Chapter 7
Precipitation Excess - Runoff Transformation

7-1. General

The transformation of precipitation excess to runoff is a key factor in flood-runoff analysis. Two approaches are described. The first employs the unit hydrograph and is based on the assumption that a watershed, in converting precipitation excess to runoff, acts as a linear, time-invariant system. The second approach is based on mathematical simulation of surface runoff using the kinematic wave approximation of the unsteady flow equations for one-dimensional open channel flow. In this chapter, the basis for the approaches, data requirements, calibration procedures, and limitations are described.

7-2. Runoff Subdivision

The methods in this chapter treat total runoff (i.e., streamflow) as being composed of two components, direct runoff and base flow. *Direct runoff* results from *precipitation excess*, which is regarded herein as that portion of storm precipitation that appears as streamflow during or shortly after a storm. *Base flow* results from subsurface runoff from prior precipitation events and delayed subsurface runoff from the current storm. The difference between total storm precipitation and precipitation excess is termed *losses* (or abstractions). This chapter deals with the calculation of direct runoff, given precipitation excess. Methods for estimating losses are described in Chapter 6. Base flow must be added to direct runoff to obtain total runoff. Base flow estimation is treated in Chapter 8. Precipitation includes both rain and snow. The methods described in this chapter are generally applied to rainfall excess. However, some applications involve the combining of rainfall excess with snowmelt excess as the basis for direct runoff determination. Chapter 5 deals with snowmelt estimation.

7-3. Unit Hydrograph Approach

a. Concepts.

(1) The unit hydrograph represents direct runoff at the outlet of a basin resulting from one unit (e.g., 1 in.) of precipitation excess over the basin. The excess occurs at a constant intensity over a specified duration. Assumptions associated with application of a unit hydrograph are the following:

(a) Precipitation excess and losses can be treated as basin-average (lumped) quantities.

(b) The ordinates of a direct runoff hydrograph corresponding to precipitation excess of a given duration are directly proportional to the volume of excess (assumption of linearity).

(c) The direct runoff hydrograph resulting from a given increment of precipitation excess is independent of the time of occurrence of the excess (assumption of time invariance).

(2) Difficulties associated with the first assumption can be alleviated by dividing a basin into subbasins so that the use of lumped quantities is reasonable. Because runoff response characteristics of watersheds are not strictly linear, the unit hydrograph used with a particular storm hyetograph should be appropriate for a storm of that magnitude. Hence, unit hydrographs to be used with large hypothetical storms should, if possible, be derived from data for large historical events. In some cases, it is appropriate to adjust a unit hydrograph to account for anticipated shorter travel times for large events. The duration of precipitation excess associated with a unit hydrograph should be selected to provide adequate definition of the direct runoff hydrograph. Generally, a duration equal to about one-fifth to one-third of the time-to-peak of the unit hydrograph is appropriate.

b. Unit hydrograph application and derivation. Application of a unit hydrograph may be performed with the following equation:

$$Q(t) = \sum_{i=1}^{n} u[\Delta t_o, t - (i - 1)] I_i \Delta t \qquad (7\text{-}1)$$

where

$Q(t)$ = ordinate of direct runoff hydrograph at time t

$u(\Delta t_o, t)$ = ordinate at time t of unit hydrograph of duration Δt_o

I_i = intensity of precipitation excess during block i of storm

n = total number of blocks of precipitation excess

Such application is represented graphically in Figure 7-1. The individual direct runoff responses to each block of precipitation excess are superimposed to produce the total direct runoff.

(1) The development of a unit hydrograph for a basin proceeds differently depending on whether a basin is gauged or ungauged. Gauged, in this case, means that there is a stream gauge at the basin outlet for which measurements during historical storms are available, and data from precipitation gauges are available with which hyetographs of basin-average precipitation can be developed for the storms. Unit hydrographs can be developed and verified with such data, as discussed later in this chapter.

(2) For ungauged basins, direct development of a unit hydrograph is not possible and techniques for estimating a unit hydrograph from measurable basin characteristics are employed. Generally, a unit hydrograph is represented mathematically as a function of one or two parameters, and these parameters are related to basin characteristics by regression analysis or other means. Several methods for representing unit hydrographs are described in the next section. Chapter 16, "Ungauged Basin Analysis," discusses the use of regional analysis for estimating unit hydrograph parameters for ungauged basins.

c. Synthetic unit hydrographs. Many methods have been devised for representing a unit hydrograph as a function of one or two parameters. The methods can be categorized as those that are strictly empirical and those that are based on a conceptual representation of basin runoff. The five methods described subsequently are the Single-linear Reservoir, Nash, Clark, Snyder, and SCS. The first three employ conceptual models of runoff; the latter two are empirical.

(1) Single-linear reservoir method. Conceptual models commonly employ one or more linear reservoirs as elements. A *linear reservoir* is a reservoir for which there is a linear relationship between storage and outflow:

$$S = KO \tag{7-2}$$

where

S = volume of water in storage in the reservoir

K = storage coefficient

O = rate of outflow from the reservoir

K has units of time and is constant for a linear system.

(a) A very simple conceptual model would represent the direct runoff from a basin with a single-linear reservoir (SLR). If such a reservoir is filled instantaneously with one unit of volume (i.e., representing one unit of depth over the basin area) and the reservoir is permitted to drain, it can be shown that the equation for the outflow is:

$$O(t) = \frac{1}{K}\, e^{-\frac{t}{K}} \tag{7-3}$$

Figure 7-2 illustrates a single-linear reservoir and the outflow hydrograph.

(b) The above equation represents an instantaneous unit hydrograph (IUH) for the basin because the duration (Δt_o) of precipitation excess is zero. The IUH can be converted to a unit hydrograph of finite duration by superposing several IUH's initiated at equal subintervals of an interval equal to the duration Δt_o and dividing the aggregate direct runoff by the number of IUH's. If Δt_o is sufficiently small (as is normally the case to provide adequate definition to a direct runoff hydrograph), the finite-duration unit hydrograph can be developed by simply averaging the ordinates of two IUH's that are separated in time by Δt_o.

(c) A unit hydrograph developed with the SLR model involves a single parameter, K. That is, once a value for K is specified, the unit hydrograph can be determined. This simple model is useful for small basins with short response times.

(2) Nash model. The Nash conceptual model (Nash 1957) represents the direct runoff response of a basin by passing a unit volume of water through a series of identical linear reservoirs, as depicted in Figure 7-3. As with the SLR, the unit volume enters the upstream-most reservoir instantaneously. The outflow from the downstream-most reservoir is the IUH for the basin. The equation for the IUH is:

$$O(t) = \frac{1}{K(n - 1)!} \left(\frac{t}{K}\right)^{n-1} e^{-\frac{t}{K}} \tag{7-4}$$

A unit hydrograph based on the Nash model has two parameters: the number of reservoirs, n, and the storage coefficient, K, which are identical for each reservoir. The model is widely used both for unit hydrograph development and for streamflow routing.

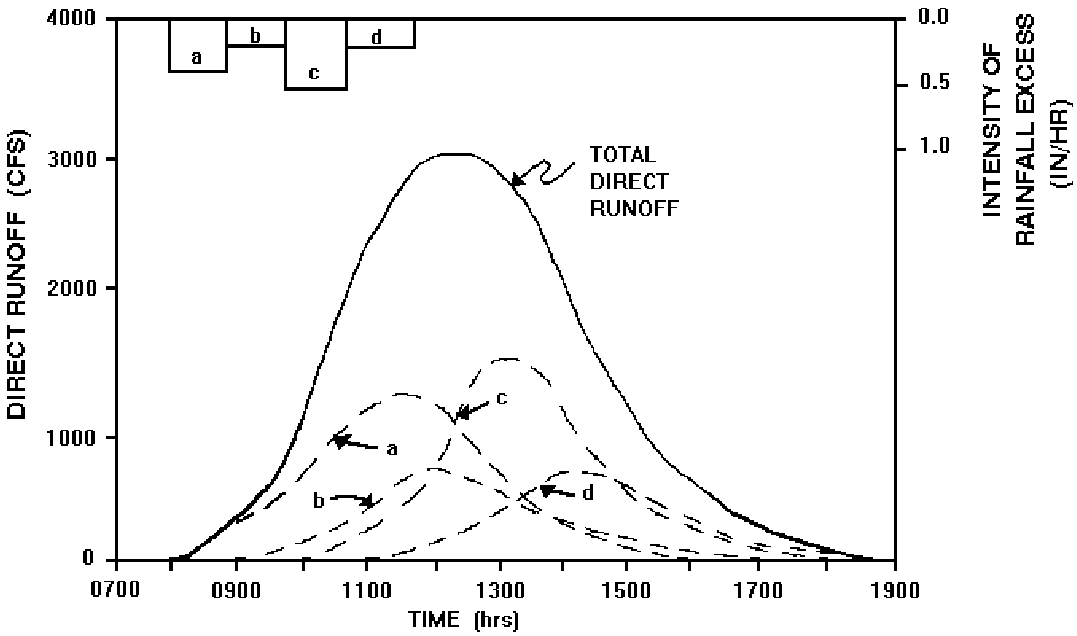


Figure 7-1. Superposition of direct runoff hydrographs

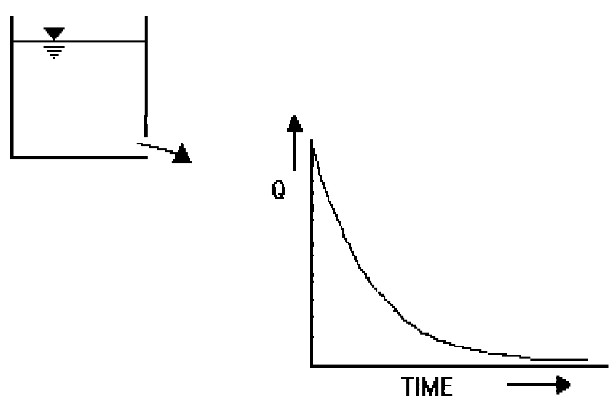


Figure 7-2. Single-linear reservoir model

(3) Clark model. The Clark conceptual model (Clark 1945) differs from the SLR and Nash models in that effects of basin shape (and other factors) on time of travel can be taken into account. As with the previous models, a unit of precipitation excess occurs instantaneously over the basin. A *translation hydrograph* at the basin outlet is developed by translating (lagging) the excess based on travel time to the outlet. The translation hydrograph is routed through a single linear reservoir, and the resulting outflow represents the IUH for the basin. Figure 7-4 illustrates the components of the Clark method.

(a) The translation hydrograph can be conveniently derived from a time-area relation, for which area is the accumulated area from the basin outlet, and time is the travel time as defined by isochrones (contours of constant time-of-travel). Such a relationship can be expressed in dimensionless form with area as a percent of total basin area and time as a percent of time of concentration (t_c). The translation hydrograph can be obtained by determining from a time-area relation the portion of the basin that contributes runoff at the outlet during each time interval after the occurrence of the instantaneous burst of unit excess. The contributing area associated with a time interval (times the unit depth and divided by the time interval) yields an average discharge. This is the ordinate of the translation hydrograph for that interval.

(b) Isochrones for use in defining the translation hydrograph may be developed by estimating, for a number of points in the basin, overland flow and channel travel times to the basin outlet. A simpler approach is to assume a constant travel velocity and base the position of isochrones on travel distance from the basin outlet, in

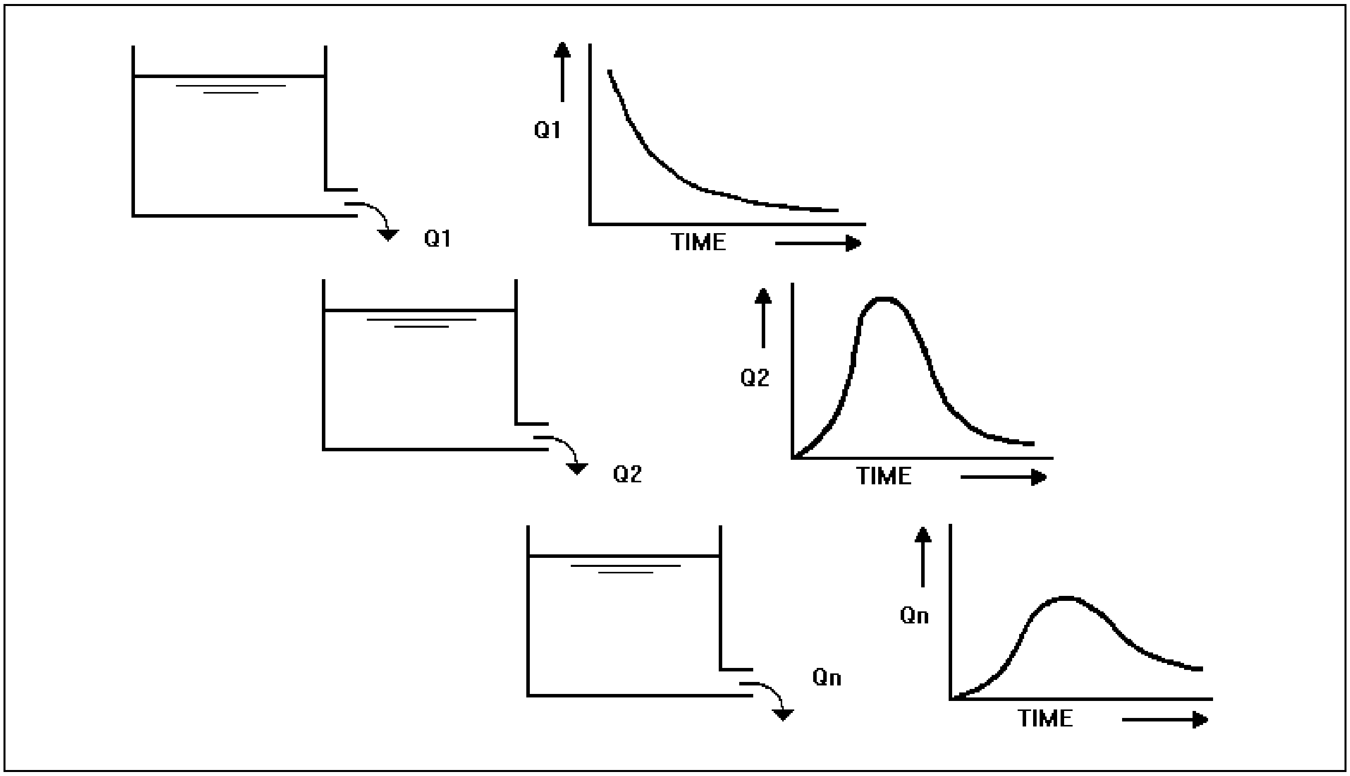


Figure 7-3. Cascade of linear reservoirs (Nash model)

which case the translation hydrograph reflects only basin shape.

(c) An even simpler approach is to use a translation hydrograph that is based on a standard basin shape, such as an ellipse. For many basins, storage effects represented by the linear reservoir cause substantial attenuation of the translation hydrograph such that the IUH is not very sensitive to the shape of the translation hydrograph. However, for a basin without a substantial amount of natural storage, such as a steep urban basin, the IUH will be much more sensitive to the shape of the translation hydrograph. For such a basin, the use of a standard shape may not be appropriate.

(d) The routing of the translation hydrograph through a linear reservoir is based on simple storage routing by solving the continuity equation. An equation for the routing is:

$$O(t) = C_a I + C_b O(t - 1) \tag{7-5}$$

The coefficients C_a and C_b are defined by:

$$C_a = \frac{\Delta t}{R + .5\Delta t}$$

and

$$C_b = 1 - C_a$$

where

O(t) = ordinate of IUH at time t

I = ordinate of translation hydrograph for interval t - 1 to t

R = storage coefficient for linear reservoir

Δt = time interval with which IUH is defined

The two parameters for the Clark method are T_c, the time of concentration (and time base for the translation hydrograph), and R, the storage coefficient for the linear

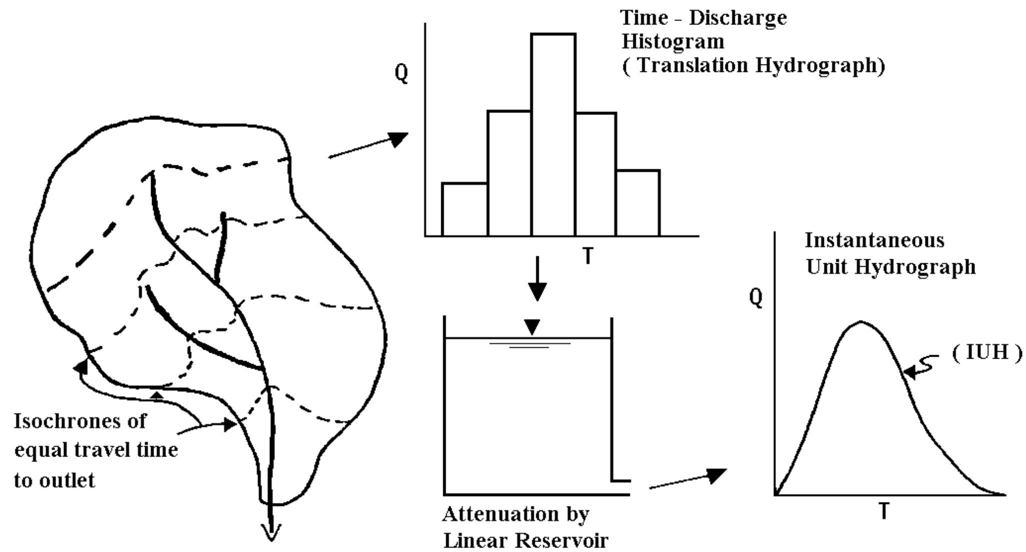


Figure 7-4. Clark model

reservoir. Values for these, along with a time-area relation, enable the determination of a unit hydrograph.

(e) To calculate direct runoff, the IUH can be converted to a unit hydrograph (UH) of finite duration. Derivation of a UH of specified duration from the IUH is accomplished using techniques similar to those employed to change the duration of a UH. For example, if a 2-hour UH is required, a satisfactory approximation may be obtained by first summing the ordinates of two instantaneous unit hydrographs, one of which is lagged 2 hr. This sum represents the runoff from 2 in. of excess precipitation; to obtain the required UH, the ordinates must be divided by 2. This procedure is illustrated in Figure 7-5.

(4) Snyder method. The Snyder method (Snyder 1938) provides equations that define characteristics of the unit hydrograph directly without the use of a conceptual model. Equations have been developed to define the coordinates of the peak and the time base of the unit hydrograph. Empirical procedures for defining the unit hydrograph width at 50 and 75 percent of the peak discharge have also been developed. Use of this method requires, as a final step, the fitting of a curve (i.e., the unit hydrograph) that has an underlying area consistent with a unit depth over the basin area.

(a) The principal equations of the Snyder method from which the peak of the unit hydrograph can be defined are:

$$t_l = C_t \left(L L_{ca}\right)^{0.3} \tag{7-6}$$

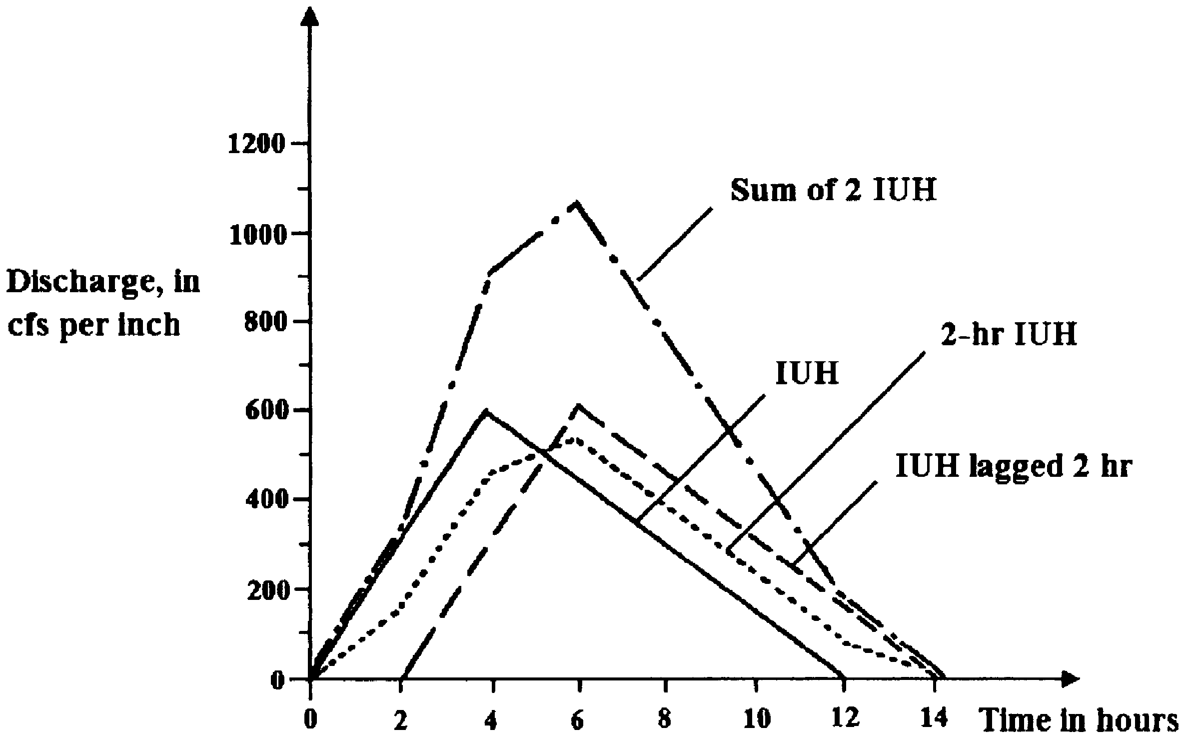
and

$$Q_p = \frac{640\, C_p A}{t_l} \tag{7-7}$$

where

t_l = lag of the "standard" unit hydrograph, in hours

C_t = empirical coefficient



Time in hours 1	IUH ordinate, in cfs per inch 2	IUH lagged 2 hr 3	Sum of Col 2 and Col 3 4	2-hr UH ord. in cfs per inch 5
0	0	-	0	0
2	300	0	300	150
4	600	300	900	450
6	450	600	1050	525
8	300	450	750	375
10	150	300	450	225
12	0	150	150	75
14	0	0	0	0

Figure 7-5. Conversion of IUH to UH with specific duration

L = length of main watercourse from basin outlet to upstream boundary of basin, in miles

L_{ca} = length of main watercourse from basin outlet to point opposite centroid of basin area, in miles

Q_p = peak discharge of "standard" unit hydrograph, in cubic feet per second

C_p = empirical coefficient

A = basin area, in square miles

The "standard" unit hydrograph is one for which the following relation holds:

$$t_r = \frac{t_l}{5.5} \tag{7-8}$$

where

t_r = duration of unit excess

t_l = time from the center of mass of the unit excess to the time of the peak of the unit hydrograph

The time at which the peak of the unit hydrograph occurs is therefore $t_l + t_r/2$. Thus, the above equations can be used to determine the coordinates of the peak of the "standard" unit hydrograph in terms of two empirical coefficients, C_t and C_p. The following equations can be used to develop the coordinates of the peak of a unit hydrograph of any other duration, t_R:

$$t_{lR} = t_l + 0.25(t_R - t_r) \tag{7-9}$$

$$Q_p = \frac{640 C_p A}{t_{lR}} \tag{7-10}$$

where

t_{lR} = adjusted lag time for unit hydrograph of duration t_R, in hours

t_l = original unit hydrograph lag time, in hours

t_R = desired unit hydrograph duration, in hours

t_r = original unit hydrograph duration, in hours

Q_p = peak discharge of unit hydrograph of duration t_R

Equations for the time base and widths of the unit hydrograph are available in several publications (Snyder 1938 and Chow, Maidment, and Mays 1988).

(b) The original development of this method and values for the coefficients C_t and C_p were made with data from the Appalachian Mountain region. Subsequent applications in other regions produced values for the coefficients that were substantially different. The coefficients should be calibrated with data from the region in which they will be applied. Indeed it is not necessary to adopt the form of the original equation for t_l; regression analysis can be used to develop expressions for t_l and C_p that take into account measurable basin characteristics other than L and L_{ca}. For example, the variable $(LL_{ca}/S^{1/2})$, where S is the slope of the main watercourse, has been found useful as an independent variable in relations for t_l. According to a number of studies, C_p tends to be fairly constant in a region.

(5) SCS dimensionless unit hydrograph. The SCS dimensionless unit hydrograph (Mockus 1957), which is shown in Figure 7-6, was derived from a large number of unit hydrographs developed with data from small rural basins. The ordinates are expressed as a ratio of the peak discharge, and the time scale is expressed as a ratio of the time-to-peak. The time base of the unit hydrograph is five times the time-to-peak.

(a) A characteristic of the dimensionless unit hydrograph is that 37.5 percent of the area under the hydrograph occurs from the origin to the peak. The rising limb of the hydrograph is well represented by a straight line. The following equation is based on an expression for the area of a triangle defined by a linear representation of the rising limb and a vertical line from the peak to the x-axis:

$$Q_p = \frac{484\ A}{t_p} \tag{7-11}$$

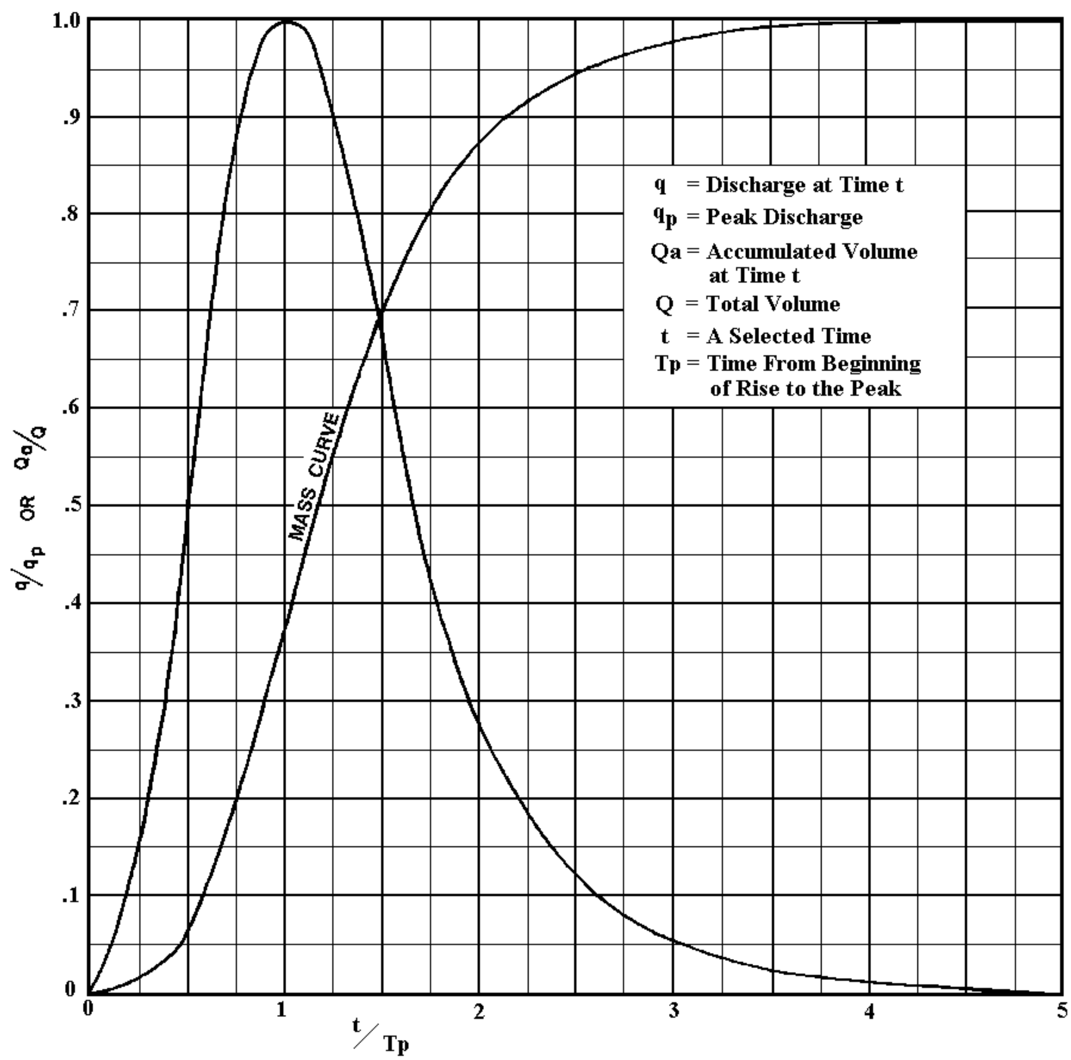


Figure 7-6. SCS dimensionless unit hydrograph

where

Q_p = peak discharge of unit hydrograph, in cubic feet per second

A = basin area, in square miles

t_p = time-to-peak of unit hydrograph, in hours

(b) A change in volume under the rising limb of the unit hydrograph would be reflected in a change in the "constant" represented by 484 in the above equation. Studies have indicated that the constant varies from about 600 for basins with steep slopes to 300 for flat swampy basins. Figure 7-6 is based on the constant 484. To utilize a constant of 300 or 600, a completely new dimensionless hydrograph must be developed.

The time-to-peak may be expressed as:

$$t_p = \frac{D}{2} + t_l \tag{7-12}$$

where

D = duration of unit excess for unit hydrograph

t_l = lag, defined as the time from the centroid of precipitation excess to the time of the peak of the unit hydrograph

The SCS developed the following empirical relation between t_l and time of concentration:

$$t_l = 0.6\,t_C \tag{7-13}$$

where t_C = time of concentration. Thus, if the time of concentration for a basin can be estimated, the above equation can be used to estimate lag, and the preceding two equations can be used to determine the time-to-peak and peak discharge. The coordinates of the dimensionless unit hydrograph can then be used to completely determine the unit hydrograph.

d. Choice of synthetic unit hydrograph method. The preceding section describes five methods for defining a unit hydrograph in terms of parameters. The SLR method employs one parameter, a storage coefficient for a linear reservoir. The SCS method is a one-parameter method if the value of 484 is adopted for the constant in the equation for peak discharge. The Nash, Clark, and Snyder methods each employ two parameters, and the Clark method, in addition, requires a time-area relation.

(1) Figure 7-7 shows a set of unit hydrographs developed by the Clark method and also a unit hydrograph developed with the SCS dimensionless unit hydrograph (based on the 484 constant). The unit hydrographs are for a drainage area of 50 sq mi and a time of concentration of 13.3 hr. The parameter that varies for the Clark unit hydrographs is the storage coefficient, R. Each of the Clark unit hydrographs is labeled with a value for the ratio $R/(t_C + R)$. This dimensionless ratio has been found in a number of studies to be fairly constant on a regional basis. For a value of this ratio of 0.1, the unit hydrograph rises steeply and might be representative of the runoff response of an urban basin. For a value of 0.7, the unit hydrograph is much attenuated and might be representative of a flat swampy basin. The point is that with two parameters, there is substantial flexibility for fitting a wide variety of runoff responses. Similar plots could be developed with the Nash and Snyder methods.

(2) If the SCS dimensionless unit hydrograph is applied as a one-parameter method (by adopting a constant of 484 in the equation for peak discharge), the result is as shown in Figure 7-7 for the given basin area and time of concentration. In this case, the unit hydrograph is approximately equivalent to a Clark unit hydrograph corresponding to a value for $R/(t_C + R)$ of about 0.25. Use of a one-parameter unit hydrograph can be very limiting with respect to ability to fit the runoff response characteristics of a basin.

(3) A number of attempts have been made to relate parameters of a synthetic unit hydrograph to measurable characteristics of an observed hydrograph. For example, the time of concentration (t_c) can be estimated as the time from the end of a burst of precipitation excess to the point of inflection on the falling limb of the direct runoff hydrograph. The storage coefficient in the Clark method can be estimated by dividing the discharge at the point of inflection by the slope of the direct runoff hydrograph at that point. The basis for these estimation procedures is that, at the point of inflection, inflow to storage has ceased, and from that time on, storage is being evacuated. At the point of inflection, the continuity equation can be stated as:

$$O_{poi} = -\left(\frac{dS_{poi}}{dt}\right) \tag{7-14}$$

where the subscript *poi* indicates “point of inflection.” Since from the storage equation, $S = RO$, then:

$$O_{poi} = -R\left(\frac{dO_{poi}}{dt}\right) \tag{7-15}$$

Solving for R:

$$R = -\frac{O_{poi}}{dO_{poi}/dt} \tag{7-16}$$

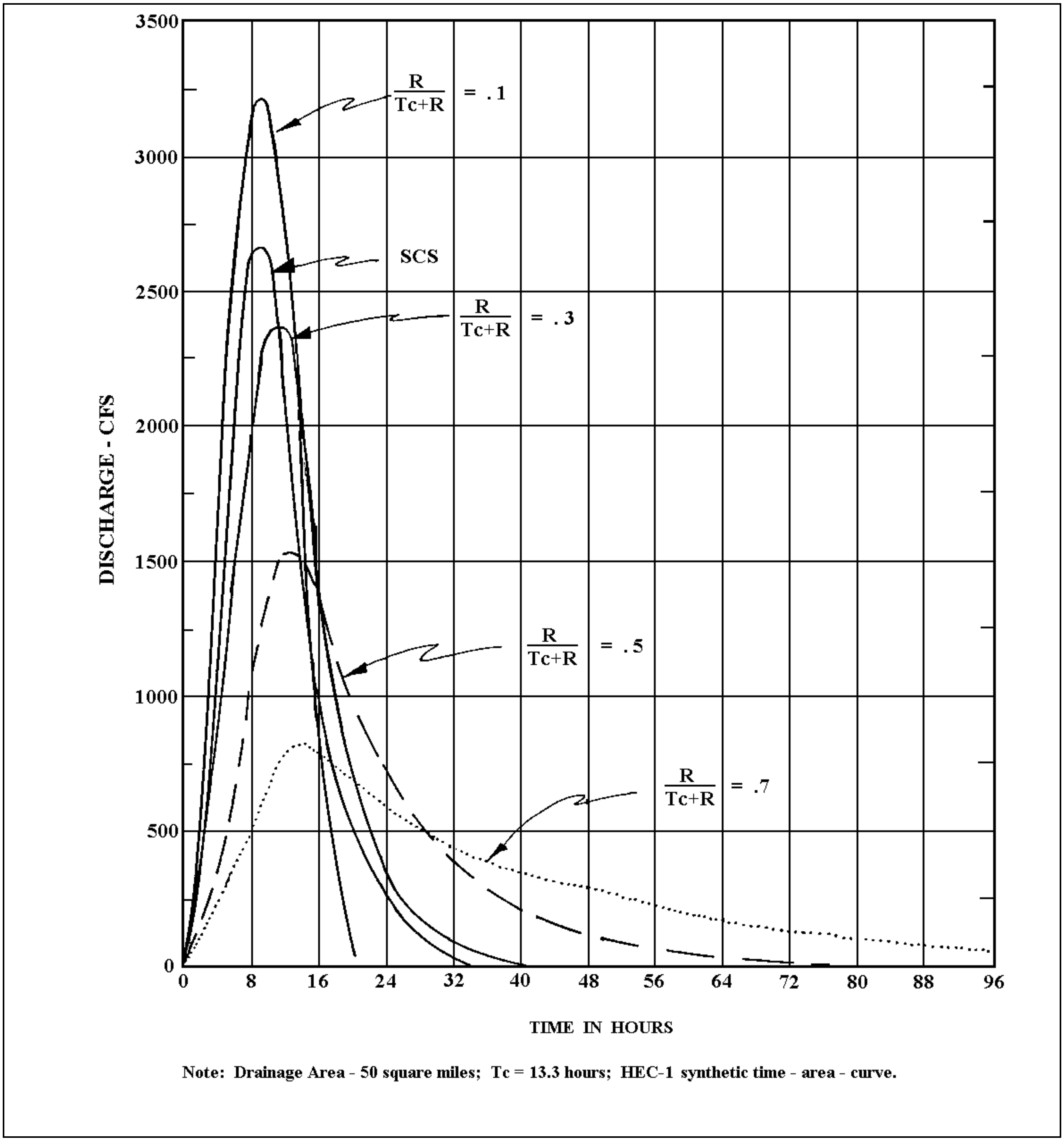


Figure 7-7. Unit hydrographs by Clark and SCS methods

Estimates obtained by such methods should not be relied on too rigorously because the conceptual models are only crude approximations at best of real-world phenomena.

(4) Any one of the two-parameter methods is adequate for describing the runoff response of most basins. The choice of method therefore can be based on other factors, such as availability of regional relations for parameters, familiarity with a method, or ease of use. Other aspects of the methods should be considered. For example, the Snyder method requires explicit curve fitting, and the Clark method permits incorporation of basin shape and timing factors through use of a time-area relation.

e. Unit hydrograph for a gauged basin. A number of methods have been developed to enable derivation of a unit hydrograph from precipitation and streamflow data. The simplest method involves the analysis of individual storms for which there are isolated blocks of significant amounts of precipitation excess. After base flow separation, the volume of direct runoff is determined and used to adjust losses to produce an equivalent volume of precipitation excess. The duration of precipitation excess is associated with a unit hydrograph that is obtained by dividing the ordinates of direct runoff by the volume of direct runoff expressed as an average depth over the basin. S-graph methods can be used to convert the unit hydrograph of a given duration to one of another duration. The "isolated storm" and S-graph methods are described in basic hydrology textbooks.

(1) Matrix methods. A unit hydrograph can be derived from a complex storm (for which there are several blocks of precipitation excess) by matrix methods. The first step is to perform a base flow separation on the observed hydrograph and to develop a precipitation excess hyetograph. Equations are written to define the ordinates of the direct runoff hydrograph as a function of hyetograph ordinates and (unknown) unit hydrograph ordinates, and these equations are solved with matrix algebra. Linear regression or optimization methods can be used to facilitate the search for a unit hydrograph that minimizes the error in the fitted direct runoff hydrograph (Chow, Maidment, and Mays 1988). A problem with such techniques is that the derived unit hydrograph may have an oscillatory shape or reflect other irregularities, and a smoothing process is commonly required.

(2) Optimization of values for unit hydrograph parameters. The methods described thus far produce a unit hydrograph defined by its ordinates. Another approach is to use a synthetic hydrograph technique and associated parameters to represent the unit hydrograph. The problem then becomes one of finding values for the parameters, generally using trial and error procedures with data from complex storms. The objective of such procedures is to obtain parameter values that enable a "best fit" of the observed hydrograph.

(a) Optimization methods have been developed for automated estimation of values for parameters. Such methods can optimize values for loss rate parameters simultaneously with values for unit hydrograph parameters (Ford, Merris, and Feldman 1980). A general scheme is shown in Figure 7-8. A quantitative measure of "best fit," termed an *objective function*, is calculated with each trial set of parameters. The optimization scheme is designed to adjust parameter values in such a way that minimization of the objective function is achieved. One such objective function is:

$$F = \left[\frac{\sum_{i=1}^{n} \left(Q_{obs_i} - Q_{comp_i} \right)^2 * WT_i}{n} \right]^{1/2} \tag{7-17}$$

where

F = objective function

Q_{obs} = ordinate of observed hydrograph

Q_{comp} = ordinate of computed hydrograph

i = ordinate number

n = total number of ordinates over which objective function is evaluated

and

$$WT_i = \frac{\left(Q_{obs_i} + Q_{avg} \right)}{2 * Q_{avg}} \tag{7-18}$$

where Q_{avg} = average of the observed-hydrograph ordinates. The purpose of the weighting function, WT_i, is to weight deviations between observed and computed ordinates more heavily for higher observed discharges. This will tend to produce a relatively good fit for high discharges compared with low discharges, which is generally desired in flood-runoff analysis.

(b) Optimization procedures also require initial values for parameters and constraints that define the acceptable range of magnitude of each parameter. The results of optimization should be reviewed carefully, both with respect to the success of the optimization and the reasonableness of optimized values.

(3) Procedure for unit hydrograph development. A procedure for developing a unit hydrograph for a gauged basin is given below. It is presumed that the analysis will be performed with the aid of a computer program that has capabilities for optimizing values of runoff parameters.

(a) Obtain precipitation and discharge data for historical storms. It is desirable for the storms to be of comparable magnitude to those to which the unit hydrograph will be applied. In the case of application with very large hypothetical storms, data for the largest storms of record will be the most useful. Ideally, it would be desirable to calibrate values for unit hydrograph parameters for about five storms and to verify the adopted values with data from about three additional storms.

(b) Determine initial streamflow conditions for each historical event and appropriate values for parameters with which to define base flow. Select an appropriate method for representing losses and a synthetic unit hydrograph method. Choice of methods will be dependent on the capabilities of computer software to be used for the analysis.

(c) Perform an optimization of values for loss and unit hydrograph parameters for each storm that has been selected for calibration. Carefully review optimization results and verify that optimized values are reasonable. Extend the analysis (for example with different initial values) as appropriate.

(d) Based on a review of values for unit hydrograph parameters that have been optimized for each calibration event, adopt a single set of values. Factors to consider in adopting values include the quality of fit of an observed hydrograph and the magnitude of the event. Events for which only a poor fit was possible would be given less weight in the adoption process. If some events are substantially larger than others, these might be given more weight, if the adopted values are intended for use with large events. The adopted values should then be used to calculate hydrographs for all of the calibration events, and the results evaluated. Additional adjustment of the values might be warranted to achieve the most satisfactory fitting of the events.

(e) If additional events for verification are available, the adopted values should be used to calculate hydrographs for these events. The quality of results will be a measure of the reliability of the adopted values. Additional adjustment of the values may be appropriate.

7-4. Kinematic Wave Approach

a. Concepts. The application of the kinematic wave method differs from the unit hydrograph technique in the following manner. First, the method takes a distributed view of the subbasin rather than a lumped view, like the unit hydrograph approach. The distributed view point allows the model to capture the different responses from both pervious and impervious areas in a single urban subbasin. Second, the kinematic wave technique produces a nonlinear response to rainfall excess as opposed to the linear response of the unit hydrograph.

(1) When applying the kinematic wave approach to modeling subbasin runoff, the various physical processes of water movement over the basin surface, infiltration, flow into stream channels, and flow through channel networks are considered. Parameters, such as roughness, slope, area, overland lengths, and channel dimensions are used to define the process. The various features of the irregular surface geometry of the basin are generally approximated by either of two types of basic flow elements: an overland flow element, or a stream- or channel-flow element. In the modeling process, overland flow elements are combined with channel-flow elements to represent a subbasin. The entire basin is modeled by linking the various subbasins together.

(2) In a typical urban system, as shown in Figure 7-9, rain falls on two types of surfaces: those that are essentially impervious, such as roofs, driveways, sidewalks, roads, and parking lots; and pervious surfaces, most of which are covered with vegetation and have numerous small depressions which produce local storage of rainfall. The contribution to the flood hydrograph of open areas (pervious surfaces) is characteristically different than that from impervious areas. An obvious difference is that the open areas can infiltrate rainfall whereas the impervious areas do not infiltrate significant amounts. A less obvious difference is that the open areas are not sewered as heavily as impervious areas, making for longer overland flow paths to major conveyances such as open channels and storm sewers. Furthermore, the open areas

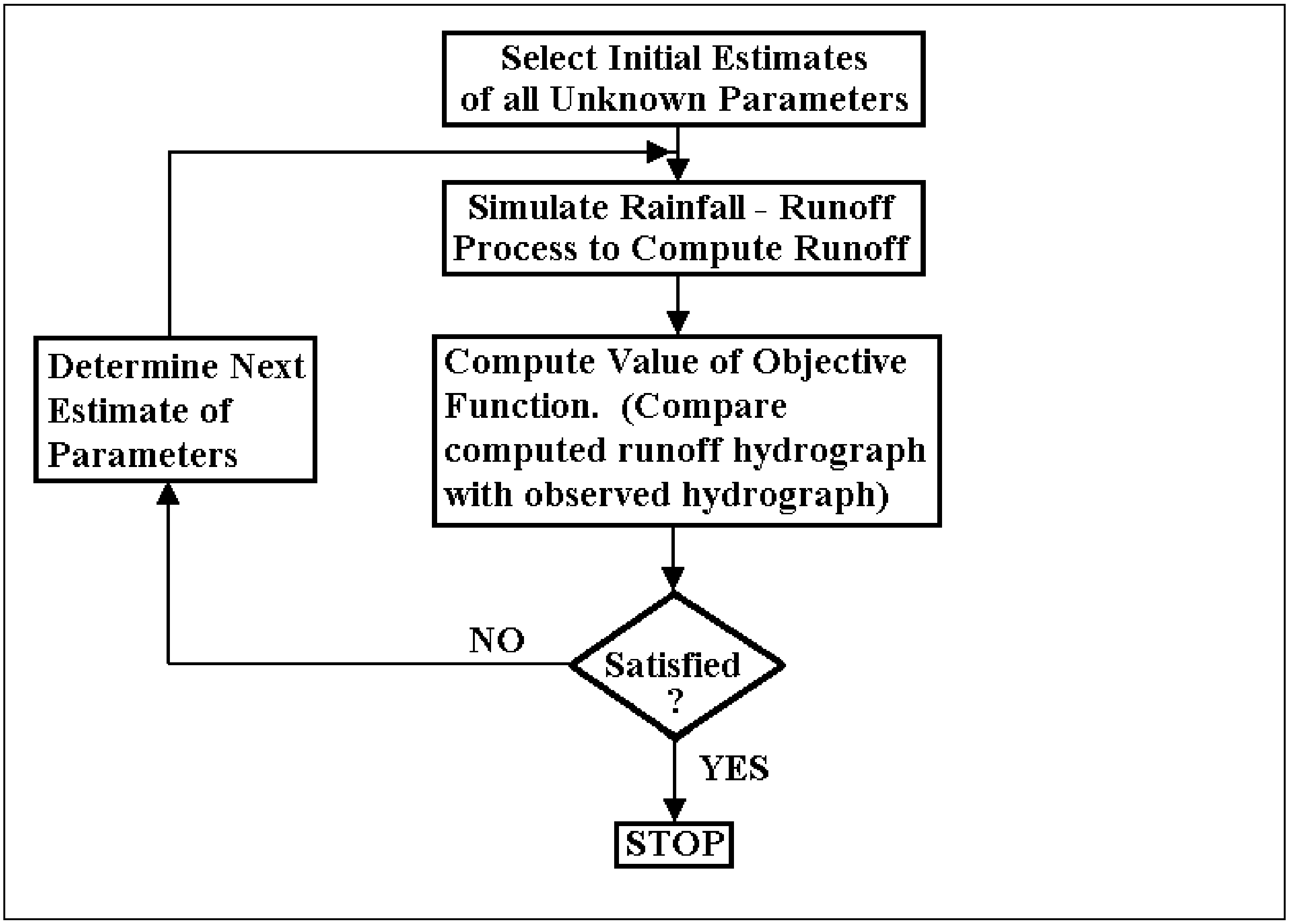


Figure 7-8. Procedure for parameter optimization

have hydraulically rougher surfaces which impedes the flow to a greater extent than the relatively smoother surface of the paved areas. The overall impact of these differences is to cause the runoff from the impervious areas to have significantly shorter times of concentration, larger peak discharges, and volumes per unit area than from open (pervious) areas.

(3) The lumped approach to modeling this type of basin (Figure 7-9) would average the runoff characteristics of both the open and impervious areas into one unit hydrograph. In performing this averaging operation, the peak runoff response of the basin will normally be underestimated when the impervious area is the dominant contributor to the runoff hydrograph. The main advantage of the kinematic wave method is that the response of both the open and impervious areas can be accounted for in a single subbasin.

b. The kinematic wave equations of motion. The kinematic wave equations are based on the conservation of mass and the conservation of momentum. The conservation principles for one-dimensional open channel flow (St. Venant equations) can be written in the following form:

<u>Conservation of mass</u>
Inflow - outflow = the rate in change of channel storage

$$A \frac{\partial V}{\partial x} + VB \frac{\partial y}{\partial x} + B \frac{\partial y}{\partial t} = q \tag{7-19}$$

<u>Conservation of momentum</u>
Sum of forces = gravity + pressure + friction
= mass x fluid acceleration

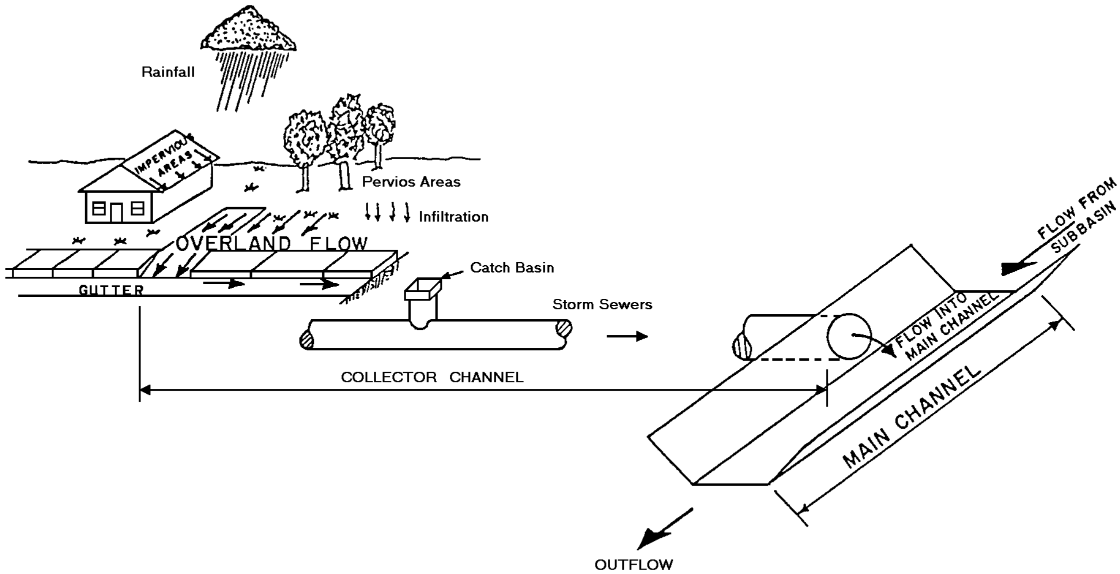


Figure 7-9. Typical urban basin flow paths

$$S_f = S_o - \frac{\partial y}{\partial x} - \frac{V}{g}\frac{\partial V}{\partial x} - \frac{1}{g}\frac{\partial V}{\partial t} \qquad (7\text{-}20)$$

where

A = cross-sectional flow area

V = average velocity of water

x = distance along the channel

B = water surface width

y = depth of water

t = time

q = lateral inflow per unit length of channel

S_f = friction slope

S_o = channel bed slope

g = acceleration due to gravity

The kinematic wave equations are derived from the St. Venant equations by preserving conservation of mass and approximately satisfying conservation of momentum. In approximating the conservation of momentum, the acceleration of the fluid and the pressure forces are presumed to be negligible in comparison to the bed slope and the friction slope. This reduces the momentum equation down to a balance between friction and gravity:

Kinematic wave conservation of momentum
Frictional forces = gravitational forces

$$S_f = S_o \qquad (7\text{-}21)$$

This equation states that the momentum of the flow can be approximated with a uniform flow assumption as described by Manning's and Chezy's equations. Manning's equation can be written in the following form:

$$Q = \alpha A^{m} \qquad (7\text{-}22)$$

where α and m are related to surface roughness and flow geometry. Since the momentum equation has been reduced to a simple functional relationship between area and discharge, the movement of a floodwave is described solely by the continuity equation. Therefore, the kinematic wave equations do not allow for hydrograph diffusion (attenuation). Hydrographs routed with the kinematic wave method will be translated in time but will not be attenuated. The kinematic wave equations are usually solved by explicit or implicit finite difference techniques. Any attenuation of the peak flow that is computed using the kinematic wave equations is due to errors inherent in the finite difference solution scheme. In spite of this limitation, the kinematic wave approximation is very good for modeling overland flow at shallow depths or channel flow in moderately steep channels. Application of the kinematic wave equations to a combination of overland flow and channel flow elements is often used in urban watershed modeling.

c. Basin representation with kinematic wave elements. The contribution to the flood hydrograph from open and impervious areas within a single subbasin is modeled in the kinematic wave method by using different types of elements as shown in Figure 7-10.

(1) The kinematic wave elements shown are an overland flow plane, collector, and main channel. In general, subbasin runoff is modeled with kinematic wave elements by taking an idealized view of the basin. Rather than trying to represent every overland flow plane and every possible collector channel, subbasins are depicted with overland flow planes and channels that represent the average conditions of the basin. Normally two overland flow planes are used, one to represent the pervious areas and one to represent the impervious areas. The lengths, slopes, and roughnesses of the overland flow planes are based on the average of several measurements made within the subbasin. Likewise, collector channels are normally based on the average parameters of several collector channels in the subbasin.

(2) Various levels of complexity can be obtained by combining different elements to represent a subbasin. The simplest combination of elements that could be used to represent an urban subbasin are two overland flow planes and a main channel (Figure 7-11). The overland flow planes are used to separately model the overland flow from pervious and impervious surfaces to the main channel. Flow from the overland flow planes is input to the main channel as a uniform lateral inflow. Urban watersheds typically have various levels of storm sewers, man-made channels, and natural streams. To model complex urban systems in a manageable fashion, the concept of typical collector channels must be employed. As shown in Figure 7-12, the complexity of an urban subbasin can be modeled by combining various levels of channel elements. An idealized overland flow, subcollector, and collector system are formulated from average parameters in the subbasin. The runoff contributing to the idealized collector system is assumed to be typical of the subbasin. The total runoff is obtained by multiplying the runoff from the idealized collector system by the ratio of the total subbasin area to the contributing area to the collector system. The total runoff is then distributed uniformly along the main channel and routed to the outlet.

d. Estimating kinematic wave parameters. Although the kinematic wave equations are used to route flow through both the overland flow planes and channels, different types of data are needed for each element because of differences in characteristic depths of flow and geometry. The depth of flow over an overland flow plane is much shallower than in the case of a channel. This results in a much greater frictional loss for overland flow than for channel flow. Frictional losses are accounted for in the kinematic wave equations through Manning's equation. Typical roughness coefficients for overland flow are about an order of magnitude greater than for channel flow. The overland flow roughness coefficients (Table 7-1) will range between 0.1 and 0.5 depending on the surface cover; whereas the roughness coefficients for channel flow are normally in the range of 0.012 to 0.10.

(1) The estimation of kinematic wave parameters for each element is an exercise in averaging slopes, lengths, roughness coefficients, and even geometry. The data for the various kinematic wave parameters can be obtained from readily available topographic, soil, sewer, and zoning maps, as well as tables of roughness coefficients. The following data are needed for each overland flow plane:

(a) Average overland flow length.

(b) Representative slope.

(c) Average roughness coefficient (Table 7-1).

(d) The percentage of the subbasin area which the overland flow plane represents.

(e) Infiltration and loss rate parameters.

Overland flow lengths for impervious surfaces are typically shorter than those for pervious surfaces. Impervious overland flow lengths range from 20 to 100 ft, while

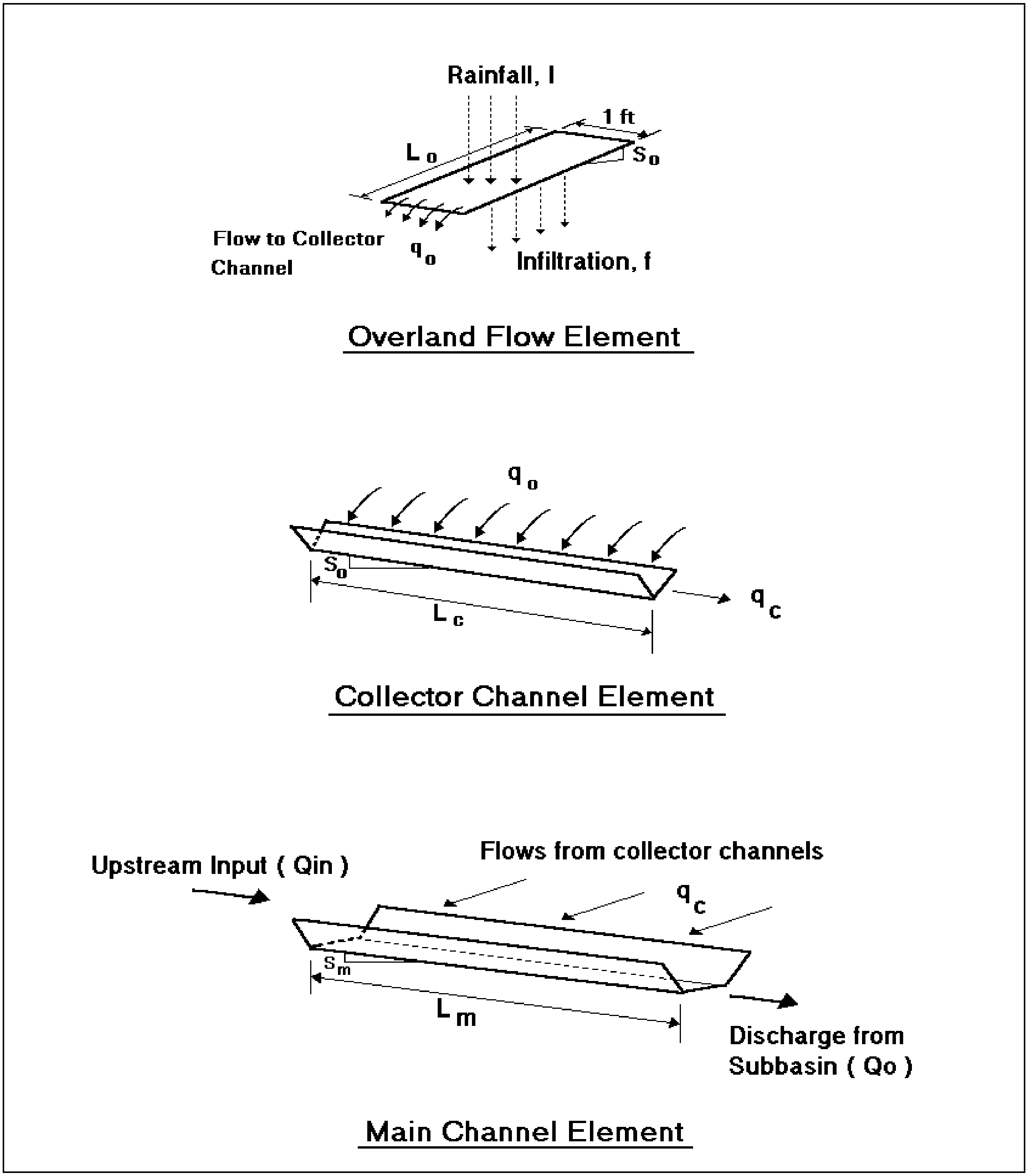


Figure 7-10. Kinematic wave elements

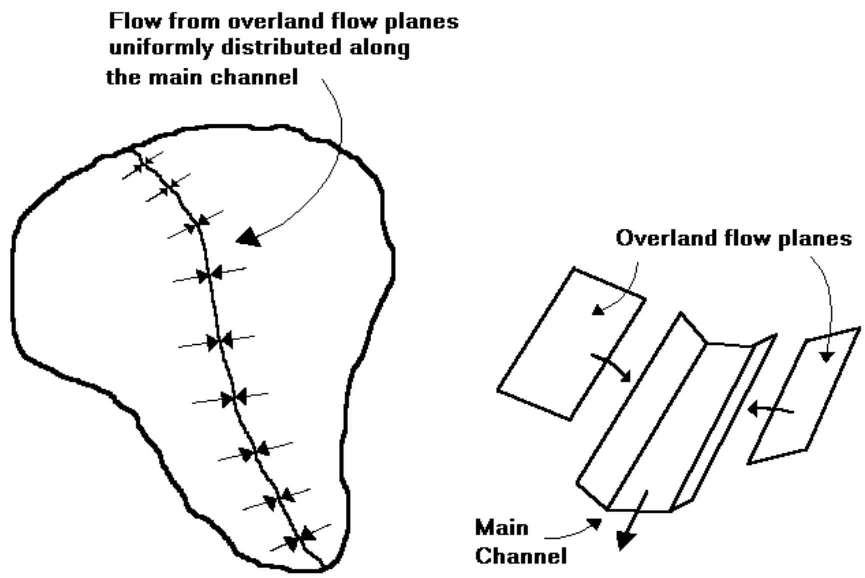


Figure 7-11. Simple kinematic wave subbasin model

pervious overland flow lengths can range from 20 to several hundred feet. Overland and channel slopes can be obtained from topographic maps. Overland slopes, as well as collector channels, should be taken as the average from several measurements made within a subbasin. The main channel slope can be measured directly. Loss rate parameters must be specified for each overland flow plane. Loss rates for impervious areas are generally restricted to a small initial loss to account for wetting the surface and depression storage. Loss rates for pervious areas are based on the soil types and surface cover. Estimating the percent of the subbasin that is actually impervious area can be quite difficult. For example, in some areas roof top downspouts are hydraulically connected to the sewers or drain directly to the driveway; whereas in other areas the downspouts drain directly into flower beds or lawns. In the former situation, the roof top acts as an impervious area, and in the latter, as a pervious area.

(2) The following data are needed to describe collector and sub-collector channels as well as the main channel:

(a) Representative channel length.

(b) Manning's n.

(c) Average slope.

(d) Channel shape.

(e) Channel dimensions.

(f) Amount of area serviced by the channel element.

For collector and subcollector channels, the representative length and slope is based on averaging the lengths of several collectors and subcollectors within the basin. The main channel length and slope should be measured directly from topographic maps. Manning's n values can be estimated from photos or field inspection of the channels. Channel shapes and dimensions are usually approximated by using simple prismatic geometry as shown in Table 7-2. Collector and subcollector channels should be based on the average of what is typical within the subbasin. The main channel shape and dimensions should be approximated as best as possible with one of the prismatic elements shown in Table 7-2.

e. Basin modeling. The assumptions made using the kinematic wave approach to model a river basin are essentially the same as those made when applying the unit

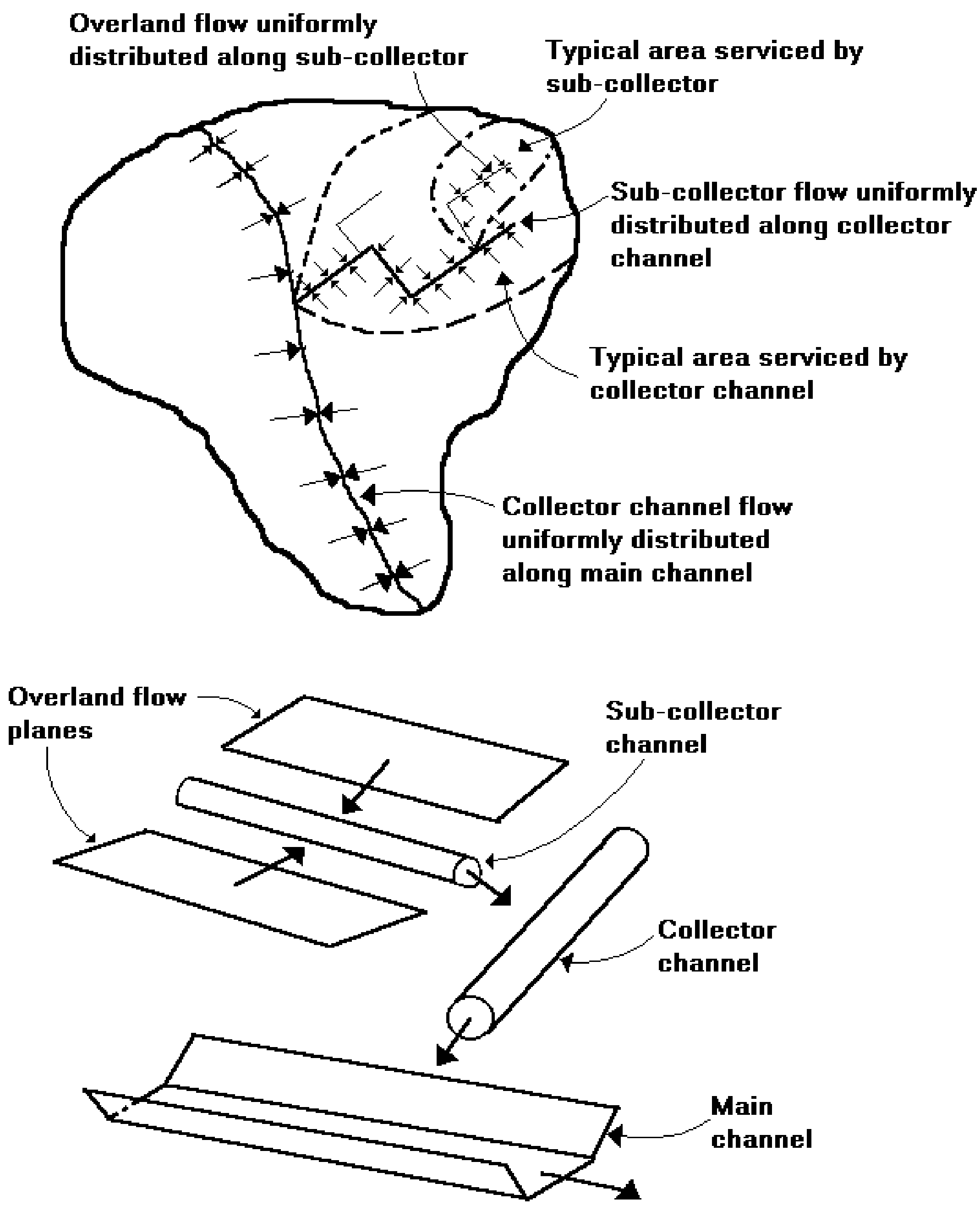


Figure 7-12. Kinematic wave representation of an urban subbasin

Table 7-1
Roughness Coefficients for Overland Flow (from Hjelmfelt 1986)

Surface	N Value	Source
Asphalt & concrete	0.05 - 0.15	a
Bare packed soil free of stone	0.10	c
Fallow - no residue	0.008 - 0.012	b
Conventional tillage - no residue	0.06 - 0.12	b
Conventional tillage - with residue	0.16 - 0.22	b
Chisel plow - no residue	0.06 - 0.12	b
Chisel plow - with residue	0.10 - 0.16	b
Fall disking - with residue	0.30 - 0.50	b
No till - no residue	0.04 - 0.10	b
No till (20 - 40 percent residue cover)	0.07 - 0.17	b
No Till (60 - 100 percent residue cover)	0.17 - 0.47	b
Sparse rangeland with debris:		
0 percent cover	0.09 - 0.34	b
20 percent cover	0.05 - 0.25	b
Sparse vegetation	0.053 - 0.13	f
Short grass prairie	0.10 - 0.20	f
Poor grass cover on moderately rough bare surface	0.30	c
Light turf	0.20	a
Average grass cover	0.40	c
Dense turf	0.17 - 0.80	a,c,e,f
Dense grass	0.17 - 0.30	d
Bermuda grass	0.30 - 0.48	d
Dense shrubbery and forest litter	0.40	a

a) Crawford and Linsley (1966).
b) Engman (1986).
c) Hathaway (1945).
d) Palmer (1946).
e) Ragan and Dura (1972).
f) Woolhiser (1975).

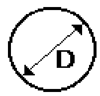
hydrograph technique. Rainfall is assumed to be uniform over any subbasin and there are no backwater effects in channel routing. The assumption that there are no backwater effects has some important ramifications for interpreting the kinematic wave results. Although the channel elements can be used to represent pipe elements, the pipes never pressurize. The kinematic wave equations are for open channel flow and cannot represent the effects of pressure flow.

(1) This is not a severe limitation when applying the kinematic wave method for design purposes. Generally speaking, sewer systems are designed to convey flow as an open channel. However, in situations where the sewer system will pressurize, flow will back up into the street gutters and flow to the nearest low point where it may enter the sewer system again. In the case where a culvert or a storm sewer pressurizes and creates a large backwater, the backwater area should be modeled separately with a technique that can handle pressure flow.

(2) The use of the kinematic wave method for main channels and large collector's should be limited to urban areas or moderately sloping channels in headwater areas. The limitation results because a hydrograph's peak discharge does not attenuate when it is routed with the kinematic wave technique. This is an adequate approximation in urban areas, or any small, quick responding basin. However, flood waves generally attenuate in most natural channels. Consequently, the kinematic wave method will tend to overestimate peak discharges in this type of stream. Therefore, in natural streams, where it is likely that hydrograph attenuation will occur, the kinematic wave method should not be used for routing. Alternative routing methods that can account for attenuation, such as the Muskingum-Cunge method, should be applied instead.

Table 7-2
Prismatic Elements for Kinematic Wave Channels

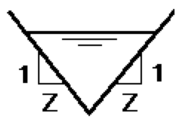
Circle



$$\alpha = \frac{.804}{n} S^{1/2} D^{1/6}$$

$$m = 5/4$$

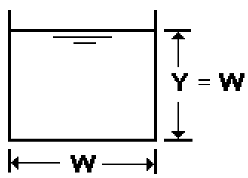
Triangle



$$\alpha = \frac{0.94}{n} S^{1/2} \left(\frac{Z}{1+Z^2}\right)^{1/3}$$

$$m = 4/3$$

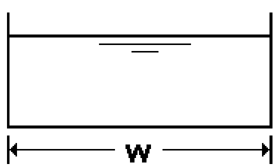
Square



$$\alpha = \frac{0.72}{n} S^{1/2}$$

$$m = 4/3$$

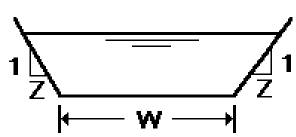
Rectangular



$$\alpha = \frac{1.49}{n} S^{1/2} W^{-2/3}$$

$$m = 5/3$$

Trapezoidal



$$Q = \frac{1.49}{n} S^{1/2} A^{5/3} \left(\frac{1}{W+2Y\sqrt{1+Z^2}}\right)^{2/3}$$