Notes on the GFDL regional atmospheric model ("ZETAC")

Stephen Garner
steve.garner@noaa.gov

1. Overview

ZETAC is a regional atmospheric model developed at GFDL. It began as a terrain-following hydrostatic model and later became nonhydrostatic and compressible. It has been used as a global model (Garner et al., *JAS* 2007) but its strength is in limited-area applications that avoid the earth's poles. It uses a latitude-longitude C-grid in the horizontal and a pure terrain-following coordinate in the vertical. The sphericity can be switched off to enable *f*-plane and β-plane simulations. It also has the capability of running in cylindrical coordinates, or with singly or doubly periodic lateral boundaries.

The model is nested in a "driving" solution defined by a time-dependent external dataset or a subroutine that creates an idealized atmosphere. The driving is accomplished by nudging model variables near the lateral boundaries or nudging spectrally smoothed variables across the full domain. Open lateral boundaries are approximated by extrapolating the normal velocity component using "phase advection" towards the grid boundary and using the driving solution to determine gradients of all other fields. An upper sponge layer damps large-scale perturbations and the residual small scales at different rates.

Acoustic-gravity waves are integrated on short time-step using a linearized form of the model equations. The basic idea is that of Klemp and Wilhelmson (*JAS* 1979) but is modified to include gravity modes and to allow monotonic and semi-lagrangian advection schemes on the long time-step. Vertical propagation of the acoustic-gravity waves is modified by semi-implicit time-stepping. Advection algorithms are chosen by the user for each model variable.

ZETAC is coupled to ocean models via the exchange grid and flux exchange module in GFDL's Flexible Modeling System (FMS). It calls the radiation, turbulence and moisture physics routines available from FMS as well as a bulk microphysics routine and a three-dimensional turbulent kinetic energy routine.

2. Coordinate system

The model's terrain-following coordinate is denoted ζ. It is assumed to range from zero to unity across the model depth. A separable mapping to height z is constructed as follows:

$$z(x, y, \zeta) = h(x, y) + Z(\zeta)[H - h(x, y)], \tag{1}$$

where $h(x, y)$ is the topography, H is the model depth, and Z varies from zero to unity. The advection operator is:

$$\boldsymbol{V} \cdot \nabla = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + \omega\frac{\partial}{\partial \zeta}, \tag{2}$$

where $\omega = d\zeta/dt$ and the horizontal derivatives are at constant ζ. In spherical and terrain-following coordinates, the divergence is

$$\nabla \cdot \boldsymbol{V} = z_\zeta^{-1}\left[\frac{\partial(uz_\zeta)}{\partial x} + (\cos\varphi)^{-1}\frac{\partial(vz_\zeta\cos\varphi)}{\partial y} + \frac{\partial(\omega z_\zeta)}{\partial \zeta}\right], \tag{3}$$

where φ is the latitude and partial derivatives are at constant ζ. The factor z_ζ is the derivative of height with respect to the model coordinate at constant x and y. The ordinary vertical velocity component w can be diagnosed from ω using

$$w = \omega z_\zeta + \boldsymbol{V}_h \cdot \nabla z, \tag{4}$$

where $\boldsymbol{V}_h = (u, v)$ and the horizontal derivatives are at constant ζ.

3. Model equations

The prognostic variables are the three-dimensional velocity, $\boldsymbol{V} = (u, v, w)$, the potential temperature θ, the specific humidity q_v, and the Exner function, $\pi = \left(\frac{p}{p_0}\right)^{R/c_p}$, where p_0 is a constant reference pressure, R is the gas constant of dry air, and c_p is the specific heat at constant pressure. Let D represent diffusion. Then the forecast equations are:

$$\frac{\partial u}{\partial t} = -\boldsymbol{V}\cdot\nabla u + \left(f + \frac{\tan\varphi}{a}u\right)v - c_p\theta_v\left.\frac{\partial\pi}{\partial x}\right|_z (1+q_c)^{-1} + D_u \tag{5}$$

$$\frac{\partial v}{\partial t} = -\boldsymbol{V}\cdot\nabla v - \left(f + \frac{\tan\varphi}{a}u\right)u - c_p\theta_v\left.\frac{\partial\pi}{\partial y}\right|_z (1+q_c)^{-1} + D_v \tag{6}$$

$$z_\zeta\frac{\partial\omega}{\partial t} = -\boldsymbol{V}\cdot\nabla w - g - c_p\theta_v z_\zeta^{-1}\frac{\partial\pi}{\partial\zeta}(1+q_c)^{-1} - \frac{\partial\boldsymbol{V}_h}{\partial t}\cdot\nabla z|_\zeta + D_\omega \tag{7}$$

$$\frac{\partial\pi}{\partial t} = -\boldsymbol{V}\cdot\nabla\pi - \frac{R}{c_v}\pi(\nabla\cdot\boldsymbol{V} - E) + \frac{R\pi}{c_v\theta_v}\frac{d\theta_v}{dt} \tag{8a}$$

$$\frac{\partial\theta}{\partial t} = -\boldsymbol{V}\cdot\nabla\theta + \frac{Q}{c_p\pi} + D_\theta \tag{9}$$

$$\frac{\partial q_v}{\partial t} = -\boldsymbol{V}\cdot\nabla q_v + E + D_q, \tag{10}$$

where q_c is the total condensate, Q is the diabatic heating/cooling, E is the source of water vapor, and $\theta_v = \theta(1+\varepsilon q_v)$, the virtual potential temperature (with $\varepsilon = 0.608$). The horizontal pressure gradient is at constant z. The constants are: the acceleration of gravity, $g = 9.81\ \mathrm{m/s^2}$; the mean earth radius, $a = 6371\ \mathrm{km}$; and the specific heat at constant volume, $c_v = c_p - R$. The vertical velocity equation (7) is obtained by differentiating (4) with respect to time and substituting for $\partial w/\partial t$.

The pressure equation (8a) can also be written in flux form, either as

$$\frac{\partial \pi}{\partial t} = \mu^{-1}\pi\left(-\frac{\nabla \cdot \pi^{\mu} \boldsymbol{V}}{\pi^{\mu}} + E + \theta_v^{-1}\frac{d\theta_v}{dt}\right), \tag{8b}$$

or as

$$\frac{\partial \pi}{\partial t} = \mu^{-1}\pi\left(-\frac{\nabla \cdot \rho \boldsymbol{V}}{\rho} + E + \theta_v^{-1}\frac{\partial\theta_v}{\partial t}\right), \tag{8c}$$

where $\mu \equiv c_v / R$. All of the pressure equations, (8a), (8b) and (8c), are derived from mass conservation,

$$\rho^{-1} d\rho / dt = -\nabla \cdot \boldsymbol{V} + E, \tag{11}$$

and time-derivatives of the equation of state, written in the form

$$\pi^{\mu} = \frac{R}{p_0}\rho\theta_v, \tag{12}$$

with ρ the moist air density.

4. Fast equations

A time-varying height-dependent reference state, denoted by an overbar, is chosen so that the following linear terms responsible for acoustic-gravity waves can be subtracted and re-evaluated on a "fast" time step:

$$\frac{\partial u}{\partial t} = -c_p\bar{\theta}_v\left.\frac{\partial \pi}{\partial x}\right|_z (1 + q_c)^{-1} \tag{13}$$

$$\frac{\partial v}{\partial t} = -c_p\bar{\theta}_v\left.\frac{\partial \pi}{\partial y}\right|_z (1 + q_c)^{-1} \tag{14}$$

$$z_\zeta\frac{\partial \omega}{\partial t} = \left[g\left(\frac{\theta_v - \bar{\theta}_v}{\bar{\theta}_v} - q_c\right) - c_p\bar{\theta}_v z_\zeta^{-1}\frac{\partial}{\partial \zeta}(\pi - \bar{\pi})\right](1 + q_c)^{-1} \tag{15}$$

$$\frac{\partial \pi}{\partial t} = -\mu^{-1}\bar{\pi}^{1-\mu}\nabla \cdot \bar{\pi}^{\mu}\boldsymbol{V} \tag{16}$$

$$\frac{\partial\theta_v}{\partial t} = -\boldsymbol{V}\cdot\nabla\bar{\theta}_v. \tag{17}$$

Here $\bar{\pi}(z)$ is in hydrostatic balance with the virtual potential temperature $\bar{\theta}_v(z)$; thus, $\frac{\partial}{\partial z}\bar{\pi} = -\frac{g}{c_p\bar{\theta}_v}$. The linearization of the pressure equation (16) is based on (8b). What remains on the right-hand side of (5) - (10) after these terms are subtracted is frozen during the fast cycle. The moisture variables q_c and q_v are also held fixed during the fast cycle.

The linear terms are subtracted at one model time level (the slow time) and re-evaluated at another (the fast time). For example, the full equation for virtual potential temperature, combining (9), (10) and (17), is

$$\frac{\partial\theta_v}{\partial t} = -\boldsymbol{V}_s\cdot\nabla\theta_v - (\boldsymbol{V}_f - \boldsymbol{V}_s)\cdot\nabla\bar{\theta}_v + \frac{Q_v}{c_p\pi}, \tag{18}$$

where “s” and “f” refer to “slow” and “fast”, and $Q_v = Q(1+\varepsilon q_v) + E\varepsilon c_p\pi\theta$ (evaluated on the slow time). This departs from the original method of Klemp and Wilhelmson (1979, KW) by including the gravity-wave terms and by retaining the full advection in the slow tendency. The latter feature makes it possible to use sophisticated transport algorithms. The size of the fast timestep is constrained by soundwave propagation. In order to relax this constraint without significantly diminishing the model’s accuracy, the vertical propagation is treated semi-implicitly, as in KW. Vertical soundwave propagation is due to the vertical derivatives in (15) and (16). The semi-implicit treatment also includes vertical advection of buoyancy, on the right-hand side of (17). This removes numerical constraints due to vertical gravity-wave propagation.

5. Mass conservation

The pressure equations (8b) and (8c) are in a convenient form for discretization and for global conservation. However, as long as density is purely diagnostic, mass cannot be conserved exactly. The most obvious remedy -- to forecast the density directly with (11) and diagnose π from (12) -- adds complexity to the momentum equation because the pressure gradient then has to be expressed in terms of ρ gradients. A good compromise is to retain the pressure forecast only for the purpose of updating the velocity, followed by overwriting the pressure with (12). To improve the consistency between the prognosis and diagnosis of pressure, it is then preferable to use (8c) instead of (8b), together with the flux form of (11):

$$\frac{\partial\rho}{\partial t} = -\nabla \cdot \rho \boldsymbol{V} + \rho E . \tag{19}$$

On the fast timestep, the updates of π and ρ are given by

$$\frac{\partial\pi}{\partial t} = -\mu^{-1}\bar{\pi}\left[\frac{\nabla \cdot \bar{\rho}(\boldsymbol{V}_f - \boldsymbol{V}_s)}{\bar{\rho}} + \frac{\boldsymbol{V}_f - \boldsymbol{V}_s}{\bar{\theta}_v} \cdot \nabla\bar{\theta}_v\right] \tag{20}$$

and

$$\frac{\partial\rho}{\partial t} = -\nabla \cdot \bar{\rho}(\boldsymbol{V}_f - \boldsymbol{V}_s) , \tag{21}$$

which are based on linearization of (8c), (17) and (19).

A linearized time-derivative of (12) is obtained by eliminating the mass divergence, $\nabla \cdot \bar{\rho}\boldsymbol{V}$, between (20) and (21) amd substituting from (17) for the temperature advection. Therefore, to the extent that the fast solution is close to the reference state, the diagnostic overwrite of π will be a small nudge. It is also a localized nudge, whereas integrations that do not use (19) or (21) directly must resort to non-local nudging to keep the total mass from drifting. The consistency of overwriting π is improved, along with the numerical stability of the overall scheme, if the reference state in (20) and (21) is updated frequently from the model solution. For the remaining linear terms, there is probably nothing to be gained from changing to a time-varying reference state.

6. Differencing scheme

Time differencing on the slow timestep is leap-frog. A Robert filter or, optionally, an occasional forward timestep, is used to remove the computational mode. On the fast time, the momentum variables are staggered in time relative to the mass (temperature and pressure) variables, except for the first and last time levels of the fast cycle. For computational stability, the three-dimensional divergence is damped on the fast timestep.

Several advection algorithms are available, including finite-volume schemes. The horizontal pressure gradient is obtained by interpolating the pressure to constant-height surfaces and evaluating centered differences. The divergence (3) is also evaluated with second-order centered differences.

Horizontal diffusion and hyperdiffusion are evaluated on the model surfaces. To avoid diffusing variations that are mainly due to the large-scale vertical stratification, a horizontal running-mean of the diffused fields is removed beforehand.

7. Lateral boundaries and nudging

The model has cyclic or open lateral boundaries. The open boundaries are designed to limit reflections of internally generated signals. Reflections are controlled by using the method of Orlanski (1983) for predicting the normal component of velocity at boundary points and then advecting all other fields with this speed. The external values of these other fields needed in the centered-difference cross-boundary advection are taken to be a linear combination of the first interior value and the value given by the imposed large-scale field. The weighting between the two is a model parameter. The outward signal speeds needed to predict the normal velocity component are bounded above and below with user-specified Courant numbers.

Nudging towards the imposed large-scale fields is strongest at the boundary, falling to zero at a specified distance from the boundary. Spectral nudging is also available.