APPENDIX C
STRESS DISTRIBUTION IN SOIL

C-1. General. Displacements in soil occur from stresses applied by loading forces. The distribution of stress in soil should be known for realistic estimates of displacements caused by applied loads on the supporting soil.

a. Effect of Foundation Stiffness. The distribution of stress in soil depends on the contact pressure between the foundation and soil, which is a function of the relative stiffness K_R between the soil and the foundation (item 3)

$$K_R = \frac{E_f(1 - \nu_s^2)}{E_s}\left[\frac{D_f}{R}\right]^3 \tag{C-1}$$

where

E_f = Young's modulus of foundation, tsf
ν_s = Poisson's ratio of foundation soil
E_s = Young's modulus of foundation soil, tsf
D_f = thickness of foundation, ft
R = radius of foundation, ft

A uniformly loaded flexible foundation where stiffness $K_R < 0.1$ causes a uniform contact pressure; whereas, a uniformly loaded rigid foundation where $K_R > 10$ causes a highly nonuniform contact pressure distribution, Figure 1-3.

(1) Embankments. Earth embankments are flexible and normally in full contact with the supporting soil.

(2) Foundations for structures. Foundations such as large mats and footings with sufficient stiffness ($K_R > 0.1$) may not always be in complete contact with the soil.

b. Limiting Contact Pressures. Contact pressures are limited to maximum pressures defined as the bearing capacity. Refer to EM 1110-2-1903, Bearing Capacity, for estimation of the soil bearing capacity.

c. Other Factors Influencing Contact Pressure. The distribution of contact pressures is also influenced by the magnitude of loading, depth of applied loads, size, shape, and method of load application such as static or dynamic applied loads.

C-2. Evaluation of Stress Distributions in Soil. The following methods may be used to estimate the stress distribution for an applied load Q at a point and a uniform contact pressure q applied to an area. Practical calculations of settlement are based on these estimates of stress distribution.

a. Approximate 2:1 Distribution. An approximate stress distribution assumes that the total applied load on the surface of the soil is distributed over an area of the same shape as the loaded area on the surface, but with dimensions that increase by an amount equal to the depth below the surface, Figure C-1. At a depth z in feet below the ground surface the total load Q in tons applied at the ground surface by a structure is assumed to be uniform-

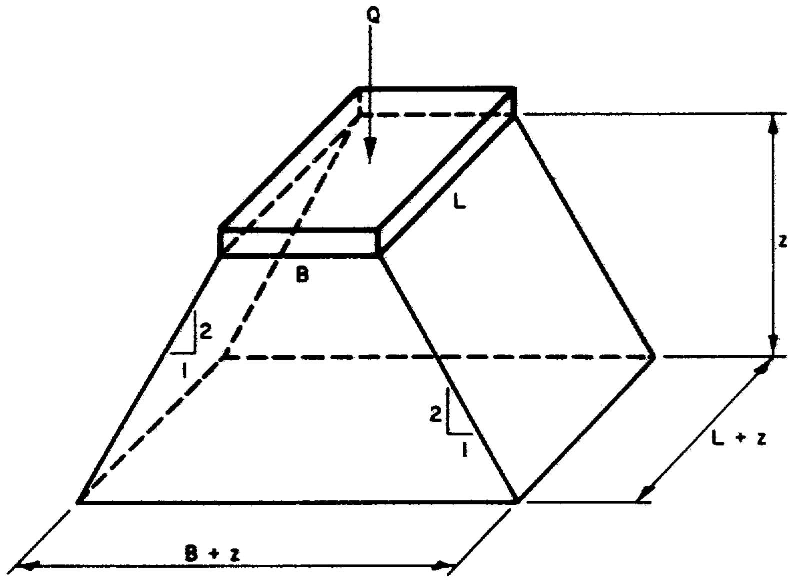


Figure C-1. Approximate stress distribution by the 2:1 method

ly distributed over an area (B + z) by (L + z). The increase in vertical pressure $\Delta\sigma_z$ in units of tsf at depth z for an applied load Q is given by

$$\Delta\sigma_z = \frac{Q}{(B + z) \bullet (L + z)} \quad \text{(C-2)}$$

where B and L are the width and length of the foundation in feet, respectively. $\Delta\sigma_z$ may be the pressure σ_{st} caused by construction of the structure. Vertical stresses calculated by Equation C-2 agree reasonably well with the Boussinesq method discussed below for depths between B and 4B below the foundation.

b. Boussinesq. The Boussinesq solution is based on the assumption of a weightless half space free of initial stress and deformation. The modulus of elasticity is assumed constant and the principle of linear superposition is assumed valid.

(1) Equations for strip and area loads. The Boussinesq solutions for increase in vertical stress $\Delta\sigma_z$ shown in Table C-1 for strip and area loads apply to elastic and nonplastic materials where deformations are continuous and unloading/reloading do not occur. Refer to EM 1110-2-1903, Bearing Capacity, and item 40 for equations of stress distributions beneath other foundation shapes.

(2) Graphical solutions. The stress distribution beneath a corner of a rectangular uniformly loaded area may be evaluated from

$$\Delta\sigma_z = I_\sigma \bullet q \quad \text{(C-3)}$$

where I_σ is the influence factor from Figure C-2 and q is the bearing pressure. Refer to TM 5-818-1 for further information on I_σ .

Table C-1

<u>Boussinesq Solutions for Increase in Vertical Stress $\Delta\sigma_z$ Beneath a Foundation</u>

a. Uniform Loads

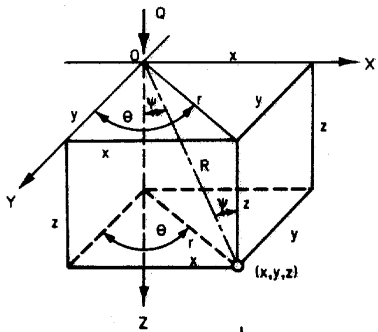
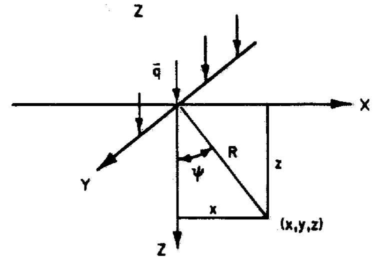
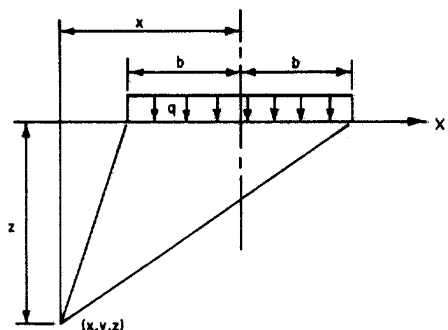
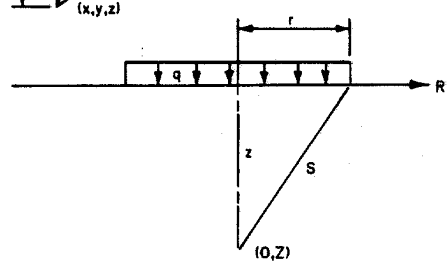
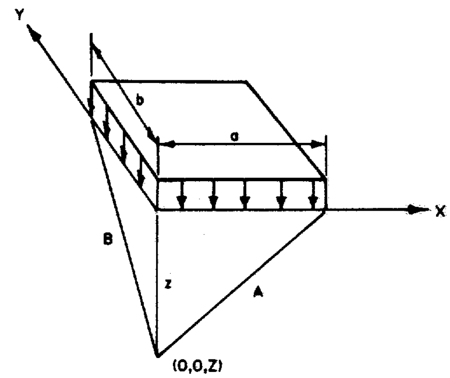
TYPE OF LOAD NORMAL TO SURFACE	EQUATION FOR $\Delta\sigma_z$	COORDINATE SYSTEM
POINT	$\frac{3Q}{2\pi R^2}\cos^3\psi$ Q = NORMAL LOAD, TONS $r^2 = x^2 + y^2$ $R^2 = r^2 + z^2$	
LINE	$\frac{2\bar{q}z^3}{\pi R^4}$ $\bar{q}$ = NORMAL LOAD, TONS/FT $R^2 = x^2 + z^2$	
STRIP	$\frac{q}{\pi}(\alpha + \sin\alpha \cos(\alpha + 2\beta))$ q = CONTACT PRESSURE, TSF $\alpha = \tan^{-1}\left(\frac{x+b}{z}\right) - \beta$, RADIANS $\beta = \tan^{-1}\left(\frac{x-b}{z}\right)$, RADIANS	
AREA (POINT UNDER CENTER CIRCULAR AREA)	$qr^2 \frac{(S^2 + 2z^2)}{2S^4}$ $S^2 = r^2 + z^2$ q = CONTACT PRESSURE, TSF	

Table C-1. Continued

TYPE OF LOAD NORMAL TO SURFACE	EQUATION FOR $\Delta\sigma_z$	COORDINATE SYSTEM
(POINT UNDER CORNER RECTANGULAR AREA)	$\frac{q}{2\pi}\left[\mathrm{TAN}^{-1}\frac{a\,b}{z\,C} + \frac{a\,b\,z}{C}\left(\frac{1}{A^2}+\frac{1}{B^2}\right)\right]$ $A^2 = a^2 + z^2$ $B^2 = b^2 + z^2$ $C = (a^2 + b^2 + z^2)^{1/2}$ q = CONTACT PRESSURE, TSF	

b. Nonuniform Strip Loads

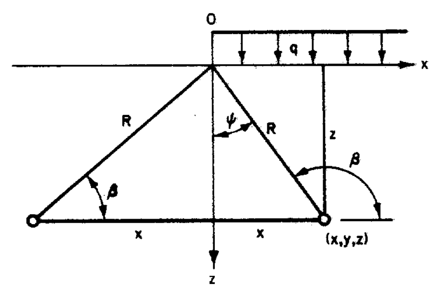
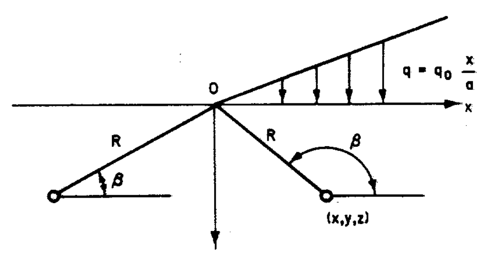
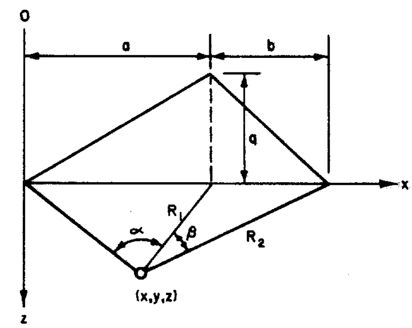
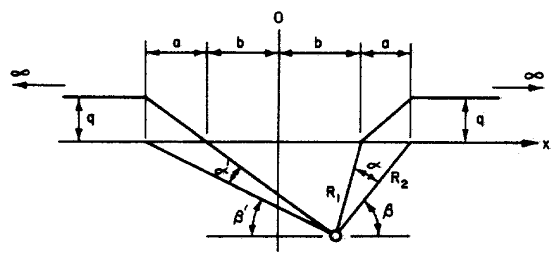
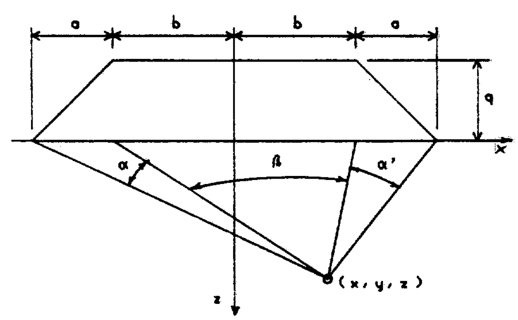
TYPE OF LOAD NORMAL TO SURFACE	EQUATION FOR $\Delta\sigma_z$	COORDINATE SYSTEM
UNIFORM ON HALF INFINITE SPACE	$\frac{q}{\pi}\left(\beta + \frac{xz}{R^2}\right)$ q = CONTACT PRESSURE, TSF β = ANGLE IN RADIANS	
LINEAR INCREASE	$\frac{q_o}{\pi a}(x\beta + z)$	

Table C-1. Concluded

TYPE OF LOAD NORMAL TO SURFACE	EQUATION FOR $\Delta\sigma_z$	COORDINATE SYSTEM
TRIANGULAR	$\frac{q}{\pi}\left[\frac{x}{a}\alpha + \frac{(a + b - x)}{b}\beta\right]$	
CONTRATRAPEZOIDAL	$\frac{q}{\pi a}\left[a(\beta + \beta') - b(\alpha + \alpha') + x(\alpha - \alpha')\right]$	
TRAPEZOIDAL	$\frac{q}{\pi a}\left[a(\alpha+\beta+\alpha')+b(\alpha+\alpha')+x(\alpha-\alpha')\right]$	

(a) The increase in stress beneath the center of a rectangular loaded area is given through the assumption of superposition of stresses as 4 times that given by Equation C-3.

(b) The increase in stress beneath the center of an edge is twice that given by Equation C-3.

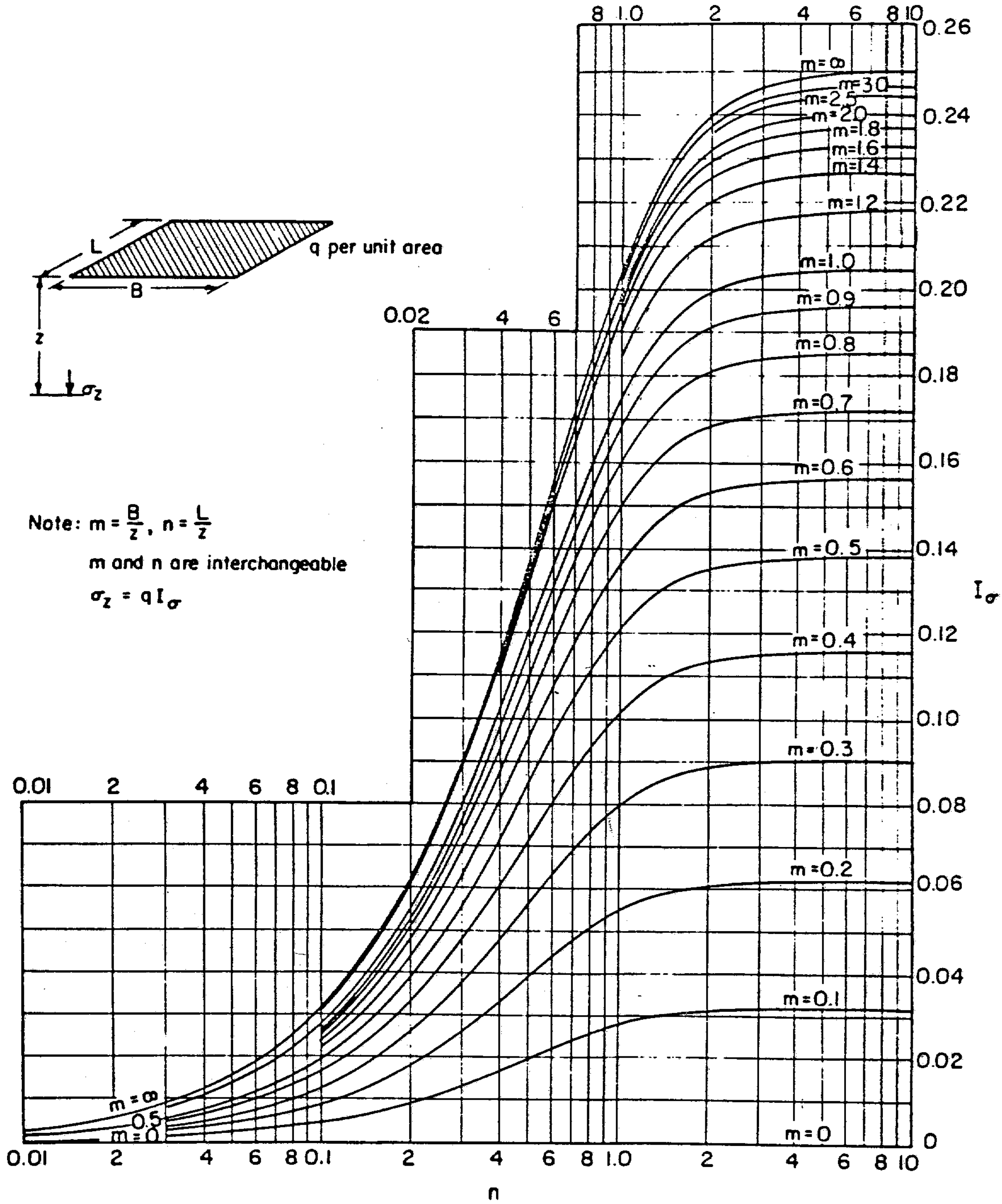


Figure C-2. Influence factor I_σ for the increase in vertical stress beneath a corner of a uniformly loaded rectangular area for the Boussinesq stress distribution (from TM 5-818-1)

(c) Refer to EM 1110-2-1903, Bearing Capacity, and TM 5-818-1, Procedures for Foundation Design of Buildings and Other Structures (Except Hydraulic Structures), for further details on estimation of stress distributions.

c. Westergaard. Soils that are stratified with strong layers may reinforce soft layers so that the resulting stress intensity at deeper depths is less than that formulated for isotropic soil after the Boussinesq approach. The Westergaard solution assumes that stratified soil performs like an elastic medium reinforced by rigid, thin sheets.

(1) Rectangular area. The increase in vertical stress beneath a corner of a rectangular uniformly loaded area may be evaluated from Equation C-3 where the influence factor I_σ is found from Figure C-3. Refer to Design Manual NAVFAC 7.1 for further information.

(2) Other areas. Item 40 provides Westergaard solutions of vertical stress beneath uniformly loaded foundations using influence charts.

C-3. Limitations of Theoretical Solutions. Boundary conditions may differ substantially from idealized conditions to invalidate solutions by elasticity theory.

a. Initial Stress. Elastic solutions such as the Boussinesq solution assumes a weightless material not subject to initial stress. Initial stress always exists in situ because of overburden weight of overlying soil, past stress history, and environmental effects such as desiccation. These initial stresses through Poisson's ratio, nonlinear elastic modulus and soil anisotropy significantly influence in situ stress and strain that occur through additional applied loads.

b. Error in Stress Distribution. Actual stresses beneath the center of shallow footings may exceed Boussinesq values by 15 to 30 percent in clays and 20 to 30 percent in sands (item 5).

c. Critical Depth. The critical depth z_c is the depth at which the increase in stresses $\Delta\sigma_z$ from foundation loads decrease to about 10 (cohesionless soil) to 20 (cohesive soil) percent of the effective vertical overburden pressure σ'_o (item 5). Errors in settlement contributed by nonlinear, heterogeneous soil below the critical depth are not significant.

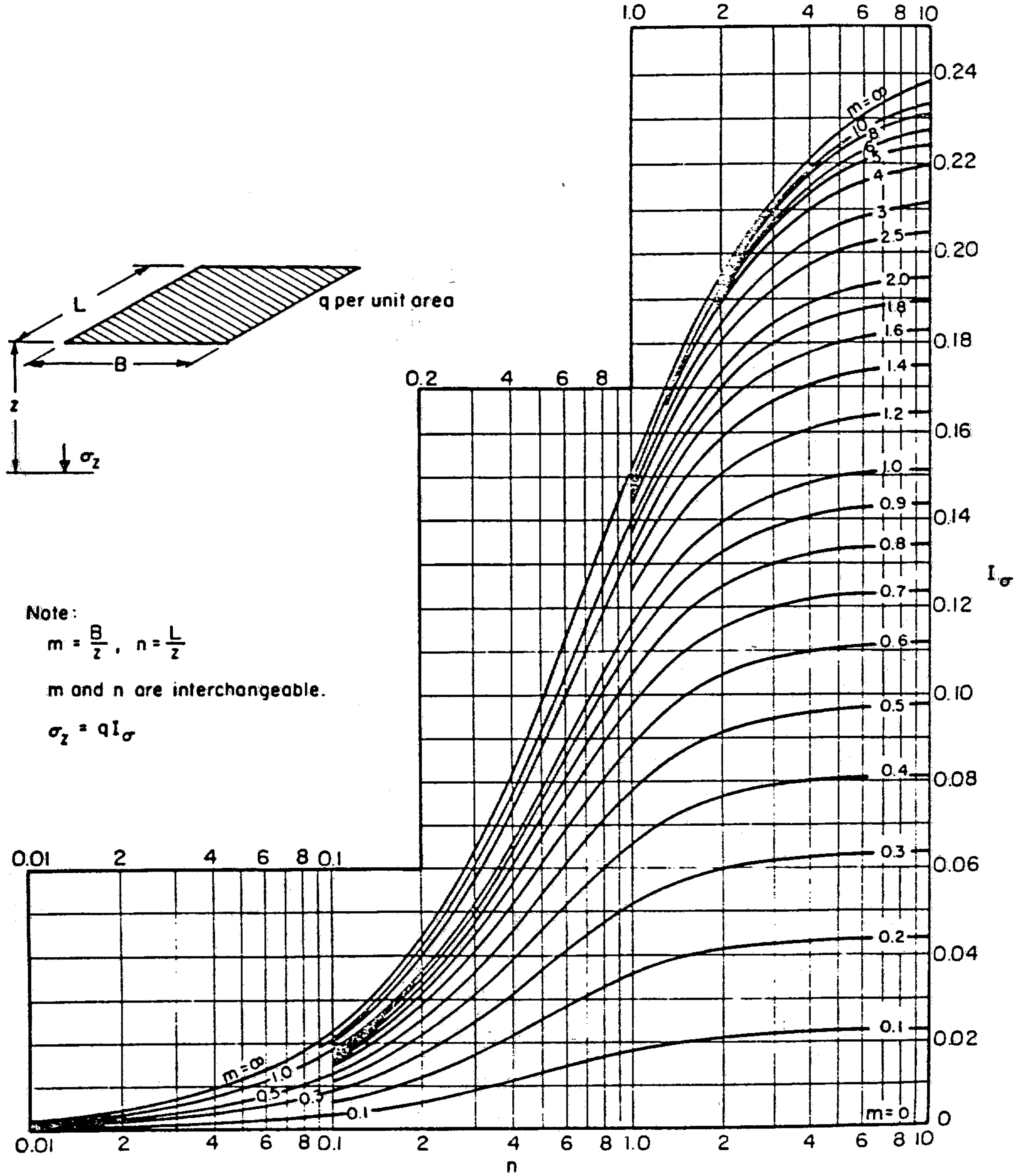


Figure C-3. Influence Factor I_σ for the increase in vertical stress beneath a corner of a uniformly loaded rectangular area for the Westergaard stress distribution (from NAVFAC DM 7.1)