Chapter 12
Frequency Analysis of Streamflow Data

12-1. General

Frequency analysis of recorded streamflow data is an important flood-runoff analysis tool. This chapter describes the role of frequency analysis and summarizes the technical procedures. EM 1110-2-1415 describes the procedures in greater detail.

a. Role of frequency analysis.

(1) The traditional solution to water-resource planning, designing, or operating problems is a deterministic solution. With a deterministic solution, a critical hydrometeorological event is selected. This event is designated the design event. Plans, designs, or operating policies are selected to accommodate that design event. For example, the maximum discharge observed in the last 40 years may be designated the design event. A channel modification may be designed to pass, without damage, this design event. If this design event is not exceeded in the next 1,000 years, the design may not be justified. On the other hand, if the discharge exceeds the design event 20 times in the next 30 years, the channel modification may be underdesigned.

(2) A probabilistic solution employs principles of statistics to quantify the risk that various hydrometeorological events will be exceeded. Risk is quantified in terms of probability. The greater the risk, the greater the probability. If an event is certain to occur, its probability is 1.00. If an event is impossible, its probability is 0.00. For flood-runoff analyses, the probability of exceedance is usually the primary interest. This is a measure of the risk that discharge will exceed a specified value. Decisions are taken so that the risk of exceedance is acceptable. For example, the channel modification described above could be designed for a discharge magnitude with an annual exceedance probability of 0.01. In that case, the risk is known and is accounted for explicitly in the decision making.

b. Definition of frequency analysis.

(1) The objective of streamflow frequency analysis is to infer the probability of exceedance of all possible discharge values (the parent population) from observed discharge values (a sample of the parent population). This process is accomplished by selecting a statistical model that represents the relationship of discharge magnitude and exceedance probability for the parent population. The parameters of the models are estimated from the sample. With the calibrated model, the hydrologic engineer can predict the probability of exceedance for a specified magnitude or the magnitude with specified exceedance probability. This magnitude is referred to as a quantile.

(2) For convenience, a statistical model may be displayed as a frequency curve. Figure 12-1 is an example of a frequency curve. The magnitude of the event is the ordinate. Probability of exceedance is the abscissa. For hydrologic engineering studies, the abscissa commonly shows “percent chance exceedance.” This is exceedance probability multiplied by 100.

(3) In some sense, frequency analysis is a model-fitting problem similar to the precipitation-runoff analysis problem described in Chapter 8. In both cases, a model must be selected to describe the desired relationship, and the model must be calibrated with observed data.

c. Summary of streamflow frequency analysis techniques. Techniques for selecting and calibrating streamflow frequency models may be categorized as graphical or numerical. With graphical techniques, historical observations are plotted on specialized graph paper and the curves are fitted by visual inspection. Numerical techniques infer the characteristics of the model from statistics of the historical observations. The procedures for both graphical and numerical analysis are presented in detail in EM 1110-2-1415 and are summarized herein for ready reference.

12-2. Frequency Analysis Concepts

a. Data requirements. Statistical models of streamflow frequency are established by analyzing a sample of the variable of interest. For example, to establish a statistical model of annual peak discharge, the sample will be a series of annual peaks observed throughout time. The procedures of statistical analysis require the following of any time series used in frequency analysis:

(1) Data must be homogeneous. That is, the data must represent measurements of the same aspect of each event. For example, daily discharge observations should not be combined with peak discharge observations. Furthermore, all sample points must be drawn from the same parent population. For example, rain-flood data and snowmelt-flood data should not be combined if they can be identified and analyzed separately. Likewise, discharge data observed after development upstream should not be combined with predevelopment data.

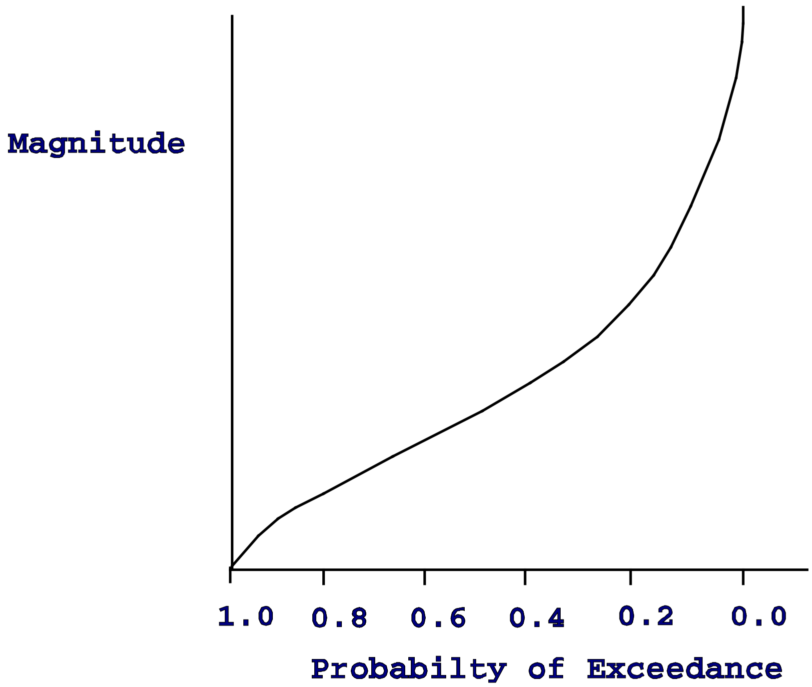

Figure 12-1. Frequency curve example

(2) Data must be spatially consistent. All data should be observed at the same location. Data observed at different locations may be used to develop probability estimates. However, these data must be adjusted to represent conditions at a common location.

(3) Time series must be continuous. Statistical analysis procedures require an uninterrupted series. If observations are missing, the missing values must be estimated, or techniques for analysis of broken records must be used.

b. Probability estimates from historical data.

(1) Streamflow probability is estimated from analysis of past occurrence. The simplest model of the relationship of streamflow magnitude and probability is a relative frequency model. This model estimates the probability of exceeding a specified magnitude as the fraction of time the magnitude was exceeded historically. For example, if the mean daily discharge at a given location exceeds 80 cfs in 6,015 of 8,766 days, the relative frequency is 0.68. The estimated probability of exceedance of 80 cfs is 0.68.

(2) Figure 12-2 is a graphical representation of the relative frequency models of mean daily flow in Fishkill Creek at Beacon, NY. Such a plot is commonly referred to as a duration curve. The abscissa of this plot shows “percent of time exceeded.” This equals relative frequency multiplied by 100, so it is consistent with the term “percent chance exceedance.”

(3) The reliability of a relative frequency model improves as the sample size increases; with an infinite sample size, relative frequency exactly equals the probability. Unfortunately, sample sizes available for streamflow frequency analysis are small by scientific standards. Thus, relative frequency generally is not a reliable estimator of probability for hydrologic engineering purposes.

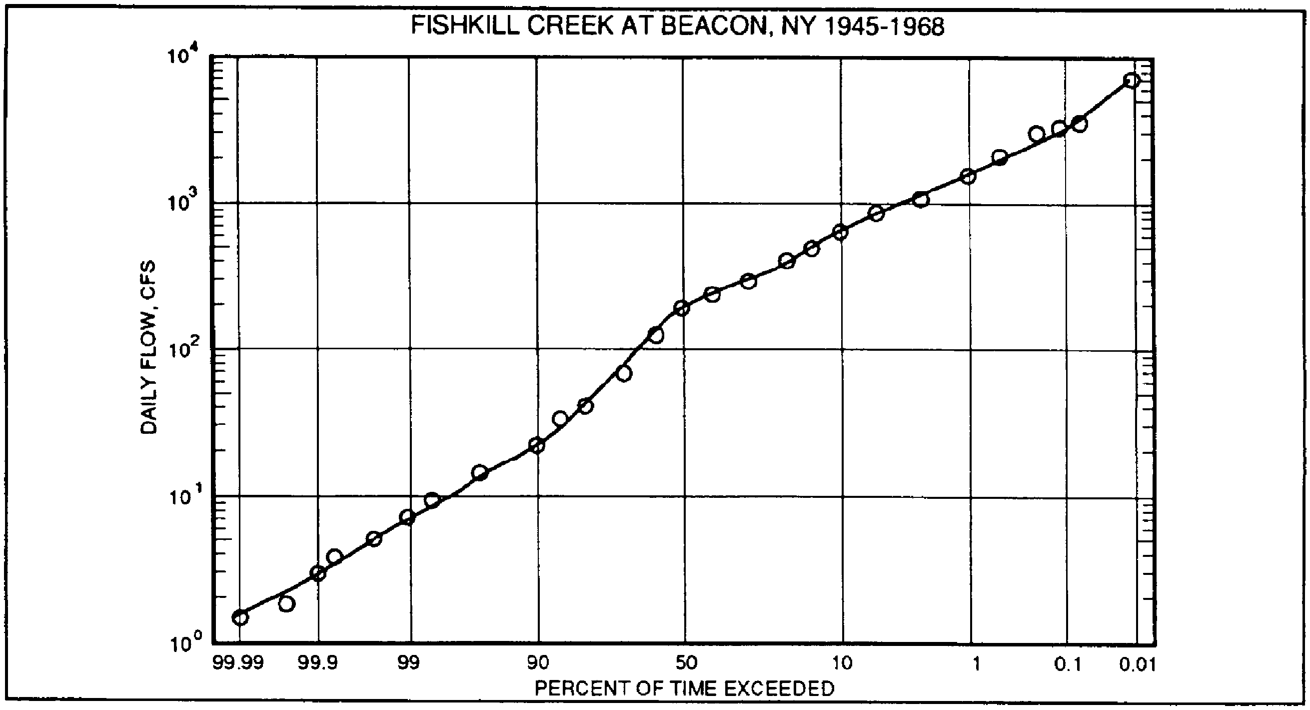

Figure 12-2. Graphical representation of relative frequency model

(4) The alternative to the empirical relative frequency model is a theoretical frequency model. With a theoretical model, the relationship of magnitude and probability for the parent population is hypothesized. The relationship is represented by a frequency distribution. A cumulative frequency distribution is an equation that defines probability of exceedance as a function of specified magnitude and one or more parameters. An inverse distribution defines magnitude as a function of specified probability and one or more parameters.

c. Distribution selection and parameter estimation. In certain scientific applications, one distribution or another may be indicated by the phenomena of interest. This is not so in hydrologic engineering applications. Instead, a frequency distribution is selected because it models well the data that are observed. The parameters for the model are selected to optimize the fit. A graphical or numerical technique can be used to identify the appropriate distribution and to estimate the parameters.

12-3. Graphical Techniques

Some of the early and simplest methods of frequency analysis were graphical techniques. These techniques permit inference of the parent population characteristics with a plot of observed magnitude versus estimated exceedance probability of that data. If a best-fit line is drawn on the plot, the probability of exceeding various magnitudes can be estimated. Also, any desired quantiles can be estimated. Graphical representations also provide a useful check of the adequacy of a hypothesized distribution.

a. Plotting-position estimates of probability.

(1) Graphical techniques rely on plotting positions to estimate exceedance probability of observed events. The median plotting position estimates the exceedance probability as:

$$P_m = \frac{(m - 0.3)}{(N + 0.4)} \qquad (12\text{-}1)$$

where:

P_m = exceedance probability estimate for the mth largest event

m = the order number of the event

N = the number of events

For example, to estimate annual exceedance probability of annual maximum discharge, *N* = the number of years of data. To express the results as percent-chance exceedance, the results of Equation 12-1 are multiplied by 100.

(2) Table 12-1 shows plotting positions for annual peak discharge on Fishkill Creek. Column 4 of the table shows the discharge values in the sequence of occurrence. Column 7 shows these same discharge values arranged in order of magnitude. Column 5 is the order number of each event. Column 8 shows the plotting position. These plotting positions are values computed with Equation 12-1 and multiplied by 100. The values in columns 7 and 8 thus are an estimate of the peak-discharge frequency distribution.

b. Display and use of estimated frequency curve.

(1) The estimated frequency distribution is displayed on a grid with the magnitude of the event as the ordinate and probability of exceedance (or percent-chance exceedance) as the abscissa. The plot thus provides a useful tool for estimating quantiles or exceedance probabilities. Specialized plotting grids are available for the display. These grids are constructed with the abscissa scaled so a selected frequency distribution plots as a straight line. For example, a specialized grid was developed by Hazen for the commonly used normal frequency distribution.

(2) The specialized normal-probability grid is a useful tool for judging the appropriateness of the normal

Table 12-1
Annual Peaks, Sequential and Ordered with Plotting Positions (Fishkill Creek at Beacon, NY)

Events Analyzed				Ordered Events			
Mon (1)	Day (2)	Year (3)	Flow, cfs (4)	Rank (5)	Water Year (6)	Flow, cfs (7)	Median Plot Pos (8)
3	5	1945	2,290.	1	1955	8,800.	2.87
12	27	1945	1,470.	2	1956	8,280.	6.97
3	15	1947	2,220.	3	1961	4,340.	11.07
3	18	1948	2,970.	4	1968	3,630.	15.16
1	1	1949	3,020.	5	1953	3,220.	19.26
3	9	1950	1,210.	6	1952	3,170.	23.36
4	1	1951	2,490.	7	1962	3,060.	27.46
3	12	1952	3,170.	8	1949	3,020.	31.56
1	25	1953	3,220.	9	1948	2,970.	35.66
9	13	1954	1,760.	10	1958	2,500.	39.75
8	20	1955	8,800.	11	1951	2,490.	43.85
10	16	1955	8,280.	12	1945	2,290.	47.95
4	10	1957	1,310.	13	1947	2,220.	52.05
12	21	1957	2,500.	14	1960	2,140.	56.15
2	11	1959	1,960.	15	1059	1,960.	60.25
4	6	1960	2,140.	16	1963	1,780.	64.34
2	26	1961	4,340.	17	1954	1,760.	68.44
3	13	1962	3,060.	18	1967	1,580.	72.54
3	28	1963	1,780.	19	1946	1,470.	76.64
1	26	1964	1,380.	20	1964	1,380.	80.74
2	9	1965	980.	21	1957	1,310.	84.84
2	15	1966	1,040.	22	1950	1,210.	88.93
3	30	1967	1,580.	23	1966	1,040.	93.03
3	19	1968	3,630.	24	1965	980.	97.13

distribution as a model of the parent population. If data drawn from a normally distributed parent population are assigned plotting positions using Equation 12-1 and are plotted on Hazen's grid, the points will fall approximately on a straight line. If the points do not, then either the sample was drawn from a population with a different distribution or sampling variation yielded a nonrepresentative sample.

(3) A specialized plotting grid has been developed also for another commonly used frequency distribution, the log-normal distribution. Figure 12-3 is an example of such a grid. The values from columns 7 and 8 of Table 12-1 are plotted on this grid, and a frequency curve is fitted. If the data are truly drawn from the distribution of a log-normal parent population, the points will fall on a straight line. The Fishkill Creek data, shown by Figure 12-3, do not fall on a straight line, so the assumption that the parent population is a log-normal distribution is suspect.

12-4. Numerical Techniques

Numerical techniques define the relationship between streamflow magnitude and probability with analytical tools, instead of the graphical tools.

a. Steps of numerical techniques. With numerical techniques, the following general steps are used to derive a frequency curve to represent the population (McCuen and Snyder 1986):

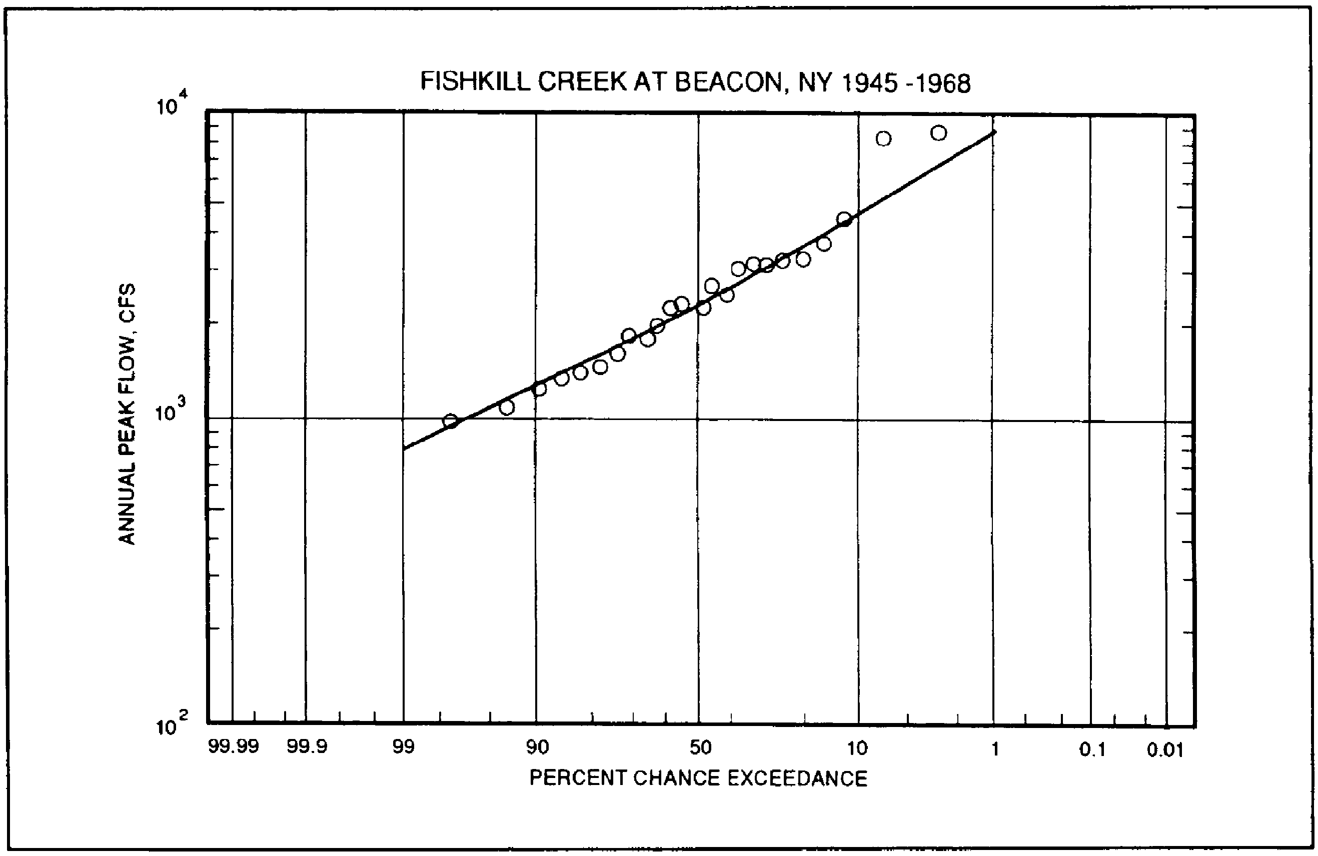

Figure 12-3. Log-normal probability grid

(1) Select a candidate frequency model of the parent population. Three distributions are commonly used for frequency analysis of hydrometeorological data: the normal distribution, the log-normal distribution, and the log Pearson type III distribution.

(2) Obtain a sample.

(3) Use the sample to estimate the parameters of the model identified in step 1.

(4) Use the model and the parameters to estimate quantiles to construct the frequency curve that represents the parent population.

b. Numerical parameter estimation.

(1) Parameters of a statistical model are commonly estimated from a sample with method-of-moments estimators. The method-of-moments parameter estimators are developed from the following assumptions:

(a) The streamflow-probability relationship of the parent population can be represented with a selected distribution. The moments (derivatives) of the distribution equation can be determined with calculus. One moment is determined for each parameter of the distribution. The resulting expressions are equations in terms of the parameters of the distribution.

(b) Moments of a sample of the parent population can be computed numerically. The first moment is the mean of the sample; the second moment is the variance; the third moment is the sample skew. Other moments can be found if the distribution selected has more than three parameters.

(c) The numerical moments of the sample are the best estimates of the moments of the parent population. This assumption permits development of a set of simultaneous equations. The distribution parameters are unknown in the equations. Solution yields estimates of the parameters.

(2) When the parameters of the distribution are estimated, the inverse distribution defines the quantiles of the frequency curve. Chow (1951) showed that with the method-of-moments estimates, many inverse distributions commonly used in hydrologic engineering could be written in the following general form:

$$Q_p = \bar{Q} + K_p S \tag{12-2}$$

where:

Q_p = the quantile with specified exceedance probability p

Q = the sample mean

S = the sample standard deviation

K_p = a frequency factor

The sample mean and standard deviation are computed with the following equations:

$$\bar{Q} = \frac{\Sigma\, Q_i}{N} \tag{12-3}$$

$$S = \left(\frac{\Sigma (Q_i - \bar{Q})^2}{(N - 1)} \right)^{0.5} \tag{12-4}$$

where:

Q_i = observed event i

N = number of events in sample

(3) The frequency factor in Equation 12-2 depends on the distribution selected. It is a function of the specified exceedance probability and, in some cases, other population parameters. The frequency factor function can be tabulated or expressed in mathematical terms. For example, normal-distribution frequency factors corresponding to the exceedance probability p $(0 < p < 0.5)$ can be approximated with the following equations (Abramowitz and Stegun 1965):

$$K_p = w - \left[\frac{(2.515517 + 0.802853\, w + 0.010328\, w^2)}{(1 + 1.432788\, w + 0.189269\, w^2 + 0.001308\, w^3)} \right] \tag{12-5}$$

$$w = \left[\ln \left(\frac{1}{P^2} \right) \right]^{0.5} \qquad (12\text{-}6)$$

where:

w = an intermediate variable, if $p > 0.5$, $(1 - p)$ is used in Equation 12-6, and the computed value of K_p is multiplied by -1.

(4) For the log-normal distribution, Equation 12-2 is written as:

$$X_p = \bar{X} + K_p S \qquad (12\text{-}7)$$

where:

X_p = the logarithm of Q_p, the desired quantile

X = mean of logarithms of sample

S = standard deviation of logarithms of sample

K_p = the frequency factor

This frequency factor is the same as that used for the normal distribution. X and S are computed with the following equations:

$$\bar{X} = \frac{\Sigma \log Q_i}{N} \qquad (12\text{-}8)$$

$$S = \left(\frac{\Sigma (\log Q_i - \bar{X})^2}{(N - 1)} \right)^{0.5} \qquad (12\text{-}9)$$

where:

Q_i = observed peak annual discharge in year i

N = number of years in sample

For the annual peak discharge values shown in Table 12-1, these values are as follows: $X = 3.3684$, and $S = 0.2456$.

c. Recommended procedure for annual maximum discharge.

(1) The U.S. Water Resources Council (USWRC) (1967, 1976, 1977) recommended the log Pearson type III distribution for annual maximum streamflow frequency studies. This recommendation is followed by USACE. Current guidelines are presented in Bulletin 17B (USWRC 1981).

(2) The log Pearson type III distribution models the frequency of logarithms of annual maximum discharge. Using Chow's (1951) format, the inverse log Pearson type III distribution is

$$X_p = \bar{X} + KS \qquad (12\text{-}10)$$

where:

X_p = the logarithm of Q_p, the desired quantile

X = mean of logarithms of sample

S = standard deviation of logarithms of sample

K = the Pearson frequency factor

X and S are computed with the Equations 12-8 and 12-9.

(3) For this distribution, the frequency factor K is a function of the specified probability and of the skew of the logarithms of the sample. The skew, G, is computed with the following equation:

$$G = \frac{\left[N \Sigma (X_i - X)^3 \right]}{\left[(N - 1)(N - 2) S^3 \right]} \qquad (12\text{-}11)$$

For the values of Table 12-1, the skew computed with Equation 12-11 is 0.7300.

(4) The log Pearson type III frequency factors for selected values of skew and exceedance probability are tabulated in Bulletin 17B (USWRC 1981) and in EM 1110-2-1415. Alternatively, an approximating

function can be used. If the skew equals zero, the Pearson frequency factors equal the normal distribution factors. Otherwise, the following approximation suggested by Kite (1977) can be used:

$$K = K_p + \left(K_p^2\right) k + \left(\frac{1}{3}\right)\left(K_p^3 - 6K_p\right) k^2 - \left(K_p^2 - 1\right) k^3 + K_p k^4 + \left(\frac{1}{3}\right) k^5 \sim \qquad (12-12)$$

where $k = G/6$.

d. Analysis of special cases.

(1) In hydrologic engineering applications, frequency analysis of annual maximum discharge is complicated by special cases. These include broken records, incomplete records, zero-flow years, outliers, historical data, and small samples. Bulletin 17B provides guidance for dealing with these cases.

(2) If 1 or more years of data are missing from a time series of annual maximum discharge due to reasons not related to flood magnitude, the record is broken. For analysis, the record segments are combined, and the combined record is analyzed as previously described.

(3) If data are missing because the events were too large to record, too small to record, or the gauge was destroyed by a large event, the record is incomplete. Any missing large events should be estimated and the estimates included in the time series. Missing small events are treated with the conditional probability adjustment recommended for zero-flow years.

(4) The log Pearson type III distribution is not suited to analysis of series which include zero-flow years. If the sample contains zero-flow years, the record is analyzed using the conditional probability procedure. With this procedure, the subseries of nonzero peaks is analyzed as described previously. The resulting frequency curve is a conditional frequency curve. The exceedance frequencies from this curve are scaled by the relative frequency of non-zero flow years. The log Pearson type III model parameters are estimated for the upper portion of the curve. With these parameters, a synthetic frequency curve is developed. Paragraph 3-6 of EM 1110-2-1415 describes the procedure.

(5) An outlier is an observation that departs significantly from the trend of the remaining data. Procedures for treating outliers require hydrologic and mathematical judgment. Bulletin 17B describes one procedure for identifying high and low outliers and for censoring the data set. High outliers are treated as historical data if sufficient information is available. Low outliers are treated as zero-flow years.

(6) Large floods outside the systematically recorded time series may be used to extend that record. The procedure recommended for analysis of these historical flows is as follows:

(a) Assemble known historic peaks and determine the historic record length.

(b) Censor the systematic record by deleting all peaks less than the minimum historical peak. Estimate the model parameters for the remaining record.

(c) Compute a weight with the following equation:

$$W = \frac{(H - Z)}{(N + L)} \qquad (12-13)$$

where:

W = the weight

H = number of years in historic record

Z = number of historic event

N = number of years in censored systematic record

L = number of zero-flow years, low outliers, missing years excluded from systematic record

(d) Adjust the model parameters with this weight. Equations for the adjustments are presented in Appendix 6 of Bulletin 17B (USWRC 1981). Compute the quantiles with these modified parameters and Equation 12-10.

(7) Small samples adversely affect the reliability of estimates of the skew. This parameter is difficult to estimate accurately from a small sample. A more reliable estimate is obtained by considering skew characteristics of all available streamflow records in a large region. An

adopted skew is computed as a weighted sum of this regional skew and the skew computed with Equation 12-11. The weights chosen are a function of the sample skew of the logs, the sample record length, the generalized skew, and the accuracy in developing the generalized values. The generalized skew can be determined from a map included in Bulletin 17B, or it can be determined from detailed analyses if additional data are available.

(8) The impact of uncertainty due to small sample size can be quantified further with the expected probability adjustment. This adjustment is based on the argument that the x percent-chance discharge estimate made with a given sample is approximately the median of all estimates that would be made with successive samples of the same size. However, the probability distribution of the estimate is skewed, so the average of the samples exceeds the median. The consequence of this is that if a very large number of estimates of flood magnitude are made over a region, more x percent-chance floods will occur than expected on the average (Chow, Maidment, and Mays 1988). For example, more "100-year floods" will occur in the United States annually than expected. Paragraph 3-4 of EM 1110-2-1415 describes how either the probability associated with a specified magnitude or the magnitude for a specified probability can be adjusted to obtain a frequency curve with the expected number of exceedances.

e. Verification of frequency estimates.

(1) The reliability of frequency estimates depends on how well the proposed model represents the parent population. The fit can be tested indirectly with a simple graphical comparison of the fitted model and the sample or with a more rigorous statistical test. The reliability can also be illustrated with confidence limits.

(2) A graphical test provides a quick method for verifying frequency estimates derived with numerical procedures. The test is performed by plotting observed magnitude versus plotting-position estimates of exceedance probability. The postulated frequency curve with best-estimate parameters is plotted on the same grid. Goodness-of-fit is judged by inspection, as described previously.

(3) Because of the complexity of the log Pearson type III distribution, no single specialized plotting grid is practical for this graphical test. Instead, the log-normal grid is used to display data thought to be drawn from a log Pearson type III distribution. The fit is judged by inspection. Figure 12-4 illustrates this. The observed peaks and plotting positions from columns 7 and 8 of Table 12-1 are plotted here. Quantiles computed with Equation 12-10 are plotted on the same grid. The estimated values of the terms of Equation 12-5 are X = 3.3684; S = 0.2456; and G = 0.700. The skew was adjusted here with a regional skew. The computed frequency curve fits well the plotted observations.

(4) Rigorous statistical tests permit quantitative judgement of goodness of fit. These tests compare the theoretical distribution with sample values of the relative frequency or cumulative frequency function. For example, the Kolmogorov-Smirnov test provides bounds within which every observation should lie if the sample actually is drawn from the assumed distribution. The test is conducted as follows (Haan 1977):

(a) For each observation in the sample, determine the relative exceedance frequency. This is given by m/N, where m = the number of observations in the sample greater than or equal to the observed magnitude, and N = the number of observations.

(b) For each magnitude in the sample, determine the theoretical exceedance frequency using the hypothesized model and the best estimates of the parameters.

(c) For each observation, compute the difference in the relative exceedance frequency and the theoretical exceedance frequency. Determine the maximum difference for the sample.

(d) Select an acceptable significance level. This is a measure of the probability that the sample is not drawn from the candidate distribution. Values of 0.05 and 0.01 are common. Determine the corresponding Kolmogorov-Smirnov test statistic. This statistic is a function of the sample size and the significance level. Test statistics are tabulated or can be computed with the following equation (Loucks, Stedinger, and Harth 1981):

$$\frac{C}{\left[n^{0.5} + 0.12 + \left(\frac{0.11}{N^{0.5}} \right) \right]} \qquad (12\text{-}14)$$

where:

C = 1.358 for significance level 0.05

C = 1.628 for significance level 0.01

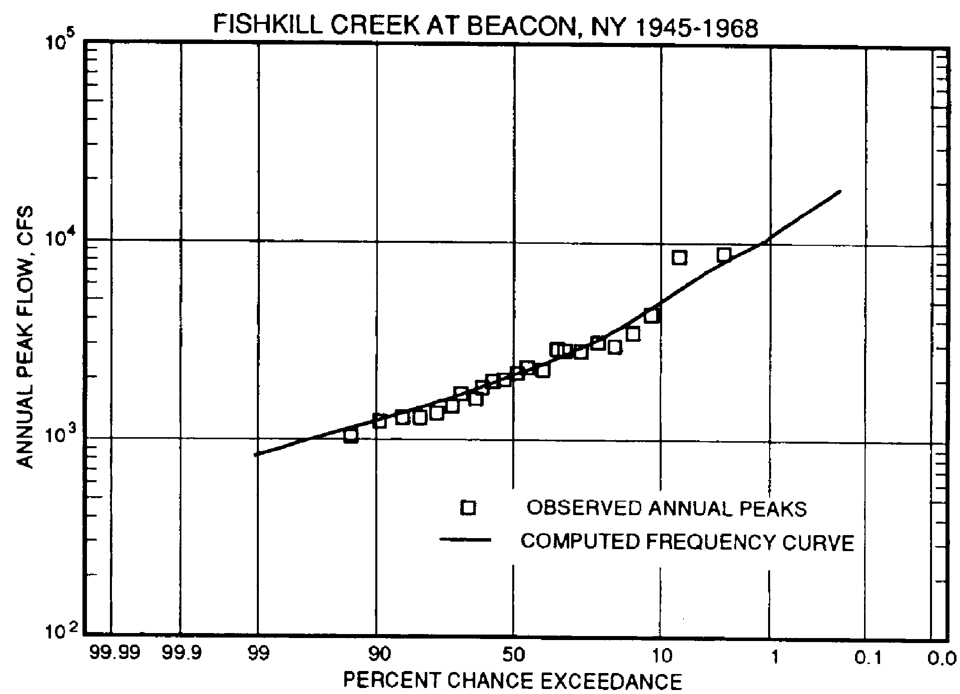

Figure 12-4. Plot for verification

(e) Compare the maximum difference determined in step c with the test statistic found in step d. If the value in step c exceeds the test statistic, the hypothesized distribution cannot be accepted with the specified significance level.

(5) The reliability of a computed frequency curve can be illustrated conveniently by confidence limits plotted on the frequency grid. Confidence limits are established considering the uncertainty in estimating population mean and standard deviation from a small sample. For convenience, Appendix 9 of Bulletin 17B (USWRC 1981) includes a table of frequency factors that permit definition of 1 percent to 99 percent confidence limits. These frequency factors are a function of specified exceedance probability and sample size. As the sample size increases, the limits narrow, indicating increased reliability.

(6) Figure 12-5 shows the 5 and 95 percent confidence limits for the Fishkill Creek frequency curve. The probability is 0.05 that the true quantile for a selected exceedance probability will exceed the value shown on the 5 percent curve. The probability is 0.95 that the true quantile will exceed the 95 percent-curve value and only 0.05 that it will be less than the 95 percent curve.

12-5. Special Considerations

a. Mixed populations. In certain cases, observed streamflow is thought to be the result of two or more independent hydrometeorological conditions. The sample is referred to as a mixed-population sample. For example, the spring streamflow in the Sacramento River, CA, is the result of both rainfall and snowmelt. For these cases, the data are segregated by cause prior to analysis, if possible. Each set can be analyzed separately to determine the appropriate distribution and parameters. The resulting frequency curves are then combined using the following equation to determine probability of union:

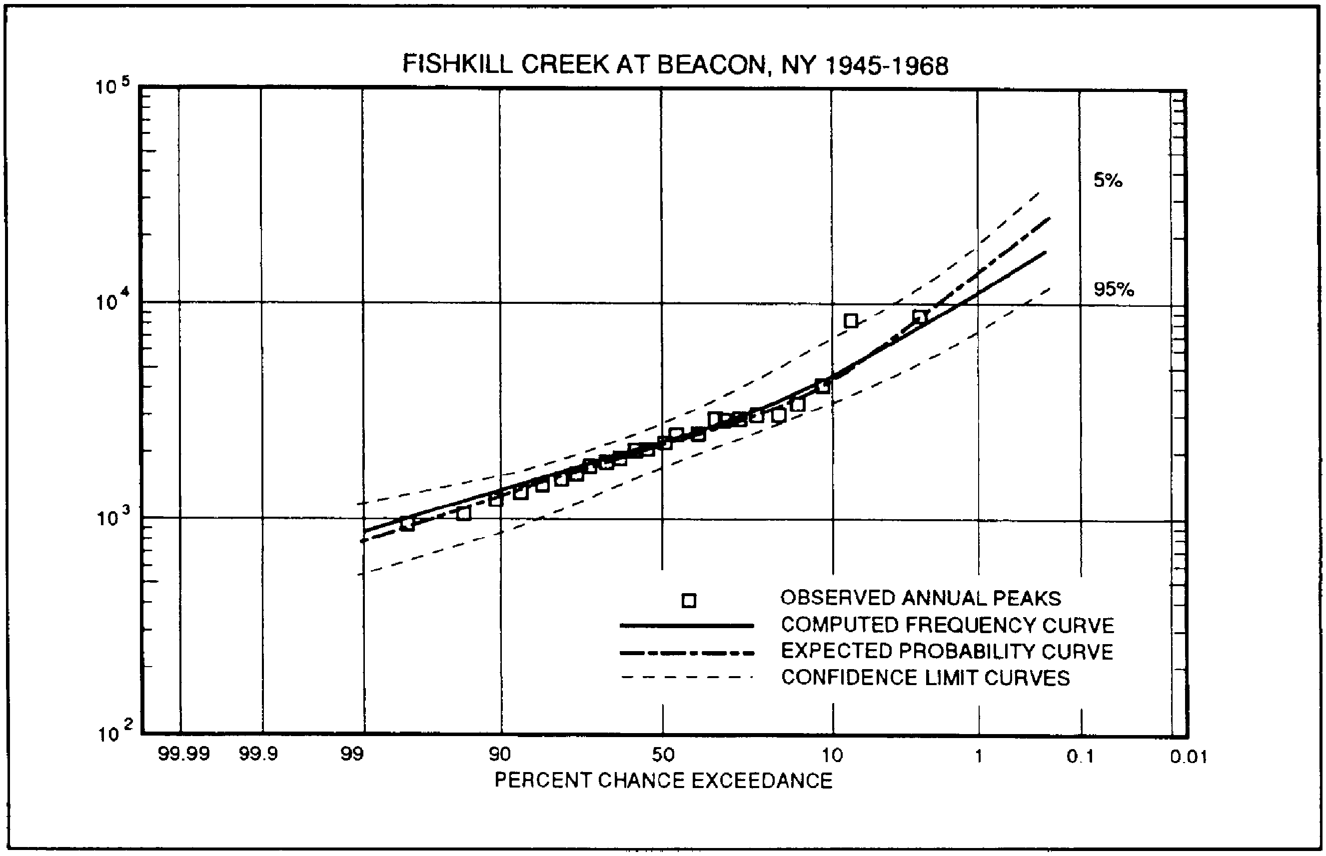

Figure 12-5. Frequency curve with confidence limits

$$P_c = P_1 + P_2 - P_1P_2 \qquad (12\text{-}5)$$

where:

P_c = annual exceedance probability of combined populations for a selected quantile

P_1 = annual exceedance probability of same magnitude for sample 1

P_2 = annual exceedance probability of same magnitude for sample 2

This assumes that the series are independent. Otherwise, coincident frequency analysis must be used.

b. Coincident frequency analysis. In some planning, designing, or operating problems, the hydrometeorological event of interest is a function of two or more random hydrometeorological events.

(1) For example, discharge at the confluence of two streams is a function of the coincident discharge in the tributary streams. The objective of coincident frequency analysis is to estimate the frequency distribution of the result if the frequency distributions of the components are known. The specific technique used depends on the mathematical form of the function relating the variables. Benjamin and Cornell (1970) describe a variety of solutions, including analytical closed-form solutions and Monte Carlo simulation.

(2) In hydrologic engineering, the variable of interest often is the sum of components. In that case, the frequency distribution of the sum can be found through conditional probability concepts. For illustration, consider the total discharge downstream of a confluence, Q_T. This is computed as the sum of tributary discharge Q_1 and tributary discharge Q_2. The frequency of Q_1 and Q_2 are established using procedures described previously. Roughly speaking, the probability that Q_T equals some

specified value, q_T, is proportional to the probability that Q_1 equals a specified value, q_1, times a factor proportional to the probability that Q_2 equals q_T - q_1. This product is summed over all possible values of Q_1. To develop a frequency curve for the sum, the process is repeated for all possible values of Q_T. Chapter 11 of EM 1110-2-1415 presents a detailed example of coincident frequency analysis.

c. Regional frequency analysis.

(1) Methods of frequency analysis described previously in this chapter apply to data collected at a single site. If a large sample is available at that site, the resulting frequency analysis may be sufficiently reliable for planning, designing, or operating civil-works projects. However, samples commonly are small. In fact, it is not unusual that risk information is required at sites for which no data are available. Regional frequency analysis techniques may be used to develop this information.

(2) Regional frequency analysis procedures relate parameters of a streamflow-frequency model to catchment characteristics. Briefly, the following general steps are followed to derive such a relationship:

(a) Select long-record sites within the region, and collect streamflow data for those sites.

(b) Select an appropriate distribution for the data, and estimate the parameters using the procedures described herein.

(c) Select catchment characteristics that should correlate with the parameters. Measure or observe these characteristics for the long-record sites. Typical characteristics for streamflow frequency model parameters include the following: contributing drainage area, stream length, slope of catchment or main channel, surface storage, mean annual rainfall, number of rainy days annually, infiltration characteristics, and impervious area.

(d) Perform a regression analysis to establish predictive equations. The dependent variables in the equations are the frequency model parameters. The independent variables are the catchment characteristics.

(3) EM 1110-2-1415 provides additional guidance in establishing regional equations.

d. Frequency of other hydrometeorological phenomena. The procedures described for discharge-frequency analysis apply to analysis of other hydrometeorological phenomena. The same general steps presented in paragraph 12-4 are followed. For example, if the variable of interest is streamflow volume, rather than discharge, the time series will be a sequence of volumes for a specified duration. The procedures for selecting, calibrating, and verifying a frequency model are the same as previously described.

e. Volume-frequency and precipitation-depth-duration-frequency analyses. These analyses present some unique problems. Because of the small samples from which parameters must be estimated, the set of frequency curves for various durations may be inconsistent. For example, the 1-day volume should not exceed the 3-day volume for all probabilities. Yet, for a small sample, the computed curves may not follow this rule. To overcome this, the computed curves may be "smoothed," adjusted by inspection of plots. Alternatively, the statistical model parameters can be adjusted to maintain consistency. Paragraph 3-8*c* of EM 1110-2-1415 describes a typical procedure.