Chapter 8
Subsurface Runoff Analysis

8-1. General

a. Subsurface runoff analysis considers the movement of water throughout the entire hydrologic cycle (Figure 8-1). The prediction of subsurface runoff is performed with models of varying complexity depending on the application requirements and constraints. The models used may be categorized as event-oriented or continuous simulation. Event-oriented models, which have been the focus of the previous chapters, utilize relatively simple techniques for estimating subsurface contributions to a flood hydrograph.

b. Continuous simulation models continuously account for the movement of water throughout the hydrologic cycle. Continuous accounting of water movement involves the consideration of precipitation, snow melt, surface loss, infiltration, and surface transport processes that have been discussed previously. Other processes that need to be considered are evapotranspiration, soil moisture redistribution, and groundwater transport. The integration of all these processes in a watershed model is usually termed as continuous soil moisture accounting (SMA). The complexity of the SMA model varies greatly depending on the degree of conceptualization employed in integrating the subsurface processes.

c. Historically, the representation of soil moisture redistribution and subsurface flows has been highly conceptualized in SMA algorithms by an interconnected system of storages. More recently, a more distributed or smaller scale representation has been attempted (e.g., the Systeme Hydrologique European SHE model, Abbott, et al. 1986). These models represent overland and subsurface flow with finite difference approximations to the St. Venant and Darcy equations. However, these techniques have not yet been widely applied and will not be covered in this manual. Instead, the focus will be on the more highly conceptualized representations of soil moisture redistribution and subsurface flow.

d. The purpose of this section is to discuss separately the continuous simulation and event oriented approaches to calculating subsurface flow. A topic important to both approaches is hydrograph recession analysis. The methods used for event-oriented modeling will be discussed initially in paragraph 8-2 because recession analysis is key to this approach.

e. The continuous simulation approach involves algorithms that consider a number of processes besides hydrograph recession. Evapotranspiration (ET) is a key element in performing continuous simulation. In paragraph 8-3, a separate discussion is provided on ET because the estimation methods vary greatly. In paragraph 8-4, a general discussion is provided of the approaches used in performing continuous simulation. In paragraph 8-5, the continuous simulation algorithms used in public domain models PRMS (U.S. Geological Survey (USGS) 1983) and SSARR (USACE 1987) are presented for example purposes. A general discussion of the techniques that might be used to estimate parameters in continuous simulation models is provided in paragraph 8-6.

8-2. Event-Oriented Methods

a. Basic model for hydrograph recession modeling. Event-oriented models do not have the capability to account for the subsurface water balance. Since the water balance is not known, these models use an empirical approach to relate model parameters to the recession characteristics of an observed hydrograph. Presumably, the recession of the hydrograph is dominated by subsurface response at the point where direct runoff from the surface and near surface ceases. The problem is identifying the point at which the direct runoff ceases.

(1) The separation of the hydrograph into direct runoff and subsurface response is termed base-flow separation. Base-flow separation methods assume a very simple model for the watershed geometry (Figure 8-2). The watershed response is assumed to be a sum of direct runoff and base flow due to aquifer discharge. The key assumption is that the aquifer is homogenous with a single characteristic response. This characteristic response should be identifiable from the hydrograph recession.

(2) The assumed characteristics of the base-flow recession are based on simplified equations for flow in a phreatic aquifer. The equations are obtained (Bear 1979) by applying the Dupuit-Forcheimer assumptions to a combination of Darcy's Law and conservation of mass which is known as the Boussinesq equation. These assumptions require the approximation that flow in the aquifer is essentially horizontal.

(3) The Boussinesq equation relates the spatial change in the square of the phreatic water surface elevation in space to its change in time. Interestingly, the Boussinesq equation results in no approximation in calculated aquifer discharge to a stream, despite the assumption

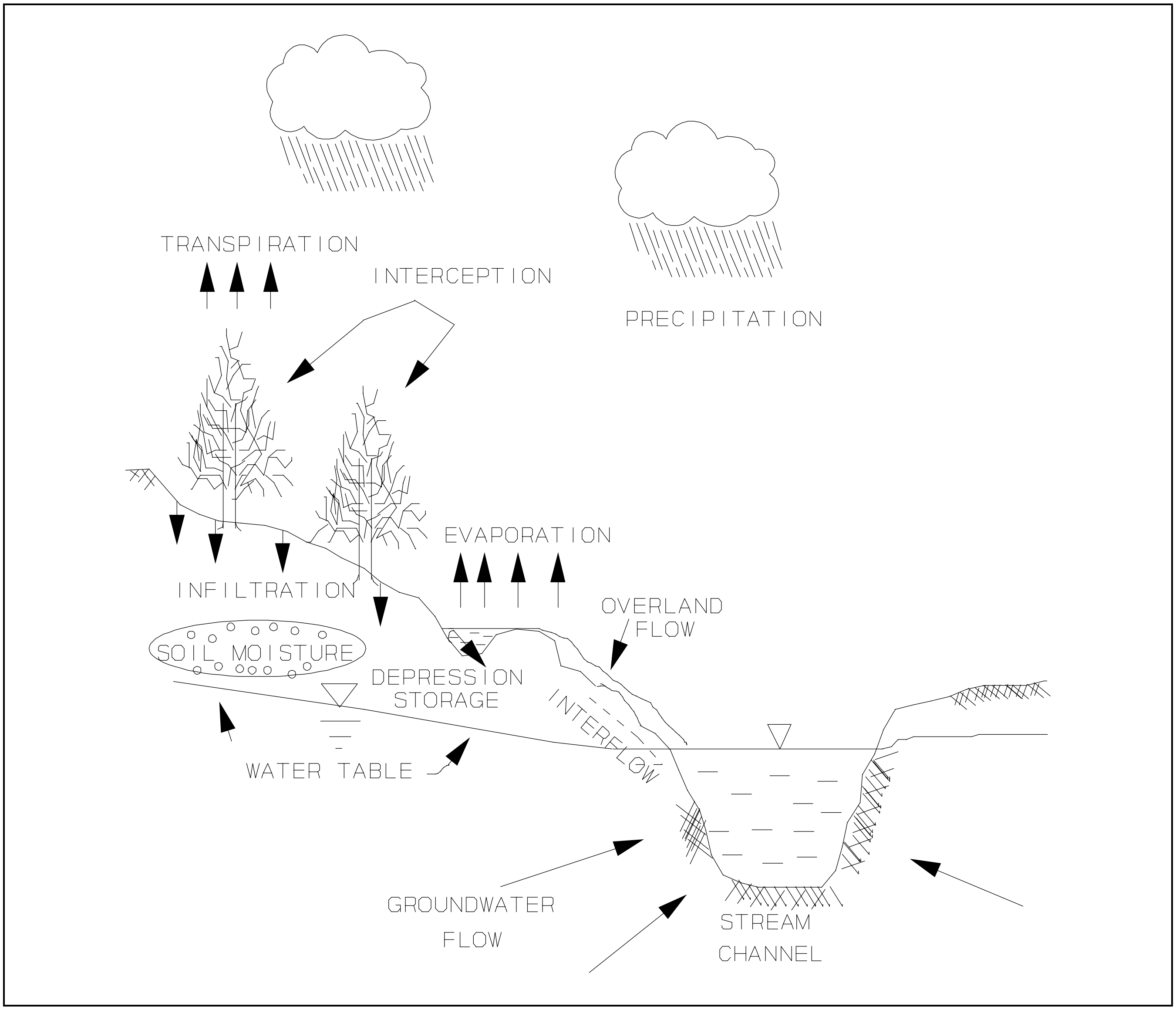

Figure 8-1. The hydrologic cycle

of horizontal flow. However, the equation does not preserve the description of the phreatic surface of the aquifer.

(4) Linearization of the Boussinesq equation for one-dimensional (1-D) flow results in the following differential equation for aquifer discharge (Figure 8-2):

$$T\frac{\partial^2 h}{\partial x^2} = S\frac{\partial h}{\partial t} \tag{8-1}$$

where

T = average aquifer transmissivity

h = phreatic surface or water table height from an arbitrary datum in the aquifer as a function of position x

S = aquifer storativity

t = time

Solution of this equation for the recession portion of the base flow or hydrograph or equivalently for a falling phreatic surface in an aquifer is of the form:

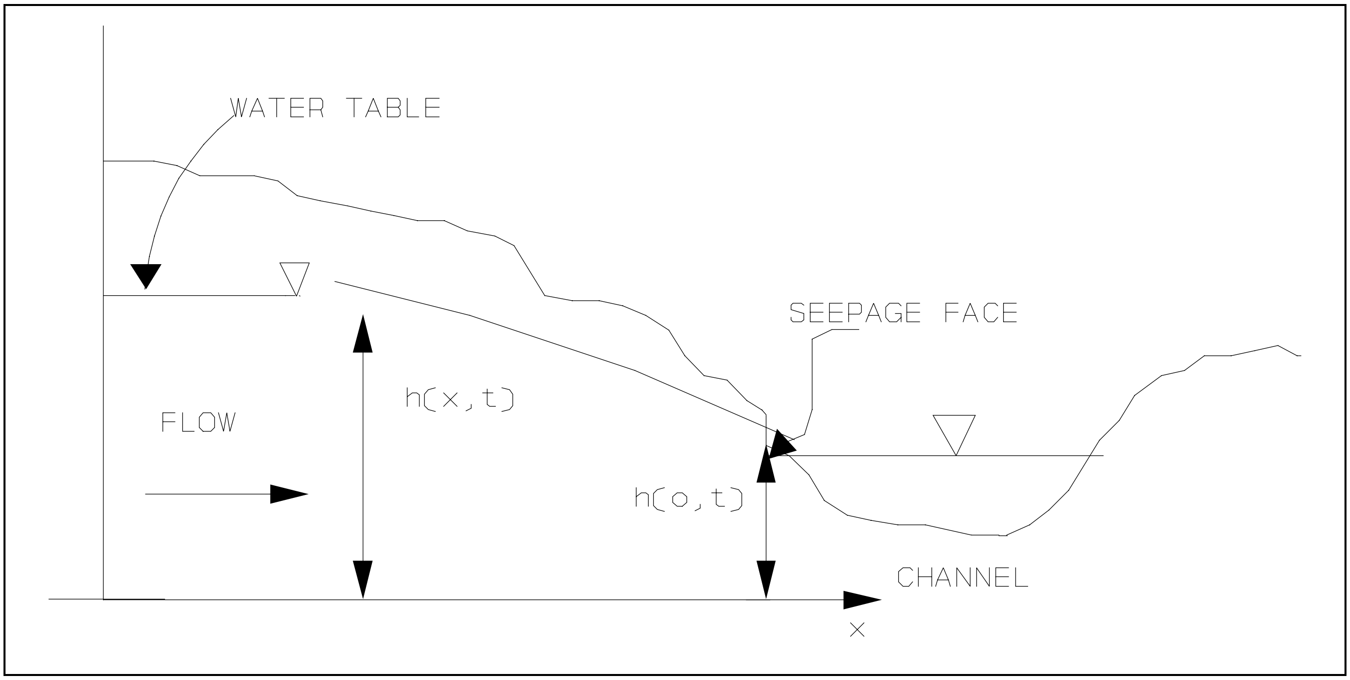

Figure 8-2. Simple groundwater model for recession analysis

$$h(0,t) \sim Ce^{-\alpha t} \tag{8-2}$$

where

$h(0,t)$ = height of the aquifer phreatic surface at the stream interface

C = constant depending on x, aquifer geometry and initial position of the phreatic surface

α = (T/S)

Since the groundwater discharge is proportional to the slope of the phreatic surface given the Dupuit-Forcheimer assumptions, the aquifer discharge or base-flow recession will also decay exponentially. Note that the decrease in flow with time or the recession is proportional to an exponential decay.

(5) The expected exponential decay in discharge is used to identify the point at which the base-flow recession begins. The standard technique is to plot log Q versus time and to determine the point at which the recession becomes a straight line.

b. Application of base-flow separation techniques. Base-flow recession analysis characterizes only the recession limb of the base-flow hydrograph (Figure 8-3). Techniques for determining the rising limb of the base-flow hydrograph vary widely. Viessman et al. (1977) describes various approaches to this problem. As an example, the approach used in the HEC-1 watershed model (USACE 1990a) will be discussed in this section.

(1) The HEC-1 model provides means to include the effects of base flow on the streamflow hydrograph as a function of three input parameters, STRTQ, QRCSN, and RTIOR. The variable STRTQ represents the initial flow in the river. It is affected by the long-term contribution of groundwater releases in the absence of precipitation and is a function of antecedent conditions (e.g., the time between the storm being modeled and the last occurrence of precipitation). The variable QRCSN indicates the flow at which an exponential recession begins on the receding limb of the computed hydrograph. Recession of the starting flow and “falling limb” follow a user specified exponential decay rate, RTIOR, which is assumed to be a characteristic of the basin. RTIOR is equal to the ratio of recession limb flow to the recession limb flow occurring 1 hr later. The program computes the recession flow Q as:

$$Q = Q_O(RTIOR)^{-n\Delta t} \tag{8-3}$$

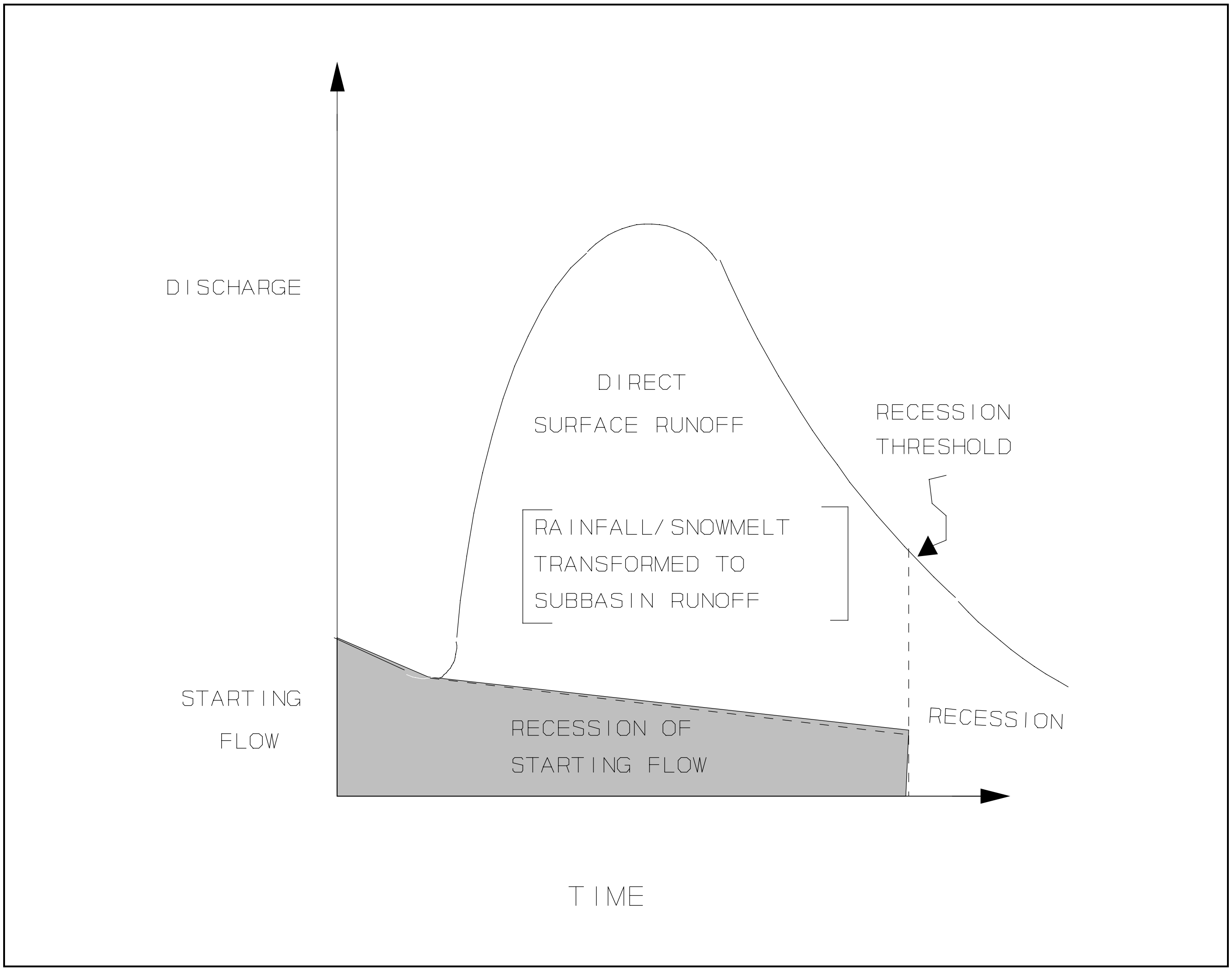

Figure 8-3. Base-flow separation diagram

where

Q_o = STRTQ or QRCSN

$n\Delta t$ = time in hours since recession was initiated

QRCSN and RTIOR can be obtained by plotting the log of observed flows versus time. The point at which the recession limb fits a straight line defines QRCSN and the slope of the straight line is used to define RTIOR. Alternatively, QRCSN can be specified as a ratio of the peak flow. For example, the user can specify that the exponential recession is to begin when the "falling limb" discharge drops to 0.1 of the calculated peak discharge.

(2) The rising limb of the streamflow hydrograph is adjusted for base flow by adding the recessed starting flow to the computed direct runoff flows. The falling limb is determined in the same manner until the computed flow is determined to be less than QRCSN. At this point, the time at which the value of QRCSN is reached is estimated from the computed hydrograph. From this time on, the streamflow hydrograph is computed using the recession equation unless the computed flow rises above the base-flow recession. This is the case of a double-peaked streamflow hydrograph where a rising limb of the second peak is computed by combining the starting flow recessed from the beginning of the simulation and the direct runoff.

(3) The values for these parameters can be established by regionalizing results from gauged basins. As an example, consider the attempts to determine base-flow

parameters for the Upper Hudson and Mohawk Rivers in New York.

(4) The starting flow, STRTQ, can be determined by plotting the initial streamflow observed prior to events versus drainage area (Figure 8-4). The recession-flow parameters were determined for each event by means of plotting the recession discharge versus time on semilog paper (Figure 8-5). QRCSN is the value of the discharge where the recession begins to plot as a straight line and RTIOR is related to the slope of this straight line. Figure 8-5 is not representative of all efforts to determine the recession parameters. In a significant number of instances, a straight line was not easily detectable on the semilog plot. Note that this study was performed with an older version of HEC-1 where RTIOR is defined as the ratio of the flow to that observed 10 time periods later rather than 1 time period later as defined in the current model.

(5) The results of the analysis indicated that RTIOR varied between 1.1 and 1.7 for the gauges and events examined. Since this range of values does not have a large affect on the recession limb, an average value of 1.3 was assumed for all subbasins. As in the case of STRTQ, QRCSN was graphically related to drainage area as shown in Figure 8-6.

8-3. Evapotranspiration

a. Introduction. The fundamental water balance relationship that a continuous simulation model must satisfy to accurately represent the hydrologic cycle is:

runoff = precipitation - evapotranspiration

Consequently, estimating ET is of major importance. This section is dedicated to describing the theory and application equations used to estimate ET in continuous simulation models.

b. Basis for computation of evapotranspiration. As in the case of infiltration, a well developed evapotranspiration (ET) theory exists for ideal conditions, i.e., conditions where the properties of the soil and the vegetative cover are well defined. However, the theory, as in the case of infiltration, is rarely implemented in a watershed model because the actual field situation deviates significantly from the ideal conditions assumed in the theory. Instead, the theory is used as a basis to develop many parametric methods that attempt to capture the essence of the evapotranspiration process.

(1) The following development is for calculating *potential* evapotranspiration (PET). PET is an estimate of the maximum amount of ET that may occur given available water. For an open water body, PET and ET are equivalent since the water supply is not limiting. Water supply is limiting in applications to bare ground or vegetative cover because available soil moisture, conductivity of the soil profile, and/or plant resistance may be limiting. Consequently, PET and ET are not equivalent in soil moisture accounting algorithms.

(2) The various parametric equations used to calculate PET have similarities that can be recognized from a rudimentary understanding of evapotranspiration theory. Consequently, the purpose of this section is to describe evapotranspiration theory so that the relationship between the parametric methods used can be related via an overall knowledge of the factors that affect ET.

(3) Evaporation theory is most easily developed by considering evaporation from a water surface and then extending these concepts to plant transpiration and evaporation from bare surfaces. Diffusion and energy budget methods have both been used to compute evaporation from a water surface. The diffusion method examines the transfer of water between water and gaseous states. Water, in a closed system, will evaporate from the water surface until the water vapor pressure above the surface reaches the saturation value. At this point, an equilibrium exists between liquid and gaseous phases of water.

(4) Practically speaking, equilibrium is not attained in the field because the atmosphere is unbounded and wind plays a major role in convecting moist air away from the water surface. The diffusion approach models this situation by assuming that a thin film of saturated air above the water surface is evaporated by convection from the wind. The rate at which wind convects water vapor from the water surface (the evaporation rate) is determined based on thermodynamic and aerodynamic principles to be proportional to:

$$E = bu\,(e_s - e) \qquad (8\text{-}4)$$

where

E = evaporation rate

b = proportionality constant

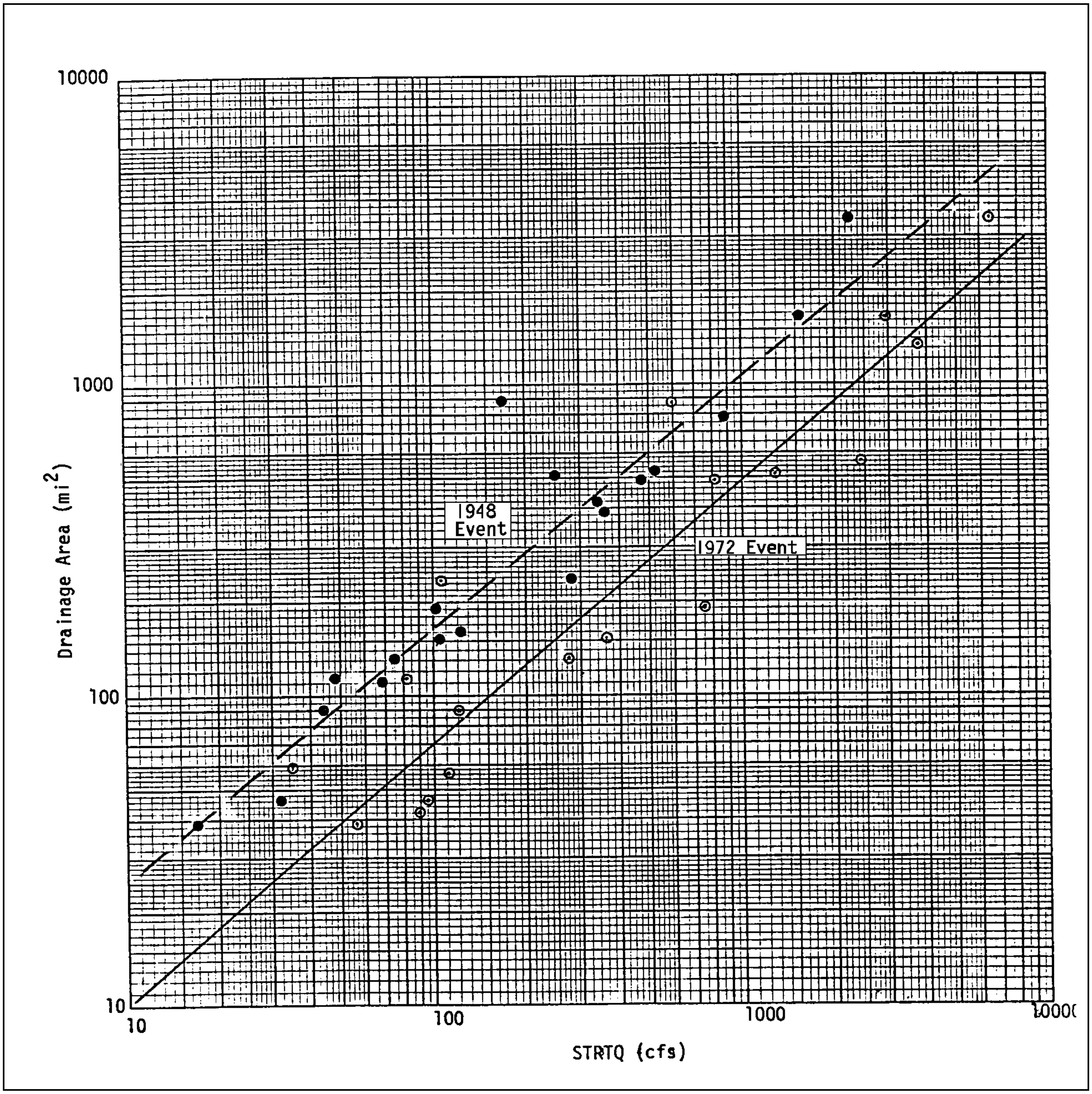

Figure 8-4. Initial flow versus drainage area Mohawk and Upper Hudson River

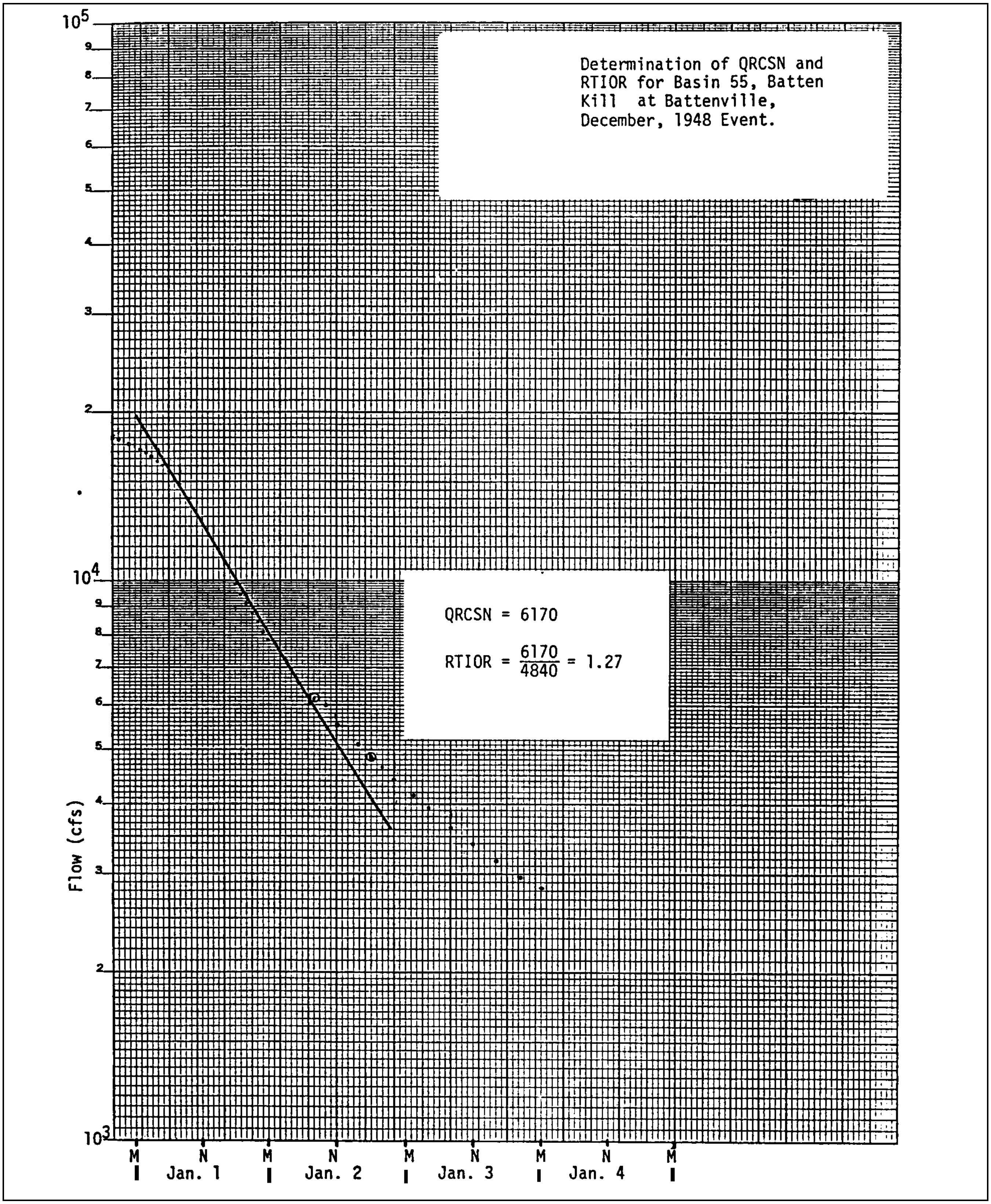

Figure 8-5. Determination of QRCSN and RTIOR for Basin 55, Batten Kill at Battenville, NY, December, 1948 Event

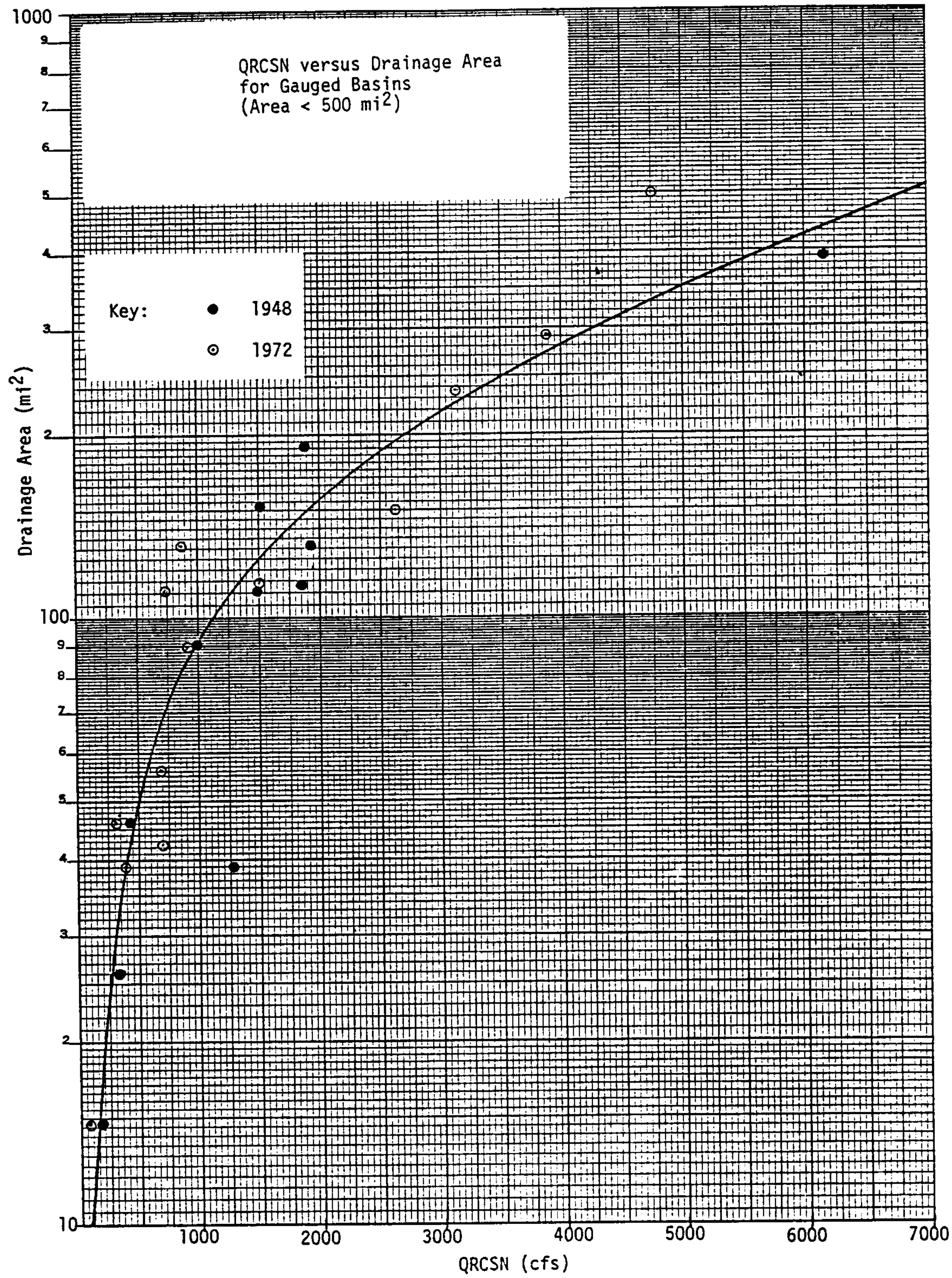

Figure 8-6. QRCSN versus drainage area for gauged basins, Upper Hudson and Mohawk Basin

e_s = water saturation vaporization pressure

e = vapor pressure at the elevation at which u, the wind speed is measured

The diffusion approach is not general because evaporation occurs in the absence of wind. Consequently, the method is modified to account for this possibility by adding a constant so that the evaporation rate is determined by:

$$E = (a + bu)(e_s - e) \tag{8-5}$$

where a and b are determined empirically from field data.

(5) An alternative approach to computing evaporation is the energy budget approach which computes the rate of increase of energy storage within the body, Q_s as:

$$Q_s = Q_i + Q_a - Q_r - Q_b - Q_e - Q_h \tag{8-6}$$

where the sources and sinks of heat are due to Q_i the incoming shortwave radiation from the sun, Q_a is the sum of all other sources of heat (due to seepage, rainfall, or other water inflows), Q_r reflected shortwave solar radiation, Q_b outgoing long wave radiation due to the “black body affects,” and Q_e is the energy utilized in evaporation (latent heat), and Q_h is the conducted and convected heat. This expression can be used to calculate evaporation rate by utilizing the Bowen ratio:

$$R = \frac{Q_h}{Q_e} \tag{8-7}$$

and relating the energy used in evaporation to the evaporation rate as:

$$Q_e = \rho_e L_e E A_s \tag{8-8}$$

where

ρ_e = density of evaporated water

L_e = the latent heat of vaporization

A_s = surface area of the water body

Substituting Equations 8-7 and 8-8 into Equation 8-6, the evaporation rate is computed as:

$$E = \frac{(Q_i - Q_r) - Q_b + (Q_a - Q_s)}{\rho_e L_e A_s (l + R)} \tag{8-9}$$

Application of this equation requires that some measurement of incoming solar radiation is available to estimate Q_i and Q_r; and the temperature of the water body and all other inflows of water be known so that the other heat terms can be computed.

(6) Penman (1948) combined the best features of both the diffusion and energy budget methods to obtain an expression similar to Equation 8-5, except that the coefficients a and b are calculable if data are available on temperature of the water body and net incoming solar radiation.

(7) Modification of methods for calculating evaporation from water surfaces to vegetative surface requires the concept of potential evapotranspiration. Unlike water bodies, water contents available in the soil via plants or bare surfaces may not be sufficient to support the capacity of the atmosphere to retain water. In this case, methods have been developed to compute the potential evapotranspiration, i.e., the evaporation that would occur if there were sufficient moisture.

(8) The Penman method was modified by Monteith (1965) to compute potential evapotranspiration. This required that a concept known as diffusion resistance (a resistance to evaporation) be incorporated into the Penman equation. The resistance to evaporation is divided into components due to atmospheric effects and plant effects. The atmospheric effects are, at least theoretically, calculable from thermodynamic and aerodynamic principles. However, the plant effects due to the resistance to moisture flux through plant leaves and the soil must be determined empirically.

(9) In summary, the calculation of potential evapotranspiration is based on the theory of evaporation from water surfaces. A significant amount of data on wind speed, net influx of solar radiation, temperature, and empirical information is needed for this calculation.

c. Empirical approaches to calculation potential evapotranspiration. Numerous empirical approaches for calculating PET exist. Most basic texts on hydrology summarize available methods (e.g., Viessman et al. 1977). The difficulty with most of these methods (and with calculations of ET in general) is that their basis is for open water bodies rather than land surfaces with vegetative cover.

(1) In this section, the empirical methods used by several continuous simulation models (PRMS, USGS 1983 and SSARR, USACE 1987) are described. PRMS allows the option of using pan evaporation, temperature, or energy-budget methods. The pan evaporation method, probably the most common and popular method for calculating PET, is estimated as:

$$PET = EPAN\ (EVC\,(MO)) \tag{8-10}$$

where

$EPAN$ = daily evaporation loss

EVC = empirical pan coefficient, less than 1.0, that varies monthly

The pan coefficient is intended to account for the differences between the thermodynamics of the pan and the prototype (e.g., a reservoir or catchment).

The temperature method by Hamon (1961) calculates PET as:

$$PET = CTS\,(MO)\ (DYL^2)\,(VDSAT) \tag{8-11}$$

where

CTS = empirical coefficient that varies monthly

DYL = possible hours of sunshine in units of 12 hours

$VDSAT$ = saturated water vapor density at the daily mean temperature in grams per cubic meter

PET = inches per day

VDSAT is computed as (Federer and Lash 1978):

$$VDSAT = 216.7\ \frac{VPSAT}{(TAVC + 273.3)} \tag{8-12}$$

where

$TAVC$ = mean daily temperature, in degrees Celsius

$VPSAT$ = saturated vapor pressure in millibars at $TAVC$

VPSAT is calculated as:

$$VPSAT = 6.108 \left[\exp\left(17.26939\ \frac{TAVC}{(TAVC + 273.3)}\right)\right] \tag{8-13}$$

The energy budget approach by Jensen and Haise (1963) calculates *PET* by:

$$PET = CTS(MO)\,(TAVF - CTX)\,(RIN) \tag{8-14}$$

where

CTS = coefficient that varies monthly

$TAVF$ = mean daily temperature, in degrees Fahrenheit

RIN = daily solar radiation, in inches of evaporation

PET = inches per day

CTX = coefficient that is a function of humidity and watershed elevation

CTS is calculated as:

$$CTS = [C1 + 13.0(CH)]^{-1} \tag{8-15}$$

where

$C1$ = elevation correction factor

CH = humidity index

$C1$ is calculated as:

$$C1 = 68.0 - \left[3.6 \left(\frac{E1}{1000}\right)\right] \tag{8-16}$$

where $E1$ = median elevation of the watershed, in feet msl. CH is calculated as:

$$CH = \frac{50}{(e_2 - e_1)} \tag{8-17}$$

where e_2 and e_1 = saturation vapor pressure (mb) for respectively the mean maximum and minimum air temperatures for the warmest month of the year. CTX in Equation 8-14 is computed as:

$$CTX = 27.5 - 0.25(e_2 - e_1) - \left(\frac{E2}{1000}\right) \tag{8-18}$$

where $E2$ = mean elevation for a particular subbasin.

(2) The SSARR model provides the capability for the user to supply *PET* values or calculate a basic *PET* via the Thornthwaite (1954) method:

$$PET = 1.6b\,(10T/I)^a \tag{8-19}$$

where

T = mean monthly temperature

b = factor to correct for the difference in days between months

I = annual heat index

a = cubic function of I

PET = monthly value

I is the sum of the monthly heat indices:

$$I = (T/5)^{1.514} \tag{8-20}$$

SSARR converts the *PET* to a daily value and then provides the capability to adjust this value for snow covered ground, month of the year, elevation of a particular snow band, and for rainfall intensity (i.e., *PET* is reduced when it is raining).

(3) In summary, empirical *PET* methods may be based on pan evaporation, mean monthly temperature, or energy budget equations. The pan evaporation approach is probably most popular and is certainly simplest. A further discussion of the importance of *ET* estimation and the corresponding choice of method will be given in paragraph 8-6 on parameter estimation.

8-4. Continuous Simulation Approach to Subsurface Modeling

a. Fundamental processes. Continuous simulation models attempt to conceptually represent the subsurface dynamics of water flow. The subsurface flow dynamics can be separated into wetting and drying phases. In the wetting phase, a wetting front of infiltrated water heads downward toward the groundwater aquifer as rainfall or snowmelt falls on the watershed surface. The aquifers of interest in this case are termed phreatic in that the aquifer surface is defined by water at atmospheric pressure. In response to this influx of infiltrated water, the groundwater levels may rise, if the influx is great enough, and the rate of water discharging from the aquifer to the stream increases. Streamflow due to aquifer discharge is usually termed base flow. The aquifer may also discharge to deep percolation depending on the permeability of soils or bedrock underlying the aquifer.

(1) For the infiltration phase of this process, in Chapter 6, the Richards equation describes an infinitely deep soil profile on infiltration. The consideration of infiltration in this instance is complicated because of the transition between unsaturated flow in the finite thickness soil profile and the saturated aquifer flow.

(2) The dynamics of the drying phase are not symmetrical with that of the wetting phase because of the affects of evapotranspiration and soil hysteresis. Soil hysteresis occurs because the unsaturated hydraulic conductivity is not a unique function of water content. The usual explanation for this curious behavior is that soil

pores do not fill and drain in the same sequence. Evaporation also affects the drying front depending on the vegetative cover and depth of the root zone.

(3) At some point during the drying phase, the aquifer levels must decrease, and the base-flow discharge must also decrease. This decrease in flow, at least theoretically, can be identified by an exponential decay.

(4) The generally accepted method for calculating the flow in this system is to simultaneously solve Richards' equation and Darcy's Law for a phreatic aquifer. However, this is a rather numerically intense exercise and is rarely performed as part of a watershed analysis.

(5) As described in Chapter 6, the overall dynamics of the direct runoff process is rather complicated by a number of factors. An additional complicating factor that had not been mentioned previously is the heterogeneity of the groundwater aquifer. These heterogeneities make it difficult to identify the characteristics of the aquifer response, particularly the identification of the exponential decay of the base flow.

(6) In summary, the dynamics of the subsurface process are complex even for an ideal soil profile and aquifer. The dynamics may be modeled using a combination of Richards' equation and Darcy's Law. Practically speaking, this is rarely done in watershed modeling. The use of these methods becomes more difficult and impractical when subsurface heterogeneities are considered.

b. Conceptual models of subsurface flow. There are a multitude of conceptual models that are available to perform continuous moisture accounting. All of these models try to capture the dynamics of subsurface flow with simple storage elements. As a precursor to discussing any of these models, a useful introduction is to construct a generic model that demonstrates the conceptual nature of the soil-moisture accounting model. Consider a model that has only rainfall as an input (Figure 8-7). To begin with, the storages represent surface effects, unsaturated zone, and saturated zone or aquifer storages (all storage shown considers volume in terms of basin-depth, e.g., basin-inches). Consider each zone separately:

(1) Surface storage. The surface storage stores water up to a maximum value of SMAX. Water leaves either by evaporation at the potential rate ES, infiltration at a rate equal to FS or via an overflow once SMAX is exceeded. The overflow volume might be routed to the stream via the unit hydrograph method.

(2) Upper zone storage. The upper zone stores water up to a maximum value UMAX. Evaporation from the zone at the rate EU models the uptake due to vegetation. Water enters the storage at the rate FS and leaves either by evaporation, infiltration to the lower zone at rate LS, or to the stream via a low-level outlet. If the assumption is made that the upper zone is a linear storage, then the outflow rate is linearly proportional to the storage.

(3) Lower zone storage. The lower zone stores water up to a maximum value LMAX. Water enters the storage from the upper zone at the rate LS and leaves via a low-level outlet as in the upper zone case or out of the system at a deep percolation rate, FD. The computation of the outflow rates is based on the following functions:

- Potential evaporation: Compute as a coefficient times the pan evaporation amount.

- Potential infiltration: The infiltration from one zone to another is based on linearly varying function of the storage receiving flow:

$$FP = FMAX \left(1 - \frac{V}{VMAX}\right) \quad VS \leq VMAX \tag{8-21}$$

where *FMAX* is the maximum infiltration rate into a storage with capacity *VMAX* and current storage *V*.

- Low-level outlet: the subsurface storages will be considered linear reservoirs where the outlet discharge is computed as:

$$O = \frac{V}{K} \tag{8-22}$$

where

O = outflow

K = linear reservoir storage coefficient

Application of this model to soil moisture accounting and runoff prediction might be done based on the following outflow rule: evaporation takes precedence over infiltration which in turn takes precedence over outflow from a low-level outlet.

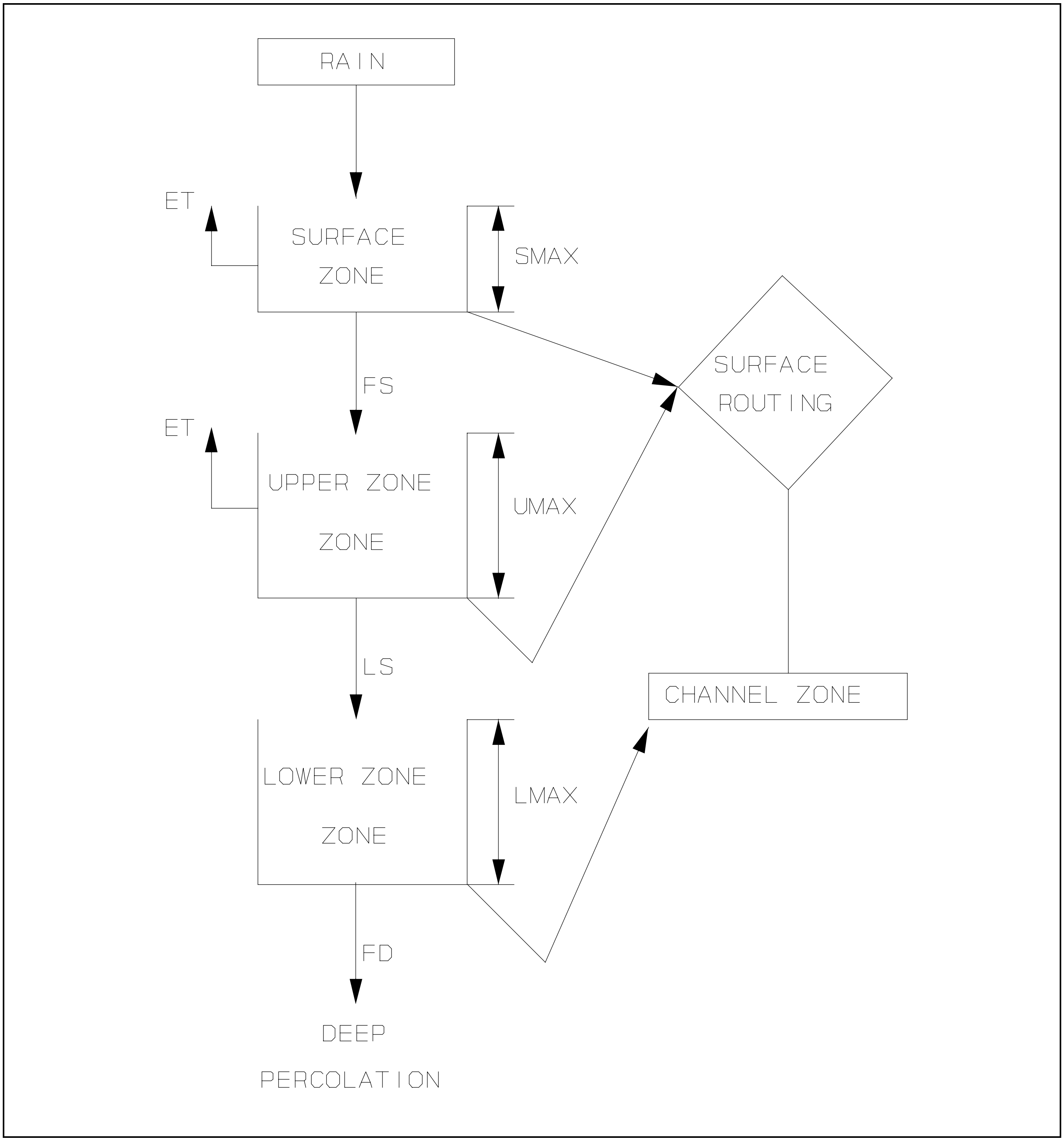

Figure 8-7. Simple example continuous simulation model

(4) Explicit solution algorithm. An explicit solution algorithm would proceed as follows given this rule for the period of duration Δt, or equivalently, between times t_i and t_{i+1}:

(a) Surface zone. Compute the available surface supply VS as:

$$VS = SZ_1 + R \tag{8-23}$$

where

SZ_i = storage at the beginning of the period

R = rainfall volume during the period

The volume left in storage after evaporation, VSE, is computed as:

$$VSE = VS\ ESP \qquad VS \geq ESP \tag{8-24}$$

or:

$$VSE = 0 \qquad VS < ESP \tag{8-25}$$

where the evaporated volume ES is lost up to the potential amount ESP if the surface storage is available. The computation of storage, VSF, after infiltration from the surface zone to the upper zone is computed in a similar manner to that of evaporation:

$$VSF = VSE - FUP \qquad VSE \geq FUP \tag{8-26}$$

$$FU = FVP \tag{8-27}$$

or:

$$FU = VSE \qquad VSE < FUP \tag{8-28}$$

$$VSF = 0 \tag{8-29}$$

where FU is the volume infiltrated to the upper zone up to the potential amount FUP if VSE is large enough. FUP can be calculated simply from the beginning of period storage in the upper zone, UZ_i. The storage at the end of the period, SZ_{i+1}, is computed as:

$$SZ_{i-1} = VSF \qquad VSF < SMAX \tag{8-30}$$

or:

$$SZ_{i-1} = SMAX \qquad VSF \geq SMAX \tag{8-31}$$

$$E = VSF - SMAX \tag{8-32}$$

where E is the excess available if the end of period storage exceeds the maximum amount SZM.

(b) Upper zone. The soil moisture accounting for the upper zone proceeds similarly to that of the surface zone except that outflow is routed based on the linear reservoir outflow relationship. The volume available for outflow, VU, is:

$$VU = UZ_i + FU \tag{8-33}$$

where UZ_i is the beginning of period storage. The volume left after evaporation, VUE, is computed as:

$$VUE = VU - EUP \qquad VU \geq EUP \tag{8-34}$$

$$EU = VUE \tag{8-35}$$

or:

$$VUE = 0 \qquad VU < EUP \tag{8-36}$$

$$EU = VU \tag{8-37}$$

where EU is the volume evaporated up to the potential amount EUP if the storage is available. The volume remaining, VUF, after infiltration from the upper zone to the lower zone is computed as:

$$VUF = VUE - FLP \qquad VUE \geq FLP \tag{8-38}$$

$$FL = FLP \tag{8-39}$$

or:

$$VUF = 0 \qquad VUE < FLP \tag{8-40}$$

$$FL = VUE \tag{8-41}$$

where FL is the volume infiltrated to the lower zone up to the potential amount FLP if the storage is available. The remaining volume is routed through the linear storage by continuity considerations:

$$OU_i + 1 = \frac{FU - FL - EU + OU_i(Ku + 0.5\Delta t)}{(Ku - 0.5\Delta t)} \tag{8-42}$$

$$UZ_{i-1} = ku(OU_{i-1}) \tag{8-43}$$

where

OU_i and OU_{i+1} = respectively the flows at the beginning and end of the period

UZ_{i+1} = storage at the end of the period

ku = linear reservoir coefficient

(c) Lower zone. The lower zone routing is similar to that of the upper zone except that no evaporation is computed. The volume available for routing through the low level outlet, VL, is simply the increase due to infiltration from the upper zone minus the constant loss due to percolation:

$$VL = LZ_i + FL - FDP \qquad (LZ_i + FL) \geq FDP \tag{8-44}$$

$$FD = FDP \tag{8-45}$$

or:

$$FD = LZ_i + FL \qquad (FL + LZ_i) < FDP \tag{8-46}$$

$$FD = 0 \tag{8-47}$$

where LZ_i is the storage at the beginning of the period, the loss due to percolation, FD may be a maximum amount up to the potential percolation loss FDP for the period. The outflow from the storage is computed as:

$$OL_i + 1 = \frac{FL - FD - OL_i(Kl + 0.5\Delta t)}{(Kl - 0.5\Delta t)} \tag{8-48}$$

where

OL_i and OL_{i+1} = outflows at the beginning and end of periods, respectively

kl = linear reservoir storage coefficient

(5) Noteworthy aspects. There are two noteworthy aspects of this model. First, the number of parameters needed is significantly larger than needed for an event oriented model:

(a) Evaporation: The adjustment of pan evaporation values will require at least seasonal coefficients which means *four* coefficients that need to be estimated.

(b) Surface zone: Parameters needed are SZM, and unit hydrograph parameters such as Clark, TC, and R, and the surface storage at the beginning of the simulation SZ_0, total *three* parameters and *one* initial condition.

(c) Upper zone: Parameters needed are UZM, FUM to calculate FUP, KU, and the initial storage UZ_0, total *three* parameters and *one* initial condition.

(d) Lower zone: Parameters needed are SZM, FLM to calculate FLP, KL, FDP, and the initial storage SZ_0, total *four* parameters and *one* initial condition.

(6) Parameter estimates. Summing these totals, the number of parameter estimates needed are *fourteen* with *three* initial conditions. This poses a significant

estimation problem for soil moisture accounting models. Furthermore, the generic model formulation ignored the problems of surface interception (water that would be stored but not free for outflow or infiltration), snowmelt and snow excess infiltration, partial area or hillslope effects, and the routing of base flow through more than a linear reservoir. If these processes were included in the model, then there would be a significant increase in the number of parameters that need to be estimated.

(7) Explicit simulation scheme. A second noteworthy aspect of the generic model is the explicit simulation scheme. The explicit simulation scheme can result in a poor simulation if the selected simulation interval, Δt, is not appropriately small. For example, computation of the infiltration loss from one zone to another is dependent on the beginning of period storage. If the storage changes greatly over the computation period, then the infiltration rate computed base on beginning of period storages will be a poor estimate of the average rate that would occur over the period. Consequently, a computation interval that is sufficiently small is needed for accurate numerical simulation with the model.

c. Summary. In summary, the purpose of this section was to introduce the concept of soil moisture accounting via a description of a simple model. Even though the model is simple, the number of parameters that must be estimated easily exceeds the number needed for event oriented estimation. The number of parameters that must be estimated poses some very significant parameter estimation problems.

8-5. Existing Continuous Simulation Models

a. Introduction. There are many different continuous simulation models available which employ different soil moisture accounting algorithms. As examples of soil moisture accounting techniques, two models in the public domain, PRMS (USGS 1983), and SSARR (USACE 1987) will be described.

b. PRMS. The Precipitation-Runoff Modeling System (USGS 1983), PRMS, soil moisture accounting algorithm is summarized in Figure 8-8. The model components represent the following watershed characteristics:

(1) Interception. Interception by vegetation is modeled as a seasonally varying process for a fraction of the basin. The fraction of the basin that has interception loss can be specified for winter and summer via parameter COVDN. The volume of water that can be stored by the vegetation, STOR, varies depending on the type of precipitation: winter snow, winter rain, or summer rain.

(2) Impervious area. This area represents the fraction of the basin that is impervious. Interception does not occur, but a surface loss, RETIP, can be specified.

(3) Snow pack. The snow pack is assumed to uniformly cover the entire basin. The assumption is made that it is a two-layer system, the surface layer being 3 to 5 in. thick. Melt water from the pack is proportioned between the pervious and impervious area based on the fraction of the area.

(4) Soil zone reservoir. This reservoir represents the active portion of the soil profile in that soil moisture redistribution is modeled. The capacity of this zone, SMAX, is defined as the difference between the field capacity and wilting point (field capacity is a loosely defined concept being generally defined as the water content of the soil after gravity drainage for some extended period from near saturation; the wilting point defines the water content at which plants can no longer extract moisture from the soil). The zone is divided into a recharge zone, capacity REMAX, and lower zone with capacity LZMX (necessarily the difference between SMAX and REMAX). The recharge zone must be full before water can move to a lower zone.

(5) Subsurface zone. This zone represents the flow from the soil's unsaturated zone to the stream and groundwater reservoir. The outflow to stream is based on the relationship:

$$\frac{d(RES)}{dt} = (INFLOW) - 0_s \tag{8-49}$$

and

$$O_s = RCF(RES) + RCP(RES)^2 \tag{8-50}$$

where

RES = storage in the reservoir

O_s = outflow

RCF and RCP = routing parameters

The outflow to the groundwater zone is determined by:

$$O_g = (RSEP)\left(\frac{RES}{RESMX}\right)^{REXP} \tag{8-51}$$

where

O_g = flow to the groundwater zone

RESMX, *RSEP*,
and *REXP* = parameters to be specified

(6) Groundwater zone. This zone represents the storage in a phreatic aquifer and outflow to the stream and deep percolation. Outflow to the stream is based on a linear reservoir assumption, requiring the estimate of a storage coefficient, RCB. Outflow to deep percolation is computed by the product of a coefficient GSNK time the current storage in the zone. Model simulation occurs at a daily computation interval if any snowpack exists or at the minimum of 5 min or a user-specified value if a snow-free ground event is occurring. The procedure for routing precipitation through the system is performed as follows:

(a) Precipitation. The form of the precipitation is determined by either of two methods: a temperature BST is specified that together with maximum and minimum daily air temperatures is used to determine if rain, snow, or a mixture of both is the form of the precipitation; or, alternatively, a temperature PAT is specified that is the threshold for rain to snow formation.

(b) Surface interception. The daily potential evapotranspiration, EPT, is computed based on one of three methods: a pan coefficient method, a method that uses daily mean temperature and daily hours of sunshine, or a method that uses daily mean air temperature and solar radiation (see paragraph 8-3). Interception is computed for the open fraction of the subbasin. The EPT demand fraction for the open portion of the basin is satisfied, if possible, from the interception storage either as evapotranspiration or snow sublimation.

(c) Snowpack growth/melt. Snowpack simulation is performed at a daily time step. The snowpack growth/melt dynamics are based on a complex energy-balance approach. A detailed discussion of this algorithm is beyond the scope of this discussion. However, as described in the previous section on snowmelt, energy budget approaches are rather data intensive.

(d) Runoff available from impervious surface. Runoff from the impervious fraction is computed by consideration of the available excess, surface storage, and EPT. The surface storage is increased by the amount of the snowmelt/rainfall excess and depleted by evapotranspiration up to the maximum amount EPT. The remaining amount in excess of surface storage RETIP becomes runoff excess.

(e) Surface runoff - daily mode. A water balance is performed on the soil zone to determine the fraction of water that contributes to subsurface storages and open-area runoff. Inflow to the soil zone is treated differently for snowpack or bare ground. Snowpack infiltration is unlimited until field capacity is reached in the recharge zone. At field capacity, the infiltration rate is limited to a constant value SRX. Snowmelt excess, including rain on the snowpack, in excess of SRX contributes to surface runoff. Surface runoff due to rain on snow is computed using a contributing area principle as:

$$SRO = CAP(PTN) \tag{8-52}$$

where *CAP* is used to factor the available rain on snowmelt into surface runoff and infiltrating volumes and *PTN* is the daily precipitation. *CAP* may be determined via a linear or nonlinear function of antecedent moisture. The linear function is:

$$CAP = SCN + \left[(SCX - SCN)\left(\frac{RECHR}{REMX}\right)\right] \tag{8-53}$$

where

SCN and *SCX* = minimum and maximum contributing watershed area, respectively

RECHR and *REMX* = storage parameters defined previously for the soil moisture zone

The nonlinear function is:

$$CAP = SCN(10^{(SC1(SMIDX))}) \tag{8-54}$$

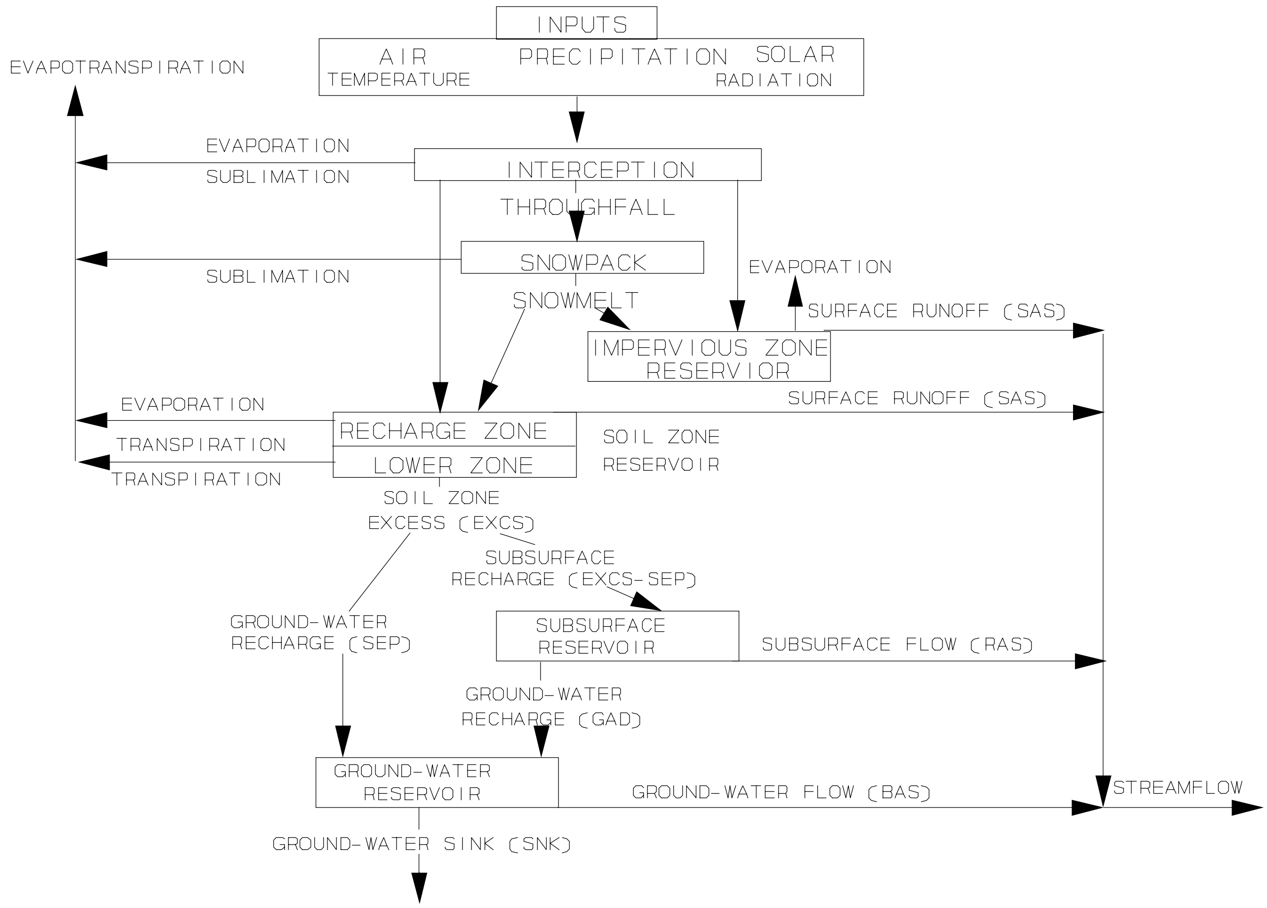

Figure 8-8. PRMS, schematic diagram of the conceptual watershed system and its inputs

where

SCN and $SC1$ = coefficients to be determined

$SMIDX$ = sum of the current available water in the soil zone (SMAV) plus one-half PTN

The coefficients of this method might be determined from soil moisture data, if available. If data are not available, then the user's manual suggests determining the coefficients from preliminary model runs. An example of the determining the coefficients for the nonlinear method as a function of an antecedent precipitation index is given in Figure 8-9. (The description in the users manual (USGS 1983) of how to establish this relationship from a preliminary model is not detailed and would seem to be very difficult).

(f) Surface runoff - event mode. Rainfall infiltration on snow-free ground is calculated from a potential infiltration rate adjusted for spatial differences in infiltration potential. The potential infiltration rate is based on a modified version of the Green and Ampt equation (Chapter 6). The modification involves multiplying the soil moisture deficit at field capacity by the product of the fraction of the storage available in the recharge zone and a user defined coefficient. The infiltration rate necessarily becomes zero when the recharge zone reaches maximum capacity. The spatial variation in infiltration properties is then accounted for as shown in Figure 8-10. Rainfall not infiltrated is then routed overland to the stream by the kinematic wave method. Infiltrated rainfall moves to the soil profile zone. Stored water is first lost to EPT that is not satisfied by surface interception from the recharge zone and then from the lower zone. In addition, water is lost from the lower zone to the groundwater zone up to a maximum rate SEP; and volume available in excess of this rate moves to the subsurface zone. Inflow from the soil zone to the groundwater and subsurface zones is routed to the stream by the equations described previously.

c. SSARR. The Streamflow Synthesis and Reservoir Regulation model (SSARR) performs continuous simulation of watershed runoff and reservoir operations. Watershed runoff simulation may be performed with either the "depletion curve" or the more general "snow band model." The more general snow band model will be discussed.

(1) Model simulation. Model simulations are performed at a user specified computation interval. Basin temperature and precipitation are input to the model as conceptualized in Figure 8-11. The model accumulates snow in different user defined elevation bands (thus the term snow-band model). The amount of snow accumulated depends on the elevation band temperature which is a function of the input temperature and elevation-temperature lapse rate. The soil moisture accounting aspect of the runoff algorithm is performed for each band. The accumulated runoff from the bands is then routed through conceptual storages to the outlet of the watershed.

(2) Differences. The model differs from PRMS, and most other conceptual continuous simulation models, in that the soil moisture accounting is **not** envisioned as an interconnected group of conceptual storages. Rather, the precipitation is routed through the system based on a set of empirical relationships, until the final routing to the basin outlet. The individual relationships are as follows:

(a) Interception. Interception is specified as total basin volume. Precipitation in excess of this amount reaches the ground surface. The intercepted volume is decreased to the potential evapotranspiration.

(b) Snowpack. The snowpack is assumed to be distributed uniformly over the watershed fraction represented by a particular elevation band.

(c) Soil moisture input zone. The soil moisture input zone accounts for the water balance in the water profile. This zone receives moisture input either from snowmelt or rainfall on bare ground. The amount of direct runoff, evapotranspiration, and percolation to the lower zone depends on an empirical index of the water content of this zone. The index ranges from a small percent representing the wilting point, to a value approaching 100 percent representing field capacity. At the wilting point there will be very little direct runoff, conversely, at field capacity, the direct runoff would approach 100 percent of available moisture. The soil moisture index varies based on the following relationship:

$$SMI_2 = SMI_1 + (MI - RGP) - \frac{PH(ETI)}{24} \qquad (8\text{-}55)$$

where

SMI_1 and SMI_2 = the soil moisture indexes at the beginning and end of a compute period, respectively

PH = compute period length, in hours

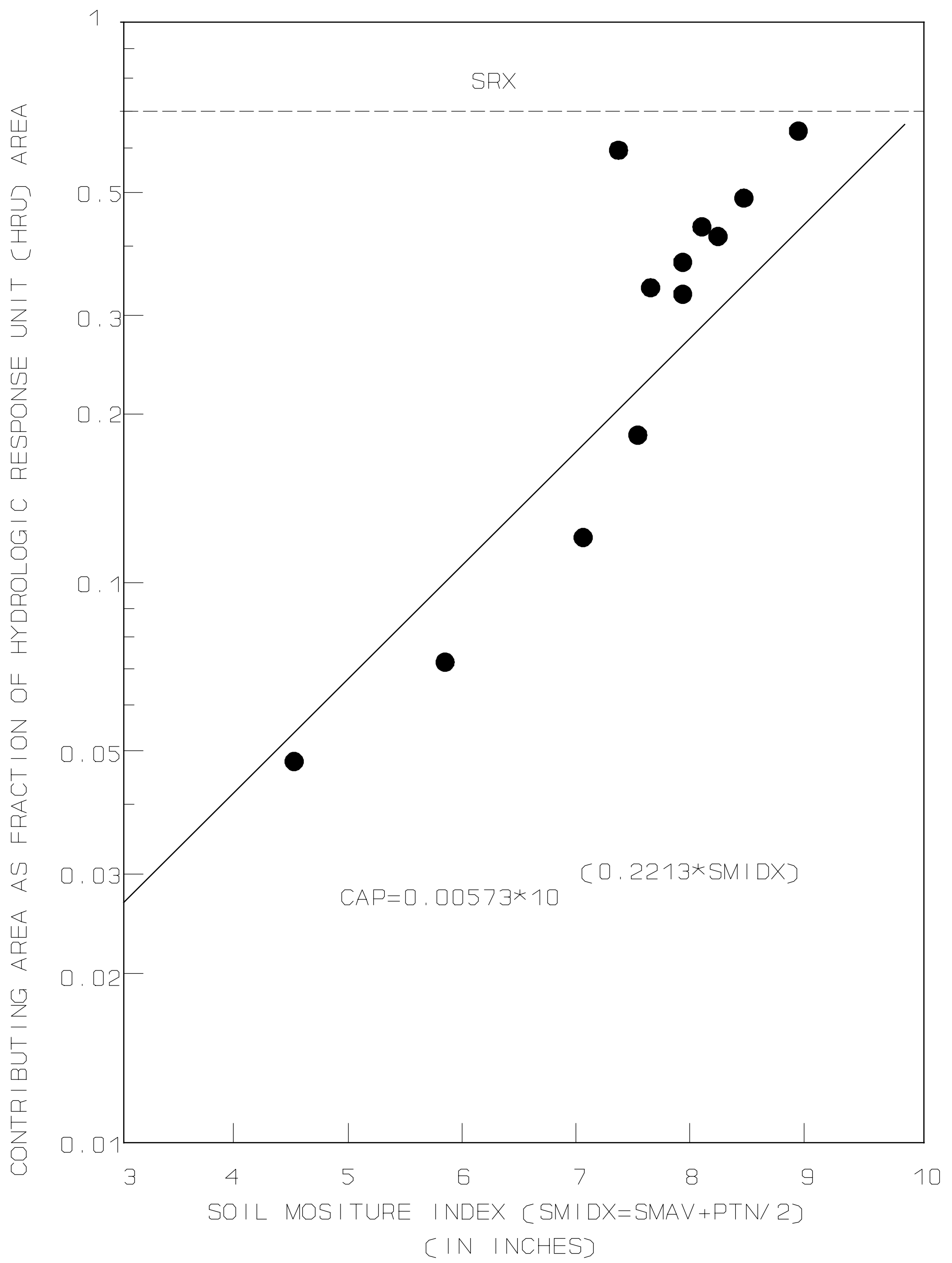

Figure 8-9. Sample PRMS partial area corrections. The relation between contributing area (CAP) and soil-moisture index (SMIDX) for Blue Creek, AL

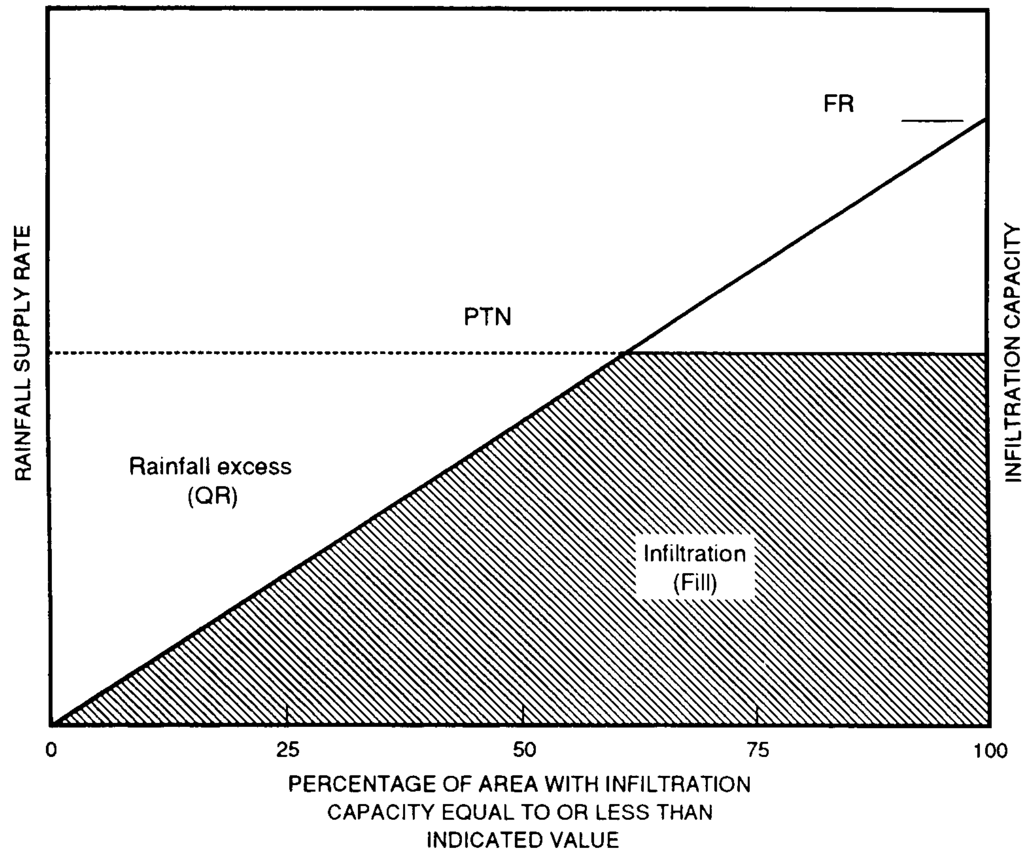

Figure 8-10. PRMS function which determines fraction of area contribution runoff due to variation in infiltration capacity

MI = available excess from snowmelt and rainfall

ETI = evapotranspiration index, in inches per day

PH = computation interval, in fractions of a day

RGP = computed surface runoff

A user estimated empirical relationship is used to calculate surface runoff from the soil moisture index. This empirical relationship may consider the intensity of the available moisture input to the zone (e.g., Figures 8-12 and 8-13). The rate of supply available for outflow is computed as:

$$RGP = ROP(MI) \tag{8-56}$$

where *ROP* = percent runoff.

(d) Base-flow separation. An empirical relation between a base-flow infiltration index and percent of runoff to base-flow is used to divide outflow from the soil moisture zone into direct runoff and base-flow (e.g., Figure 8-14). The base-flow infiltration index is computed as:

$$BII_2 = BII_1 + 24\left(\frac{RGP}{PH} - BII_1\right)\frac{PH}{BIITS + \frac{PH}{2}} \tag{8-57}$$

$$BII_2 \leq BIIMX$$

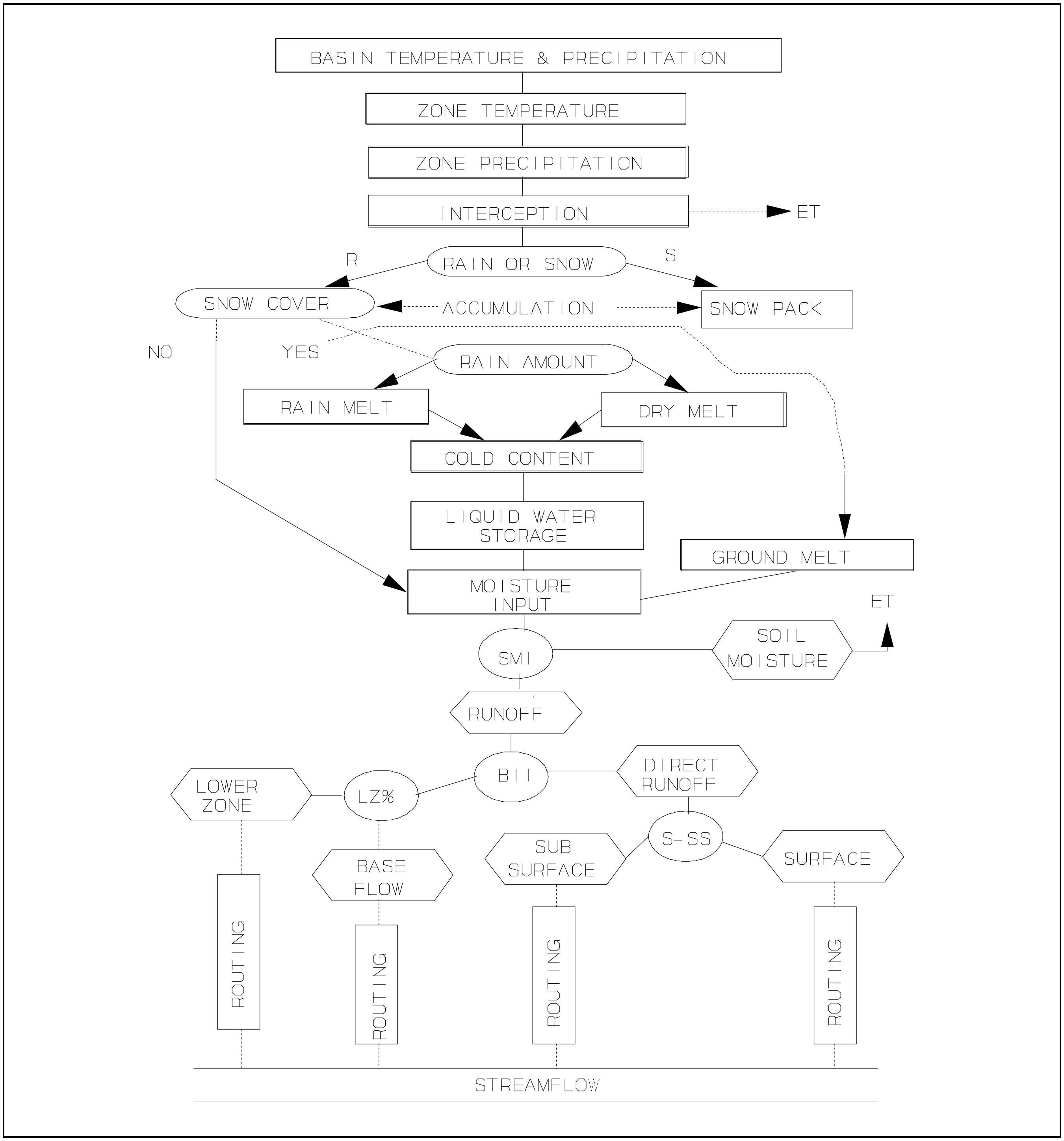

Figure 8-11. SSARR “snowbank” watershed model

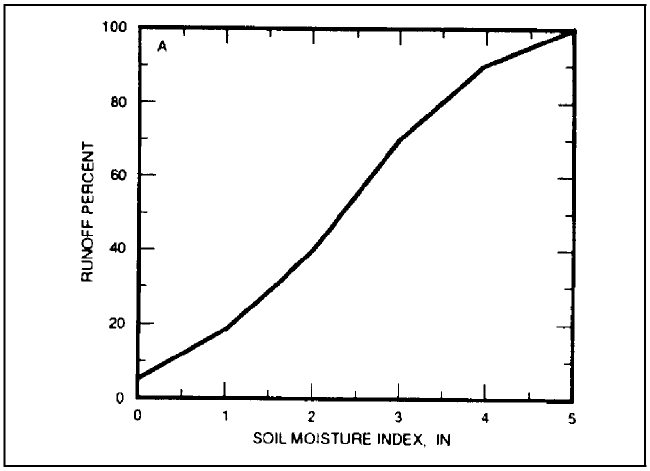

Figure 8-12. SSARR SMI versus runoff percent

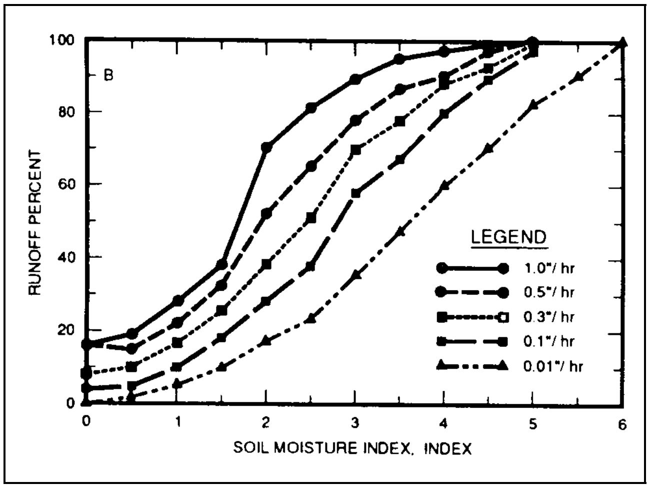

Figure 8-13. SSARR SMI versus precipitation intensity and runoff percent

or

$$BII_2 = BIIMX \qquad BII_2 > BIIMX \tag{8-58}$$

where

BII_1 and BII_2 = base-flow indexes at the beginning and ending of the computational period

$BIITS$ = time delay or time of storage

$BIIMX$ = limiting value for the index

The rate of inflow to the lower and base-flow zone is then computed as:

$$TBF = BFP\left(\frac{RGP}{PH}\right) \tag{8-59}$$

where BFP is determined from Figure 8-14 using BII.

(e) Lower zone versus base flow. The lower zone and base-flow components are separated based on a user-defined factor $PBLZ$:

$$LZ = TBF(PBLZ) \tag{8-60}$$

where LZ is the inflow rate to the lower zone, up to a value DGWLIM. The difference between LZ and TBF is the contribution to base flow.

(f) Direct runoff. The inflow to direct runoff is the difference between the outflow from the soil moisture zone and the inflow to the base-flow zone:

$$RGS = RG - TBF \tag{8-61}$$

Surface and subsurface runoff are distinguished by a user-specified empirical relationship (e.g., Figure 8-15). The SSARR user's manual provides guidelines for developing this relationship.

(g) Routing flows to outlet. Surface, subsurface, lower zone, and base flows are routed to the outlet via linear reservoir routing. The user may separately specify the number of linear storages for each outflow component.

8-6. Parameter Estimation for Continuous Simulation Models

a. Parameter estimation. Parameter estimation for continuous simulation models is much more difficult than for event-oriented models. The reason for this is that a continuous simulation model must represent the entire hydrologic cycle. This representation requires an increase in model complexity and, correspondingly, an increase in the number of parameters to be estimated. The parameter estimation process requires an extensive amount of data and user experience. A totally ungauged parameter estimation procedure is not practical or advisable.

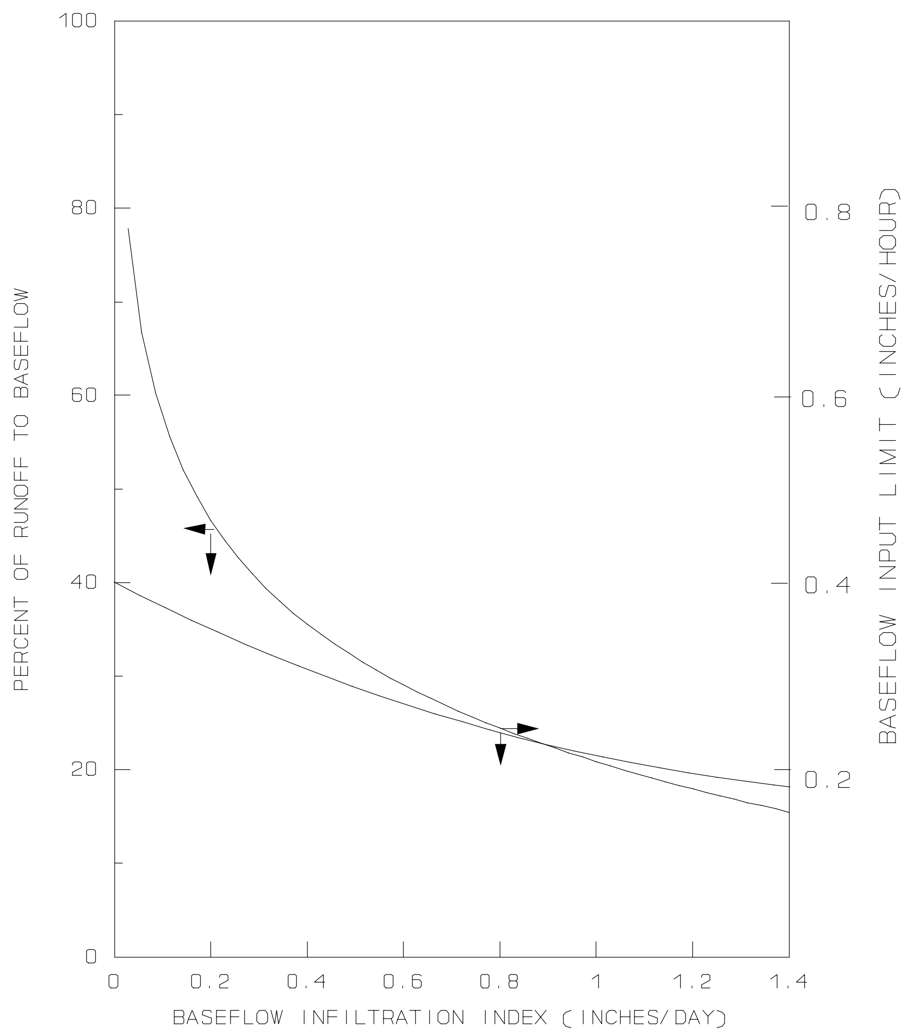

Figure 8-14. SSARR base-flow infiltration index (BII) versus base-flow percent (BFP)

b. Conceptual model. A conceptual model which is applicable to all watersheds does not exist. The subsurface characteristics of watersheds, and consequently the base-flow response, will vary. This variation will require different model representations to capture the subsurface response. Consequently, the conceptualization of the base-flow response by the number of storage zones or tanks in the model is, in some sense, a parameter estimation decision. A single subsurface tank may be sufficient for small watersheds with limited base-flow response, and multiple zones or tanks might be necessary for watersheds that have a complicated base-flow response. At the very least, a particular conceptual model should allow flexibility in the number of subsurface zones

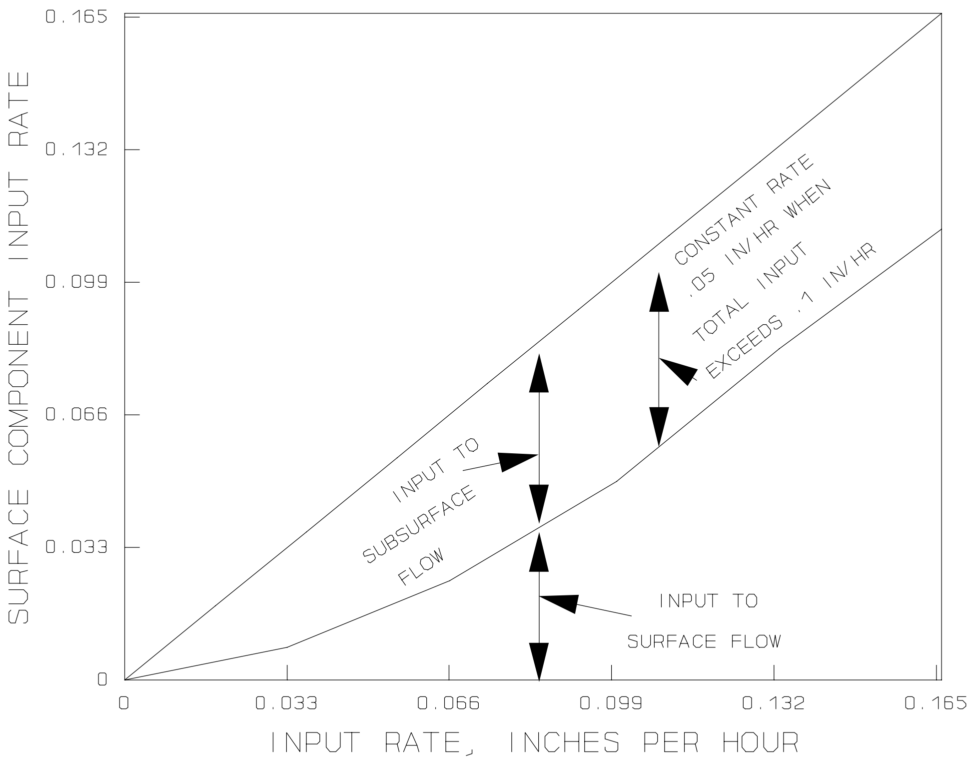

Figure 8-15. Surface - subsurface separation

that can be used to model subsurface response. The engineer would be well advised to find a model that has been successfully calibrated for a watershed that is similar to the one under investigation and subject to the same meteorologic conditions. Previous experience will help in selecting the appropriate structure for the model.

c. Previous experience. If no previous experience exists, then the structure of the model required depends on hydrograph recession analysis. The hydrograph recession analysis is an important aspect of an overall parameter estimation procedure which will be discussed subsequently.

(1) A general procedure for estimating parameters is to examine the hydrometeorologic record for errors, perform a water balance to determine ET, estimate parameters based on event analysis and watershed physical characteristics, and apply automatic parameter estimation to fine tune parameters. An automatic parameter estimation procedure, if available, can only be used to estimate a handful of parameters, eight at the very most, preferably four or less. The automatic procedure is very useful when the number of parameters is limited, as in the case of event-oriented modeling. However, the large number of parameters available for continuous models requires that most of these parameters be estimated prior to application of an automatic procedure.

(2) Many of the continuous model parameters have a similar effect on the predicted hydrograph. An optimization procedure cannot distinguish between these parameters for this reason. The impact of each parameter must be examined in context with the physics of the process

affecting hydrographs. Available automatic parameter estimation algorithms have not been developed which can consider the physics of the problem as part of the fitting procedure.

d. Experience in applying model. Burnash (1985), who developed and has had extensive experience in applying the Sacramento Model (a conceptual continuous simulation model), recommends the first three steps mentioned when estimating parameters. Although his recommendations were directed toward the Sacramento model, they are equally applicable to other continuous models.

(1) Examination of hydrometeorologic record for errors. Burnash is convinced that the major deficiency in hydrometeorologic record is the potential underestimation of rainfall by raingauges due to wind effects. The underestimation is on the order of 10 to 15 percent. The error may not be consistent and is likely to affect large events where wind speeds are the greatest. Other factors that contribute to errors in the record are change in gauge location, gauge type, or in the environment surrounding the gauge which changes local wind patterns.

(a) Burnash makes some suggestions to identify and correct this problem. For these reasons and others, a careful application of the Sacramento Watershed Model or, for that matter, the basic water balance equation requires a continuous comparative analysis of rainfall and runoff records to describe an unusual pattern which may be a result of data inconsistencies rather than a true event. Implicit in these comments is the notion that the rainfall input should be scaled to arrive at a consistent rainfall-runoff record.

(b) Discharge measurements, particularly for large flows, may have large errors due to ill-defined rating curves. Although not explicitly stated, Burnash seems to be warning against accepting streamflow measurements that are inconsistent with the rest of the record which, in turn, would distort model parameters in the estimation process.

(2) Water balance preservation. A successful parameter estimation procedure depends on preserving the fundamental water balance equation:

Runoff = Precipitation - Evapotranspiration

Estimation of evapotranspiration is difficult because the most common indicator used is evaporation, most commonly estimated by evaporation pans. Evaporation is a very different process from ET and a poor indicator as well. Burnash cautions against using evaporation as the final arbitrator of ET; evaporation may be used as an aid in preserving the fundamental water balance equation.

(3) Parameters from event analysis. The key to estimating continuous simulation model parameters is to identify circumstances in the hydrologic record where the individual parameter has the most effect. This may be accomplished by examining different events or an aspect of the hydrograph where a particular parameter is of first-order importance.

(a) The impervious area fraction of the basin may be identified by examining direct runoff when antecedent precipitation conditions are extremely dry. The direct runoff in these circumstance would be due to the impervious fraction.

(b) As antecedent precipitation increases, there will be an increase in direct runoff from a larger portion of the watershed. The maximum fraction of area that contributes to direct runoff will occur under the wettest conditions. The partial area correction, the relationship between basin contribution to direct runoff and basin moisture conditions, can be developed from examining the basin response from wet to dry antecedent conditions.

(c) The soil profile zone capacity can be estimated by examining prediction errors when the soil moisture deficit should be small. Presumably, an overprediction of runoff will indicate that the soil profile capacity has been underestimated.

(d) The subsurface response characteristics are determined by performing hydrograph recession analysis as discussed in paragraph 8-2 on event-oriented modeling of base flow. However, the recession analysis tends to be more detailed than in the event case. The continuous simulation analysis endeavors to identify different levels of aquifer response characteristics by identifying straight line segments on a log-discharge versus time plot. Burnash cautions that deviations from the straight line recession may occur due to channel losses or riparian vegetation ET. The impact of channel losses may be discerned by examining the deviations from a straight line during periods when ET is low. The recession can then be corrected for channel loss and then used to examine the impact of ET on the recession during high ET periods.

(e) Burnash does not discuss the use of automatic parameter estimation or optimization algorithms for estimating parameters. However, his recommended estimation techniques should be used to reduce the number of parameters that will be used when estimating parameters via an optimization approach. Optimization techniques are only useful when the number of parameters are limited to less than eight and preferably less than four. Consequently, optimization or automatic parameter estimation will probably be used to fine tune parameter estimates obtained by event analysis and application of the water balance equation.