APPENDIX D
ELASTIC PARAMETERS

D-1. General. The magnitudes of soil elastic distortion or immediate settlement for practical applications are evaluated from the elastic soil parameters Young's modulus E_s, shear modulus G_s and Poisson's ratio ν_s. For most practical applications the foundation soil is heterogeneous or multilayered in which the elastic parameters can vary significantly from layer to layer.

D-2. Elastic Young's Modulus. Young's elastic modulus is commonly used for estimation of settlement from static loads. Suitable values of the elastic modulus E_s as a function of depth may be estimated from empirical correlations, results of laboratory tests on undisturbed specimens and results of field tests.

a. Definition. Materials that are truly elastic obey Hooke's law in which each equal increment of applied uniaxial stress σ_z causes a proportionate increase in strain ε_z

$$\varepsilon_z = \frac{1}{E} \bullet \sigma_z \qquad \text{(D-1)}$$

where E is Young's modulus of elasticity, Table D-1. Figure D-1 illustrates the stress path for the uniaxial (UT) and other test methods. An elastic material regains its initial dimensions following removal of the applied stress.

(1) Application to soil. Hooke's law, which is applicable to homogeneous and isotropic materials, was originally developed from the observed elastic behavior of metal bars in tension. Soil is sometimes assumed to behave linearly elastic under relatively small loads. A partially elastic material obeys Hooke's law during loading, but this material will not gain its initial dimensions following removal of the applied stress. These materials are nonlinear and include most soils, especially foundation soil supporting heavy structures that apply their weight only once.

(2) Assumption of Young's elastic modulus. Soils tested in a conventional triaxial compression (CTCT) device under constant lateral stress will yield a tangent elastic modulus E_t equivalent with Young's modulus. The soil modulus E_s is assumed approximately equal to Young's modulus in practical applications of the theory of elasticity for computation of settlement.

(3) Relationship with other elastic parameters. Table D-2 relates the elastic modulus E with the shear modulus G, bulk modulus K and constrained modulus E_d. These parameters are defined in Table D-1.

b. Empirical Correlations. The elastic undrained modulus E_s for clay may be estimated from the undrained shear strength C_u by

$$E_s = K_c C_u \qquad \text{(D-2)}$$

where

E_s = Young's soil modulus, tsf
K_c = correlation factor, Figure D-2
C_u = undrained shear strength, tsf

Table D-1

Laboratory Tests for Evaluation of Elastic Parameters (Refer to Figure D-1)

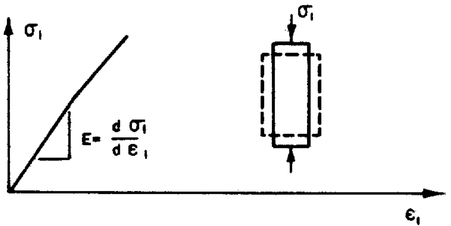
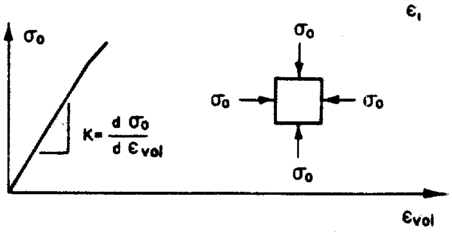
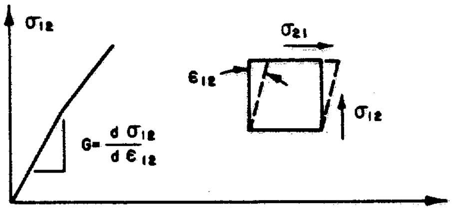
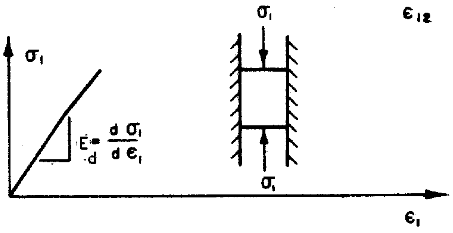
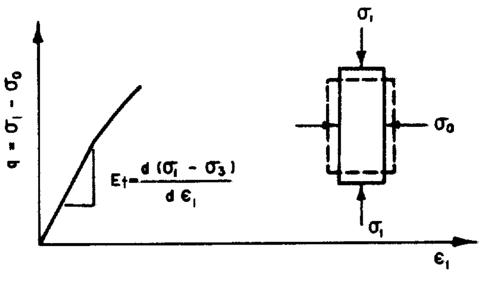
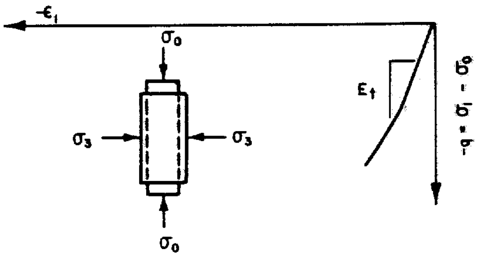
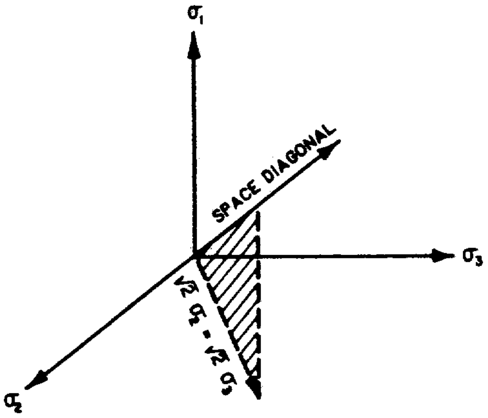
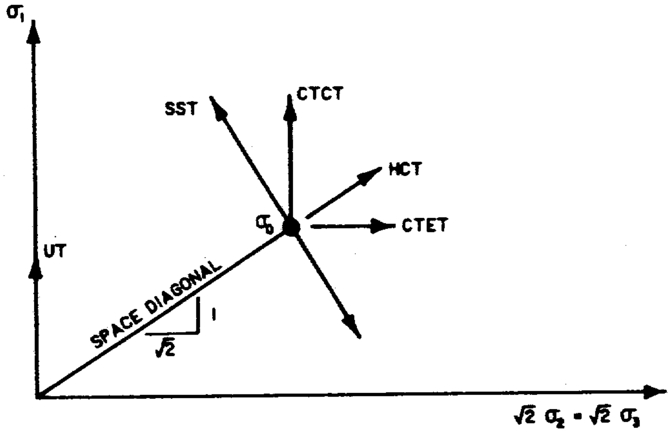
TYPE OF TEST	DESCRIPTION	DIAGRAM
UNIAXIAL STRESS (UT)	LOADING σ_1 ON A SINGLE VERTICAL AXIS. $\sigma_2 = \sigma_3 = 0$. YOUNG'S MODULUS E IS DETERMINED.	
HYDROSTATIC COMPRESSION (HCT)	LOADING OCCURS ALONG THE SPACE DIAGONAL IN EQUAL INCREMENTS $\sigma_0 = \sigma_1 = \sigma_2 = \sigma_3$ AND $\epsilon_{vol} = \epsilon_1 + \epsilon_2 + \epsilon_3$ BULK MODULUS K IS DETERMINED.	
SIMPLE SHEAR (SST)	AFTER HYDROSTATIC LOADING TO $\sigma_0 = \sigma_{oct}$ $(\sigma_1 + \sigma_2 + \sigma_3)/3$ KEPT CONSTANT, BUT TWO OF THREE STRESS AXES VARIED; i.e., $\Delta\sigma_1 = -\Delta\sigma_3$, $\Delta\sigma_2 = 0$. SHEAR MODULUS G IS DETERMINED.	
CONFINED COMPRESSION (CCT)	LOADING σ_1 WHEN $E_2 = E_3 = 0$ (CONSOLIDATION TEST) CONSTRAINED MODULUS E_d IS DETERMINED.	
CONVENTIONAL TRIAXIAL COMPRESSION (CTCT)	AFTER HYDROSTATIC LOADING TO σ_0, σ_1 INCREASED WHILE $\sigma_2 = \sigma_3$ KEPT CONSTANT AT σ_0. TANGENT MODULUS E_t IN COMPRESSION DETERMINED.	

Table D-1. Concluded

TYPE OF TEST	DESCRIPTION	DIAGRAM
CONVENTIONAL TRIAXIAL EXTENSION (CTET)	AFTER HYDROSTATIC LOADING TO σ_0, $\sigma_2 = \sigma_3$ INCREASED WHILE σ_1 KEPT CONSTANT AT σ_0. TANGENT MODULUS E_t IN EXTENSION DETERMINED.	



a. THE TRIAXIAL PLANE



b, STRESS PATHS

Figure D-1. Examples of stress paths for different tests
(Refer to Table D-1 for descriptions of tests)

Table D-2

Relationships Between Elastic Parameters

Parameter	Relationship
Shear modulus G , tsf	$\frac{E}{2(1 + \nu)}$ ν = *Poisson's ratio*
Bulk Modulus K , tsf	$\lambda_L + \frac{2G}{3}$ λ_L = *Lames's constant*
Constrained modulus E_d , tsf	$\frac{E(1 - \nu)}{(1 + \nu)(1 - 2\nu)}$

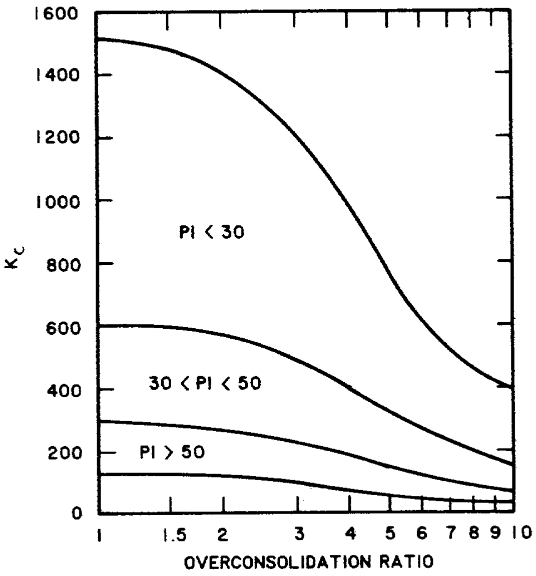


Figure D-2. Chart for estimating constant K_c to determine the elastic modulus $E_s = K_cC_u$ from the undrained shear strength (after Figure 3-20, TM 5-818-1)

The values of K_c as a function of the overconsolidation ratio and plasticity index PI have been determined from field measurements and are therefore not affected by soil disturbance compared with measurements on undisturbed soil samples. Table D-3 illustrates some typical values for the elastic modulus.

Table D-3

Typical Elastic Moduli

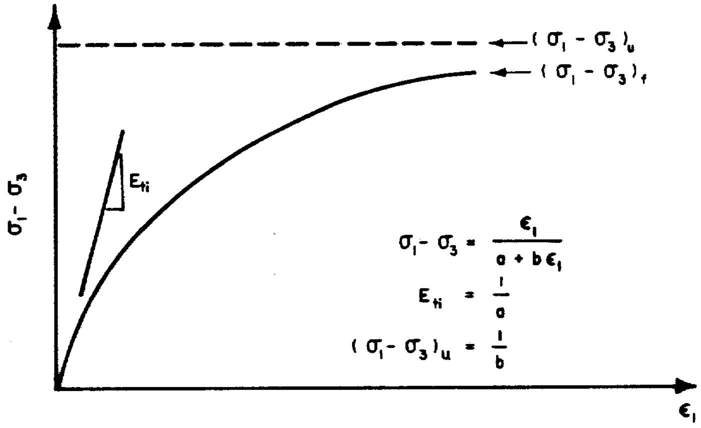
Soil	E_s, tsf
Clay	
Very soft clay	5 - 50
Soft clay	50 - 200
Medium clay	200 - 500
Stiff clay, silty clay	500 - 1000
Sandy clay	250 - 2000
Clay shale	1000 - 2000
Sand	
Loose sand	100 - 250
Dense sand	250 - 1000
Dense sand and gravel	1000 - 2000
Silty sand	250 - 2000

c. Laboratory Tests on Cohesive Soil. The elastic modulus is sensitive to soil disturbance which may increase pore water pressure and, therefore, decrease the effective stress in the specimen and reduce the stiffness and strength. Fissures, which may have little influence on field settlement, may reduce the measured modulus compared with the in situ modulus if confining pressures are not applied to the soil specimen.

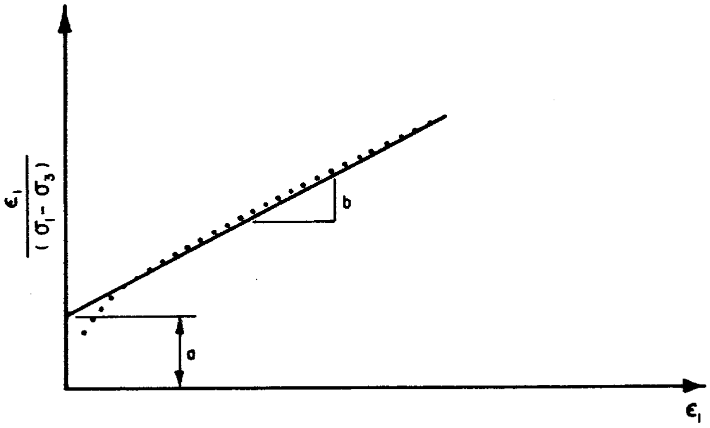
(1) Initial hyperbolic tangent modulus. Triaxial unconsolidated undrained (Q or UU) compression tests may be performed on the best available undisturbed specimens at confining pressures equal to the total vertical overburden pressure σ_o for that specimen when in the field using the Q test procedure described in EM 1110-2-1906, Laboratory Soils Testing. An appropriate measure of E_s is the initial tangent modulus $E_{ti} = 1/a$ where a is the intercept of a plot of strain/deviator stress versus strain, Figure D-3 (item 14).

(2) Reload modulus. A triaxial consolidated undrained (R or CU) compression test may be performed on the best available undisturbed specimens. The specimen is initially fully consolidated to an isotropic confining pressure equal to the vertical overburden pressure σ_o for that specimen in the field. The R test procedure described in EM 1110-2-1906 may be used except as follows: stress is increased to the magnitude estimated for the field loading condition. The axial stress may then be reduced to zero and the cycle repeated until the reload curve shows no further increase in slope. The tangent modulus at 1/2 of the maximum applied stress is determined for each loading cycle and plotted versus the number of cycles, Figure D-4. An appropriate measure of E_s is the reload tangent modulus that approaches the asymptotic value at large cycles.

d. Field Tests. The elastic modulus may be estimated from empirical and semiempirical relationships based on results of field soil tests. Refer to



a. HYPERBOLIC RELATIONSHIP



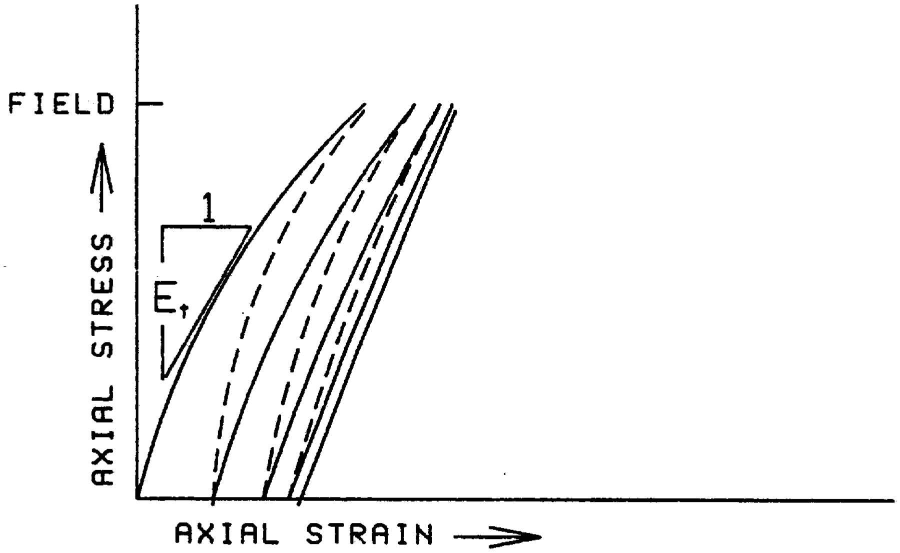
b. EVALUATION OF HYPERBOLIC PARAMETERS a, b

Figure D-3. Hyperbolic simulation of stress-strain relationships

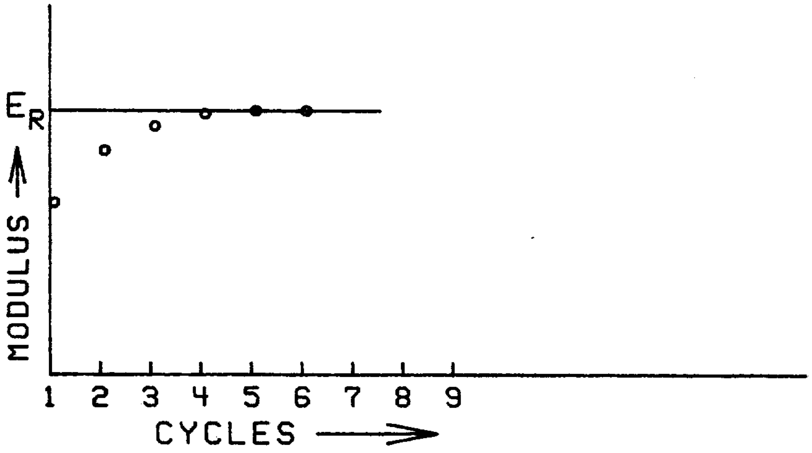
EM 1110-1-1804, Geotechnical Investigations, for more information on in situ tests.

(1) Plate load test. The plate load test performed in accordance with ASTM Standard Test Method D 1194, "Bearing Capacity of Soil for Static Loads on Spread Footings" is used to determine the relationship between settlement and plate pressure q_p, Figure D-5. The elastic modulus E_s is found from the slope of the curve $\Delta\rho/\Delta q_p$

$$E_s = \frac{(1 - \nu_s^2)}{\frac{\Delta\rho}{\Delta q_p}} \bullet B_p \bullet I_w \tag{D-3}$$



a. TANGENT MODULUS AT 1/2 MAXIMUM APPLIED STRESS



b. TANGENT RELOAD MODULUS VERSUS CYCLES

Figure D-4. Elastic modulus from cyclic load tests

where

E_s = Young's soil modulus, psi
ν_s = Poisson's ratio, 0.4
$\frac{\Delta \rho}{\Delta q_p}$ = slope of settlement versus plate pressure, inches/psi
B_p = diameter of plate, inches
I_w = influence factor, $\pi/4$ for circular plates

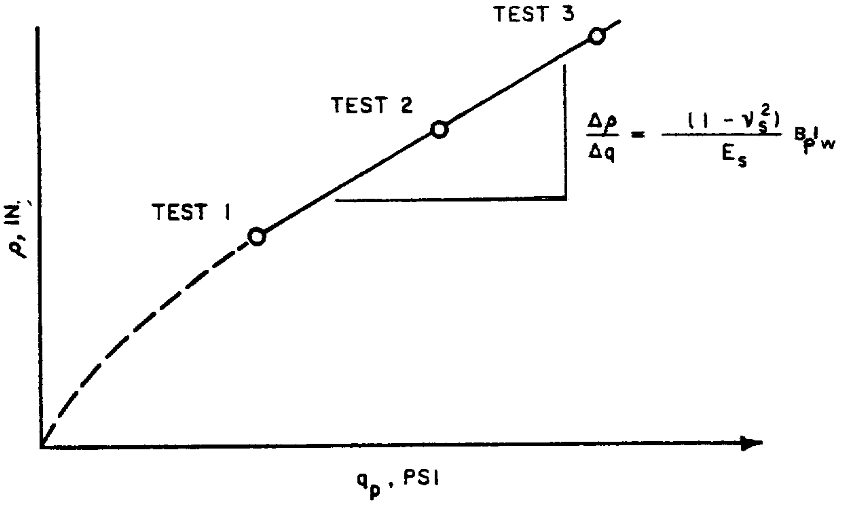


Figure D-5. Graphical solution of soil elastic modulus E_s from the plate load test. $I_w = \pi/4$ for circular rigid plate of diameter B_p . ν_s = Poisson's ratio

This elastic modulus is representative of soil within a depth of $2B_p$ beneath the plate.

(2) Cone penetration test (CPT). The constrained modulus E_d has been empirically related with the cone tip bearing resistance by

$$E_d = \alpha_c \cdot q_c \tag{D-4}$$

where

E_d = Constrained modulus, tsf
α_c = correlation factor depending on soil type and the cone bearing resistance, Table D-4
q_c = cone tip bearing resistance, tsf

A typical value for sands is $\alpha_c = 3$, but can increase substantially for overconsolidated sand. A typical value for clays is $\alpha_c = 10$ when used with the net cone resistance $q_c - \sigma_o$ where σ_o is the total overburden pressure. The undrained shear strength C_u is related to q_c by

$$C_u = \frac{q_c - \sigma_c}{N_k} \tag{D-5}$$

where

C_u = undrained shear strength, tsf
q_c = cone tip resistance, tsf
σ_o = total overburden pressure, tsf
N_k = cone factor

The cone factor often varies from 10 to 20 and can be greater.

Table D-4

Correlation Factor α_c (Data from Item 44)

Soil	Resistance q_c , tsf	Water Content, percent	α_c
Lean clay (CL)	<7		3 to 8
	7 to 20		2 to 5
	>20		1 to 2.5
Silt (ML)	<20		3 to 6
	>20		1 to 3
Plastic silt clay (CH,MH)	<20		2 to 6
Organic silt	<12		2 to 8
Organic clay	<7	50 to 100	1.5 to 4
peat		100 to 200	1 to 1.5
		>200	0.4 to 1
Sand	<50		2 to 4
	>100		1.5
			$1 + D_r^2$
Clayey sand			3 to 6
Silty sand			1 to 2
Chalk	20 to 30		2 to 4

* Note: D_r = relative density, fraction

(3) Standard penetration test (SPT). The elastic modulus in sand may be estimated directly from the blow count by (item 60)

$$E_s = 9.4 N^{0.87} \sqrt{B} \left(1 + 0.4 \frac{D}{B}\right) \tag{D-6}$$

where

E_s = Young's soil modulus, tsf
N = average blow count per foot in the stratum, number of blows of a 140 pound hammer falling 30 inches to drive a standard sampler (1.42" ID, 2.00" OD) one foot. Sampler is driven 18 inches and blows counted the last 12 inches.
B = width of footing, ft
D = depth of embedment of footing, ft

Equation D-6 was developed from information in the literature and original settlement observations without consideration of the energy of the hammer. An alternative method of estimating the elastic modulus for footing foundations on clean sand or sand and gravel is (after item 12)

Preloaded sand: $E_m = 420 + 10N_{ave}$ (D-7a)

Normally loaded sand or sand and gravel: $E_m = 194 + 8N_{ave}$ (D-7b)

where

E_m = deformation modulus, $\frac{E_s}{1 - \nu_s^2}$, tsf

N_{ave} = average measured blow count in depth H = B below footing, blows/ft

(4) Pressuremeter test (PMT). The preboring pressuremeter consists of a cylindrical probe of radius R_o containing an inflatable balloon lowered into a borehole to a given depth. The pressure required to inflate the balloon and probe against the side of the borehole and the volume change of the probe are recorded. The self-boring pressuremeter includes cutting blades at the head of the device with provision to permit drilling fluids to circulate and carry cuttings up to the surface. The self-boring pressuremeter should in theory lead to a less disturbed hole than the preboring pressuremeter. The pressure and volume change measurements are corrected for membrane resistance and volume losses leading to the corrected pressuremeter curve, Figure D-6. The preboring pressuremeter curve indicates a pressuremeter modulus E_i that initially increases with increasing radial dimensional change, $\Delta R/R_o$, as shown in Figure D-6. The self-boring pressuremeter curve is characteristic of

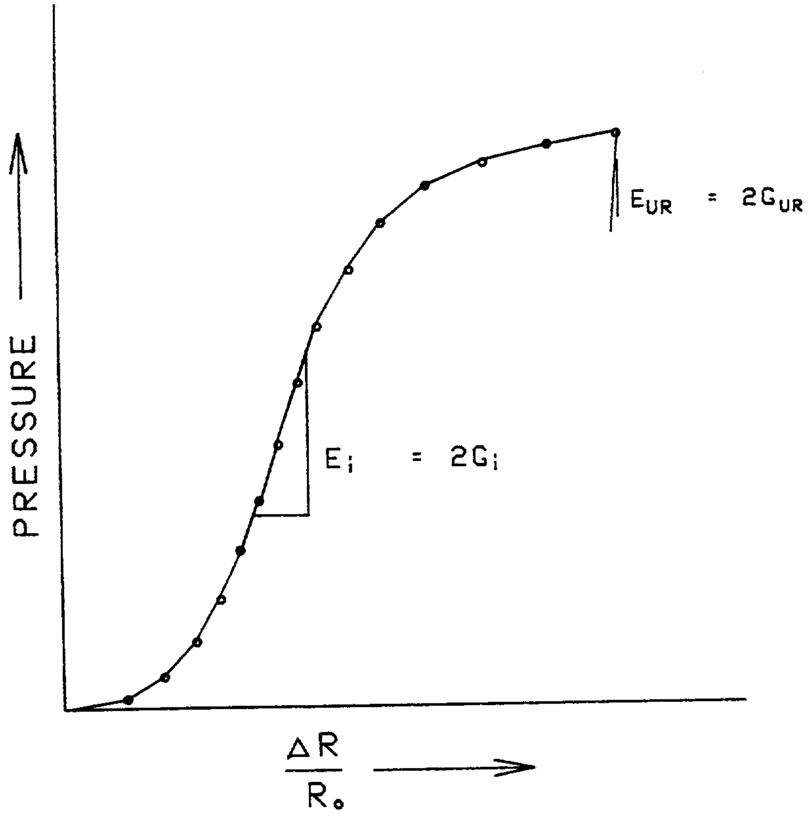


Figure D-6. Example corrected preboring pressuremeter curve an initially high pressuremeter modulus E_i that decreases with increasing volume change without the initial increasing modulus shown in the figure. The pressuremeter modulus is a measure of twice the shear modulus. If the soil is perfectly elastic in unloading, characteristic of a sufficiently small unload-reload cycle, the gradient will be $2G_{UR}$ (item 23). The unload-reload modulus should be determined on the plastic portion of the pressuremeter curve. The pressuremeter modulus may be evaluated from the gradient of the unload-reload cycle by (ASTM 4719)

$$E_p = \frac{(1 + \nu_s) \bullet \Delta P \bullet (R_{po} + \Delta R_{pm})}{\Delta R_p} \qquad \text{(D-8)}$$

where

ν_s = soil Poisson’s ratio, 0.33
ΔP = change in pressure measured by the pressuremeter, tsf
R_{po} = radius of probe, inches
ΔR_{pm} = change in radius from R_{po} at midpoint of straight portion of the pressuremeter curve, inches
ΔR_p = change in radius between selected straight portions of the pressuremeter curve, inches

e. <u>Equivalent Elastic Modulus</u>. The following two methods are recommended for calculating an equivalent elastic modulus of cohesive soil for estimating settlement of mats and footings.

(1) Kay and Cavagnaro approximation. The equivalent elastic modulus E_s^* may be calculated by (item 31)

$$E_s^* = \frac{2 \bullet q \bullet R \bullet (1 - \nu_s^2)}{\rho_c} \qquad \text{(D-9)}$$

where

E_s^* = equivalent elastic modulus, tsf
q = bearing pressure, tsf
R = equivalent mat radius,
L = length of mat, ft $\sqrt{LB/\pi}$, $L < 2B$, ft
B = width of mat, ft
ρ_c = center settlement from the Kay and Cavagnaro method, Figure 3-10, ft

(2) Semiempirical method. The equivalent elastic modulus of a soil with elastic modulus increasing linearly with depth may be estimated by

$$E_s^* = \frac{2 \bullet k \bullet R \bullet (1 - \nu_s^2)}{0.7 + (2.3 - 4 \bullet \nu_s) \bullet \log n} \qquad \text{(D-10)}$$

where

k = constant relating soil elastic modulus E_s with depth z
$E_s = E_o + kz$, tons/ft^3
D = depth of foundation below ground surface, ft
n = $kR/(E_o + kD_b)$
E_o = elastic soil modulus at the ground surface, tsf

Equation D-10 was developed from results of a parametric study using Equation D-9 (item 29).

(3) Gibson model. The equivalent modulus of a soil with elastic modulus increasing linearly with depth and $E_o = 0$ is (item 19)

$$E_s^* = \frac{Bk}{2} \qquad \text{(D-11)}$$

where B is the minimum width of the foundation, ft.

D-3. Shear Modulus. The shear modulus G may be used for analysis of settlement from dynamic loads.

a. Definition. Shear stresses applied to an elastic soil will cause a shear distortion illustrated by the simple shear test (SST), Table D-1.

b. Evaluation by Dynamic Tests. The shear modulus may be evaluated from dynamic tests after methodology of Chapter 17, TM 5-818-1, Procedures for Foundation Design of Buildings and Other Structures (Except Hydraulic Structures).

c. Relationships with Other Parameters. Table D-2 illustrates the relationship of the shear modulus with Young’s elastic E and bulk modulus K .

D-4. Poisson’s Ratio. A standard procedure for evaluation of Poisson’s ratio for soil does not exist. Poisson’s ratio ν_s for soil usually varies from 0.25 to 0.49 with saturated soils approaching 0.49. Poisson’s ratio for unsaturated soils usually vary from 0.25 to 0.40. A reasonable overall value for ν_s is 0.40. Normal variations in elastic modulus of foundation soils at a site are more significant in settlement calculations than errors in Poisson’s ratio.