Appendix B
Basis of Certain Charts in Paragraphs 5-4 and 5-5

B-1. Example of Allowable Velocity-Depth Data for Granular Materials

The chart in Figure 5-5 has been developed using a variety of sources in an attempt to consolidate allowable mean velocities for no erosion of granular materials over a wide range of grain sizes. An earlier version appeared in Roads and Transportation Association of Canada (1973) for use in checking the adequacy of bridge waterways to avoid general scour. Figure 5-5 should be taken as indicative of trends only and not as definitive guidance for flood control channels. Channels with significant bed sediment inflows will be found to tolerate higher velocities without bed erosion. On the other hand, bank erosion may occur at considerably lower velocities than shown, particularly at channel bends. The development of the chart can be explained briefly as follows:

a. Coarse sizes (generally larger than 10 mm).

(1) The Shields number criterion for coarse sizes applies, strictly speaking, to a static flat bed condition. A Shields number value of 0.045 is adopted, corresponding to effective beginning of sediment transport but not to absolute stability. The bed roughness, expressed in terms of the grain roughness k, is assumed to be three times the median grain size D, which implies a particular type of grain size distribution.

(2) The algebraic development of the Shields number is as follows:

$$\frac{dS}{(s - 1)D} = 0.045 \qquad \text{(B-1)}$$

where

d = depth

S = slope

s = dry specific gravity

D = median grain size

(3) The Manning formula for mean velocity V, assuming a wide channel, is converted to replace n with k in the form (Ackers 1958)

$$\frac{V}{\sqrt{gdS}} = 8.45 \left(\frac{d}{k}\right)^{1/6} \qquad \text{(B-2)}$$

where g is the gravitational acceleration.

(4) Equations B-1 and B-2 are combined to eliminate S. Then, assuming $k = 3D$ and $s = 2.6$, mean velocity is derived in terms of grain size and depth as

$$V = 10.7 \, D^{1/3} \, d^{1/6} \qquad \text{(B-3)}$$

where V is in feet per second and D and d are in feet.

b. Fine sizes (generally smaller than 2 mm).

(1) Allowable mean velocities for the finer sizes are difficult to develop in the same way as for the coarser sizes because the flat bed assumptions underlying the Shields relationship are not even roughly applicable to field channels.

(2) A comparison of published velocity-depth data for the finer sizes shows considerable discrepancies between experimental beginning-of-movement data (e.g., Sundborg 1956), empirical (“regime”) data based on field experience of stable sand-bed canals (e.g., Blench 1957), and semitheoretical data for stable channels (e.g., White, Paris, and Bettess 1981b).

(3) The curves for the fine size range in Figure 5-5 generally indicate higher allowable velocities than experimental beginning-of-movement data, but lower velocities than regime canal data. They are reasonably comparable with the semitheoretical predictions of White, Paris, and Bettess (1981b) for live-bed channels with a relatively low bed sediment concentration, in the order of 40 parts per million by weight.

B-2. Tentative Guide to Width-Discharge Relationships for Erodible Channels

a. The chart in Figure 5-9 is based on a general relationship first formulated by Lacey (1929-30) whereby, comparing one channel with another, bank-full width or wetted perimeter varies as the square root of a discharge parameter, that is $W = C\,Q^{0.5}$, where W is the width, Q is the discharge, and C is a coefficient. The discharge parameter is variously given in the literature as dominant discharge, channel-forming discharge, or bank-full discharge. Numerous subsequent investigations of channels in different environments have confirmed the approximate applicability of the Lacey relationship, although a generally accepted theoretical explanation is lacking. Figure B-1 shows a consolidated data plot by Kellerhals and Church (1989) that covers an extremely wide range of discharges, of which the middle part closely follows the Lacey relationship.

b. The factors that affect the coefficient C in the Lacey relationship are not well defined. In general, channels with easily erodible banks, and with higher transport of bed material, tend to be wider. The curves in Figure 5-9 make allowance for bank erodibility but not for sediment transport. Coefficients are varied from 2.7 to 1.6 according to the nature of the channel banks. Curve 3 ($C = 2.7$) corresponds approximately to Lacey's original equation for channels in sandy alluvium. Curve 2 ($C = 2.1$) corresponds closely to an equation by Simons and Albertson (1963) for channels with cohesive bed and banks. Curve 1 ($C = 1.6$) is close to a relationship by Kellerhals (1967) for lake-outlet channels with gravel-paved or cobbled bed and banks.

c. In a set of similar curves presented by Hey and Thorne (1986) for gravel-bed channels in the United Kingdom (Figure B-2), variation in the Lacey coefficient C is associated with type of bank vegetation rather than with type of bank material. Vegetation is defined generally in terms of the percentage of tree-shrub cover, and their fitted C values, converted to ft-sec units, range from 2.34 to 4.33. It is evident that this basis for discrimination would not be generally applicable in arid climates. (Another basis that has been suggested is the percentage of silt/clay in bank materials.)

B-3. Tentative Guide to Depth-Discharge Relationships for Alluvial Channels

a. The chart in Figure 5-10 is based loosely on a comparable chart presented in a previous report (Northwest Hydraulic Consultants 1982), assuming wide channels with mean depth equivalent to hydraulic radius. Figure 5-10 should be taken as indicative of trends only for channels with low bed sediment transport, and not as definitive guidance for the design of flood control channels.

b. The source chart (Figure B-3) was based on selected relationships in the literature for a range of channel materials. Figure B-3 can be summarized as follows. Curves 1 and 2 are based on Lacey's (1929-30) original equations, with "silt factors" for medium and very fine sand respectively. Curves 3, 4, and 5 are based on Simons and Albertson's equations as quoted by USDA (1977) for (3) sand bed and banks, (4) sand bed and cohesive banks, and (5) cohesive bed and banks. Curves 6 and 7 are based on Kellerhals' (1967) equation for stable gravel-paved channels, using D_{90} values of 0.1 ft and 1 ft, respectively. (Curve 8 is irrelevant to the present discussion.)

B-4. Tentative Guide to Slope-Discharge Relationships for Erodible Channels

a. Figure 5-11 should be taken as indicative only for channels with low bed-sediment transport, and not as definitive guidance for the design of flood control channels.

b. The curves for gravel and cobble materials with median grain sizes from 20 to 200mm are based on combining the Shields criterion for beginning of movement with a Lacey-type width relationship and the Manning formula. The algebraic development, assuming a trapezoidal cross section, is as follows:

(1) Shields Number

$$\frac{dS}{(s - 1)\,D} = 0.045 \qquad \text{(B-1 bis)}$$

For $s = 2.6$, Equation B-1 transforms to

$$S = 0.072\,\frac{D}{d} \qquad \text{(B-4)}$$

(2) Lacey width relation

$$b = 1.8\,Q^{1/2} \qquad \text{(B-5)}$$

where b is the mean width in feet and Q is in cubic feet per second.

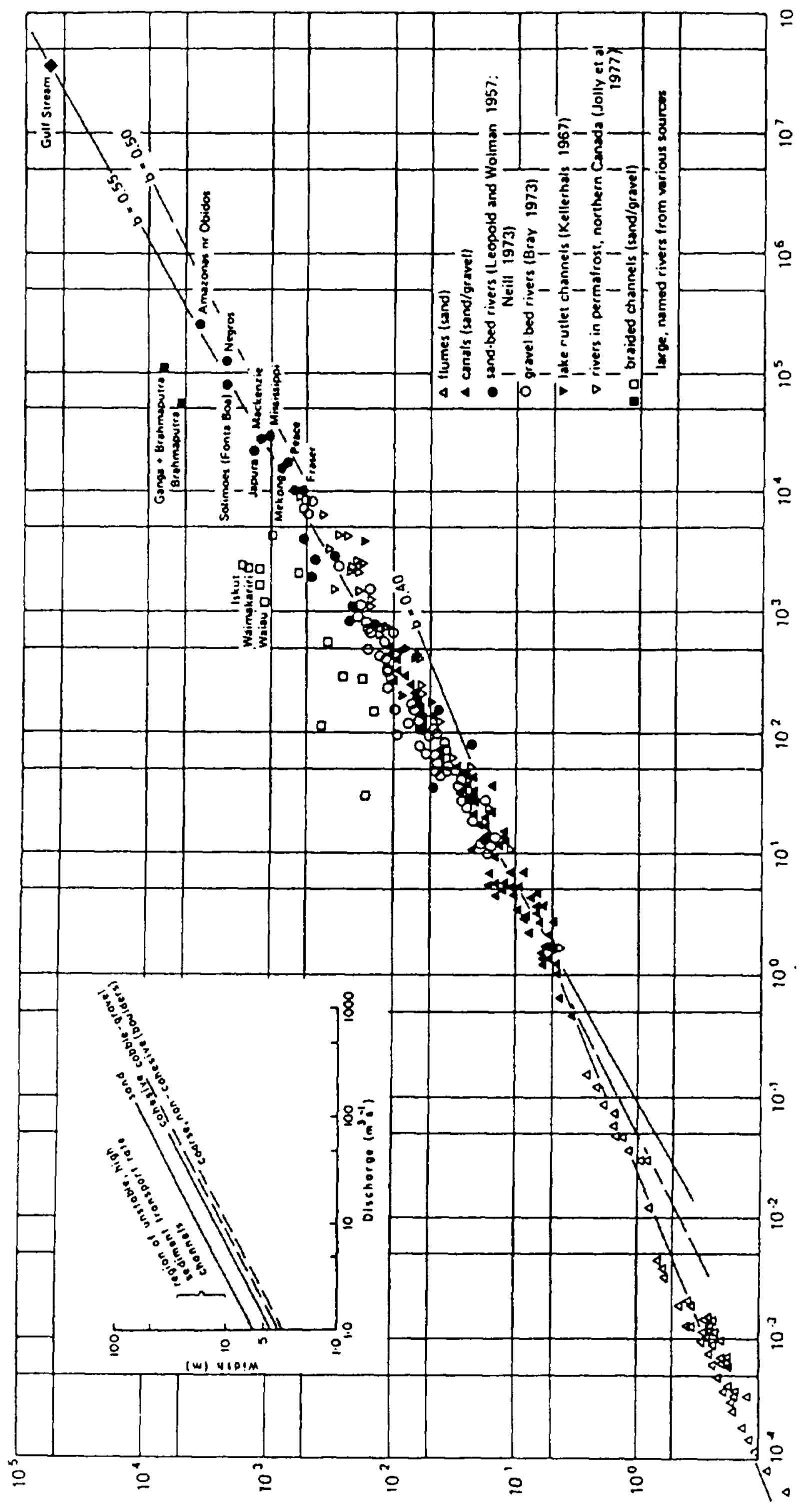

Figure B-1. Consolidated width discharge plot by Kellerhals and Church (1989)

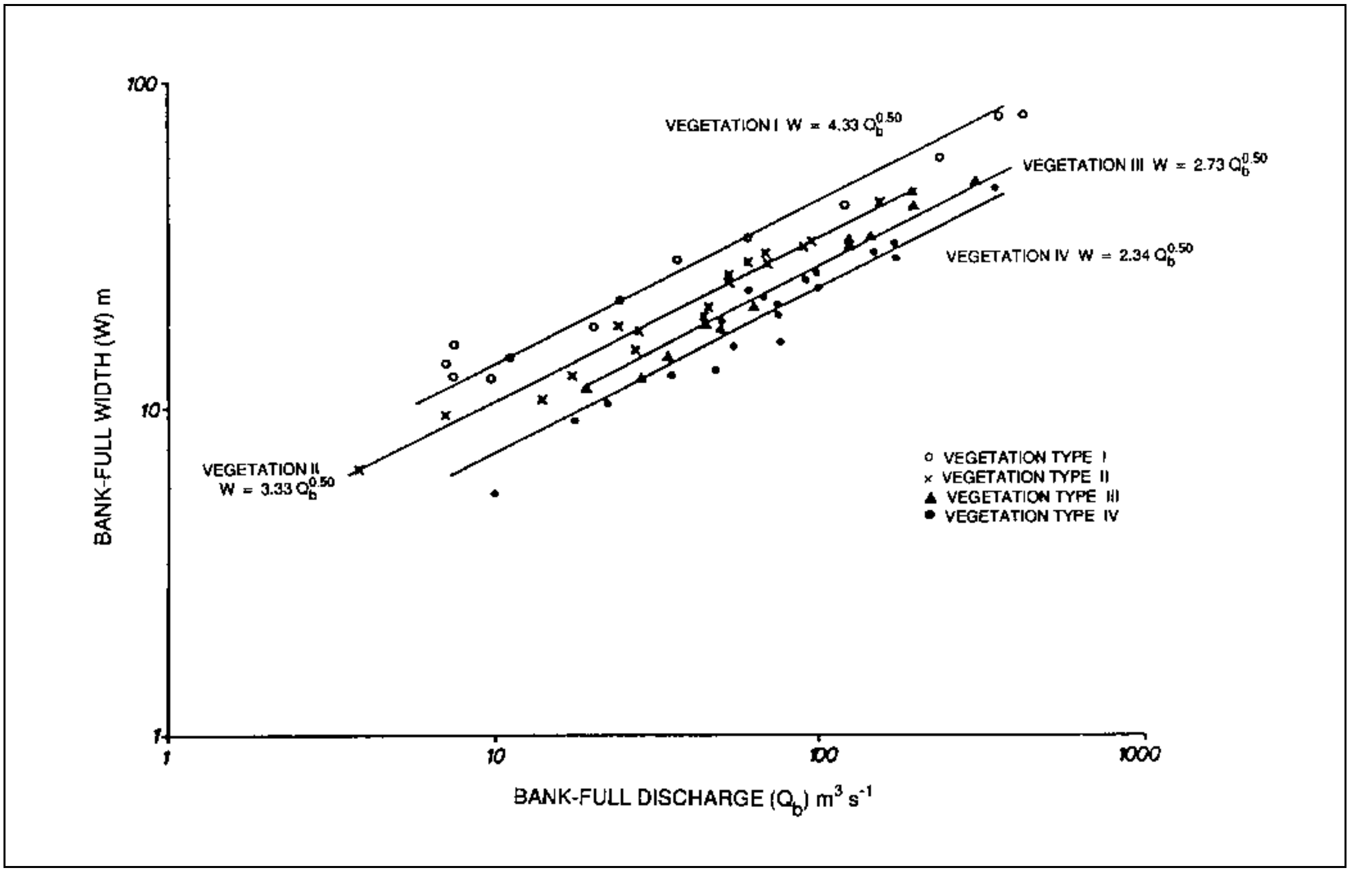

Figure B-2. Width-discharge plot by Hey and Thorne (1986), using bank vegetation as basis of discrimination; courtesy of American Society of Civil Engineers

(3) Manning formula

$$V = \frac{1.486}{n} R^{2/3} S^{1/2} \qquad \text{(B-6)}$$

where n is Manning's roughness and R is hydraulic radius. Assume roughness $k = 3D$, and $n = k^{1/6}/32$ where k is in feet, then Equation B-6 transforms to

$$V = 40 \frac{R^{2/3} S^{1/2}}{D^{1/6}} \qquad \text{(B-7)}$$

With the further assumption that $R = 0.9d$, Equations B-4, B-5, and B-7 may be combined with the equation of continuity, $Q = bdV$, to yield beginning-of-movement slope in terms of grain size and discharge:

$$S = 0.854 \frac{D^{1.286}}{Q^{0.429}} \qquad \text{(B-8)}$$

c. The curve for medium sand is based on Lacey's formula for sandy alluvial canals

$$S = \frac{0.000547}{Q^{1/6}} \qquad \text{(B-9)}$$

but multiplied by 1.3 to accord better with data for flat-slope sand-bed rivers. The curve for fine sand is drawn to give slopes about 60 percent of those for medium sand. The curves for coarse sand and for 10-mm material are interpolated arbitrarily.

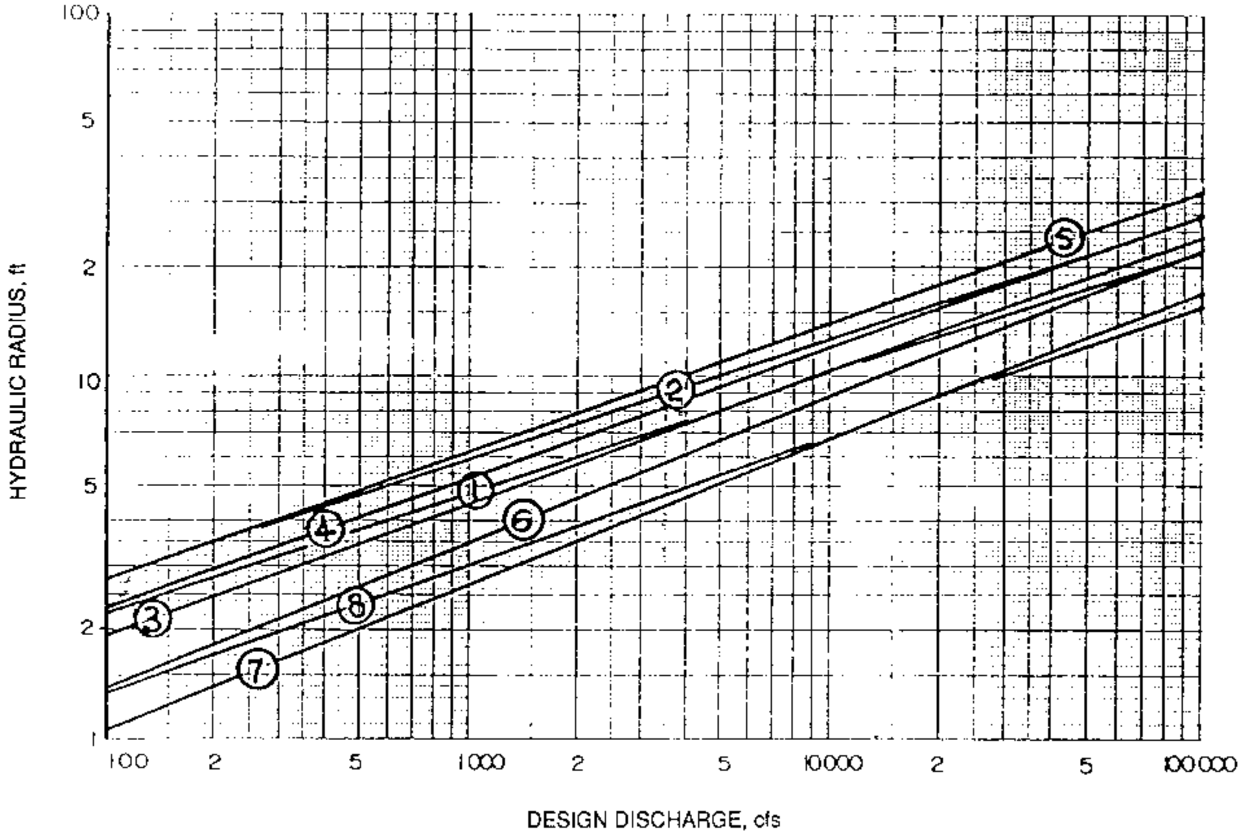

Figure B-3. Chart used as partial basis for Figure 5-10 (from Northwest Hydraulic Consultants 1982)