Absolute Ionosphere Slant Delays From Ambiguous Carrier Phase Data

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BIOGRAPHY

Dr. Dru Smith is a research geodesist with NOAA's National Geodetic Survey (NGS). He received his Ph.D. in geodesy from Ohio State before joining NGS in 1995. He was the principle investigator for geoid modeling from 1996 until 2001, at which time he went to work for the Executive Secretariat for the Interagency GPS Executive Board, working with GPS policy makers. In 2002 he returned to NGS and has been working on a method for using the Continuously Operating Reference Station (CORS) network to model total electron content in the ionosphere. He is a member of the American Geophysical Union, the International Association of Geodesy, the Institute of Navigation and serves on the Board of Directors for the American Association for Geodetic Surveying.

ABSTRACT

A new method for computing absolute (unambiguous) levels of Total Electron Content (TEC) in the ionosphere and subsequently the L1 and L2 phase advances of GPS is presented, dubbed "ICON" (for its primary purpose of modeling the Ionosphere over CONus). Unlike previous computational methods, this method relies solely upon dual-frequency, ambiguous carrier phase data with no reliance on pseudo-range, a-priori values or other external information. The only requirements for this method are that mapping functions between sufficiently close slant-views of the ionosphere are available, and that the GPS data come from a network of ground stations, geographically separated so as to allow satellites to be viewed by a variety of stations at overlapping times. This method can be applied either through independent least squares adjustments (such as one day of data at a time), or may be applied in an on-the-fly mode, where new data are adjusted into pre-adjusted values epoch-by-epoch.

Sensitivity analyses are presented, showing both the advantages and current limitations of this method. Additionally, comparisons between ICON and the IGTEC and MAGIC models of the ionosphere are presented and discussed. Plans to improve the method, as well as its application toward forecasting of the ionosphere are also included.

INTRODUCTION

Dual-frequency carrier phase data is currently the most accurate data for computing precise positions with the Global Positioning System. However, both integer ambiguities and the phase advance (caused by Total Electron Content, TEC) along the receiver-satellite path are frequency dependent. These two unknowns are generally inseparable without either lengthy observation sessions or else significantly more accurate pseudo-ranges. As such, it stands to reason that if one were given an independent estimate of the TEC (and thus the phase advance), that a faster determination of integer ambiguities (and thus centimeter level positioning) would be possible [1,2]. With this idea in mind, the National Oceanic and Atmospheric Administration (NOAA), through their National Geodetic Survey (NGS) embarked upon research to independently model the ionosphere over the Conterminous USA (CONUS) region.

Many methods already exist for getting around the "ionosphere problem", from either removing it mathematically, fitting carrier phase data to pseudo-range data, or computing both ambiguities and ionosphere delay using lengthy observing sessions. Additionally, in many existing studies of the GPS related ionosphere (such as [1], [2]), the presumption is that data, including the ionosphere, are processed in double difference mode.

However, in this paper, an entirely new method for unambiguously computing the absolute level of TEC, from a network of ground stations and only using ambiguous carrier phase data is being presented. For the sake of simplicity, this method will be referred to as the "ICON" method (named for its primary purpose of modeling the Ionosphere over CONUS). The method presented is fast, and can be kept updated over time using a sequential least squares adjustment as the data change (as new satellites rise and old ones set). ICON was first outlined in [3], however the method was in its infancy at that point, and this paper attempts to clarify and expand upon the both the method's advantages and limitations.

ASSUMPTIONS AND DEFINITIONS

Certain mathematical and physical assumptions are required for ICON to work. The **first assumption**, and most fundamental is that a network of GPS receivers exist, geographically spaced so that they view various satellites from various geographic locations throughout the day. This has the effect of "sampling" the ionosphere at various locations at a variety of angles every epoch. (Station spacing is left purposefully vague, as a variety of spacings would suffice for this method to work).

The **second assumption** as that each receiver receives both the L1 and L2 carriers for extended periods of time (again, being vague; as long as loss of lock and cycle slips can be kept to a minimum, ICON will work). The successive data over time (without loss of lock) for any given receiver/satellite combination will be referred to as a "track".

The **third assumption** is that the phase advances (due to the scintillating effects of the ionosphere), in cycles, on L1 and L2 (called I1 and I2, respectively) can be directly related to the Total Electron Content along the receiver-satellite vector (called TECS) at any given epoch by the following equations:

$$I_1 = -\frac{\kappa}{f_1\, c} TECS = (0.853) TECS \tag{1}$$

$$I_2 = -\frac{\kappa}{f_2\, c} TECS = (1.095) TECS \tag{2}$$

where:

I_1, I_2 = cycles of phase advance in L1 and L2 respectively
c = speed of light [299,792,458 m / s]
f_1, f_2 = frequency of L1 and L2 respectively (1,575,420,000 and 1,227,600,000 cyc / s)
TECS = density of electrons in receiver-satellite direction in TECU (10^{16} elec / m^2)
κ = 40.3 x 10^{16} (m cyc^2 / TECU / s^2)

Equations 1 and 2 can be found in [4], equation 6.58. The assumptions which led to these equations are more fully discussed in [5]. For the purposes of this paper it is assumed that the approximations used to derive these equations are far smaller in magnitude (generally) than other approximations of ICON.

The **fourth assumption** of ICON is that any two TECS values "sufficiently close" (defined later) to one another (in space and time) may be accurately related to one another through some mapping function. That is, for two different tracks, with TECS samples close in space and within δi epochs in time, one may write:

$$\begin{aligned} &TECS(\text{track } 1, \text{epoch i}) = \\ &f(TECS(\text{track } 2, \text{epoch } (\text{i} \pm \delta\text{i}))) \end{aligned} \tag{3}$$

The actual mapping function used to validate ICON will be discussed in a later section. It should be pointed out though that any reliable mapping function could be used in equation 3.

Further assumptions will be discussed as they come up, but these four are generally all that is required for ICON to work.

CONNECTING RINEX DATA TO TECS

At any epoch "i", the relationship between the geometric range (${}^{i}d$) and the measured range (${}^{i}r$) from a GPS receiver to satellite can be expressed (for L1 and L2 respectively) as:

$${}^{i}r_1 = {}^{i}d + c\,{}^{i}\Delta t + {}^{i}I_1 / \lambda_1 + {}^{i}T + {}^{i}m_1 \tag{4}$$

$${}^{i}r_2 = {}^{i}d + c\,{}^{i}\Delta t + {}^{i}I_2 / \lambda_2 + {}^{i}T + {}^{i}m_2 \tag{5}$$

where

r_1, r_2 = Measured range to satellite on L1 and L2 carriers respectively
d = Geometric range to satellite
$c\Delta t$ = Range error due to clock errors
λ_1, λ_2 = Wavelengths of L1 and L2
T = Range error due to troposphere
m_1, m_2 = Range error due to carrier phase multipath error on L2 and L2

Now, under our initial assumption, each GPS receiver will be tracking carrier phase data on L1 and L2. At acquisition of L1 and L2, it will compare its internal oscillator with the number of L1 and L2 cycles received and report a more or less nonsensical (i.e. "ambiguous") number of cycles (called ${}^{i}\phi_1$ and ${}^{i}\phi_2$ respectively, for epoch i; these are the values usually found as the L1 and L2 observables in a RINEX file). Presuming the receiver does not lose lock, it will continue to generate an internal L1and L2 cycle count, and compare the changes in cycles of its internal count to those cycles received from the GPS satellite. The difference between received and internal cycles can be interpreted as the change in measured range (r) due to satellite motion. Equivalently it could be considered the change in geometric range (d) plus the change to all error sources. Thus between two epochs i and j:

$$
\begin{aligned}
&{}^{i,j}\Delta\varphi_1\lambda_1 = {}^{i,j}\Delta r_1 = \\
&{}^{i,j}\Delta d + c\,{}^{i,j}\Delta\Delta t + {}^{i,j}\Delta I_1/\lambda_1 + {}^{i,j}\Delta T + {}^{i,j}\Delta m_1
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
&{}^{i,j}\Delta\varphi_2\lambda_2 = {}^{i,j}\Delta r_2 = \\
&{}^{i,j}\Delta d + c\,{}^{i,j}\Delta\Delta t + {}^{i,j}\Delta I_2/\lambda_2 + {}^{i,j}\Delta T + {}^{i,j}\Delta m_2
\end{aligned} \tag{7}
$$

where ${}^{i,j}\Delta\phi_1$ and ${}^{i,j}\Delta\phi_2$ are the differences, in cycles, between the number of L1 or L2 cycles generated inside the receiver with the number of cycles received from the GPS satellite, over the time period from epoch i to epoch j. (This is just the difference between any two RINEX values of L1 or L2 between two epochs). Now, if one subtracts equation 7 from equation 6 then only frequency dependent terms will remain:

$$
\begin{aligned}
&{}^{i,j}\Delta\varphi_1\lambda_1 - {}^{i,j}\Delta\varphi_2\lambda_2 = \\
&({}^{i,j}\Delta I_1/\lambda_1 - {}^{i,j}\Delta I_2/\lambda_2) + {}^{i,j}\Delta\Delta m_{1,2}
\end{aligned} \tag{8}
$$

Note that by arriving at equation 8, the geometric range (d) has been removed, and thus so have the unknown integer ambiguities. Unfortunately (for the time being) it also removes the absolute TECS value as well, leaving only TECS gradients.

The assumption will now be made that m_1 and m_2 are small, and also that the difference between m_1 and m_2 is small enough to be neglected [6,7]. Then, applying equations 1 and 2 to equation 8 and solving for the TECS term we end up with the following:

$$
\begin{aligned}
&{}^{i,j}\Delta TECS = {}^{j}TECS - {}^{i}TECS = \\
&\left(\frac{1}{\kappa}\right)\left(\frac{1}{f_1^2} - \frac{1}{f_2^2}\right)^{-1}\left({}^{i,j}\Delta\varphi_1 - {}^{i,j}\Delta\varphi_2\right)
\end{aligned} \tag{9}
$$

This equation shows that if we take the difference (in time) of the difference (in frequency) of L1 and L2 cycle changes (between internal and received) we have an unambiguous measure of the difference (in time) of the electron density as seen along the receiver-satellite path (TECS). To state it more clearly, if we plot a curve of TECS versus time for a particular receiver-satellite combination, we know the *shape* of TECS for that track, but we are *missing a single bias* which would define the *absolute values* of TECS for that track. This is exemplified in Figure 1 where five (of an infinite number) of TECS curves have been drawn for one particular track. Note that all 5 curves have the same shape, but with an unknown absolute value.

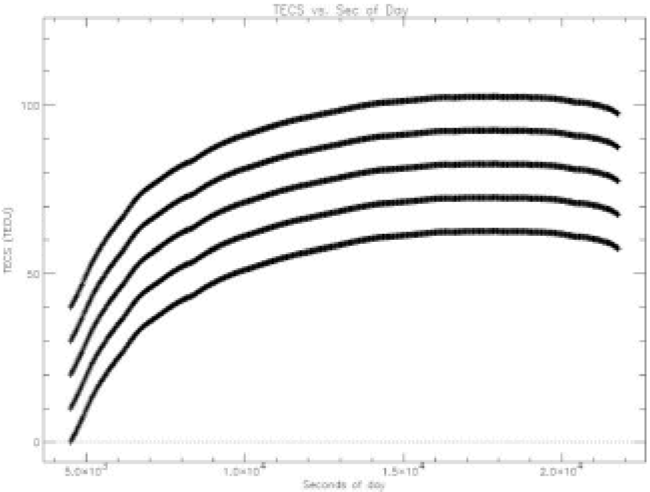

Fig. 1. Five possible versions of TECS for the same track showing identical shape, but variable absolute values.

One point that bears emphasizing is that, independent of how energetic the ionosphere is, a GPS receiver tracking L1 and L2 without loss of lock will successfully yield a ∂TECS/∂t curve that is highly accurate. This would only break down if either equations 1 or 2 were not reliable, a situation generally not considered in most literature [4,5,7]. As such, the goal of ICON is to retain the full shape of each TECS track and find the one unknown bias for each track of data, and thus have a full set of reliable TECS values. That data may then be put into various forms for distribution, but the raw information comes from finding the absolute TECS value for each track. Because it would be desirable to have sub-cycle accuracies (for the fastest ambiguity resolution), and based on equations 1 and 2, the accuracy goal of ICON is sub-TECU. It will be shown that, at this time, the method may reach this goal under certain circumstances.

MATHEMATIC MODEL

Assuming we have many tracks of TECS data (from multiple ground receivers observing multiple GPS satellites), the question arises as to how to solve for the unknown biases on each track.

Consider any given GPS track of TECS data. For each epoch, pick a point of convenience (or “POC”; somewhere that two TECS values can be reliably mapped into one another). These points can be fixed in height or varying (and should not be confused with pierce points on a fixed height shell; points of convenience can vary in height and are merely “convenient points” on two TECS vectors whose locations can be compared to one another to see if two TECS vectors are “sufficiently close” to one another to allow them to be mapped accurately into one another). If one plots these points geographically (assuming the height of the points doesn’t change significantly from one epoch to the next), one can see they form a smooth series of points from first acquisition (satellite rise, for example) to loss of lock (satellite set, for example). One such track, (using a fixed height of 300 km for simplicity), is plotted in Figure 2.

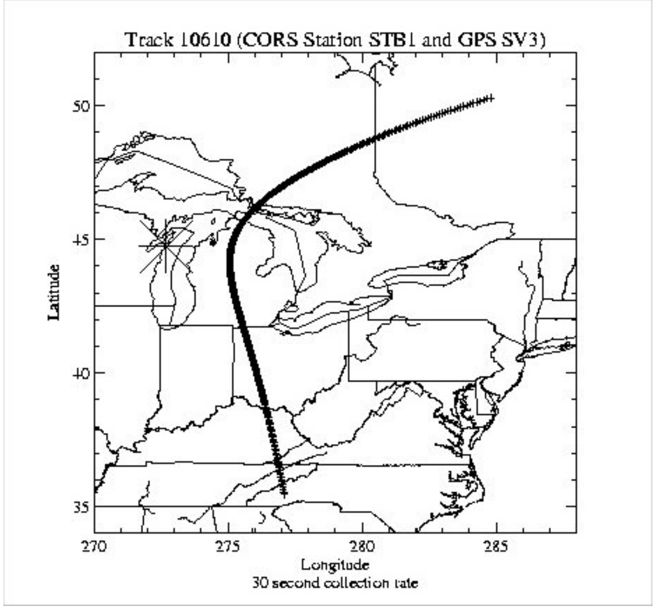

Fig. 2. An example plot of the points of convenience for one track (formed when station SBT1 tracks GPS SV#3 from rise to set, with points of convenience at a fixed height of 300 km on July 12 of 2002). Station SBT1 is shown by the large asterisk.

Consider next any two tracks, where at some epoch, the satellite-receiver vectors of the two different tracks are "sufficiently close" to the same POC. We will call such an occurrence a "<u>crossover</u>". See Figure 3.

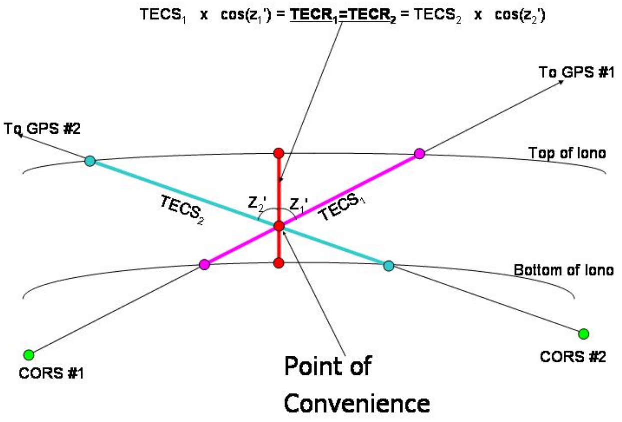

Fig. 3. Side view cross-section of the ionosphere showing a crossover (two different satellite-receiver vectors crossing at a single point of convenience).

Now as per assumption 4, $TECS_1$ must be mappable into $TECS_2$. For the purposes of this paper, the mapping function to be invoked for testing ICON will rely upon the familiar cos z' mapping between slant (TECS) and vertical (TECR) views of TEC. See equation 10:

$$TECS = TECR / \cos z' \tag{10}$$

Because the cos z' mapping is often associated with a "thin shell", ICON may seem to rely on such a shell, but the only true assumption is that some suitable mapping function (fulfilling equation 3) can be found to map "sufficiently close" TECS values into one another. <u>No direct requirement that the ionosphere lies on a "shell" is actually called for!</u> (This contradicts previous statements of [3], and corrects that error.)

Now, notice that (see Figure 3) both $TECS_1$ and $TECS_2$ may be mapped into a TECR value at the point of convenience using their respective z' angles. Note further that, because the two TECS values are "sufficiently close" to each other (and to the POC), that if the mapping of TECS into TECR at both tracks is accurate, that they must generate the same value of TECR. That is, $TECR_1=TECR_2$. Expanding this, the mapping of TECS1 into TECS2 can be written as:

$$TECS_1 = \cos z'_2 TECS_2 / \cos z'_1 \tag{11}$$

thus fulfilling assumption 4. This means that for every crossover that occurs, we have:

1) One constraint ($TECR_1=TECR_2$)
2) Two unknowns (unknown TECS bias for each track)

Standing by itself, one crossover can not solve for two unknowns. However, if a situation existed where the number of unknowns was equal to (or exceeded by) the number of crossovers, then the situation would be solvable.

Such situations can (and do) exist. Consider the case of 3 GPS tracks (call them 1, 2 and 3), generated by 3 GPS/receiver combinations, and distributed in such a way that the 3 tracks have 3 crossovers (call them A, B and C), effectively forming a "triangle" in the ionosphere (see Figure 4).

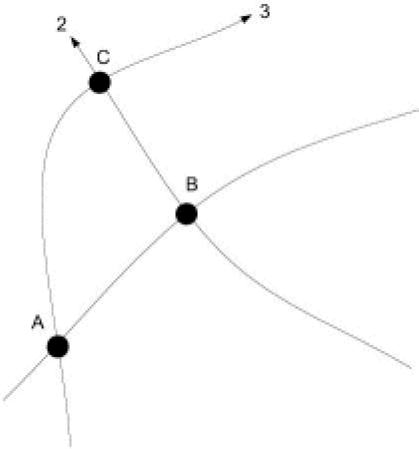

Fig. 4. Top view of a "triangle" formed by 3 tracks and 3 crossovers.

As previously stated, we know the shape of the TECS curve (∂TECS/∂t) along each of the three tracks, but do not know the absolute value. Put another way, each of the 3 tracks (1, 2 and 3) has 1 unknown bias (b_1, b_2 and b_3). In fact, one can define b_i as the actual value of TECS on track i at the very first point on that track. If we know that bias, we could compute the absolute values of TECS (because we know ΔTECS along the track). The three crossovers at A, B and C provide the following constraints on the system:

$$ {}^{A}_{1}TECR = {}^{A}_{3}TECR (= {}^{A}TECR) \tag{12}$$

$$ {}^{B}_{1}TECR = {}^{B}_{2}TECR (= {}^{B}TECR) \tag{13}$$

$$ {}^{C}_{2}TECR = {}^{C}_{3}TECR (= {}^{C}TECR) \tag{14}$$

That is, the TECR values at a crossover must be unique, independent of track. Notice that while all of our actual track information is in the form of ΔTECS values, the actual constraints on the system are on TECR. So, in the notation used above, let's write out the conversion from TECR to TECS and see how the unknown biases fit into the picture:

$$ \begin{aligned} &{}^{A}_{1}TECR = {}^{A}_{1}TECS \cos{}^{A}_{1}z' = \\ &(b_1 + {}^{A}_{1}\Delta TECS)\cos{}^{A}_{1}z' \end{aligned} \tag{15}$$

$$ \begin{aligned} &{}^{A}_{3}TECR = {}^{A}_{3}TECS \cos{}^{A}_{3}z' = \\ &(b_3 + {}^{A}_{3}\Delta TECS)\cos{}^{A}_{3}z' \end{aligned} \tag{16}$$

$$ \begin{aligned} &{}^{B}_{1}TECR = {}^{B}_{1}TECS \cos{}^{B}_{1}z' = \\ &(b_1 + {}^{B}_{1}\Delta TECS)\cos{}^{B}_{1}z' \end{aligned} \tag{17}$$

$$ \begin{aligned} &{}^{B}_{2}TECR = {}^{B}_{2}TECS \cos{}^{B}_{2}z' = \\ &(b_2 + {}^{B}_{2}\Delta TECS)\cos{}^{B}_{2}z' \end{aligned} \tag{18}$$

$$ \begin{aligned} &{}^{C}_{2}TECR = {}^{C}_{2}TECS \cos{}^{C}_{2}z' = \\ &(b_2 + {}^{C}_{2}\Delta TECS)\cos{}^{C}_{2}z' \end{aligned} \tag{19}$$

$$ \begin{aligned} &{}^{C}_{3}TECR = {}^{C}_{3}TECS \cos{}^{C}_{3}z' = \\ &(b_3 + {}^{C}_{3}\Delta TECS)\cos{}^{C}_{3}z' \end{aligned} \tag{20}$$

Substituting equations 15-20 into equations 12-13, and converting to matrix (Y=A X) form yields:

$$ \begin{bmatrix} {}^{A}_{1}\Delta TECS\cos{}^{A}_{1}z' - {}^{A}_{3}\Delta TECS\cos{}^{A}_{3}z' \\ {}^{B}_{1}\Delta TECS\cos{}^{B}_{1}z' - {}^{B}_{2}\Delta TECS\cos{}^{B}_{2}z' \\ {}^{C}_{2}\Delta TECS\cos{}^{C}_{2}z' - {}^{C}_{3}\Delta TECS\cos{}^{C}_{3}z' \end{bmatrix} = \begin{bmatrix} -\cos{}^{A}_{1}z' & 0 & +\cos{}^{A}_{3}z' \\ -\cos{}^{B}_{1}z' & +\cos{}^{B}_{2}z' & 0 \\ 0 & -\cos{}^{C}_{2}z' & +\cos{}^{C}_{3}z' \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \tag{21}$$

This system of linear equations has a similarity to those used, for example, in solving altimetric crossovers or geodetic leveling problems. In the case of altimetry and geodetic leveling however, the A matrix contains only +1, -1 and 0 and is thus non-invertible without additional constraints. In contrast, the cosines in our A matrix allow for an inversion. In simplest terms, this means that even though we only know ΔTECS (that is, the shape or temporal changes of TECS) on a track and we only know that TECR must be identical at the crossovers (independent of track), we can compute one unique absolute value of TECS at the beginning of each track (b_1, b_2 and b_3). Knowing this value, we are then able to compute all TECS values for the entire track (and subsequently all absolute TECR values for the entire track if vertical, rather than slant, delays are desired.)

Therefore, within the confines of our opening assumptions, an unambiguous solution for TECS, based solely on ambiguous carrier phase data, has been found. It is entirely due to the existence of a mapping function (in this case, cos z'_2/cos z'_1) between TECS values at crossovers that this is possible. As it turns out, this mathematical relationship holds for any number of tracks that form a closed polygon, not just for triangles, since the number of sides (tracks) of a polygon must equal the number of vertices (crossovers), and thus would provide a unique and solvable system.

However, while the above triangular example offers a *unique* solution, it offers no *redundancy*. But if the situation is made just slightly more complex, say by adding a 4th track, crossing tracks 1 and 2 at points D and E respectively (forming another triangle, with common vertex B) we begin to build up redundancy; see Figure 5.

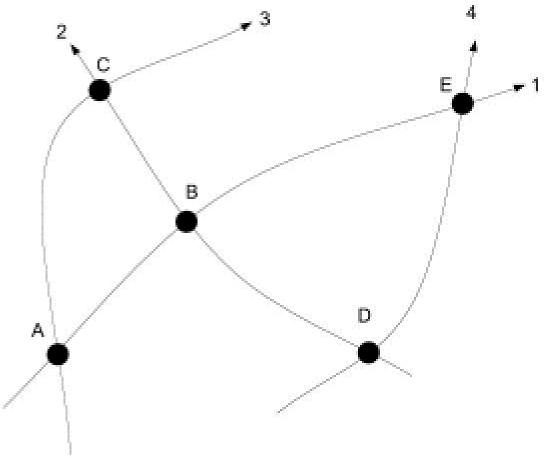

Fig. 5. Adjoining “triangles” showing 4 tracks (unknowns) and 5 crossovers (observations) and thus redundancy

In this case, we now have 4 tracks (with unknowns b_1, b_2, b_3 and b_4) but 5 crossovers (constraints), and thus a Least-Squares Adjustment can be performed on this redundant system. This buildup of redundancy grows as the number of tracks grows, since each new track can be expected to cross more and more existing tracks, adding multiple crossovers (constraints) to the system while only adding 1 unknown bias for the new track.

Given the fact that a complex network of tracks should yield more crossovers than tracks, it seems that the ideal application of ICON is toward a large network of dual-frequency carrier phase GPS receivers, which are continuously operating and spaced geographically so that their tracks cross, but offer some wide spatial distribution in the ionosphere. (Thus our initial assumption that one needs a large network of receivers, since forming crossovers is the key to this method.)

One important point to consider is that, once a large network of data has been adjusted, and biases computed, that future tracks of data can be adjusted into the pre-adjusted network. This application is not the approach currently taken in production; daily, independent networks are being computed at NGS under the name “ICON”, but nothing prevents an iterative least squares adjustment from computing biases on an epoch by epoch basis, as new tracks form new crossovers and enter into the network adjustment.

THE CORS NETWORK

The National Geodetic Survey of NOAA, coordinates a network of continuously operating reference stations (CORS) that provide GPS carrier phase and code range measurements in support of 3-dimensional positioning activities throughout the United States and its territories (see Figure 6).

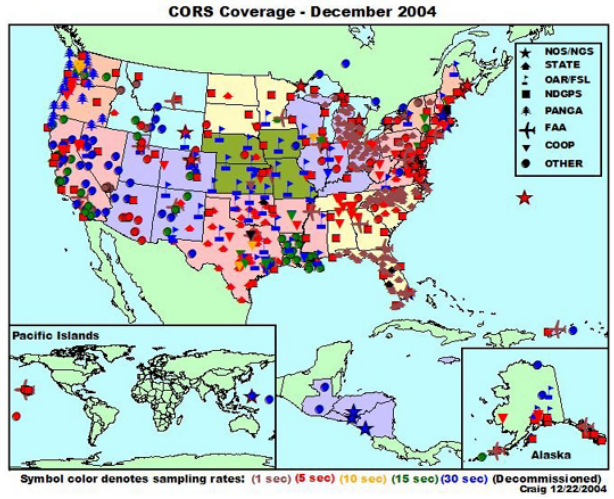

Fig. 6. The CORS Network

The CORS system benefits from a multi-purpose cooperative endeavor involving many government, academic, commercial and private organizations. New sites are evaluated for inclusion according to established criteria. The data are collected at various rates (1, 5, 10 or 30 seconds) and then transmitted (with varying latencies) to NGS for coordinate computation, data distribution and archival. However, these data are also perfect for modeling the ionosphere using the ICON procedure outlined in the previous section.

TESTING THE MODEL

As shown in [3], extensive testing was done on the ICON method of modeling TECS values. The initial tests were on small networks, and proved the mathematical rigor of the method itself. Large scale, full-day tests were then performed, showing that an entire day of data could be adjusted (after cleaning cycle slips) in mere seconds.
New tests have since been performed, and are discussed below.

OHIO STATE UNIVERSITY TESTS

The Department of Civil and Environmental Engineering and Geodetic Science at The Ohio State University have been actively testing ICON and its applicability. In order to facilitate their testing, various nominal values were chosen (such as crossover limitations, cycle slip detection and repair methods, location of POCs, etc) and various days of data provided for analysis. Details of these tests can be found in [8], [9], and [10], all of which compare and contrast ICON against a variety of other ionosphere modeling methods. The predominant conclusions of those tests were positive, but a summary highlights follows.

According to these OSU studies, ICON appears to very accurately model the shape of TECS data in double-difference mode, to within a few tenths of a TECU for most cases, except for the following issues that were found:

1) The method used to detect and repair cycle slips at NGS differed from that used to generate "truth" values at OSU, and therefore the comparison between the two occasionally yields a "step function" between "truth" and this method which can be on the order of 1-2 cm (~0.1 TECU).
2) Occasionally a track has so few crossovers that its bias is poorly constrained in the adjustment and causes the double-differenced version of the ionosphere to be in error by many TECUs (both positively and negatively, allowing or rare (<0.1%) cases where the TECS estimates turn out negative).

Considering the sheer amount of data processed, and the reliance solely upon crossovers to control this adjustment, the generally good results found by OSU is a validation of ICON with the caveats that further controls on data outliers need to be instituted, and a deeper understanding of the sensitivity of the method to mapping functions needs to be understood.

SENSITIVITY ANALYSES

Although nominal values were found to work for the initial tests of ICON, a sensitivity analysis was performed at NGS in order to quantify the stability of the method and its dependence upon such nominal values.

Two major nominal values were tested: location of the POCs and definition of how close two satellite-receiver vectors need to be to constitute a crossover.

The points of convenience, in theory, could occur anywhere on the TECS vectors, so long as a reliable mapping between two "sufficiently close" TECS values is established at those points. (In fact, the points of convenience themselves could actually be done away with, and an entirely generic mapping of one TECS vector to another could be used. Points of convenience are simply an aid to defining "sufficiently close" TECS vectors.) Ostensibly, points of convenience could change over time, though in this study, no allowances are put in ICON for the location of the points of convenience to be unknowns (variable and known, yes; truly unknown, no). Some function must be known to describe their locations.

For the sake of simplicity, points of convenience have been defined as being at a constant height in this study. During the OSU tests, a nominal value of 300 km was chosen for the heights of the points of convenience. Sensitivity tests were conducted using constant heights of 250, 300, 350, 400 and 450 km. A specific date (day 298 of 2004) was chosen as a nominal test day, and TECS estimates were formed using the 5 different definitions of the POCs. The individual values of TECS for every combination epoch/receiver/satellite were compared for the five different heights of the POCs. The maximum value and minimum value computed were differenced for all epoch/receiver/satellite combinations, and the statistics of those spreads are shown in Table 1.

Table 1: Statistics of the deviation (spread) between TECS values computed for 5 different POC locations

Number of TECSs	Ave	STD	RMS	Min	Max
8,204,539	1.98	1.85	2.71	0.00	36.05

What table 1 shows is that there is a sensitivity to POC location (at about a 2 TECU level). This sort of sensitivity will enter directly into the absolute determination of TECS values themselves. This is about an order of magnitude larger than would be desirable for a high-accuracy ionosphere model.

A similar sensitivity analysis was performed for 5 different methods of defining a crossover. Specifically what criteria were used to define if two POCs were "sufficiently close". These are shown in Table 2 (Note that crossover type #1 was the nominal choice for data sent to OSU for testing.)

Table 2: Definitions of 5 different types of crossovers

Crossover Type	Max $\Delta\phi$	Max $\Delta\lambda$	Max Δt
1	0.1°	0.1°	60 s
2	0.2°	0.2°	300 s
3	0.05°	0.05°	30 s
4	0.15°	0.15°	150 s
5	0.01°	0.01°	10 s

Using these 5 different criteria, and now holding the POC definition as fixed at 300 km height, the same process was applied. The first thing to point out is that, even with over 400 CORS stations seeing almost 30 satellites over CONUS, there were not enough crossovers for crossover type 5 to even form an adjustment. As such, while a strict crossover criteria would mean a more reliable equality of the TECR values at the POC, the fact remains that tightening that definition too much yields too little useful information. Of 22,052 total tracks in the whole CORS network for this day, the number of tracks solved by crossover adjustment are shown in Table 3.

Table 3: Number of adjusted tracks for day 298 of 2004

Crossover Type	Number (of 22,052 total) tracks adjusted
1	12,698
2	14,657

3	9,129
4	13,941
5	0

Not unexpectedly, a larger crossover definition would allow more crossovers and thus more tracks to enter the adjustment. As such, crossover type 2 has the largest number of tracks adjusted, and track 5 has the least.

So, considering only those tracks solved from crossover types 1 through 4, a sensitivity analysis was done in a similar fashion to that of the POC definition sensitivity. Table 4 shows the statistics of crossover definition sensitivity.

Table 4: Statistics of the deviation (spread) between TECS values computed for 4 different crossover definitions

Number of TECSs	Ave	STD	RMS	Min	Max
8,233,223	1.26	1.87	2.25	0.00	26.73

As can be seen, the definition of crossover is also a point upon which the final TECS values depend (less so than POC definition, but still much larger than the ultimate goal of a few tenths of a TECU).

What these sensitivities mean is clear: while the mathematics of ICON are internally consistent, their reliance upon the simple cos z'_2/cos z'_1 mapping function between TECS values can only yield absolute accuracies of TECS to one or two TECUs at best, with the potential for outliers in the tens of TECUs.

Nonetheless, there is light at the end of the tunnel. First, it must be pointed out that in differential accuracy, the studies of [3], [8], [9] and [10], all point out that this method yields results that are generally good at the sub-TECU level. But the goal of this method was not to simply find differential accuracies at that level, but actual absolute accuracies. As such, more investigation into how best to perform and constrain the crossover adjustment, over and above the assumptions of this paper must be investigated.

COMPARISON WITH IGTEC DATA

Although ICON has been shown to be mathematically stable and internally consistent, it also needs validation in comparison with other independent methods of determining the ionosphere. One such readily available data set are the IGTEC data files [11] compiled daily by the International GPS Service (IGS). Although the accuracies stated in each IGTEC file are in the 2-8 TECU range (see the header of any IGTEC file), these data should provide a basic check on the reliability of the data produced by ICON.

Data for almost a month (around Dec 2004) were compared, and the results analyzed. The comparison was done track-by-track by determining an IGTEC value of TECS (by computing the piercing location at their given shell height of 450 km, interpolating their TECR value from the IGTEC grid and dividing by the cos z' at that pierce point) and subtracting the TECS value from ICON.

The first noticeable issue was that ICON occasionally yields "outliers" – tracks where the TECS bias is estimated at an unreasonable value relative to the surrounding tracks (either being negative in value, or tens to hundreds of TECUs off of neighboring values). These outliers are almost always caused by the outlying track's reliance upon a single, questionable crossover with another track. As such, future work must be done to remove these outliers from the least squares adjustment process. After evaluating daily files, and removing the few outlying tracks, the results were tabulated. The results are shown in Table 5. The statistics of each day were computed after outlying tracks had been removed from the comparison. Note that even on the worst day (342) only 35 of 12,306 tracks were identified as outliers (0.28%).

Table 5: Statistics of the difference between ICON and IGTEC TECS values for part of 2004. A "*" indicates no IGTEC data was available for that day. A "!" indicates no solution was available for ICON for that day.

Day of Year	# Tks	Ave	STD	RMS	Min	Max	# outliers
330	*	*	*	*	*	*	*
331	*	*	*	*	*	*	*
332	*	*	*	*	*	*	*
333	11,424	-4.21	2.78	5.04	-17.4	9.4	5
334	12,214	-4.89	2.60	5.54	-16.7	8.0	8
335	13,318	-4.81	2.53	5.43	-17.1	7.7	8
336	12,601	-4.46	2.67	5.20	-15.2	8.8	22
337	11,482	-4.85	2.70	5.55	-17.1	8.6	16
338	10,439	-4.93	2.36	5.46	-15.9	6.7	10
339	11,485	-5.19	2.26	5.66	-16.4	5.8	10
340	11,913	-5.76	2.69	6.36	-18.2	7.6	17
341	12,373	-4.65	3.90	6.07	-21.9	14.8	31
342	12,306	-4.65	3.78	5.99	-22.0	14.2	35
343	12,779	-4.58	3.22	5.60	-19.5	11.4	22
344	13,376	-4.49	3.20	5.51	-19.3	11.2	10
345	13,332	-4.01	3.17	5.11	-18.5	11.8	9
346	13,145	-4.00	3.09	5.06	-18.5	10.9	14
347	13,212	-3.74	3.10	4.85	-18.5	11.4	16
348	13,528	-4.10	2.23	4.66	-14.9	7.0	17
349	14,044	-3.73	2.57	4.52	-16.1	9.1	18
350	13,952	-4.15	2.67	4.99	-16.9	9.6	25

351	13,934	-4.41	2.54	5.09	-15.1	8.2	32
352	13,960	-3.89	2.83	4.81	-16.8	10.1	27
353	12,808	-3.47	2.55	4.31	-13.5	9.1	32
354	12,242	-3.47	2.76	4.43	-14.0	10.1	20
355	!	!	!	!	!	!	!
356	13,742	-4.35	2.96	5.26	-16.5	10.2	26
357	13,257	-4.67	3.24	5.68	-20.1	11.3	10
358	13,417	-4.03	3.35	5.24	-20.0	12.7	33
359	13,357	-4.10	2.97	5.06	-17.0	10.7	23
360	13,623	-4.93	2.78	5.66	-17.5	8.9	26

A number of clear conclusions can be drawn from Table 5. First, the number of outliers, relative to the number of adjusted tracks, is quite small, indicating internal consistency of ICON. Second, it is obvious that, at least for this month of data, a consistent bias of 3-4 TECUs exists between ICON and IGTEC. The source of this bias is not clear at this time, but a brief investigation (see below) was performed to see if height of points of convenience or method of cross-over determination was the cause.

Because day 298 of year 2004 had been previously studied for sensitivity (see previous section), the results of those tests were compared to the IGTEC data. The results of those comparisons are shown in Tables 6 and 7.

Table 6: Statistics of the difference between IGTEC and ICON TECS values for various heights of the points of convenience on day 298 of year 2004.

Height of POC (km)	# Tks	Ave	STD	RMS	Min	Max	# outliers
250	12,041	-7.30	3.47	8.90	-22.8	9.3	7
300	12,698	-7.49	3.13	8.12	-20.6	8.0	19
350	12,680	-7.54	2.90	8.08	-18.3	5.8	8
400	12,905	-7.82	2.89	8.34	-22.0	5.3	7
450	13,044	-7.93	2.75	8.39	-20.6	5.4	14

Table 7: Statistics of the difference between IGTEC and ICON TECS values for various methods for selecting crossovers on day 298 of year 2004. (See Table 2 for definitions of various crossover criteria).

Crossover Criteria	# Tks	Ave	STD	RMS	Min	Max	# outliers
2	14,657	-7.59	3.12	8.20	-19.3	7.7	2
4	13,941	-7.55	3.10	8.16	-22.9	7.9	9
1	12,698	-7.49	3.13	8.12	-20.6	8.0	19
3	9,129	-7.73	3.91	8.67	-27.1	11.6	22
5	0	-	-	-	-	-	-

Looking at the results of Tables 6 and 7, one may immediately conclude that the choice of the height used in the points of convenience has no significant impact on closing the gap between IGTEC and ICON. Interestingly, choosing 450 km as the POC height (which matches the "shell height" listed in IGTEC) makes the bias 0.6 TECU *worse* than the 250 km height *but* it does reduce the standard deviation of errors around that larger bias. The conclusion is that by matching the POC height to the shell height of IGTEC the two methods more closely resemble each other in shape, while maintaining an unresolved bias between the methods.

Returning to Table 5, one can see that the standard deviations range from 2.30 TECU up to 3.78 TECU around the bias. Considering that IGTEC lists its formal error estimates around 2-8 TECU, this number is encouraging. If the source of the bias between IGTEC and this method were discovered and alleviated, this consistently small standard deviation would indicate an overall reliability of this method. In order to further understand how this method compares to IGTEC, a particular day was chosen (day 334/year 2004) and the differences in TECS values by epoch were plotted. For the purposes of completeness, outliers were left in this plot. See Figure 7.

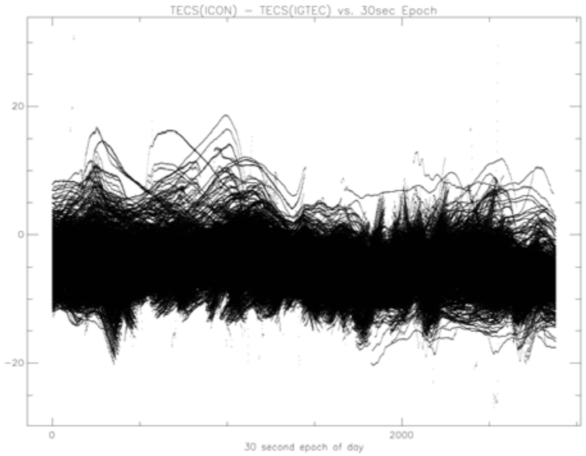

Fig. 7. 7,261,965 differences between TECS (ICON) and TECS (IGTEC) on day 334, year 2004, plotted by 30 second epoch number (1 to 2881)

Recall from Table 5 that the differences for this day average -4.75 +/- 2.89 TECU, which is clear from Figure 7. There are a few other points of interest in Figure 7. First, it's clear that there is a slight downward trend in the data throughout the day, but nothing that could be considered a significant time dependent error. Second, the spread of data, while having a standard deviation of 2.89 TECU shows plenty of differences (visually) from around -11 to +6 TECU. Theoretically, the cause for this spread can be traced to the difference in resolution of the models, in addition to errors in the models themselves (including POC choice and crossover definition sensitivities; see earlier). Specifically, for any given track of data from ICON, the entire shape of the TECS is retained at very fine resolution, as opposed to interpolating off of the IGTEC grid which is at a grid spacing of 2.5º x 5º x 2 hours (and therefore very coarse relative to the amount of information contained in all the tracks of data.)

To further validate ICON, and compare with the spread of data seen in Figure 7, one other data set was checked against this method, albeit briefly.

COMPARISON WITH MAGIC DATA

NOAA's Space Environment Center (SEC) in collaboration with NGS, produced a method for modeling the ionosphere over CONUS, using CORS phase and pseudo-range data, discrete radial empirical orthonormal functions and a Kalman filter under the model name "MAGIC". Both MAGIC and ICON went into prototype daily productions on November 1, 2004 and are distributed via the NGS web site at (www.ngs.noaa.gov/ionosphere). However, most MAGIC data is currently unavailable so a detailed comparison between this method and the MAGIC method was not possible; however, for the day 334 of year 2004 (exemplified in Figure 7), data from both methods were made available and they were compared to one another. The results are shown in Figure 8. Again, for purposes of exemplifying the outliers (3 of 4,778 tracks), they are left in this plot.

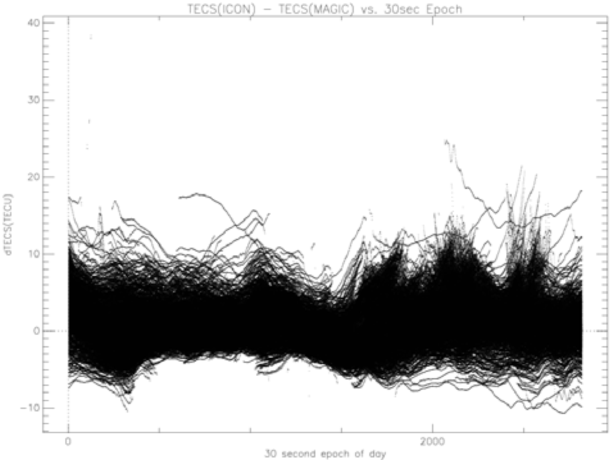

Fig. 8. 2,716,181 differences between TECS (ICON) and TECS (MAGIC) on day 334, year 2004, plotted by 30 second epoch number (1 to 2881)

Because MAGIC only uses a sub-set of all available CORS data, there are significantly less points in Figure 8 than Figure 7. Statistically, for this day, the differences average +1.00 +/- 2.91 TECU (after outlier removal). Note that the standard deviation about the average is about the same as that when compared to IGTEC data, while the bias has switched to +1.00. Because MAGIC was set up specifically to model the ionosphere over CONUS at CORS stations (like ICON), it is gratifying to see a small bias in this comparison. However, similar to IGTEC, the MAGIC model is not specifically built around the retention of the exact shape of TECS curves the way ICON is. As such, differences in final resolution of track data as well as model errors lead to the ~3 TECU standard deviation of the differences. However, MAGIC's method prevents the sort of outliers seen in this method by constraining the ionosphere through use of the orthonormal functions and fitting to the International Reference Ionosphere, making it likely that some combination of MAGIC and ICON will be tested at NGS in future research.

DATA PRODUCTION

While the daily production of ICON yields TECS values at all CORS stations over a day, these values are difficult to interpret in a network sense without a visual aid. As such, animated GIFs are produced daily as part of ICON's production. These GIFs have been found invaluable for identifying outliers on tracks, as well as observing the differences between calm and stormy ionosphere days.

For example, Figures 9 through 12 show 4 different views of the vertical TECR values (implied by taking ICON's TECS values and mapping them to TECR at 300 km) on a grid for day 311 of 2004 at GPS times 0h, 6h, 12h and 18h (being local times over CONUS 4-7pm, 10pm-1am, 4-7am and 10am-1pm).

Two things should be noted from Figures 9-12. First, one can occasionally see an outlying track causing a local "bump" in the TECR values (such as in the northwest of Figure 9). But also, this particular day is fairly "normal" in that most maps of the TECR look about like these figures. However, on the very next day (312, Nov 8), an ionosphere storm began (and continued to affect the ionosphere for about 4 days). Plotting hours 0, 6, 12 and 18 for day 312 in Figures 13-16, it is interesting how similar they are during the night, but in mid day (Figure 16), the effect of the ionosphere storm is highly pronounced.

On the positive side, these results show that ionosphere storms can be detected through the ICON method. Unfortunately, the intensity of this particular storm was so high that the accuracy of the mapping functions used in ICON failed in their day-to-day accuracy, and no successful least squares solution could be solved for the days of highest intensity (Nov 10-12). This further points out the need to investigate more accurate mapping functions for ICON to remain stable in all conditions.

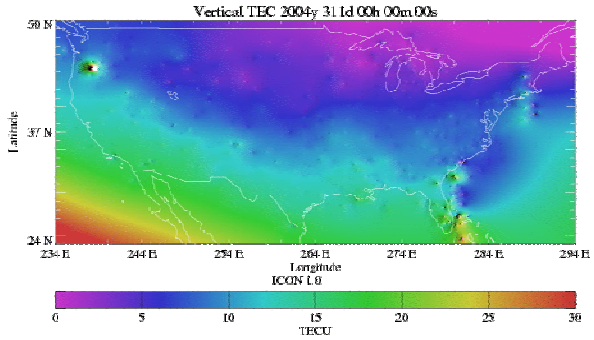

Figure 9: ICON based TECR map for GPS time 0h on day 311 (Nov 7) 2004. (Local EST: Nov 6, 7:00 p.m.)

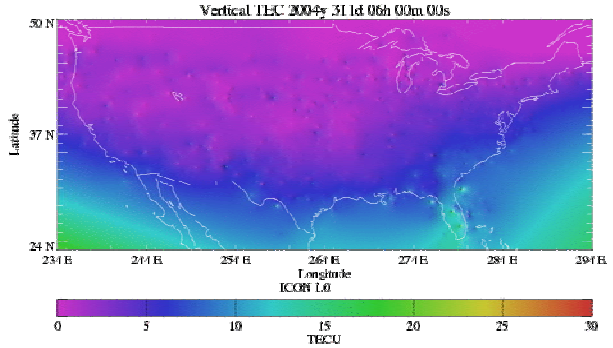

Figure 10: ICON based TECR map for GPS time 6h on day 311 (Nov 7) 2004. (Local EST: Nov 7, 1:00 a.m.)

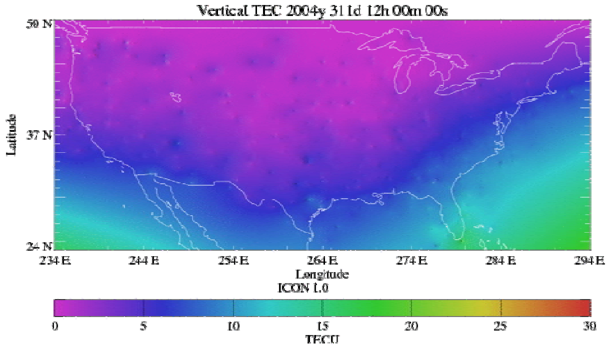

Figure 11: ICON based TECR map for GPS time 12h on day 311 (Nov 7) 2004. (Local EST: Nov 7, 7:00 a.m.)

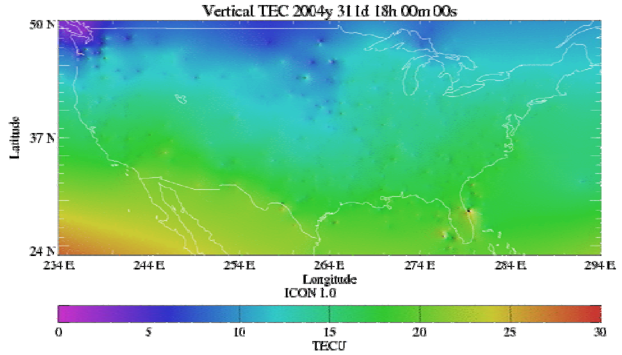

Figure 12: ICON based TECR map for GPS time 18h on day 311 (Nov 7) 2004. (Local EST: Nov 7, 1:00 p.m.)

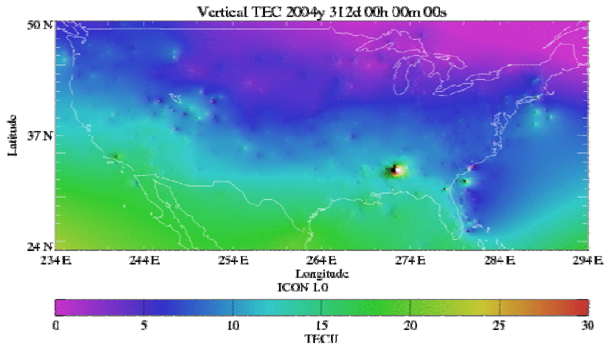

Figure 13: ICON based TECR map for GPS time 0h on day 312 (Nov 8) 2004. (Local EST: Nov 7, 7:00 p.m.)

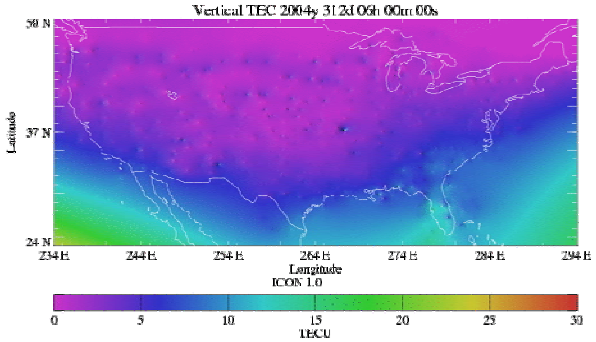

Figure 14: ICON based TECR map for GPS time 6h on day 312 (Nov 8) 2004. (Local EST: Nov 8, 1:00 a.m.)

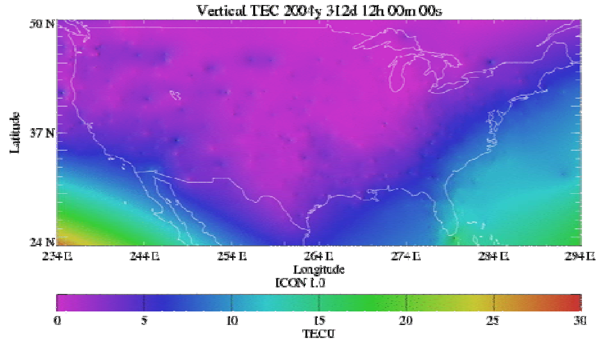

Figure 15: ICON based TECR map for GPS time 12h on day 312 (Nov 8) 2004. (Local EST: Nov 8, 7:00 a.m.)

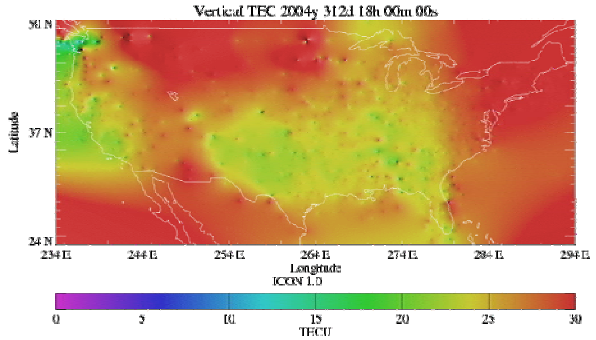

Figure 16: ICON based TECR map for GPS time 18h on day 312 (Nov 8) 2004. (Local EST: Nov 8, 1:00 p.m.)

CONCLUSIONS AND FUTURE WORK

A new method for computing absolute TEC values along GPS receiver-satellite vectors using only ambiguous dual-frequency carrier phase data has been presented. This method, dubbed "ICON", relies upon the idea that two different skew views of the ionosphere (TECS) that are near one another (a crossover) can both be successfully mapped into one another with reliability and a network of such crossovers provides a unique solution in absolute TECS space. Significant study has been done on ICON, showing that in double-difference applications, this method is reliable to sub-TECU levels. But ICON's strong reliance upon the simple cos z'_2/cos z'_1 mapping function means that even minor, relatively small changes to certain nominal values, such as where and how crossovers are defined, introduces uncertainty of the absolute TECS values at the +/- 2 TECU level. This is about one order of magnitude greater than the target for using this method to get instantaneous ambiguity resolution.

Comparisons between ICON and other data (IGTEC and MAGIC) show that ICON is able to match those data at the level +/- 3 TECUs off of a bias, but the errors in all three methods don't allow for a strong conclusion about which is nearest to the true absolute TECS values.

Thus ICON is seen as a reasonable first step toward modeling the absolute ionosphere, though recognizing that it remains in its infancy with much work yet to be done. To encourage further testing, this method is being applied in a prototype production mode on a daily basis and distributed via the NGS web site at (www.ngs.noaa.gov/ionosphere). The data is being produced and released for free to the public, while research into improving the method is ongoing.

Future work on this model will include ways to make the least squares adjustment more robust and less sensitive to the choices made for nominal values. These methods include introducing more physically realistic constraints on the mapping of TECS values into one another at crossovers, identification and removal of outlying tracks, and possibly introducing constraints on the overall network. Additionally, direct, in-situ measurements of the ionosphere need to be obtained and compared directly to ICON to help quantify its ability to accurately model true absolute ionosphere delays.

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