GPS Precision with Carrier Phase Observations:
Does Distance and/or Time Matter?

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The Global Positioning System (GPS) has dramatically changed the way that surveyors, GIS/LIS professionals, engineers, and others measure positional coordinates. These practitioners can now determine the 3D coordinates of a new point with centimeter-level accuracy relative to a control point located several hundred kilometers away. That control point, moreover, may already be associated with a GPS receiver that is being continuously operated by some institution for any of several diverse applications. The National and Cooperative CORS (Continuously Operating Reference Station) network comprises such a set of 'active' control points (See: *www.ngs.noaa.gov/CORS/*). To address the practicality of using either a CORS or a 'passive' control point—such as those comprising the Federal Base Network (FBN)—for providing accurate positioning control, we studied how the precision of an observed 3D relative position between two GPS antennas depends on the distance between these antennas and on the duration of the observing session.

For our experiment, we processed 10 days of dual-frequency, carrier phase observations for each of 11 baselines formed by pairs of sites in the National CORS network. These 11 baselines range in length from 26 km to 300 km, and are widely distributed throughout the coterminous United States (**Figure 1**). The data for each baseline comprised 10 non-overlapping 24-hr sessions that were further subdivided into 20 non-overlapping 12-hr sessions, 30 non-overlapping 8-hr sessions, 40 non-overlapping 6-hr sessions and, finally, 60 non-overlapping 4-hr sessions. Moreover, the data for each baseline and each session was processed independently from the data of other baselines and other sessions.

In addition to the length of the baseline and the duration of the observing session, positioning precision will depend on several other factors, including the methodology and the software used for processing GPS data. Here, we used the "static-mode, ionosphere-free, double-difference-carrier-phase" option as encoded in the PAGES (Program for the Adjustment of GPS Ephemeris) software developed by the National Geodetic Survey (NGS). Also, we used the 'final' precise satellite orbits (ephemerides) disseminated by the International GPS Service (IGS), and we fixed phase ambiguities to integer values whenever we were sufficiently confident in these values.

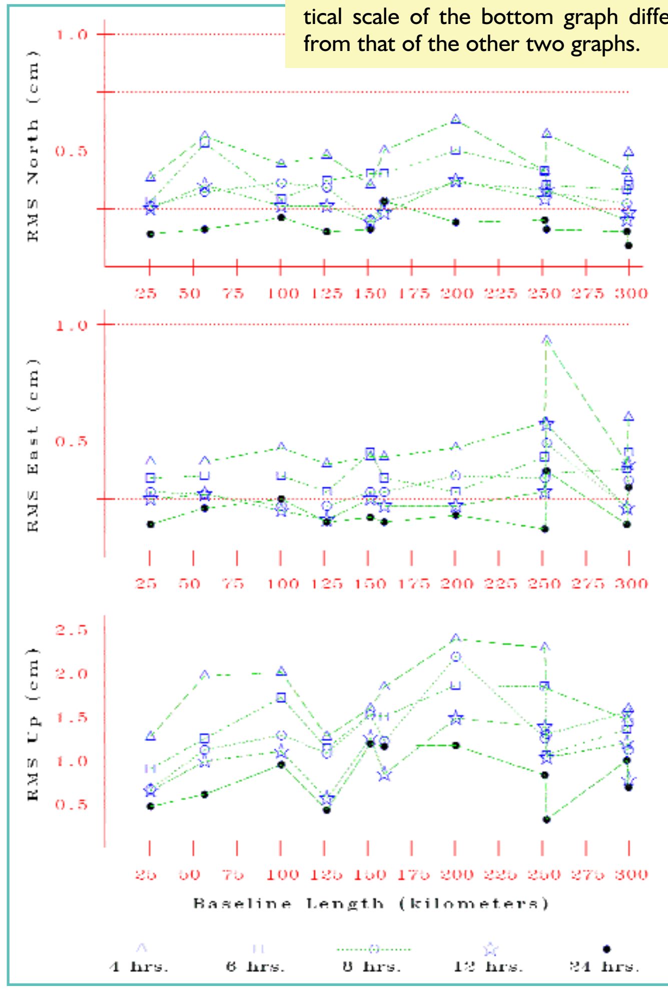


Figure 2: RMS values for each baseline and each value of T. Note that the vertical scale of the bottom graph differs from that of the other two graphs.

Figure 1: Baselines of the National CORS network involved in this study. Baseline lengths are given in km.

We treated one station of each baseline as the control point, denoted C, and fixed its 3D positional coordinates to the official values adopted by NGS. We then computed corresponding coordinates for the other (unknown) point, denoted P. For the 'true' position of P, we adopted the average of the ten non-overlapping 24-hr solutions. Then for each baseline, we determined the differences—in the north, east, and up dimensions—between the 'true' position of P and the computed position of P for each observing session. We then computed the RMS (root mean square) value for the collection of positional differences for each baseline, each duration, and each positional component. Recall that the

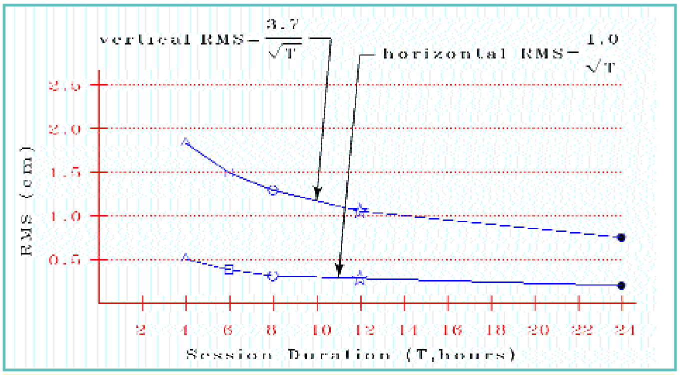


Figure 3: Predicted RMS values for the vertical and horizontal components of relative position as a function of session duration.

RMS value measures the scatter among a set of numbers. The computed RMS values are plotted in **Figure 2** as a function of baseline length. In this figure, the graph for each of the three dimensions—north, east, and up—includes five paths, with each path connecting all RMS values corresponding to a common time duration. Each path visually indicates that the corresponding RMS values do not grow significantly as baseline length increases. Indeed, we conducted rigorous statistical tests that corroborate this visual interpretation. These statistical tests do not negate the possibility that RMS values grow as baseline length increases, they simply say that any such growth is statistically negligible.

We attribute this negligible growth to the fact that we used IGS precise orbits, as opposed to the GPS broadcast orbits, for processing our data. IGS orbits provide satellite positions that have a 1-sigma uncertainty smaller than 0.1 m, whereas broadcast orbits provide satellite positions that have a 1-sigma uncertainty larger than 2 m. The fact that relative positioning error is essentially independent of baseline length also indicates that the meteorological effects on the accuracy associated with relative GPS positioning are statistically the same for baseline lengths ranging between 26 and 300 km. This is probably not the case for baselines shorter than 26 km. That is, as baselines become much shorter than 26 km, the relative meteorological effects should approach zero and the corresponding RMS values should be smaller than those presented in Figure 2.

While distance doesn't matter under the conditions of our experiment, the results in Figure 2 indicate that the duration of the observing session does. The RMS values for a longer observing session are generally smaller than those for a shorter observing session. To quantify this pattern, we averaged the results for the 11 baselines to compute a single RMS value for each of the five considered durations and for each of the three components of relative position. These averages should be more reliable than the results for the individual baselines if, indeed, baseline length does not matter. **Figure 3** displays how these averaged RMS values decrease as the duration of the observing session increases. We empirically fit these averaged RMS values (when expressed in cm) to an equation of the form

$$RMS = \frac{k}{\sqrt{T}} \quad \begin{cases} k = 1.0; & \text{horizontal (north \& east)} \\ k = 3.7; & \text{vertical (up)} \end{cases}$$

where T denotes the duration expressed in hours and k is a free parameter in units of cm$\sqrt{\ }$hour. The usefulness of the above equation rests in its ability to predict RMS values for other possible durations in the range between 4 hours and 24 hours. The curves in Figure 3 illustrate such predicted RMS values. **Figure 4** illustrates, in a different manner, the effect of increasing the duration of the observing session by comparing the scatter in the computed horizontal positions for all 6-hr sessions with that for all 24-hr sessions. In accordance with our equation, the scatter (or RMS) is reduced by a factor of about two by increasing the duration by a factor of four.

The results of our study, based on sessions from 4 to 24 hours, indicate that highly accurate positional coordinates can be obtained using CORS as control points even though the distance to these sites may be a few hundred kilometers. One drawback, however, is the need for rather long observing sessions to realize

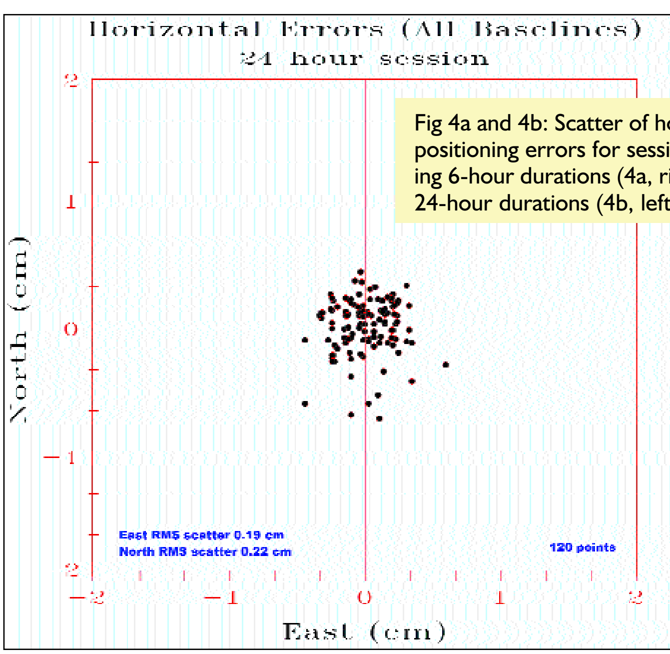
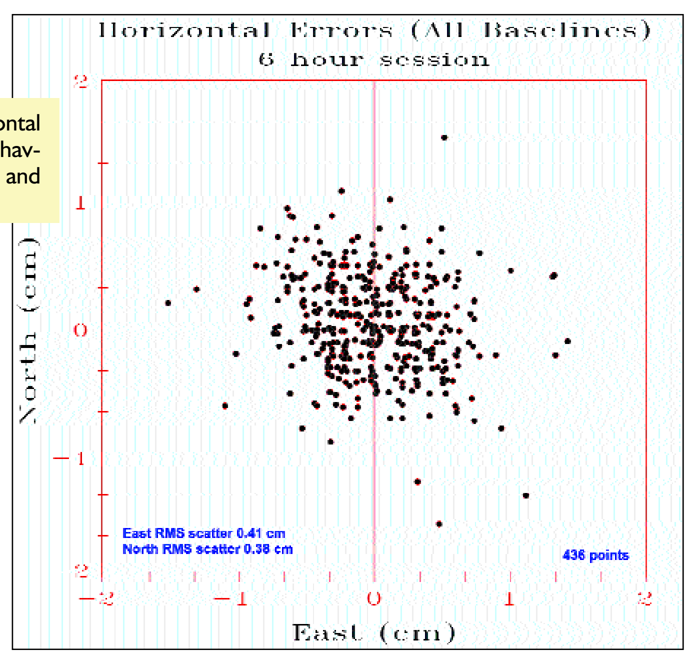


Fig 4a and 4b: Scatter of horizontal positioning errors for sessions having 6-hour durations (4a, right) and 24-hour durations (4b, left).



these accuracies. It is costly to remain at a single site for several hours, and this cost may be prohibitive if several tens of sites need to be positioned. A remedy is available if two or more GPS receivers are on hand and if the points to be positioned are clustered within a few kilometers of a site where one of these receivers can be placed as a local base ('hub') station. This scenario would allow the remaining GPS receivers to accurately position the various new points relative to the hub station by spending a relatively short time at each new point. Meanwhile, the GPS data being collected at the hub station can also be used to accurately position this base station relative to one or more existing CORS. Consequently, all visited points can be effectively positioned relative to the CORS network. When excessive spacing occurs among the proposed survey points, then users should follow the guidelines contained in the Technical Memorandum, NGS TM 58.

In this article, we have considered only that precision associated with measuring the relative position between two GPS antennas. The total error involved in positioning a new point also depends on:

- the accuracy of the vertical offset measurements that relate each point's position to the position of the GPS antenna placed above it, and
- the accuracy of the positional coordinates of the control point.

The use of fixed-height poles will help to obtain reliable offset measurements. To help mitigate errors associated with control point coordinates, we recommend that the new point be positioned in a statistical manner relative to two or more control points. For example, perform a network adjustment involving each baseline connecting the new point to a distinct control point, and (in this adjustment) constrain the positional coordinates for these control points to their adopted values. This strategy has been encoded into the OPUS (Online Positioning User Service) software that allows a user to submit GPS data for a point to NGS via the Internet. This data set will then be automatically processed on an NGS computer using the PAGES software. In this process, OPUS computes three separate estimates of the new point's coordinates by using GPS data from each of three suitable CORS. OPUS then averages these three positional estimates and emails the resulting positional coordinates (with appropriate documentation) to the user-specified address. Additional information about OPUS can be found at *www.ngs.noaa.gov/OPUS/*. Additional information about our studies can be found in the December, 2001, issue of the *Journal of Geodesy*.

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