Chapter 8
Analytical Aerotriangulation

8-1. General

Since ground control is a significant expense in any mapping project, aerotriangulation bridging or control extension methods are often used to reduce the amount of field surveying required by extending control to each stereomodel photogrammetrically. The number of control points required to scale and level each stereomodel does not change on small projects of a few stereomodels. Ground control requirements for projects with only a few stereo pairs are minimal, and conventional photo control as indicated in Figure 6-2, Chapter 6, may be suitable and time and cost efficient. However, as the arial extent of a project increases, and thereby the number of stereomodels, aerotriangulation becomes an efficient method of extending a sparse field survey control network. This chapter emphasizes fully analytical aerotriangulation methods since these methods are most appropriate for modern instruments and large-scale mapping requirements.

8-2. Aerotriangulation Principles

a. Definition. Aerotriangulation is the simultaneous space resection and space intersection of image rays recorded by an aerial mapping camera. Conjugate image rays projected from two or more overlapping photographs intersect at the common ground points to define the three-dimensional space (3-D) coordinates of each point. The entire assembly of image rays is fit to known ground control points in an adjustment process. Thus, when the adjustment is complete, ground coordinates of unknown ground points are determined by the intersection of adjusted image rays.

b. Purpose. The purpose of aerotriangulation is to extend horizontal and vertical control from relatively few ground survey control points to each unknown ground point included in the solution. The supplemental control points are called pass points, and they are used to control subsequent photogrammetric mapping. Each stereomodel is scaled and leveled using the adjusted coordinate values of the pass points located in the stereomodel.

c. Relationship to ground control. Aerotriangulation is essentially an interpolation tool, capable of extending control points to areas between ground survey control points using several contiguous uncontrolled stereomodels. An aerotriangulation solution should never be extended or cantilevered beyond the ground control. Ground control should be located at the ends of single strips and along the perimeter of block configurations. Within a strip or block, ground control is added at intervals of several stereomodels to limit error propagation in the adjusted pass point coordinates. Extending control by aerotriangulation methods is often referred to as *bridging* since the spatial image ray triangulation spans the gap between ground control.

8-3. Softcopy Methods

a. Definition. Aerotriangulation procedures that involve softcopy workstations must necessarily include fully analytical aerotriangulation software and high-resolution scanners. Diapositives are not required and all interior, exterior, and control point mensuration are read from the scanned images. The elimination of diapositives removes the process of identifying and drill marking the points for mensuration. **For the purpose of this manual, the processes and standards for softcopy aerotriangulation will be assumed identical to analytical stereo plotter methods once the orientation process begins.**

b. Softcopy Process. Accurate softcopy aerotriangulation requires equipment and materials not necessarily required for analytical stereoplotter aerotriangulation procedures. Softcopy aerotriangulation must follow procedures and utilize equipment that will allow the operator the ability to ascertain feature resolution at a level that will achieve the aerotriangulation accuracy required. A major advantage to softcopy aerotriangulation is that the software is generally interactive and thus provides excellent quality control. The results of point selection, measurements, and weighting are shown to the operator immediately. Successful softcopy aerotriangulation planning must begin at the image acquisition phase. Issues for consideration are listed below.

(1) High-resolution film must be used.

(2) Extreme care must be taken in the processing of the film to ensure maximum **clarity**.

(3) Processed film must be handled in a manner to minimize dust and scratches prior to scanning.

(4) Scanning must be accomplished with a scanner capable of scanning between 7 and 25 microns.

(5) Color scanning may be accomplished by a single-pass or three-pass scanning system.

(6) Software and hardware must be capable of model orientation in both stereo and monoscpoic modes, capable of interior, relative, and absolute orientation, as well as single photo resection.

Once acceptable scanned images are created, the aerotriangulation process is similar to the process followed using an analytical stereoplotter. Latitude should be used in allowing contractors to use specific expertise in softcopy aerotriangulation as it relates to the number of artificially marked pass points. Softcopy processes make the identification of these points relatively easy and some contractor experience indicates that more artificially marked pass points can improve the solution in some cases. The minimum number of artificially marked pass points stated in this manual for analytical stereoplotter processes should be the minimum used for softcopy processes.

8-4. Pass Points

Pass point requirements are related to type of point used, location, and point transfer and marking requirements. **Softcopy aerotriangulation processes do not require diapositives, nor the identification or drill marking of pass points.**

a. Type of points. Pass points may be artificially marked points, targeted points, or natural images. However, since pass points must lie at or very near the center line of the triple overlap area, artificially marked points designed from the photography taken should be used. Premarked targets are too expensive and too difficult to align with the triple overlap areas. Natural images are not always suitable for precise pointing.

b. Marking artificial points. Artificially marked pass points must be well-defined symmetrical patterns drilled, punched, or otherwise marked in the emulsion using a suitable marking instrument such as a Wild PUG or equivalent. Only the aerotriangulation/compilation positives should be marked. The original negatives should not be marked.

c. Location. A minimum of three pass points must be marked along the center line of each triple overlap area. One pass point must lie near the photo principal point, and two as wing points in the sidelap with adjacent flight lines. To better control error skew, the wing points could be in pairs. Pass point locations are to be selected by examining the photographic prints with a stereoscope. Pass points must be located on unobscured level ground in accordance with the characteristics for vertical photo control. All pass point locations must be symbolized and labeled on the control photographs.

d. Point transfer for monoscopic measurements. Artificial pass points are typically marked in stereoscopic correspondence on all photographs showing the site of the point, using a stereoscopic transfer and point marking device such as a Wild PUG or equivalent. For the minimum of three pass points in each triple overlap area, this operation will result in a minimum of nine pass points on each photo. This method is required when photo coordinates are to be measured on a monocomparator. This method may be used when pass points are to be measured stereoscopically, either as photo coordinates on a stereocomparator or analytical stereo plotter or as model coordinates on any stereoplotter. Stereoscopic point transfer and marking should be done by a highly experienced operator using utmost care in choosing the site and in the marking of each pass point.

e. Point transfer for stereoscopic measurements. When pass points are to be measured stereoscopically, either as photo coordinates on a stereocomparator, softcopy workstation, analytical stereoplotter, or as model coordinates on any stereoplotter, artificial pass points need be marked on the center photograph only using a suitable point marking device. Viewing the marked point stereoscopically with adjacent photographs will accomplish the point transfer of the pass point location to the overlapping photo as part of the measurement process. When parallel flight lines of photography are used, tie points should be transferred from one flight line to each adjacent flight line using a stereoscopic transfer and point marking device such as a Wild PUG or equivalent. Artificial points are typically not superimposed on images of targets.

f. Softcopy pass points. Aerotriangulation with softcopy simplifies the procedure of pass points processing. After the photos are scanned, two successive photo images are displayed on the monitor screen. The operator then selects arbitrary pass points on one image, then the computer automatically assigns appropriate photo coordinates of that point on each photo. Images of adjacent flight line photos can then be displayed and the position of pass points in the sidelap area are stored in the computer's database. This operation eliminates the necessity of manual pugging, plate reading, and transferring as discussed above. Refer to Figure 8-1 for a schematic of this concept.

8-5. Ground Control Points

Ground control requirements are related to targeting, control location, and survey accuracy requirements.

a. Targeting. Whenever feasible, ground control points for Classes 1 through 3 aerotriangulation and subsequent mapping should be ground panels placed at the appropriate locations prior to the flight. Targeting should be in accordance with paragraph 6-4. It is understood that for many projects and many reasons it is necessary to establish photo identifiable features to be used ground control targets. All ground control photo identifiable and panel points should be approved by the Government prior to use in a project.

b. Control location. Final control location and bridging distances used should be the decision of the Contractor, but the following guidelines should be applied:

(1) Single strips.

(a) Along a single flight line of photography (Figure 8-1), horizontal control points should be in pairs at the strip ends within the terminal stereomodel, one on each side of the flight line approximately opposite each other. For each single flight line, additional horizontal control points should be located at intervals along the strip that do not exceed the maximum allowable bridging distance. Horizontal basic control points should be no more than one-third of the width of coverage of a photograph from the flight line.

(b) Along a single flight line of photography, vertical control points should be in pairs, one on each side of the flight line approximately opposite each other and at a distance from the flight line of between one-fourth and one-third of the width of coverage of a photograph. For each single flight line, the pairs of vertical control points should be located at the strip ends within the terminal stereomodel and at intervals along the strip that do not exceed the allowable bridging distance for the class of mapping.

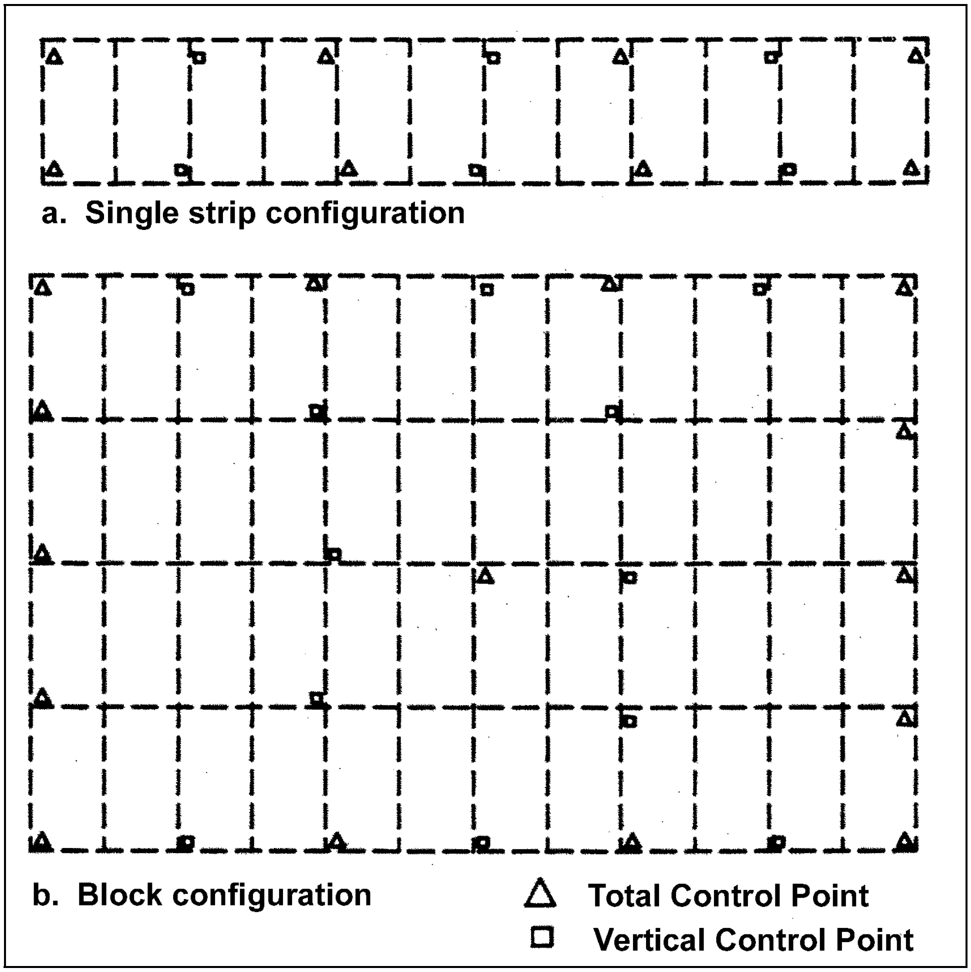

Figure 8-1. Typical strip and block control configurations

(2) Blocks. A block of photography, consisting of two or more flight lines of photography (Figure 8-1), should have control points spaced approximately equally around the periphery following the same spacing and location guidelines as for strips in paragraph 8-4*b*. There should be at least one horizontal control point and two vertical control points near the center of any block.

(a) Additional horizontal control should be located in the center of the block such that horizontal control falls in alternating strips at an interval not to exceed two times the allowable horizontal bridging distance. There should be at least one horizontal control point and two vertical control points near the center of any block.

(b) Additional vertical control should be located in the center of the block such that vertical control falls in each strip at an interval not to exceed two times the allowable vertical bridging distance.

c. Bridging distance. Table 8-1 lists typical allowable bridging distances that may be used as a guide in estimating control requirements for a project. **These guidelines apply to softcopy methods.** These are minimum guidelines, and many contractors will design a more dense control pattern. For example, a horizontal control points every five stereomodels regardless of block size. When ground control is attained with GPS procedures, an XYZ coordinate is derived. Hence, the lesser of the horizontal/vertical stipulations in Table 8-1 should be used.

Table 8-1
Allowable Bridging Distances

Map Class (ASPRS)	Allowable Bridging Distance (Stereomodel Basis): Maximum Horizontal Control Spacing	Maximum Vertical Control Spacing
1	4	3
2	5	4
3	6	5

d. Ground control accuracy. Ground control accuracy for aerotriangulation should be more stringent than for a project fully controlled by field survey points.

The aerotriangulation solution will contribute to the propagated error in the pass point ground control values. Since the pass point coordinates should meet the accuracy required for photo control, the photo identifiable ground control points used to control the aerotriangulation should be more typical of the accuracy of the basic control survey.

e. GPS control. The GPS is an effective method of establishing basic project control and photo control. GPS is an especially effective way to connect the project area surveys to existing national network stations outside the project region. Kinematic GPS methods are also being used to position the camera at the time of exposure. The use of kinematic GPS methods should be evaluated carefully. Unless the terrain is inaccessible for ground targeting, terrestrial GPS surveys to establish control points and normal flying procedures may be more cost-effective and accurate for large-scale mapping.

8-6. Other Points

Coordinates can be established by aerotriangulation for additional points located on the photography by targets or artificially marked as pass points. For example, these points may be aerotriangulation checkpoints, stereo-plotter test points, or cadastral points to be located on the map. The Government should specify all points required to be included in the aerotriangulation in addition to the control points and standard pass point pattern. If a supplemental pass point is required for checking stereomodel flatness at compilation time, it should be located near the center of the stereomodel within a circle whose diameter is the central third of the airbase.

8-7. Instrumentation

Precise photo coordinate measurements are required for fully analytical aerotriangulation. A softcopy workstation or analytical stereoplotter are usually utilized by contemporary photogrammetric mapping companies.

8-8. Accuracy and Quality Control Criteria

The contractor is responsible for designing the aerotriangulation scheme that will meet the requirements of the photogrammetric product. Table 8-2 summarizes the guidelines for evaluating aerotriangulation methods. However, since meeting required pass point accuracies is dependent on the photogrammetric system, the contractor should be allowed some latitude in meeting criteria for these intermediate results.

a. Photo coordinate measurements. Photo coordinate measurement is the most critical factor contributing to the accuracy of aerotriangulation results. The contractor should be especially careful to control the quality of point transfer, point marking, and point measurement. The measurement stage(s) of the softcopy workstation, stereo plotter or the monocomparator should have a least count of 0.001 mm or less. The viewing, pointing, and digitizing components of these instruments should enable the operator to group multiple readings

Table 8-2
Guidelines for Evaluating Analytical Aerotriangulation

Analytical Aerotriangulation Procedures	**Criteria**
Photo Coordinate Measurements: Softcopy System or Analytical Stereoplotter	
Least Count of Stage Coordinate	0.001 mm
Interior Orientation: Transformation to Fiducial Coordinates	
Minimum (Recommended)	**4 (8)**
Maximum Residual (after Affine Transformation)	0.020 mm
Preliminary Sequential Strip Formation and Adjustment	
Stereomodel Relative Orientation	
Minimum Number of Points, Y-Parallax Residuals	6
RMSE	0.005 mm
Maximum	0.015 mm
Stereomodel Joins	
Minimum Number of Points	3
X,Y Pass Point Coordinate Discrepancy	
RMSE	H/12,000 ft
Maximum	H/6,000 ft
Z Pass Point Coordinate Discrepancy	
RMSE	H/10,000 ft
Maximum	H/5,000 ft
Polynomial Strip Adjustment	
X,Y Control Point Coordinate Residual	
RMSE	H/10,000 ft
Maximum	H/6,000 ft
Z Control Point Coordinate Residual	
RMSE	H/7,000 ft
Maximum	H/6,000 ft
Simultaneous Bundle Adjustment	
Rmse of Photo Coordinate Residual	0.004 mm
Maximum Variance Factor Ratio	1.5
(See Also Table 8-3)	

on any well-defined target or marked pass point within a maximum spread of 0.004 mm. Multiple readings are of more value if they are not consecutive; however, this reading scheme is often not practical. If multiple readings are made consecutively, the operator must move off the image and repoint between each reading. The instrument used should be capable of measuring a photo coordinate with an RMSE not greater than 0.003 mm. It is mandatory to end the measuring of a photograph with a reading on the first point measured (usually a fiducial mark) to assure that the instrument encoders have not drifted or skipped counts.

b. Interior orientation. Interior orientation refers to the geometric relationship between the image plane and the perspective center of the lens.

(1) The initial step is to transform the measured stage coordinates into the photo coordinate system defined by the calibrated fiducial coordinates. An affine transformation, which accounts for differential image scale and shear, is typically used to establish the photo coordinate system. The transformation parameters are determined by a least squares adjustment using at least four fiducials (**eight is recommended**).

(2) After the photo coordinate system is established, the image measurements must be corrected for systematic errors. This procedure is called photo coordinate refinement. Corrections are applied for principal point offset, radial lens distortion, tangential lens distortion, and atmospheric refraction.. Photo coordinate refinement may be performed by the analytical stereo plotter software or the aerotriangulation software. Typically, the interior orientation and refinement parameters are considered known based on the calibration report. Then the photo coordinate refinement is performed before the photo coordinates are used for the

aerotriangulation adjustment. If self-calibration aerotriangulation software is used, the camera interior orientation parameters are considered to be approximations, and they are adjusted as a parameter in the aerotriangulation solution.

c. Preliminary sequential aerotriangulation. This process (Table 8-2) refers to the sequential assembly of independent stereomodels to form a strip unit and the polynomial strip adjustment into the ground coordinate system. The sequential procedure is a preliminary adjustment that develops initial approximations for the final simultaneous bundle adjustment. The sequential procedure also serves as a quality control check of the photo and ground coordinate data. The guidelines listed in Table 8-2 are not rigorously enforced, but they are used to evaluate the building blocks of a larger strip or block configuration.

(1) Relative orientation of each stereo pair is performed by a least squares adjustment using the collinearity equations. The stereomodel is created in an arbitrary coordinate system, and the adjustment is unconstrained by ground coordinate values. Therefore, the photo coordinate residuals should be representative of the point transfer and measuring precision. The photo coordinate residuals should be examined to detect misidentified or poorly measured points. The minimum number of points that will uniquely determine a relative orientation is six. The six-point minimum recommended in Table 8-2 results if the standard nine pass points per photo configuration is used. Typically, more than the minimum six pass points are available (field survey points and pass points from adjacent flight lines). The RMSE and maximum residual values listed will more likely be reached when larger numbers of pass points are used.

(2) When stereomodels are joined to form a strip, the pass points shared between models will have two coordinate values, one value in the strip coordinate system and one value in the transformed model coordinate system. The coordinate differences or discrepancies between the two values can be examined to evaluate how well the models fit to one another. Horizontal coordinate discrepancies will typically be smaller than vertical discrepancies since the image ray intersection geometry is weaker in the vertical direction. As the stereomodels are transformed into the strip, one after the other, the pass point coordinate discrepancies should be uniform and no outliers should be observed. The coordinate discrepancy criterion is expressed as a fraction of the flying height above terrain because the magnitude of the discrepancy in ground units is dependent on the photo scale.

(3) Polynomial strip adjustment is a preliminary adjustment that produces initial ground coordinate values for all the pass points in a strip. Pass point coordinate values will be adjusted again by the final bundle adjustment. Polynomial correction curve is fit to the coordinate errors at the control point locations using a least squares adjustment. Residuals of the least squares curve fit can be examined to evaluate the adequacy of the polynomial adjustment. The residual criterion is expressed as a fraction of the flying height above terrain because the magnitude of the discrepancy in ground units is dependent on the photo scale. Evaluation of the polynomial residuals is the least critical check in the aerotriangulation process, and a great deal of latitude can be allowed in meeting these criteria. From project to project, large variations in the residuals may occur because of the number of stereomodels in the strip, the polynomial function used, and the distribution of the control points. It is more important to check the X, Y, and Z error curves after the linear transformation of the strip into the ground coordinate system and before the polynomial correction. These error curves should be smooth Second- or Third-Order curves. Outliers from a smooth continuous curve are an indication of a blunder in the photogrammetric value or the ground survey value at a control point.

d. Simultaneous bundle adjustment. Fully analytical aerotriangulation must be adjusted by a weighted least squares adjustment method. Adjustment software must form the collinearity condition equations for all the photo coordinate observations in the block and solve for all photo orientation and ground point coordinates in each iteration until the solution converges.

(1) The exterior coordinate system used for the adjustment should be a local rectangular coordinate system as defined in Chapter 3. This coordinate system contains no earth curvature or map projection distor-

tions. These effects may be judged to be negligible for small project areas and low flying heights, but they are significant factors for large project areas and high flying heights.

(2) Least squares adjustment results should be examined to check the consistency of the photo coordinate measurements and the ground control fit. Residuals on the photo coordinates should be examined to see that they are representative of the random error expected from the instrument used to measure them. Residuals should be randomly plus or minus and have a uniform magnitude. Residuals should be checked carefully for outliers and systematic trends. Standard deviation of unit weight computed from the weighted adjusted residuals should not be more than 1.5 times the reference standard deviation used to compute the weights for the adjustment. A large computed reference variance indicates inflated residuals and possible systematic errors affecting the adjustment. For example, if photo coordinates were judged to have an overall measurement standard deviation of 0.005 mm and this value was used to compute observation weights, the standard deviation of unit weight computed by the adjustment should not exceed 0.0075 mm.

(3) Accuracy of aerial analytical triangulation should be measured by the RMSE and the maximum error in each coordinate (X, Y, and Z) direction for the combined checkpoints. The maximum allowable error should be checked at the midpoint of the bridging distance between ground control points using checkpoints or test drop points surveyed for this purpose. Table 8-3 lists the accuracy criteria suggested for each class of mapping. These criteria are the final and most important check of the aerotriangulation results.

Table 8-3
Aerotriangulation Accuracy Criteria (for 6-in. Focal Length Photography)

		Allowable RMSE at Test Points[1]	
Map Class	**Aerotriangulation Method**	**Horizontal[2]**	**Vertical[2]**
1	Fully Analytical	H/10,000	H/9,000
2	Fully Analytical	H/8,000	H/6,000
3	Fully Analytical or Semianalytical	H/6,000	H/4,500

Notes:
[1] The maximum allowable error is 3 RMSE.
[2] One-sigma level.

8-9. Stereoplotter Settings

Fully analytical aerotriangulation determines the six camera exterior orientation parameters for each photograph, camera position (X_L, Y_L, and Z_L), and angular orientation. By relating these parameters to the flight line between each two successive camera stations and scaling to the stereomodel, data are obtained for setting up the stereoscopic model in the stereoplotter.

8-10. Deliverables

Unless otherwise modified by the contract specifications, the following materials will be delivered to the Government upon completion of the aerotriangulation:

a. General report about the project and procedures used including description of the project area, location, and extent; description of the instrumentation used for pass point transfer and marking, and photo coordinate measurement; and description of the aerotriangulation methods and software used including version numbers.

b. One set of paper prints showing all control points and pass points used. The points should be symbolized and named on the image side, and the exact point location should be pinpricked through the print.

c. A list of the computed coordinates of all points specified by the Government.

d. A report of the accuracies attained and listing discrepancies in each coordinate direction at control points and checkpoints separately, a justification for any control points or pass points omitted from the final adjustment, and the RMSE and maximum error (in relation to ground surveyed coordinates) in each coordinate direction (X, Y, and Z) for the control points and checkpoints as a group.

e. Complete copies of all computer printouts.

f. A list of stereoplotter orientation settings, if specified.