The Development of a New Ocean Circulation Model in the Sigma Coordinate System and its Application to North Atlantic Ocean

Gary A. Zarillo, Ph.D., PG
Department of Marine and Environmental Systems
Florida Institute of Technology
and
Melbourne, FL USA

Current Problems of Models in the Sigma-coordinate System

Over Steep Bottom Topography

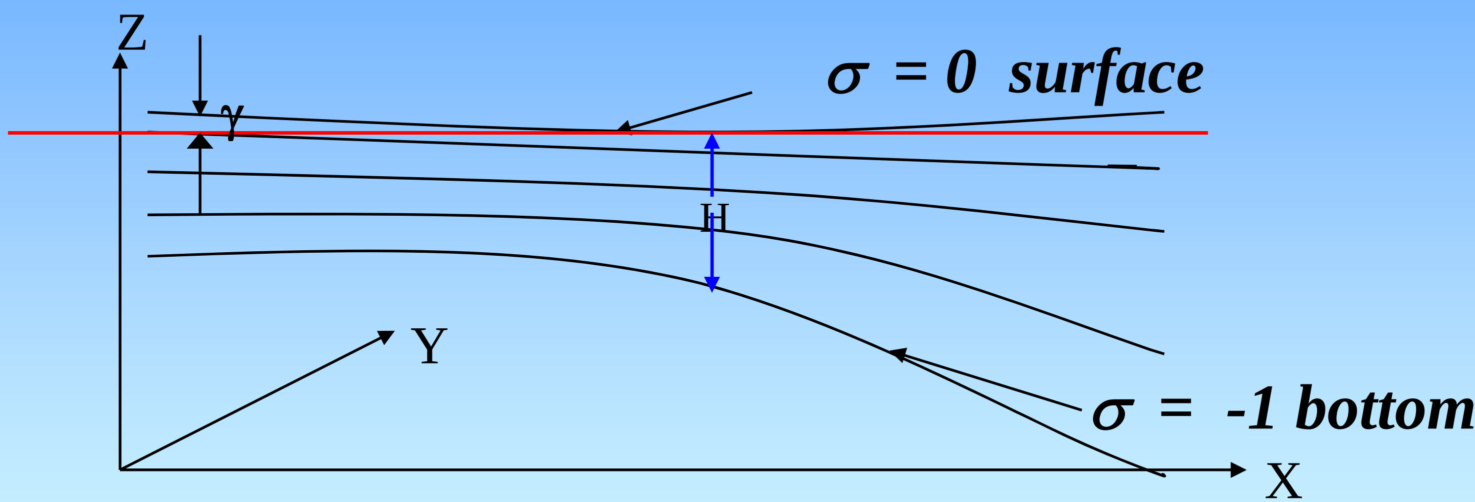
1. Numerical Instability

2. Diapycnal Mixing Process

3. Baroclinic Forces

4. Unusual Oscillations

The Sigma-Coordinate System



$$\sigma(x,y,z,t)=\frac{z-\eta(x,y,t)}{H(x,y)+\eta(x,y,t)}$$

Formulation of the Florida Ocean Model

Goals

Reduce numerical truncation error
Long-term stable runs over high relief topography
Utilize full values of scalar ocean properties
Prevent diapycnal processes in scalar properties arising from bottom slope

Formulation of the Florida Ocean Model

Primitive Equation Formulation

$$\frac{\partial u}{\partial t}+\frac{\partial uu}{\partial x}+\frac{\partial uv}{\partial y}+\frac{\partial \omega u}{D\partial \sigma}-u\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial \omega}{D\partial \sigma}\right)-fv+g\frac{\partial \eta}{\partial x}=$$

$$\frac{\partial}{D^2\partial \sigma}A_V\left(\frac{\partial u}{\partial \sigma}\right)+\frac{\partial}{\partial x}\left(2A_H\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}A_H\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)$$

$$-gD\int_{\sigma}^{0}\left(\frac{\partial \rho}{\partial x}-\frac{\sigma}{D}\frac{\partial D}{\partial x}\frac{\partial \rho}{\partial \sigma}\right)d\sigma$$

$$\frac{\partial \theta}{\partial t}+\frac{\partial \theta u}{\partial x}+\frac{\partial \theta v}{\partial y}+\frac{\partial \theta\omega}{D\partial \sigma}-\theta\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial \omega}{D\partial \sigma}\right)=-\frac{\partial R}{D\partial \sigma}$$

$$+Sflux\Big|_{\sigma=0}+\frac{\partial}{D^2\partial \sigma}K_v\left(\frac{\partial \theta}{\partial \sigma}\right)+\frac{\partial}{\partial x}\left(K_h\frac{\partial \theta}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_h\frac{\partial \theta}{\partial y}\right)$$

Formulation of the Princeton Ocean Model

$$\frac{\partial \eta}{\partial t} + \frac{\partial uD}{\partial x} + \frac{\partial vD}{\partial y} + \frac{\partial \omega}{\partial \sigma} = 0$$

(continuity)

$$\frac{\partial uD}{\partial t} + \frac{\partial uuD}{\partial x} + \frac{\partial uvD}{\partial y} + \frac{\partial u\omega}{\partial \sigma} - fvD =$$

$$\frac{\partial}{D\partial \sigma} A_V (\frac{\partial u}{\partial \sigma}) + \frac{\partial}{\partial x}(2A_H D \frac{\partial u}{\partial x}) + \frac{\partial}{\partial y} A_H D(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y})$$

$$- gD\frac{\partial \eta}{\partial x} - gD^2 \int_{\sigma'}^{0} (\frac{\partial(\rho - \overline{\rho})}{\partial x} - \frac{\sigma'}{D}\frac{\partial D}{\partial x}\frac{\partial(\rho - \overline{\rho})}{\partial \sigma'}) d\sigma'$$

(x-momentum)

$$\frac{\partial \theta D}{\partial t} + \frac{\partial u\theta D}{\partial x} + \frac{\partial v\theta D}{\partial y} + \frac{\partial \omega\theta}{\partial \sigma} = -\frac{\partial R}{D\partial \sigma} - Sflux|_{\sigma=0}$$

$$\frac{\partial}{D\partial \sigma}(K_V \frac{\partial \theta}{\partial \sigma}) + \frac{\partial}{\partial x}(K_h D \frac{\partial(\theta - \overline{\theta})}{\partial x}) + \frac{\partial}{\partial y} K_h D(\frac{\partial(\theta - \overline{\theta})}{\partial y})$$

(Tracer)

Calculating Numerical Truncation Errore

$$POMError\ (t,\ell)=\frac{1}{48}\{\frac{\partial^3 FVD}{\partial \ell^3}\Delta\ell^2+\frac{\partial^3 F\omega}{\partial\sigma^3}\Delta\sigma^2+\frac{\partial^3 FD}{\partial t^3}\Delta t^2\}$$

$$=O(\Delta\ell^2,F,D,V)+O(\Delta t^2,F,D)$$

$$FOMError(t,l)=\frac{1}{48}\{(\frac{\partial^3 FV}{\partial\ell^3}-F\frac{\partial^3 V}{\partial\ell^3})\Delta\ell^2+\frac{1}{D}(\frac{\partial^3 F\omega}{\partial\sigma^3}-F\frac{\partial^3\omega}{\partial\sigma^3})\Delta\sigma^2+\frac{\partial^3 F}{\partial t^3}\Delta t^2\}$$

$$=O(\Delta\ell^2,F,V)+O(\Delta t^2,F)$$

Numerical Scheme using FDM

- Leap-Frog Scheme in Time

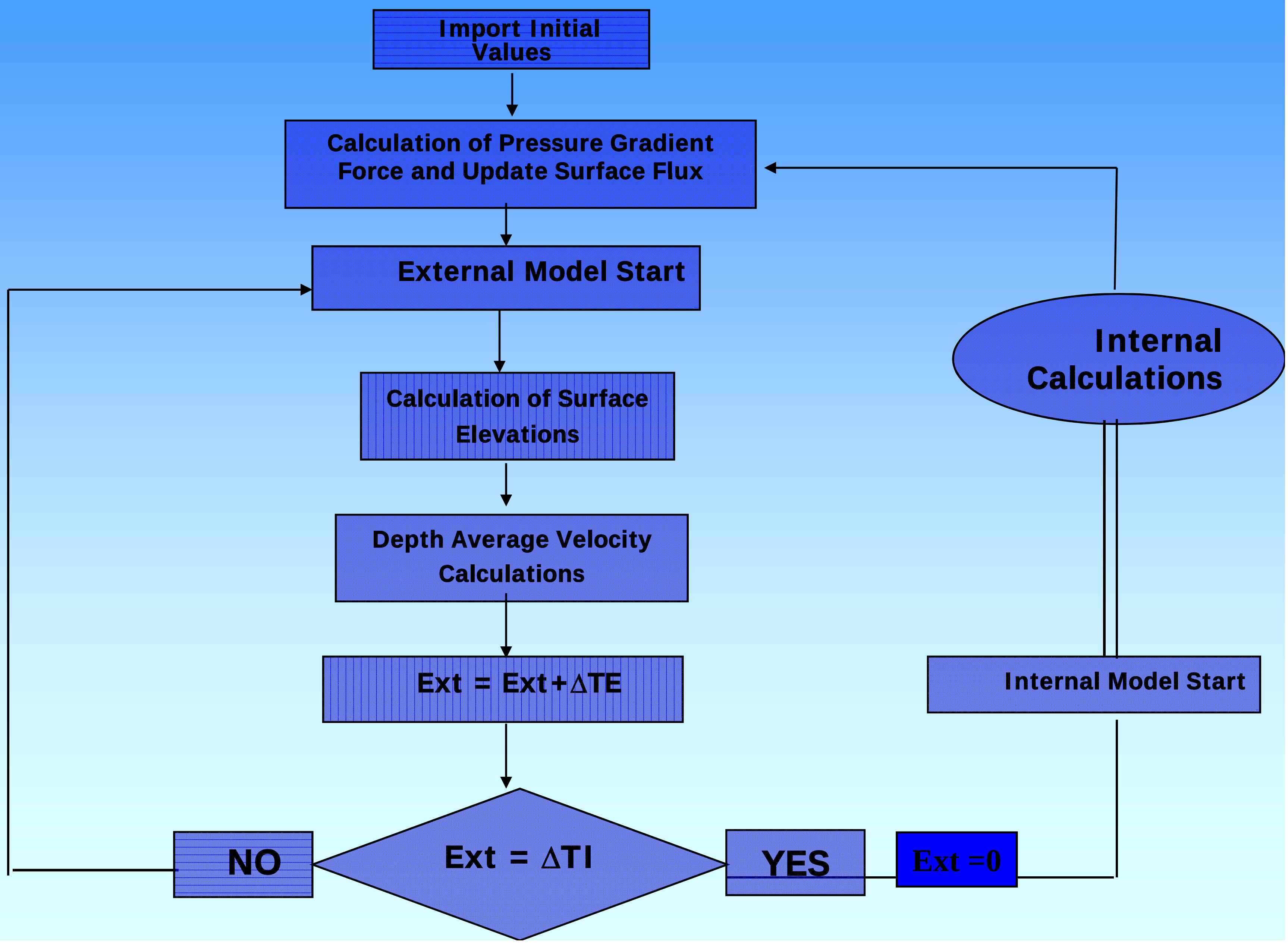
- Centered Scheme in Space

-Weak Numerical Filter

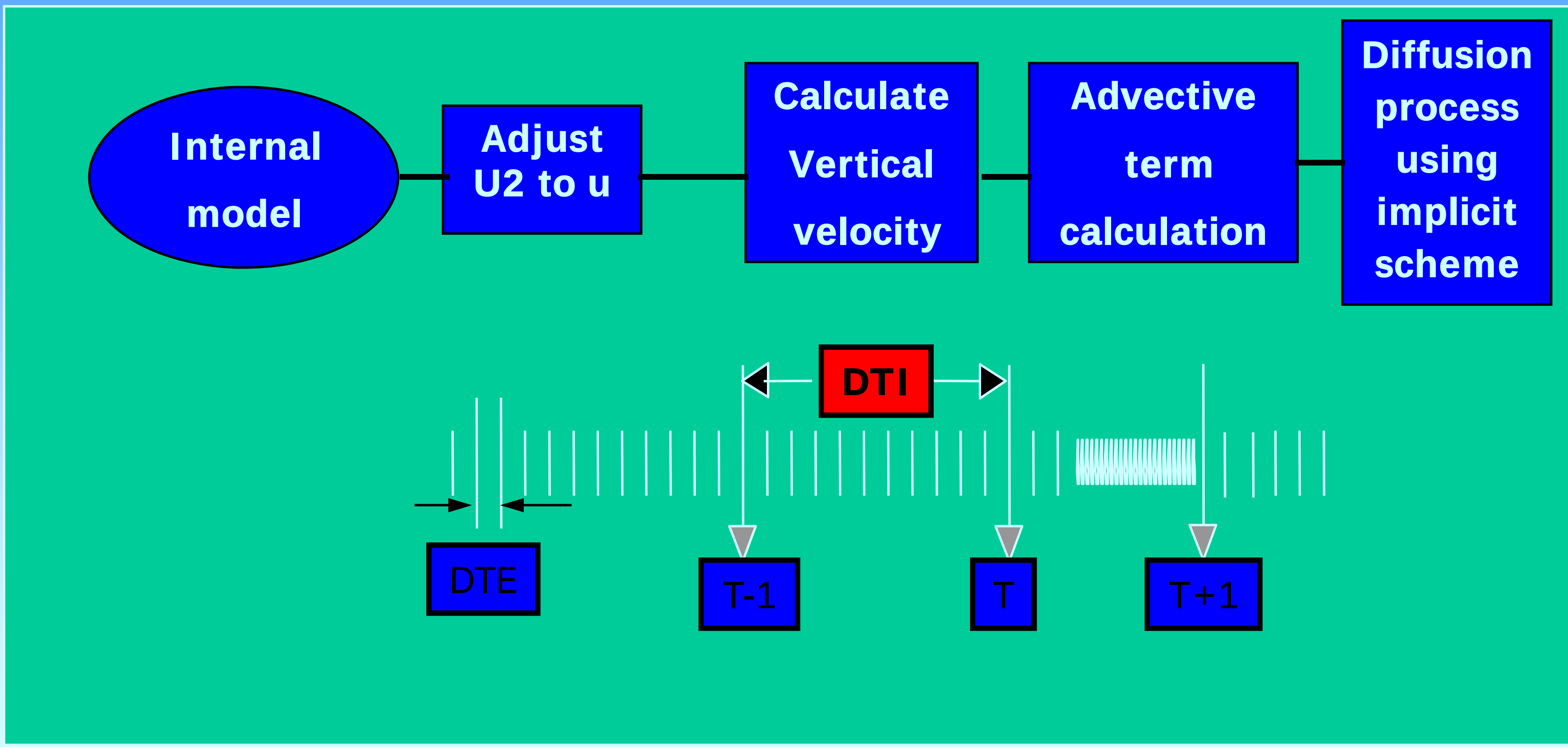
- Implicit Scheme for Vertical Diffusion

  and Surface Flux

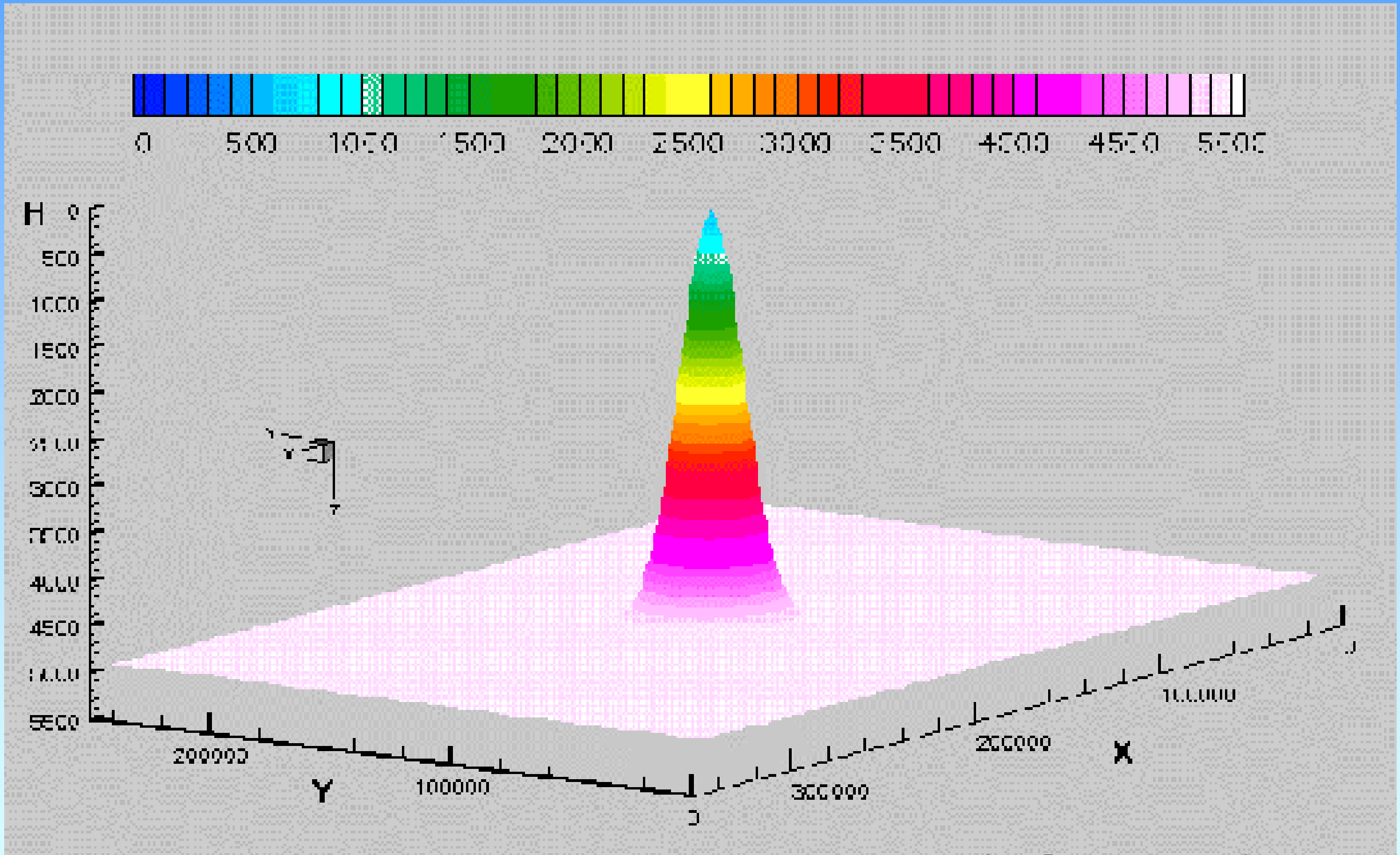
Schematic Flow Chart of Model



Split Method Between 2-D and 3-D



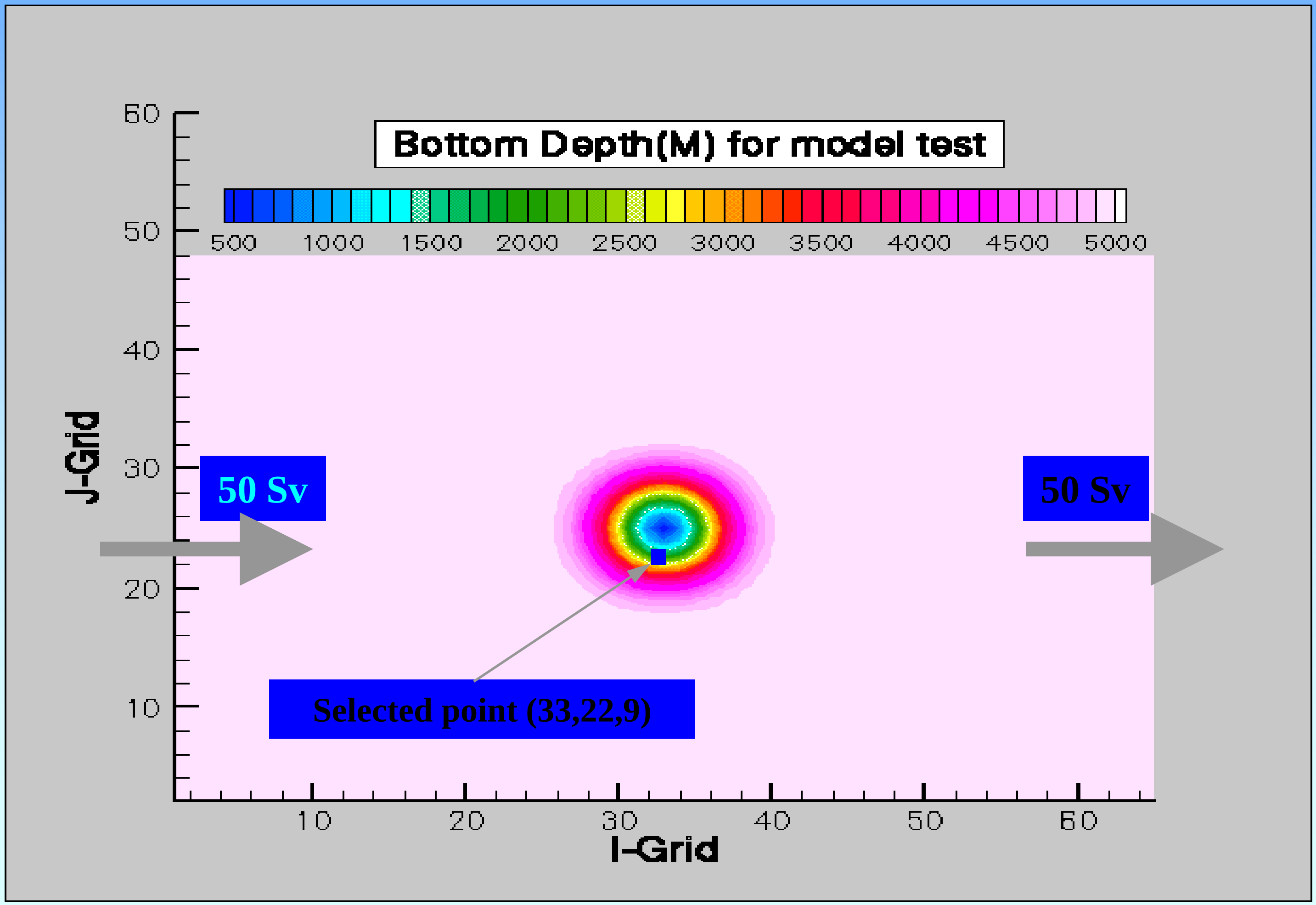
Model Tests Comparing POM and FOM



Numerical Test Basin

Bottom contours & O.B.C. of Model

Grid (I=65,J=48) **Mean Vel.@O.B.C. =4.0 cm/s**



Initial Conditions

Salinity=35.0 constant

T_0=25.0 °C; Surface T_b=1.0 °C; Bottom

There is theoretically no baroclinic force by density gradient.

Comparison of Results Between FOM and POM

Case 1:

Typical Experiment

Case 2:

Climatology

Case 3:

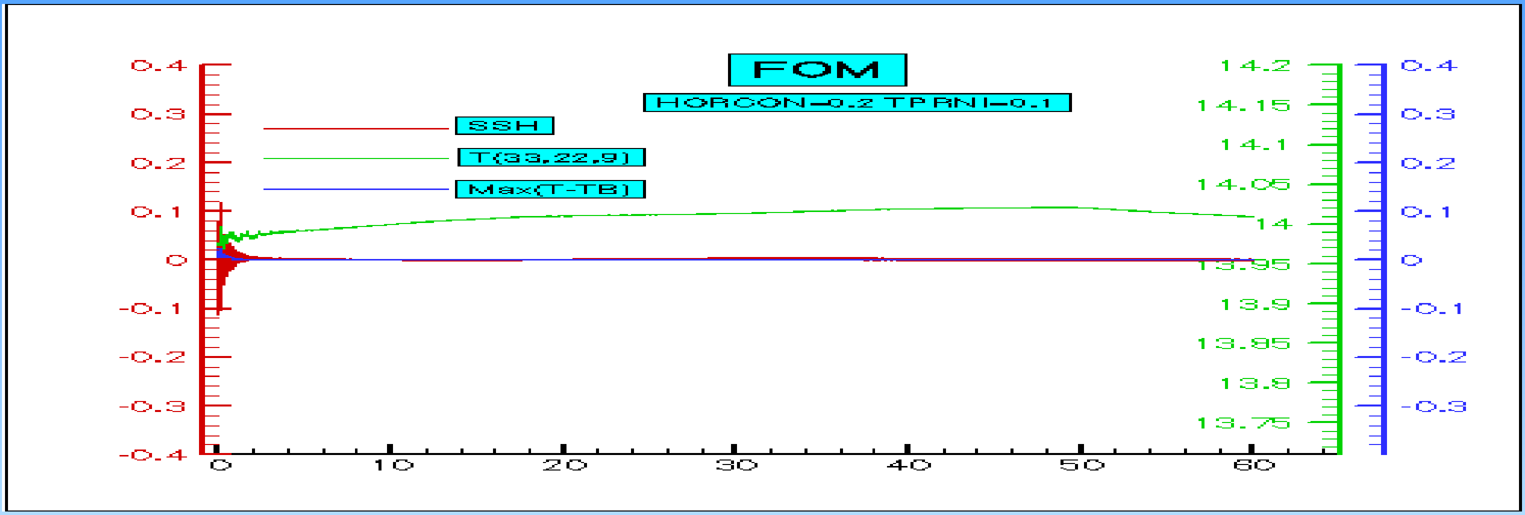
Large Viscosity(Clim.=0. & Horcon=1)

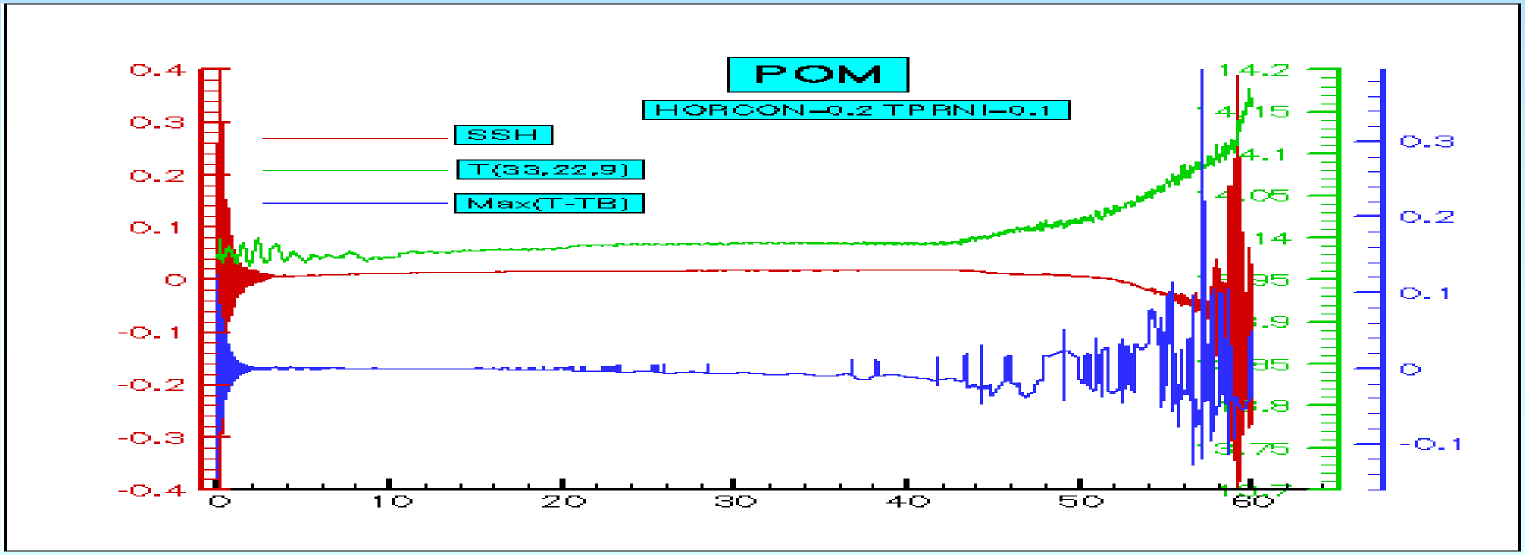
Case 4:

Diagnostic Experiment in Closed Basin

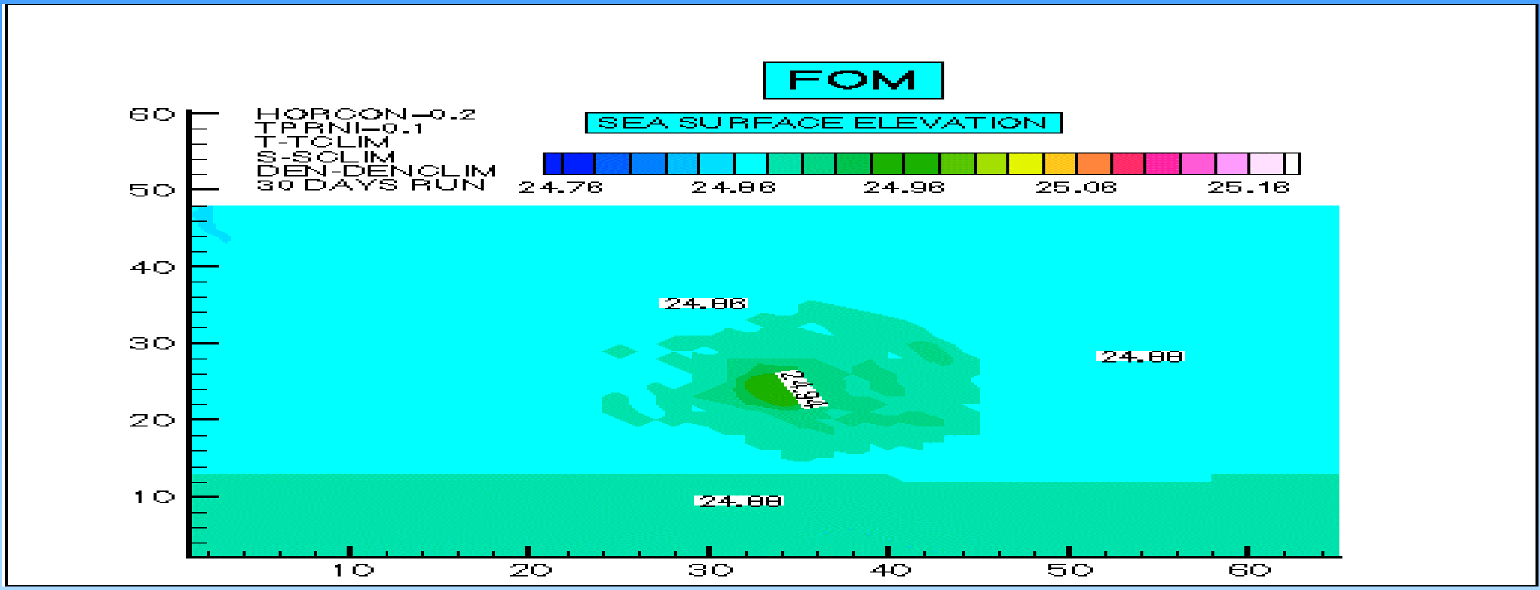
Case 1: Typical Experiment

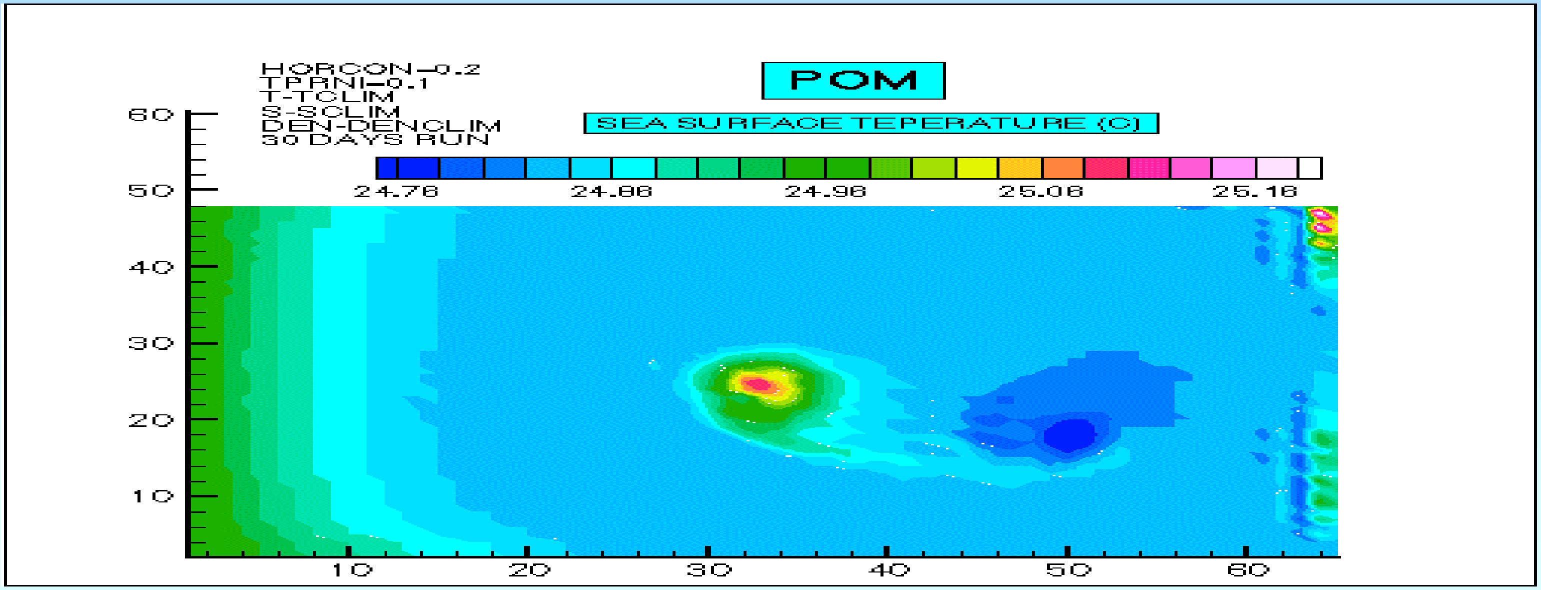
Time Series of SSH, Temperature (33,22,9) and Max. Dev. Temp



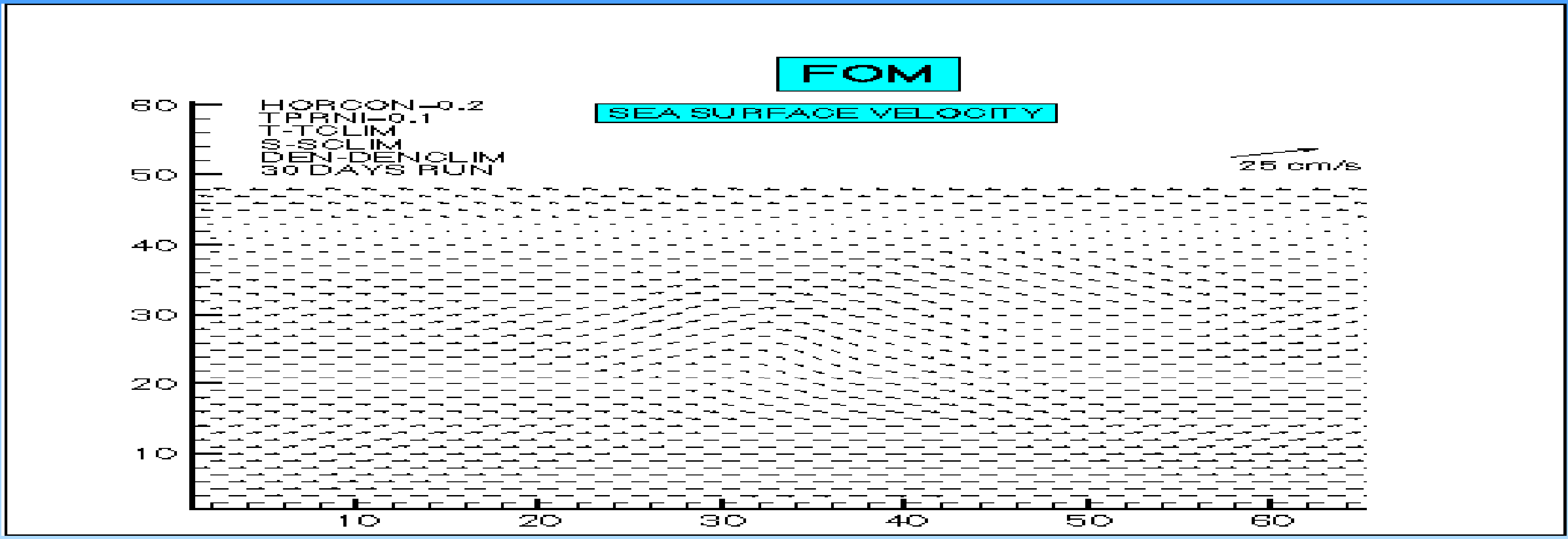


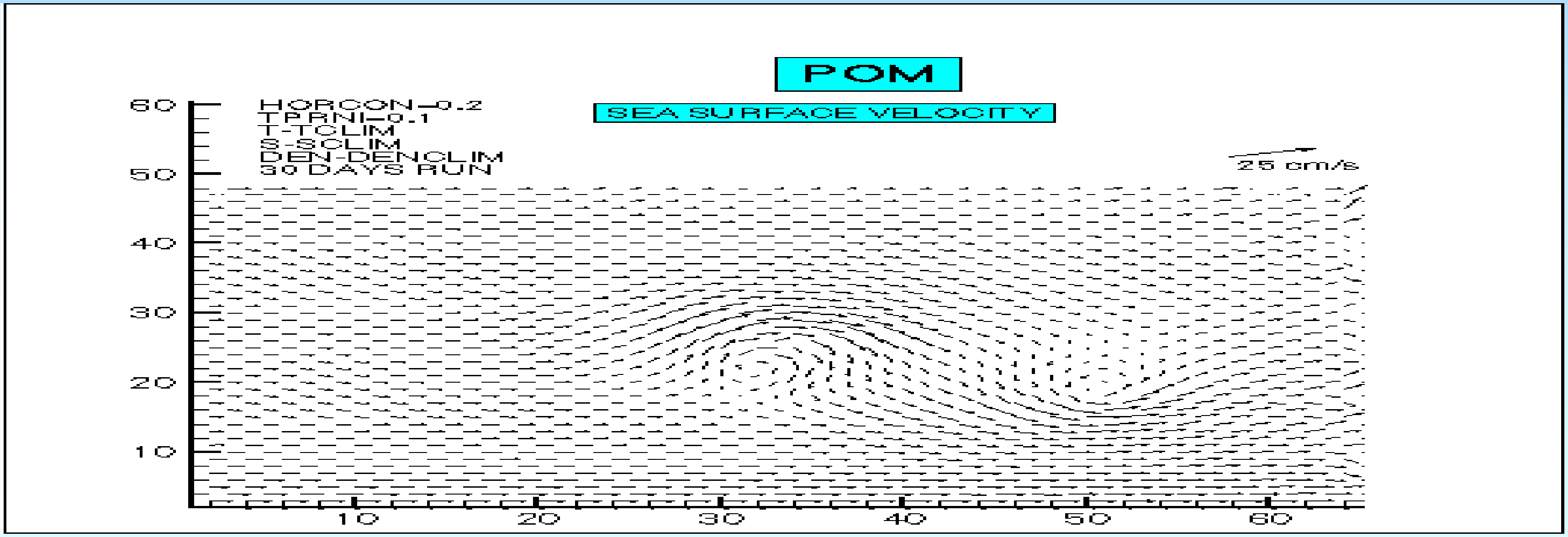
Case 1: Typical Experiment:SST



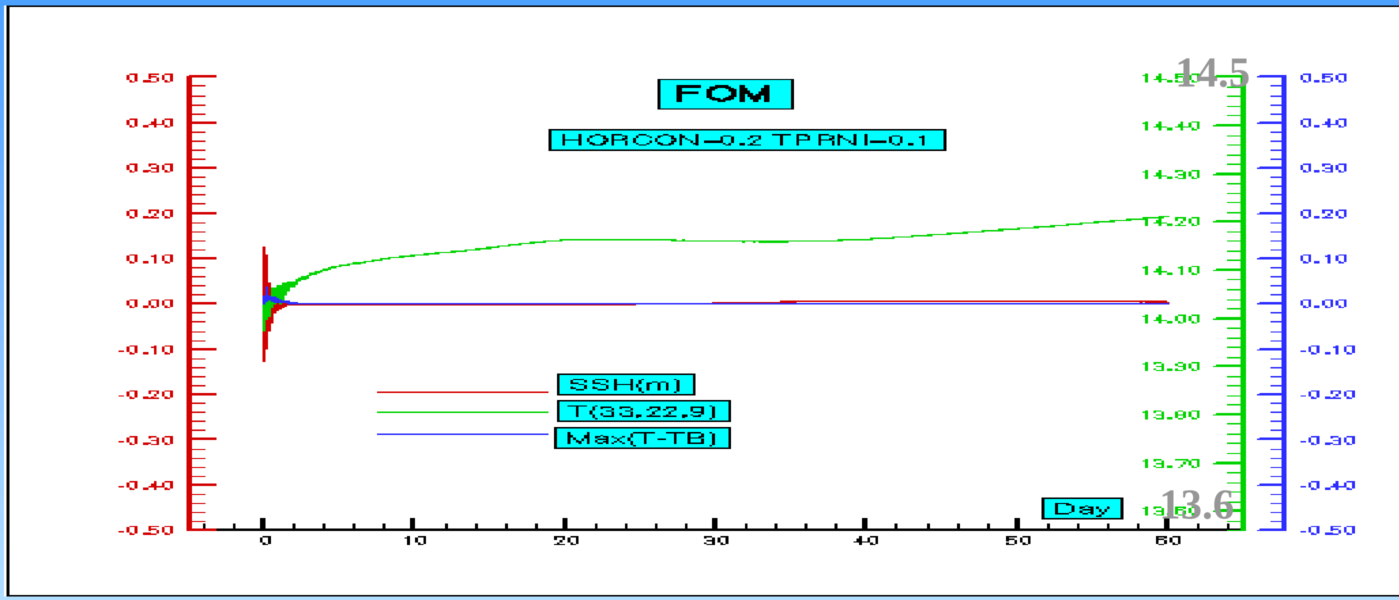


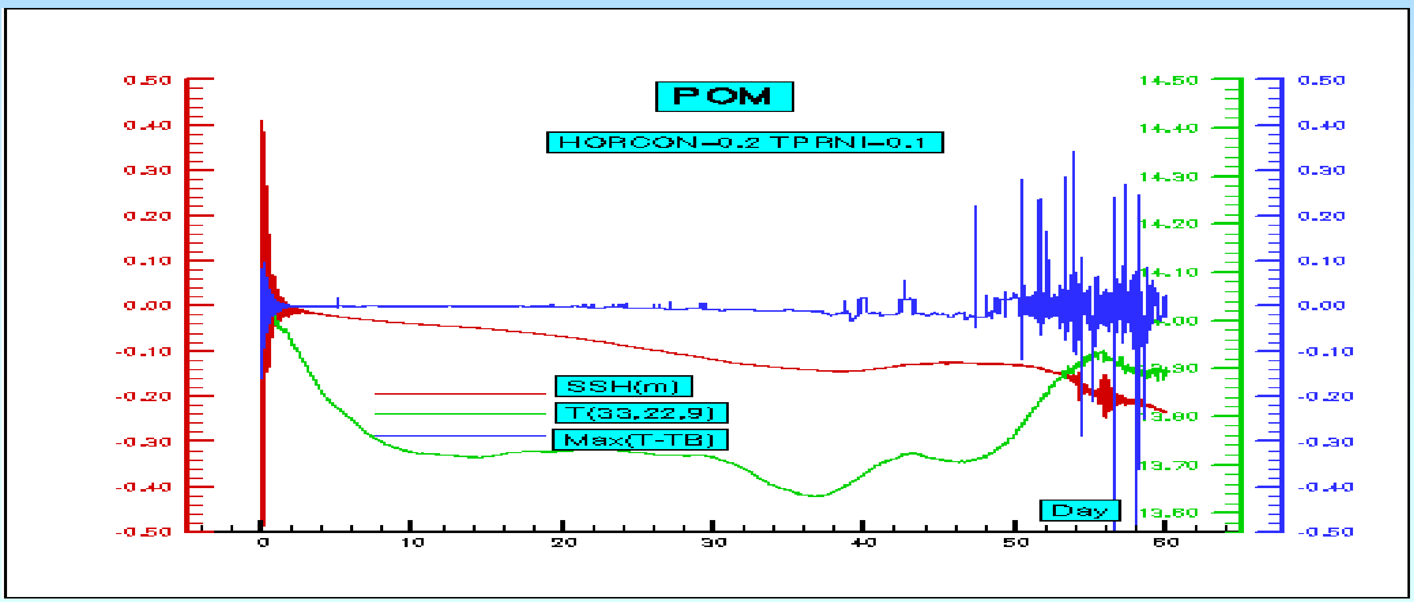
Case 1: Typical Experiment:Velocity(20m)



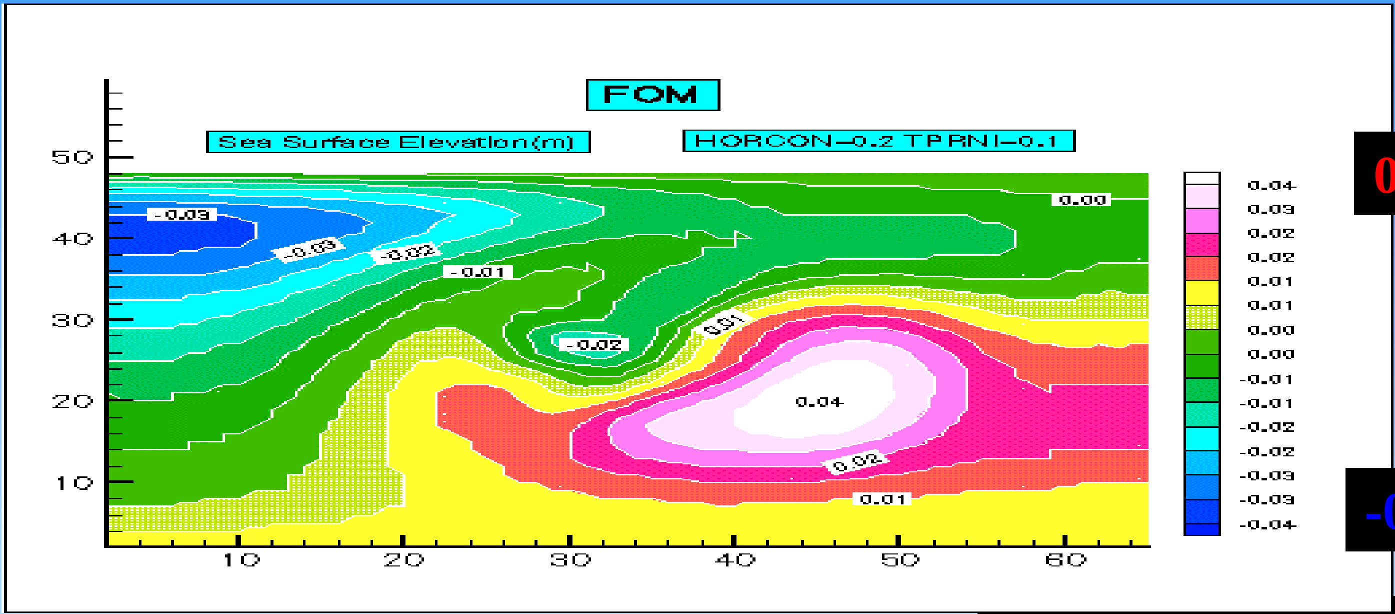


Case 2: Time series at atoll





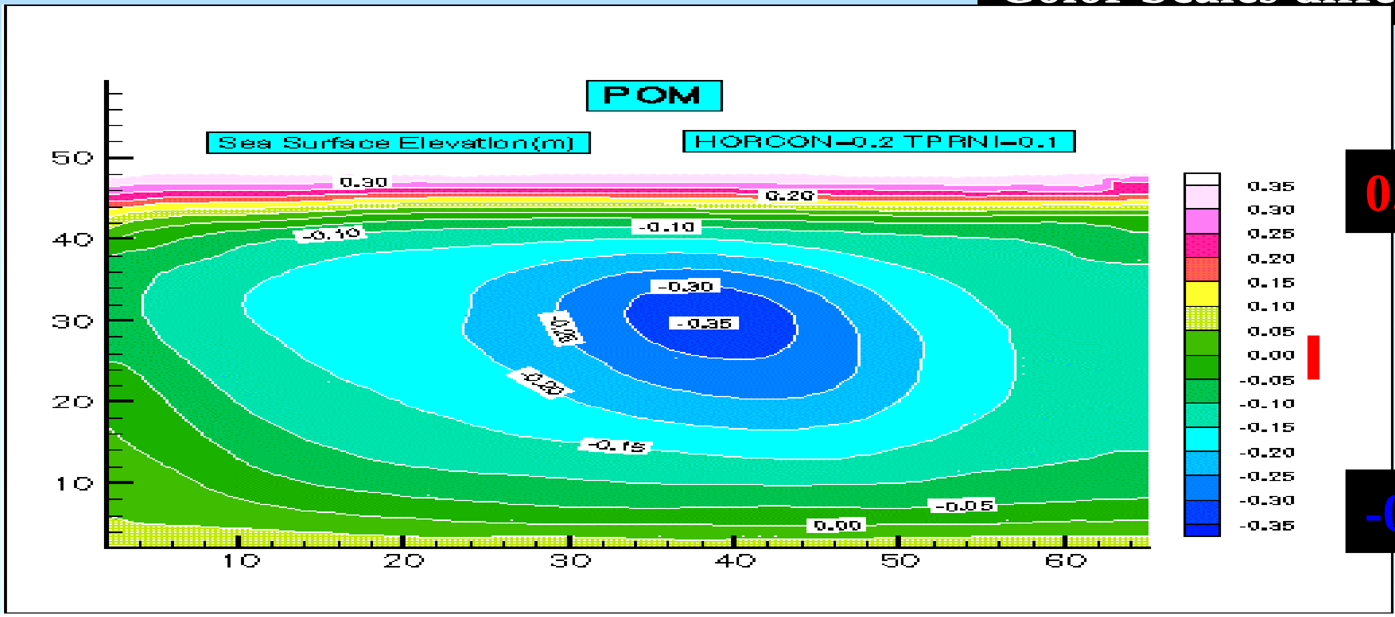
Case 2: Surface elevation after 60 days



0.04m

-0.04m

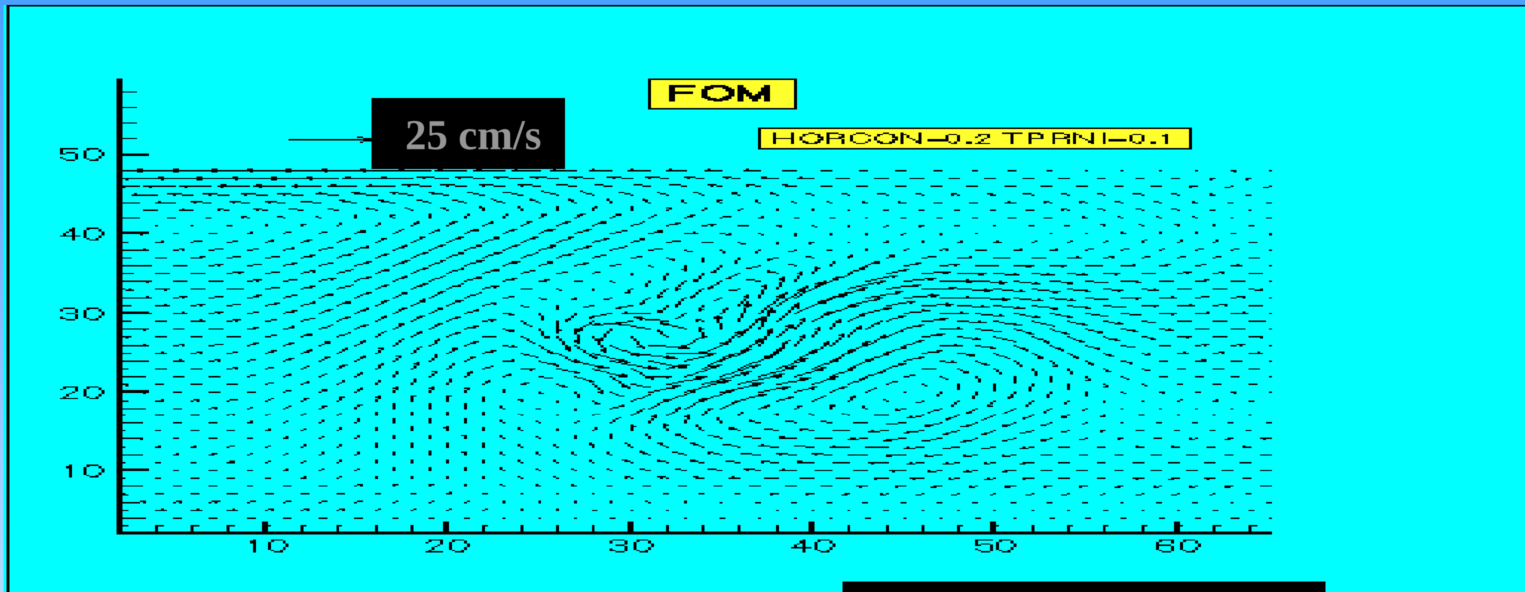
Color Scales differ



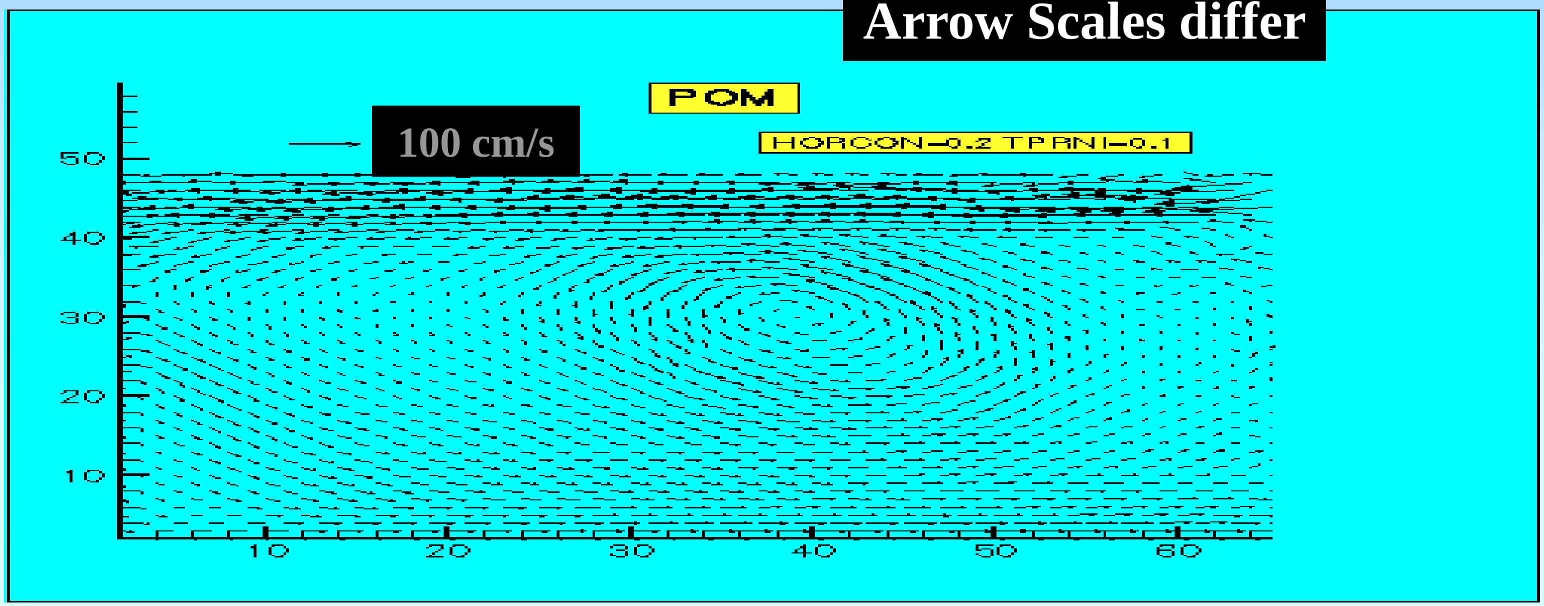
0.35m

-0.35m

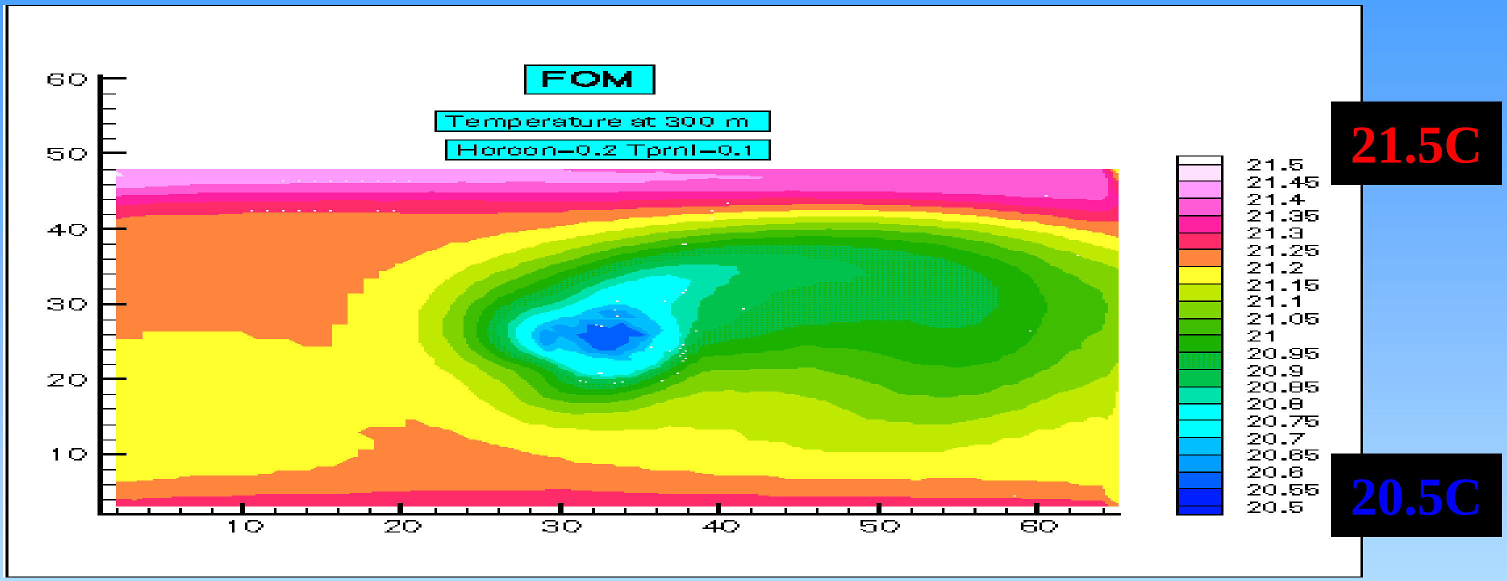
Case 2: Surface Velocity after 60 days



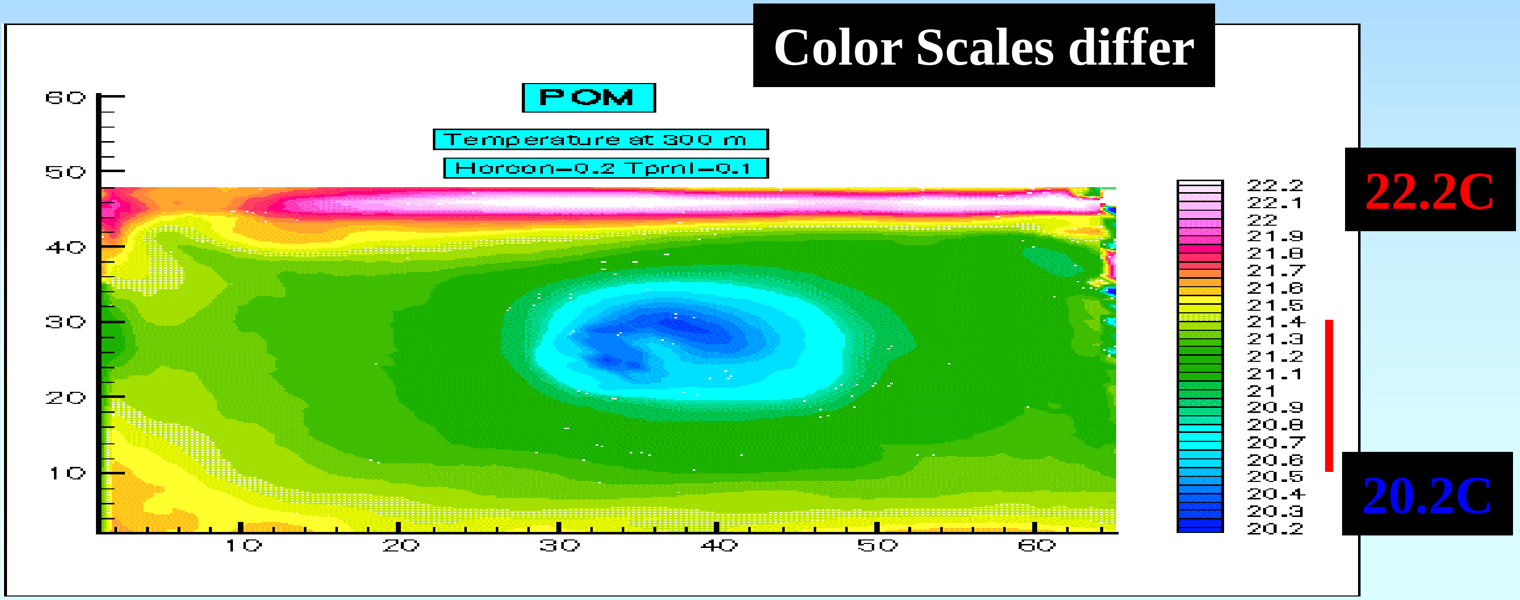
Arrow Scales differ



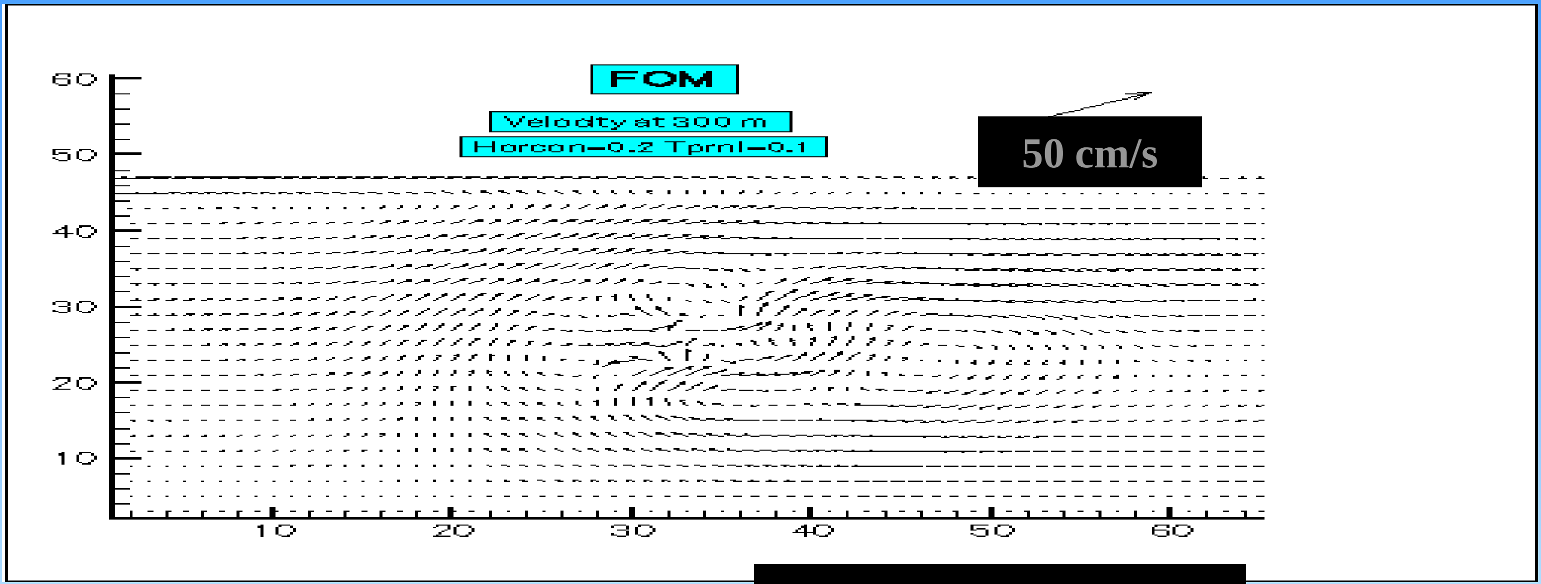
Case 2: Temperature at 300m after 60 days



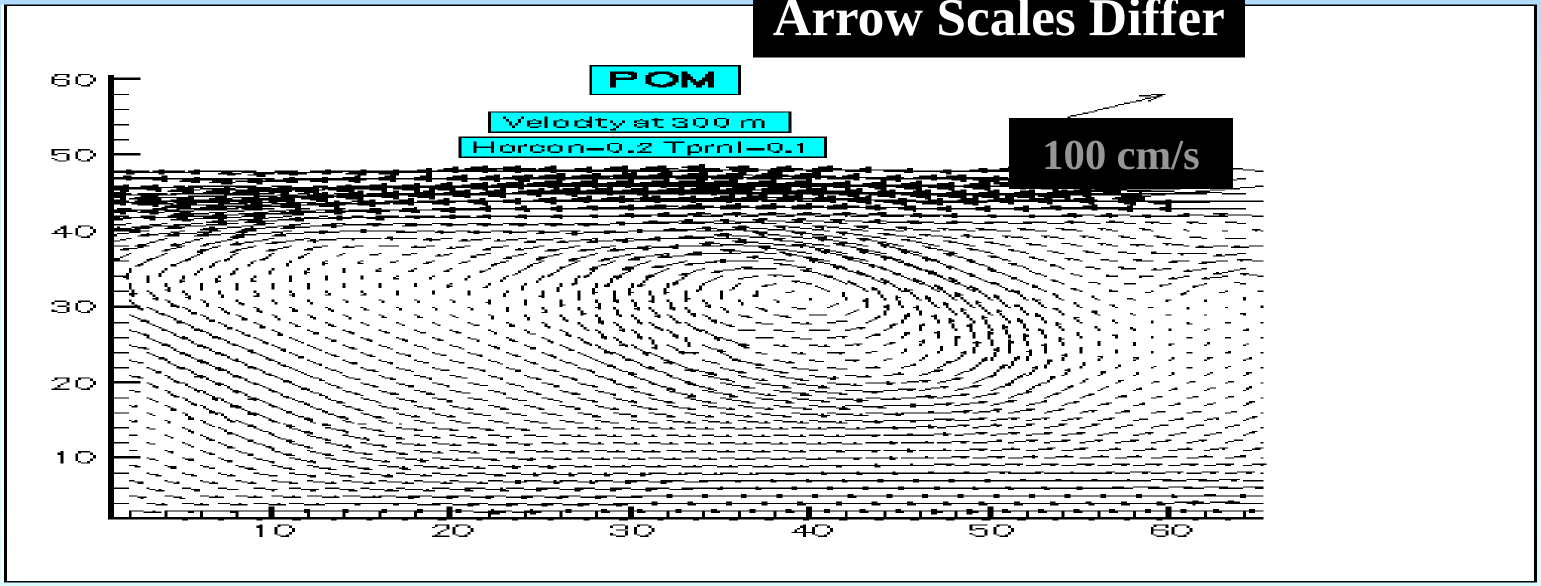
Color Scales differ



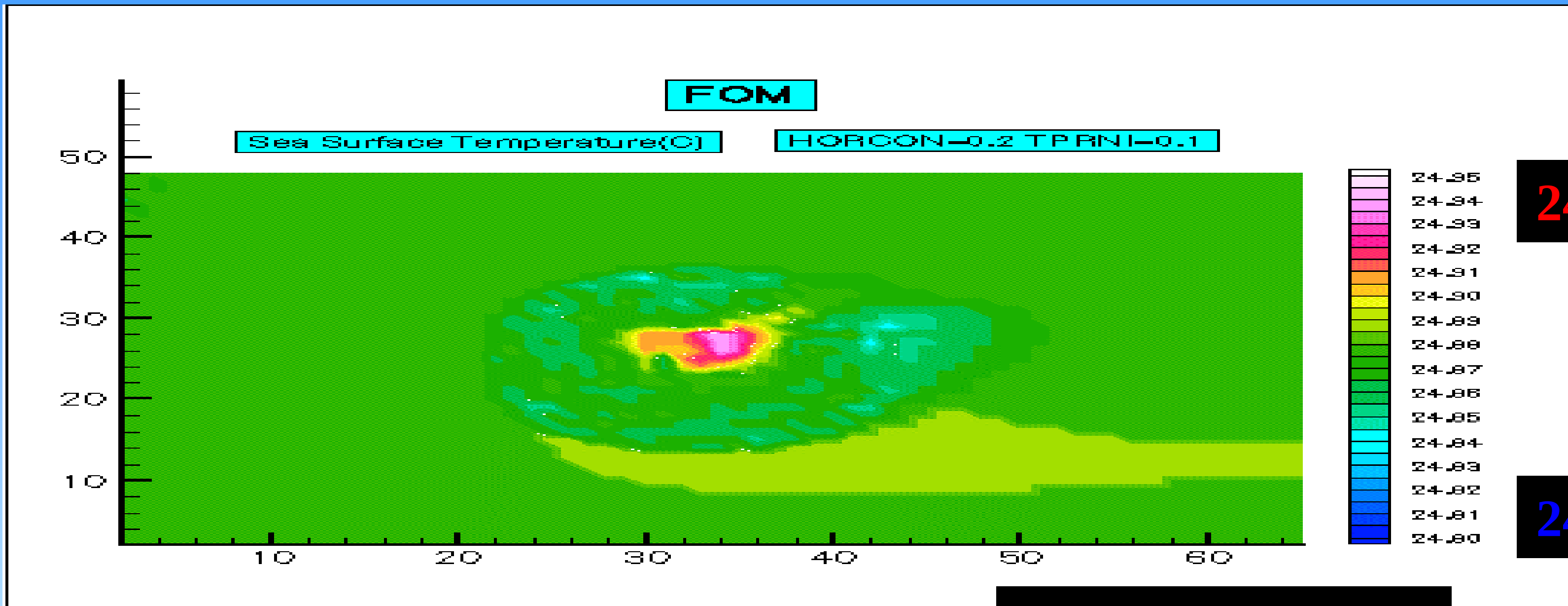
Case 2: Velocity at 300m after 60 days



Arrow Scales Differ



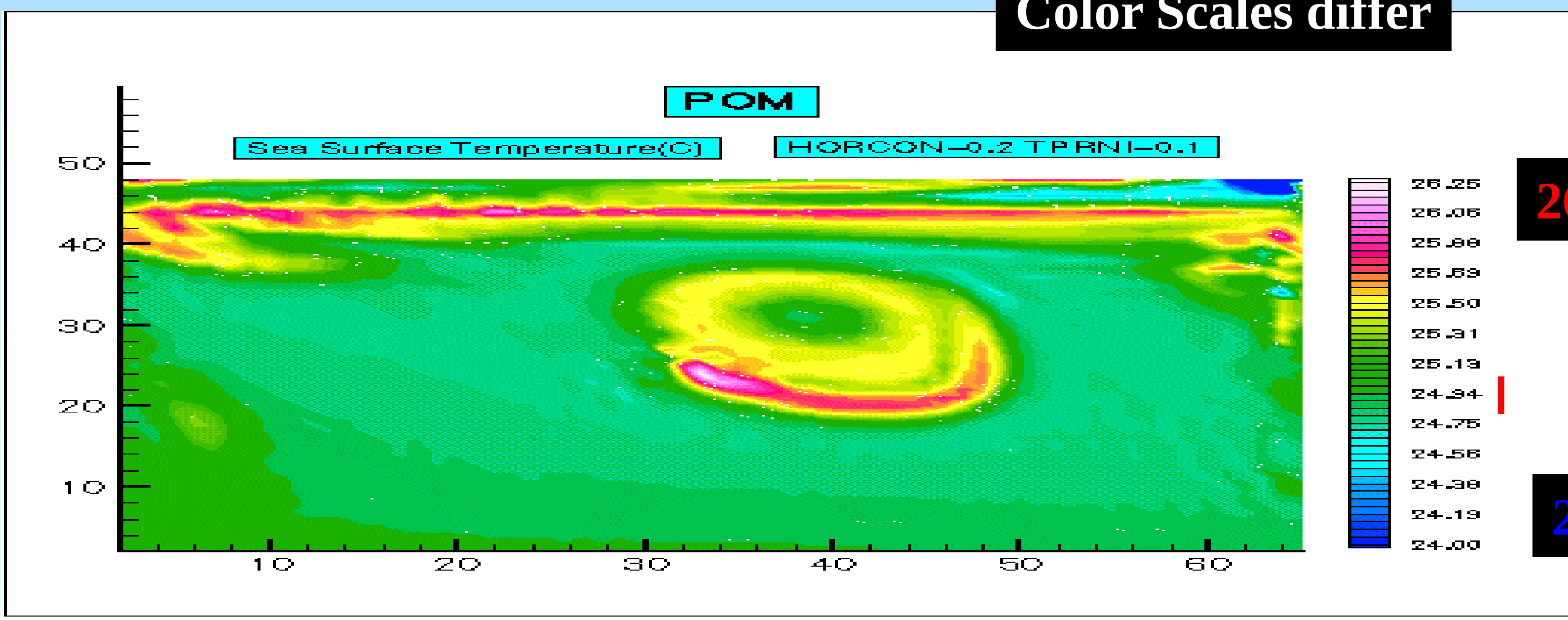
Case 2: Surface Temperature after 60 days



24.95C

24.80C

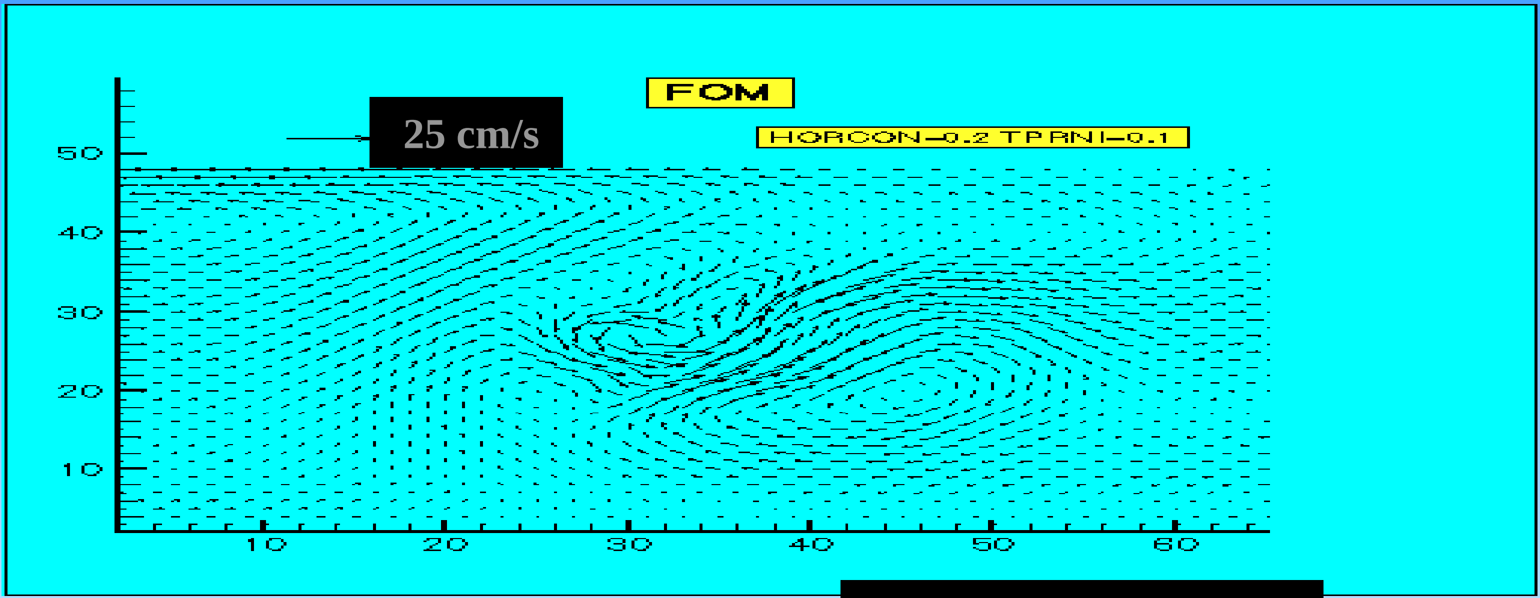
Color Scales differ



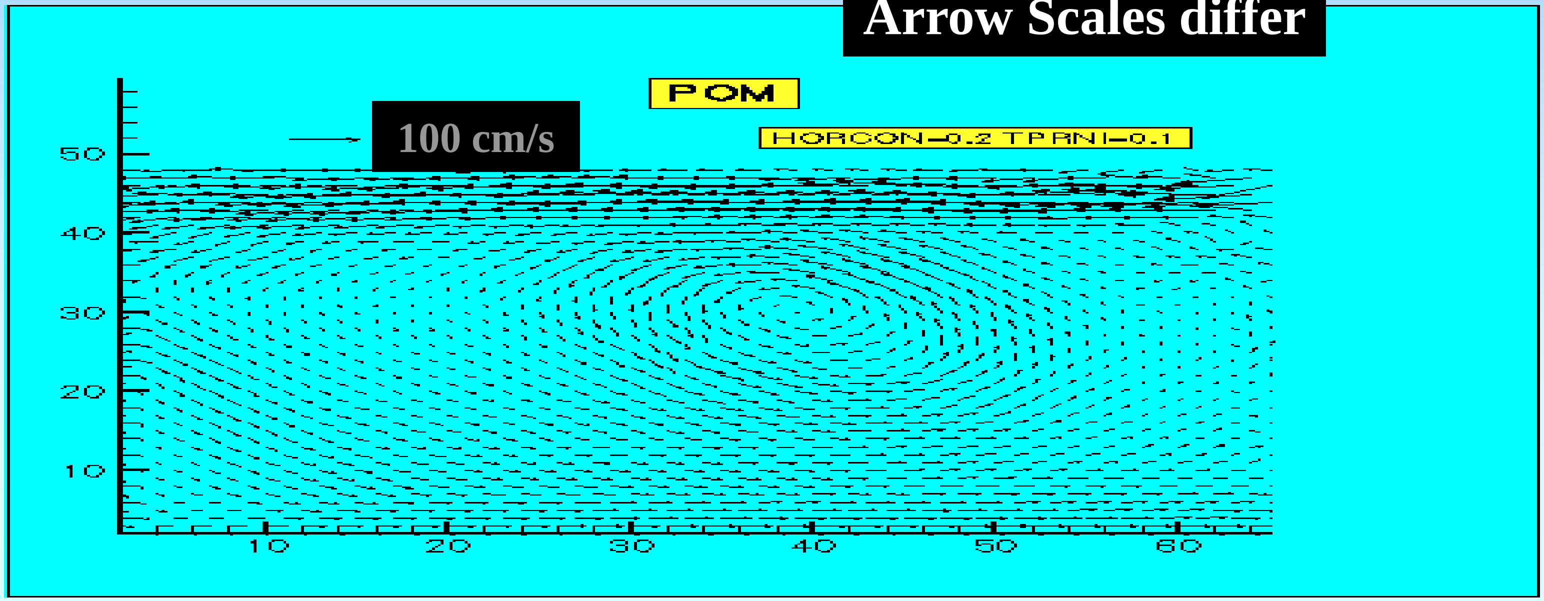
26.25C

24.00C

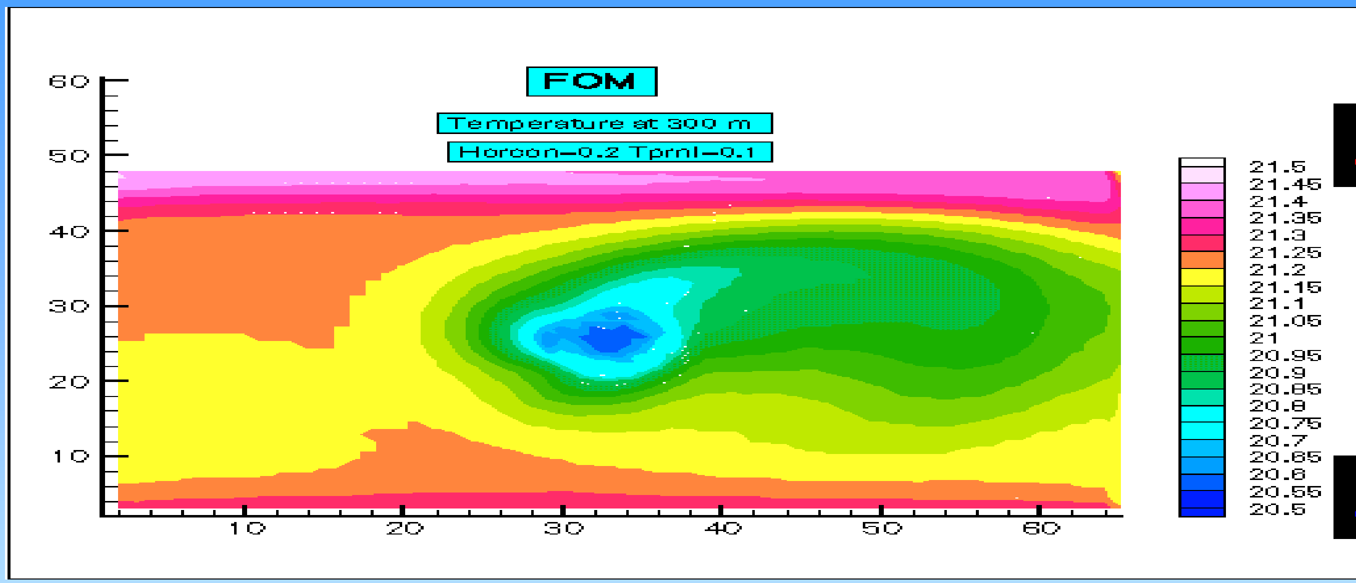
Case 2: Surface Velocity after 60 days



Arrow Scales differ



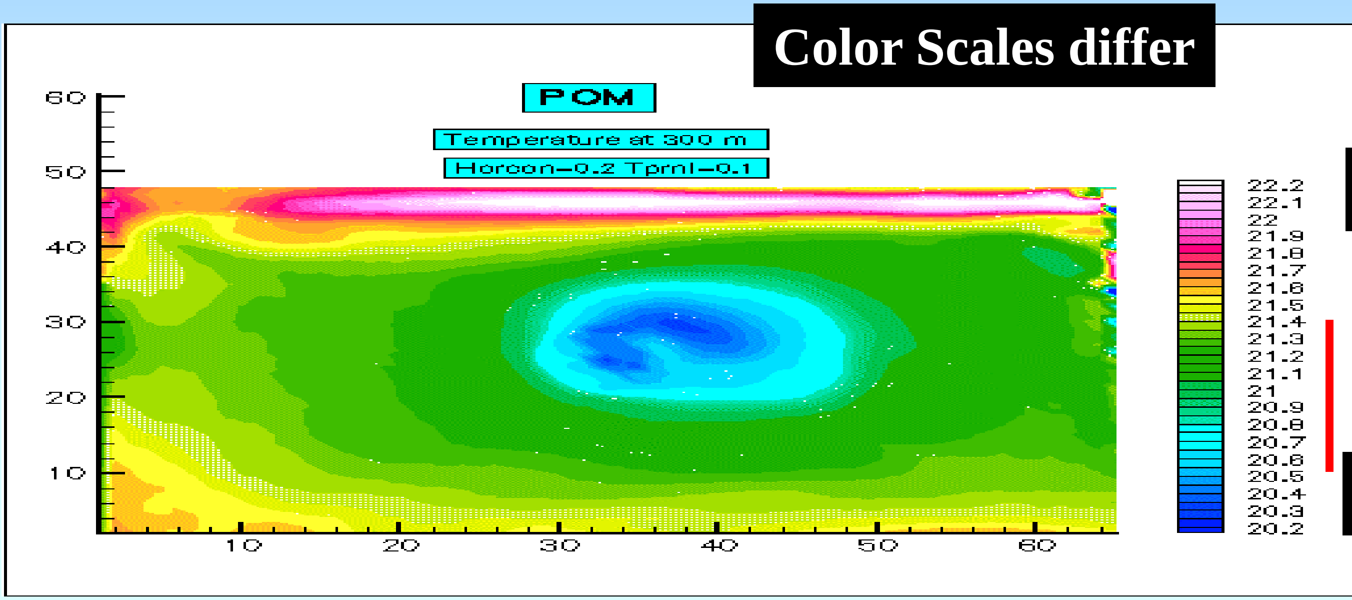
Case 2: Temperature at 300m after 60 days



21.5C

20.5C

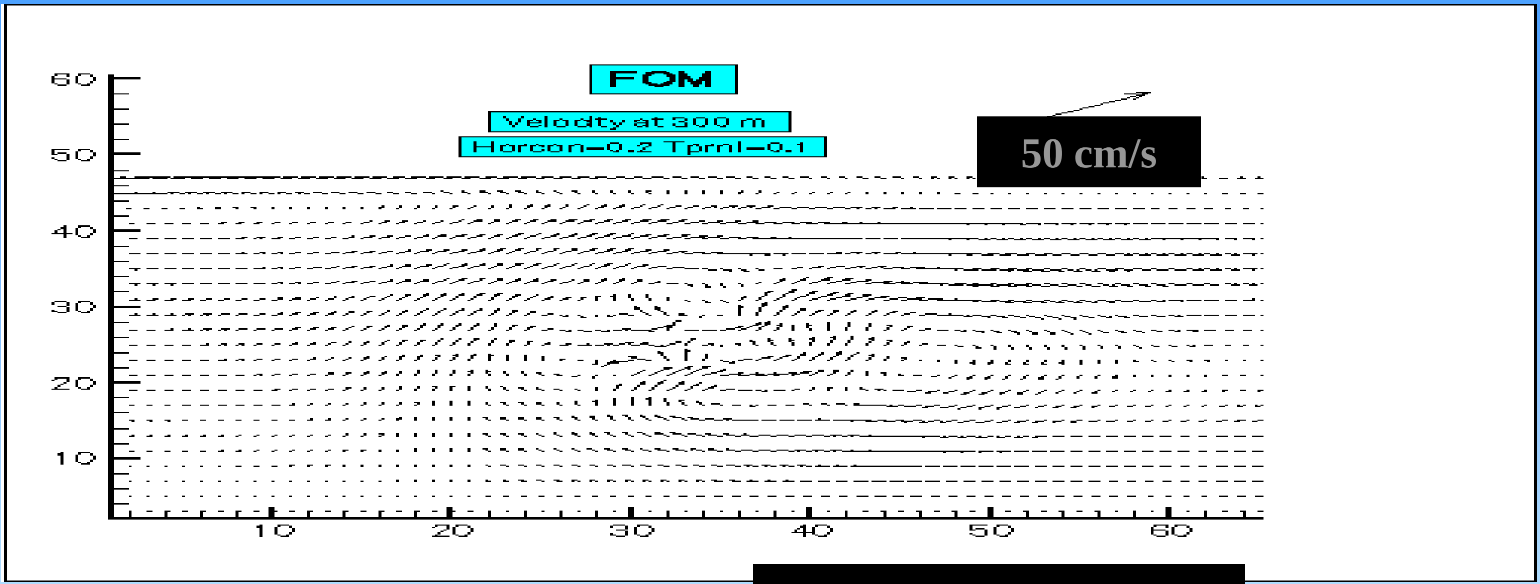
Color Scales differ



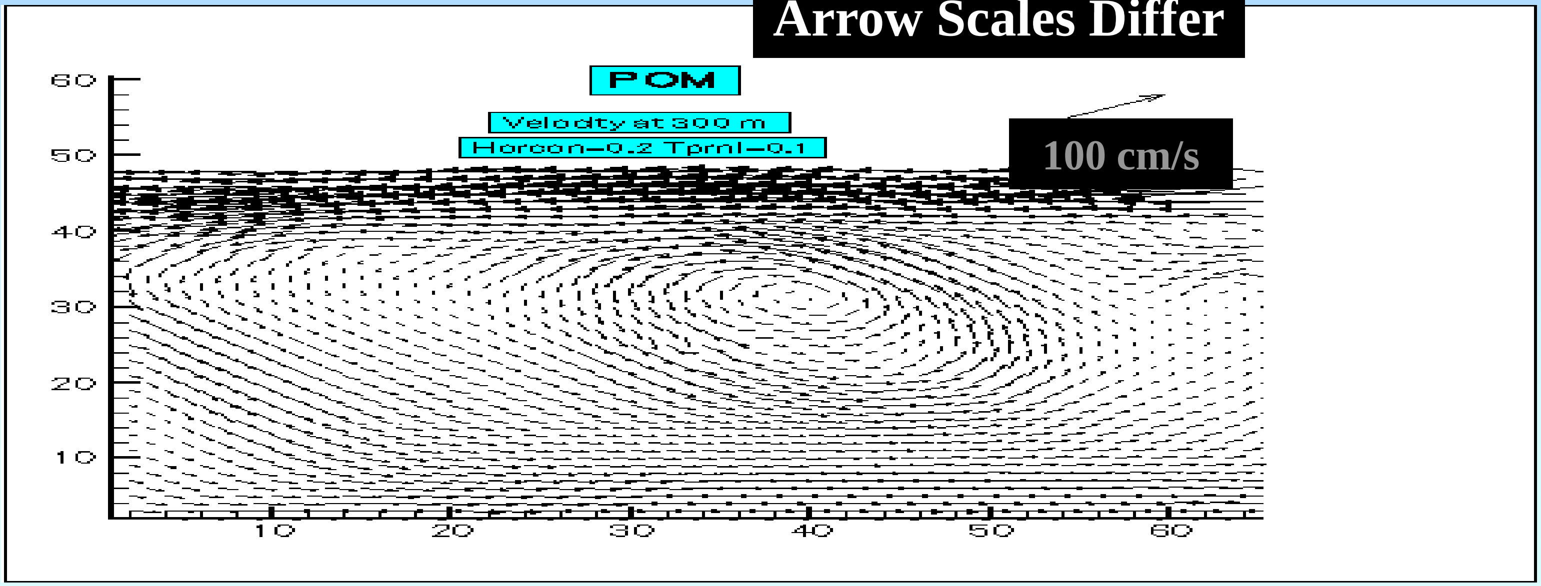
22.2C

20.2C

Case 2: Velocity at 300m after 60 days

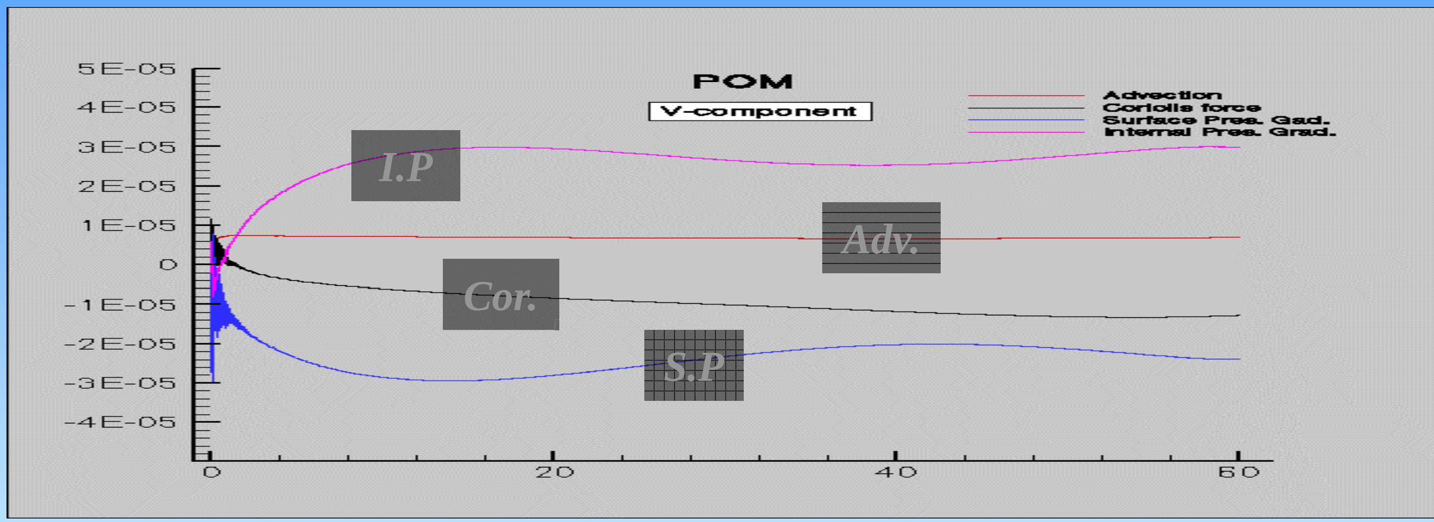


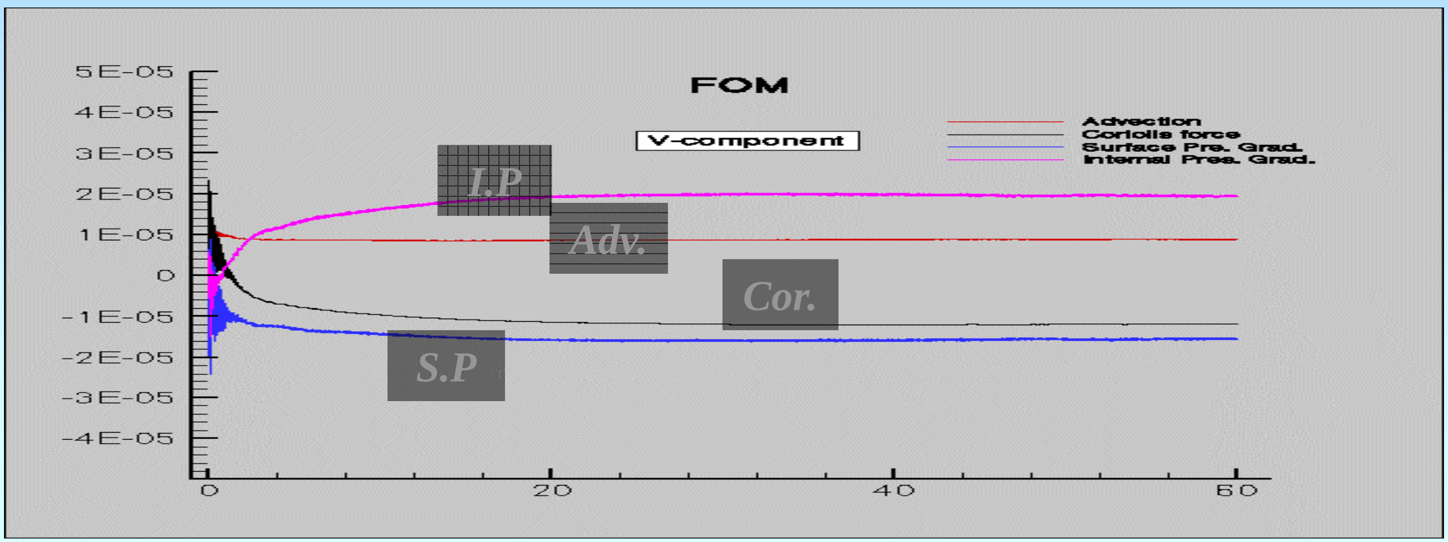
Arrow Scales Differ



Comparison of Major Forces - Case 2

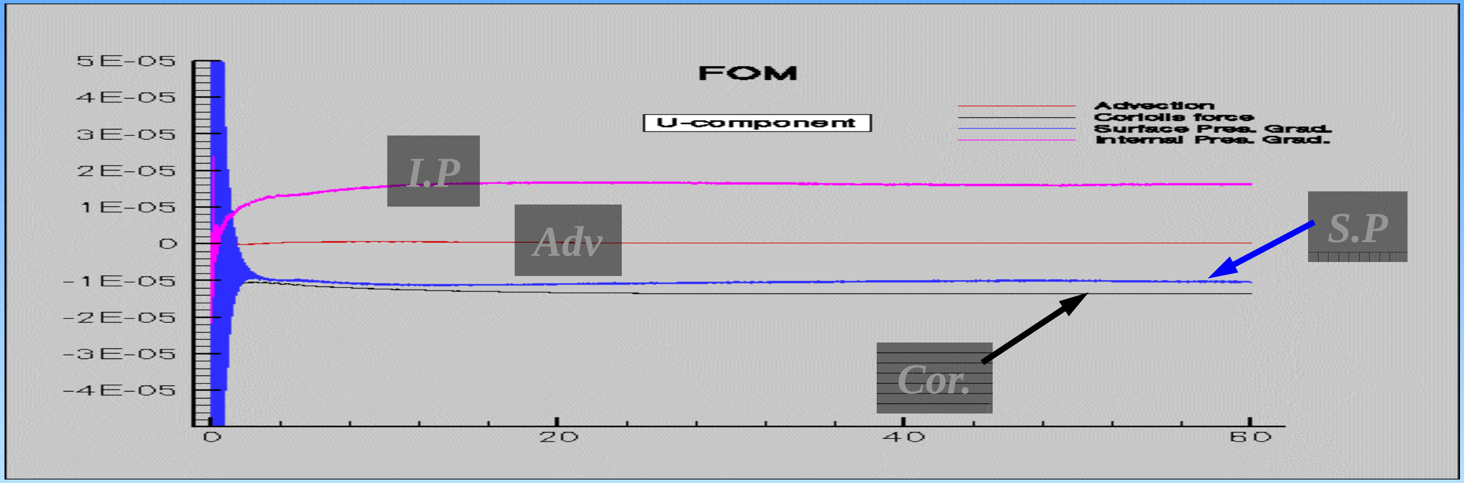
V-Compomemt

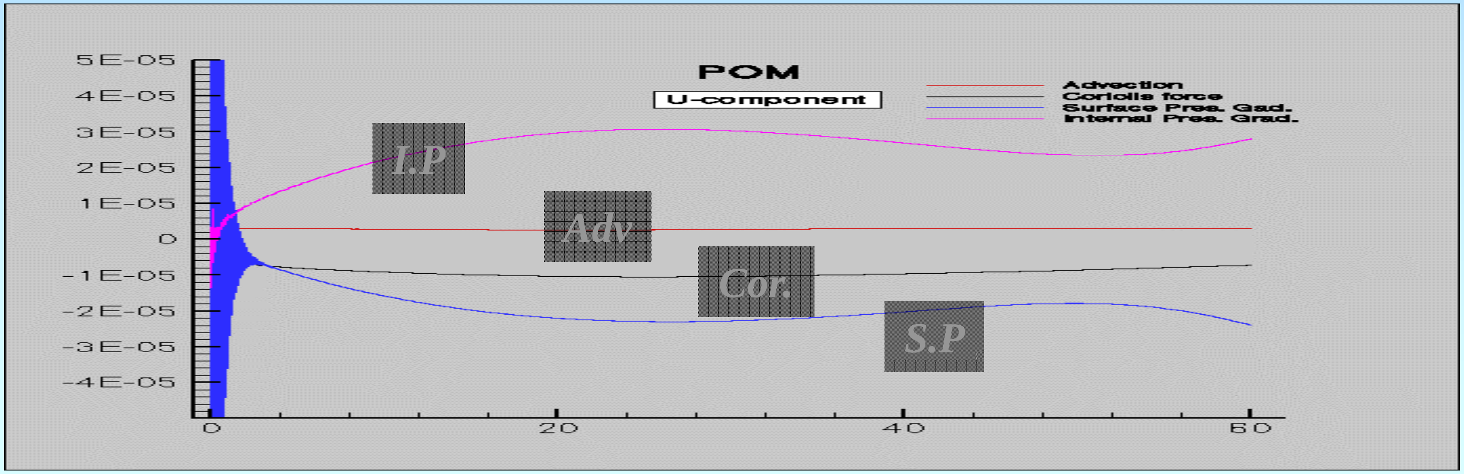




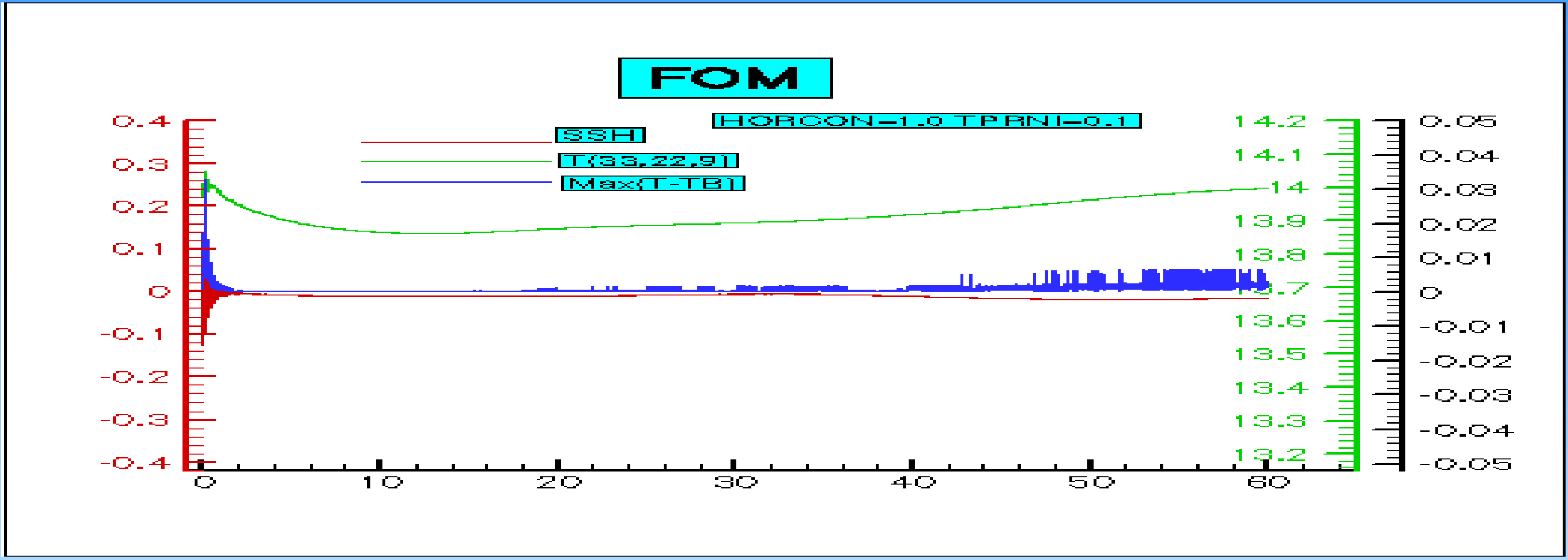
Comparison of Major Forces - Case 2

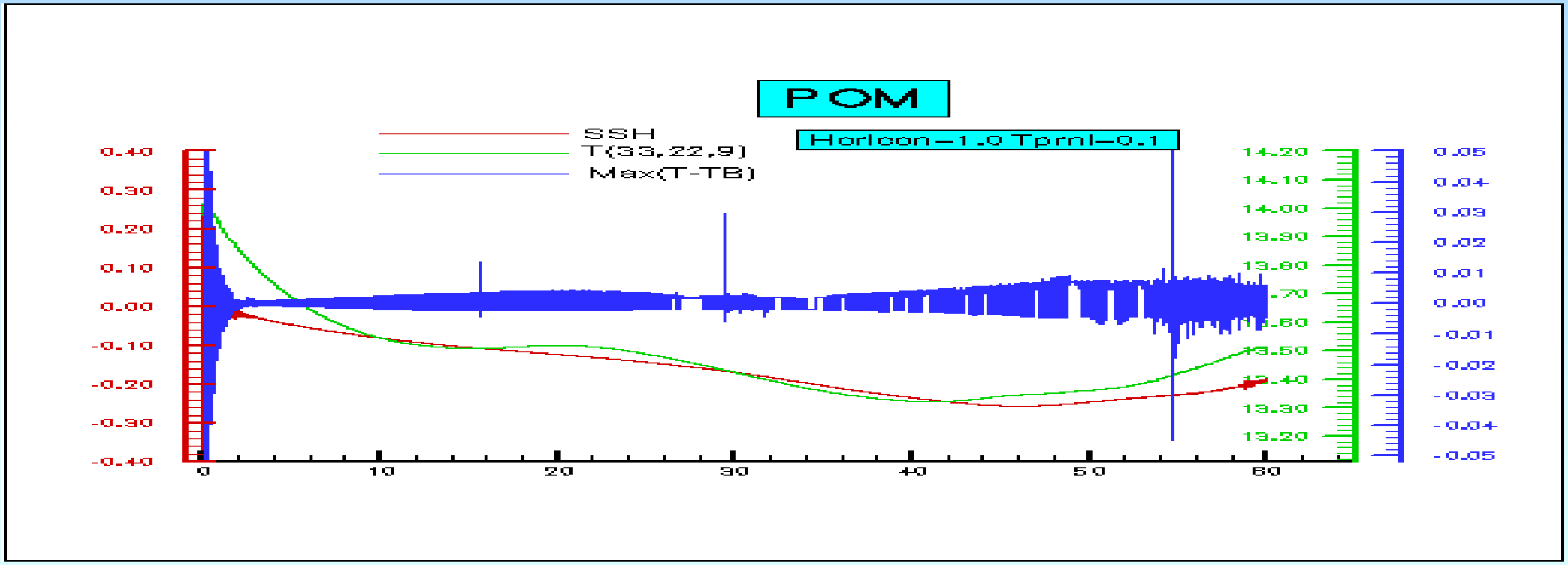
U-Compomemt



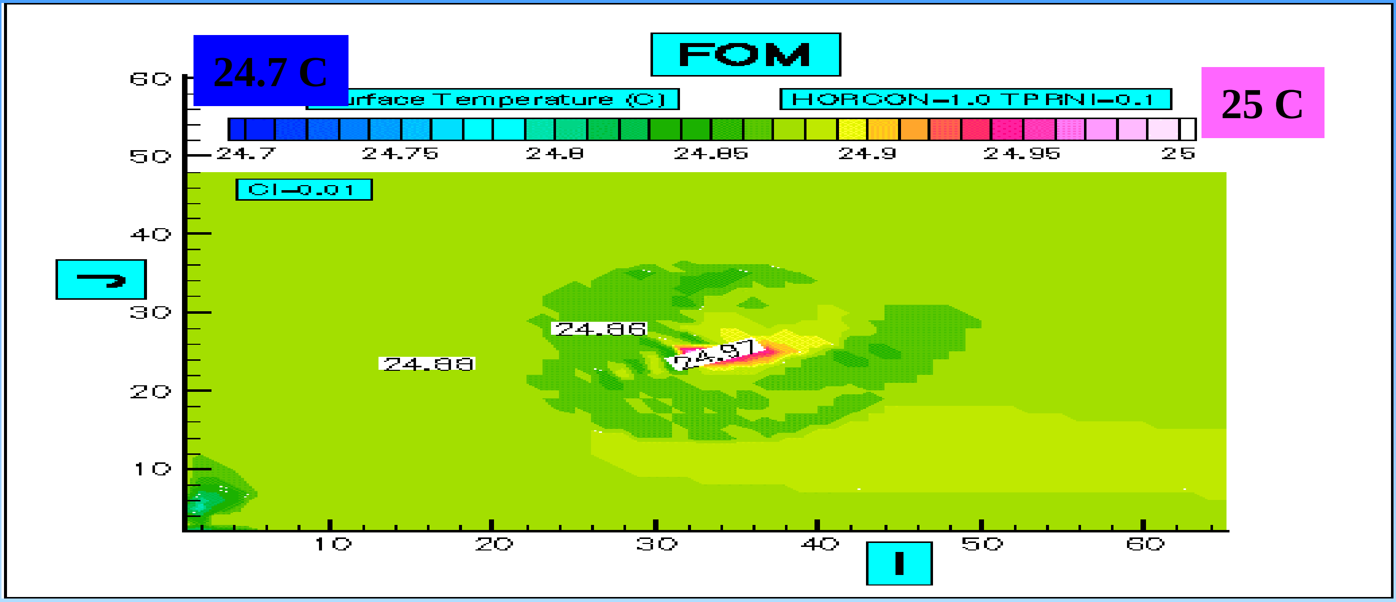


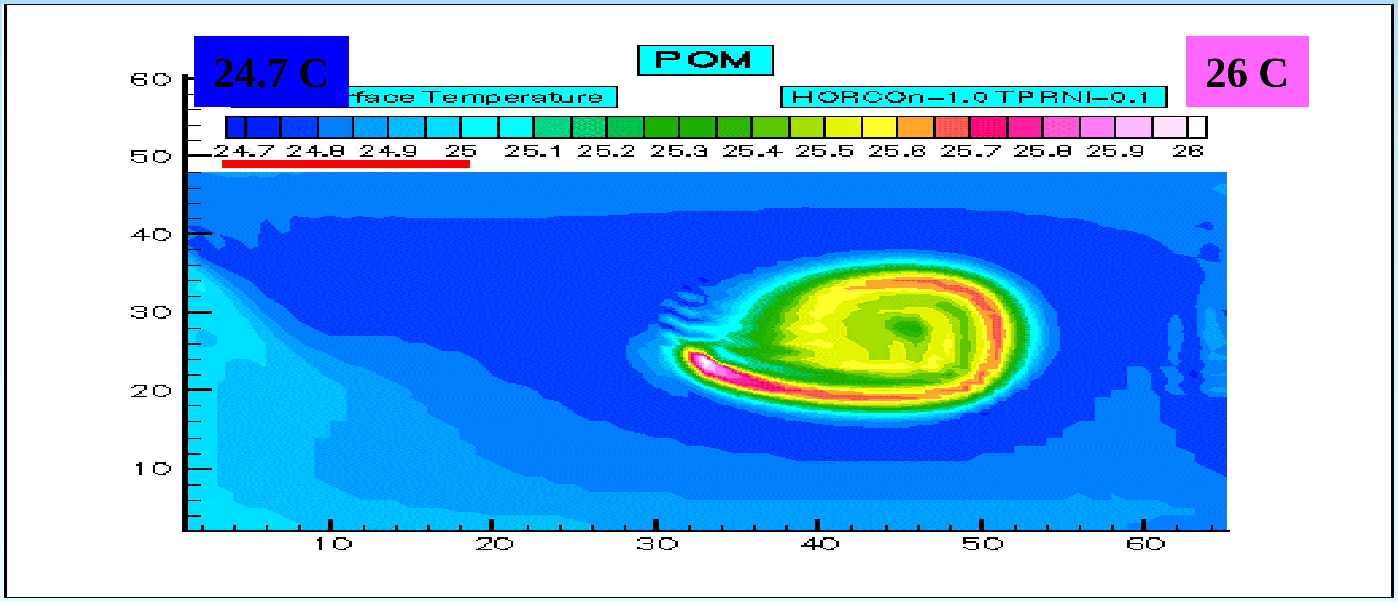
Case 3: Large viscosity(HORCON=1.0)



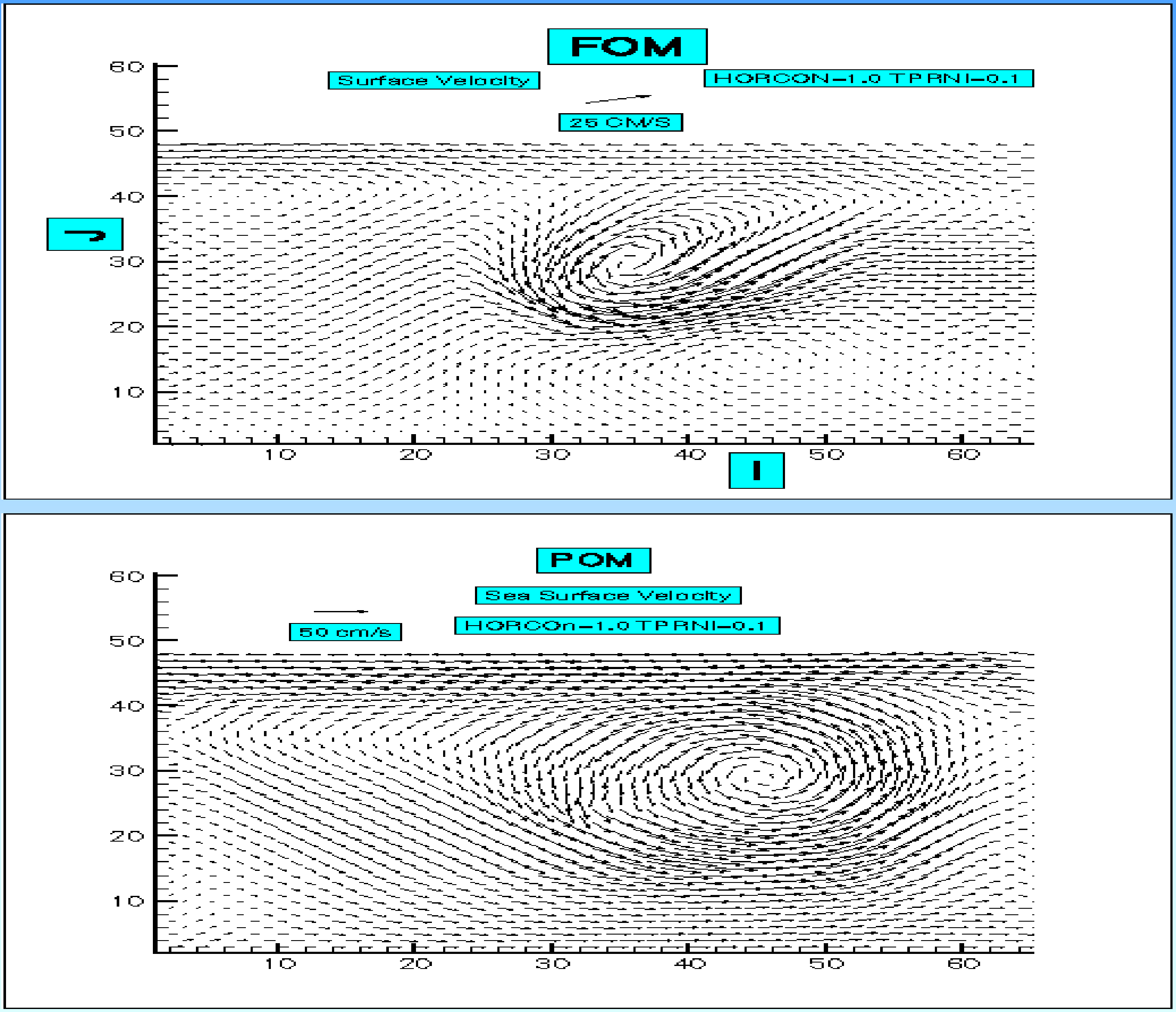


Case 3: Surface Temperature after 60 days

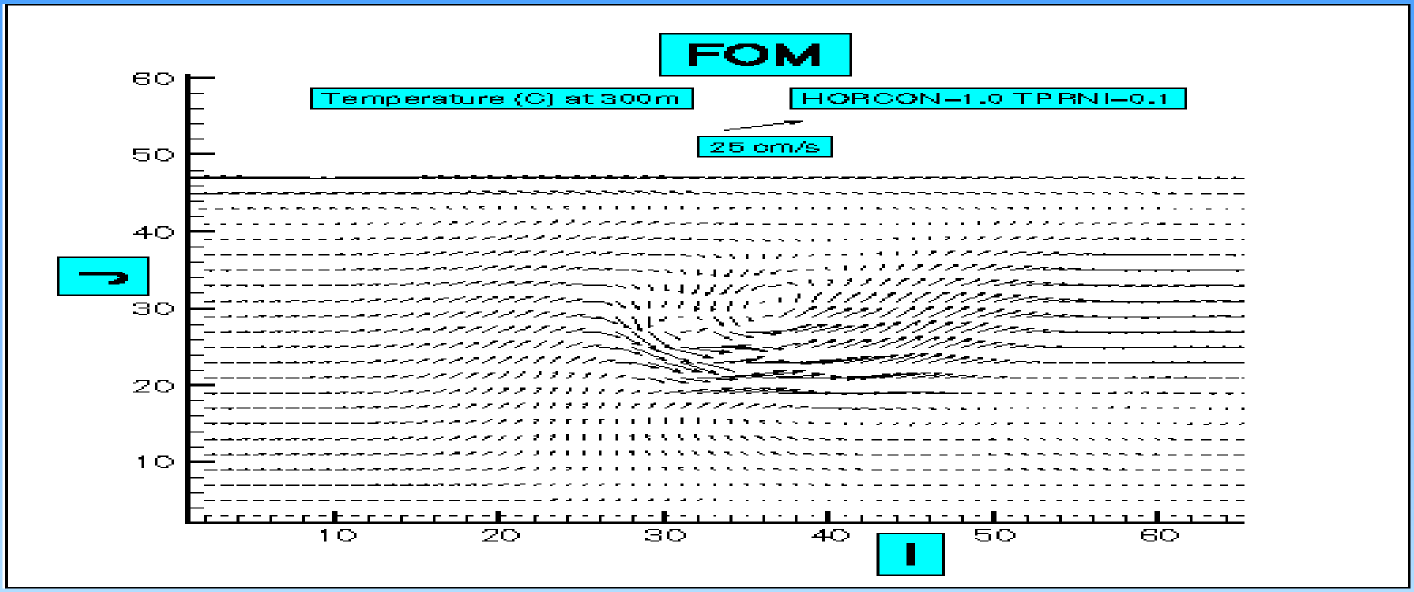


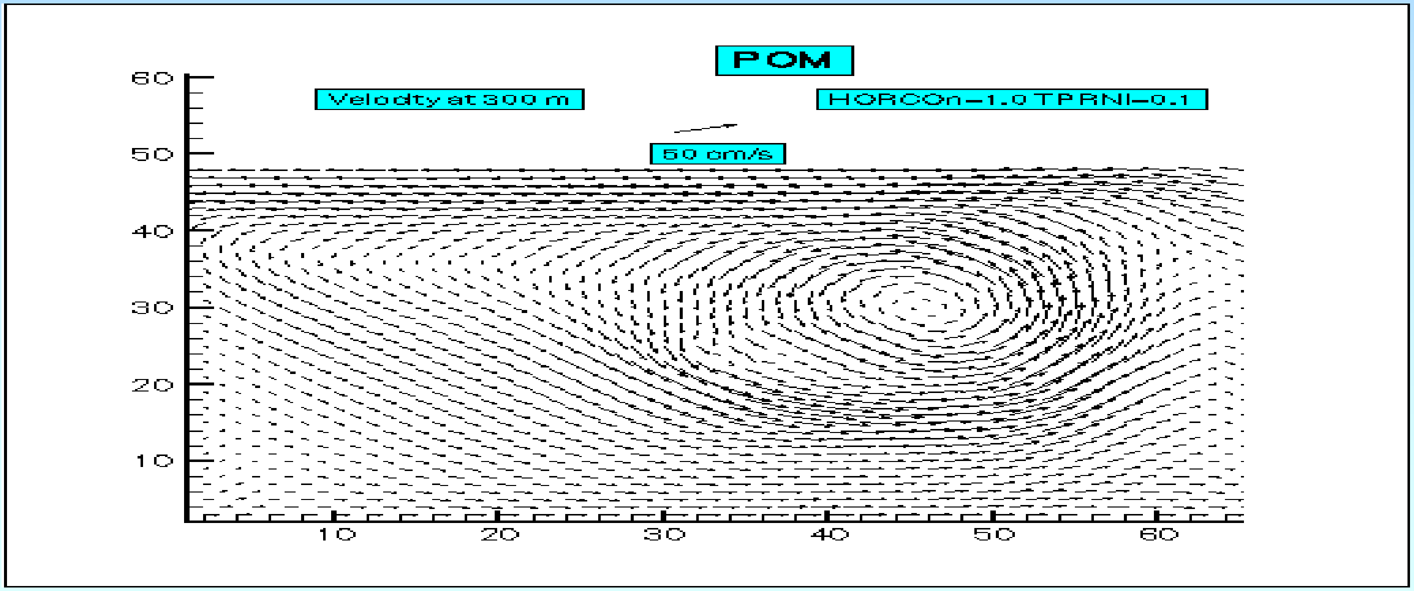


Case 3: Velocity at surface after 60 days

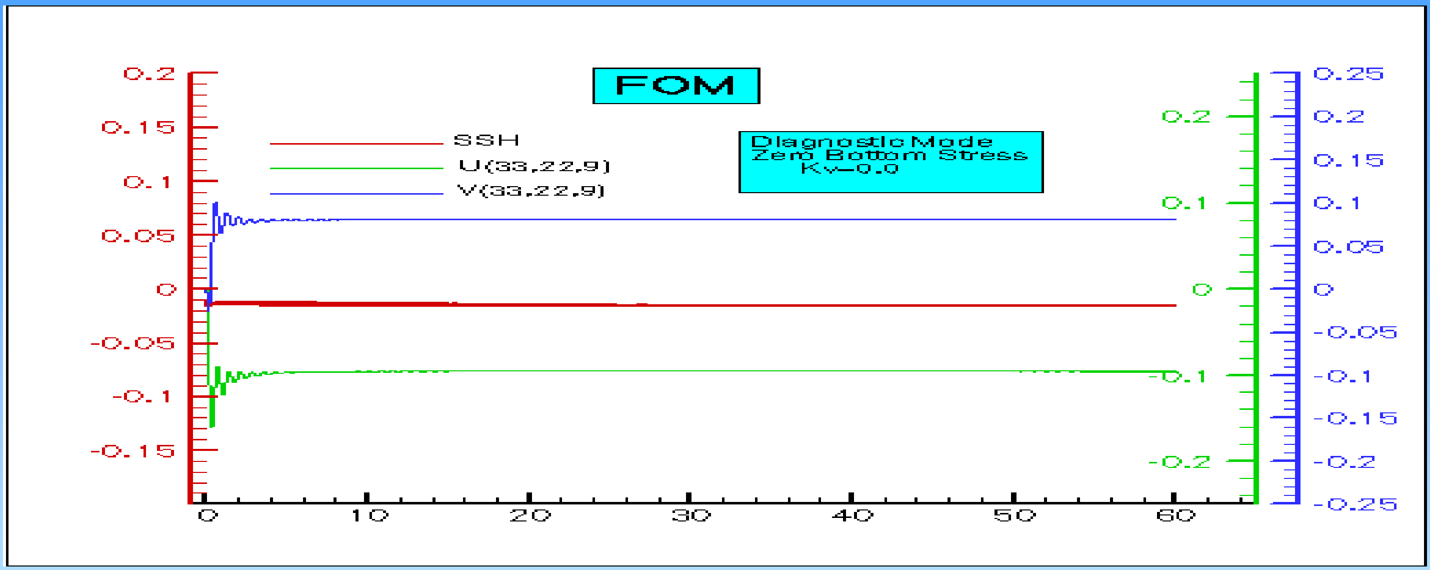


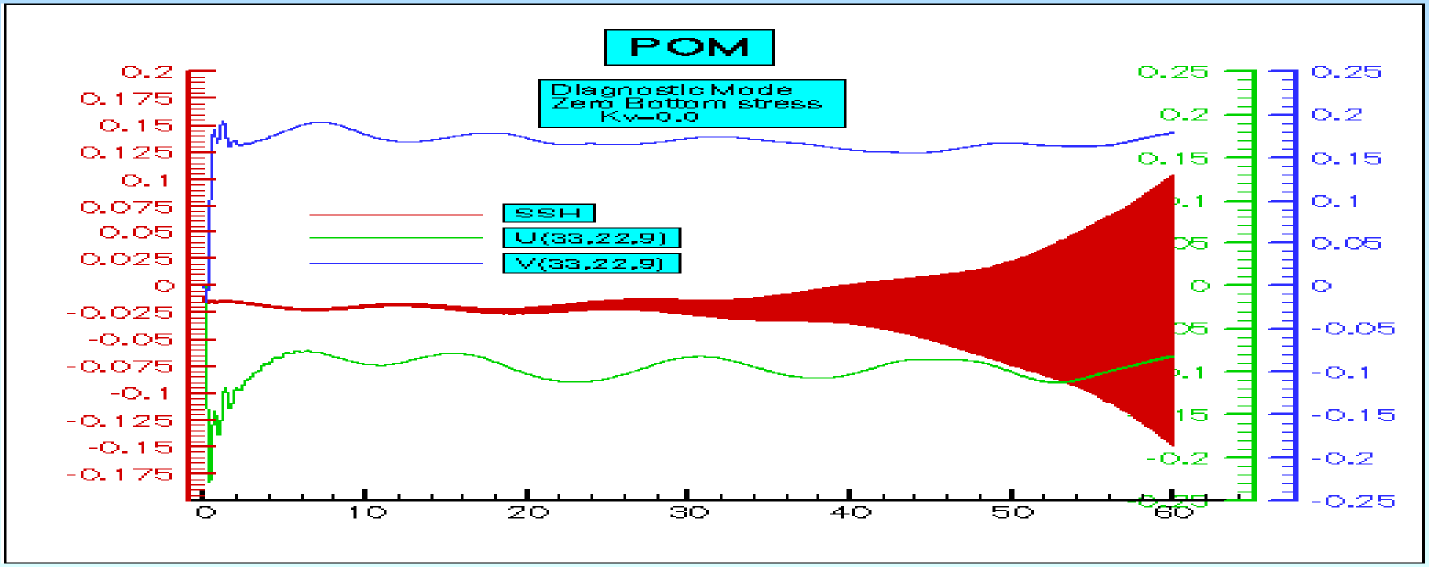
Case 3: Velocity at 300m after 60 days



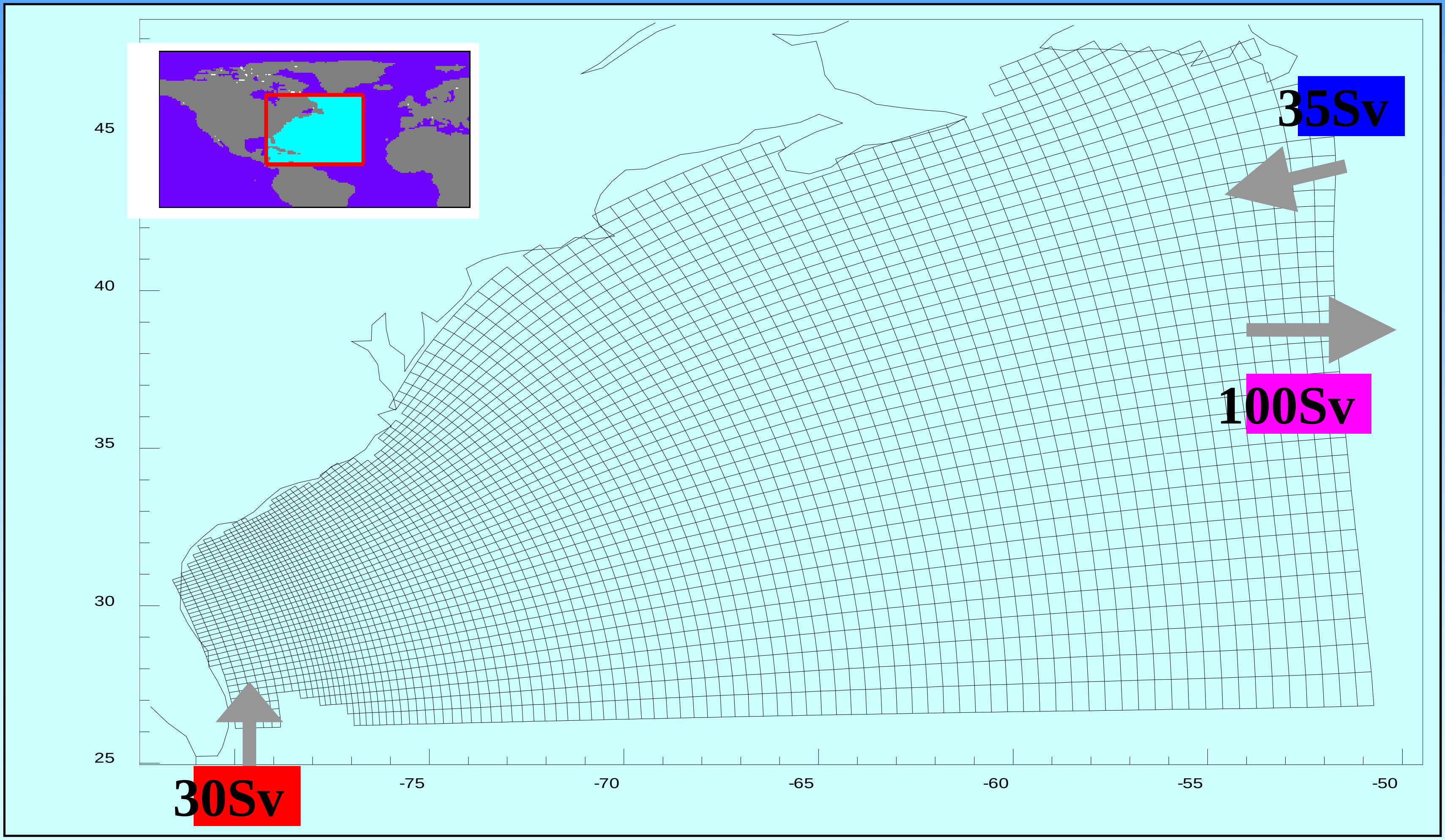


Case 4: Diagnostic run in closed basin

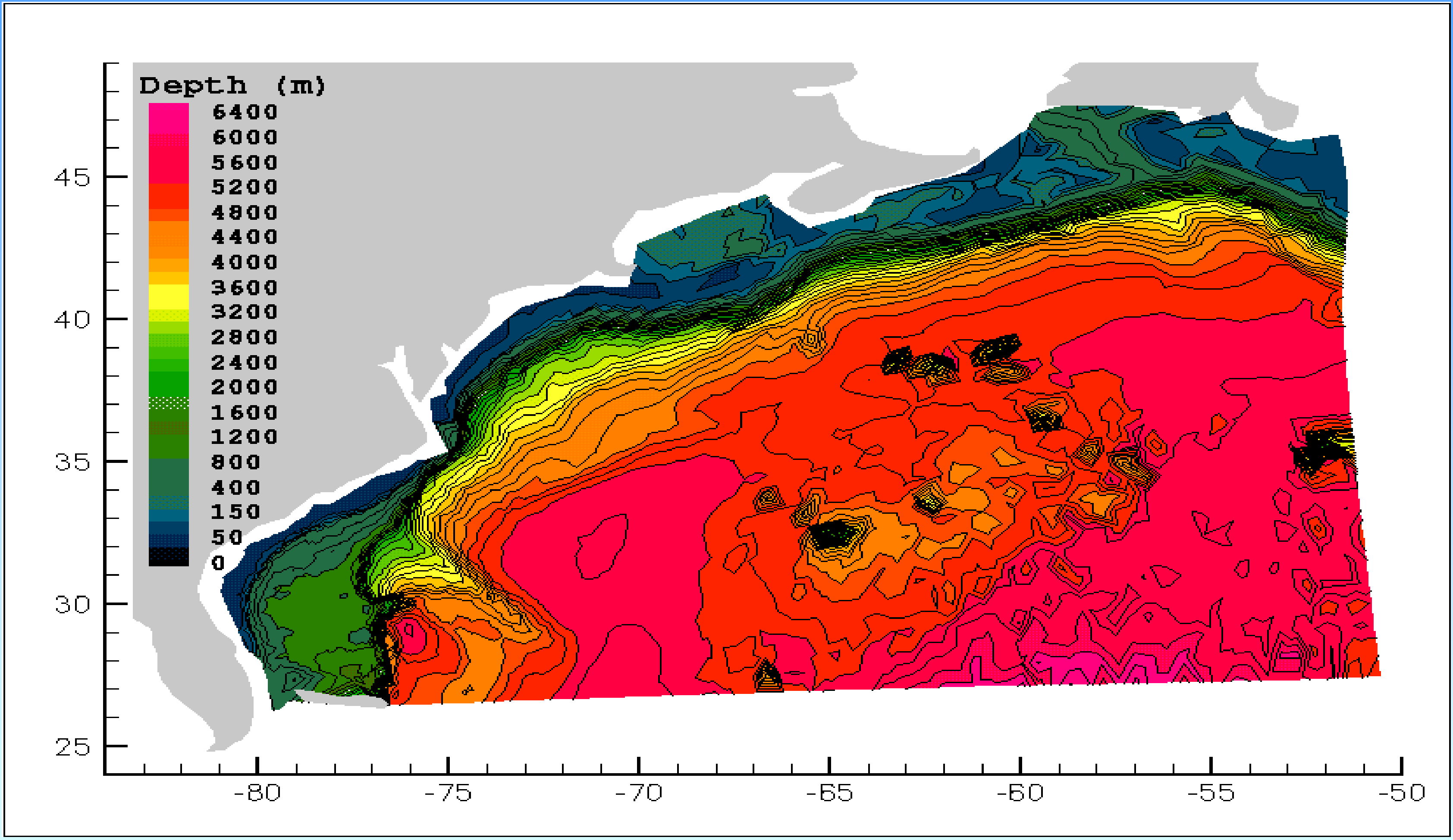




Curvilinear Orthogonal Gird

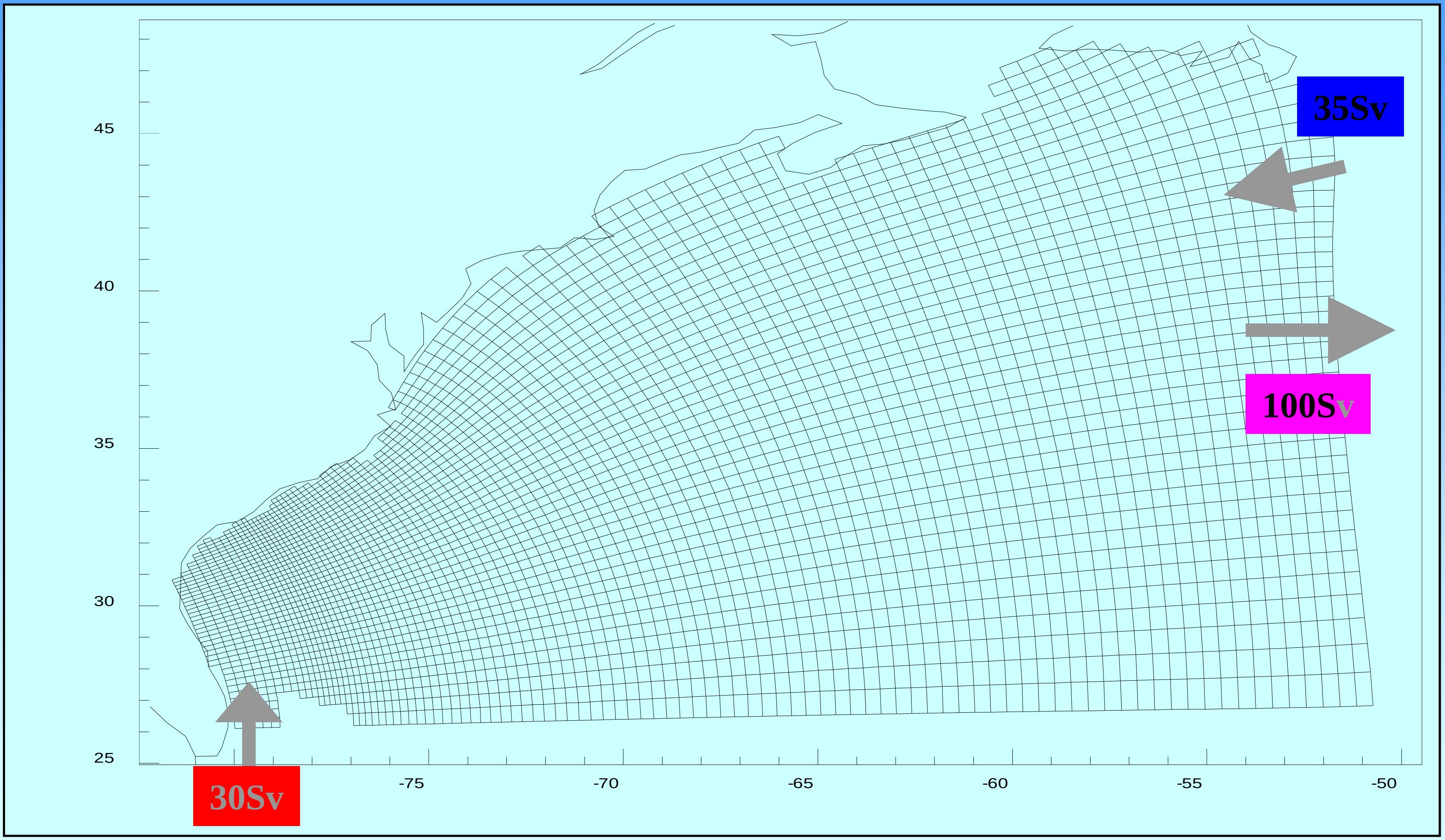


Topography



Depth (m)
6400
6000
5600
5200
4800
4400
4000
3600
3200
2800
2400
2000
1600
1200
800
400
150
50
0
45
40
35
30
25
-80
-75
-70
-65
-60
-55
-50

Model Grid



Initial Data

Initial Input Data of Model

-.Temperature and Salinity: World Ocean Atlas

(WOA 98) 1°× 1° Grid data

-. Bottom Bathymetry: ETOP5 5min.

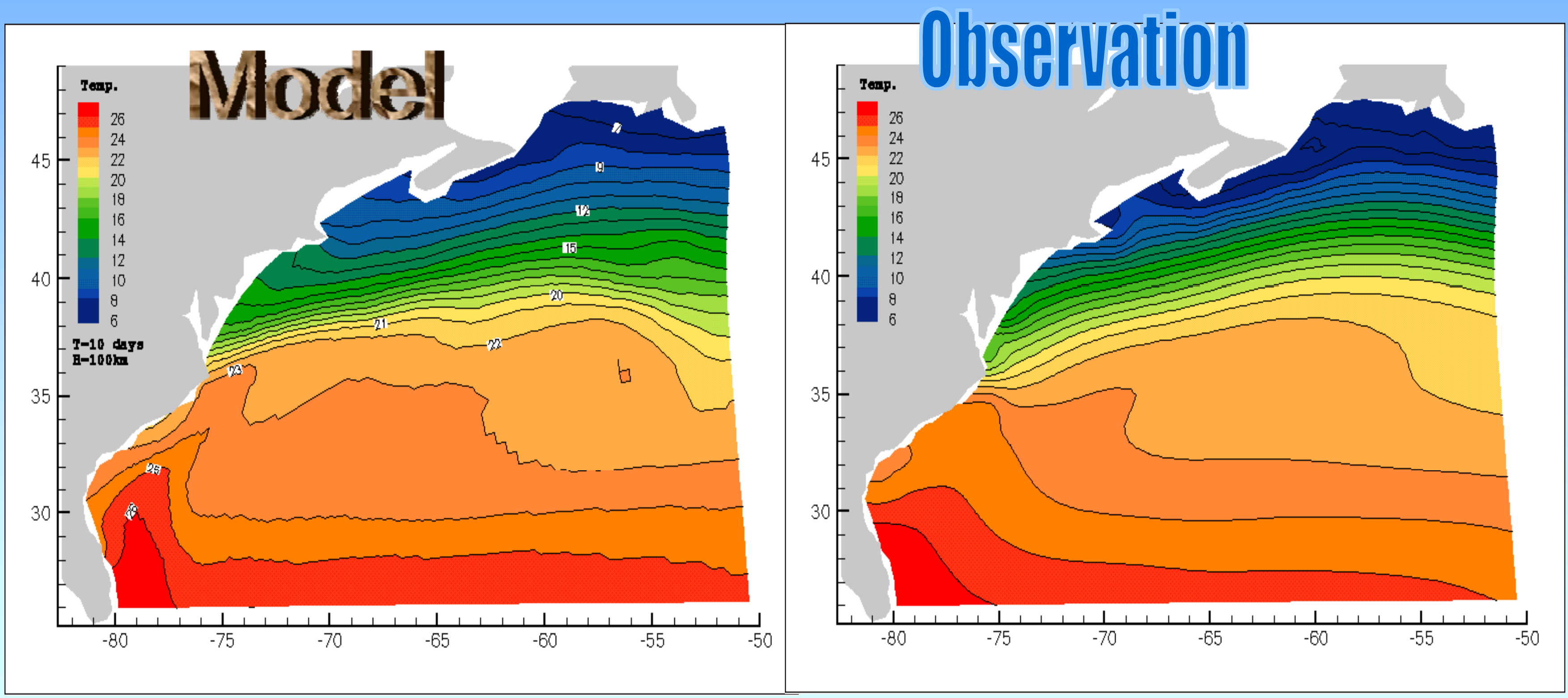
Surface Boundary Condition

Surface Wind Stress: Hellerman, S. and M. Rosenstein(1983)

1°× 1° Grid data

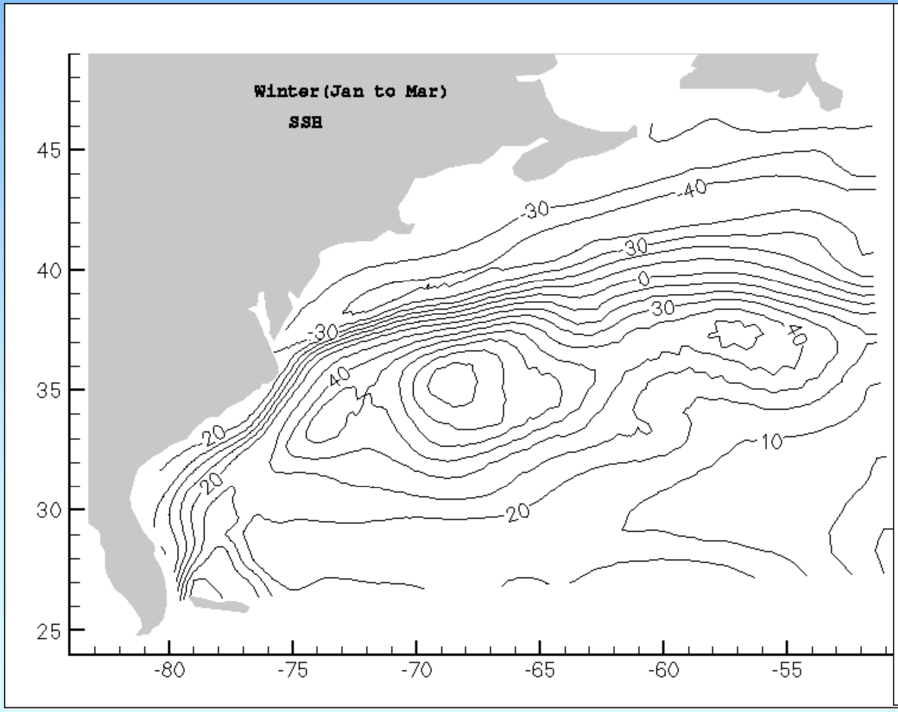
Surface Heat Flux: Comprehensive Ocean-Atmosphere Data Set (COADS) analyzed by Oberhuber (1988) **2°× 2°**

One-Year Mean SST

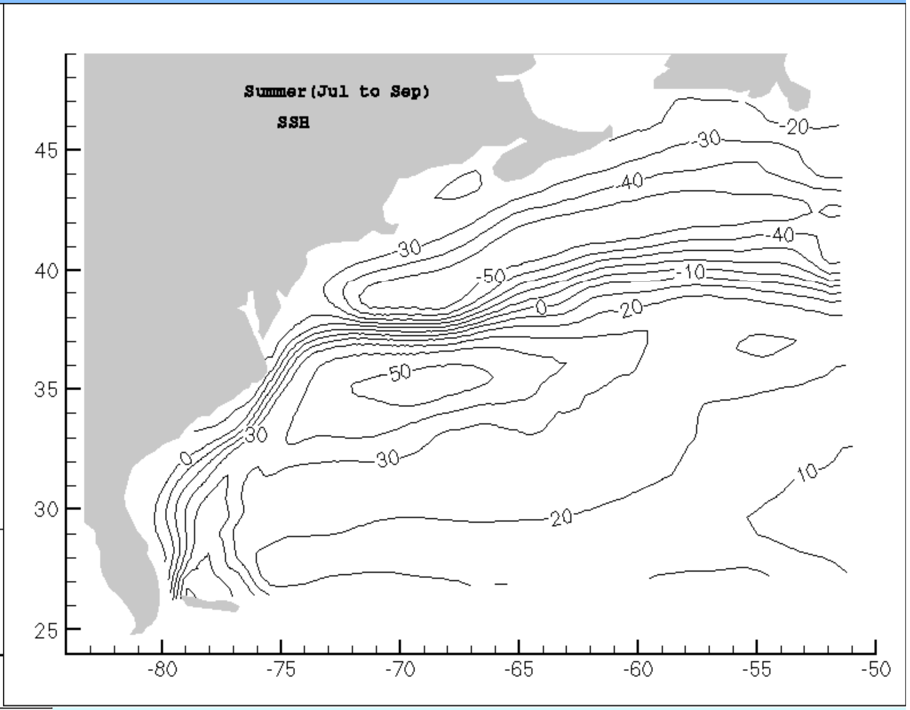


Surface Elevation

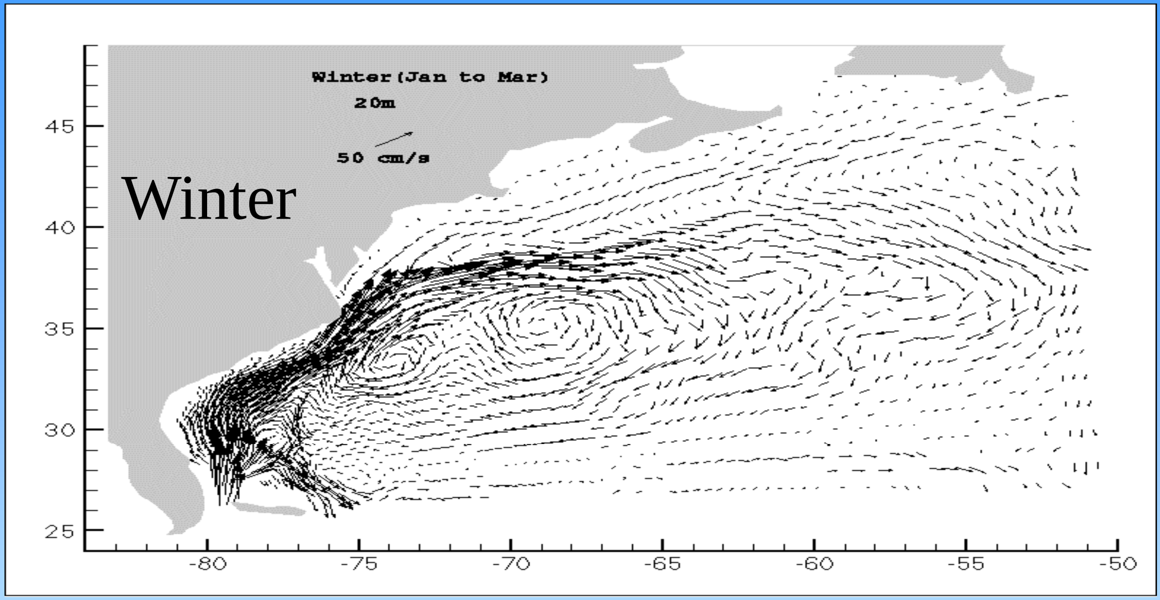
Winter

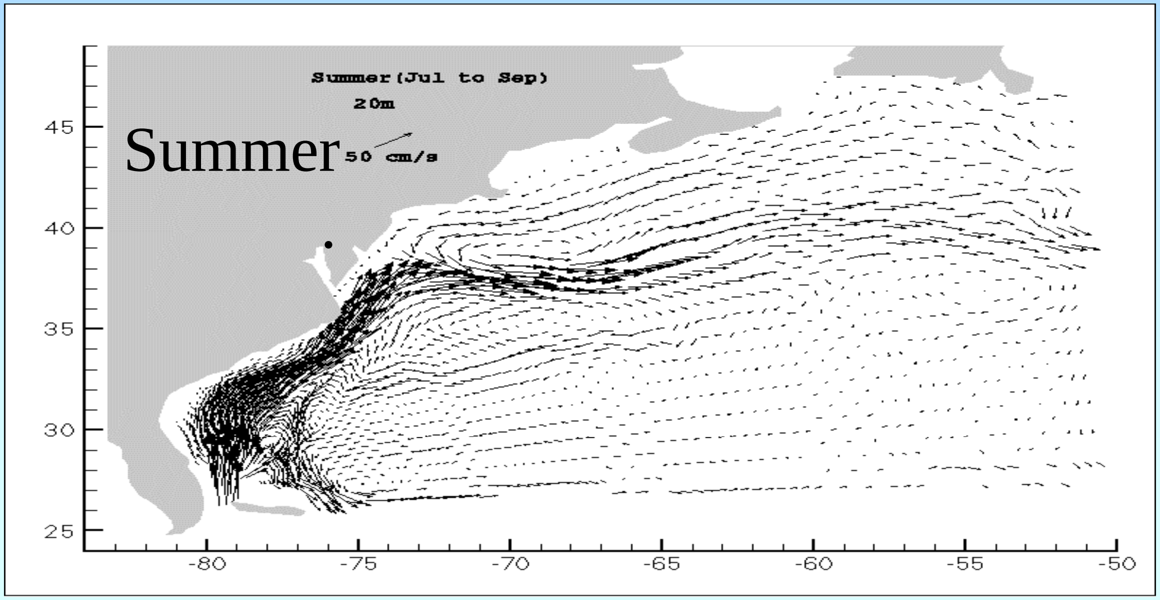


Summer

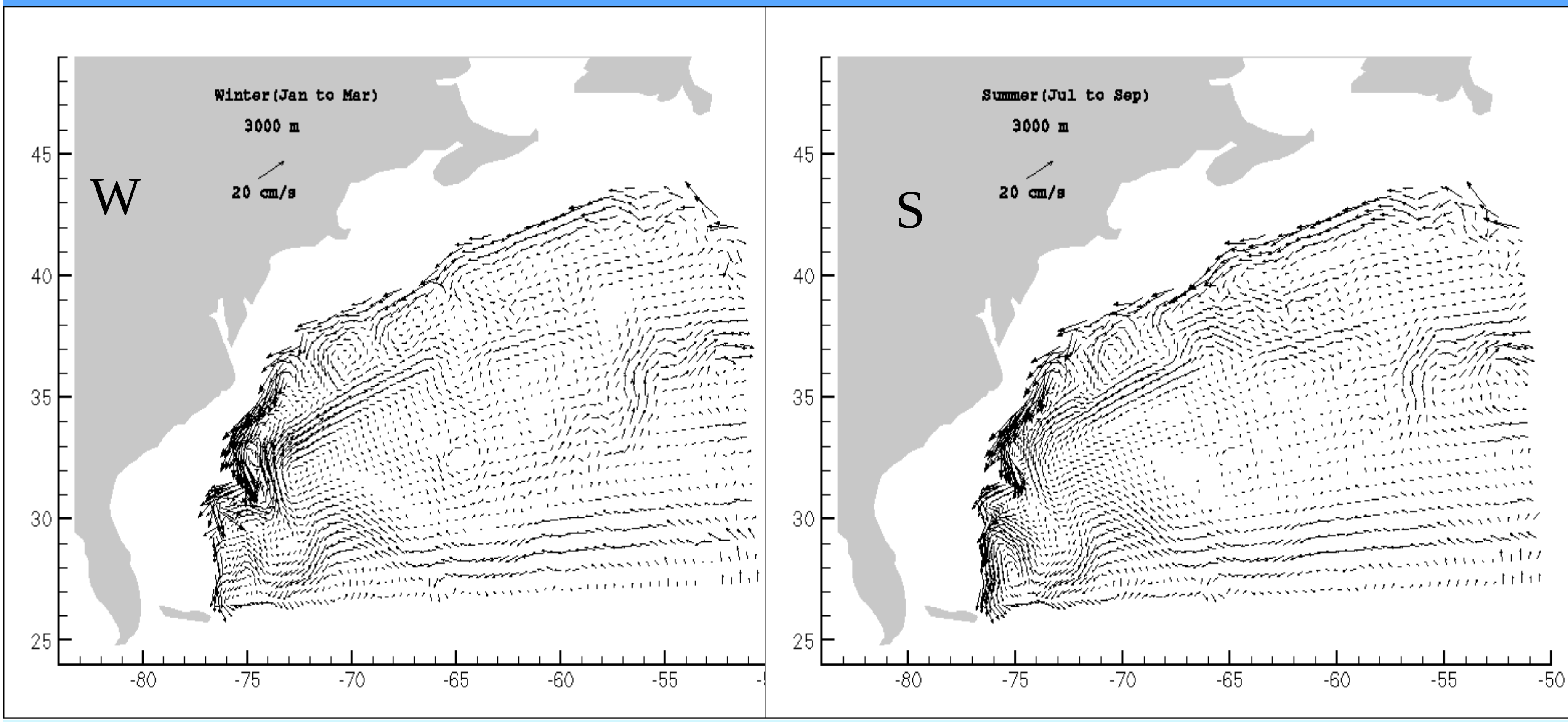


Model Surface Velocity

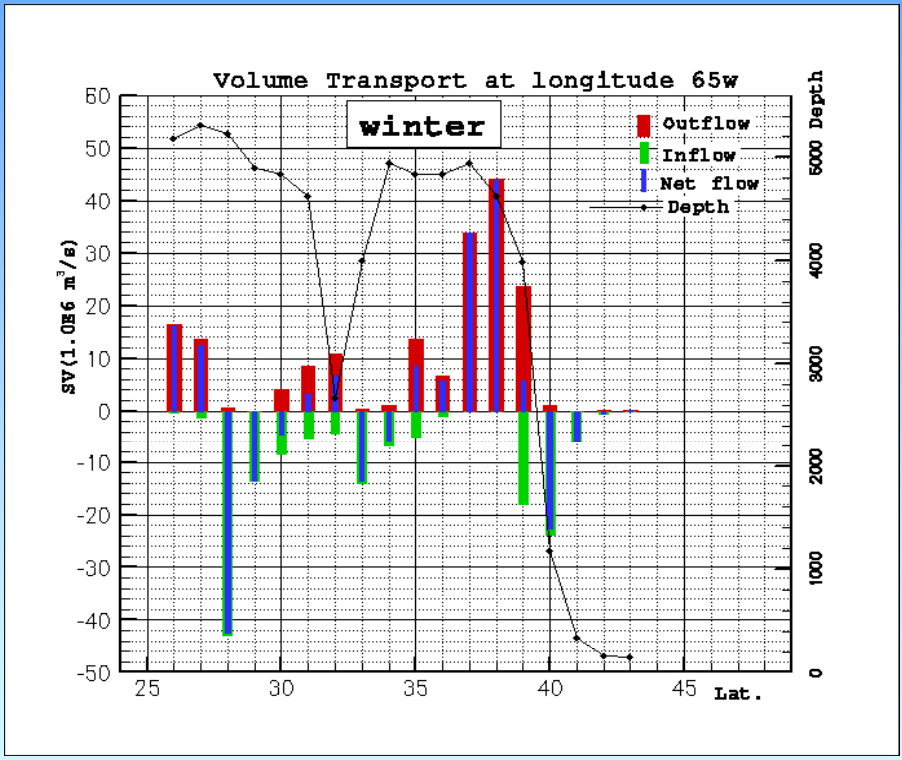


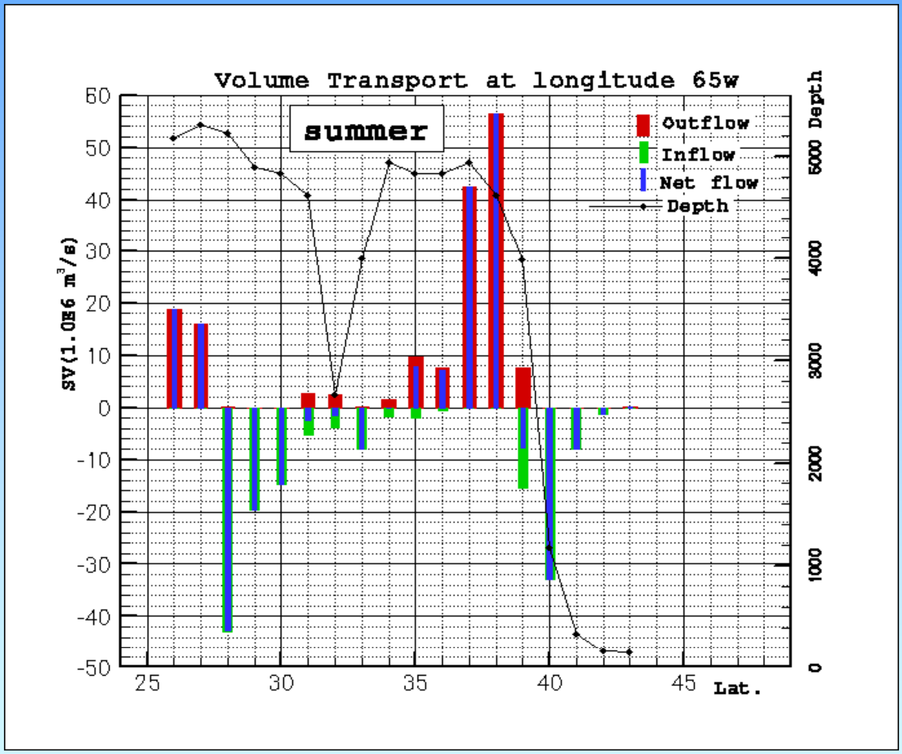


Deep Water Current

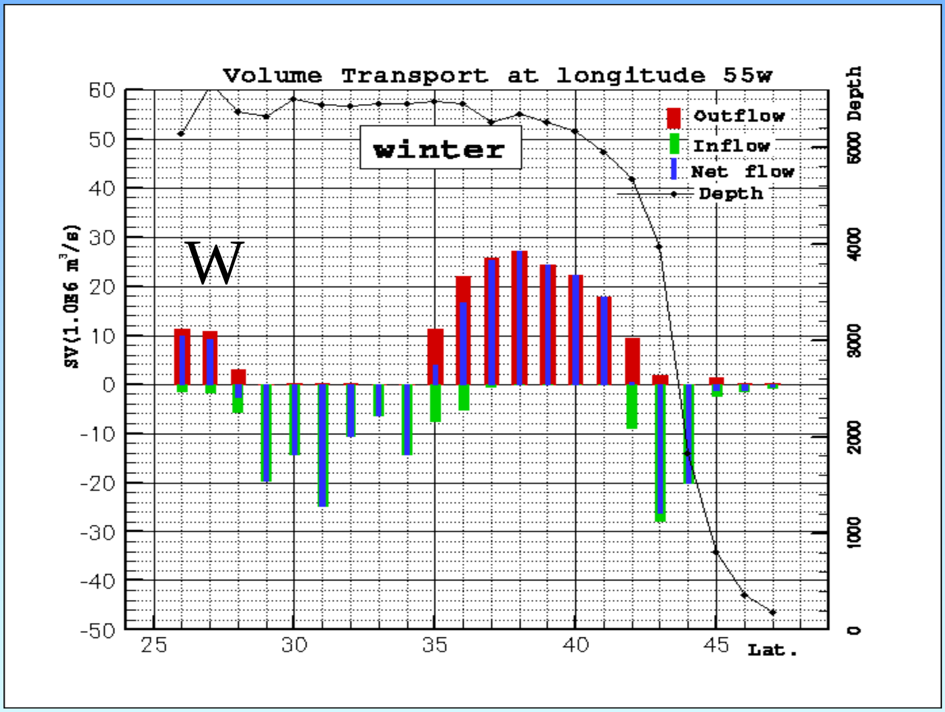


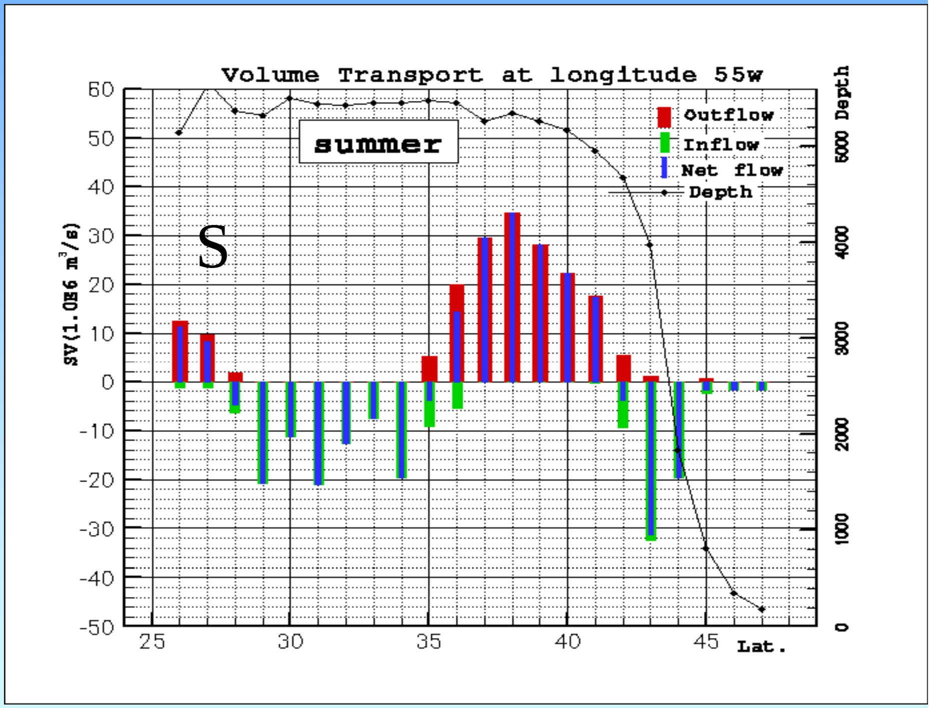
Zonal Volume Transport – 65 W





Zonal Volume Transport – 65 W

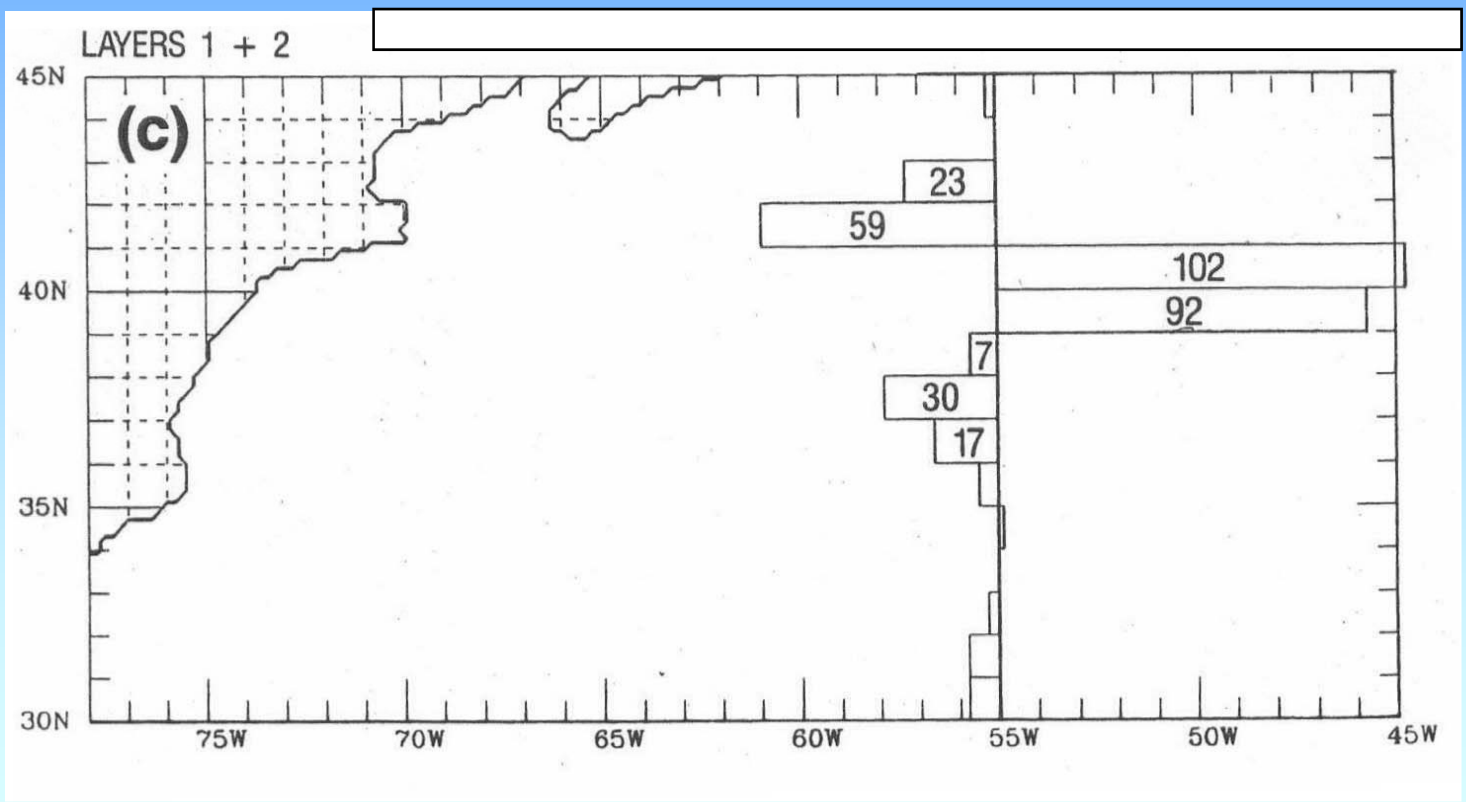




Zonal Volume Transports

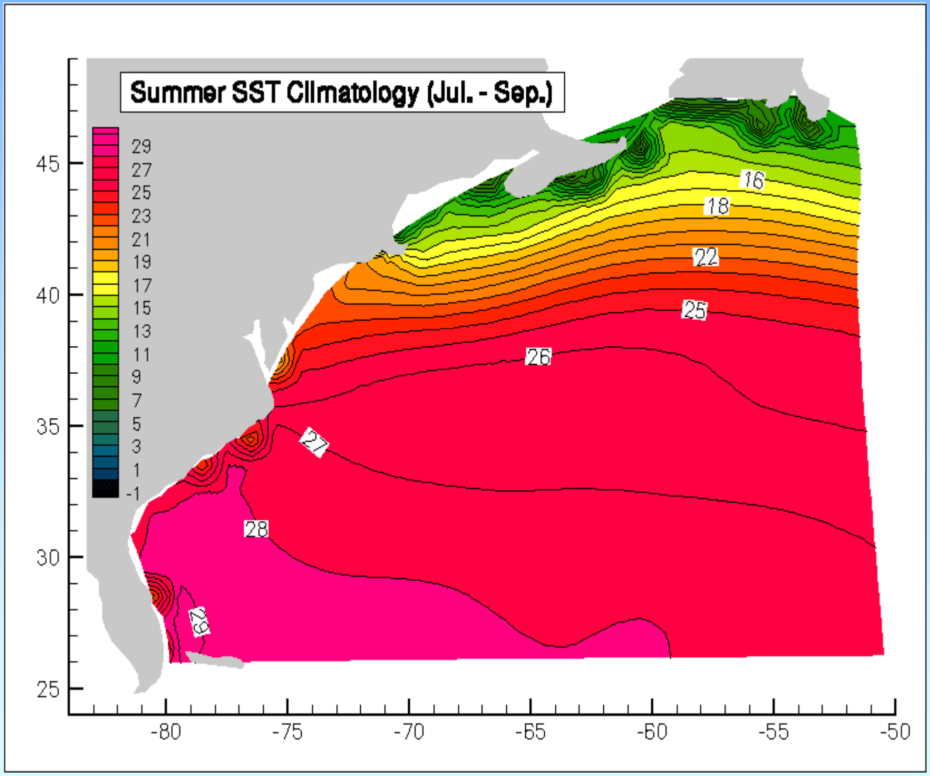
by Thompson & Schmitz Jr.

Along 55W

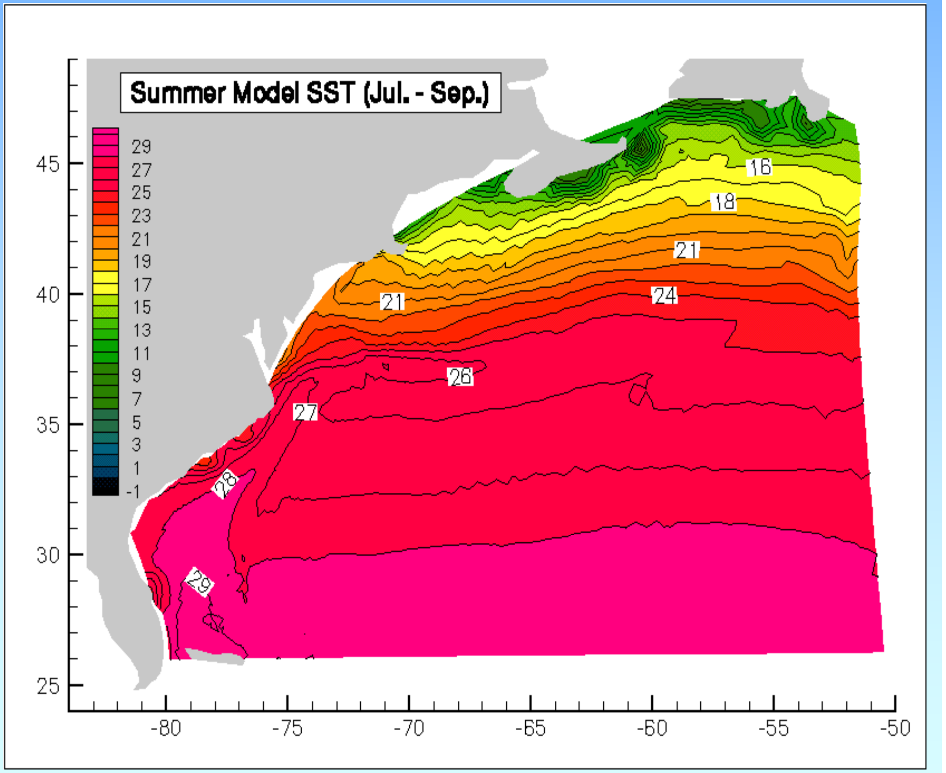


SST

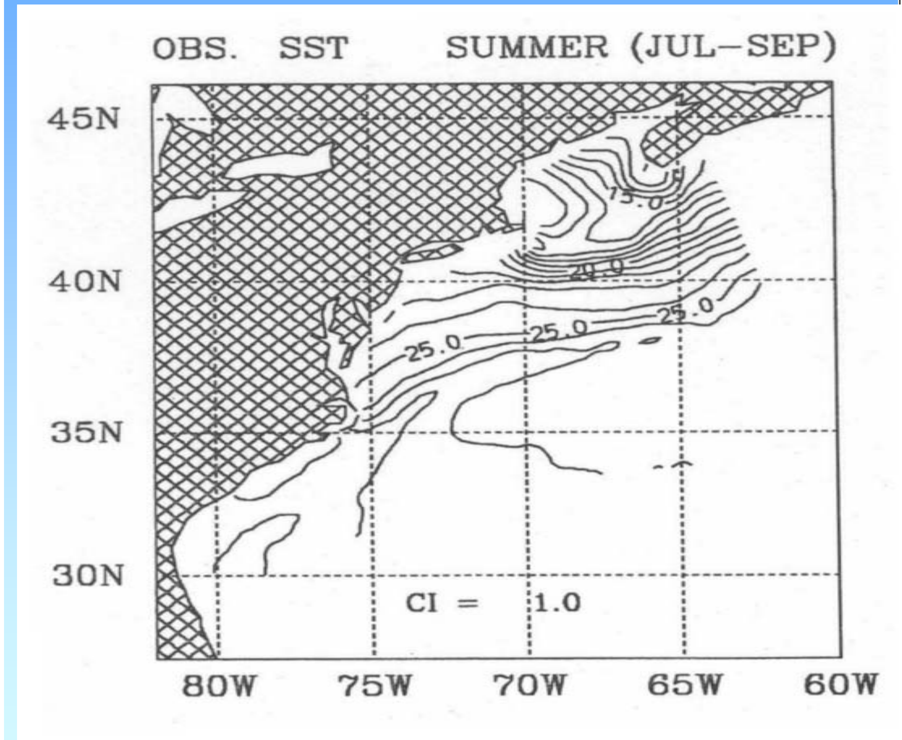
Climatology

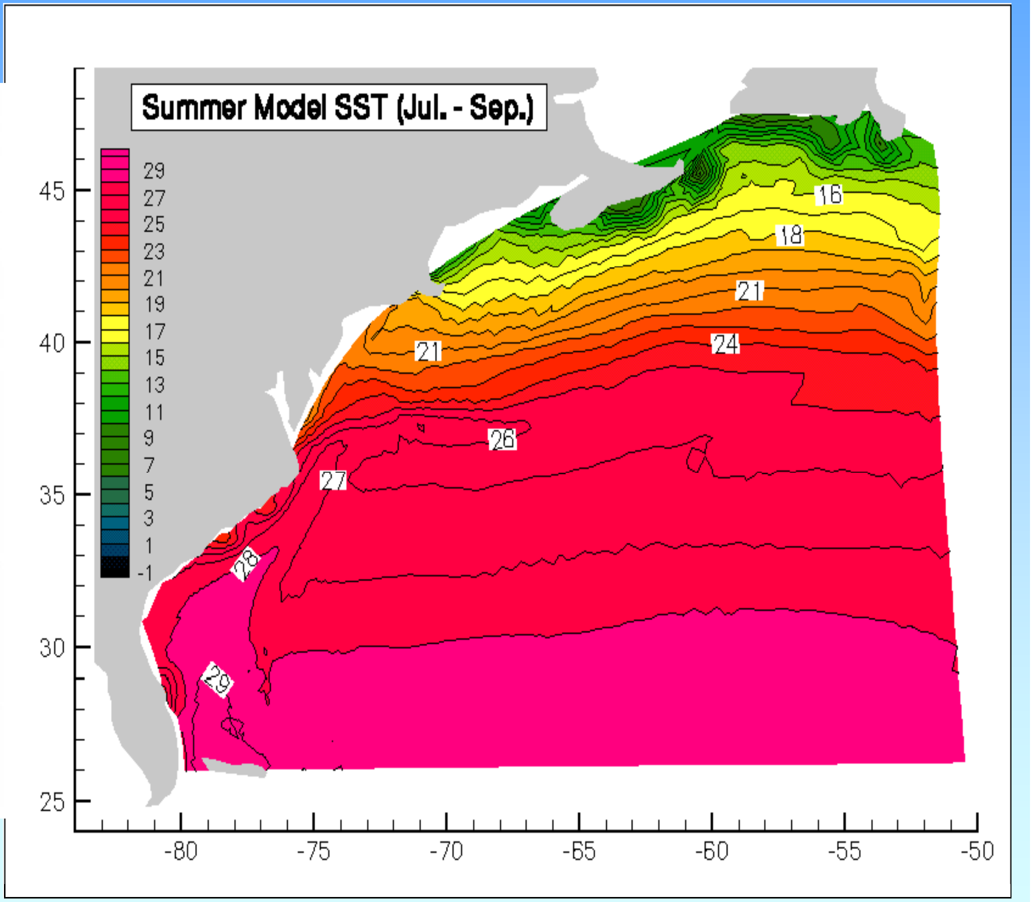


Model



SST Comparison In Summer.



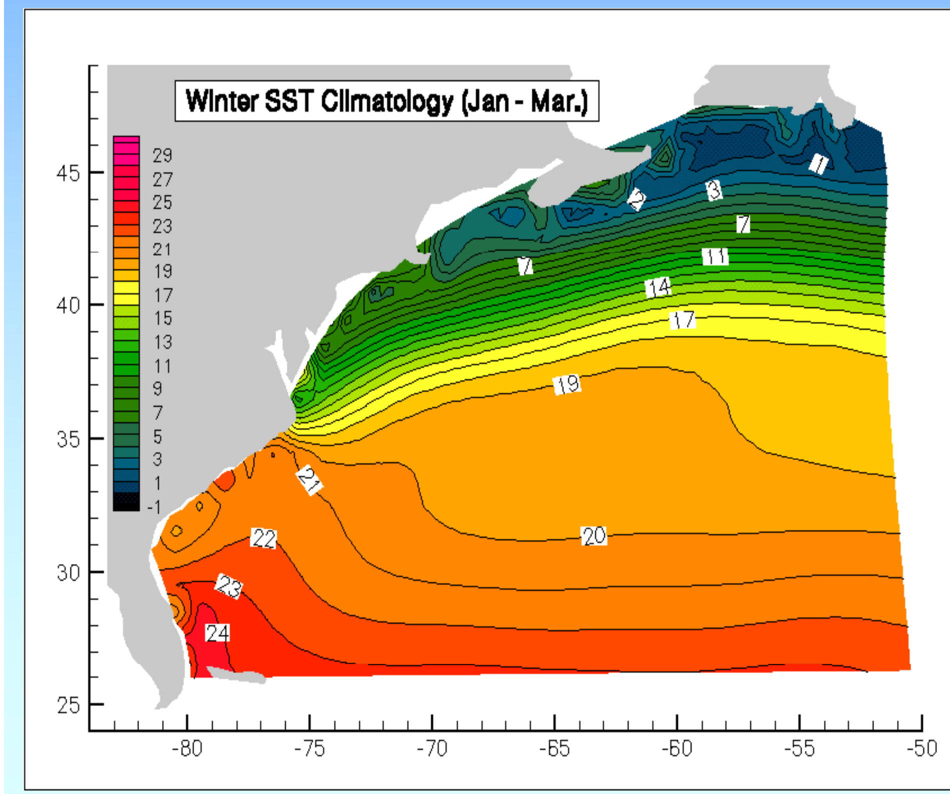


AVHRR

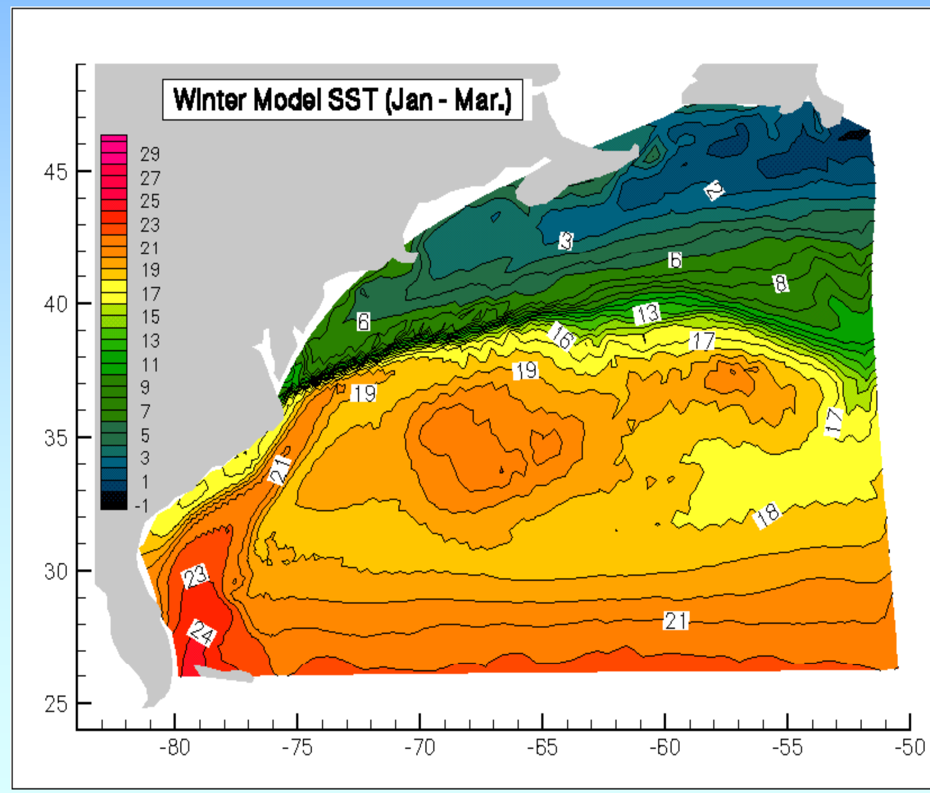
Model

SST Comparison Winter

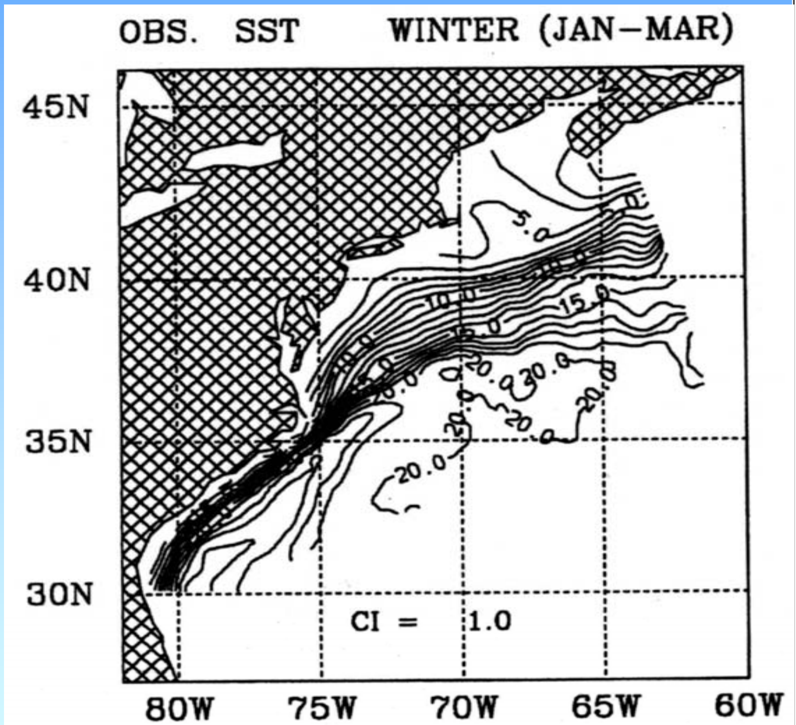
Climatology



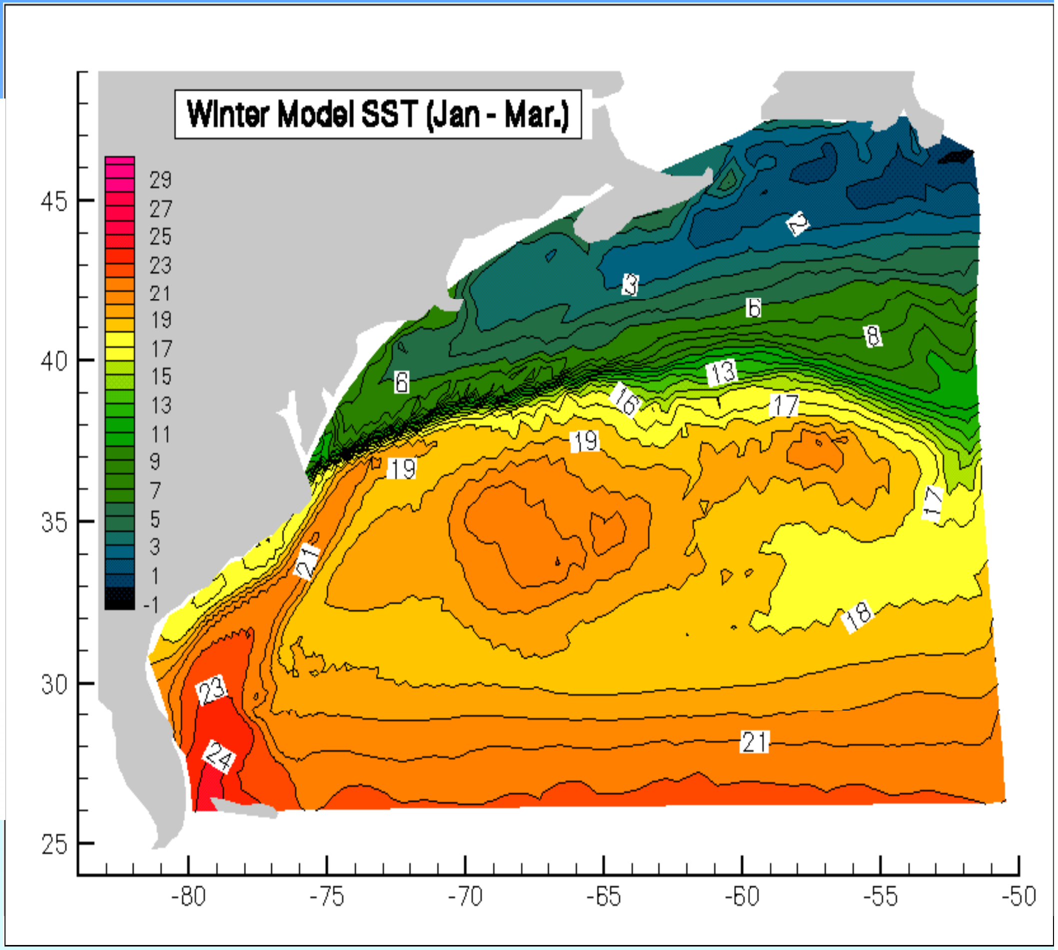
Model



SST Comparison Winter



AVHRR



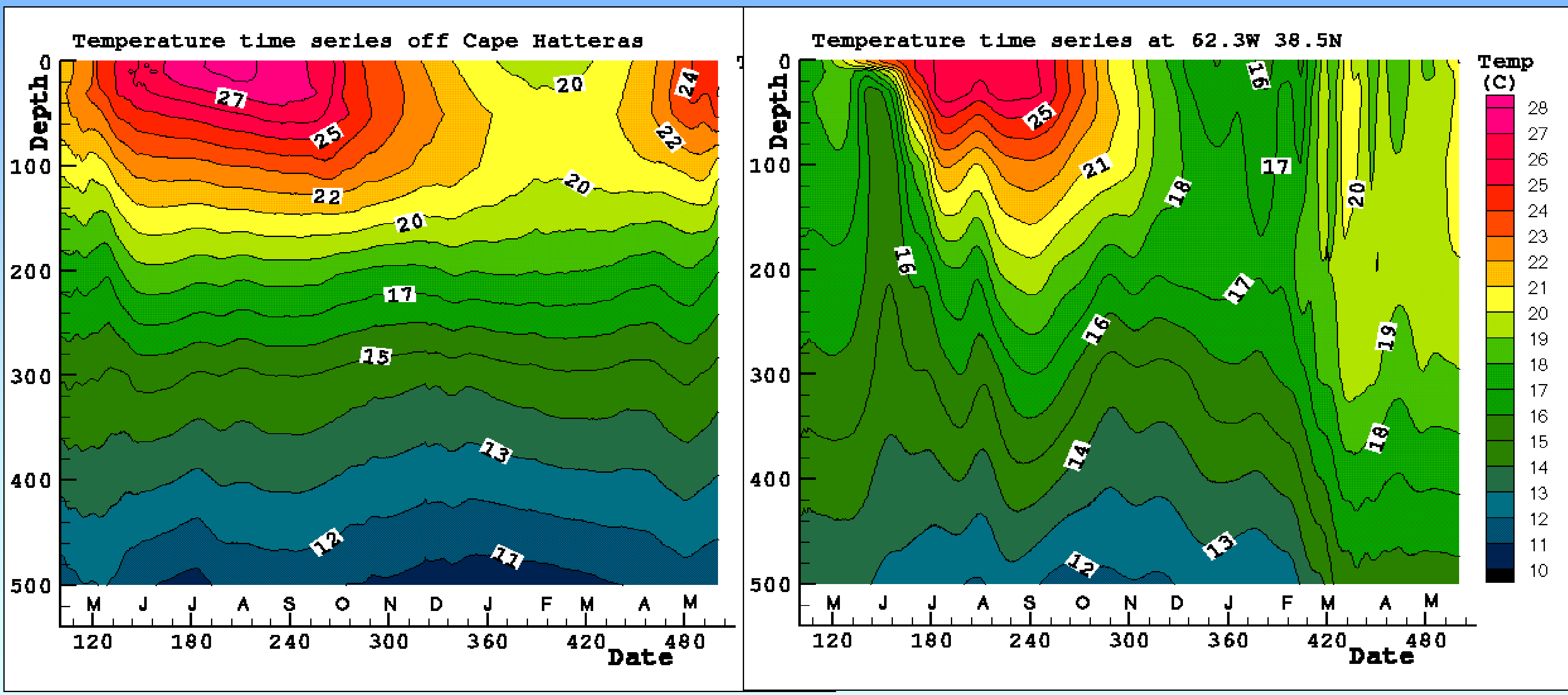
Model

Vertical Temperatrue Time Series

Off Cape Hatteras

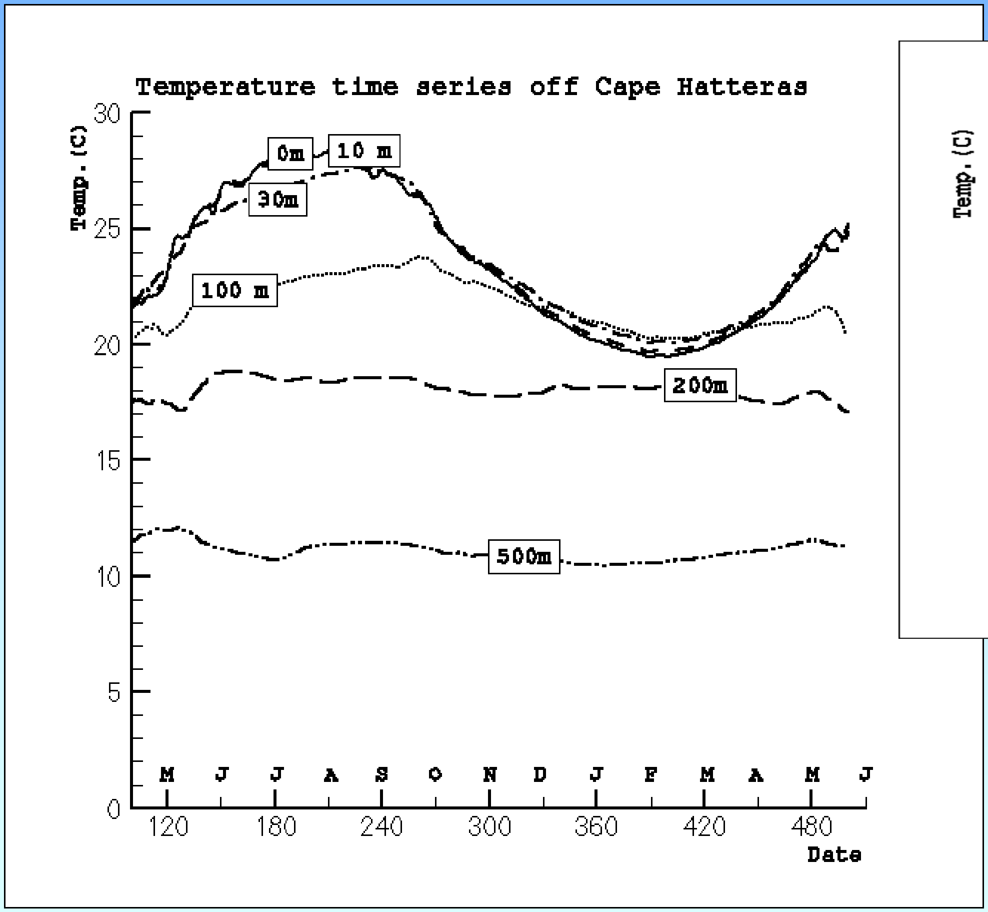
75.08 W 35.50N

62 W 38.5N

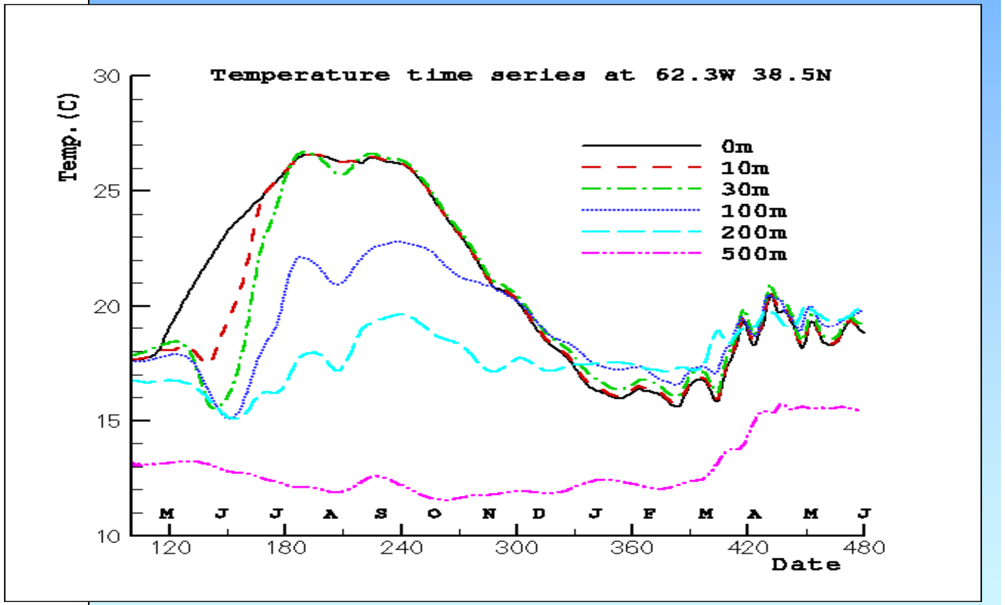


Temp. Time Series at Selected Depths

75.08 W 35.5N

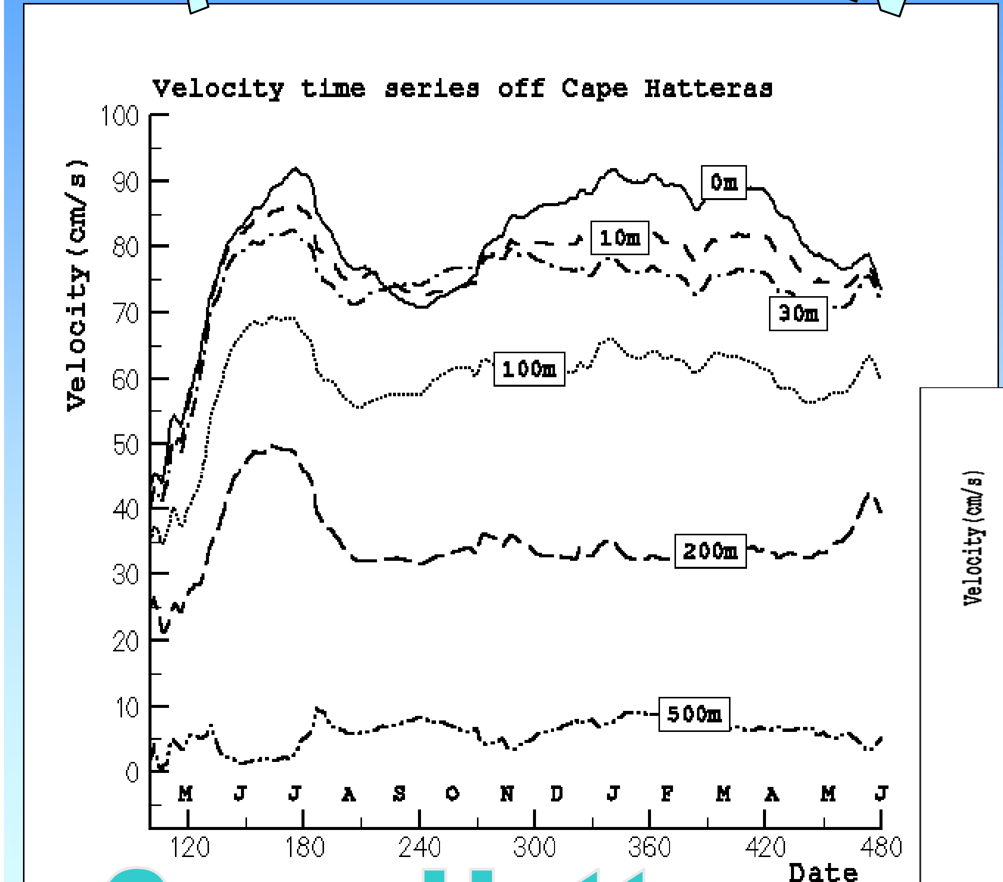


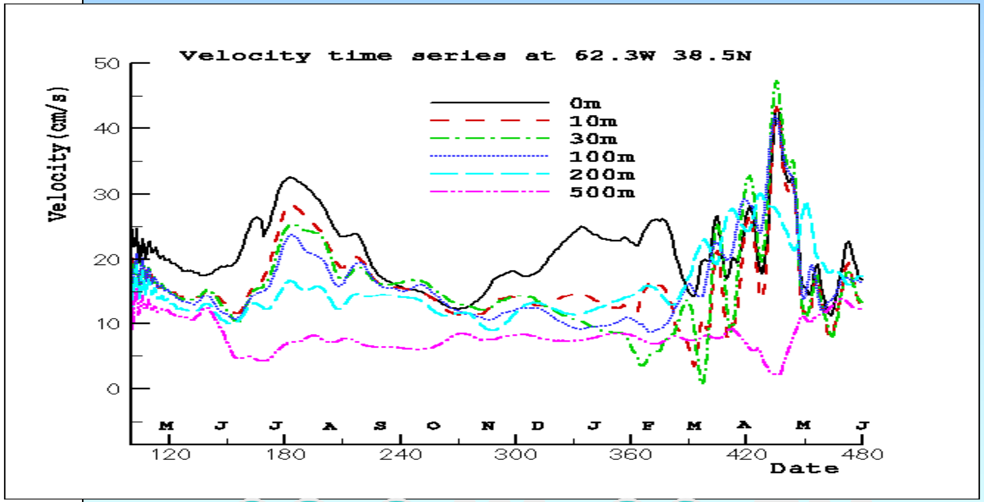
62 W 38.5N



Velocity Time Series at Selected depth

75.08 W 35.5N

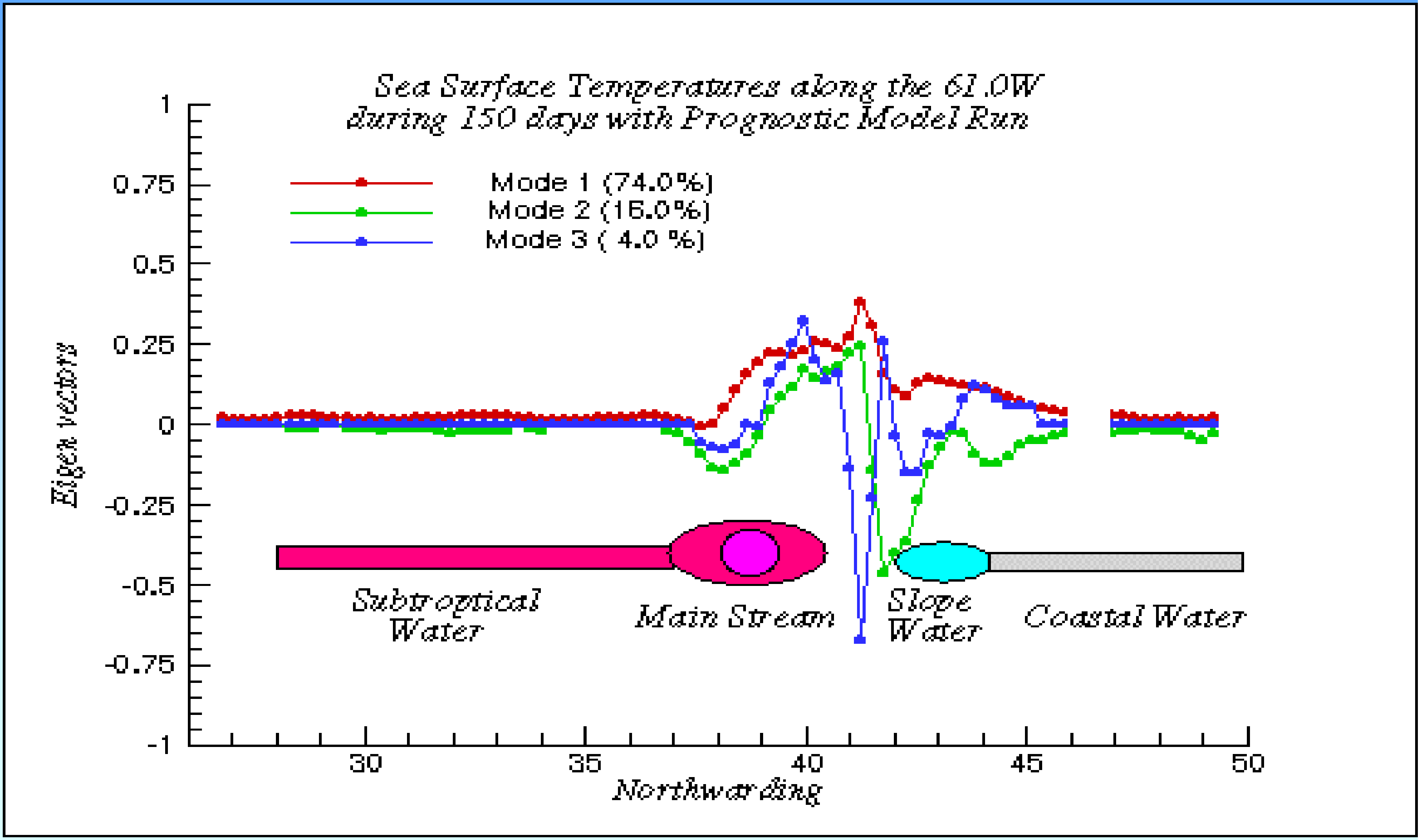




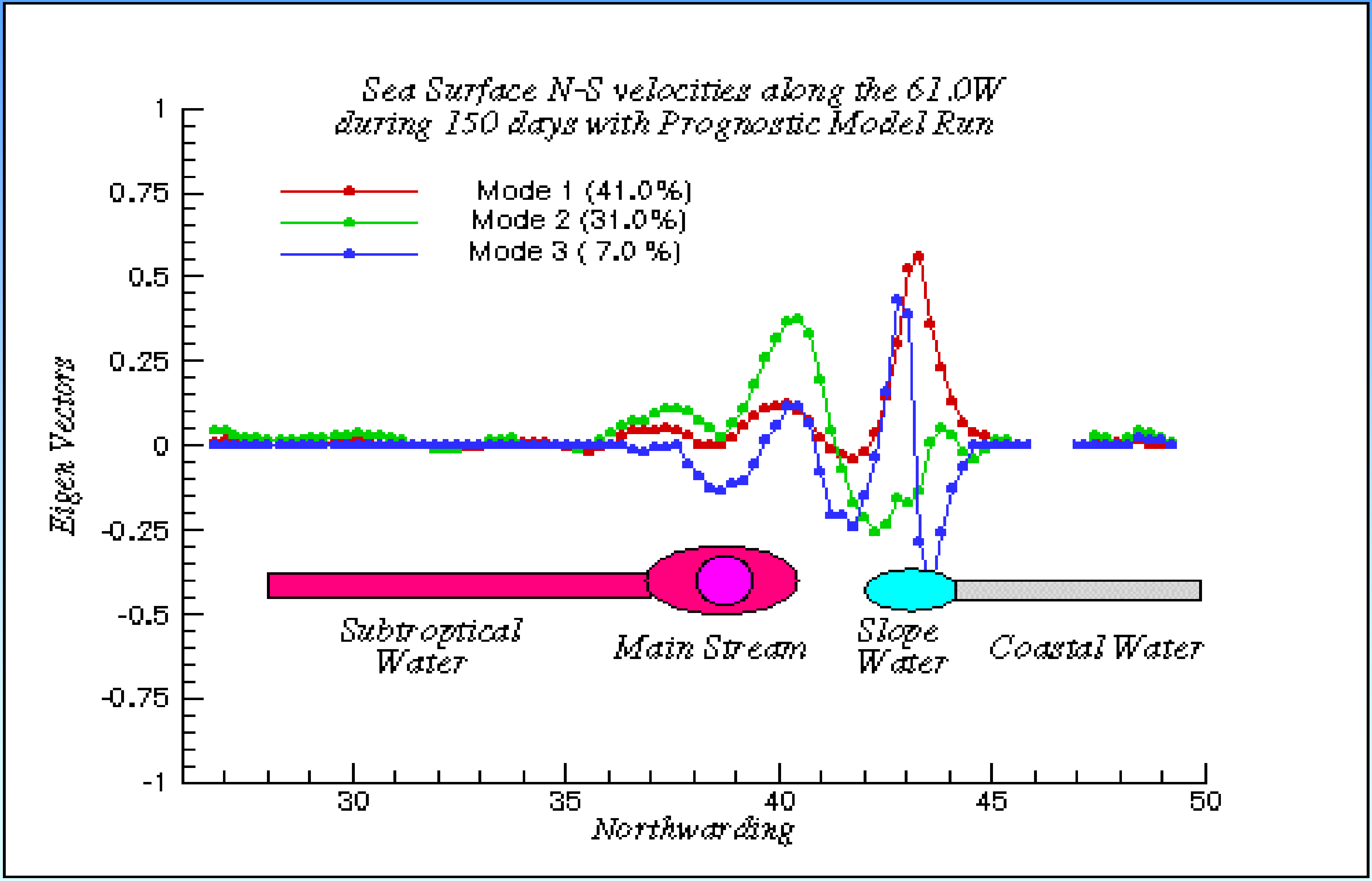
Cape Hatteras

62.3 W, 38.5N

EOF Analysis for SST Along 61° W



EOF Analysis for N-S Velocity Along 61° W



Summary and Conclusion

- *The numerical integration over steep bottom topography shows that FOM is numerically more stable than POM.*
- *The POM simulation over the steep bottom may misrepresent ocean physics without using climatology due to diapycnal mixing along sigma levels, which causes strong velocity fields due to overestimation of pressure forces.*
- *The POM simulated an unrecognizable prediction of temperature,over predicting at the in surface under no heat flux condition*
- *The FOM is much less sensitive to the bottom slope than the POM because the FOM is free from excessive numerical truncation error over steep bottom topography.*

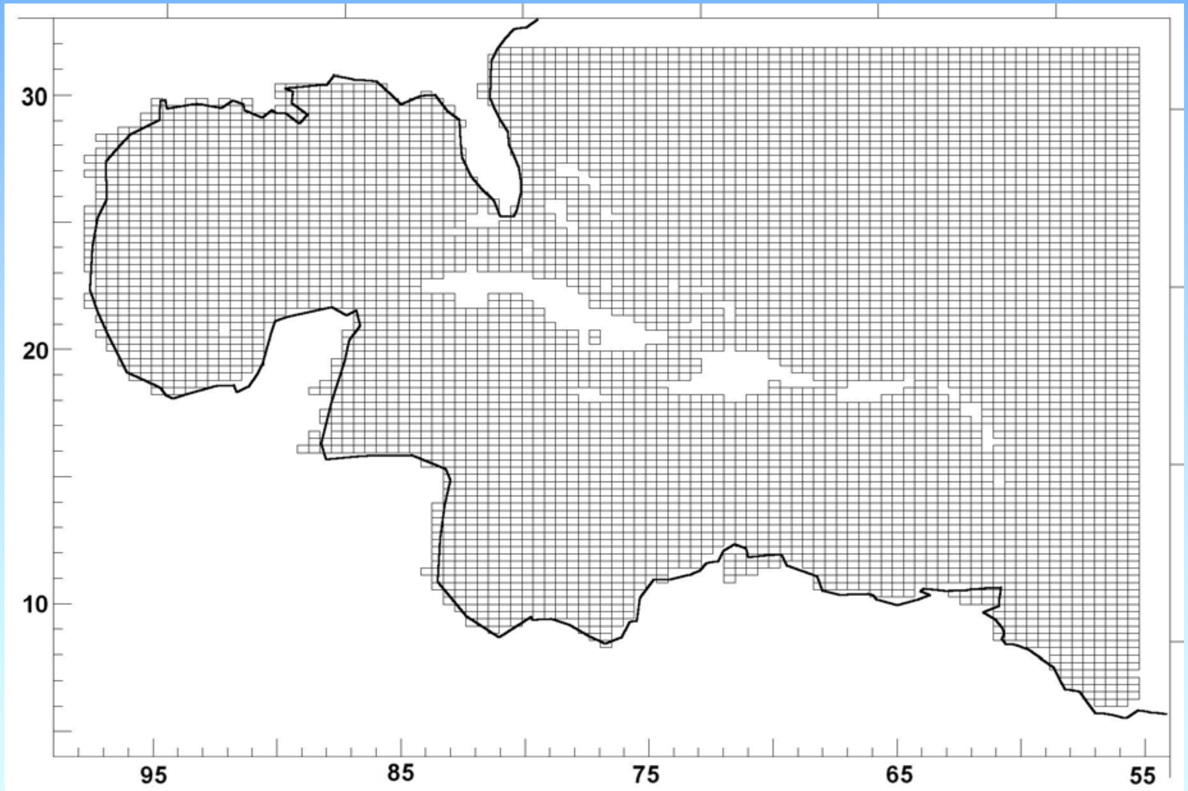
Summary and Conclusion

Application to the Gulf Stream

- ***The FOM resolved Major Currents, FC, SW,GS,RCCS, and DWBC.***
- ***The Maximum Volume Transport at 65° W was simulated by the FOM at about 134 Sv in the summer and 112 Sv in the winter,***
- ***Surface Temperatures were predicted to be comparable with the AVHRR temperatures in the summer and in the winter, even when using surface heat fluxes having low resolution.***
- ***The EOF analysis of model outputs provided information about interaction among current regimes.***

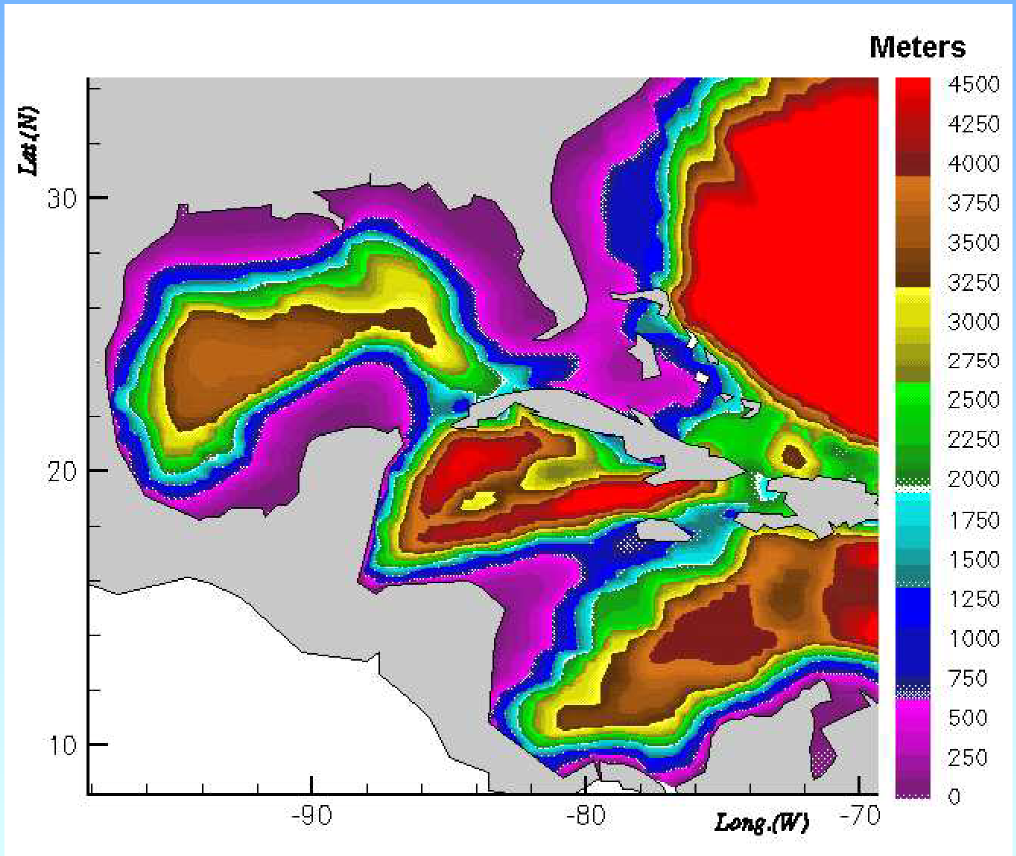
Model Setup for the Gulf of Mexico and Caribbean Basin

- Grid Generation - Curvilinear Orthogonal



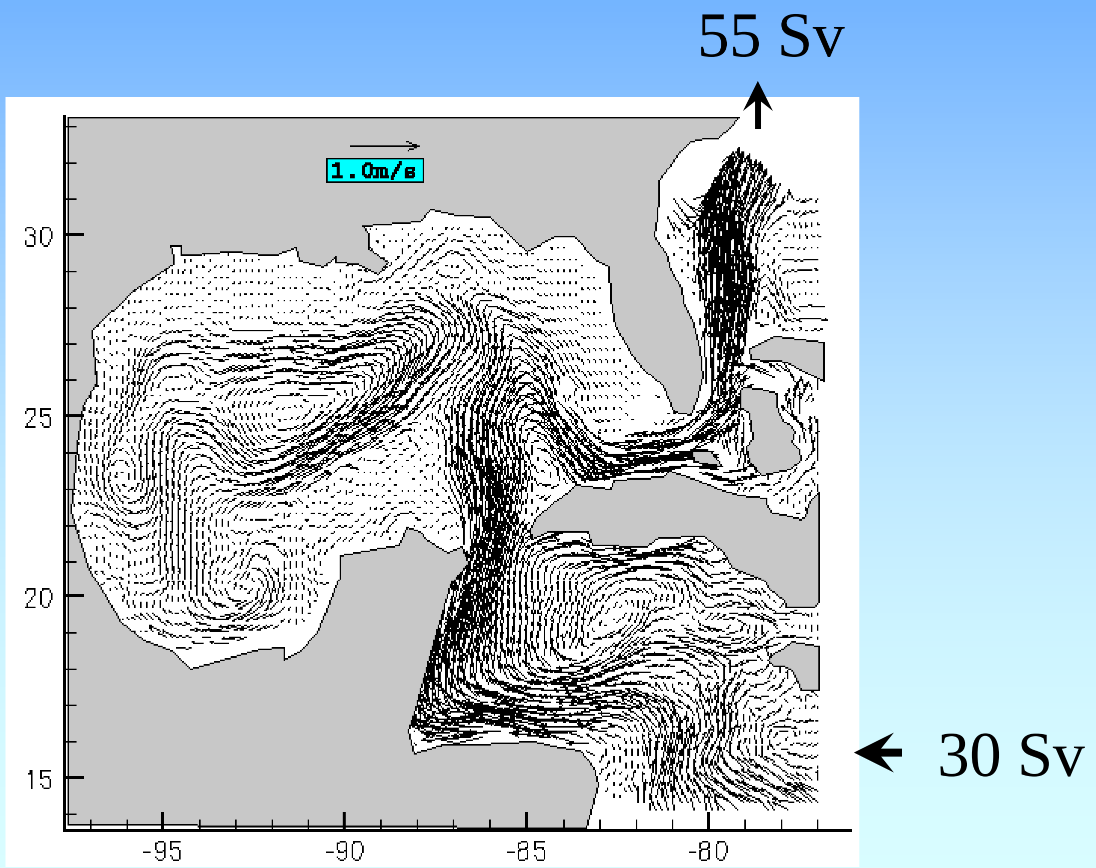
Model Setup for the Gulf of Mexico and Caribbean Basin

- **Bottom Topography- ETOP 5'**

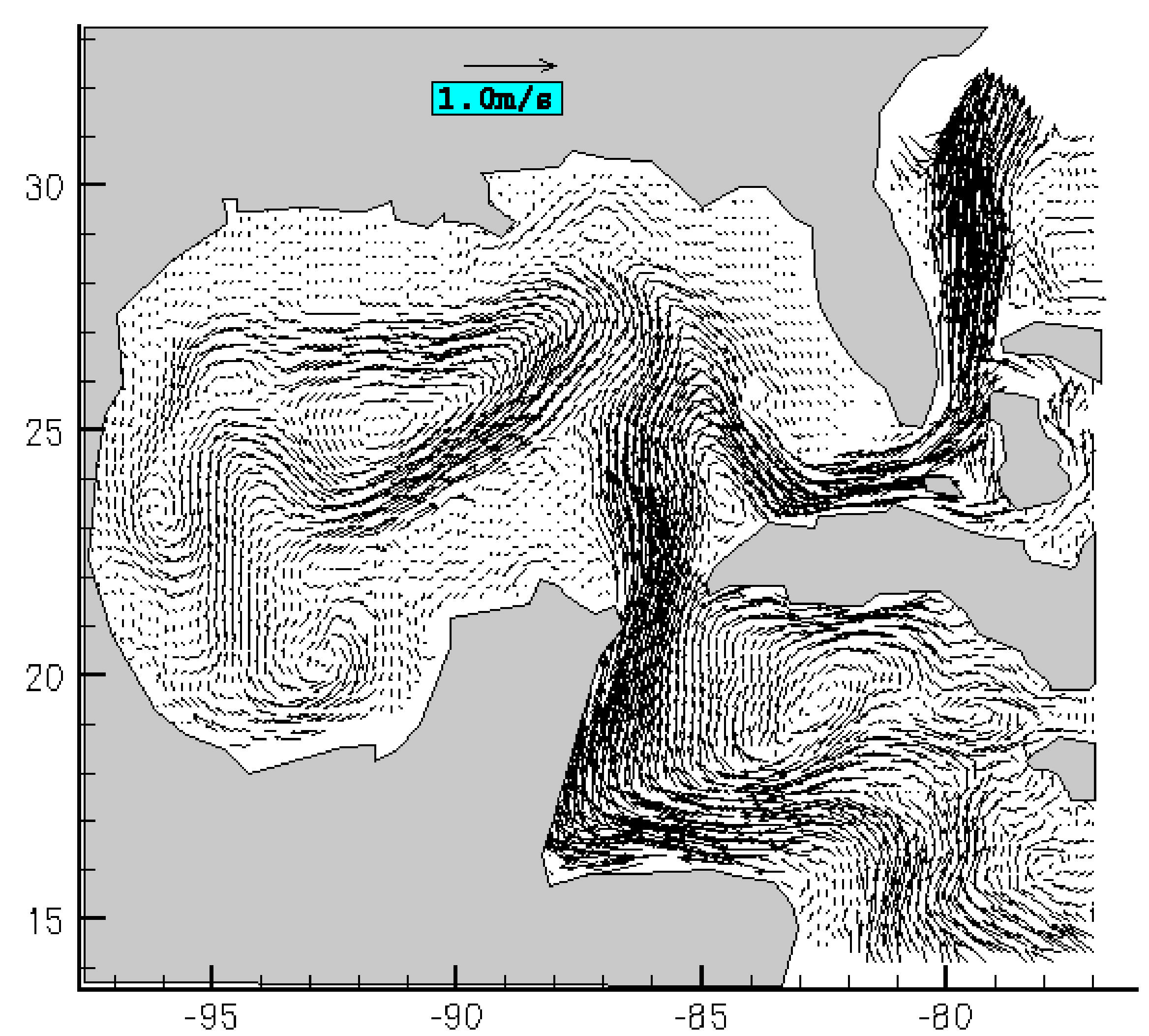


Model Setup for the Gulf of Mexico

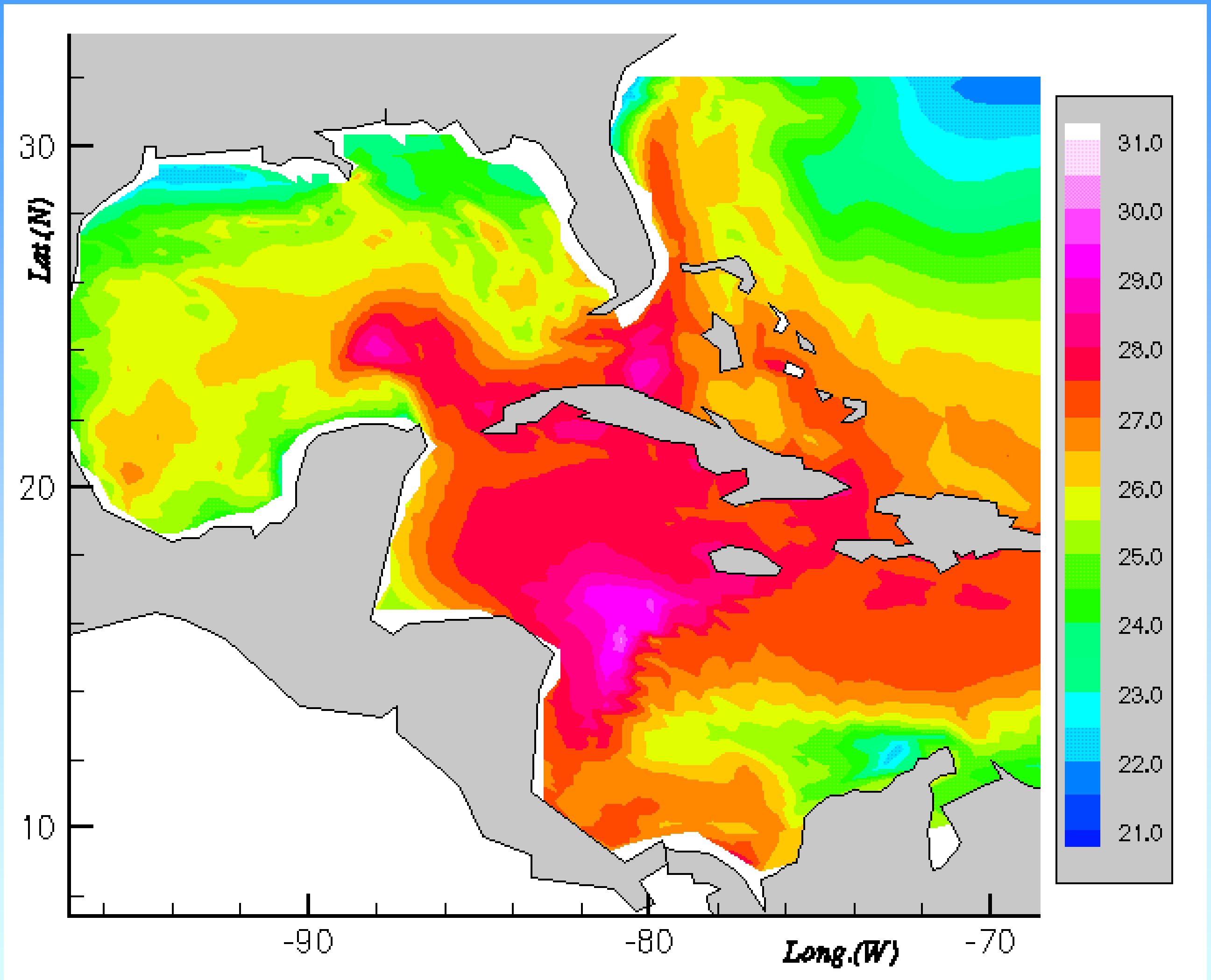
- **Boundary Conditions - Applied Flows**



Visualization of Model Hindcast



Visualization of Model Hindcast



Visualization of Model Hindcast