$
- An SU(5) adjoint (which may be contained in the SU(12) adjoint) breaks to the SM:
$$SU(5) \otimes U(1)^7 \to SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)^7. \tag{5}$$
- The U(1)'s other than the hypercharge can be broken at the SUSY breaking scale with a set of singlet VEV's

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An explicit SU(12) model

Requirements for Three Family Unification

1. Set of anomaly-free irreps analogous to $3(\mathbf{10}+\overline{\mathbf{5}})$ for three families in SU(5).
2. Breaking the set to SU(5) yields $\mathbf{10}$s, $\overline{\mathbf{10}}$s, $\mathbf{5}$s, $\overline{\mathbf{5}}$s and $\mathbf{1}$s. Only an excess of three $\mathbf{10}$s over $\overline{\mathbf{10}}$s and three $\overline{\mathbf{5}}$s over $\mathbf{5}$s yields a chiral three-family model.
3. All others pair up like $\mathbf{10}+\overline{\mathbf{10}}$ or $\mathbf{5}+\overline{\mathbf{5}}$ or are SU(5) singlets with a Dirac mass term and become heavy. Three SU(5) singlets can be assigned to Majorana neutrinos.

SU(12) Model

1. Anomaly free set of irrep in SU(12):

$$6(\mathbf{495})+4(\overline{\mathbf{792}})+4(\overline{\mathbf{220}})+(\overline{\mathbf{66}})+4(\overline{\mathbf{12}}) \tag{6}$$

2. Branching rules SU(12) $\to$ SU(5):

$$\begin{aligned}
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\end{aligned} \tag{7}$$

3. Three chiral families on the SU(5) level and pairs of fermions that get massive:

$$3(\mathbf{10}+\overline{\mathbf{5}})+238(\mathbf{5}+\overline{\mathbf{5}})+211(\mathbf{10}+\overline{\mathbf{10}})+487(\mathbf{1}) \tag{8}$$

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Particle Assignments

Reproducing the Standard Model Mass and Mixing Phenomenology

- Assign the three families to the SU(12) irreps according to their $\mathbf{10}$, $\overline{\mathbf{5}}$ and $\mathbf{1}$ content.
- Assign Higgs with the EW vev to an SU(12) irrep.
- Introduce SU(12) Higgs fields with an SU(5) singlet VEV about 50 times smaller than the SU(12) unification scale M_{GUT} defining $\epsilon = \langle 1 \rangle_{\mathrm{SU(5)}} / M_{\mathrm{GUT}} \sim 1/50$.
- Assign SU(12) adjoint Higgs containing an SU(5) adjoint for symmetry breaking.
- Introduce vectorlike massive fermions for tree-level diagrams at M_{GUT}.

SU(12) Model

- First Family: $(\mathbf{10})\mathbf{495}_1 \to u_{\mathrm{L}}, u^c_{\mathrm{L}}, d_{\mathrm{L}}, e^c_{\mathrm{L}}$ $(\overline{\mathbf{5}})\overline{\mathbf{66}}_1 \to d^c_{\mathrm{L}}, e_{\mathrm{L}}, \nu_{1,\mathrm{L}}$ $(\mathbf{1})\overline{\mathbf{792}}_1 \to N^c_{1,\mathrm{L}}$
  Second Family: $(\mathbf{10})\overline{\mathbf{792}}_2 \to c_{\mathrm{L}}, c^c_{\mathrm{L}}, s_{\mathrm{L}}, \mu^c_{\mathrm{L}}$ $(\overline{\mathbf{5}})\overline{\mathbf{792}}_2 \to s^c_{\mathrm{L}}, \mu_{\mathrm{L}}, \nu_{2,\mathrm{L}}$ $(\mathbf{1})\overline{\mathbf{220}}_2 \to N^c_{2,\mathrm{L}}$
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- Assign the three families to the SU(12) irreps according to their **10**, $\overline{\mathbf{5}}$ and **1** content.
- Assign Higgs with the EW vev to an SU(12) irrep.
- Introduce SU(12) Higgs fields with an SU(5) singlet VEV about 50 times smaller than the SU(12) unification scale M_{GUT} defining $\epsilon = \langle 1 \rangle_{\mathrm{SU(5)}} / M_{\mathrm{GUT}} \sim 1/50$.
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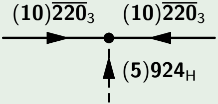
Yukawa Interactions

Setup and Requirements for the Yukawa Interactions

- Require dim 4 top quark Yukawa interaction and all others of higher dimension.
- Compute tree-level Yukawa interactions via mass insertions with heavy fermions.
- Integrate out heavy fermions $\rightsquigarrow$ higher dimensional effective Yukawa interactions.
- Perform the spontaneous symmetry breaking.

SU(12) Yukawa Interactions

- Dim-4 top quark Yukawa interaction (**U33**):



- Example of tree-level Yukawa interaction:

D33:

(9)

or in dense notation:

$$\text{D33:}\ (10)\overline{220}_3.(\overline{5})924_\mathrm{H}.(\overline{5})\overline{220}\times(5)220.(1)66_\mathrm{H}.(\overline{5})\overline{792}_3, \tag{10}$$

and after integrating out the massive fermions:

$$\text{D33:}\ (10)\overline{220}_3(\overline{5})924_\mathrm{H}(1)66_\mathrm{H}(\overline{5})\overline{792}_3 \tag{11}$$

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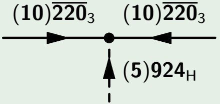

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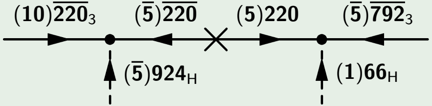

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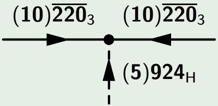

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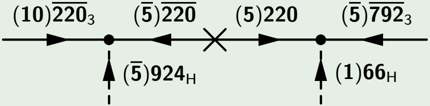

(9)

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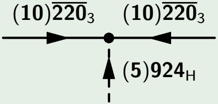

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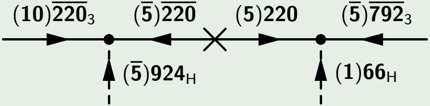

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Mass-Term Diagrams

Leading Order Up-Type Quark Mass-Term Diagrams

Dim 4:

U33: $(10)\overline{220}_3.(5)924_H.(10)\overline{220}_3$

Dim 5:

U23: $(10)\overline{792}_2.(1)66_H.(\overline{10})220\times(10)\overline{220}.(5)924_H.(10)\overline{220}_3$

U32: $(10)\overline{220}_3.(5)924_H.(10)\overline{220}\times(\overline{10})220.(1)66_H.(10)\overline{792}_2$

Dim 6:

U13: $(10)495_1.(1)220_H.(\overline{10})792\times(10)\overline{792}.(1)66_H.(\overline{10})220\times(10)\overline{220}.(5)924_H.(10)\overline{220}_3$

U31: $(10)\overline{220}_3.(5)924_H.(10)\overline{220}\times(\overline{10})220.(1)66_H.(10)\overline{792}\times(\overline{10})792.(1)220_H.(10)495_1$

U22: $(10)\overline{792}_2.(1)66_H.(\overline{10})220\times(10)\overline{220}.(5)924_H.(10)\overline{220}\times(\overline{10})220.(1)66_H.(10)\overline{792}_2$

Dim 7:

U12: $(10)495_1.(1)220_H.(\overline{10})792\times(10)\overline{792}.(1)66_H.(\overline{10})220\times(10)\overline{220}$
$.(5)924_H.(10)\overline{220}\times(\overline{10})220.(1)66_H.(10)\overline{792}_2$

U21: $(10)\overline{792}_2.(1)66_H.(\overline{10})220\times(10)\overline{220}.(5)924_H.(10)\overline{220}\times(\overline{10})220$
$.(1)66_H.(10)\overline{792}\times(\overline{10})792.(1)220_H.(10)495_1$

Dim 8:

U11: $(10)495_1.(1)220_H.(\overline{10})792\times(10)\overline{792}.(1)66_H.(\overline{10})220\times(10)\overline{220}$
$.(5)924_H.(10)\overline{220}\times(\overline{10})220.(1)66_H.(10)\overline{792}\times(\overline{10})792.(1)220_H.(10)495_1$

Mass-Term Diagrams

Leading Order Down-Type Quark Mass-Term Diagrams

Dim 5:

D32: $(10)\overline{220}_3.(\bar{5})924_H.(\bar{5})\overline{220}\times(5)220.(1)66_H.(\bar{5})\overline{792}_2$

D33: $(10)\overline{220}_3.(\bar{5})924_H.(\bar{5})\overline{220}\times(5)220.(1)66_H.(\bar{5})\overline{792}_3$

Dim 6:

D31: $(10)\overline{220}_3.(\bar{5})924_H.(\bar{5})\overline{220}\times(5)220.(1)66_H.(\bar{5})\overline{792}\times(5)792.(1)\overline{220}_H.(\bar{5})\overline{66}_1$

D22: $(10)\overline{792}_2.(1)66_H.(\overline{10})220\times(10)\overline{220}.(\bar{5})924_H.(\bar{5})\overline{220}\times(5)220.(1)66_H.(\bar{5})\overline{792}_2$

D23: $(10)\overline{792}_2.(1)66_H.(\overline{10})220\times(10)\overline{220}.(\bar{5})924_H.(\bar{5})\overline{220}\times(5)220.(1)66_H.(\bar{5})\overline{792}_3$

Dim 7:

D12: $(10)495_1.(1)220_H.(\overline{10})792\times(10)\overline{792}.(1)66_H.(\overline{10})220\times(10)\overline{220}$
$.(\bar{5})924_H.(\bar{5})\overline{220}\times(5)220.(1)66_H.(\bar{5})\overline{792}_2$

D21: $(10)\overline{792}_2.(1)66_H.(\overline{10})220\times(10)\overline{220}.(\bar{5})924_H.(\bar{5})\overline{220}\times(5)220$
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Quark Masses and Mixings

Up-Type and Down-Type Quark Mass Matrices

$$M_{\mathrm{U}} = \begin{pmatrix} h^{\mathrm{u}}_{11}\epsilon^4 & h^{\mathrm{u}}_{12}\epsilon^3 & h^{\mathrm{u}}_{13}\epsilon^2 \\ h^{\mathrm{u}}_{12}\epsilon^3 & h^{\mathrm{u}}_{22}\epsilon^2 & h^{\mathrm{u}}_{23}\epsilon \\ h^{\mathrm{u}}_{13}\epsilon^2 & h^{\mathrm{u}}_{23}\epsilon & h^{\mathrm{u}}_{33} \end{pmatrix} v\,, \quad M_{\mathrm{D}} = \begin{pmatrix} h^{\mathrm{d}}_{11}\epsilon^4 & h^{\mathrm{d}}_{12}\epsilon^3 & h^{\mathrm{d}}_{13}\epsilon^3 \\ h^{\mathrm{d}}_{21}\epsilon^3 & h^{\mathrm{d}}_{22}\epsilon^2 & h^{\mathrm{d}}_{23}\epsilon^2 \\ h^{\mathrm{d}}_{31}\epsilon^2 & h^{\mathrm{d}}_{32}\epsilon & h^{\mathrm{d}}_{33}\epsilon \end{pmatrix} v\,, \quad M_{\mathrm{L}} = M_{\mathrm{D}}^T\,. \tag{12}$$

with the EW vev $v = 174\,\mathrm{GeV}$ and the so-called prefactors $h^{\mathrm{u,d}}_{..}$ of $\mathcal{O}(1)$.

Up-type and Down-type Quark Masses and CKM Matrix

- Diagonalizing $M_{\mathrm{U}}M_{\mathrm{U}}^\dagger$ and $M_{\mathrm{D}}M_{\mathrm{D}}^\dagger$ yields the up-type and down-type quark masses:

$$\mathrm{diag}(m_u^2, m_c^2, m_t^2) = U_{\mathrm{U}}^\dagger M_{\mathrm{U}} M_{\mathrm{U}}^\dagger U_{\mathrm{U}}\,, \tag{13}$$

$$\mathrm{diag}(m_d^2, m_s^2, m_b^2) = U_{\mathrm{D}}^\dagger M_{\mathrm{D}} M_{\mathrm{D}}^\dagger U_{\mathrm{D}} \tag{14}$$

- The CKM matrix encodes the mismatch of the mass eigenstates of the up-type and down-type quarks:

$$V_{\mathrm{CKM}} = U_{\mathrm{U}}^\dagger U_{\mathrm{D}}\,, \tag{15}$$

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DN33: $(\overline{5})\overline{792}_3.(5)924_H.(1)\overline{12}_3$

Dim 5:

DN13: $(\overline{5})\overline{66}_1.(1)\overline{220}_H.(5)792\times(\overline{5})\overline{792}.(5)924_H.(1)\overline{12}_3$

DN22: $(\overline{5})\overline{792}_2.(1)66_H.(5)220\times(\overline{5})\overline{220}.(5)924_H.(1)\overline{220}_2$

DN32: $(\overline{5})\overline{792}_3.(1)66_H.(5)220\times(\overline{5})\overline{220}.(5)924_H.(1)\overline{220}_2$

Dim 6:

DN12: $(\overline{5})\overline{66}_1.(1)\overline{220}_H.(5)792\times(\overline{5})\overline{792}.(1)66_H.(5)220\times(\overline{5})\overline{220}.(5)924_H.(1)\overline{220}_2$

DN21: $(\overline{5})\overline{792}_2.(1)66_H.(5)220\times(\overline{5})\overline{220}.(5)924_H.(1)\overline{220}\times(1)220.(1)66_H.(1)\overline{792}_1$

DN31: $(\overline{5})\overline{792}_3.(1)66_H.(5)220\times(\overline{5})\overline{220}.(5)924_H.(1)\overline{220}\times(1)220.(1)66_H.(1)\overline{792}_1$

Dim 7:

DN11: $(\overline{5})\overline{66}_1.(1)\overline{220}_H.(5)792\times(\overline{5})\overline{792}.(1)66_H.(5)220\times(\overline{5})\overline{220}$
$.(5)924_H.(1)\overline{220}\times(1)220.(1)66_H.(1)\overline{792}_1$

Mass-Term Diagrams

Leading Order Majorana-Neutrino Quark Mass-Term Diagrams

Dim 4:

- **MN11**: $(1)\overline{792}_1.(1)\overline{66}_H.(1)\overline{792}_1$
- **MN33**: $(1)\overline{12}_3.(1)66_H.(1)\overline{12}_3$

Dim 5:

- **MN12**: $(1)\overline{792}_1.(1)\overline{66}_H.(1)\overline{792}\times(1)792.(1)\overline{66}_H.(1)\overline{220}_2$
- **MN21**: $(1)\overline{220}_2.(1)\overline{66}_H.(1)792\times(1)\overline{792}.(1)\overline{66}_H.(1)\overline{792}_1$

Dim 6:

- **MN13**: $(1)\overline{792}_1.(1)\overline{66}_H.(1)\overline{792}\times(1)792.(1)\overline{66}_H.(1)\overline{220}\times(1)220.(1)\overline{66}_H.(1)\overline{12}_3$
- **MN31**: $(1)\overline{12}_3.(1)\overline{66}_H.(1)220\times(1)\overline{220}.(1)\overline{66}_H.(1)792\times(1)\overline{792}.(1)\overline{66}_H.(1)\overline{792}_1$
- **MN22**: $(1)\overline{220}_2.(1)\overline{66}_H.(1)792\times(1)\overline{792}.(1)\overline{66}_H.(1)\overline{792}\times(1)792.(1)\overline{66}_H.(1)\overline{220}_2$

Dim 7:

- **MN23**: $(1)\overline{220}_2.(1)\overline{66}_H.(1)792\times(1)\overline{792}.(1)\overline{66}_H.(1)\overline{792}\times(1)792$
  $.(1)\overline{66}_H.(1)\overline{220}\times(1)220.(1)\overline{66}_H.(1)\overline{12}_3$
- **MN32**: $(1)\overline{12}_3.(1)\overline{66}_H.(1)220\times(1)\overline{220}.(1)\overline{66}_H.(1)792\times(1)\overline{792}$
  $.(1)\overline{66}_H.(1)\overline{792}\times(1)792.(1)\overline{66}_H.(1)\overline{220}_2$

Neutrino Mass Matrices

Majorana and Dirac Neutrino Mass Matrices

$$M_{\mathrm{DN}} = \begin{pmatrix} h^{\mathrm{dn}}_{11}\epsilon^3 & h^{\mathrm{dn}}_{12}\epsilon^2 & h^{\mathrm{dn}}_{13}\epsilon \\ h^{\mathrm{dn}}_{21}\epsilon^2 & h^{\mathrm{dn}}_{22}\epsilon & h^{\mathrm{dn}}_{23} \\ h^{\mathrm{dn}}_{31}\epsilon^2 & h^{\mathrm{dn}}_{32}\epsilon & h^{\mathrm{dn}}_{33} \end{pmatrix} v\,, \quad M_{\mathrm{MN}} = \begin{pmatrix} h^{\mathrm{mn}}_{11} & h^{\mathrm{mn}}_{12}\epsilon & h^{\mathrm{mn}}_{13}\epsilon^2 \\ h^{\mathrm{mn}}_{12}\epsilon & h^{\mathrm{mn}}_{22}\epsilon^2 & h^{\mathrm{mn}}_{23}\epsilon^3 \\ h^{\mathrm{mn}}_{13}\epsilon^2 & h^{\mathrm{mn}}_{23}\epsilon^3 & h^{\mathrm{mn}}_{33} \end{pmatrix} \Lambda_{\mathrm{R}}\,, \quad M_{\mathrm{l}} = M_{\mathrm{D}}^T \quad (16)$$

The right-handed scale Λ_{R} coincides with the SU(5) singlet VEV $\langle 1 \rangle_{\mathrm{SU(5)}}$.

Seesaw Mechanism and Light-Neutrino Mass Matrix

- Light neutrino mass matrix via the Type-I Seesaw machanism:

$$M_\nu = -M_{\mathrm{DN}} M_{\mathrm{MN}}^{-1} M_{\mathrm{DN}}^T\,, \quad (17)$$

yields to lowest order in ϵ for each matrix element: $M_\nu \approx \frac{v^2}{\Lambda_{\mathrm{R}}} \times$

$$\times \begin{pmatrix} \epsilon^2 \left(\frac{h^{\mathrm{dn}\,2}_{12} h^{\mathrm{mn}}_{11}}{h^{\mathrm{mn}\,2}_{12} - h^{\mathrm{mn}}_{11} h^{\mathrm{mn}}_{22}} - \frac{h^{\mathrm{dn}\,2}_{13}}{h^{\mathrm{mn}}_{33}} \right) & \epsilon \left(\frac{h^{\mathrm{dn}}_{12} h^{\mathrm{dn}}_{22} h^{\mathrm{mn}}_{11}}{h^{\mathrm{mn}\,2}_{12} - h^{\mathrm{mn}}_{11} h^{\mathrm{mn}}_{22}} - \frac{h^{\mathrm{dn}}_{13} h^{\mathrm{dn}}_{23}}{h^{\mathrm{mn}}_{33}} \right) & \epsilon \left(\frac{h^{\mathrm{dn}}_{12} h^{\mathrm{dn}}_{32} h^{\mathrm{mn}}_{11}}{h^{\mathrm{mn}\,2}_{12} - h^{\mathrm{mn}}_{11} h^{\mathrm{mn}}_{22}} - \frac{h^{\mathrm{dn}}_{13} h^{\mathrm{dn}}_{33}}{h^{\mathrm{mn}}_{33}} \right) \\ \epsilon \left(\frac{h^{\mathrm{dn}}_{12} h^{\mathrm{dn}}_{22} h^{\mathrm{mn}}_{11}}{h^{\mathrm{mn}\,2}_{12} - h^{\mathrm{mn}}_{11} h^{\mathrm{mn}}_{22}} - \frac{h^{\mathrm{dn}}_{13} h^{\mathrm{dn}}_{23}}{h^{\mathrm{mn}}_{33}} \right) & \frac{h^{\mathrm{dn}\,2}_{22} h^{\mathrm{mn}}_{11}}{h^{\mathrm{mn}\,2}_{12} - h^{\mathrm{mn}}_{11} h^{\mathrm{mn}}_{22}} - \frac{h^{\mathrm{dn}\,2}_{23}}{h^{\mathrm{mn}}_{33}} & \frac{h^{\mathrm{dn}}_{22} h^{\mathrm{dn}}_{32} h^{\mathrm{mn}}_{11}}{h^{\mathrm{mn}\,2}_{12} - h^{\mathrm{mn}}_{11} h^{\mathrm{mn}}_{22}} - \frac{h^{\mathrm{dn}}_{23} h^{\mathrm{dn}}_{33}}{h^{\mathrm{mn}}_{33}} \\ \epsilon \left(\frac{h^{\mathrm{dn}}_{12} h^{\mathrm{dn}}_{32} h^{\mathrm{mn}}_{11}}{h^{\mathrm{mn}\,2}_{12} - h^{\mathrm{mn}}_{11} h^{\mathrm{mn}}_{22}} - \frac{h^{\mathrm{dn}}_{13} h^{\mathrm{dn}}_{33}}{h^{\mathrm{mn}}_{33}} \right) & \frac{h^{\mathrm{dn}}_{22} h^{\mathrm{dn}}_{32} h^{\mathrm{mn}}_{11}}{h^{\mathrm{mn}\,2}_{12} - h^{\mathrm{mn}}_{11} h^{\mathrm{mn}}_{22}} - \frac{h^{\mathrm{dn}}_{23} h^{\mathrm{dn}}_{33}}{h^{\mathrm{mn}}_{33}} & \frac{h^{\mathrm{dn}\,2}_{32} h^{\mathrm{mn}}_{11}}{h^{\mathrm{mn}\,2}_{12} - h^{\mathrm{mn}}_{11} h^{\mathrm{mn}}_{22}} - \frac{h^{\mathrm{dn}\,2}_{33}}{h^{\mathrm{mn}}_{33}} \end{pmatrix} \quad (18)$$

$\rightsquigarrow$ milder hierarchy than quark and charged-lepton mass matrices.

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Neutrino Sector

Lepton Masses and PMNS Matrix

- Diagonalizing $M_\mathsf{L} M_\mathsf{L}^\dagger$ and $M_\nu M_\nu^\dagger$ yields the charged-lepton and light-neutrino masses:

$$\begin{aligned} \mathrm{diag}(m_e^2, m_\mu^2, m_\tau^2) &= U_\mathsf{L}^\dagger M_\mathsf{L} M_\mathsf{L}^\dagger U_\mathsf{L}, \\ \mathrm{diag}(m_{\nu_1}^2, m_{\nu_2}^2, m_{\nu_3}^2) &= U_\nu^\dagger M_\nu M_\nu^\dagger U_\nu. \end{aligned} \tag{19}$$

- Lepton Mixing Matrix (PMNS matrix) as mismatch of charged lepton and neutrino mass eigenstates:

$$V_\mathsf{PMNS} = U_\mathsf{L}^\dagger U_\nu. \tag{20}$$

Fit to Phenomenology

Motivation

- Naturalness predicts prefactors h^{u}_{ij}, h^{d}_{ij}, h^{ℓ}_{ij}, h^{dn}_{ij} and h^{mn}_{ij} of $\mathcal{O}(1)$ at the SU(12) unification scale $M_{\mathrm{SU(12)}}$. A fit to phenomenology with the prefactors as fit parameters serves as conformance test.
- The theoretical predictions of masses and mixings from the fit exhibit the predictive power and limitations of the SU(12) model.

Inputs from Phenomenology $n_{\mathrm{data}}=30$

Up-type masses
$m_{\mathrm{u}} = 2.2$ MeV
$m_{\mathrm{c}} = 600$ MeV
$m_{\mathrm{t}} = 166$ GeV

Down-Type masses
$m_{\mathrm{d}} = 3.8$ MeV
$m_{\mathrm{s}} = 75$ MeV
$m_{\mathrm{b}} = 2.78$ GeV

CKM Matrix
$$\begin{pmatrix} 0.974 & 0.225 & 0.003 \\ -0.225 & 0.973 & 0.041 \\ 0.009 & -0.040 & 0.999 \end{pmatrix}$$

Ch. Lepton masses
$m_{\mathrm{e}} = 0.501$ MeV
$m_{\mu} = 104$ MeV
$m_{\tau} = 1.75$ GeV

Neutrino Mass Diff.
$|\Delta_{21}| = 7.6\times10^{-5}\,\mathrm{eV}^2$
$|\Delta_{31}| = 2.4\times10^{-3}\,\mathrm{eV}^2$
$|\Delta_{32}| = 2.4\times10^{-3}\,\mathrm{eV}^2$

PMNS Matrix
$$\begin{pmatrix} 0.824 & 0.547 & -0.145 \\ -0.500 & 0.582 & -0.641 \\ -0.267 & 0.601 & 0.754 \end{pmatrix}$$

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Fit to Phenomenology

Inputs Quark Sector

Up-type masses

$m_u = 2.2$ MeV
$m_c = 600$ MeV
$m_t = 166$ GeV

Down-Type masses

$m_d = 3.8$ MeV
$m_s = 75$ MeV
$m_b = 2.78$ GeV

CKM Matrix

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Theoretical Predictions Quark Sector

Up-type masses

$m_u = 2.1$ MeV
$m_c = 600$ MeV
$m_t = 166$ GeV

Down-Type masses

$m_d = 2.7$ MeV
$m_s = 90.7$ MeV
$m_b = 2.32$ GeV

CKM Matrix

$$\begin{pmatrix} 0.974 & 0.227 & 0.003 \\ -0.227 & 0.973 & 0.042 \\ 0.007 & -0.042 & 0.999 \end{pmatrix}$$

Remarks

- Quark and charged lepton masses evaluated at the top-quark mass as common scale
- No scaling for CKM and PMNS matrix considered
- CKM RGE running for the mixings of the first two families is very small and mixing with the third family not too large either.

Fit to Phenomenology

Inputs Lepton Sector

Lepton masses

$m_e = 0.501$ MeV
$m_\mu = 104$ MeV
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Mixing Angles

$\sin^2\theta_{12} = 0.306$
$\sin^2\theta_{23} = 0.420$
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$\sin^2\theta_{13} = 0.021$

Light Neutrinos

$m_1 = 0.0$ meV
$m_2 = 8.65$ meV
$m_3 = 49.7$ meV

Heavy Neutrinos

$M_1 = 1.67\times10^{12}$ GeV
$M_2 = 6.85\times10^{13}$ GeV
$M_3 = 5.30\times10^{14}$ GeV

Fit Results

Results for Fit Parameters

Using a fixed $\epsilon = 1/6.5^2 = 0.0237$ we find for $n_{\text{params}} = n_{\text{prefactors}} + 1 = 26$ fit parameters:

$$M_U = \begin{pmatrix} -1.1\epsilon^4 & 7.1\epsilon^3 & 5.6\epsilon^2 \\ 7.1\epsilon^3 & -6.2\epsilon^2 & -0.10\epsilon \\ 5.6\epsilon^2 & -0.10\epsilon & -0.95 \end{pmatrix} v, \quad M_D = \begin{pmatrix} -6.3\epsilon^4 & 8.0\epsilon^3 & -1.9\epsilon^3 \\ -4.5\epsilon^3 & 0.38\epsilon^2 & -1.3\epsilon^2 \\ 0.88\epsilon^2 & -0.23\epsilon & -0.51\epsilon \end{pmatrix} v,$$
$$M_{DN} = \begin{pmatrix} h_{11}^{\text{dn}}\epsilon^3 & 0.21\epsilon^2 & -2.7\epsilon \\ h_{21}^{\text{dn}}\epsilon^2 & -0.28\epsilon & -0.15 \\ h_{31}^{\text{dn}}\epsilon^2 & 2.1\epsilon & 0.086 \end{pmatrix} v, \quad M_{MN} = \begin{pmatrix} -0.72 & -1.5\epsilon & h_{13}^{\text{mn}}\epsilon^2 \\ -1.5\epsilon & 0.95\epsilon^2 & h_{23}^{\text{mn}}\epsilon^3 \\ h_{13}^{\text{mn}}\epsilon^2 & h_{23}^{\text{mn}}\epsilon^3 & 0.093 \end{pmatrix} \Lambda_R,$$
$$M_\nu = \begin{pmatrix} -81.\epsilon^2 & -4.3\epsilon & 2.4\epsilon \\ -4.3\epsilon & -0.25 & 0.28 \\ 2.4\epsilon & 0.28 & -1.1 \end{pmatrix} \frac{v^2}{\Lambda_R}, \tag{21}$$

and the right-handed scale fit to $\Lambda_R = 7.4 \times 10^{14}$ GeV.
Fit Stats: $\chi^2 = 0.239$, $n_{\text{dof}} = n_{\text{data}} - n_{\text{params}} = 4 \rightsquigarrow \chi^2/n_{\text{dof}} = 0.060$, $\text{prob}(0.239, 4) = 0.993$

SU(12) Unification Scale

With $\epsilon = \langle 1 \rangle_{SU(5)}/M_{GUT} = 1/6.5^2$ and $\Lambda_R = \langle 1 \rangle_{SU(5)}$ the SU(12) unification scale is:

$$M_{GUT} = \frac{\langle 1 \rangle_{SU(5)}}{\epsilon} = \frac{\Lambda_R}{\epsilon} = 7.4 \times 10^{14}\ \text{GeV} \cdot 6.5^2 = 3.1 \times 10^{16}\ \text{GeV} \tag{22}$$

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Limitations and Ongoing Research

Advantages

- No discrete flavor symmetries $\rightsquigarrow$ avoids problems with breaking by gravity, domain walls and explanation of its origin
- The phenomenology of the Standard Model can be reproduced by the right assignments and choices of prefactors.

Problems and Limitations

- The SU(12) gauge group is a very large $\rightsquigarrow$ prediction of a host of heavy fermions and heavy singlet fermions.
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Outline

Model Scan

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We have computerized a brute-force scan of SU(N)'s that can

- find all anomaly free sets of irreps that have three chiral families,
- loop over all fermion, Higgs and massive fermion assignments and combinations including right-handed neutrinos and compute the resulting mass matrices,
- fit the Yukawa couplings of order one in the mass matrices to Standard-Model and neutrino phenomenology to test the model in the loop.

Computational Requirements

- Branching rules SU(N) $\to$ (SU($N-k$)$\times$SU(k)$\times$U(1) $\to$) SU(5)
- Determination of singlets in SU(N) and SU(5) tensor products
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LieART

LieART: Lie Algebras and Representation Theory

- A Mathematica package to compute tensor products and branching rules of irreps of all classical and exceptional Lie algebras.
- Originally intended as part of the computerized scan to compute tensor products and branching rules of SU(N) only.
- LieART is easy to use and displays irreps using the nomenclature physicists prefer, e.g., $\overline{\mathbf{10}}$ instead of the Dynkin label (0010).
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Features

- Root systems of algebras, Weyl group orbits of weights, weight systems of irreps
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LieART

LieART: Lie Algebras and Representation Theory

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- Higher dimensional Yukawa interactions involving Higgs with SU(5) singlet vevs amount for the mass and mixing hierarchy.
- Three chiral families with the right masses and mixings can be unified in SU(12) without additional discrete flavor symmetries.
- Despite the limitations of our analysis we consider the model as proof of concept for an alternative approach to unification of families and flavor.
- The spin-off LieART brings Lie algebras and representation theory to Mathematica.

Outlook

- Reintroducing discrete symmetries might find a balance of gauge group and flavor symmetry group size.
- Scan for alternative models in all SU(N)s with and without discrete symmetries.
- Many parameters of the scan and requirements for a suitable model can be changed, i.e. the set of Higgs and massive fermion irreps to choose from. All of these possibilities should be explored.
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