Mass-deformed conformal field theories on the lattice

Ethan T. Neil (FNAL)
Fermilab Theory Seminar
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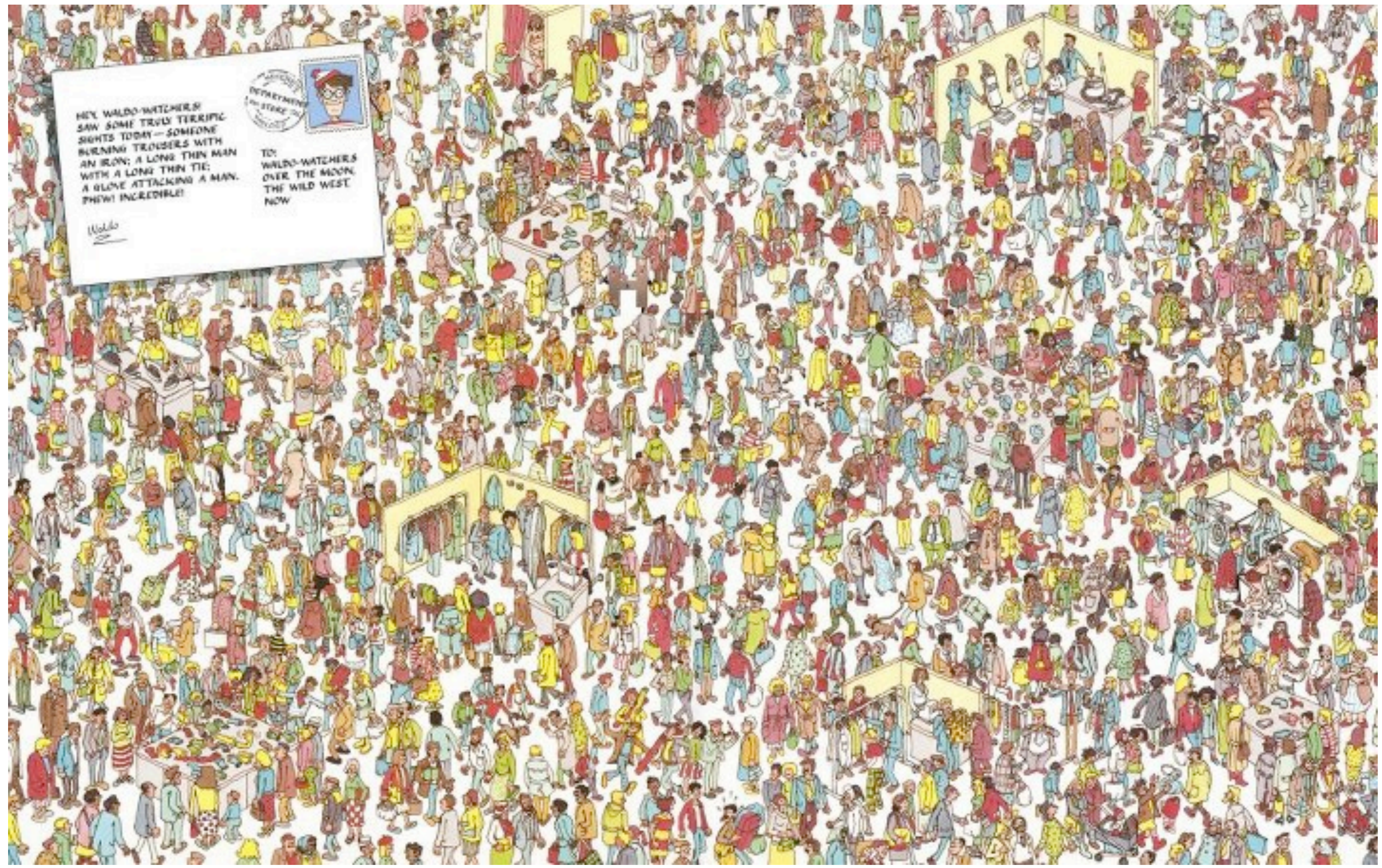
arXiv:1106.2148
arXiv:1204.6000
(and in progress)

Motivation

(credit Jerome Sprecher & symmetry magazine)

- We found a Higgs boson! But how "standard-model-like" is it? Could it be composite?

- "Traditional" technicolor is firmly dead, since f_0 (600) in QCD is too broad. But scaled-up QCD didn't work anyway.



- Can argue for composite Higgs-like states in more exotic strongly-coupled gauge theories: dilaton if approximate scale-invariance, or PNGBs with different gauge groups/irreps

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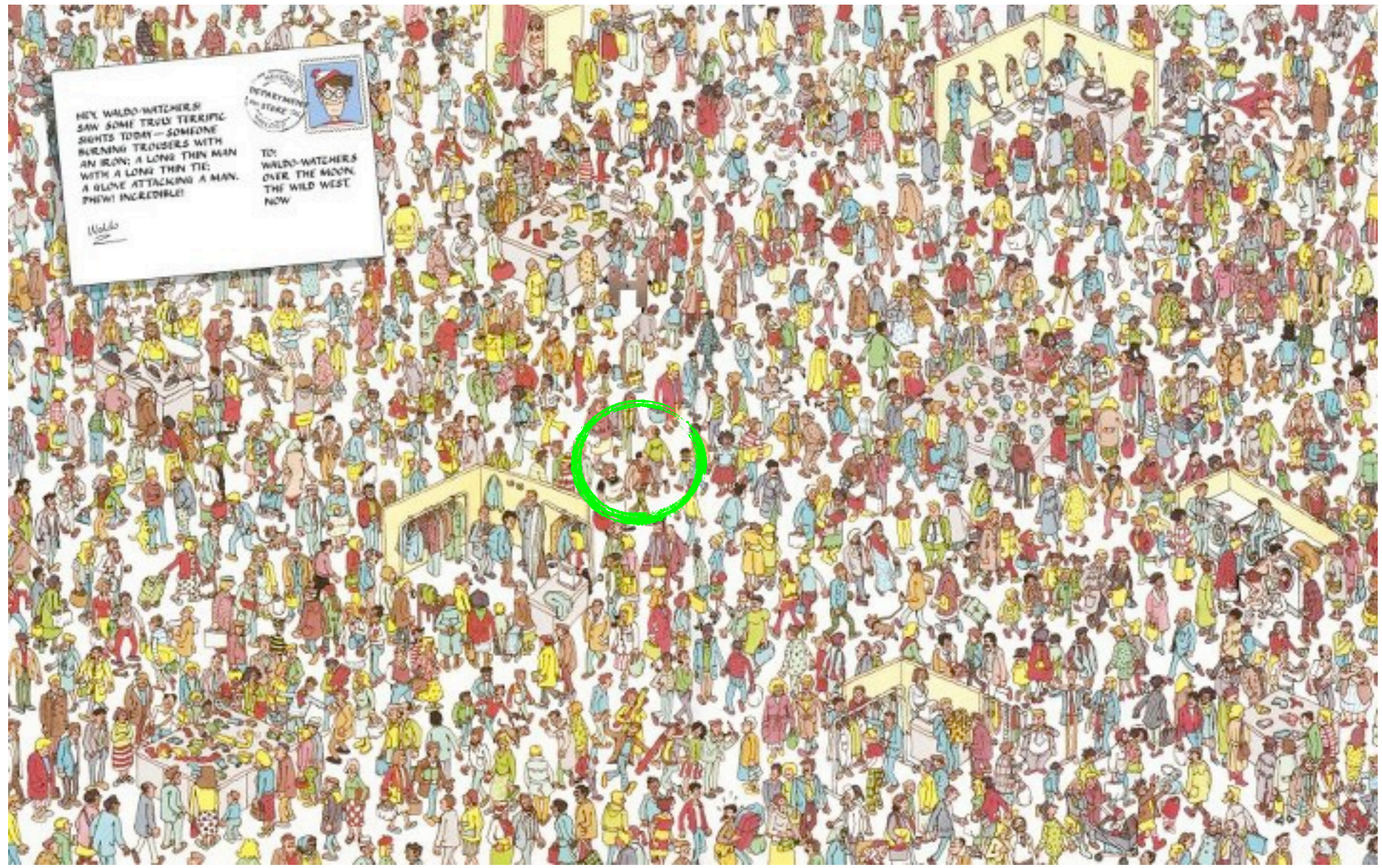


- Can argue for composite Higgs-like states in more exotic strongly-coupled gauge theories: dilaton if approximate scale-invariance, or PNGBs with different gauge groups/irreps

Overview

- We know there is a transition in strongly-coupled gauge theories where scale invariance is restored in the IR. Lattice is the best way to study this transition, lots of groups working on it now.

- Conformal symmetry is hard to see on the lattice! for technical reasons, we usually work at finite mass, which breaks scale invariance. However, dependence of the theory on this mass scale is very different from QCD - no PNGBs, all states collapse to zero as mass is removed. Maybe we can turn a bug into a feature and study the IR CFT using mass dependence?

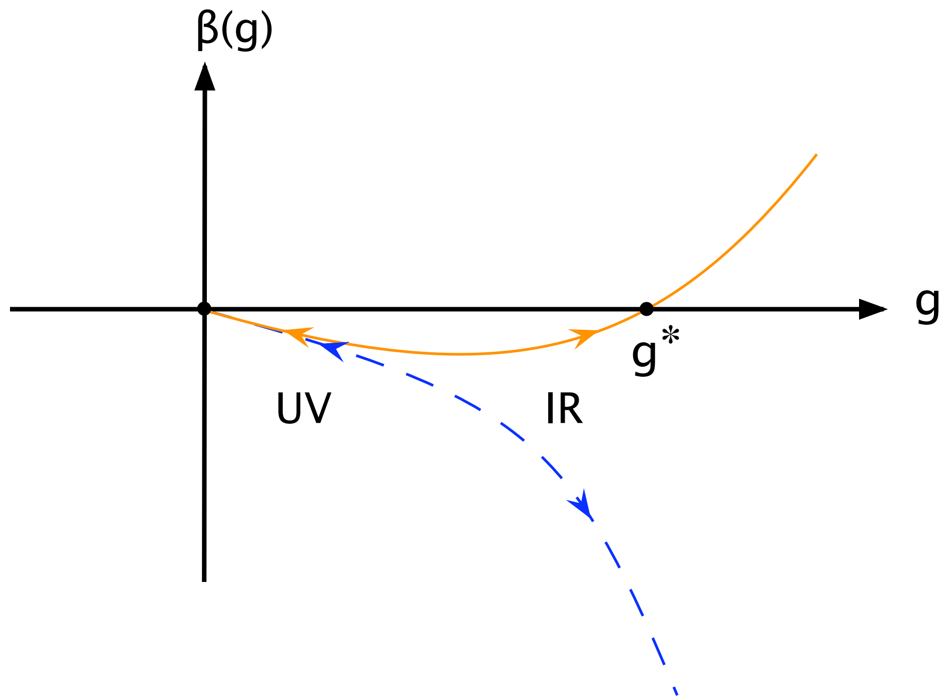
- I will introduce a general framework for mass-deformed CFTs, then show case studies for three different theories: SU(2) with 2 adjoint (conformal), SU (3) with 12 fundamental (controversial), SU(3) with 10 fundamental (near the edge?)

From confining to conformal

[1]Phys.Rev.Lett. 33 (1974) 244
[2]Nucl.Phys. B196 (1982) 189

- Due to Caswell[1], Banks and Zaks[2], we have an example of a Yang-Mills gauge theory vastly different from QCD.

- Examine scale dependence of the theory via the gauge coupling (β-function).



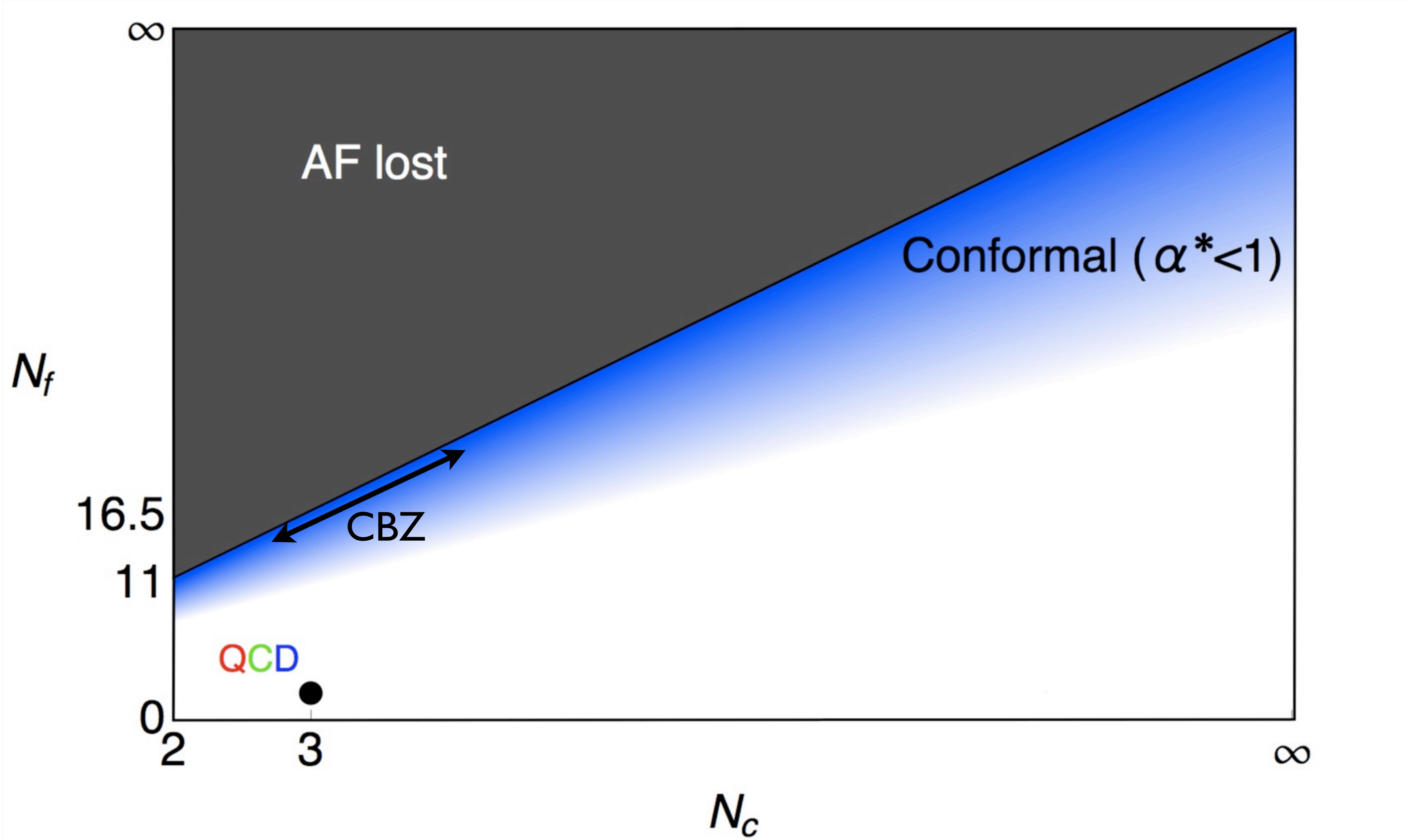
- For large enough N_f, β-function has a second zero! Weak coupling at all scales (note: m=0 only). Conformal symmetry in the IR limit.

$$\beta(\alpha) \equiv \frac{\partial \alpha}{\partial(\log \mu^2)} = -\beta_0 \alpha^2 - \beta_1 \alpha^3 - \ldots$$

$$\beta_0 = \frac{1}{4\pi}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)$$

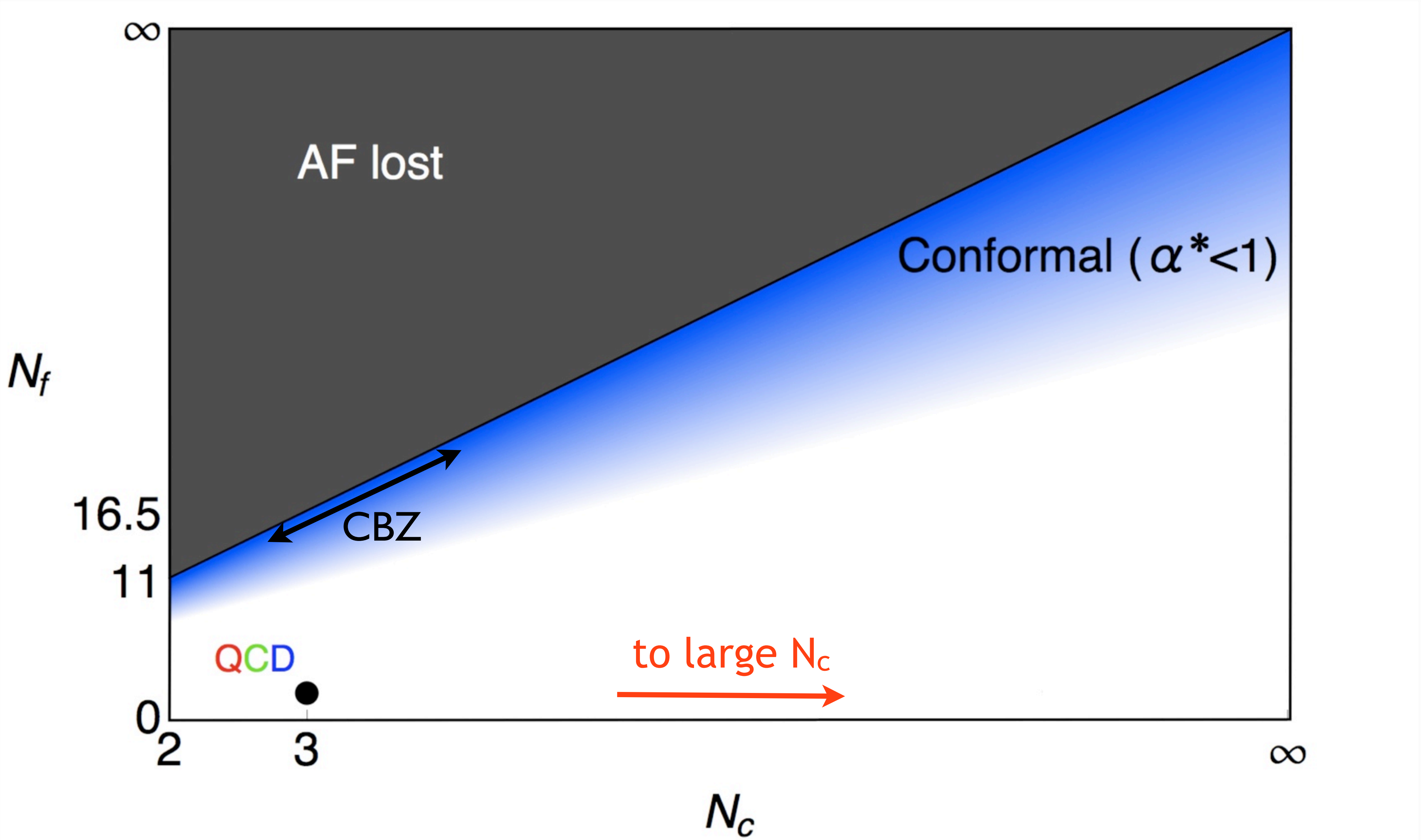
$$\beta_1 = \frac{1}{16\pi^2}\left(\frac{34}{3}N_c^2 - \left[\frac{13}{3}N_c - \frac{1}{N_c}\right]N_f\right)$$

(R=fundamental)



Theories with IRFP lie in the conformal window.

(Note: massless fermions, fundamental rep.)



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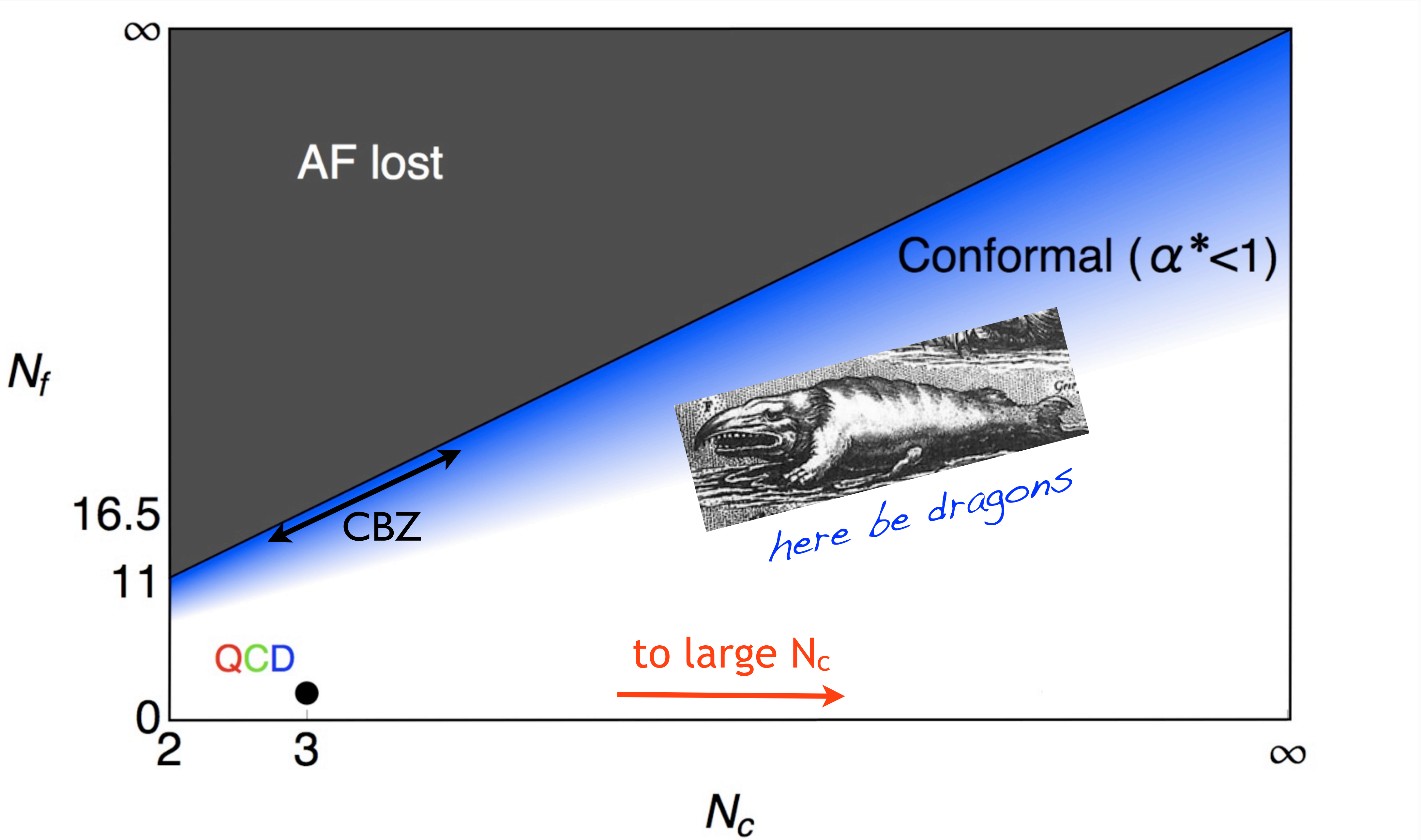

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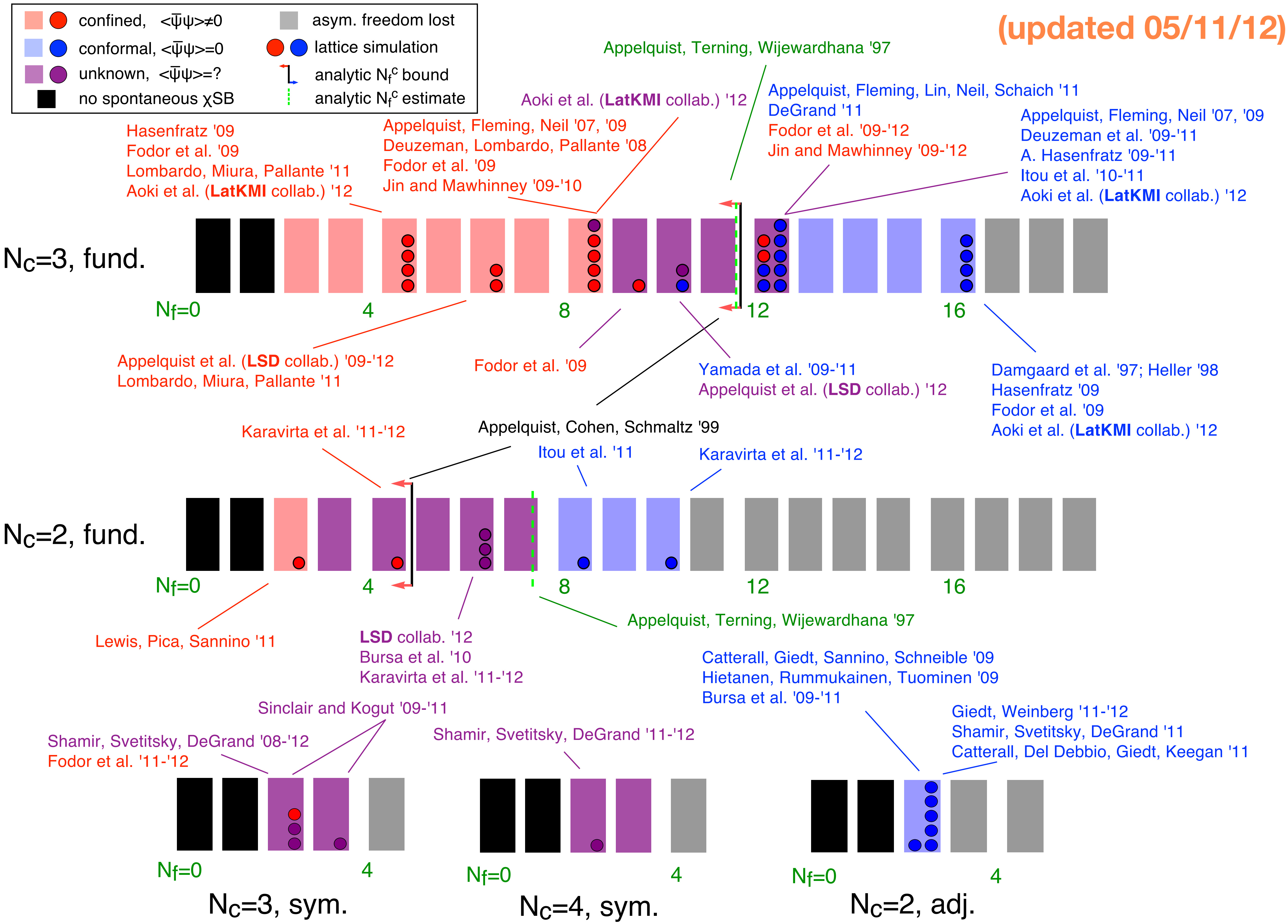


A Conformal Window Roadmap (see 1205.4706 for details)

Mass anomalous dimension

- The mass anomalous dimension gives the RG flow of the mass operator:

$$\gamma_m = \frac{d(\log m)}{d(\log \mu^2)}$$

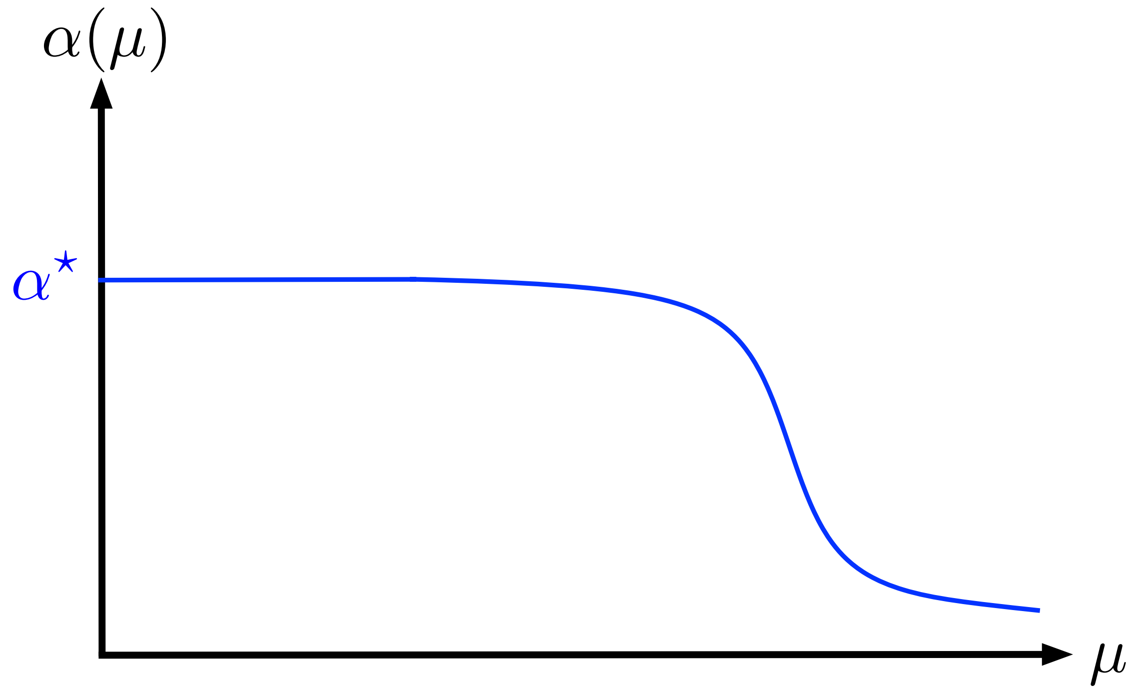
- Function of the underlying couplings, like the beta-function; takes a constant, scheme-independent value at an IRFP. Crucial to understanding what happens when we perturb the IRFP with a small mass!

- Also related to the scaling of the chiral condensate:

$$m\bar{\psi}\psi \sim \text{const.} \Rightarrow \langle \bar{\psi}\psi \rangle \sim \exp\left(\int \frac{\gamma_m(\mu)}{\mu} d\mu \right)$$

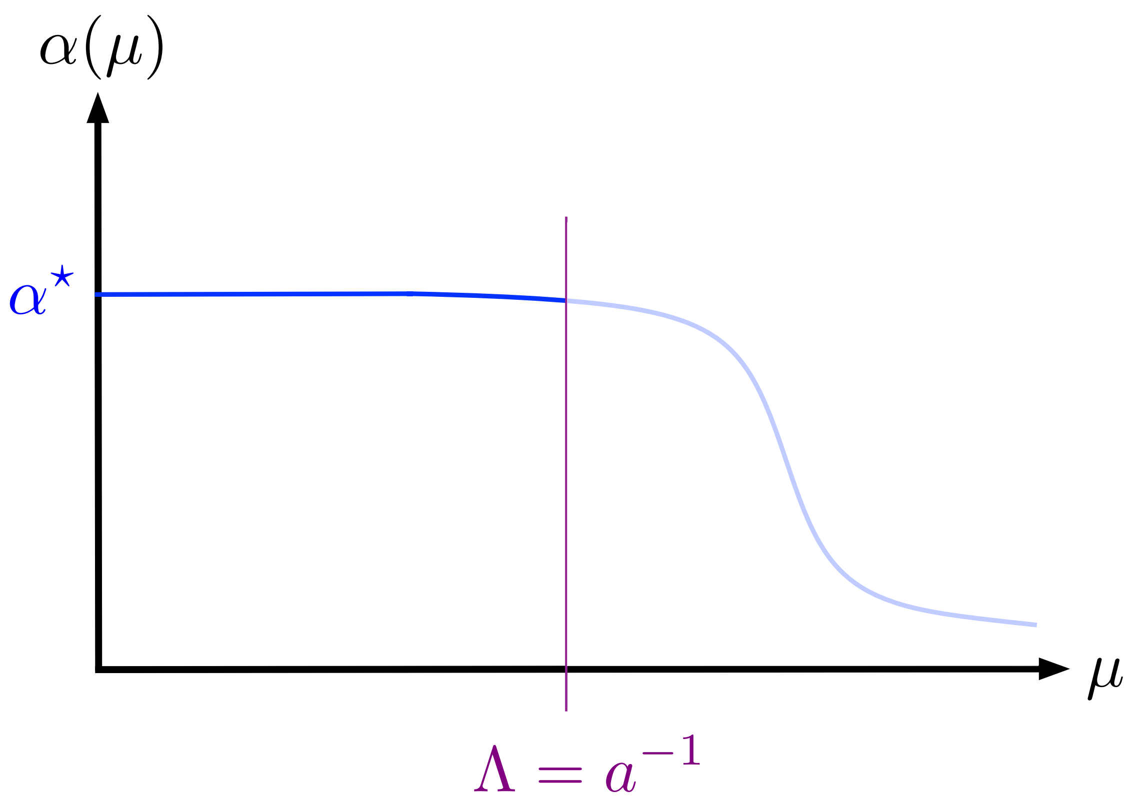
- In the context of composite Higgs theories, this condensate will generally be important for generation of fermion masses (like Higgs vev.) Large anomalous dimension is generally more interesting - can give large scale separation from confinement scale.

Mass deformation



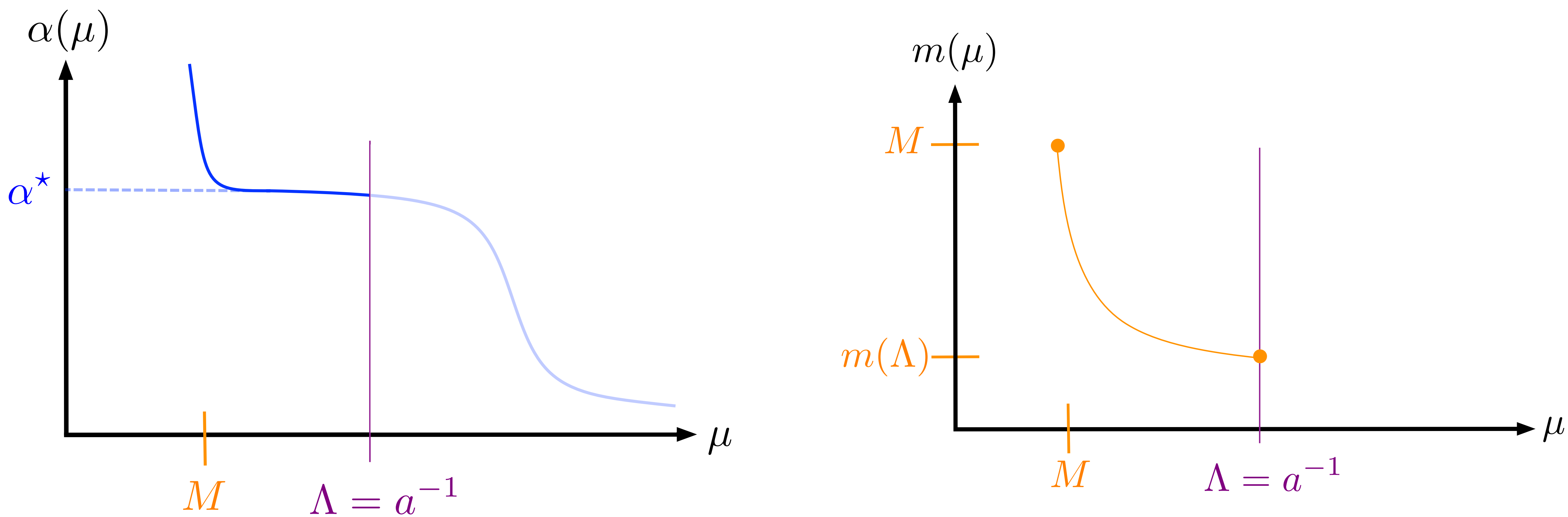
1) Start with a theory in the conformal window, with massless fermions.

Mass deformation



2) Tune the lattice cutoff so that the fixed-point coupling governs physics in the box (ignoring finite-volume effects.)

Mass deformation

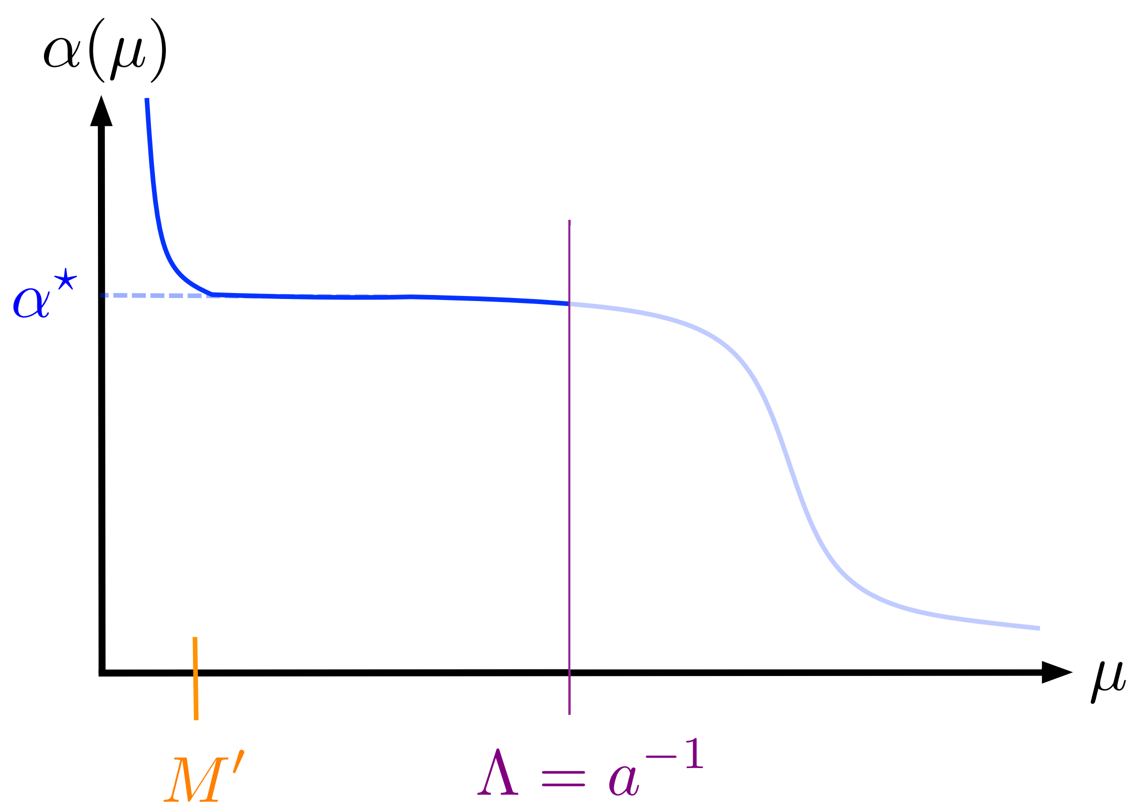


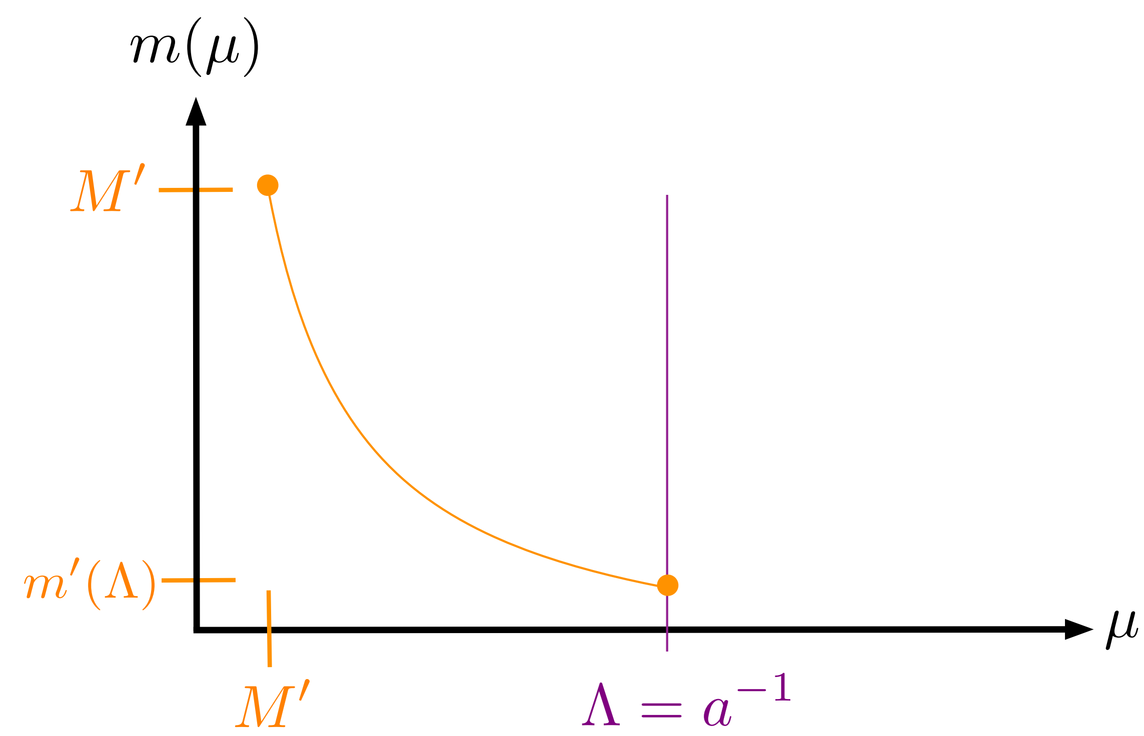
3) Add a small seed mass *m*, which will evolve as

$$m(\mu) = m(\Lambda)\left(\frac{\Lambda}{\mu}\right)^{\gamma^\star}$$

governed by the fixed-point anomalous dimension.
Fermions screen out at m(M) = M, inducing confinement.

Mass deformation





4) Bound-state masses are set by M, as in QCD-like theory.
Three major differences here:

- No Goldstones - PS state scales like everything else.
- M is <u>controlled</u> by m: $M \sim m^{1/(1+\gamma^\star)}$
- Expansion in *am*, as opposed to $aM_\pi^2/(4\pi F_\pi)^2$ for χPT

More on mass deformation

- No spontaneous xSB means the pseudoscalar meson is no longer a Goldstone - scales like everything else. Still the lightest non-singlet meson due to QCD inequalities (e.g. Weingarten PRL 51, 1983.)

- To zeroth order all bound states scale with the "induced confinement scale" *M*, but (as in QCD) there should be corrections in fermion mass *m*:

$$M_X = C_X m^{1/(1+\gamma^\star)} + D_X m + \mathcal{O}(m^2)$$

- Up to corrections, ratios of bound-state masses are <u>independent of m</u> - very different from QCD with small m! (However, can look similar to heavy-quark limit...)

Another perspective: “hyperscaling”

(Del Debbio and Zwicky, 1005.2371 and 1009.2894)

- Analyze theory around the fixed point using RG scaling. Start with a correlator for operator H:

$$C_H(t; g, m, \mu) = \int d^3x \langle H(x,0) H^\dagger(x,t) \rangle \to_{t\to\infty} e^{-M_H t}$$

- RG blocking by factor b:

$$\mu = b\mu' \qquad m' = b^{y_m} m \quad C_H(t; g, m, \mu) = b^{-2\gamma_H} C_H(t; g', m', \mu')$$
$$y_m = (1+\gamma_m) = (1+\gamma^\star)$$

- Now rescale all mass units by b, then pick b = (m’)$^{-1}$:

$$C_H(t; g, m, \mu) = b^{-2d_H} C_H(tb^{-1}; g', m'b, \mu) = c_H \mathcal{F}(tm^{1/y_m}; \mu)$$

- So the correlator is a function of the “scaling variable” $x = tm^{1/(1+\gamma^\star)}$

- Furthermore, we can pick off the behavior of the mass at small m:

$$M_H \sim c_H \mu m^{1/(1+\gamma^\star)}$$

- Finally, similar derivations for decay constants and the condensate follow:

$$F_H \sim m^{1/(1+\gamma^\star)} \qquad \langle \bar\psi\psi \rangle \sim m^{\frac{3-\gamma^\star}{1+\gamma^\star}}$$

Volume scaling and curve collapse

- In the preceding, finite-size scaling can be considered in the same way, with a similar conclusion: the correlator (and thus the mass) becomes a smooth function of a single "scaling variable",

$$M_H = L^{-1} f(L^{1+\gamma^\star} m)$$

- Should recover the right mass scaling as L is taken to infinity, so as $x \to \infty$

$$f(x) \sim x^{1/(1+\gamma^\star)}$$

- This suggests that hyperscaling can be seen through "curve collapse" - for the correct value of gamma, data for any volume and mass will fall on a smooth curve (assuming L "large enough" and m "small enough".)

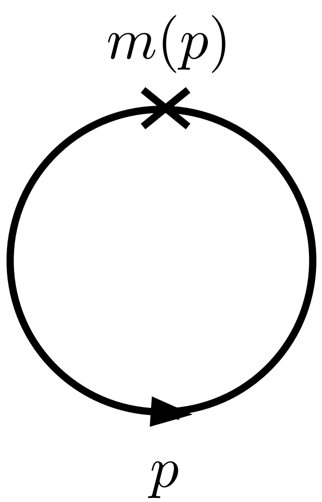
- We can also try to expand out the volume dependence in correction terms. Assuming the corrections are analytic in 1/ML, we have

$$M_X = C_X m^{1/(1+\gamma^\star)} \left[1 + \frac{z_x}{m^{1/(1+\gamma^\star)} L}\right] + D_X m + ...$$

More on the chiral condensate

- The chiral condensate is special - much more sensitive to the ultraviolet than other observables. In particular, on the lattice we pick up UV divergences, e.g. $\langle\bar{\psi}\psi\rangle = A_C m/a^2 + ...$

- We can analyze the mass dependence explicitly in terms of running mass:



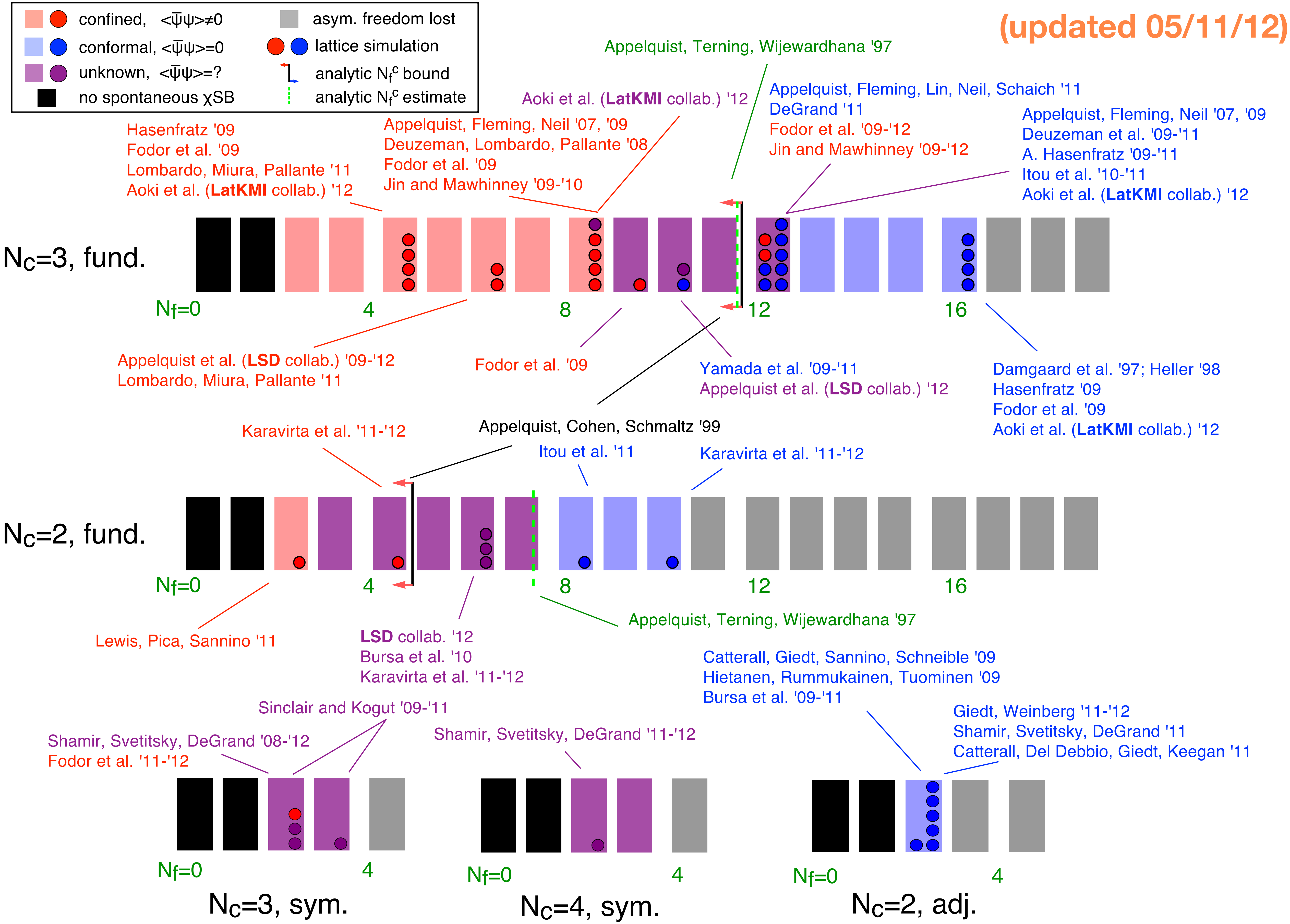
$$\propto \int_M^{a^{-1}} d^4k \frac{[m(k)+\Sigma(k)]}{k^2} = \int_M^{a^{-1}} d^4k \left[\frac{M(M/k)^{\gamma^\star}}{k^2} + \frac{M(M/k)^{2-\gamma^\star}}{k^2}\right]$$

$$= \int_M^{a^{-1}} d^4k \left[mk^{-(\gamma^\star+2)} + m^{\frac{3-\gamma^\star}{1+\gamma^\star}} k^{(\gamma^\star-4)}\right]$$

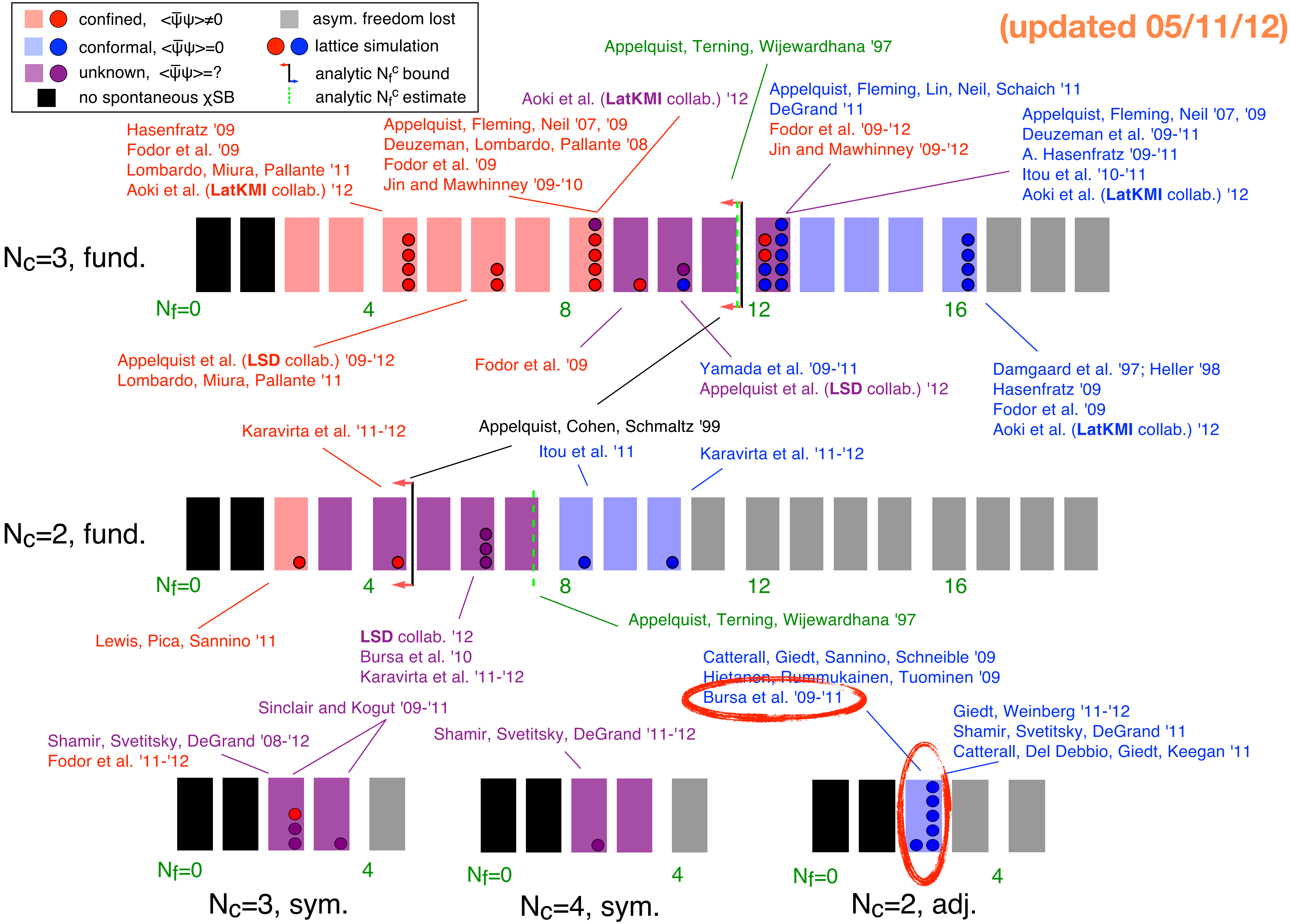
$$= (\ldots)M^3 + (\ldots)m^{\frac{3-\gamma^\star}{1+\gamma^\star}} + (\ldots)m$$

- From the above, we expand the condensate as shown to the right:

$$\langle\bar{\psi}\psi\rangle = A_C m + B_C m^{[(3-\gamma^\star)/(1+\gamma^\star)]}$$
$$+ C_C m^{[3/(1+\gamma^\star)]} + D_C m^3.$$



A Conformal Window Roadmap (see 1205.4706 for details)



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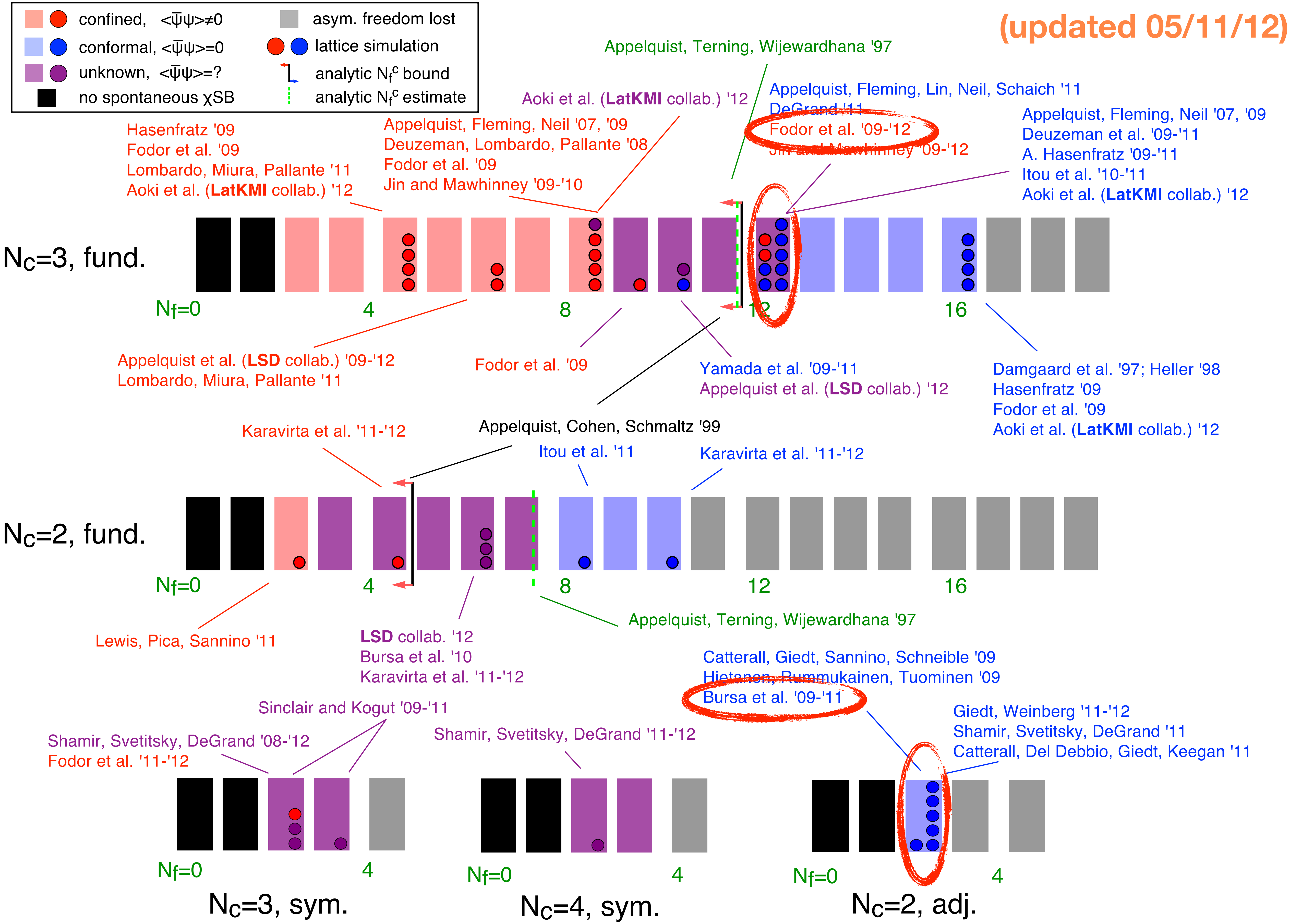

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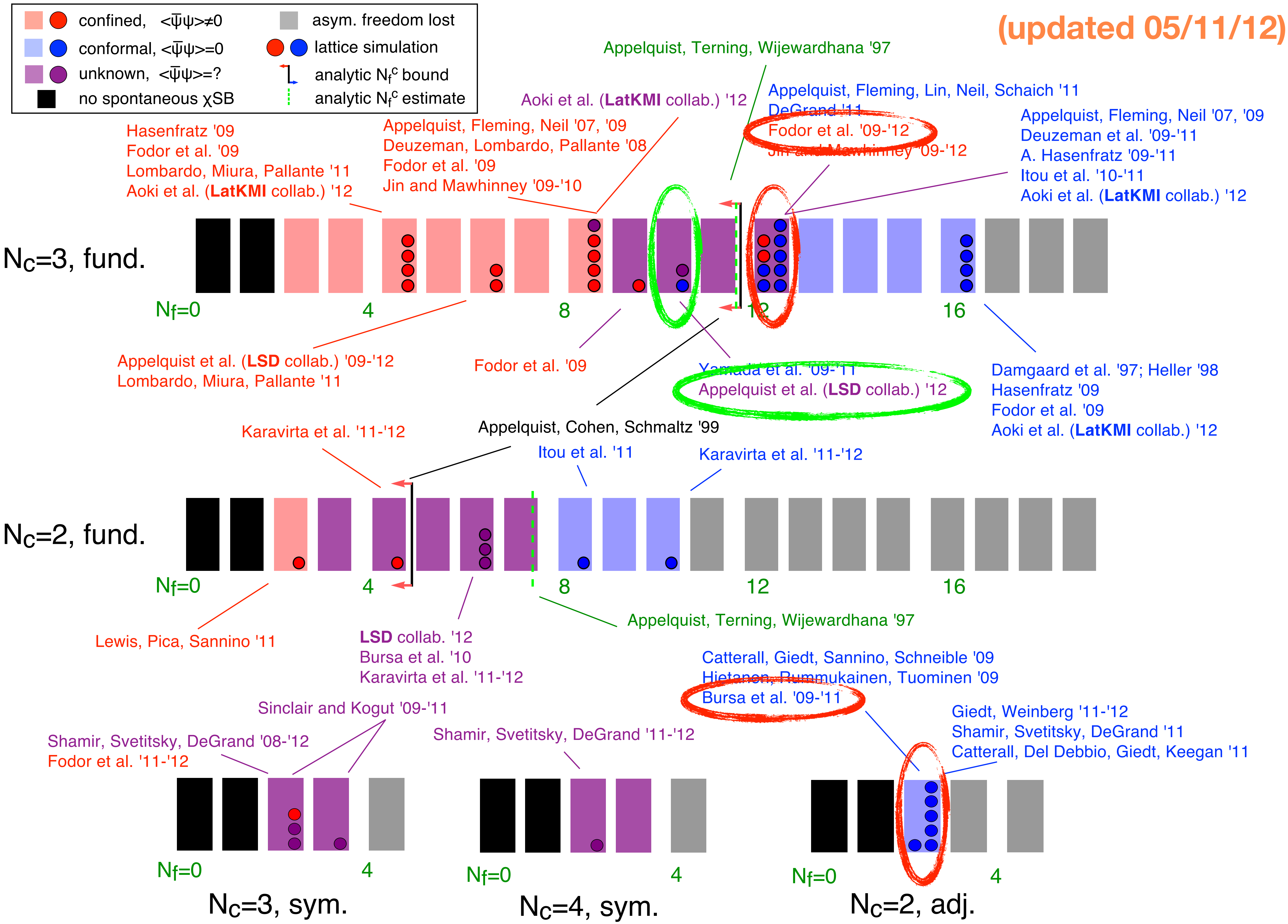
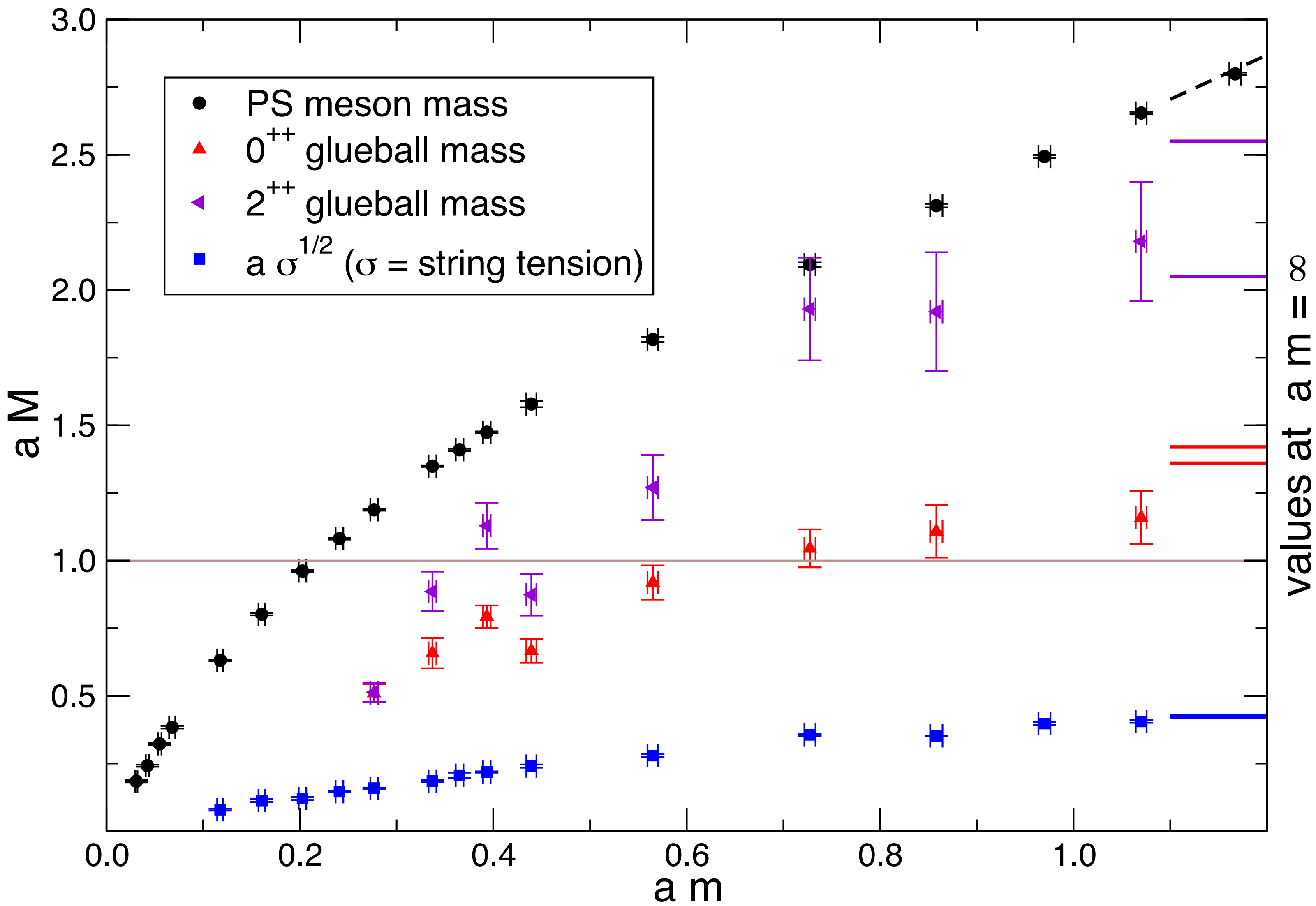


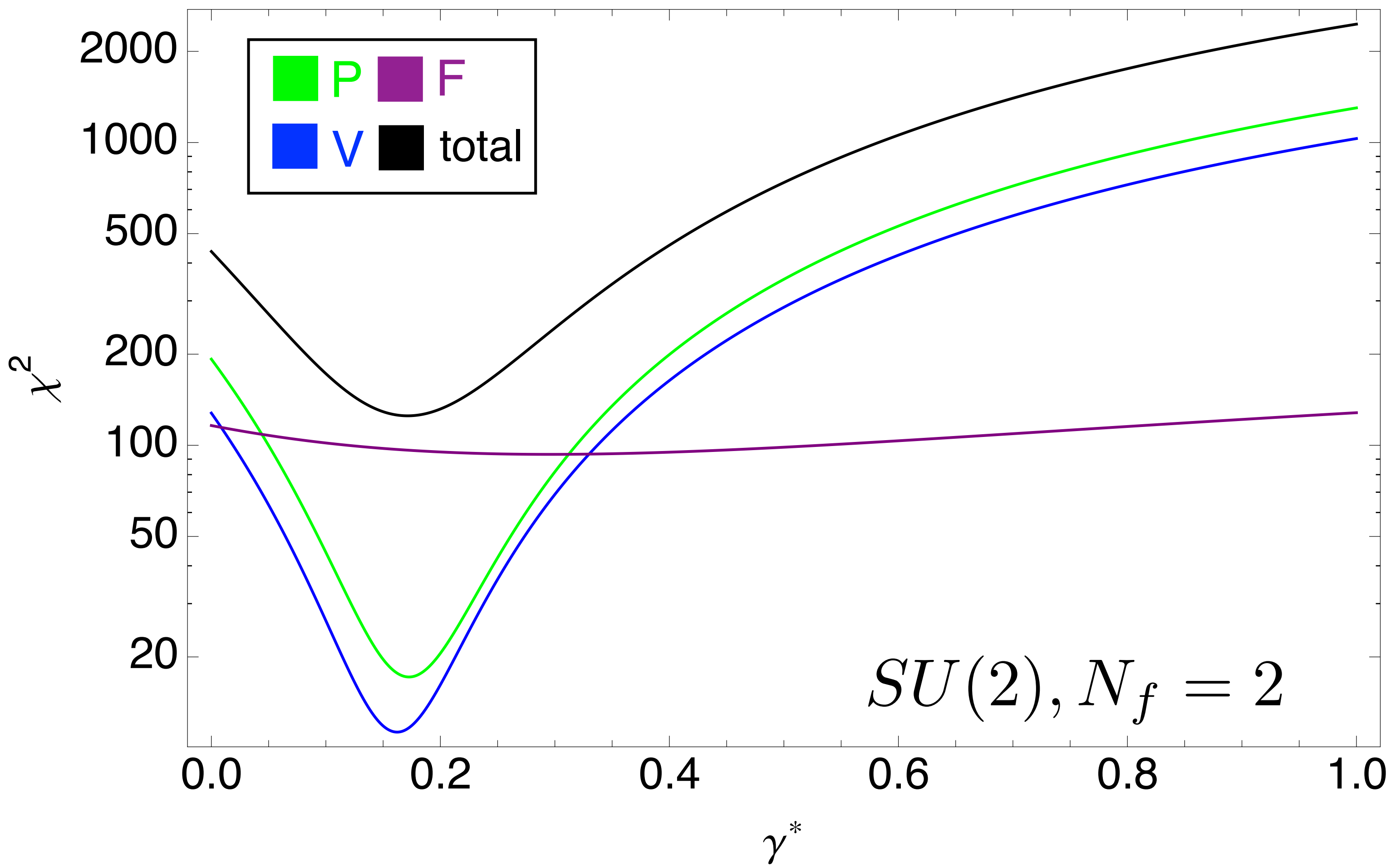
A Conformal Window Roadmap (see 1205.4706 for details)

SU(2) with 2 adjoint fermions

- Many studies of this theory (originally dubbed "minimal walking technicolor".) General consensus: IR conformal, anomalous dimension small (~< 0.5.)
- Will consider data set of Bursa et al., arXiv:1104.4301 - sample shown on the right.
- Novel feature - glueballs lighter than pions! Predicted in mass-deformed weakly-coupled IRFP (V. Miransky, hep-ph/9812350) True for entire mass range?

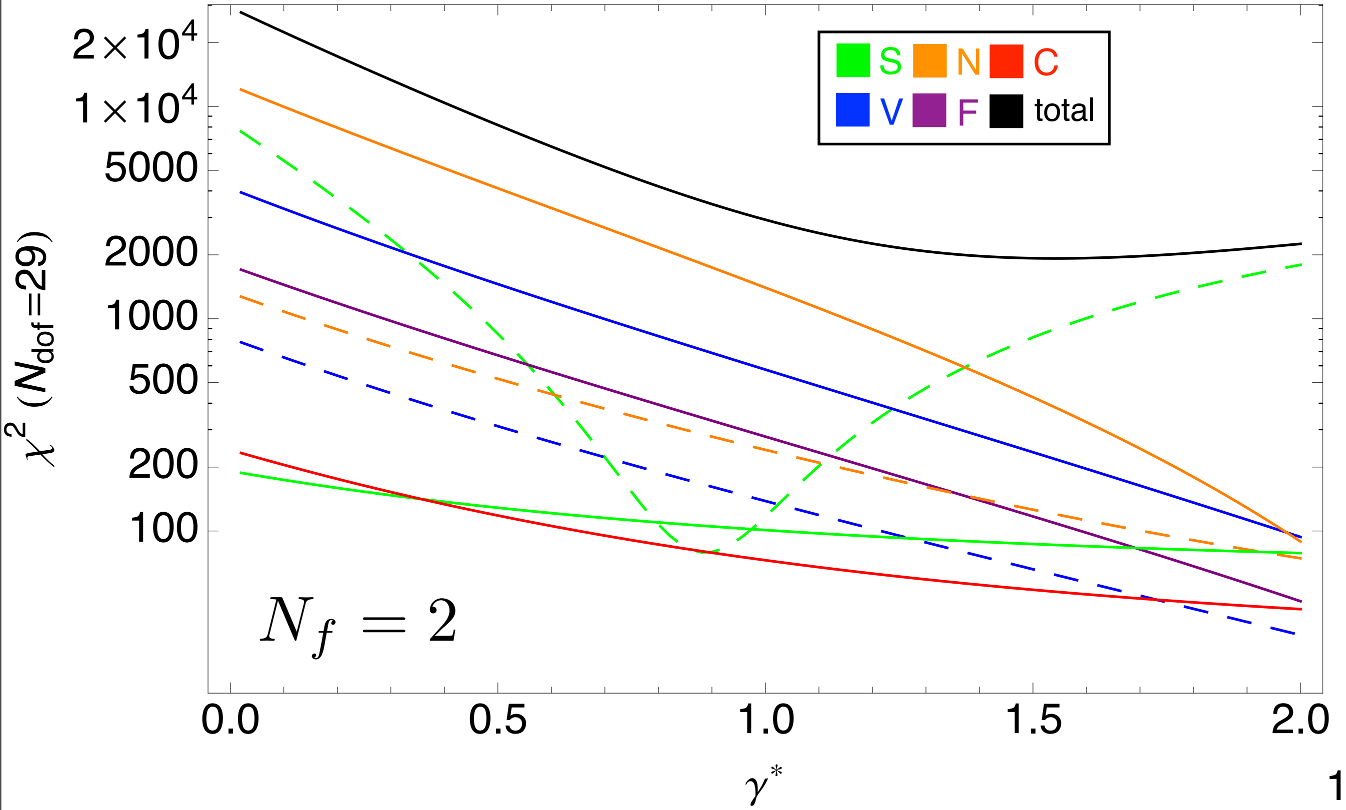


Scaling fit results, SU(2) 2-adjoint



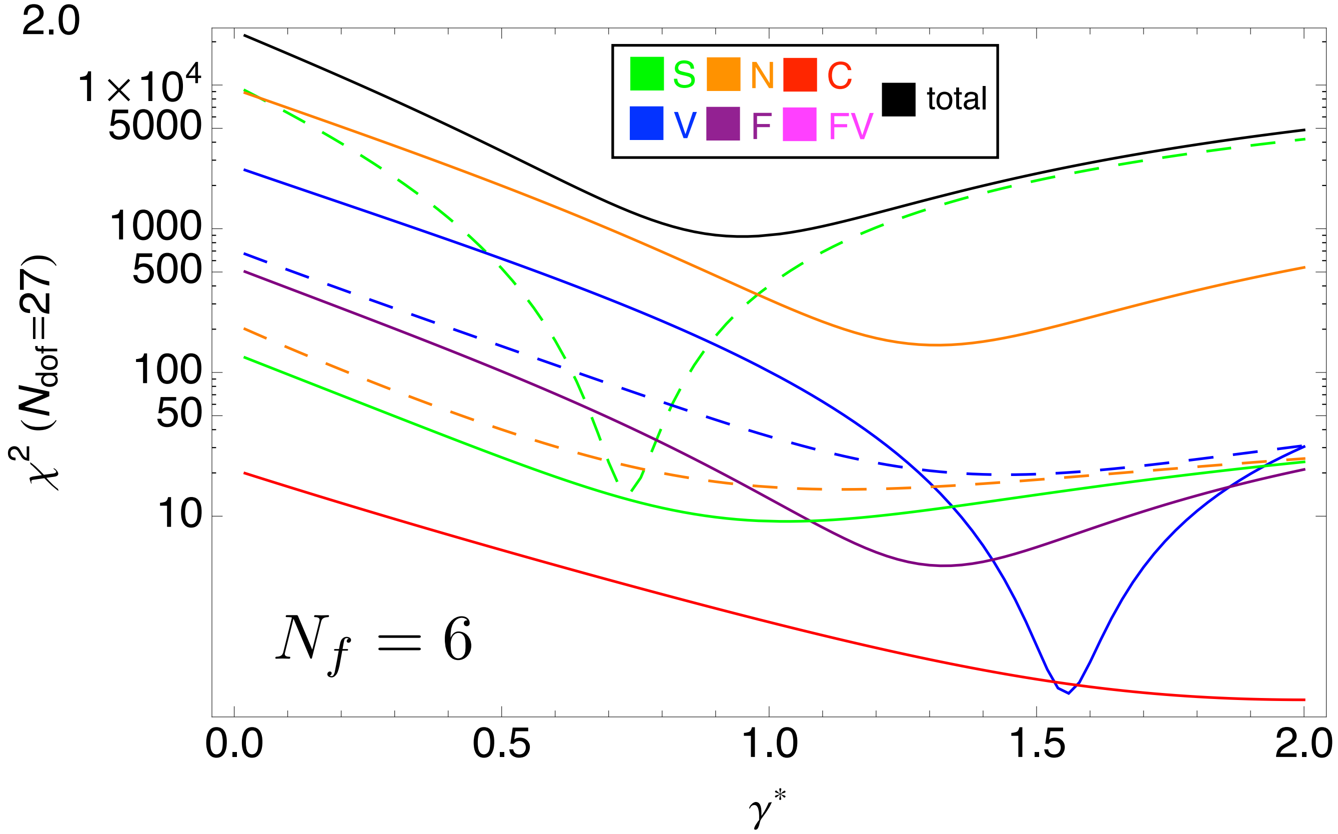
- With gamma fixed, no non-linearity in the other parameters, can fit easily
- Showing chi-squared contours in gamma makes it easy to split by channel, read off best-fit point and error on gamma
- P,V channels point to gamma ~ 0.2, in agreement with other determinations. F likely has underestimated errors.

A deliberately bad example



- Lack of minimum in most channels (except pion), enormous overall chi-squared

- Can get spurious minima, too...tends to favor large gamma, poor consistency



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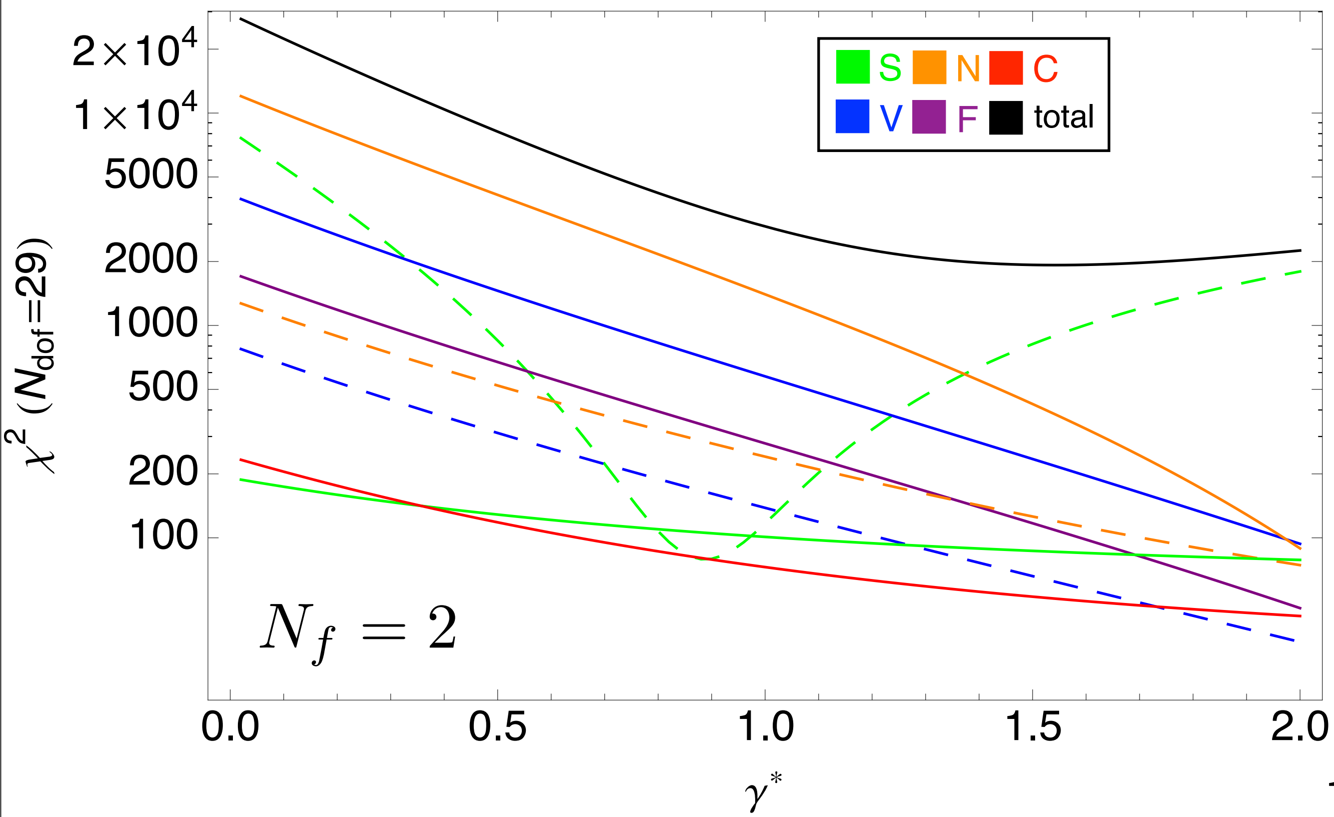

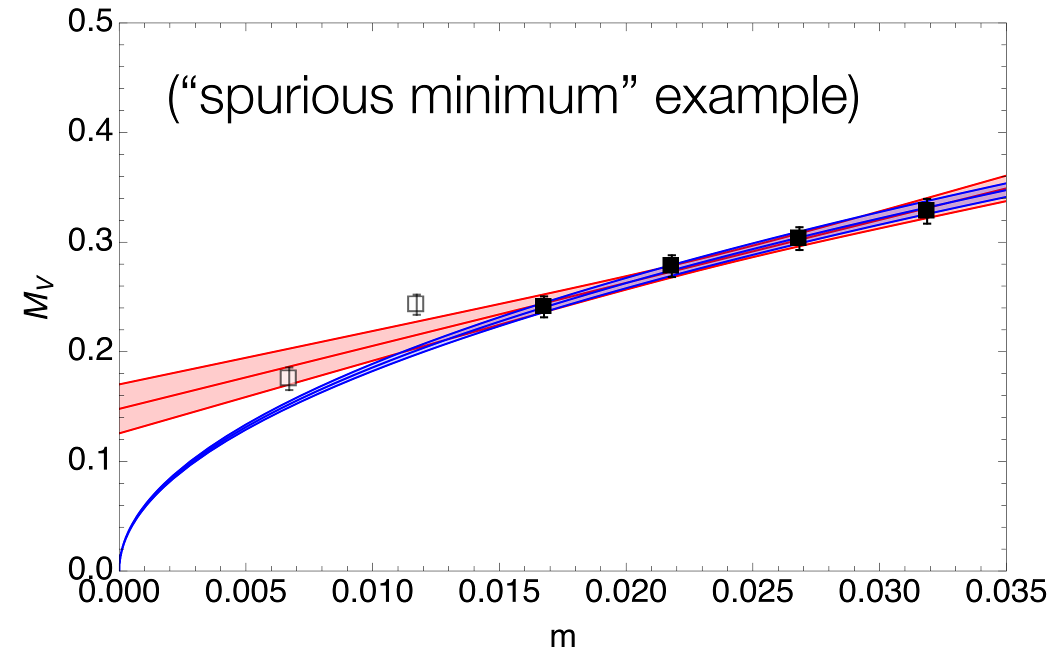

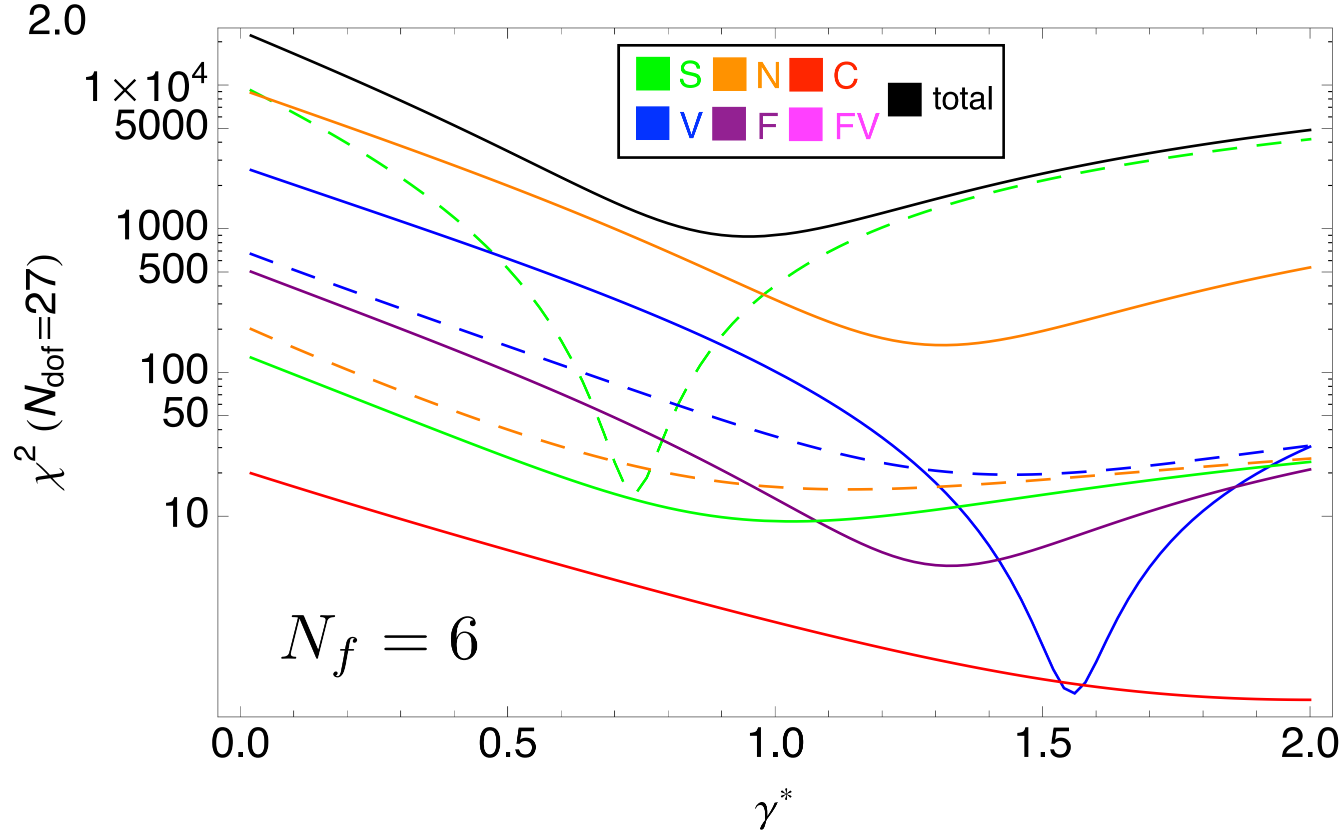

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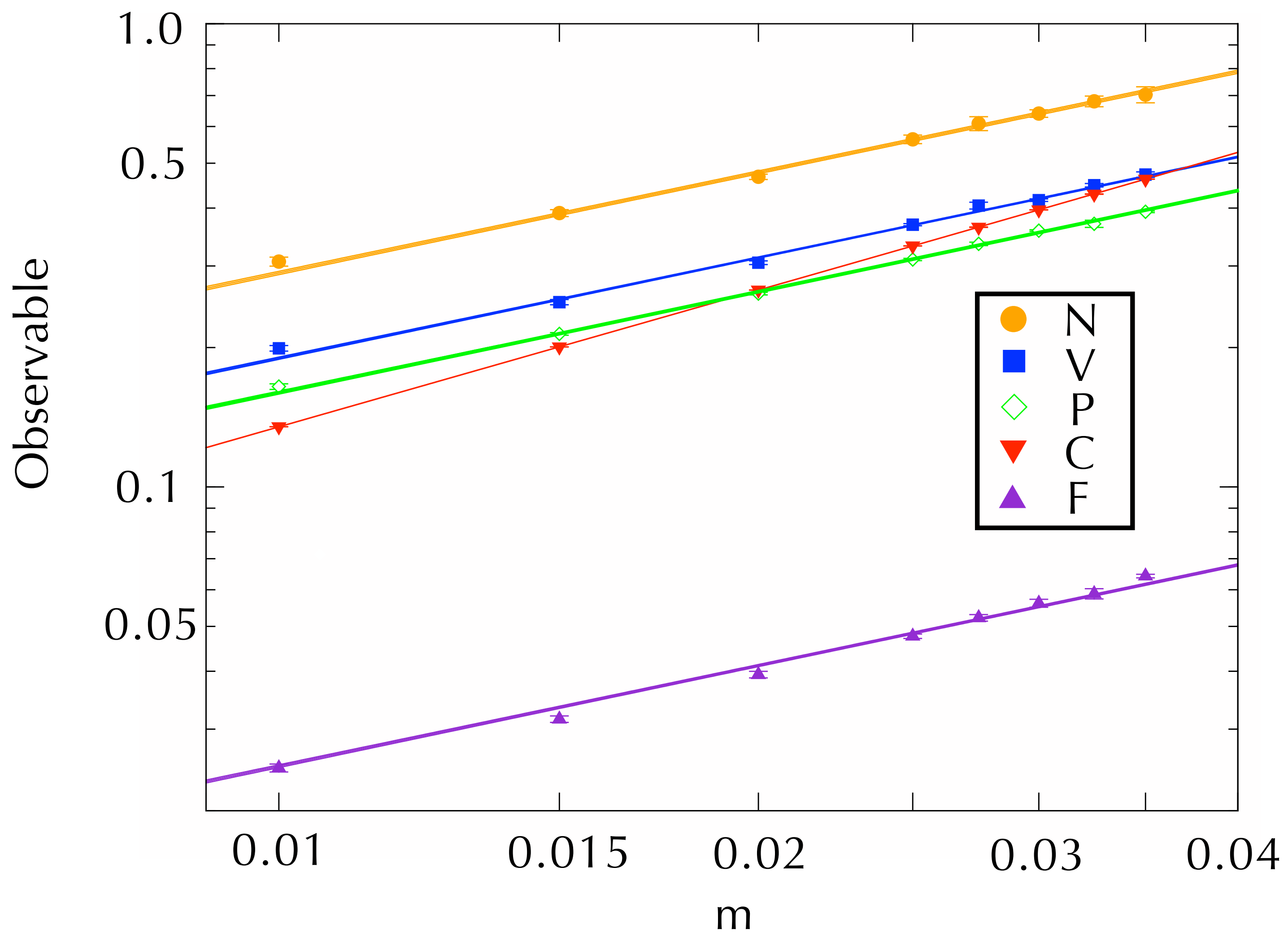
SU(3) with 12 fundamental fermions

(arXiv:1104.3124)

Twelve massless flavors and three colors below the conformal window

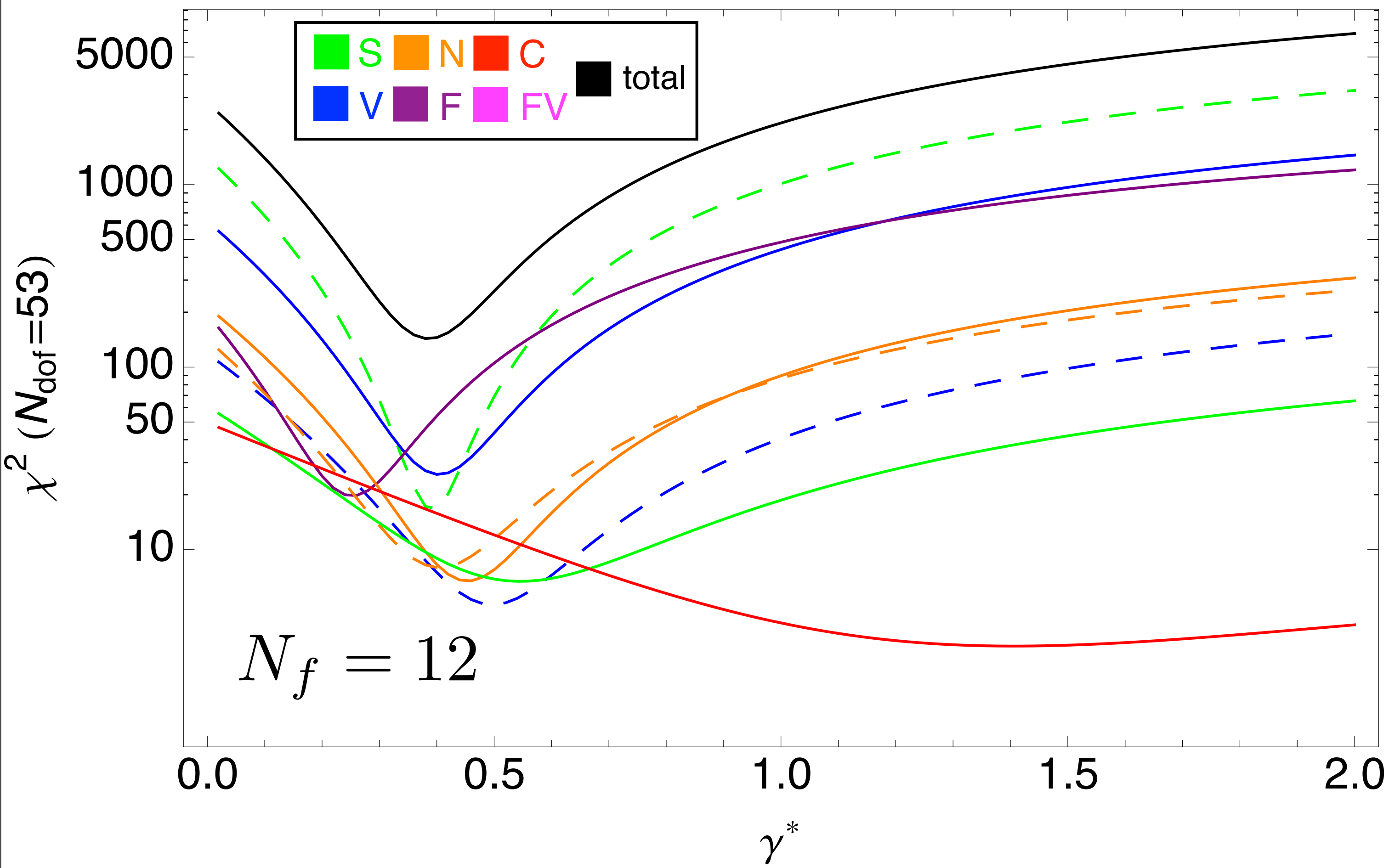
Zoltán Fodor[a,e], Kieran Holland[b], Julius Kuti[*,c], Dániel Nógrádi[e], Chris Schroeder[d]

- We reanalyze the extensive data set of Fodor et al (above), coming to a different conclusion
- Fodor et al. consider mass-deformed fits in their paper; main difference with us is over correction terms, relative importance of channels



Scaling fit results, SU(3) Nf=12

(no D or z terms here)



$$M_X = C_X m^{1/(1+\gamma^\star)} \left[1 + \frac{z_x}{m^{1/(1+\gamma^\star)} L}\right] + D_X m + \ldots$$

Scaling fit results, SU(3) Nf=12

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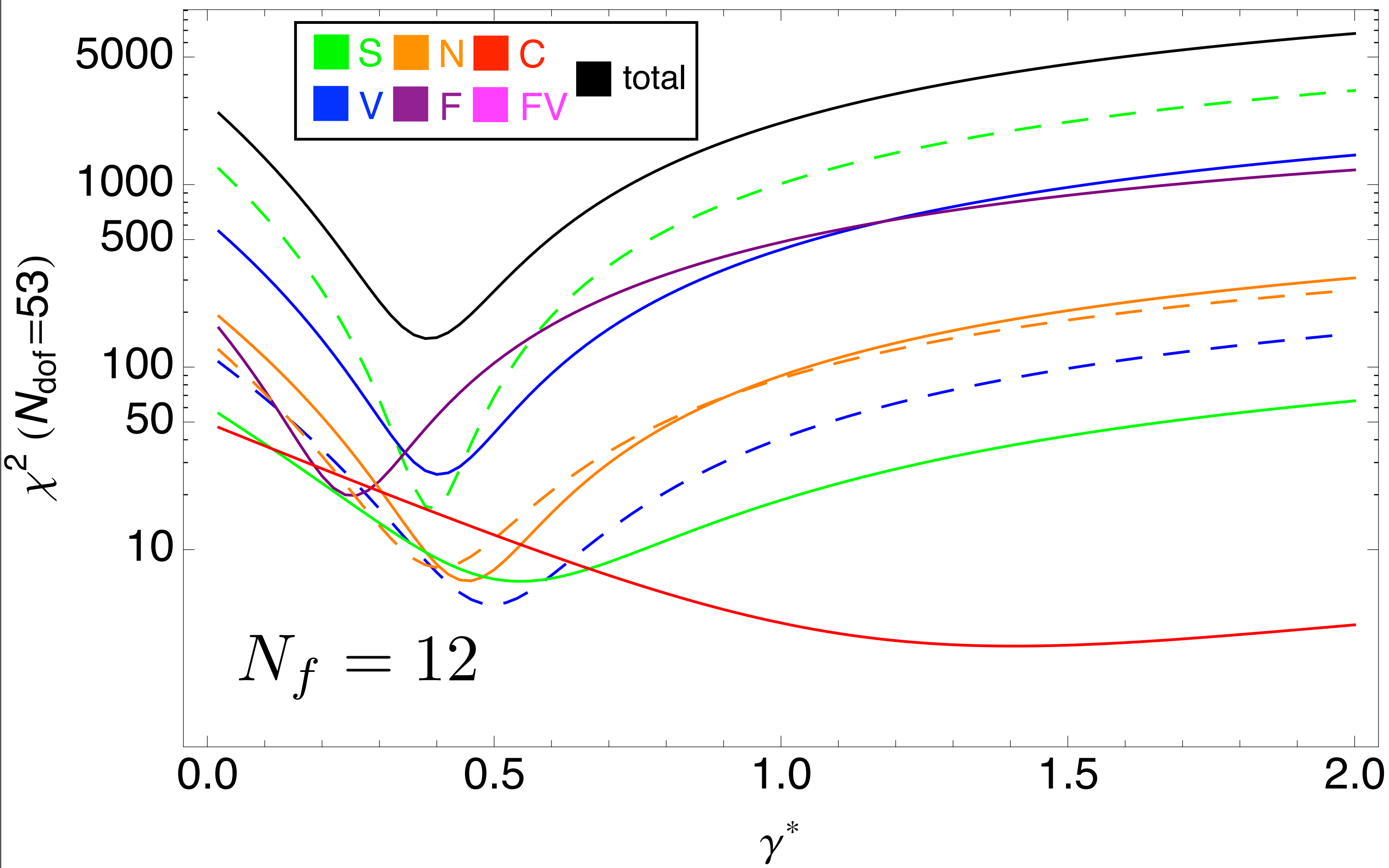
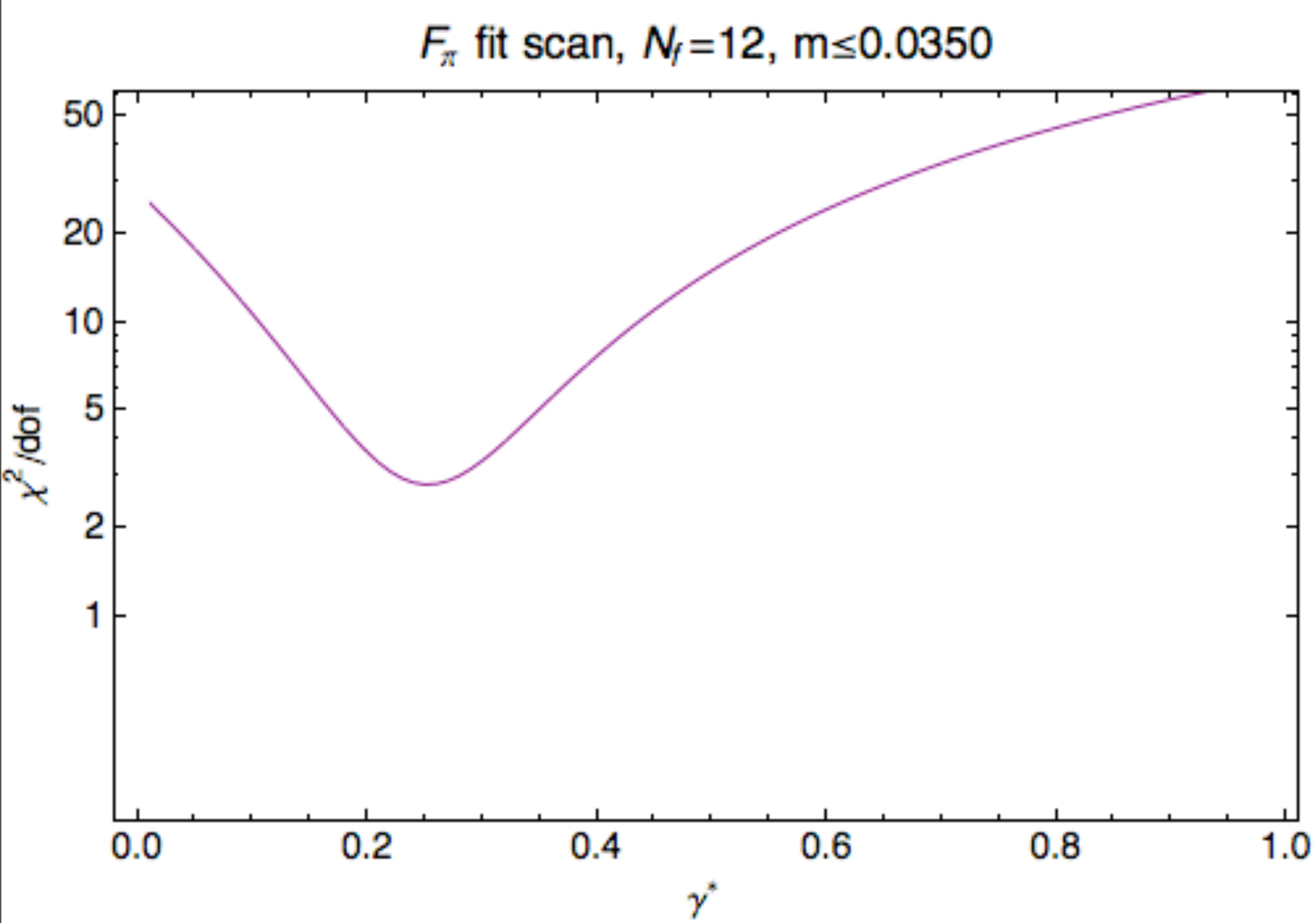


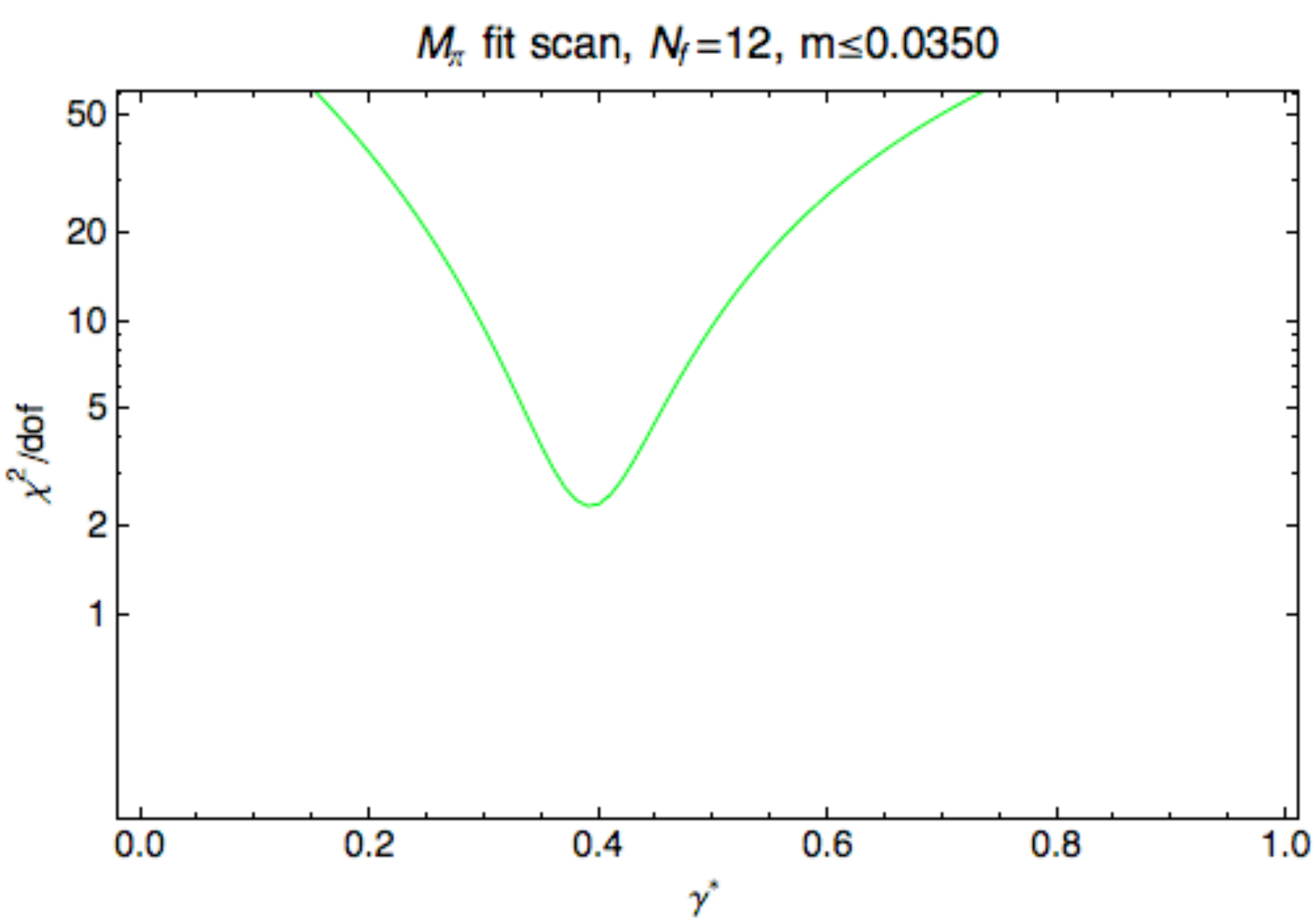
Obs.	$D_X = 0$	$D_F \neq 0, z_X \neq 0$		
$\gamma^\star$	0.3858(98)	0.403(13)		
C_P	4.445(83)	4.267(85)	z_P	0.209(64)
C_S	5.99(14)	5.75(14)	z_S	0.63(45)
C_V	5.26(10)	5.05(10)	z_V	0.319(88)
C_A	6.68(15)	6.41(15)	z_A	0.50(30)
C_N	8.04(17)	7.70(17)	z_N	0.35(18)
$C_{N^\star}$	8.06(17)	7.73(17)	$z_{N^\star}$	0.49(24)
C_F	0.692(13)	0.455(39)	z_F	0.61(27)
D_F	—	0.61(10)		
A_C	13.898(28)	13.926(31)	z_C	-0.036(43)
B_C	-50.8(5.5)	-42.2(4.8)		
C_C	94(11)	79.0(9.5)		
χ^2/dof	133/53	42/44		

$$M_X = C_X m^{1/(1+\gamma^\star)} \left[1 + \frac{z_x}{m^{1/(1+\gamma^\star)} L}\right] + D_X m + \ldots$$

A closer look at the decay constant

- F favors a small minimum compared to other channels...full fit likes a larger mass correction D-term. Some hint of this if we drop the heavier points:

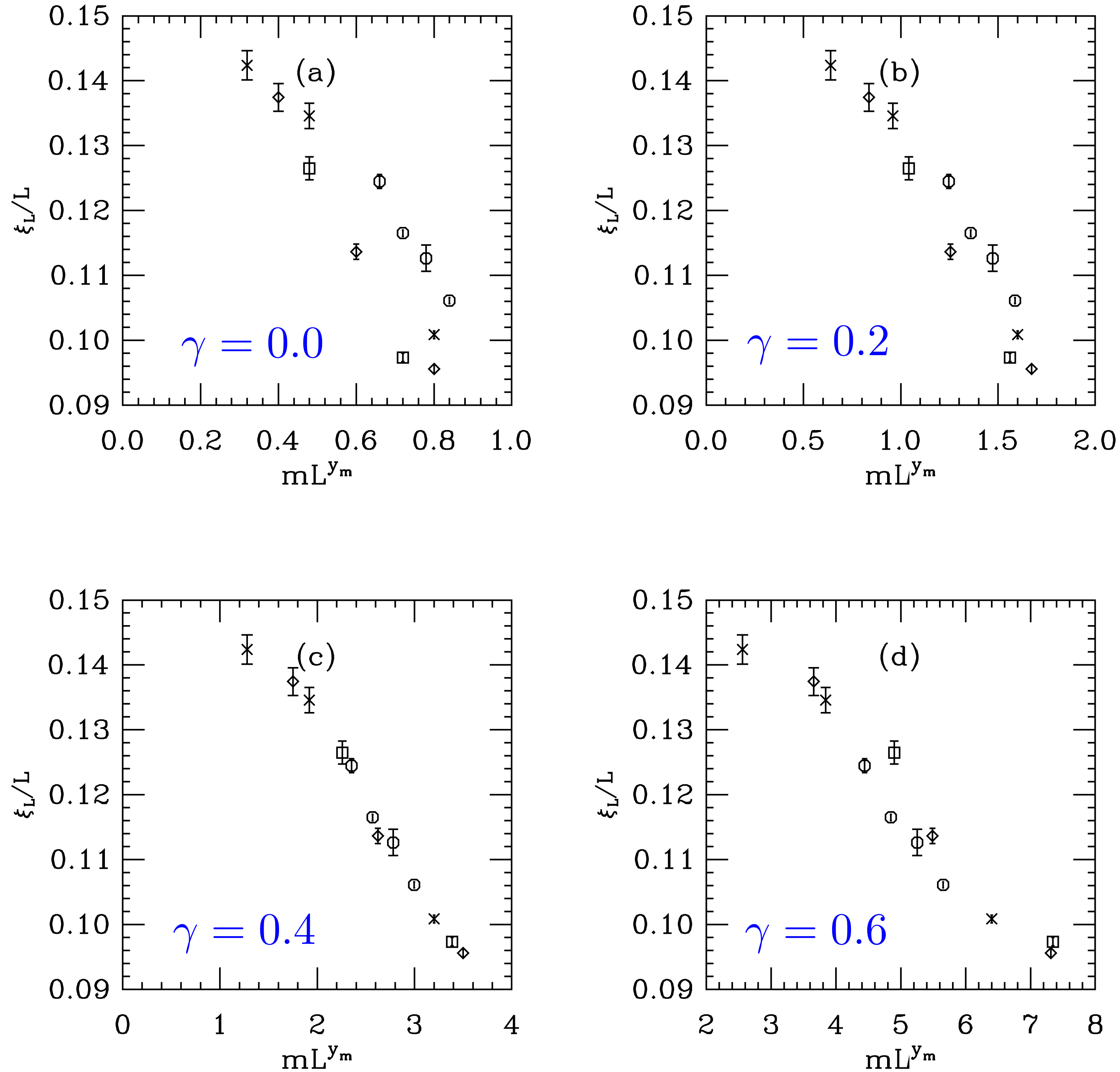




Other analyses

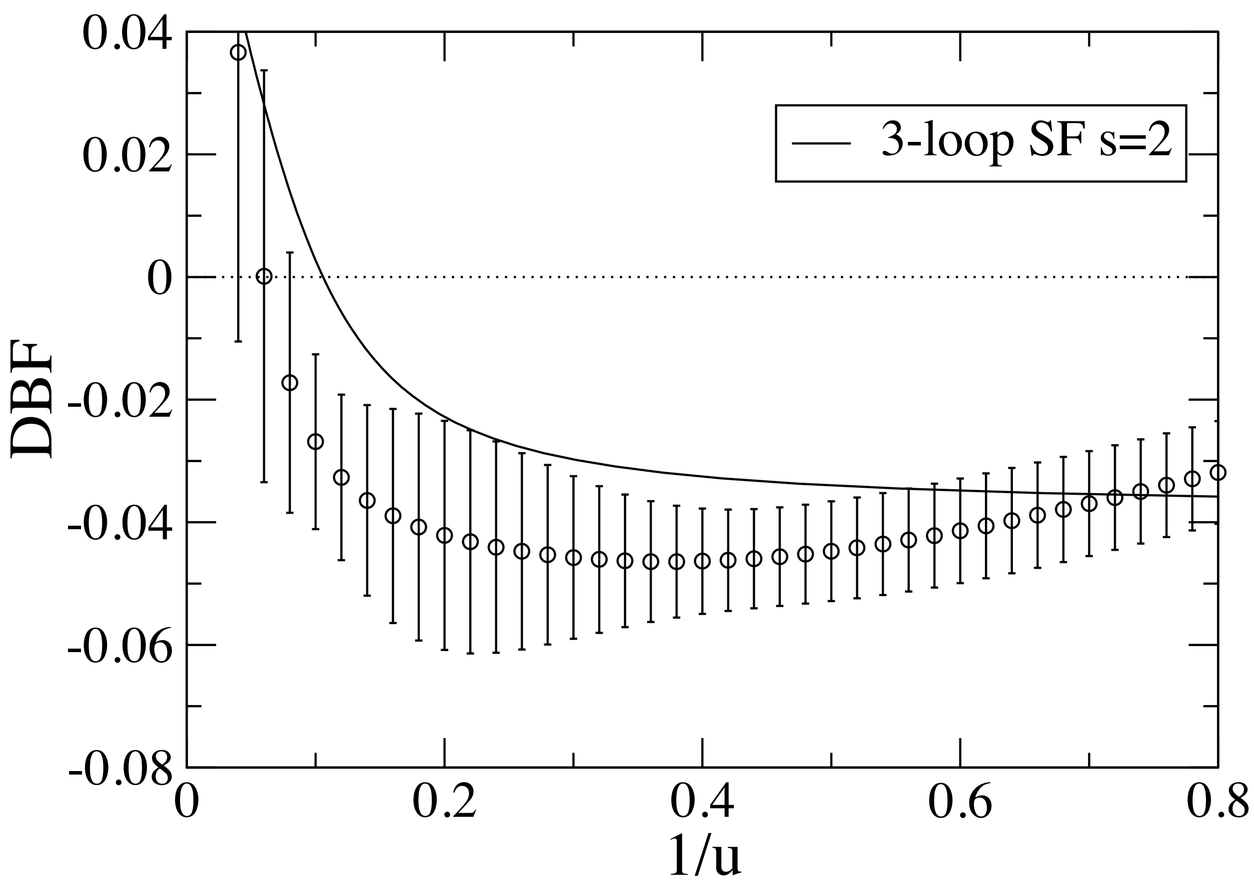
- DeGrand also re-analyzed the same data set using the "curve collapse" method, also finds $\gamma^\star \sim 0.4$
- New paper from Fodor, Kuti et al. shows additional data, similar fits, concludes that conformal fits don't work. Most problematic channel is once again F_P...D terms? Or something we don't understand?

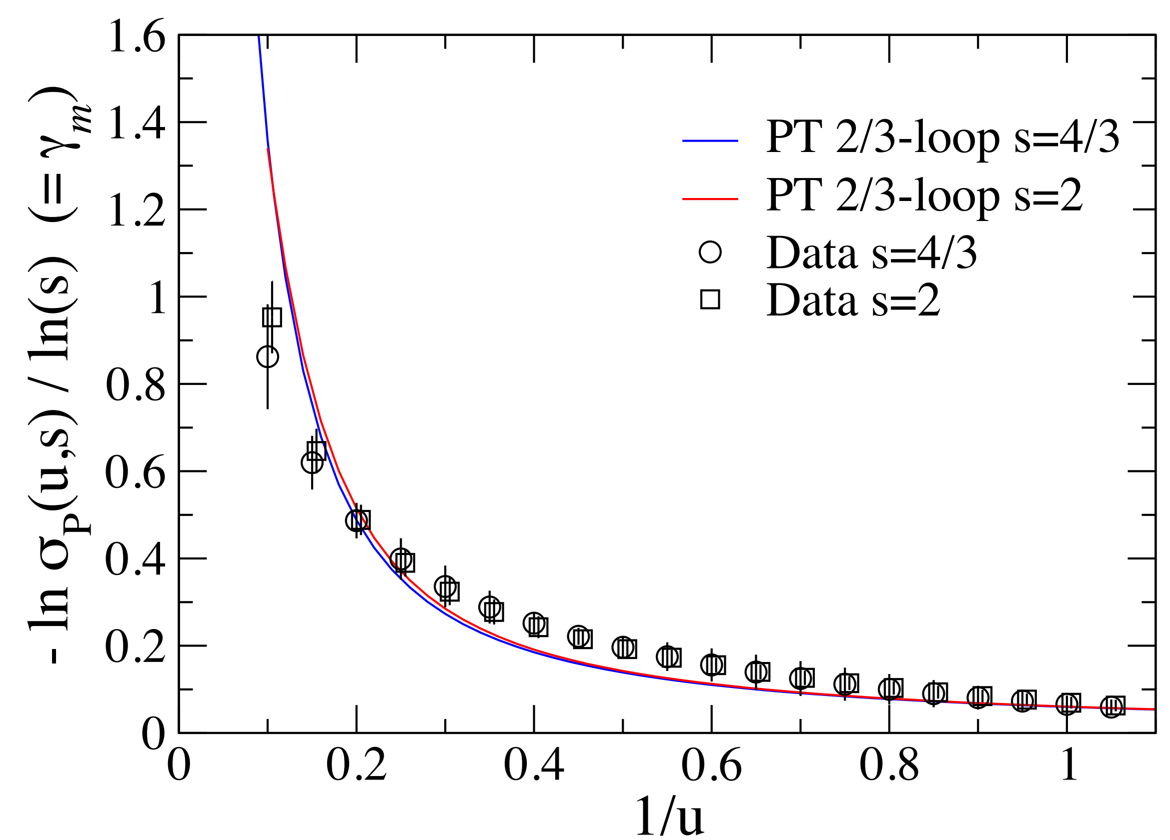
(from T. DeGrand, arXiv:1109.1237)



SU(3) with 10 fundamental fermions

- On to **10 flavors** - probably near the edge of the conformal window for SU(3), one way or another.
- One other study in addition to us, running coupling + direct anomalous dimension. Sees strong fixed point and gamma near 1!





(Talk by N. Yamada, KMI mini-workshop 03/12)

Lattice Strong Dynamics Collaboration

James Osborn
Heechang Na

Rich Brower
Michael Cheng
Claudio Rebbi
Oliver Witzel

David Schaich

Ethan Neil

Sergey Syritsyn

Mike Buchoff
Chris Schroeder
Pavlos Vranas
Joe Wasem

Joe Kiskis

Tom Appelquist
George Fleming
Meifeng Lin
Gennady Voronov

Saul Cohen

(IBM Blue Gene/L supercomputer at LLNL)

(Cray XT5 “Kraken” at Oak Ridge)

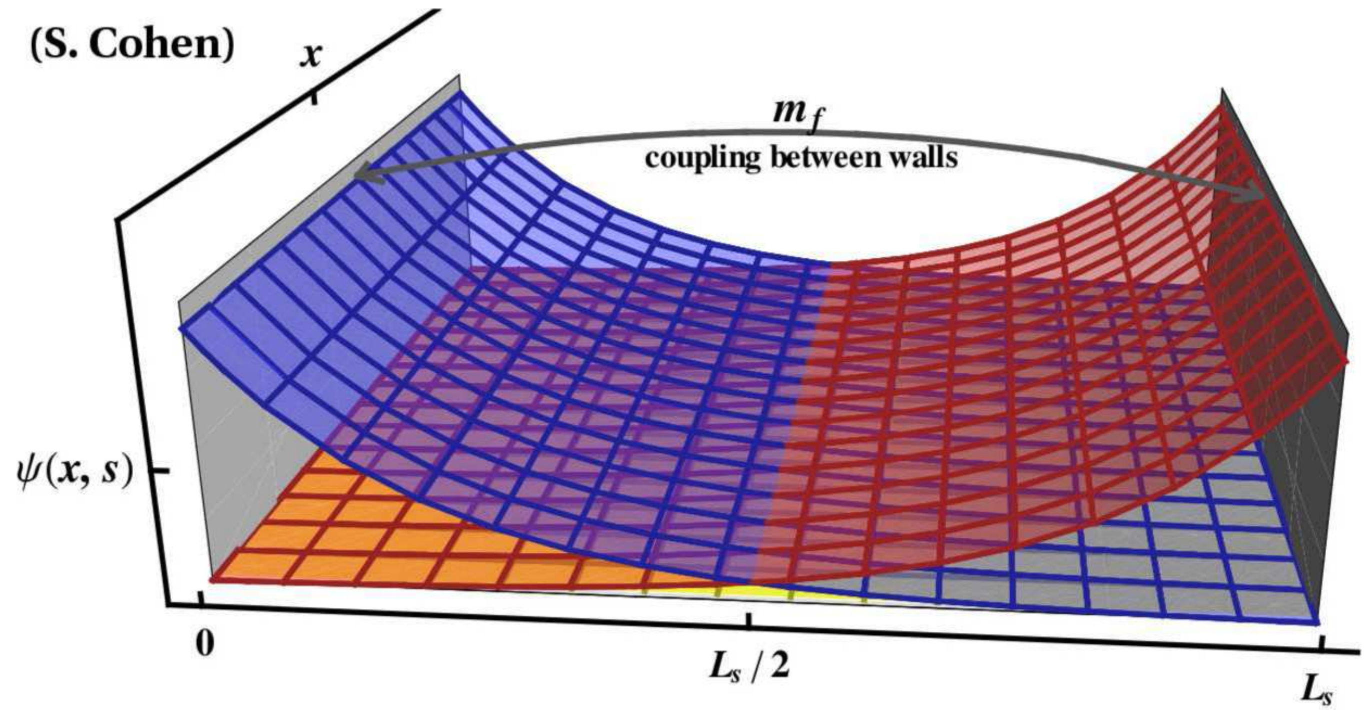
Results to be shown are state-of-the-art for lattice simulation - O(100 million) core-hours for full program

Many thanks to the computing centers and funding agencies (DOE through USQCD and LLNL, NSF through XSEDE)

(Computing cluster “7N” at JLab)

Simulation details

- Iwasaki gauge action + domain-wall fermions, fermion masses from m_f=0.005 to m_f=0.03, one volume (32^3x64).
- Residual chiral symmetry breaking reasonably small, m_{res}~0.002. All chiral extrapolations in $m=m_f+m_{res}$.
- To address autocorrelation, start every gauge ensemble from both “ordered” (free) and “disordered” (random).



m_f	start	m_I	τ	$\delta\tau$	# trajs.
0.005	ord				1310
0.005	mix				910
0.010	ord	0.1	1	0.0667	1370
0.010	dis	0.1	1	0.0667	1260
0.015	ord				1200
0.015	dis	0.1	1	0.0667	1450
0.020	ord				1240
0.020	dis				1220
0.025	ord	0.1	1	0.0625	1250
0.025	dis	0.1	1	0.0667	1480
0.030	ord	0.1	1	0.0667	1410
0.030	dis	0.1	1	0.0667	1220

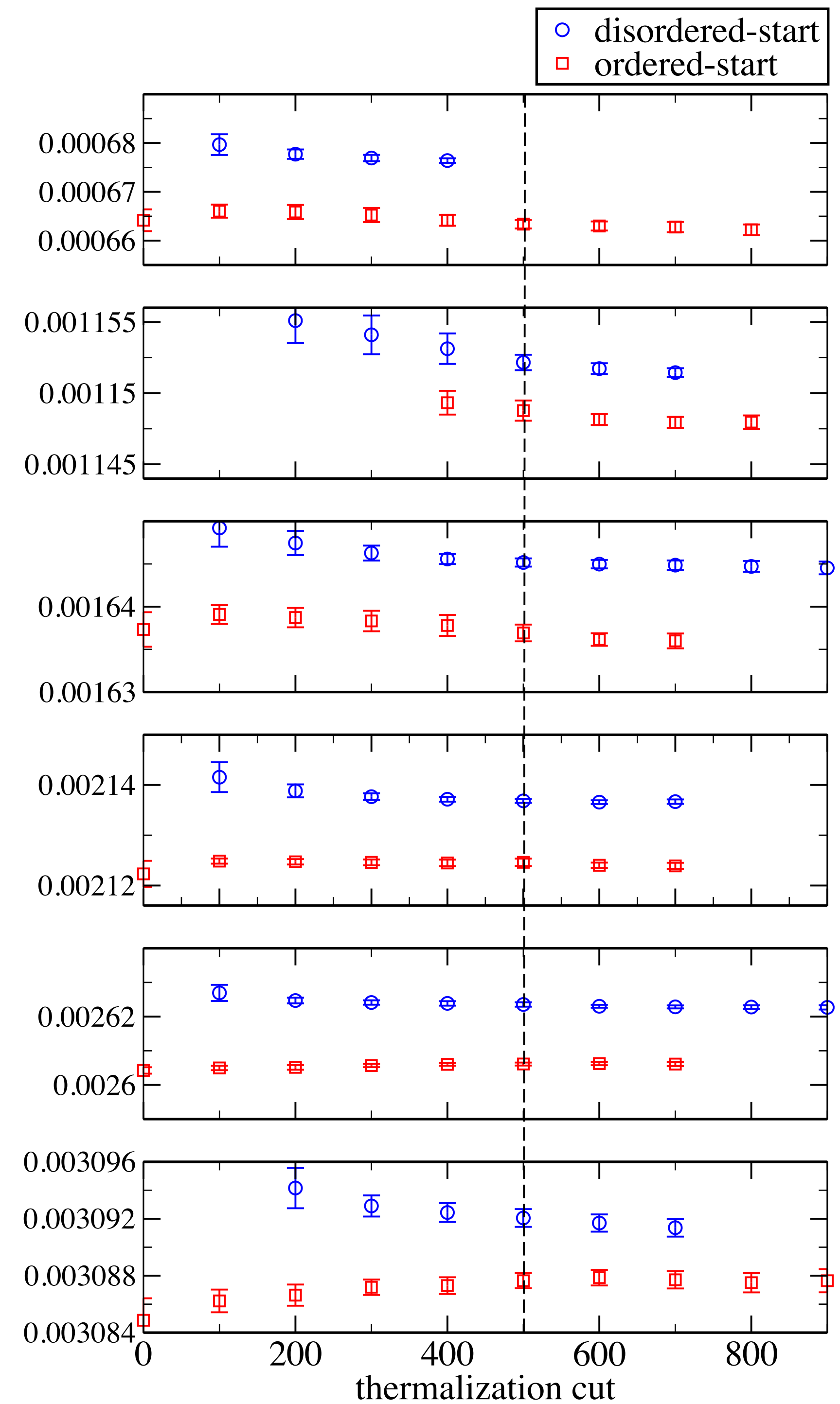
Initial states and data combination

- Persistent ord/dis difference (right) treated as systematic effect, averaging procedure is used. Conservative error estimate including both statistical error and "spread" between two ensembles:

$$p(x) \propto \int_{-\infty}^{\infty} U(\mu|\bar{\mu}, \delta) \left[G(x|\mu, \sigma_1) + G(x|\mu, \sigma_2) \right],$$

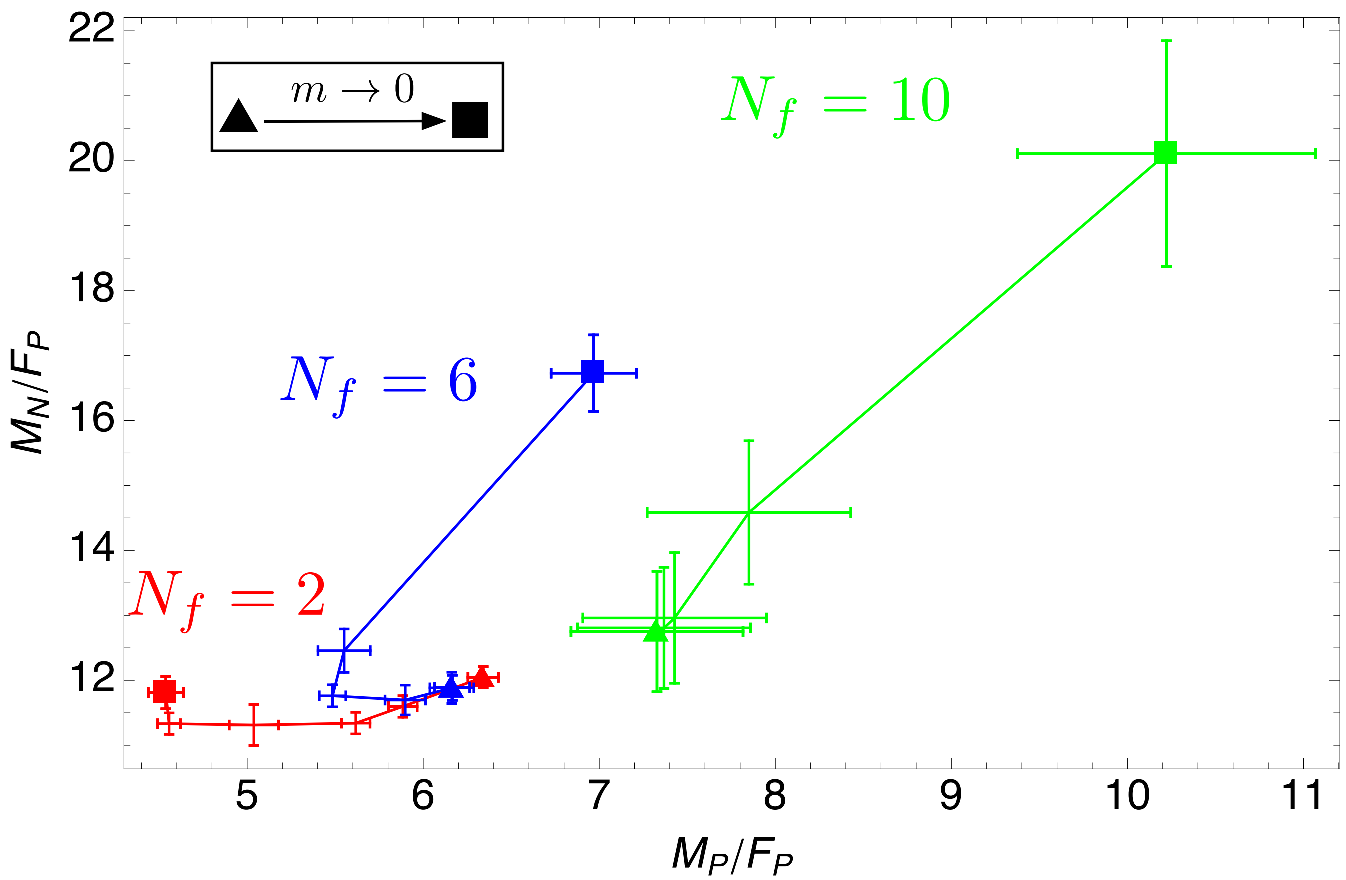
- U and G are uniform/Gaussian pdfs, respectively. Width of uniform estimated from (ord-dis) fractional difference, avg over all ensembles.

- m=0.005 excluded from analysis (systematic differences too large, plus finite-volume concerns.)



(chiral condensate)

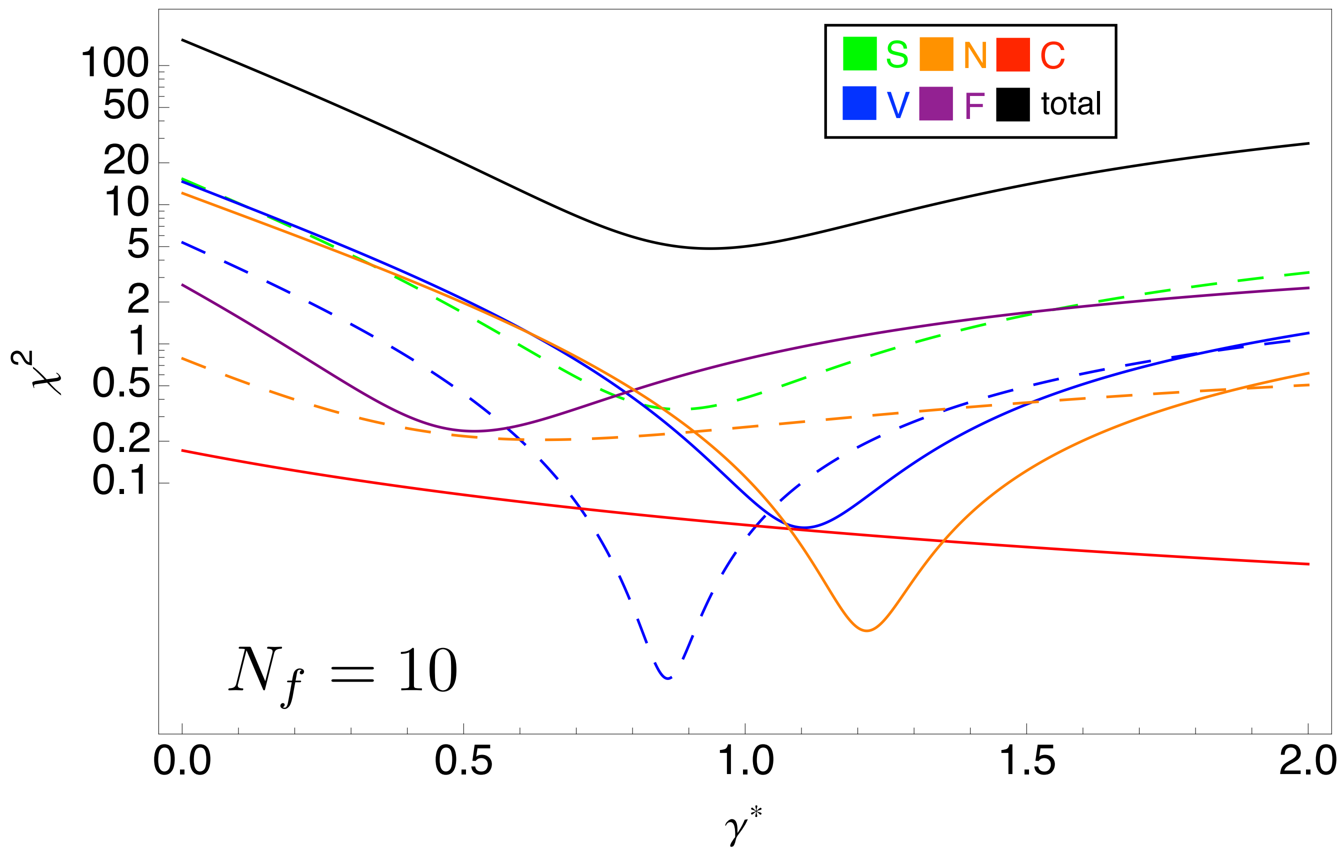
Finite-volume effects at Nf=10



- We only have one useful simulation volume, so we have to be extra-careful about finite-volume effects
- "Edinburgh plot" (left) provides a way to check - FV corrections increase mass but decrease F, so send all points up and right on this plot
- 2, 6 flavors show expected scaling, i.e. M_P/F_P becoming small. 10 has both ratios constant - until m=0.015 and (especially) m=0.010

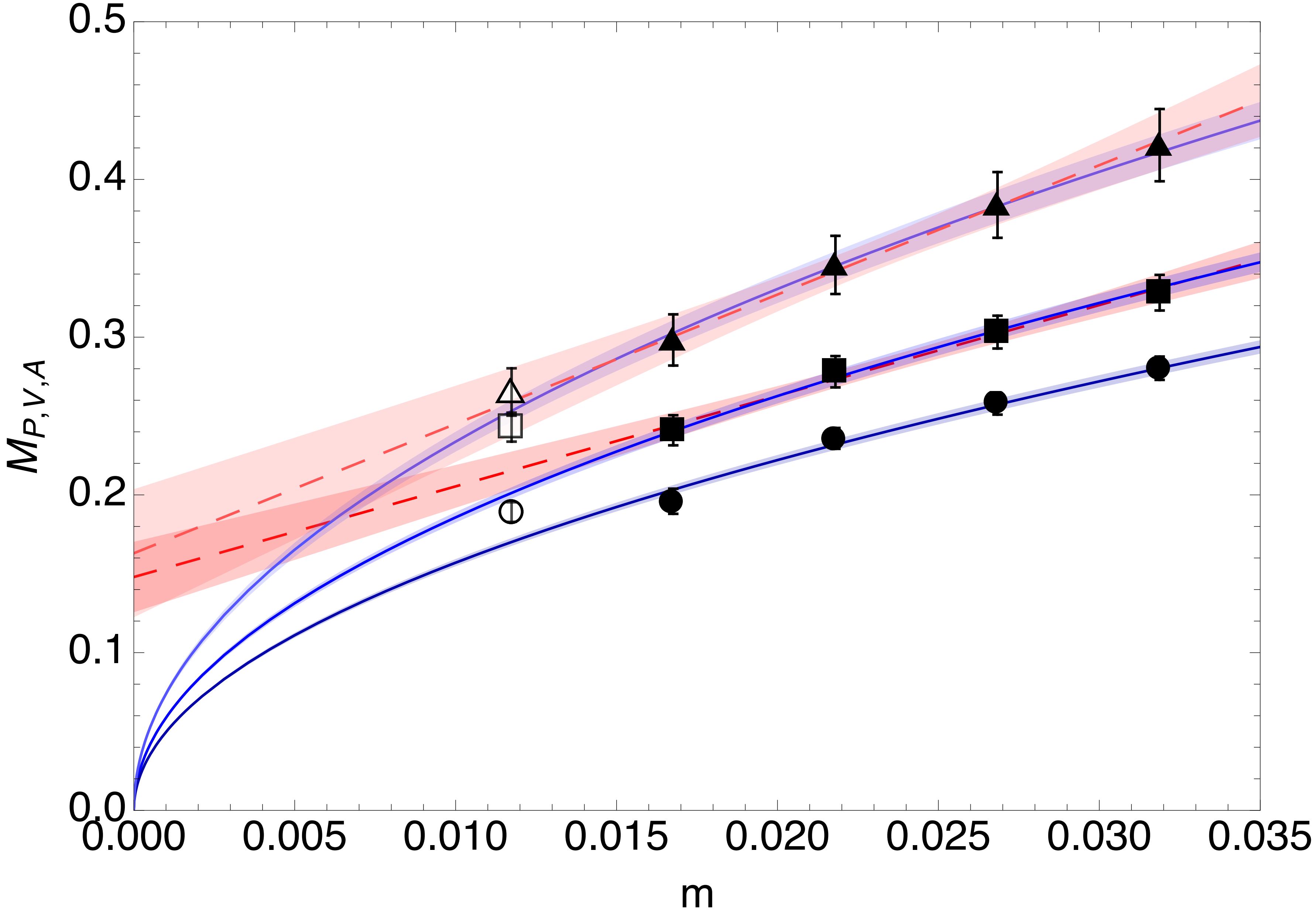
Scaling fit results, Nf=10

(shown for mf>=0.015



Obs.	$m_f \geq 0.010$	$m_f \geq 0.015$	$m_f \geq 0.020$
$\gamma^\star$	*1.69(16)*	1.10(17)	1.35(47)
[68% CI]	*[1.54,1.86]*	[0.95,1.27]	[1.06,1.73]
[95% CI]	*[1.40,2.06]*	[0.82,1.46]	[0.83,2.27]
C_P	*0.98(9)*	1.44(21)	1.21(37)
C_V	*1.17(10)*	1.70(25)	1.42(44)
C_A	*1.43(13)*	2.14(32)	1.79(56)
C_N	*1.75(16)*	2.53(37)	2.10(65)
$C_{N^\star}$	*2.23(25)*	3.35(55)	2.87(92)
C_{FP}	*0.121(12)*	0.190(28)	0.164(51)
C_{FV}	*0.165(15)*	0.238(35)	0.195(60)
C_{FA}	*0.136(13)*	0.192(28)	0.154(48)
χ^2/d.o.f.	*69/31*	14/23	3.1/15

Taking a closer look



- Can't decisively tell power-law fits (blue) from pure linear (red) over the range of masses available...but the common power law $m^{1/2}$ working for all states is suggestive at least.

One more thing...

- I've mostly ignored the coefficients C_X so far, because it's hard to interpret exactly how meaningful they are, and can't compare between different theories (buried dependence on mass scale, gamma, lattice cutoff.)

- However, if I start taking ratios of those coefficients, the junk cancels and a strange pattern emerges: (caveat: no correlations included)

ratio	$N_f = 10$	$N_f = 12$
C_V/C_P	$1.18(24)$	$1.18(4)$
C_A/C_P	$1.49(31)$	$1.50(5)$
C_N/C_P	$1.76(36)$	$1.81(6)$
C_{N*}/C_P	$2.33(51)$	$1.81(6)$
C_{FP}/C_P	$0.132(27)$	$0.156(5)$

- Completely different lattice actions, different simulation groups, different volumes, and most importantly different theories! Why do these ratios seem to agree?

Conclusion

- Theories other than QCD is a very active topic of current lattice research, and new ground is being explored. Original attempts to investigate the conformal window frustrated by need for mass term, but starting to realize that it can be used to our advantage

- Framework for mass-deformed CFT is very young and not completely rigorous, especially for certain channels. Still, it can be used to accurately describe lattice spectrum results, and resulting extraction of mass anomalous dimension shows good overlap with more standard approach

- SU(2) 2-flavor adjoint and SU(3) 12-flavor fund. shown as examples for theories where the framework can be applied, but the mass dimension is small. For SU(3) 10-flavor, our simulation results still need refinement, but preliminary indications of **gamma ~ 1**, in agreement with other lattice work.

- 10f update in progress - more volumes, better combination, glueballs.