FLAVORED LEPTOGENESIS

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HIGGS AT 125 GEV?

- Evolution of Higgs quartic coupling
- SM meta-stable up to Planck scale

Elias-Miro, Espinosa, Giudice, Isidori, Riotto, Strumia, 2011

SM VALID TO VERY HIGH SCALES?

What about:

Dark Matter?

Neutrino Masses?

Baryogenesis?

SM VALID TO VERY HIGH SCALES?

What about:

Dark Matter?

See-Saw!

Leptogenesis!

OUTLINE

- Introduction
- Non-Equilibrium Quantum Field Theory Approach
  - Thermal corrections
  - Flavor effects
- Conclusions

based on:
NPB 838 (2010) 1-27
NPB 843 (2011) 177-212
with M. Beneke, B. Garbrecht, C. Fidler, M. Herranen

SEE-SAW MODEL OF NEUTRINO MASSES

Minkowski 1977, Yanagida 1979, ...

- Right-handed neutrinos ψ_{Ni} are neutral singlets

- Can have Majorana mass term:

$$\mathcal{L} = \frac{1}{2}\bar{\psi}_{Ni}(\mathrm{i}\partial\!\!\!/ - M_i)\psi_{Ni} + \bar{\psi}_\ell \mathrm{i}\partial\!\!\!/ \psi_\ell - Y_i^* \bar{\psi}_\ell \phi^\dagger P_{\mathrm{R}} \psi_{Ni} - Y_i \bar{\psi}_{Ni} P_{\mathrm{L}} \phi \psi_\ell$$

- Mass matrix: $\begin{pmatrix} 0 & Y_i v \\ Y_i^* v & M_i \end{pmatrix}$

- Eigenvalues:

$$\lambda_+ \approx M_1 \qquad \lambda_- \approx |Y^2| \frac{v^2}{M_1}$$

Neutrino masses done!

What about Baryogenesis?

HOW TO GENERATE A BARYON ASYMMETRY?

Sakharovs conditions (1967): **SM:**

- Baryon number violation ☺
- CP violation 😐
- Departure from equilibrium ☹

SEE-SAW MODEL OF NEUTRINO MASSES

Minkowski 1977, Yanagida 1979, ...

- Right-handed neutrinos ψ_{Ni} are neutral singlets
- Can have Majorana mass term:

new CP violation

$$\mathcal{L} = \frac{1}{2}\bar{\psi}_{Ni}(\mathrm{i}\partial\!\!\!/ - M_i)\psi_{Ni} + \bar{\psi}_\ell \mathrm{i}\partial\!\!\!/ \psi_\ell - Y_i^* \bar{\psi}_\ell \phi^\dagger P_\mathrm{R} \psi_{Ni} - Y_i \bar{\psi}_{Ni} P_\mathrm{L} \phi \psi_\ell$$

- Majorana mass violates lepton number
- Out of equilibrium decay of N_1 if couplings satisfy

$$\Gamma_{N_1} \propto \sum |Y_{1i}|^2 M_1 < H\big|_{T\approx M_1}$$

HOW TO GENERATE AN ASYMMETRY?

Sakharovs conditions:	SM + See Saw
• Baryon number violation	☺
• CP violation	☺
• Departure from equilibrium	☺

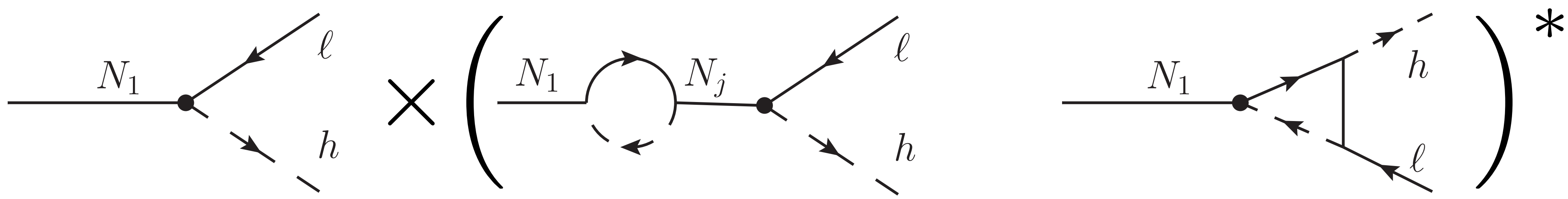
Leptogenesis

Fukugita & Yanagida, 1986

USUAL WAY TO PREDICT ASYMMETRY:

- Calculate CP asymmetry in decays

e.g. Covi et al, 1996

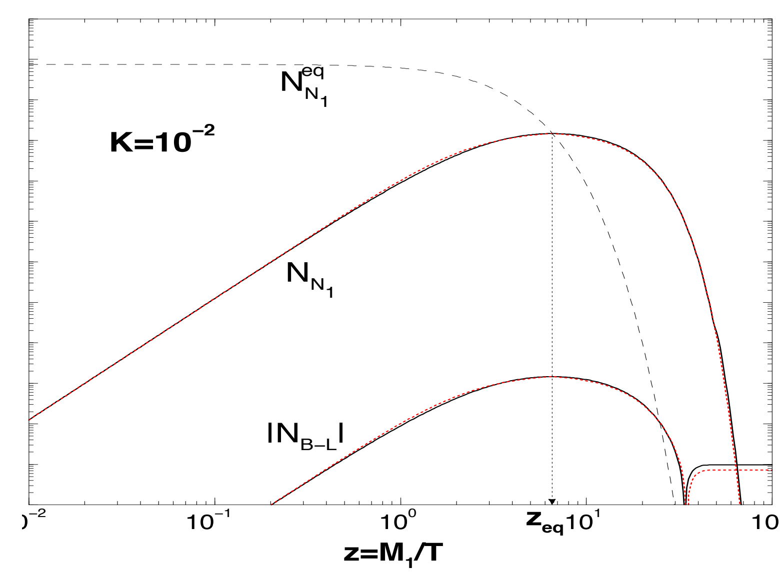


- Plug into Boltzmann equation

$$\partial_\eta f_{\ell-\bar{\ell}} = C_D[f_{\ell-\bar{\ell}}] + C_S[f_{\ell-\bar{\ell}}]$$

- Solve (with approximations)

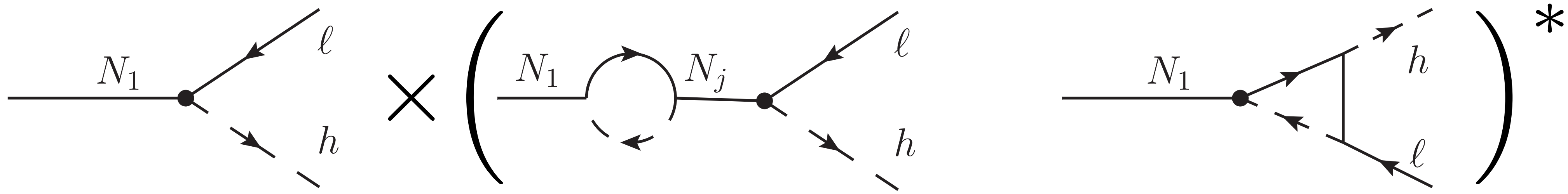
e.g. Pedestrian: Buchmuller, di Bari, Plumacher, 2000



VALID APPROACH?

- Calculate CP asymmetry in decays

Quantum Effect



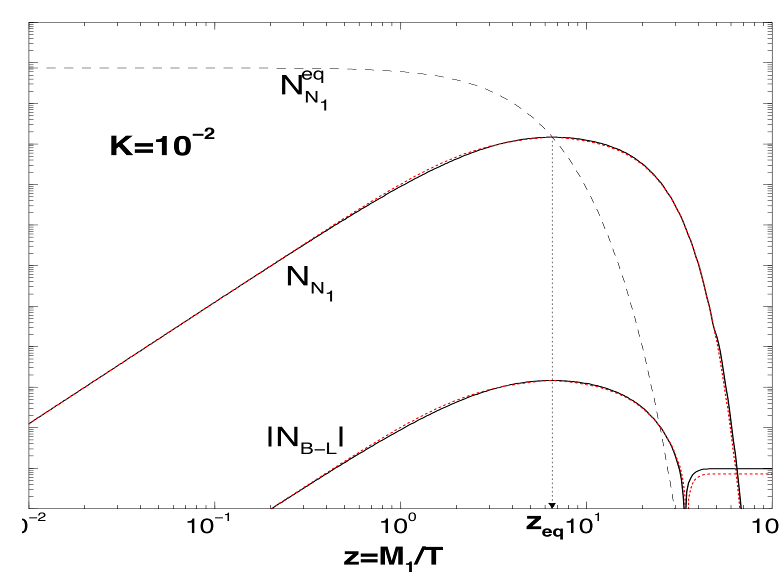
- Plug into Boltzmann equation

Classical Equation

$$\partial_\eta f_{\ell-\bar{\ell}} = C_D[f_{\ell-\bar{\ell}}] + C_S[f_{\ell-\bar{\ell}}]$$

- Solve (with approximations)
  e.g. Pedestrian: Buchmuller, di Bari, Plumacher, 2000

something missed?



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earlier work:
Buchmuller, Fredenhagen
Riotto, de Simone

QUANTITIES OF INTEREST

- Equilibrium distributions: $f_B^{\rm eq}(E,T) = \frac{1}{e^{E/T}-1}$ $\quad f_F^{eq}(E,T) = \frac{1}{e^{E/T}+1}$

- N_1 distribution: $f_{N_1} - f_{N_1}^{\rm eq} = \delta f_{N_1}$ $\quad$ Deviation from th. equilibrium

- Lepton/anti-lepton distributions:

$$\delta f_\ell = f_\ell - f_{\bar{\ell}}$$

Lepton asymmetry

- Derive time evolution equations for $\delta f_\ell \,, \quad \delta f_{N_1}$

WHERE DO f_X APPEAR IN FIELD THEORY?

- Mode expansion for free scalar field:

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_k} \left(a_{\mathbf{k}} e^{ikx} + a^\dagger_{\mathbf{k}} e^{-ikx} \right)$$

- Expectation values of two point functions:

$$\mathcal{F}\mathcal{T} \langle \phi^\dagger(x) \phi(y) \rangle \sim \langle a^\dagger_{\mathbf{k}} a_{\mathbf{k}} \rangle \sim f(\mathbf{k})$$

- Start with EOM for two point Greens functions

$$iS_\ell(x,y) = \langle \psi_\ell(x) \bar{\psi}_\ell(y) \rangle \qquad iS_{N_1}(x,y) = \langle \psi_{N_1}(x) \bar{\psi}_{N_1}(y) \rangle$$

NEQFT FOR LEPTOGENESIS

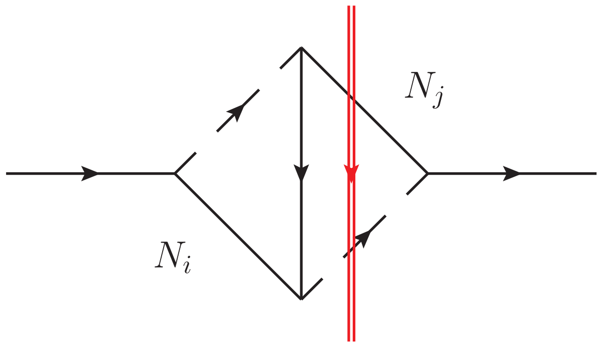
- Dyson-Schwinger equation

$$i\partial\!\!\!/_u S^{ab}(u,v) = a\delta_{ab}\delta^4(u-v) + \sum_c \int d^4w \Sigma^{ac}(u,w) S^{cb}(w,v)$$

1PI self energies

$$\frac{d}{d\eta} f_{N1}(\mathbf{k}) = D(\mathbf{k})$$

$$n_\ell(t) = \int d^3k f_\ell(t,k) \qquad \frac{d}{d\eta}(n_\ell - \bar{n}_\ell) = W + S\,.$$



OUT OF EQUILIBRIUM?

- In th. equilibrium: $iS_\ell(x,y) = iS_\ell(x-y)$
- Out of equilibrium: $iS_\ell(x,y) = iS_\ell(X,r)$

$$X = \tfrac{1}{2}(x+y) \qquad r = x-y$$

- Separate microscopic r and macroscopic X scales using "Wigner transformation"

$$iS(X,k) = \int d^4r e^{ikr} iS(X,r)$$

KADANOFF BAYM EQUATIONS

$$i\partial\!\!\!/_u S^{ab}(u,v) = a\delta_{ab}\delta^4(u-v) + \sum_c \int d^4w\Sigma^{ac}(u,w)S^{cb}(w,v)$$

- After Wigner transformation:

$$(-ik\!\!\!/ + \partial\!\!\!/_X)\, iS(X,k) + ie^{-i\diamond}\{\Sigma_h\}\{iS(X,k)\} = \mathcal{C}$$

- Gradient expansion: $e^{-i\diamond} \to 1$

$$(k\!\!\!/ - \Sigma_h)iS(X,k) = 0$$

Constraint equation: Quasi-particle poles

$$\partial\!\!\!/_X iS(X,k) = \mathcal{C}$$

Kinetic equation: Time evolution

LEFT HAND SIDE

- From constraint equation:

$$iS(X,k) = \not{k} 2\pi\delta(k^2)\left[\theta(k^0)f(X,\mathbf{k}) - \theta(-k^0)(1-\bar{f}(X,\mathbf{k}))\right]$$

- Plug into kinetic equation: $\not{\partial}_X iS(X,k) = \mathcal{C}$

- Take Dirac trace and integrate over k^0:

$$\frac{\partial}{\partial X^\mu} f(X,\mathbf{k}) = \int dk^0 \mathcal{C}$$

QUANTUM BOLTZMANN EQUATIONS

- Spatial homogeneity: $\partial_{X^\mu} \to \partial_{X^0} = \partial_t$
- Expansion of universe: $t \to \eta$ (conformal time)
- Quantum Boltzmann equations

$$\frac{d}{d\eta} f_{N1}(\mathbf{k}) = D(\mathbf{k})$$

$$\frac{d}{d\eta}(n_\ell - \bar{n}_\ell) = W + S\,.$$

$$n_\ell(t) = \int d^3k f_\ell(t,k)$$

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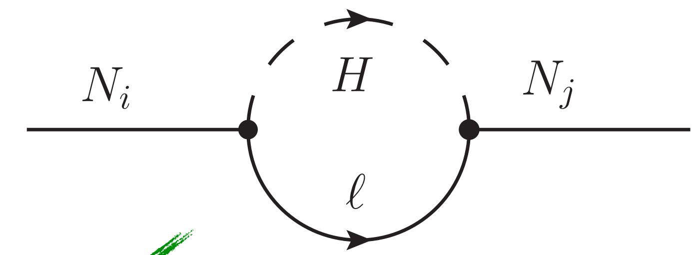
NOW THE COLLISION TERM

- N_1 decay/inverse decay

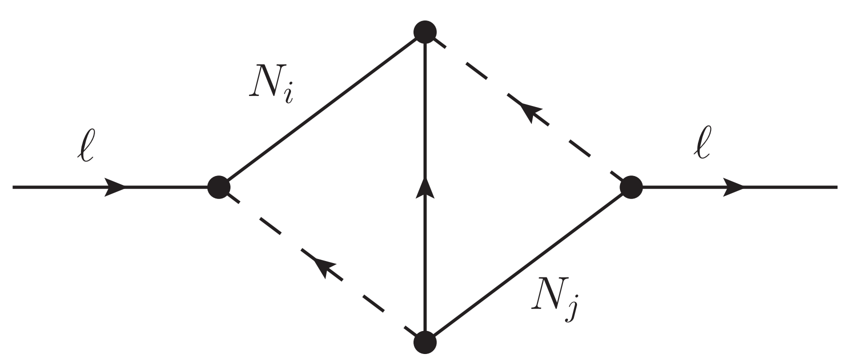
  decay rate Γ_{N_1}

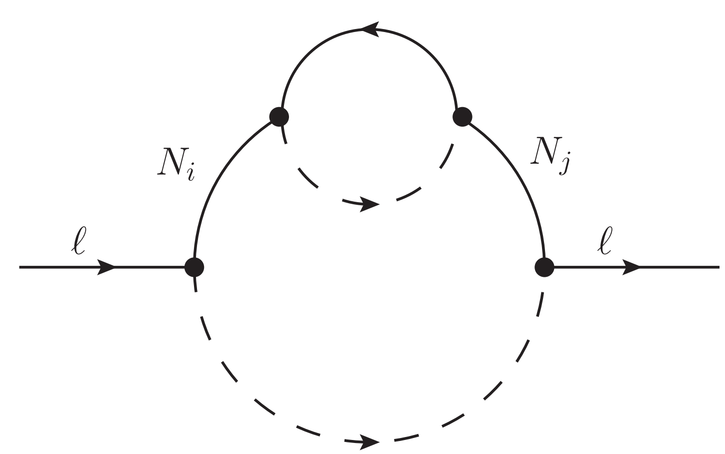


$$\frac{d}{d\eta} f_{N1}(\mathbf{k}) = D(\mathbf{k})$$

$$\frac{d}{d\eta}(n_\ell - \bar{n}_\ell) = W + S\,.$$

- CP source





CP SOURCE

- Source term in hierarchical limit ($M_2 \gg M_1$):

$$S = 3\,\mathrm{Im}[Y_1^2 Y_2^{*2}] \left(-\frac{M_1}{M_2}\right) \int \frac{d^3k'}{(2\pi)^3 2\,\omega_{k'}} \delta f_N(\mathbf{k}') \Sigma_{N\mu}(\mathbf{k}') \Sigma_N^\mu(\mathbf{k}')$$

$$f_N - f_N^{eq}$$

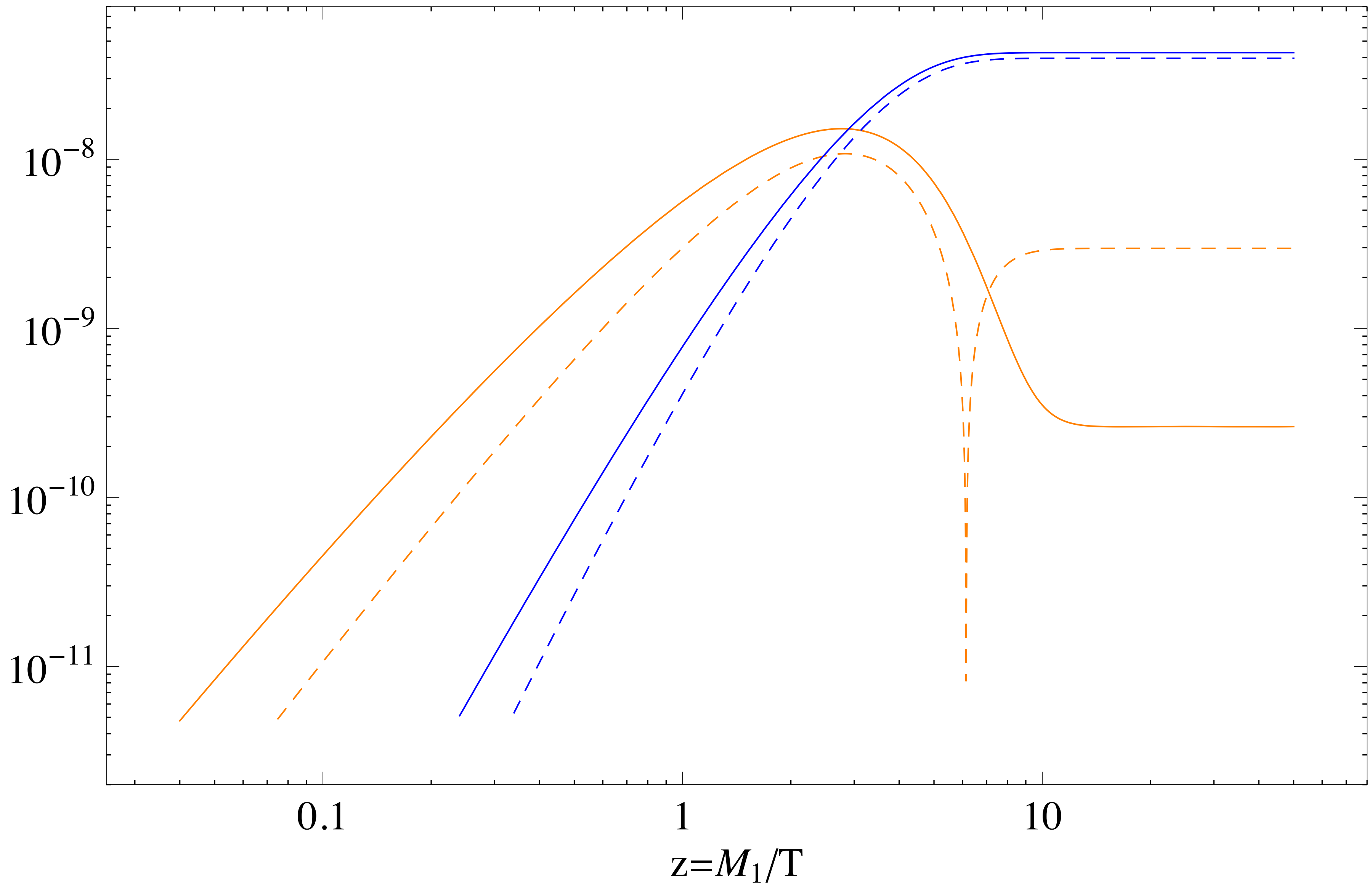
no asymmetry in equilibrium

$$\Sigma_N^\mu(k) = \int_{p,q} \delta^4(k-p-q)\, p^\mu \left(1 - f_\ell^{\mathrm{eq}}(\mathbf{p}) + f_\phi^{\mathrm{eq}}(\mathbf{q})\right)$$

finite density corrections

In vacuum QFT: $\Sigma_N^\mu(k) = \dfrac{k^\mu}{16\pi}$

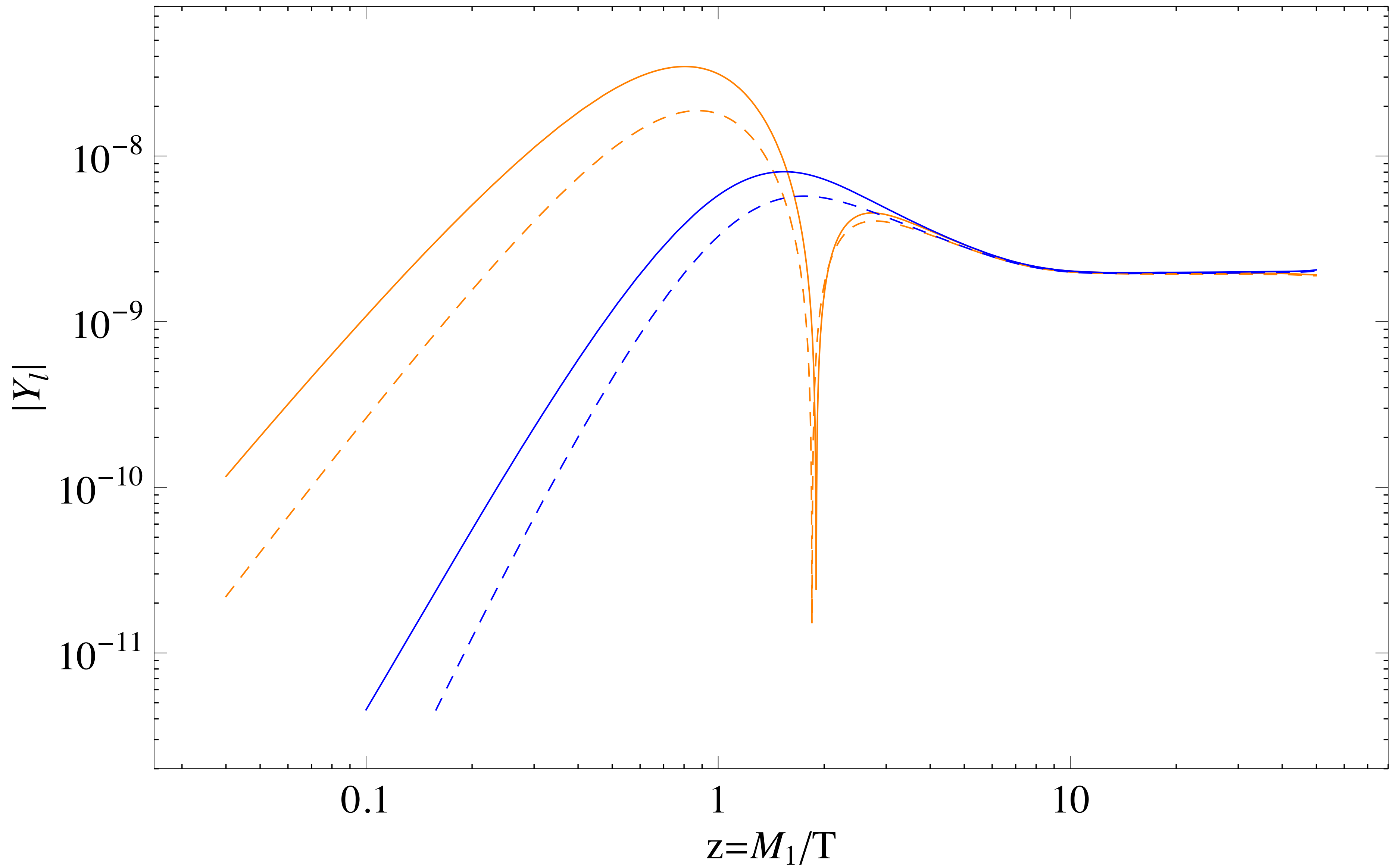
THERMAL EFFECTS: WEAK WASHOUT



blue: thermal initial N_1 density
red: zero initial N_1 density

can be sizable!

THERMAL EFFECTS: STRONG WASHOUT



blue: thermal initial N_1 density
red: zero initial N_1 density

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LEPTON FLAVORS

Barbieri et al, 2000; Endoh et al, 2004; Abada et al, 2006; Nardi et al, 2006;

- Neutral and charged Lepton Yukawa couplings in general not aligned

$$\mathcal{L} = Y_{ia}\bar{\psi}_{Ni}\phi\psi_{\ell a} + h_{ab}\bar{\psi}_{Ra}\phi^{\dagger}\tau\psi_{\ell b} + \text{h.c.}$$

- Leptogenesis usually dominated by N_1 decays

- Decay into linear combination of $e,\ \mu,\ \tau$

$$N_1 \to \phi\ell\,, \qquad \ell \sim \alpha_e \ell_e + \alpha_\mu \ell_\mu + \alpha_\tau \ell_\tau$$

MODIFICATION OF WASHOUT RATES

- Assume tau Yukawa in thermal equilibrium
- ℓ Projected onto states ℓ_τ and $\ell_\perp$ by flavor sensitive interactions (denote as $\ell_{1,2}$)
- Boltzmann E: $$\frac{d}{d\eta}\Delta n_{\ell i} = W_i + S_\ell$$
- Small washout in one flavor can largely increase the asymmetry (over 100%)

DEPENDENCE ON LEPTOGENESIS SCALE

- Expansion of Universe: $H = 1.66\sqrt{g_\star}\frac{T^2}{M_{\rm pl}}$

- Charged Higgs Yukawa interactions $\Gamma^{\rm fl} \propto h_\tau^2 T$ in equilibrium below 10^{12} GeV

- Now three rates are relevant: $H, \quad \Gamma_{N_1}, \quad \Gamma^{\rm fl}$

- Basis dependent treatment not sufficient

NEW TERMS IN EVOLUTION EQUATION

- Promote propagators to flavor matrices:

$$iS_\ell(k) \sim \begin{pmatrix} f_{11}(k) & f_{12}(k) \\ f_{21}(k) & f_{22}(k) \end{pmatrix}$$

- Flavor sensitive (thermal) masses: Oscillations!

- Flavor sensitive interactions: Dissipation

$$\frac{\partial}{\partial \eta} \delta n^{\pm}_{\ell ab} = \mp \Delta\omega^{\text{eff}}_{\ell ab} \delta n^{\pm}_{\ell ab} - [W, \delta n^{\pm}_{\ell}]_{ab} \pm S_{ab} - \Gamma^{\pm \text{fl}}_{\ell ab}$$

FLAVOR BLIND GAUGE INTERACTIONS

- Fast, always in equilibrium
- Enforce kinetic equilibrium, lepton anti-lepton pair annihilation $-\Gamma^{\mathrm{bl}}(\delta n^{+}_{\ell ab} + \delta n^{-}_{\ell ab})$
- Physical picture: Assume $n^{+}_{eq} = n^{-}_{eq} = 1000$

Asymmetry: 6

$n^{+} = 1006$

$n^{-} = 1000$

Γ^{bl} → kinetic equilibrium

Asymmetry: 6

$n^{+} = 1003$

$n^{-} = 997$

FULL EOM

$$\frac{\partial}{\partial\eta}\delta n^{\pm}_{\ell ab} = \mp\Delta\omega^{\rm eff}_{\ell ab}\delta n^{\pm}_{\ell ab} - [W,\delta n^{\pm}_{\ell}]_{ab} \pm S_{ab} - \Gamma^{\pm{\rm fl}}_{\ell ab}$$

$$-\Gamma^{\rm bl}(\delta n^{+}_{\ell ab} + \delta n^{-}_{\ell ab})$$

SUPPRESSION OF OSCILLATIONS

- Toy Model

$$d(\delta^+)/dt = -\mathrm{i}\omega\,\delta^+ - \Gamma^{\mathrm{bl}}[\delta^+ + \delta^-]$$
$$d(\delta^-)/dt = +\mathrm{i}\omega\,\delta^- - \Gamma^{\mathrm{bl}}[\delta^+ + \delta^-]$$

- Flavor blind interactions $\Gamma^{\mathrm{bl}} \sim g_2^4\, T$ (kinetic equilibrium)
- Oscillations $\Delta\omega \sim h_\tau^2\, T << \Gamma^{\mathrm{bl}}$ (from thermal masses)
- Last term enforces $\delta^+ = -\delta^- + \mathcal{O}(\omega/\Gamma^{\mathrm{bl}})\delta^-$
- Oscillations suppressed by large Γ^{bl}

Note: Not Quantum-Zeno!

FLAVORED EVOLUTION EQUATION

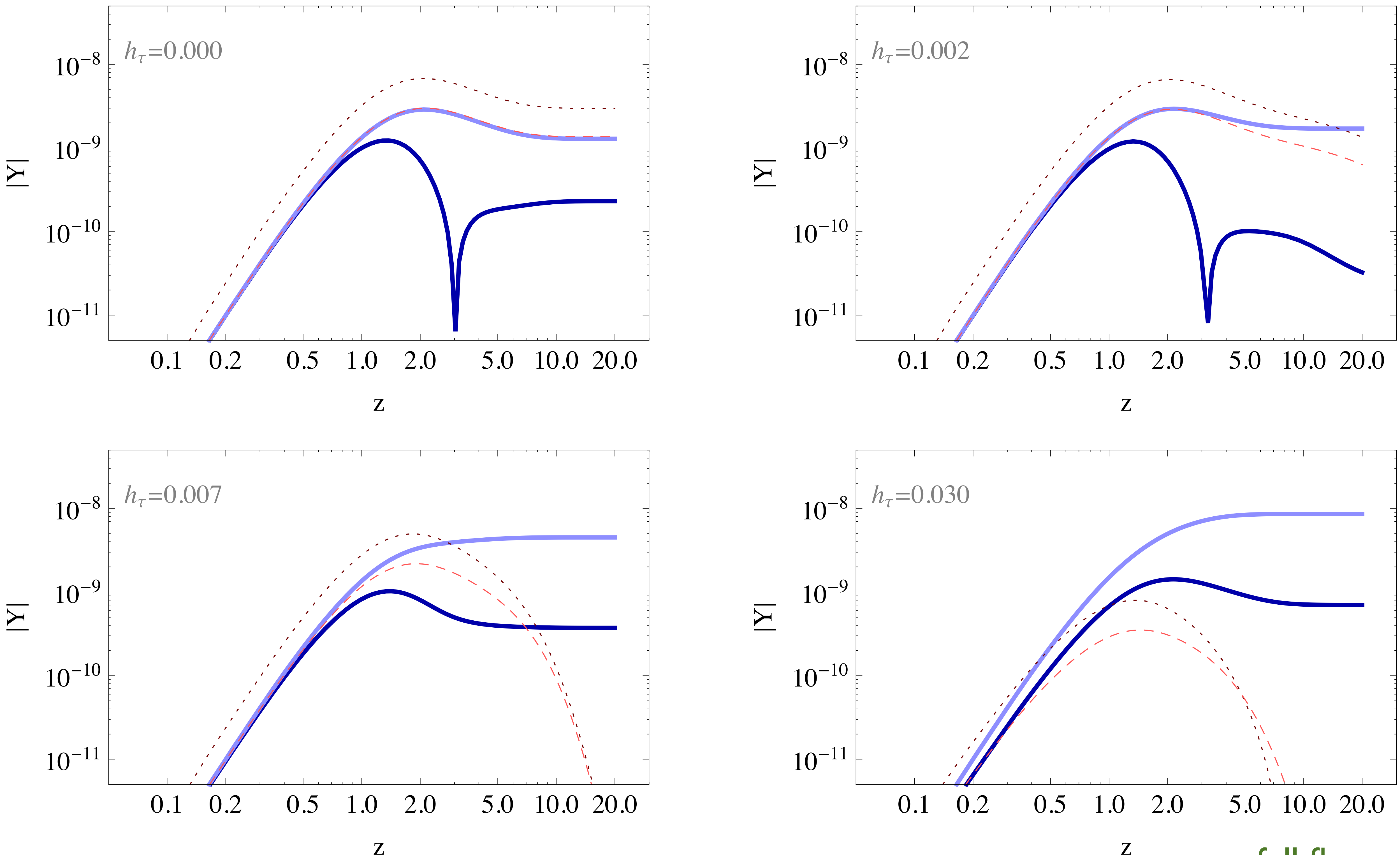
- Evolution equation:

$$\frac{\partial \Delta n_{\ell ab}}{\partial \eta} = -\{W, \Delta n_\ell\}_{ab} + 2S_{ab} - \Gamma^{\text{fl}}_{\ell ab}$$

- Total asymmetry: $\mathrm{Tr}(\Delta n_\ell)$

NUMERICS (CHARGED LEPTON FLAVOR BASIS)

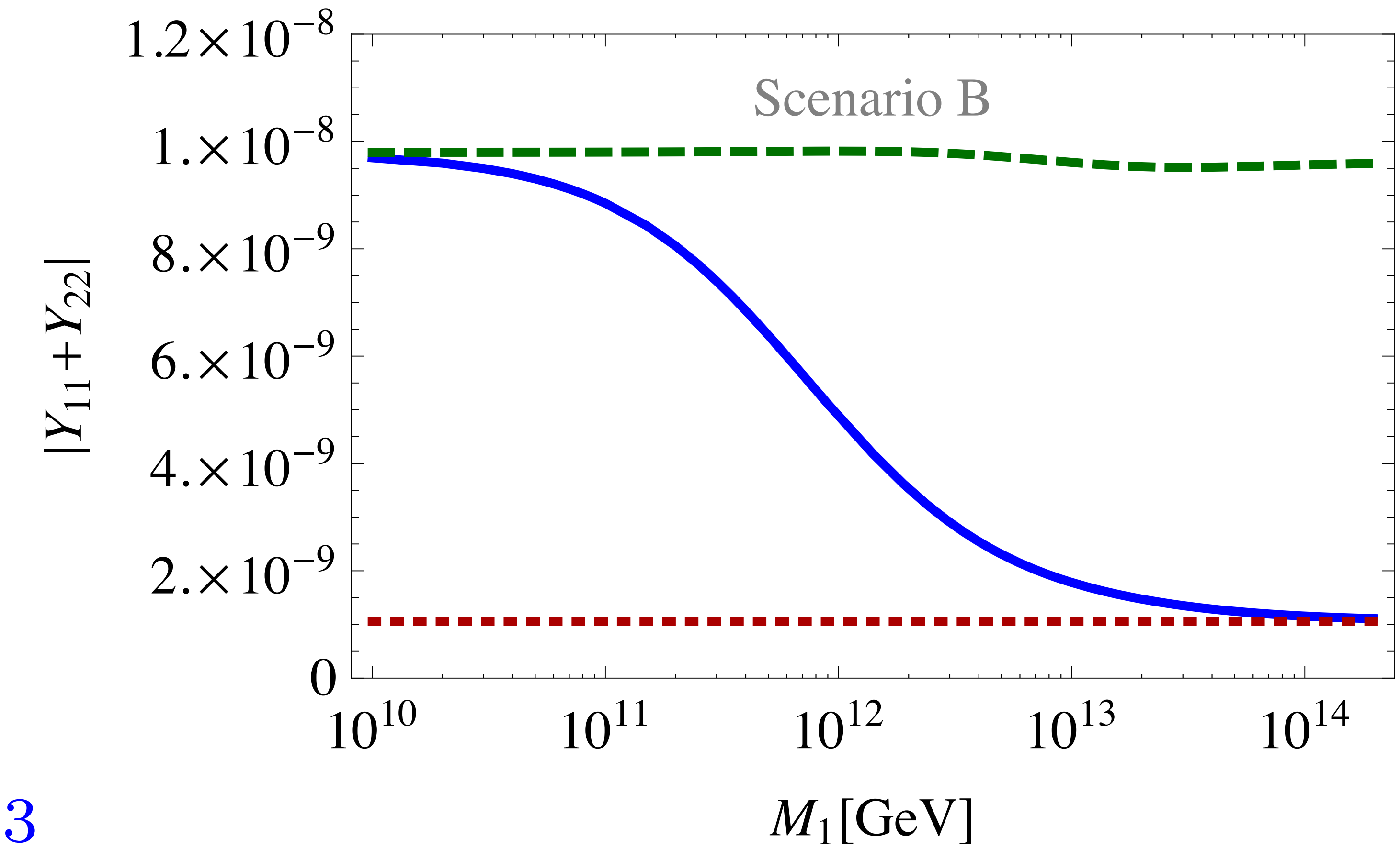
no flavor



full flavor

IMPORTANCE OF FLAVOR

- Total asymmetry as function of the Leptogenesis scale



- Unflavored: $M_1 > 10^{13}$

- Fully Flavored: $M_1 < 10^{11}$

blue: full solution
red: unflavored
green: fully flavored

CONCLUSIONS

- Consistent framework to derive evolution equations for lepton (and other) asymmetries
  - Finite density corrections
  - Basis independent equation for flavored leptogenesis
  - Suppression of flavor oscillations
  - Precise description of "intermediate" regime
  - Systematic calculation of actual leading order interaction rates

Thank you!

DEPARTURE FROM TH. EQ.

- N_1 distribution in thermal equilibrium

$$f_{N_1}^{\mathrm{eq}}(\mathbf{k}) = \frac{1}{e^{\sqrt{\mathbf{k}^2+M_1^2}/T} + 1} \xrightarrow{M_1 \gg T} e^{-M_1/T}$$

- If decay rate is close to or smaller than expansion rate of the universe: $\delta f_{N_1} \neq 0$

- Note: Non-thermal initial conditions also possible

WHEN ARE FLAVOR EFFECTS IMPORTANT?

- Three regimes (neglecting muon, electron Yukawas and assuming that flavors are not aligned)
- Unflavored: Single flavor approximation is good
- Fully Flavored: Off-diagonal densities can be neglected
- Intermediate: Full evolution equation needs to be solved

SUPPRESSION OF OSCILLATIONS

- Flavor blind interactions $\Gamma^{\rm bl} \sim g_2^4\, T$ (kinetic equilibrium)
- Oscillations $\Delta\omega \sim h_\tau^2\, T << \Gamma^{\rm bl}$ (from thermal masses)
- Toy Model:

$$d(\delta^+)/dt = -{\rm i}\omega\, \delta^+ - \Gamma^{\rm bl}[\delta^+ + \delta^-]$$

$$d(\delta^-)/dt = +{\rm i}\omega\, \delta^- - \Gamma^{\rm bl}[\delta^+ + \delta^-]$$

- Last term enforces $\delta^+ = -\delta^- + \mathcal{O}(\omega/\Gamma^{\rm bl})\delta^-$
- Oscillations suppressed by large $\Gamma^{\rm bl}$

NONEQUILIBRIUM QFT

- Conventional QFT: Calculate "in - out" correlators (S-matrix elements)

$$_{\rm in}\langle A|B\rangle_{\rm out} = \langle A|U(-t,t)|B\rangle_{t\to\infty} = \langle A|S|B\rangle$$

- NEQFT: Know the "in" state $\rho(t_0)$, want to predict the time evolution of operator:

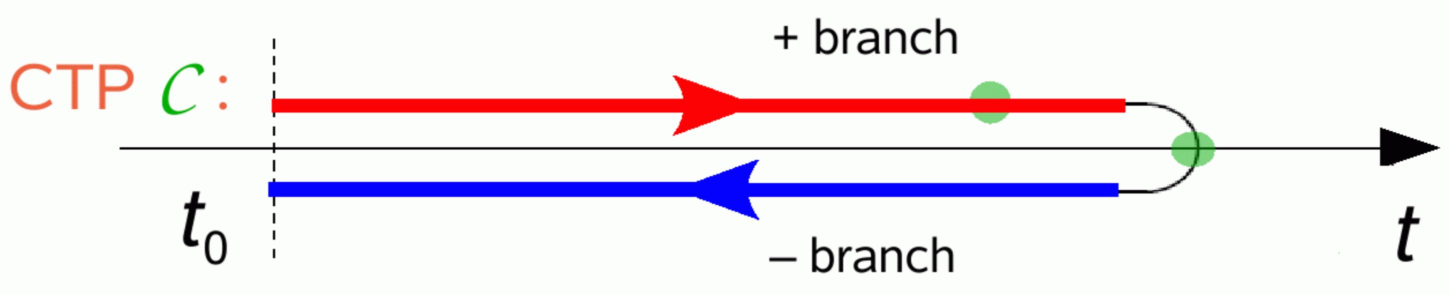
$$\langle t|\mathcal{O}|t\rangle = \mathrm{Tr}[\rho(t_0)U^\dagger(t,t_0)\mathcal{O}U(t,t_0)]$$

CTP FORMALISM

Schwinger, 1961; Keldysh, 1964, ...

- Instead of "in-out" correlators: Calculate "in-in" expectation values

- Possible using conventional QFT methods if we let time coordinate on Closed Time Path



- Fields get additional index $\phi^a(t,x)$ that indicates the position of the time coordinate $a = \pm$

CTP FORMALISM

- Relevant information contained in 2-point functions for bosons $\Delta(u,v)$ and fermions $S(u,v)$
- become 2x2 matrices $\begin{pmatrix} G^{++} & G^{+-} \\ G^{-+} & G^{--} \end{pmatrix} = \begin{pmatrix} G^{T} & G^{<} \\ G^{>} & G^{\bar{T}} \end{pmatrix}$
- Time evolution from Dyson-Schwinger equation:

$$i\partial\!\!\!/_u S^{ab}(u,v) = a\delta_{ab}\delta^4(u-v) + \sum_c \int d^4w \Sigma^{ac}(u,w) S^{cb}(w,v)$$

1PI self energy

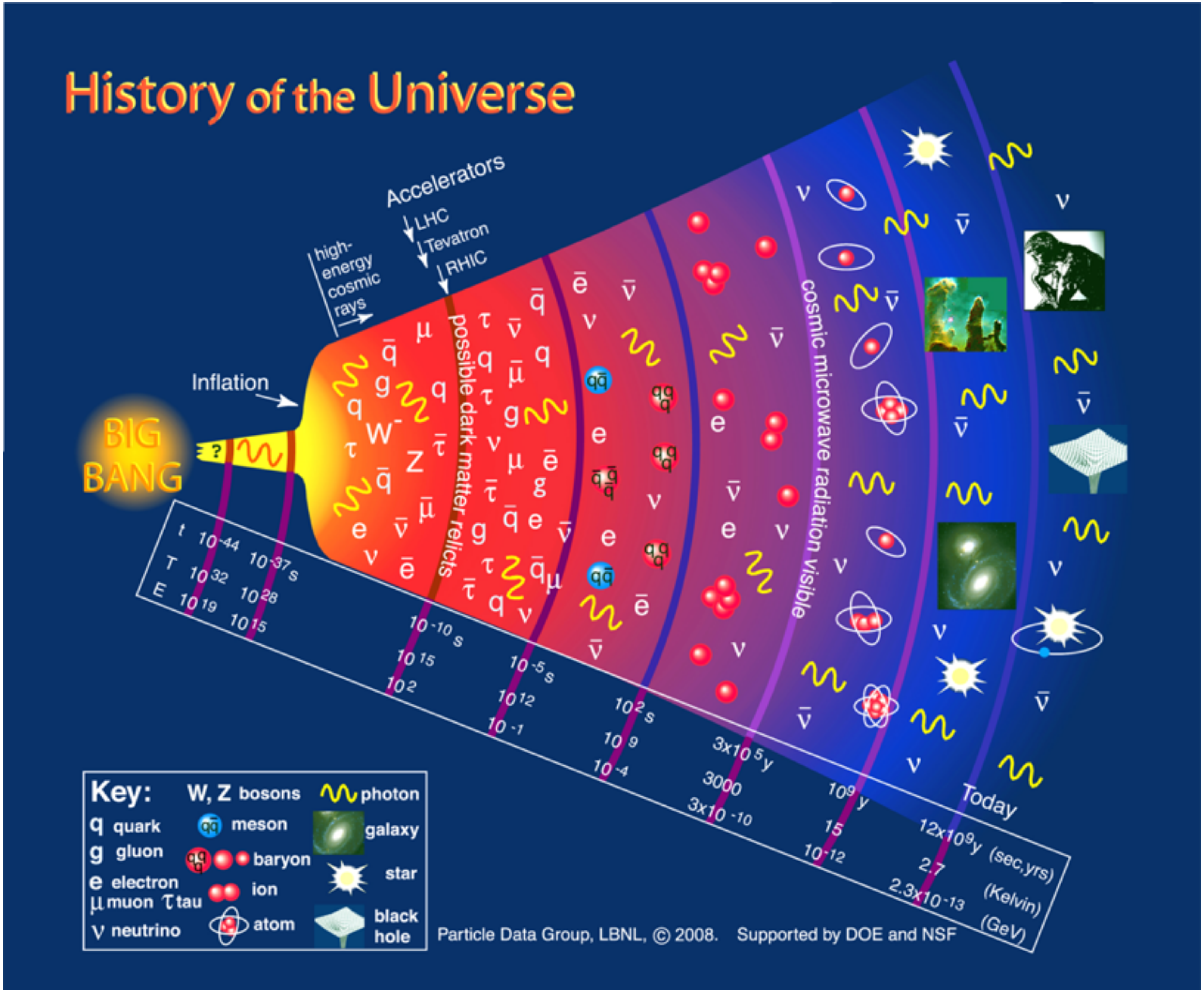
THE BARYON ASYMMETRY

- The number we have to explain is

$$Y_{\Delta B} = \frac{n_B - n_{\bar{B}}}{s} = (8.75 \pm 0.23) \times 10^{-11}$$

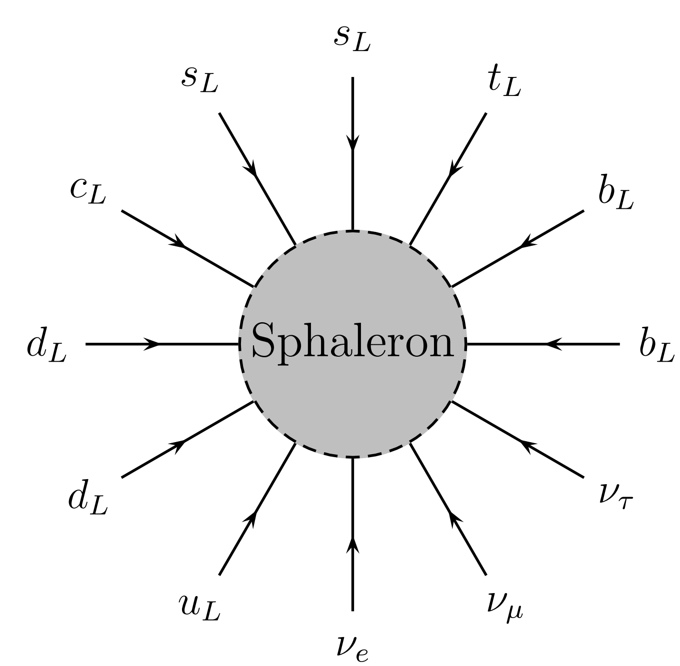
- Entropy $s = g_\star (2\pi^2/45) T^3$ is conserved, related to photon density: $s = 7.04\, n_\gamma$

- Measured using BBN (deuterium abundance) and CMB anisotropies (temperature fluctuations)


History of the Universe
BIG BANG
Inflation
high-energy cosmic rays
Accelerators
LHC
Tevatron
RHIC
possible dark matter relicts
cosmic microwave radiation visible
Today
t 10-44 10-37 s 10-10 s 10-5 s 102 s 3x105 y 109 y 12x109 y (sec,yrs)
T 1032 1028 1015 1012 109 3000 15 2.7 (Kelvin)
E 1019 1015 102 10-1 10-4 3x10-10 10-12 2.3x10-13 (GeV)
Key:
q quark
g gluon
e electron
μ muon τ tau
ν neutrino
W, Z bosons
meson
baryon
ion
atom
photon
galaxy
star
black hole
Particle Data Group, LBNL, © 2008. Supported by DOE and NSF

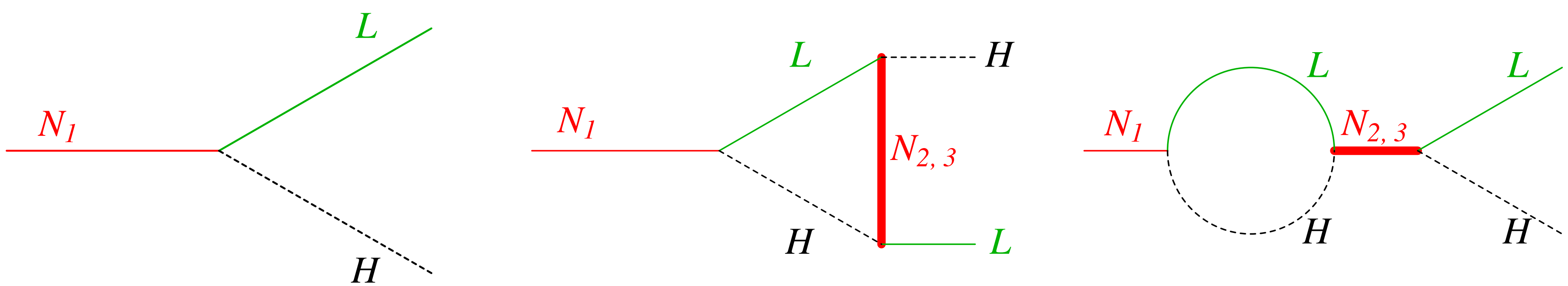
THE ELECTROWEAK SPHALERON

- $B + L$ current is anomalous in the SM
- At $T = 0$: Tunneling between configurations with different $B + L$ highly suppressed
- At $T \gtrsim \mathrm{TeV}$: In equilibrium
- have $\Delta B = \Delta L = 3$: no proton decay



CP VIOLATION

- Must be able to distinguish particles from anti-particles
- In Leptogenesis: CP violated in decays of heavy right-handed neutrinos:



CP VIOLATION II

- QM: Observables are expectation values of operators

$$\Gamma(N_1 \to H\ell^+) = |\langle N_1|\mathcal{H}_{\rm int}|H\ell^+\rangle|^2 = |\mathcal{A}|^2$$

- Asymmetry:

$$Y_L = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}$$

- Simplest case:

$$\mathcal{A} = h_1 A_0$$
$$\bar{\mathcal{A}} = h_1^* A_0$$

No asymmetry since

$$\bar{\Gamma} = \Gamma$$

CP VIOLATION III

- Add one loop correction

$$\mathcal{A} = h_1 A_0 + h_1^*(h_2)^2 A_1$$
$$\bar{\mathcal{A}} = h_1^* A_0 + h_1 (h_2^*)^2 A_1$$

- Asymmetry proportional to interference term

$$Y_L \propto \Im(h_1 h_1 h_2^* h_2^*) \Im(A_0 A_1)$$

- Note: Requires complex couplings and complex A_1