



Operational Implementation of 4D-VAR Assimilation for the U.S. Navy

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With acknowledgments to Roger Daley, Andrew Bennett, Ed Barker, and Jim Goerss



Outline

- **Background**
- **NAVDAS-AR¹ (AR) Design**
- **Basic Components of AR**
- **AR Validation Test (VT)**
- **AR Observation Impact**
- **Additional Capabilities**
- **Planned New Capabilities**
- **References**

¹ **NRL Atmospheric Variational Data Assimilation System – Accelerated Representer**



Recent Operational Transitions to FNMOC

- **NAVDAS-AR (observation-space 4D-Var) replaced NAVDAS¹ (observation-space 3D-Var) to provide the analysis for the Navy Operational Global Atmospheric Prediction System (NOGAPS) on 12Z September 23, 2009 at FNMOC².**
- **As part of this transition, a NOGAPS/T239L42 (42 vertical levels) model replaced NOGAPS/T239L30, increasing the model top to 0.04 hPa, for better assimilation of satellite radiances.**
- **An adjoint-based observation impact monitoring system, NAVOBS³, became operational to monitor the impact of assimilated individual observations (or satellite channels) and observation types on short term global forecast error at the same time.**

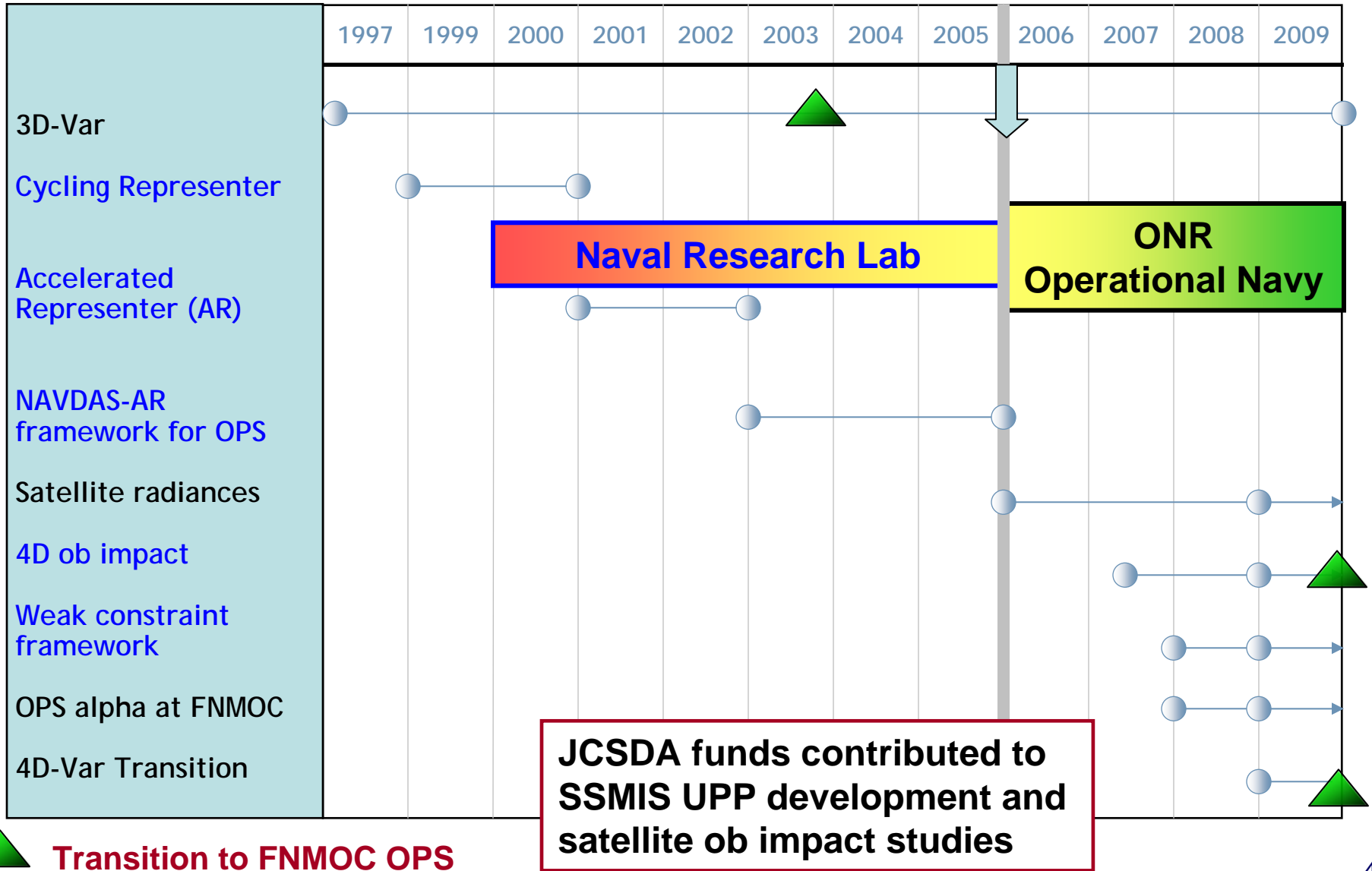
¹ **NRL Atmospheric Variational Data Assimilation System**

² **Fleet Numerical Meteorology and Oceanography Center**

³ **NAVDAS-AR adjoint Observation monitoring System**

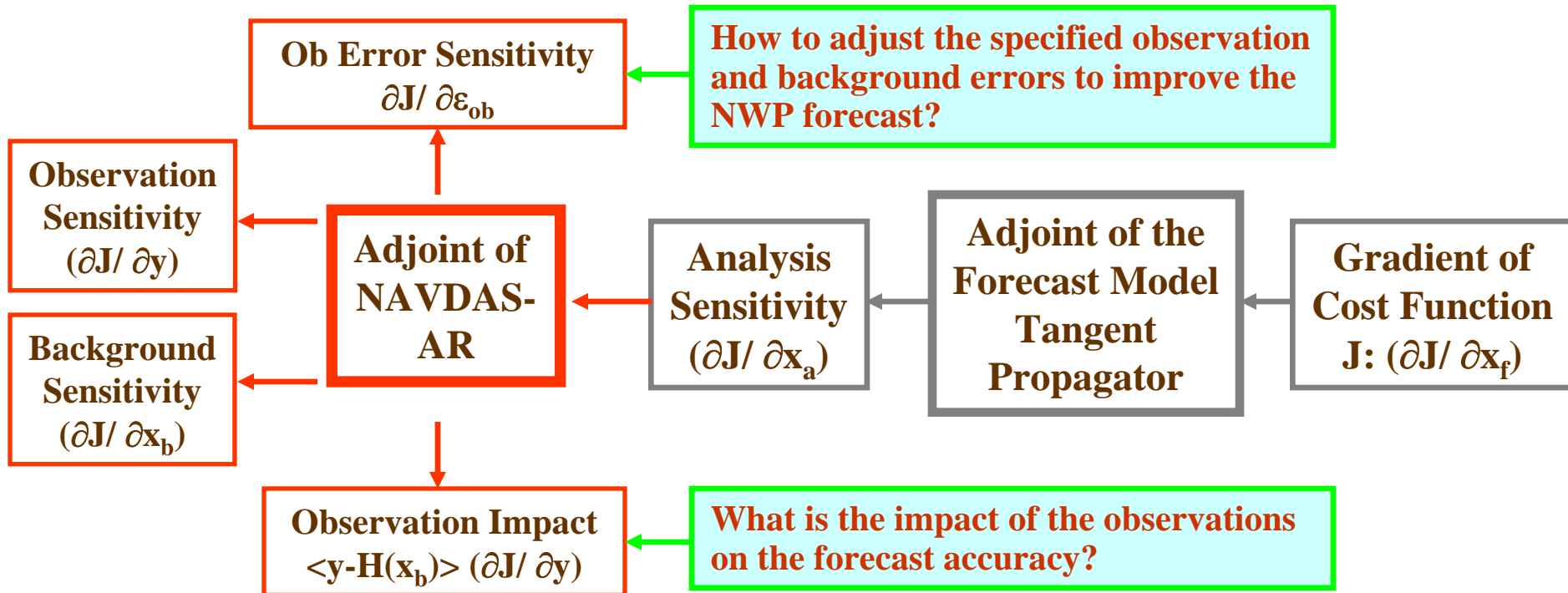
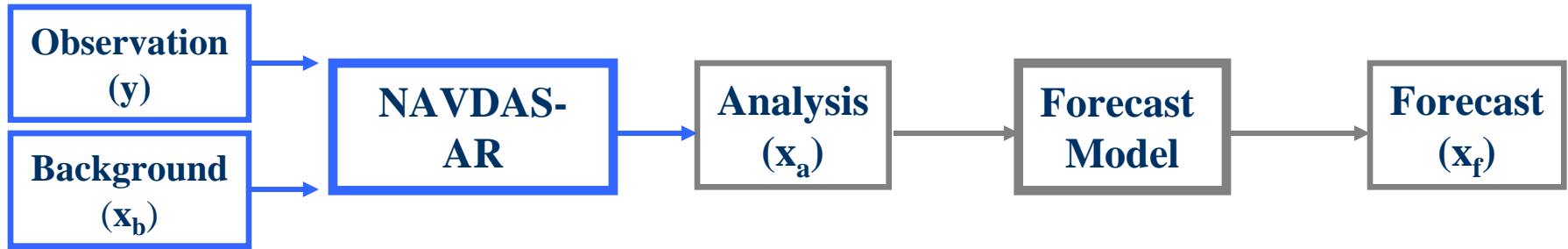


NRL Var Data Assimilation Timeline





NAVDAS-AR Data Assimilation System





Ob-Space Weak Constraint AR Formulation

$$J = J_0^b + J^q + J^r$$

$$\begin{cases} J_0^b = \frac{1}{2} [\mathbf{x}_0^b - \mathbf{x}_0]^T [\mathbf{P}_0^b]^{-1} [\mathbf{x}_0^b - \mathbf{x}_0], \\ J^q = \frac{1}{2} \sum_{n=1}^N \sum_{n'=1}^N [\mathbf{x}_n - \mathcal{M}(\mathbf{x}_{n-1})]^T \mathbf{Q}_{nn'}^{-1} [\mathbf{x}_n - \mathcal{M}(\mathbf{x}_{n-1})], \\ J^r = \frac{1}{2} [\mathbf{y} - \mathcal{H}(\mathbf{x})]^T \mathbf{R}^{-1} [\mathbf{y} - \mathcal{H}(\mathbf{x})]. \end{cases} \quad (1)$$

Euler-Lagrange Equations,

$$\begin{cases} \vec{\lambda}_n - \mathbf{M}_n^T \vec{\lambda}_{n-1} = \left\{ \mathbf{H}^T \mathbf{R}^{-1} [\mathbf{y} - \mathcal{H}(\mathbf{x}^a)] \right\}_n, \text{ subject to } \vec{\lambda}_{N+1} = 0, \\ \mathbf{x}_n^a - \mathcal{M}(\mathbf{x}_{n-1}^a) = \sum_{n'=1}^N \mathbf{Q}_{nn'} \vec{\lambda}_n, \text{ subject to } \mathbf{x}_0^a = \mathbf{x}_0^b + \mathbf{P}_0^b \mathbf{M}_0^T \vec{\lambda}_0. \end{cases} \quad (2)$$

Assuming \mathcal{M} and \mathcal{H} are linear, we can then formally write the analysis as,

$$\mathbf{x}^a = \mathbf{x}^b + \mathbf{P}^b \mathbf{H}^T [\mathbf{H} \mathbf{P}^b \mathbf{H}^T + \mathbf{R}]^{-1} [\mathbf{y} - \mathbf{H} \mathbf{x}^b]. \quad (3)$$

1. We first solve $\beta = [\mathbf{H} \mathbf{P}^b \mathbf{H}^T + \mathbf{R}]^{-1} [\mathbf{y} - \mathbf{H} \mathbf{x}^b]$, the solver.
2. We then post-multiply the solution by $\mathbf{P}^b \mathbf{H}^T$, the post-multiplication.
3. $\mathbf{P}^b \mathbf{H}^T$ is the background error covariance between errors in model and observation spaces.
4. $\mathbf{H} \mathbf{P}^b \mathbf{H}^T$ is the background error covariance in observation-space, is also known as the **Representer Matrix**.

(from Xu, Rosmond, and Daley, 2005)

In both the solver and post-multiplication, we need to evaluate the following matrix/vector multiply operation many times during each inner loop,

$$\mathbf{g} = [\mathbf{P}^b \mathbf{H}^T] \mathbf{z}, \quad (4)$$

where \mathbf{z} is a known vector of length K in observation space and \mathbf{g} is a vector of length $N \cdot I$ in model space, which is the result of the matrix/vector multiply. Following are the steps to obtain \mathbf{g} through the use of tangent linear and adjoint models.

1. Define $\mathbf{z}^T = (\mathbf{z}_1^T \dots \mathbf{z}_n^T \dots \mathbf{z}_N^T)^T$ and $\mathbf{g}^T = (\mathbf{g}_1^T \dots \mathbf{g}_n^T \dots \mathbf{g}_N^T)^T$, where \mathbf{z}_n^T and \mathbf{g}_n^T are vectors of length k_n and I , respectively.
2. $\mathbf{H}^T = (\mathbf{H}_1^T \dots \mathbf{H}_n^T \dots \mathbf{H}_N^T)$ is a $K \times N \cdot I$ block diagonal matrix with N blocks, each of size $k_n \times I$.

3. Introduce a vector of length I , \mathbf{f}_n , which is defined for each time t_n . Let \mathbf{f}_n be the output from the backward adjoint model, \mathbf{M}_n^T , starting at time t_N and with forcing $\mathbf{H}_n^T \mathbf{z}_n$,

$$\mathbf{f}_n = \mathbf{M}_n^T \mathbf{f}_{n+1} + \mathbf{H}_n^T \mathbf{z}_n, \text{ subject to } \mathbf{f}_{N+1} = \mathbf{0}. \quad (5)$$

We refer to (5) as the **backward sweep**.

4. The matrix/vector multiplication at time t_n , and \mathbf{g}_n is simply the output from the forward tangent linear model, \mathbf{M}_{n-1} , starting at time t_0 ,

$$\mathbf{g}_n = \mathbf{M}_{n-1} \mathbf{g}_{n-1} + \sum_{n'=1}^N \mathbf{Q}_{nn'} \mathbf{f}_{n'}, \text{ subject to } \mathbf{g}_0 = \mathbf{P}_0^b \mathbf{f}_0. \quad (6)$$

We refer to (6) as the **forward sweep**.

5. It requires one **single** backward and forward sweep to calculate $\mathbf{P}^b \mathbf{H}^T \mathbf{z}$.



Special Characters of the AR Formulation

- **The 4D-Var cost function is not explicitly calculated in the system.**
- **It searches for the minimum in the observation space.**
- **The strong constraint (perfect model assumption) is only a special case, where model error variance is set to zero, of the weak constraint (allows for model errors).**
- **A linearized (instead of a nonlinear) full resolution prediction model is used in the additional outer loops.**
- **The size of the control variables of the minimization problem equals the number of observations to be assimilated.**
- **The coding of the adjoint of NAVDAS-AR is very simple.**



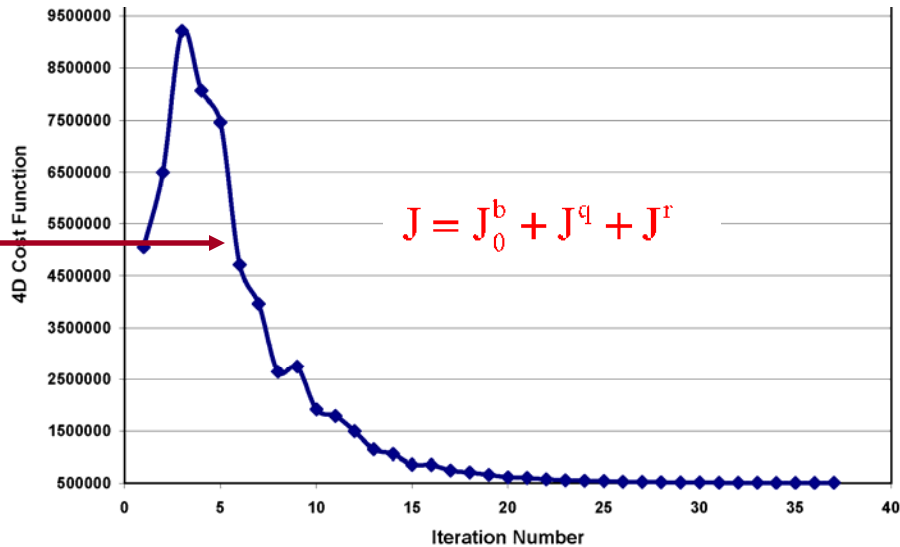
Basic Components of NAVDAS-AR

- **Nonlinear NOGAPS (T239/L42):**
 - Calculate innovation and low resolution basic state trajectory required in TLM/Adjoint models.
- **Observation pre-process:**
 - Provide quality-controlled and bias-corrected observations to be used in the Solver (differs from NAVDAS).
- **Adjoint and tangent linear models of NOGAPS (T119/L30):**
 - Calculate special Matrix/Vector multiplication in both the Solver & Post-multiplier.
- **Observation operators and associated Jacobians (CRTM):**
 - Calculate innovation vector and to provide gradient information in both the Solver and Post-multiplier.
- **A Flexible Conjugate Gradient (FCG) Solver:**
 - Find iterative solution to the Solver.
- **Error covariance model:**
 - Specify background error covariance.



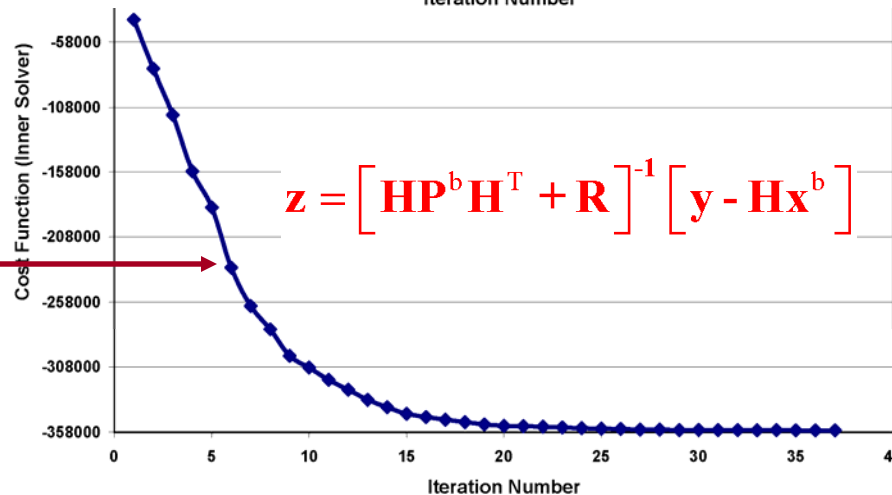
Convergence Behavior of NAVDAS-AR

4D-Var cost function to be minimized. It is not explicitly calculated in AR.



- Calculated from a 6 h data assimilation window centered at 2009041506.
- The number of assimilated observations was 812,277.

4D-Rep cost function associated with the FCG “SOLVER”. It is not explicitly calculated in AR either.

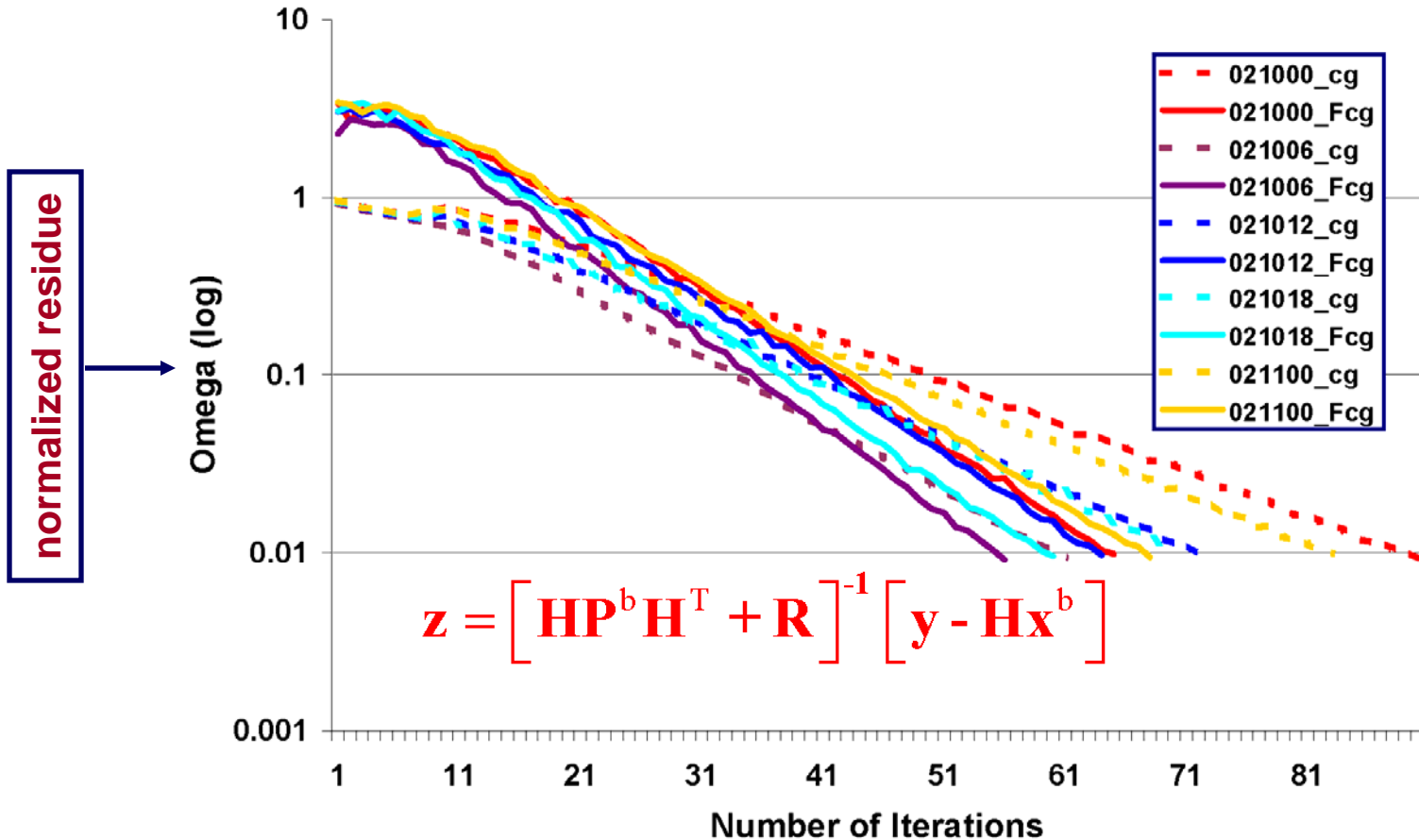


(from Chua and Xu)

Both cost functions are converged ~ 23 iterations!



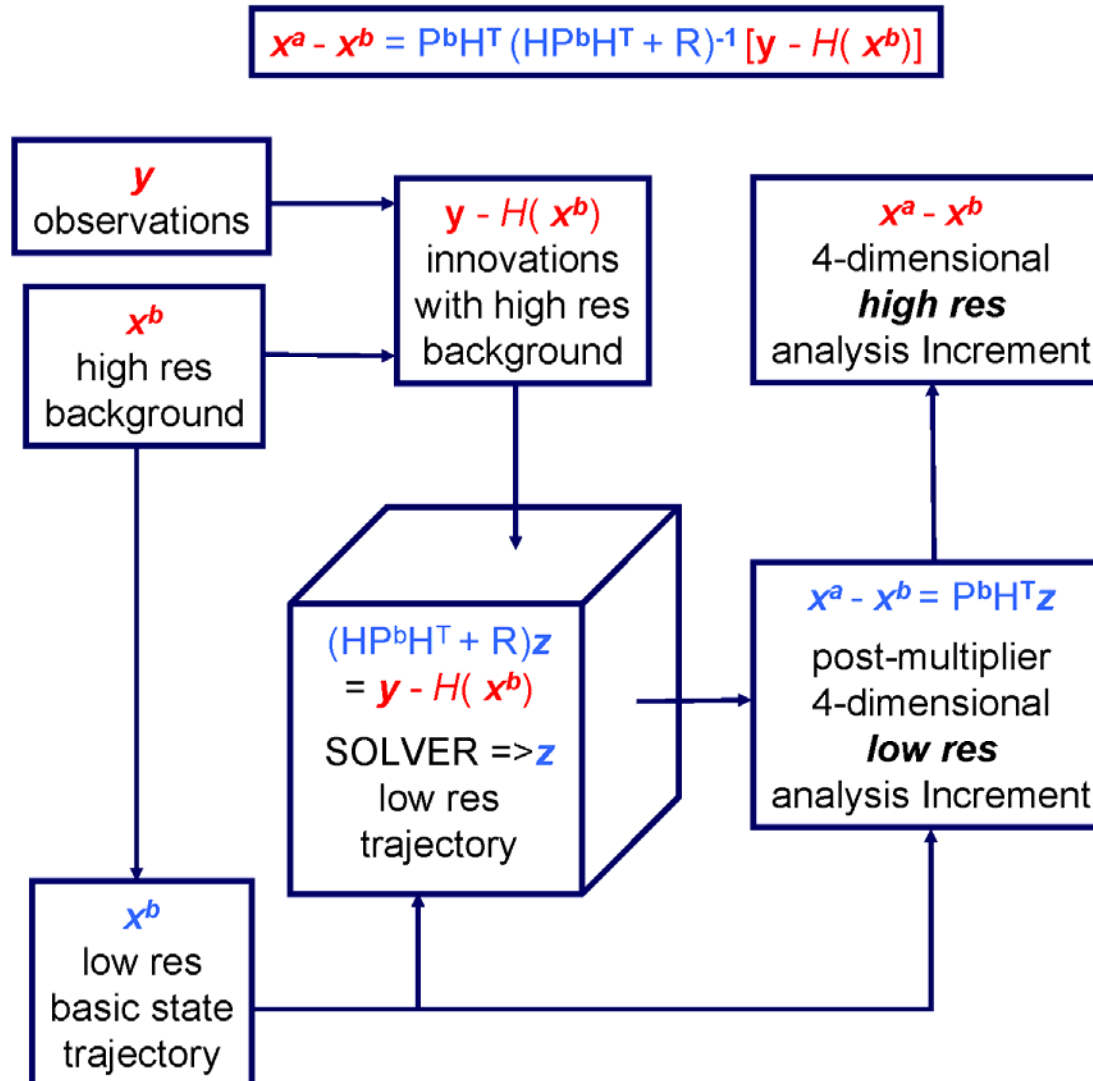
Standard CG vs. Flexible CG (FCG)



FCG is a "must have" in finding z accurately and efficiently!



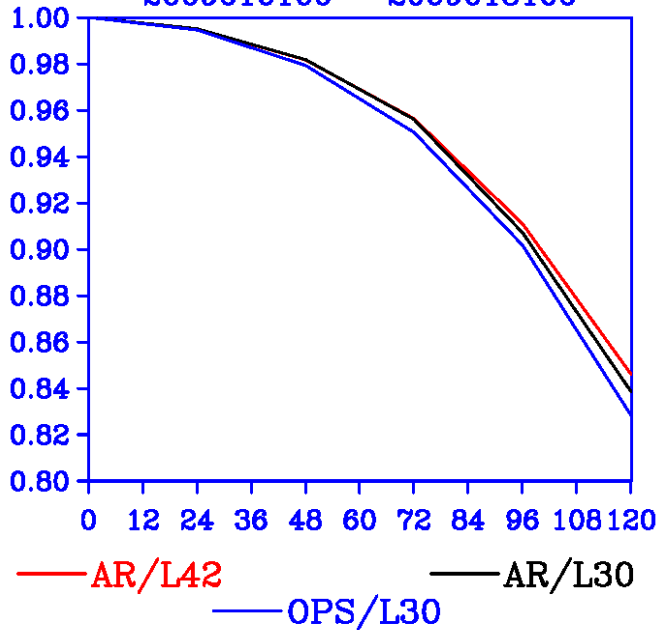
Flow Chart of NAVDAS-AR



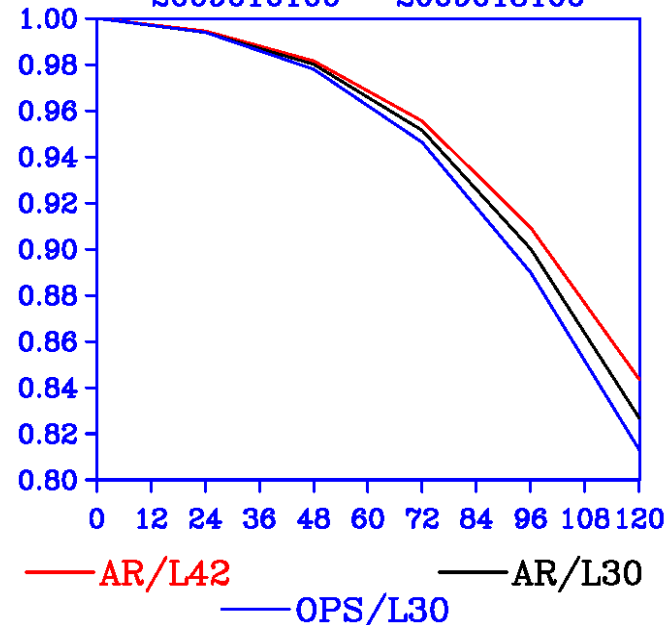


NAVDAS/L30, AR/L30, and AR/L42

GLOBAL MODELS COMPARISON
500 MB NORTH HEM HEIGHT ANOMALY COR
2009010100 - 2009013100



GLOBAL MODELS COMPARISON
500 MB SOUTH HEM HEIGHT ANOMALY COR
2009010100 - 2009013100



Comparison of NAVDAS (OPS/L30), NAVDAS-AR with 30 vertical levels (AR/L30) and NAVDAS-AR with 42 vertical levels (AR/L42) and model top of 0.04 hPa. AR/L42 allows for better radiance assimilation.



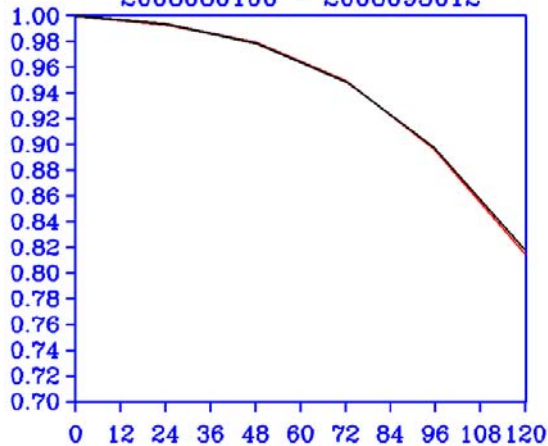
NAVDAS-AR Validation Test (VT)

- **Run a 6-h update cycle for both NAVDAS-AR/NOGAPS (alpha test) and NAVDAS/NOGAPS (OPS) data assimilation systems on FNMOCC's operational Linux cluster since October 2008.**
- **Initiated each run from the same set of NOGAPS initial conditions and allowed them to "spin-up" using its respective data assimilation system for 6-7 days before any test forecasts were made.**
- **Run five-day NOGAPS forecasts at 00Z for each data assimilation system from August-September 30, 2008 and from February–March, 2009.**
- **Verified the resulting NOGAPS one-day to five-day forecasts against their respective 00Z analyses and against 00Z rawinsonde observations from August-September 2008 and February-March 2009.**

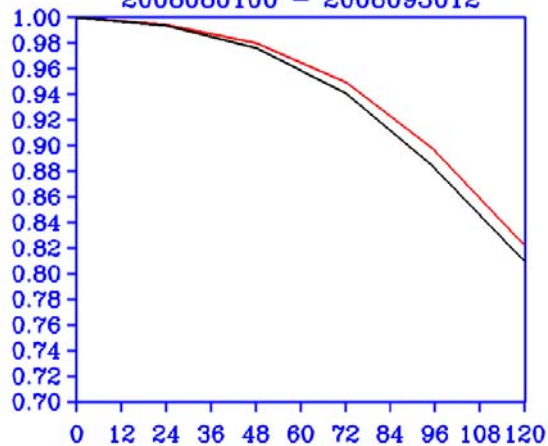


NAVDAS-AR VT Results - Summer 08

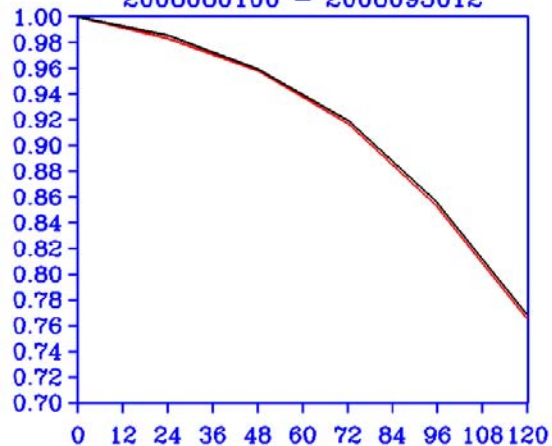
NOGAPS DATA ASSIMILATION TEST
500 MB NORTH HEM HEIGHT ANOMALY COR
2008080100 - 2008093012



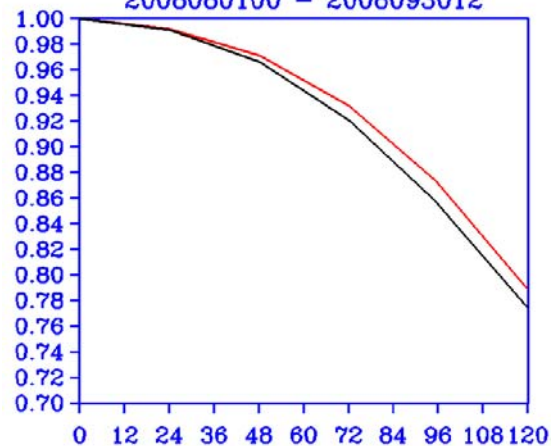
NOGAPS DATA ASSIMILATION TEST
500 MB SOUTH HEM HEIGHT ANOMALY COR
2008080100 - 2008093012



NOGAPS DATA ASSIMILATION TEST
1000 MB NORTH HEM HEIGHT ANOMALY COR
2008080100 - 2008093012



NOGAPS DATA ASSIMILATION TEST
1000 MB SOUTH HEM HEIGHT ANOMALY COR
2008080100 - 2008093012



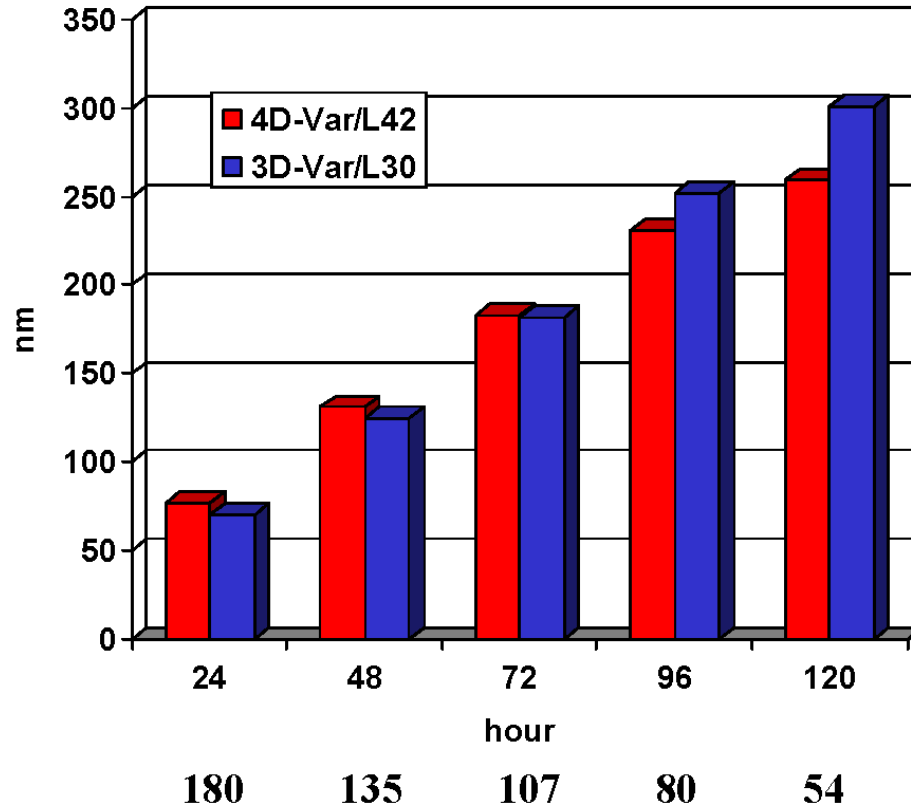
—AR/L42 —NAVD/L30

—AR/L42 —NAVD/L30

(from Hogan)



TC Track Errors - Summer 08



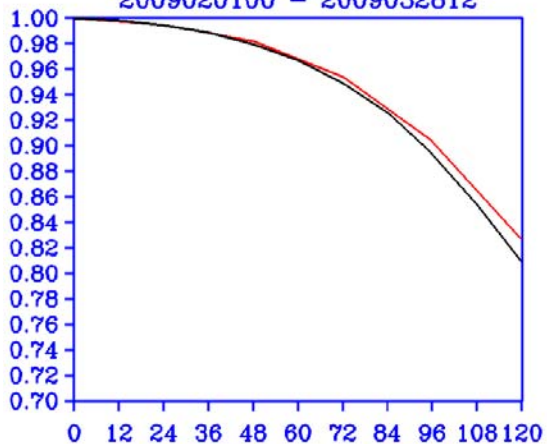
(from Hogan 2009)

Homogeneous comparison of Tropical Cyclone (TC) forecast track error (nm) for 4D-Var/L42 NAVDAS-AR (red) and 3D-Var/L30 NAVDAS (blue). Number of verifying forecasts is given below each forecast length (hour), August 1 - September 30, 2008.

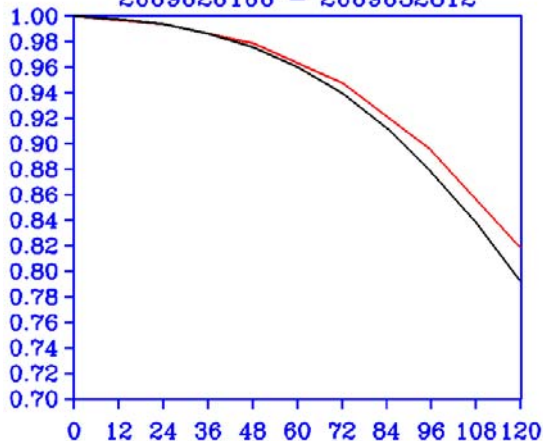


NAVDAS-AR VT Results - Winter 09

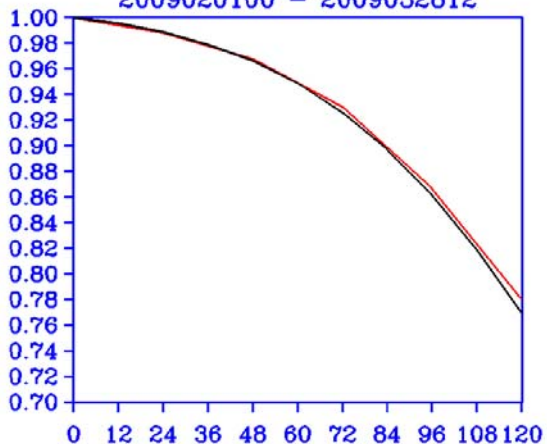
GLOBAL MODEL DATA ASSIMILATION
500 MB NORTH HEM HEIGHT ANOMALY COR
2009020100 - 2009032812



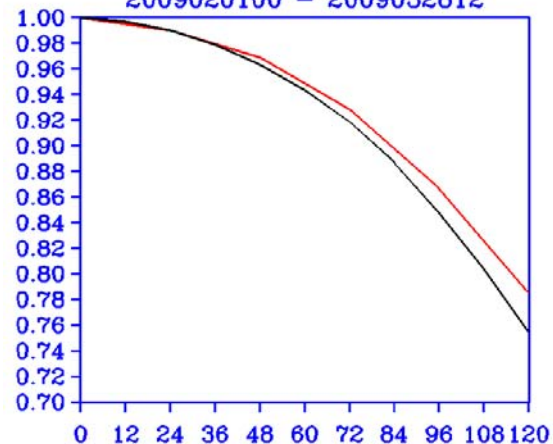
GLOBAL MODEL DATA ASSIMILATION
500 MB SOUTH HEM HEIGHT ANOMALY COR
2009020100 - 2009032812



GLOBAL MODEL DATA ASSIMILATION
1000 MB NORTH HEM HEIGHT ANOMALY COR
2009020100 - 2009032812



GLOBAL MODEL DATA ASSIMILATION
1000 MB SOUTH HEM HEIGHT ANOMALY COR
2009020100 - 2009032812



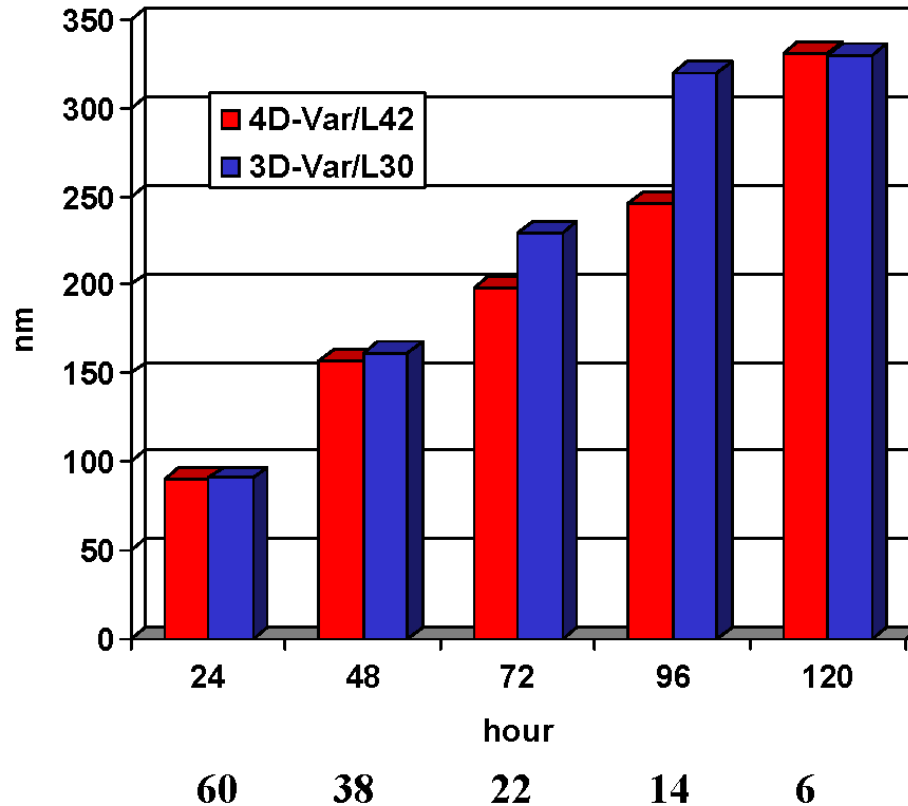
— AR/L42 — NAVD/L30

— AR/L42 — NAVD/L30

(from Hogan)



TC Track Errors - Winter 09



(from Hogan 2009)

Homogeneous comparison of Tropical Cyclone (TC) forecast track error (nm) for 4D-Var/L42 NAVDAS-AR (red) and 3D-Var/L30 NAVDAS (blue). Number of verifying forecasts is given below each forecast length (hour), February 1 - March 31, 2009.



Computational Aspect of NAVDAS-AR

- **NAVDAS-AR/L42, assimilating ~ 800,000 obs, runtime is about 25 min on 90 processors (NAVDAS/L30, assimilating ~ 500,000 obs, is about 15 min on 60 processors) on FNMOC's Linux cluster.**
- **NAVDAS-AR outer loop resolution is T239L42, while inner loop resolution is T119/L42.**
- **The iterative Solver (FCG) stops when the residue is less than 5% of the original innovation (~ 42 inner loop iterations). This stopping criteria is very conservative since the 4D-Var cost function converges at about 23 iterations.**
- **The number of inner loop iterations appears not very sensitive to the number of additional observations being assimilated so far.**
- **The matrix/vector multiplication of initial background error covariance with the adjoint sensitivity field at initial time uses about 35% of the total CPU during the minimization.**



Observations Being Assimilated in AR

• **Satellite observations**

- AMSU-A (4 NOAA satellites)
- AQUA AIRS/AMSU
- MetOp IASI*/AMSU
- Geostationary satellite winds – vis, IR and WV
- MODIS and AVHRR polar winds (including direct broadcast winds)
- DMSP SSMI and SSMIS wind speed and TPW
- QuikScat, ERS, and ASCAT scatterometer wind vectors
- DMSP F16 and F17 SSMIS sounder radiances (UPP)
- GOES Rapid Scan Atmospheric Motion Vectors
- WindSat ocean surface wind vectors and total precipitable water

* temporarily in monitoring status for OPS; due to FNMOC OPS runtime environment issues

• **Conventional observations**

- Raobs, pibals, dropsondes
- Surface ship, fixed and drifting buoys, land stations
- Aircraft obs, including Predator UAV capability
- Australian synthetic SLP obs, tropical cyclone synthetics obs (based on warning messages)

- Number of assimilated observations ~ 800,000 at FNMOC
- Number of assimilated observations ~ 1, 200,000 at NRL
- Number of iterations required in the “Solver” is not sensitive to the increase of number of observations to be assimilated

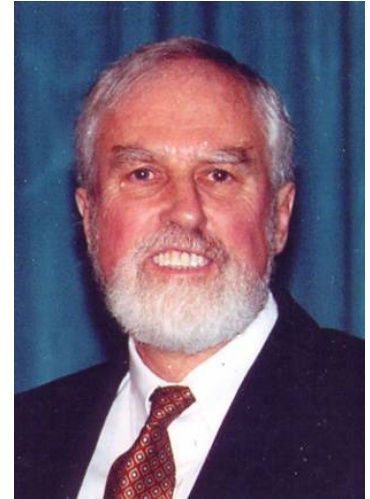


Observation Sensitivity Equation

NAVDAS(-AR) adjoint

$$\frac{\partial J}{\partial \mathbf{y}} = \overbrace{[\mathbf{HP}_b \mathbf{H}^T + \mathbf{R}]^{-1} \mathbf{HP}_b} \frac{\partial J}{\partial \mathbf{x}_a}$$

$$\frac{\partial \mathbf{x}^a}{\partial \mathbf{y}} = \mathbf{K}^T = (\mathbf{HP}^b \mathbf{H}^T + \mathbf{R})^{-1} \mathbf{HP}^b$$



The results of targeted observing field programs can be interpreted by extending the adjoint sensitivity vector into observation space --- Roger Daley

(from Baker 2000; Baker and Daley 2000; Langland and Baker 2004)

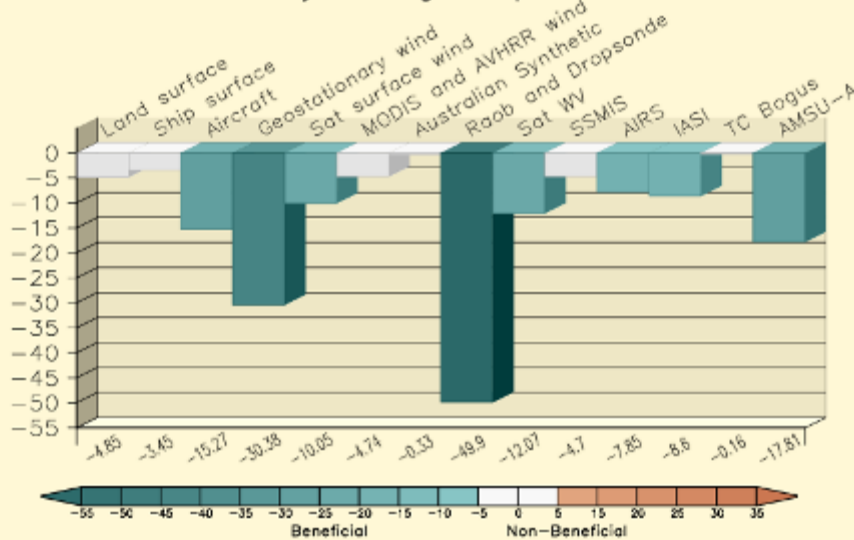


AR & NAVDAS Observation Impact

Developmental NAVDAS-AR
Impact Sum by Instrument Type

Impact of 00UTC observations on 24h global forecast error – moist total energy norm ($J\ kg^{-1}$)

30-days ending 08 Apr 2009



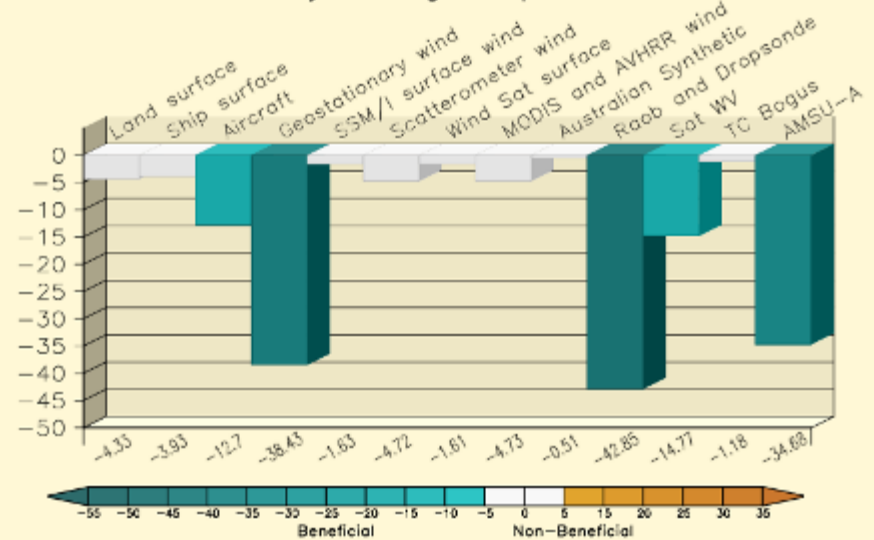
NAVDAS-AR

New satellites: SSMIS, AIRS, IASI

NRL NAVDAS
Impact Sum by Instrument Type

Impact of 00UTC observations on 24h global forecast error – moist total energy norm ($J\ kg^{-1}$)

30-days ending 09 Apr 2009



NAVDAS

<http://www.nrlmry.navy.mil/obsens/>



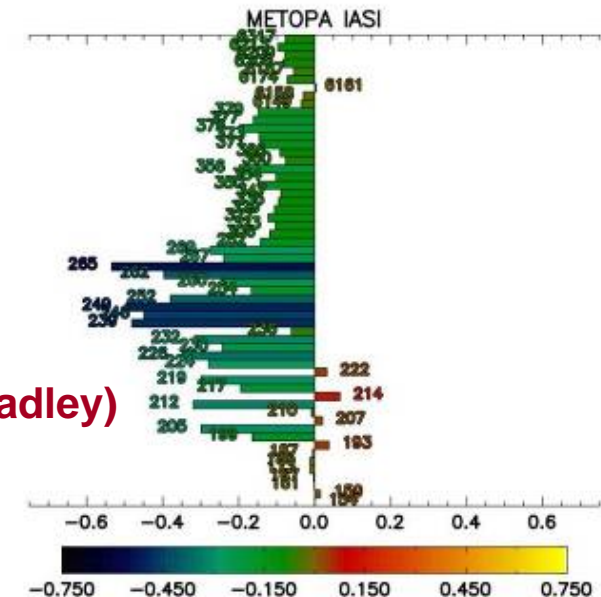
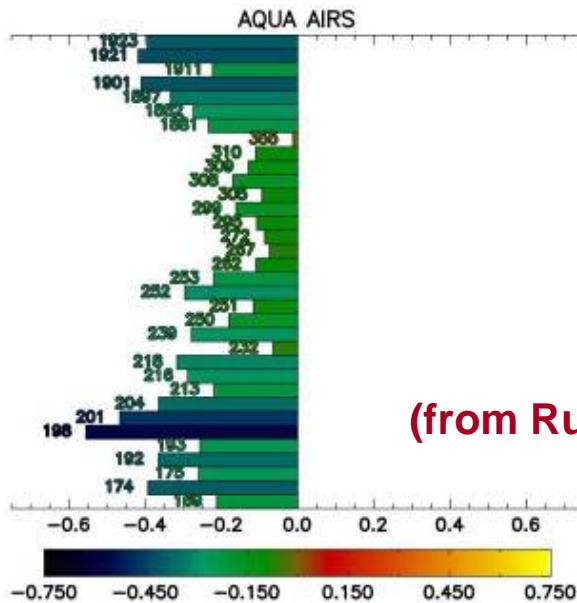
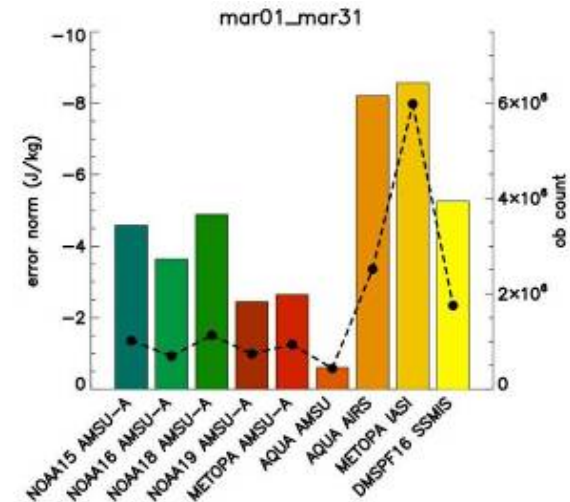
Impact of Advanced Sounders

Observation Sensitivity

Hyperspectral sounders contribute:

- large volume of data
- large reduction in forecast error norm

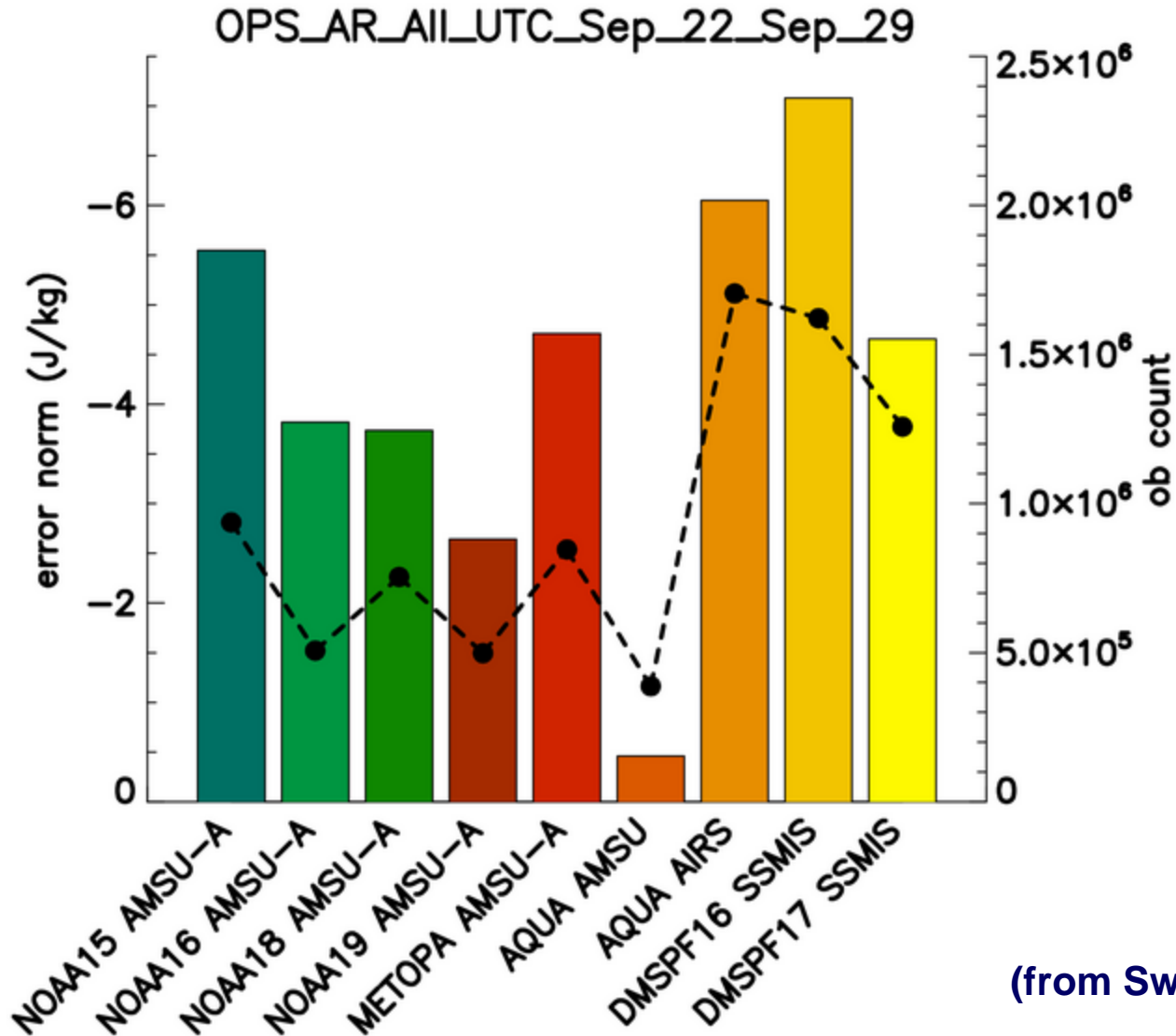
Complex channel interactions; observation sensitivity powerful tool discriminating impacts of additional channels



(from Ruston and Swadley)



Impact of Advanced Sounders ...



(from Swadley)



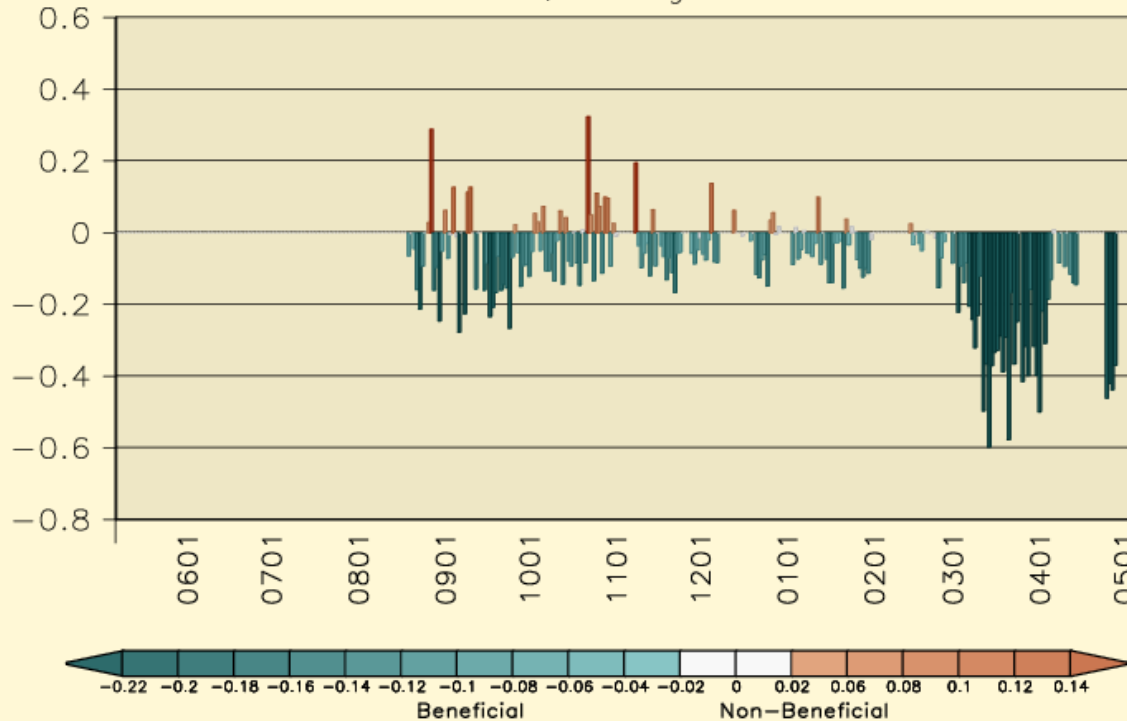
Satellite Data Impact (JCSDA)

Impact Sum for IASI

Impact of 00UTC observations on 24h global forecast error – moist total energy norm ($J\ kg^{-1}$)

1-year ending 06 May 2009

Sum = -19.5, Average = -0.101



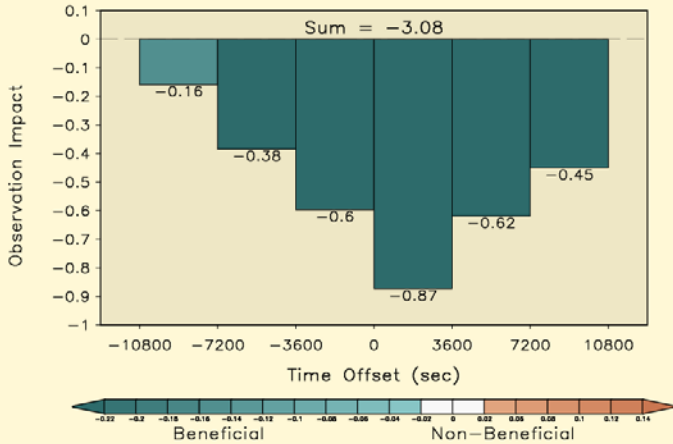
Improved impact of IASI radiance data from channel selection diagnostics and other changes

Reduction of 24h forecast error

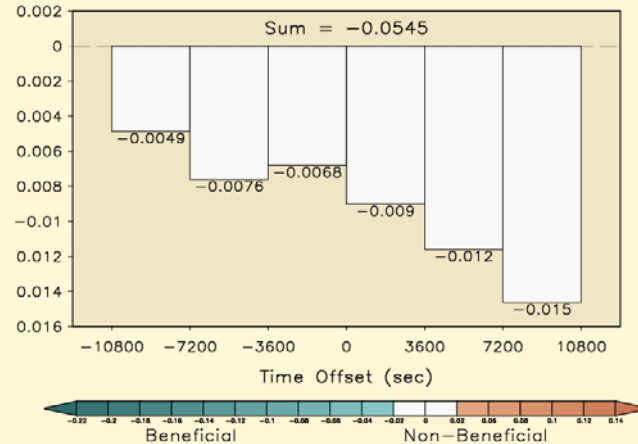


Operational Observation Impact

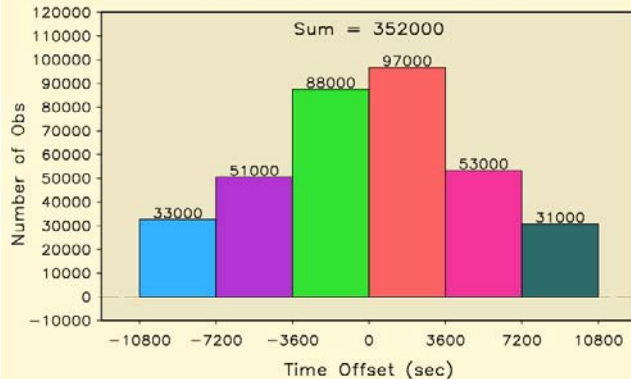
Global Observation Impact Sum
 Mean AMSU TB OB T[C]
 All Satellites Chan 7
 30-days ending 28 SEP 2009



Observation Impact / Ob [*1000]
 Mean AMSU TB OB T[C]
 All Satellites Chan 7
 30-days ending 28 SEP 2009



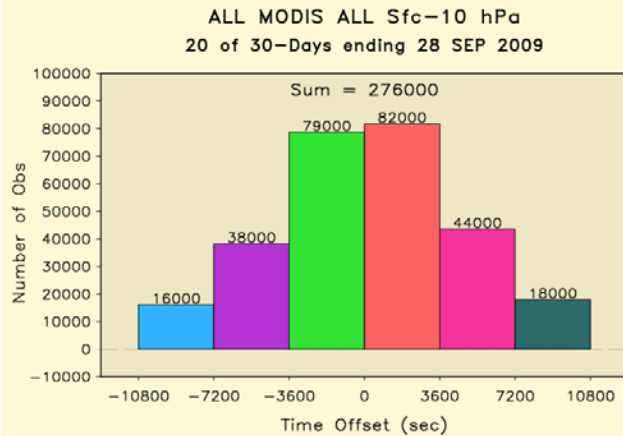
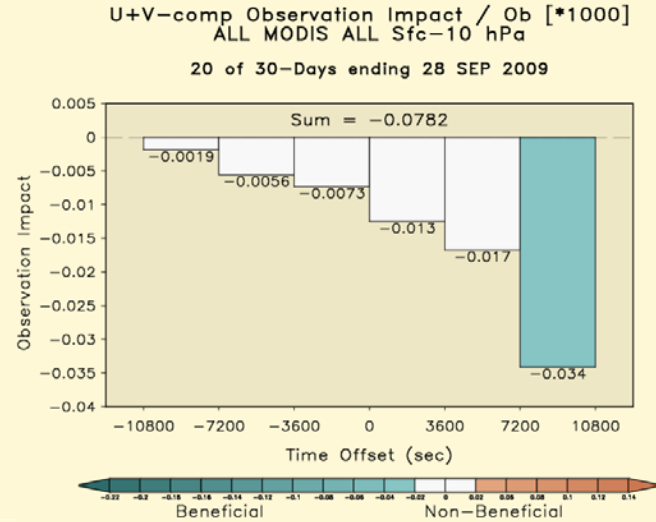
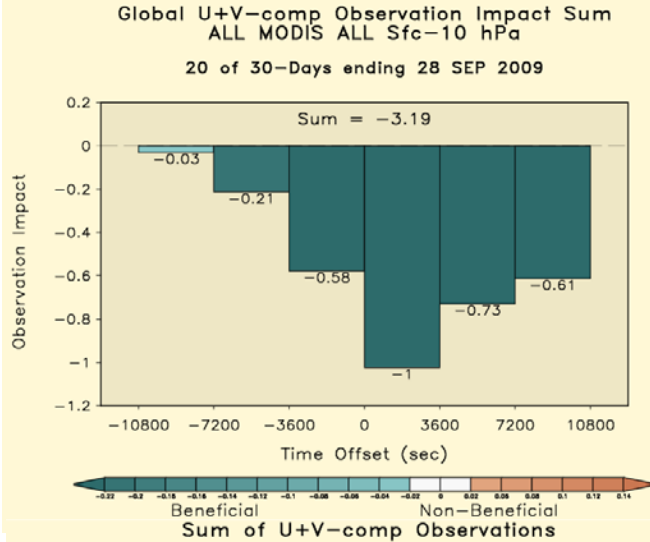
Sum of Observations
 Mean AMSU TB OB T[C]
 All Satellites Chan 7
 30-days ending 28 SEP 2009



- Ob impact for all amsu-a, channel 7 (mid trop), current FNMOC AR Shows later arriving data has more impact per ob.
- Total impact, shows same trend up to the point where the ob count decreases (obs haven't arrived at FNMOC yet).



Operational Observation Impact ...



- Ob impact for all MODIS winds, current FNMOC AR Shows later arriving data has more impact per ob.
- Total impact, shows same trend up to the point where the ob count decreases (obs haven't arrived at FNMOC yet).



Recently Implemented AR Capabilities

- **Weak constraint option:** allows us to treat not only the spatially correlated model errors but also the temporally correlated model errors.
- **Two additional outer loop options:** allow us to account for the non-linearity in both the NWP model and the observation operators.
- **Adaptive observation error variance adjustment:** provides us an efficient way to dynamically adjust the observation error variance (Desroziers and Ivanov, 2001).
- **A multi-purpose preconditioner:** allows us to efficiently calculate: the adjoint of NAVDAS-AR; adaptive observation error variance adjustment; and the additional outerloop(s) of the minimization.
- **Variational bias correction of radiances.**
- **Large scale precipitation and cumulus parameterization:** provides us more accurate TLM/ADJ models.
- **Chemistry component:** allows us to assimilate ozone.

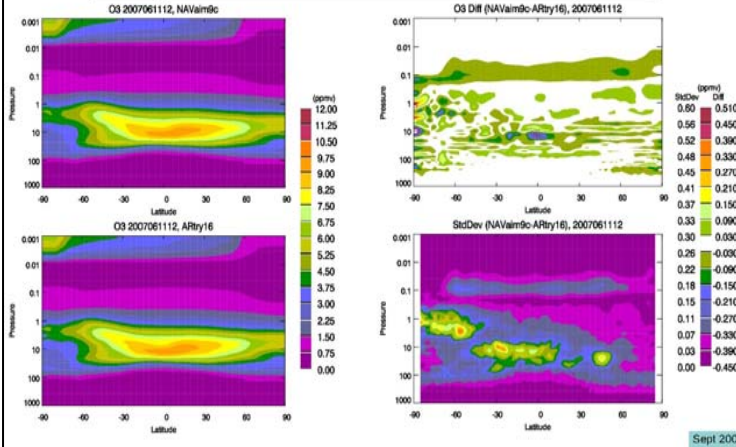


Example of Recent AR Development

Ozone Assimilation

Comparison between 3DVar & 4DVar Analysis of 11-Jun-2007

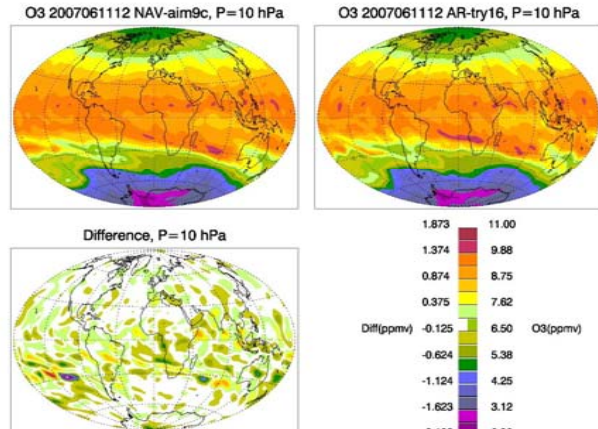
The zonal mean differences and standard deviation are very small, indicating that the NAVDAS and NAVDAS-AR analysis are nearly identical



Sept 2009

Comparison between 3DVar & 4DVar Ozone Analysis

Synoptic comparison shows nearly identical horizontal distribution

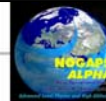


Sept 2009



First Results: NAVDAS-AR Ozone Assimilation

Karl Hoppel, NRL Code 7227



NAVDAS 3DVar Analysis (denoted NAVaim9c):

- NAVDAS ozone-only assimilation
- MLS Ozone observations, 215 – 0.02 hPa
- NOGAPS-ALPHA T79L68 with ozone chemistry

Eckermann et al., High-Altitude Data Assimilation System Experiments for the Northern Summer Mesosphere Season of 2007, JASTP 2009

NAVDAS-AR 4DVar Analysis (denoted ARtry16):

- NAVDAS-AR SVN version ~142 plus...
- Tom Rosmond's new chemistry additions w/ Karl Hoppel's debugging/testing
- All other aspects the same as the NAVDAS analysis
- **** This is a preliminary "first try" ****

Sept 2009

Conclusions

- Ozone assimilation with NAVDAS-AR works!
- Results appear similar to 3DVar, for similar setup
- No obvious degradation in the temp/wind analysis with ozone included

(from Karl Hoppel)

Sept 2009



Planned AR New Capabilities

- **Calculation of impact of initial background error variance specified in NAVDAS-AR on NOGAPS short term forecasts.**
- **Calculation of impact of observation error variance specified in NAVDAS-AR on NOGAPS short term forecasts.**
- **Estimation of state-dependent background error covariance.**
- **Estimation of model error variance needed in the weak constraint option.**
- **Estimation of analysis error variance/covariance using ensemble of NAVDAS-AR.**
- **Assimilation of new sensors.**



NAVDAS-AR Related Publications ...

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- Rosmond, T., L. Xu, N. Baker, B. Campbell, C. Blankenship, J. Goerss, B. Ruston, and P. Pauley, 2006: Direct radiance assimilation with NAVDAS-AR. *The Seventh International Workshop on Adjoint Applications in Dynamic Meteorology*, Obergurgl, Austria, 9-13 October 2006.
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A sunset scene with a bright sun low on the horizon, casting a warm orange glow. The sky is filled with dark, dramatic clouds, some of which are illuminated from below by the sun. The foreground is a dark, silhouetted landscape. The text "Thank You" is written in a white, cursive font with a thin black outline, centered in the middle of the image.

Thank You