

An Experimental Comparison of Flexible and Tiered Package Bidding

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Federal Communications Commission

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Executive Summary

Laboratory experiments were run to evaluate a modified form of flexible package bidding (MPB) and bidding on hierarchically structured packages, or “tiered package bidding” (TPB). The simultaneous, multi-round auction (SMR) without package bidding was included to provide a benchmark. The auction procedures were implemented with the *jAuctions* program developed by Jacob Goeree at Caltech. Each laboratory session consisted of a series of auctions in which participants were assigned “national” or “regional” bidder roles. Bidders were provided with randomly generated license values and information about the extent to which combinations of licenses were worth more than the sum of the individual license values (complementarities).

This report is based on a series of auctions conducted in a laboratory setting with financially motivated bidders who had gained experience in similar experiments. The environment was such that there were significant synergies associated with acquiring adjacent licenses, but in which pre-specified regional packages under TPB did not match the preferred packages for half of the regional bidders. Both package formats, MPB and TPB, performed better than SMR in terms of efficiency and revenue, which reinforces the findings of previous studies (e.g. Goeree, Holt, and Ledyard, 2006) regarding the advantage of package bidding in terms of solving the “exposure problem” that arises from value complementarities. Furthermore, SMR required more rounds of bidding and resulted in more unsold licenses than the package bidding formats.

The main results of this study are that TPB outperforms MPB both in terms of economic efficiency and auction revenue. In addition, there were fewer rounds of bidding and fewer unsold licenses with TPB. These differences are statistically significant and substantial from an economic point of view, given the potential scale of upcoming FCC auctions.

We had initially expected full flexibility to confer a performance advantage, and hence, the better performance of package bidding with constrained packages that fit into a non-overlapping hierarchical structure came as a surprise. This result reflects the simplicity and transparency of the package definitions and pricing under TPB, which helps regional bidders to coordinate and more effectively compete with the national bidder, as evidenced by the fact that regional bidders won more licenses under TPB as compared with the more flexible MPB. The difficulty of coordinating with flexible package bids is further reflected by the fact that MPB auctions required 20% more rounds of bidding than was the case under TPB.

I. Introduction

This project involved running and analyzing laboratory experiments designed to evaluate alternative methods of simultaneously auctioning off multiple spectrum licenses. The auctions were run in a laboratory with networked PCs, using Caltech subjects who were financially motivated to bid carefully and who had gained experience with the environment in earlier experiments. The contract work consisted of 1) software development, 2) refinement of laboratory procedures and experiment design using pilot experiments, and 3) five waves of laboratory sessions for each of the auction formats (SMR, MPB, TPB, and TPB with rotated regional bidder interests).

The experiments were conducted using *jAuctions*, which has been developed at Caltech by Jacob Goeree. The *jAuctions* software consists of a flexible suite of Java-based auction programs designed to handle a wide range of auction formats and bidding environments, including combinatorial auctions with bid-driven or clock-driven prices, private and common valuations, alternative pricing rules, etc.

II. Experimental Design

The design that we came up with and cleared with FCC staff involves 7 bidders and 18 licenses. There are two types of bidders: small or “regional” bidders (labeled 1 through 6) and one “national” bidder (labeled 7). A graphical representation of bidders’ interests is shown in Figure 1.

Bidders’ Interests:

The large circle (licenses A through L) on the left contains licenses of interest to the national and regional bidders. For example, license A is of interest to regional bidders 1 and 6 and to the national bidder 7. Note that each regional bidder has an interest in 4 adjacent licenses, with partial overlap in these interests. The smaller circle on the right (licenses M through R) contains licenses only of interest to regional bidders, e.g. license P is of interest to regional bidders 3 and 4. Activity and purchase limits were such that regional bidders can acquire at most four licenses, and the national bidder can acquire up to 12 licenses on the larger circle. One useful feature of the smaller circle was to reduce earnings inequities among experimental subjects in cases where the national bidder managed to win a national license or a large share of licenses, which was expected to happen more frequently with package bidding.

Value Distributions:

For the national bidder, the baseline draw distributions are uniform on the range $[0, 10]$ for licenses A-D and I-L and uniform on the range $[0, 20]$ for licenses E-H. For regional bidders, the baseline draw distributions are uniform on the range $[0, 20]$ for licenses A-D and I-L and uniform on the range $[0, 40]$ for licenses E-H. Finally, for licenses M-R the baseline draw distributions are uniform on the range $[0, 20]$. In other

words, the E-H region of the national circle is, on average, worth more to the national and regional bidders. This asymmetry allows us to measure the impact of “pre-packaging” on regional bidders of different strengths. In particular, the regional bidder with an interest in the high-value licenses E-H would often be a strong competitor with the national bidder, and a threshold problem could arise if the other regional bidders drop out of the bidding without coordinating a strong response to an aggressive national bid.

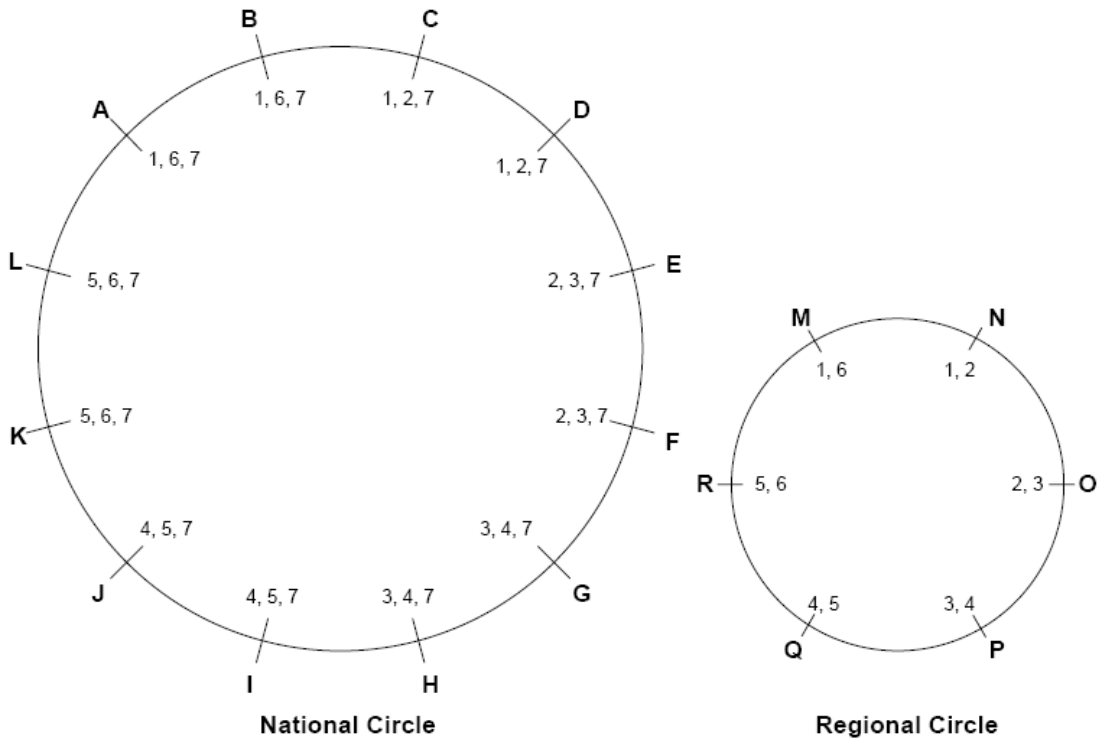


Figure 1. Bidders' Interests

Regional bidders 1-6 are interested in 4 licenses from the national circle and 2 licenses from the regional circle. National bidder 7 is interested in all 12 licenses from the national circle.

Complementarities:

For both the national and regional bidders, each license acquired goes up in value by 20% (with two licenses), by 40% (with three licenses), by 60% (with four licenses), etc., and by 220% if the national bidder wins all twelve licenses A-L. In waves 1-4 (explained below) complementarities occur among all licenses while in wave 5 complementarities occur only among licenses from the national circle. For example, if bidder 1 wins the combination ABM, then the value synergies would apply to licenses A and B and M in waves 1-4 and only to A and B in wave 5. The national bidder can acquire up to twelve licenses and has value complementarities for all licenses in all five waves. These numbers were chosen such that all bidders (1-7) would be included in the

optimal allocation in some auctions; see Appendix E for a list of optimal allocations by auction.

Packages:

Bidders cannot bid on packages under SMR, and they have full flexibility in bidding on packages under MPB, subject to activity constraints. Under TPB, the admissible packages have a hierarchical structure, with a single national package ABCDEFGHIJKL and three non-overlapping regional packages: ABCD, EFGH, and IJKL in our TPB-odd design and ABKL, CDEF, and GHIJ in our TPB-even design. In other words, in TPB-odd the odd numbered regional bidders can bid on their preferred packages but not the even numbered bidders, while in TPB-even the reverse is true.¹

In all three formats, bidders were able to create “custom” packages in order to see the values associated with winning combinations of licenses. Under MPB, bidders could bid on these self-created packages. In contrast, under SMR, these custom packages would be shown but could not be placed into the bidding basket. Finally, under TPB, bidders could submit bids for pre-defined packages but not for custom packages. Figure 2 shows a screen shot of the bidder interface in a TPB-odd auction, where the pre-defined package ABCD shown in white in the information table (on the left) has been placed in the bidding basket (on the right). The custom created package BCDM is shown in gray to indicate that it could not be added to the bidding basket, but the bidder is able to observe its value in case this combination of licenses is won on the basis of bidding on individual licenses and its current price.² The lower-right corner of the bidder interface shows the history of all bids, with provisionally winning bids shown in light green.

Sessions:

We conducted a total of 20 sessions. Five sessions with the SMR and MPB formats and ten sessions with the TPB format: five sessions with TPB-odd (in which the odd numbered bidders can bid on the package they prefer) and TPB-even (in which the even numbered bidders can bid on the package they prefer). Each session consisted of three practice auctions and a series of 6 auctions for cash payments. The auction type was unchanged for all auctions in each session, but the randomly generated value draws changed from one auction to the next in the same session. We used different sets of value draws for each of the five sessions with the same institution (five “waves”), but the same set of draws across the different auction institutions (SMR, MPB, TPB-odd, and TPB-even). Bidder roles were assigned randomly in order to attenuate earnings differences across national and regional bidders and to help bidders understand the strategic considerations faced by both types of bidders. Average earnings were \$36 for the experiment, which lasted about an hour and a half.

¹ With this choice of pre-packaging we are able to create environments in which the package preferences of 50% of the regional bidders are not coincident with the pre-established packages. In addition, these bidders have valuations such that at least some of them are included in the efficient assignment.

² Note that the value for the BCDM package (62) is equal to the sum of the individual license values (15+11+8+5) plus an additional 60% that reflects synergies.

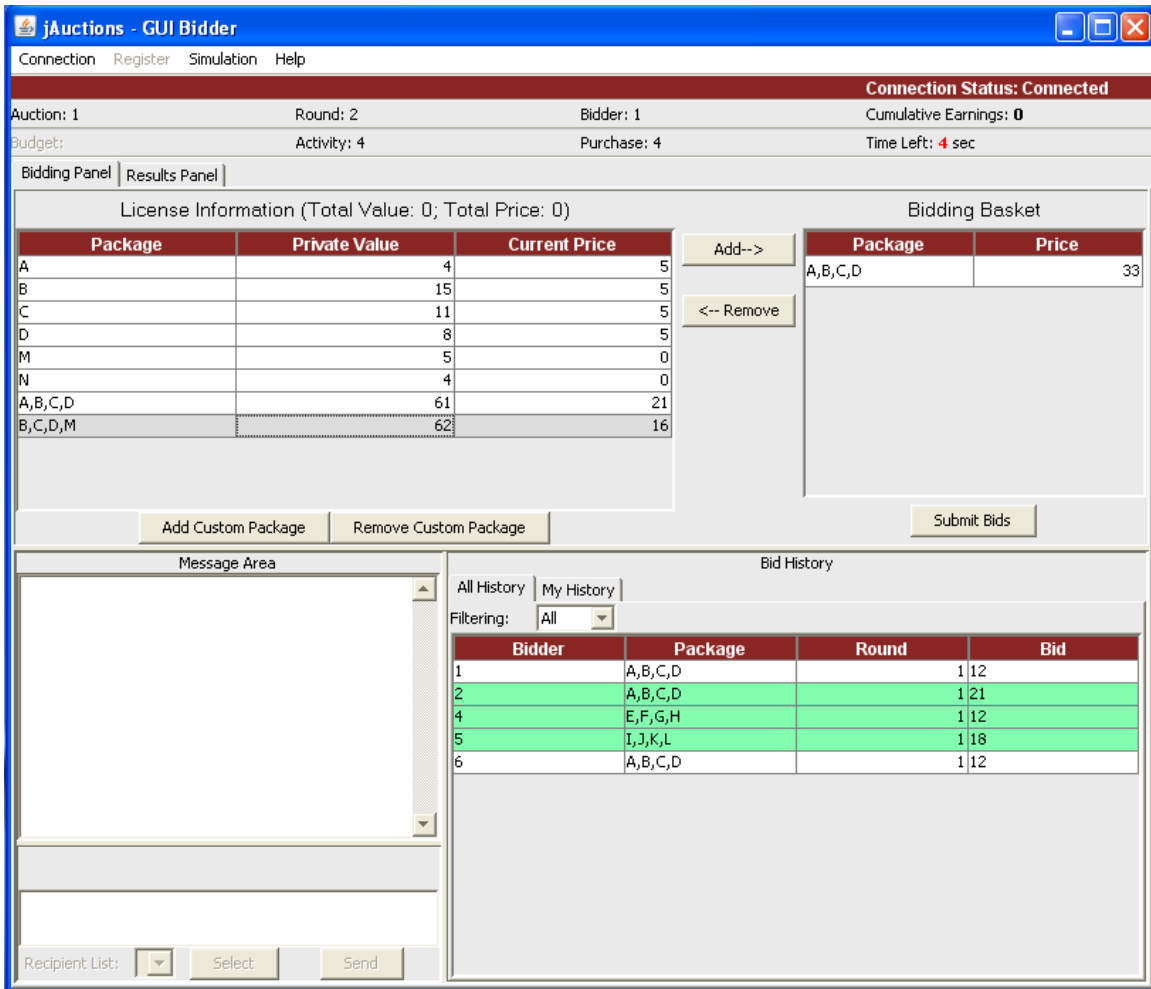


Figure 2. Screen Shot of Bidder Interface (TPB-odd)

License Information: Packages in white (gray) are predefined (custom made) and can (cannot) be bid on
 Bid History: Licenses/packages shown in light-green are provisionally winning
 Bidding Basket: Licenses and predefined packages can be included

III. Auction Formats

The three auction formats are described in detail in Appendices A, B, and C. The main differences are:

- **Provisionally Winning Bids:** With SMR, the highest bids submitted for each license in a round become the provisionally winning bids, whereas with MPB and TPB the provisionally winning bids for licenses or packages follow from a constrained revenue-maximization procedure. In general, under MPB the revenue maximization problem is “NP hard” to do: the number of possible ways to

allocate grows exponentially and with many objects for sale there is no guarantee the best allocation is found in a reasonable amount of time. This is commonly referred to as the computational complexity problem.³ Under TPB, the revenue maximization process is clearer to bidders: first calculate a regional winner as the highest bid on that regional package or the sum of the highest bids on the licenses in that region, whichever is greater. Then compare the sum of the regional winners with the highest bid on the national package to determine the revenue maximizing allocation, see Rothkopf, Pekeč, and Harstad (1998) and Appendix B for more details.

- **Prices:** Under SMR, prices are simply equal to the highest bids for the licenses. Under MPB, the prices for licenses are set so that the losing bids on licenses (or packages) are less than or equal to the corresponding license prices (or sum of prices for individual licenses in that package) and the winning bids are equal to the corresponding prices. Although the intuition behind these “allocation constraints” is clear, the presence of complementarities in license values may preclude the existence of such prices. In this case, prices are approximated in a manner that minimizes some measure of the extent to which the constraints are not satisfied. In addition, there may be multiple solutions to the constraints and a second constrained optimization problem is run to resolve the indeterminacy, e.g. maximizing the minimum price or minimizing the sum of squared deviations from previous round prices.⁴ Prices under TPB are computed in a simple recursive manner. If a bid on an individual license is provisionally winning then the price for that license is simply equal to this bid. If a regional package bid is provisionally winning, then the individual licenses each receive their “share” of the shortfall so that the license prices in that region sum to the winning package bid. With multiple hierarchy levels or “tiers,” this same process is repeated at the next level up. For example, if a bid on the national package is provisionally winning then the difference between that bid and the sum of the bids for the regional winners determines the shortfall allocated to regions. These amounts are in turn distributed across individual licenses so that license prices end up being scaled up by “regional shares” and “national shares.” The result is that license and package prices convey to bidders how high the bids must be to unseat the provisionally winning bids at any level, see Appendix B for details.
- **Bid Increments:** New bids at the start of a round must exceed provisionally winning bids under SMR by at least one bid increment, whereas new bids under the two package bidding formats must exceed the price of a license or sum of prices for licenses in a package by at least one bid increment for each license in

³ In practice, there may be strategies to manage computational issues such as limiting the time the solver runs to approximate the optimal allocation. Even if the allocation problem is solved in a reasonable amount of time, it seems realistic to assume that between rounds bidders may not be able to reproduce the outcome to understand why their bids did (not) win.

⁴ The constrained optimization procedures underlying MPB need to be adapted to correct for the number of bidding units since otherwise small bidders may be pushed out of the auction after an aggressive package bid, see Appendix C for details.

the package. Under MPB, after a round in which new bids were submitted but revenue did not increase, the bid increment needs to be raised to avoid cycling.⁵

- **Activity:** In general, activity is specified in terms of the number of bidding units that bidders are allowed to bid on. For simplicity, in the experiment, each license counts as one bidding unit. Therefore, for all three formats, activity is computed as the number of different licenses for which a bidder is a provisional winner or for which a bid is submitted, either individually or as part of a package (under MPB and TPB). That is, in the activity calculation, a given license is counted only once when a bidder places a bid on the license individually and bids on a package containing the license. If a bidder's activity falls, it cannot rise in subsequent rounds.⁶
- **Withdrawals:** Since package bidding protects bidders from the exposure problem, withdrawals were not permitted under MPB and TPB. Under SMR bidders have limited opportunities to withdraw (in at most two rounds of an auction), subject to penalties that compensate the seller for lower prices obtained.⁷
- **Considered Bids:** Under SMR, new and current provisionally winning bids are considered to determine provisional winners for the next round. Under MPB and TPB all bids received are considered (to prevent cycles).⁸

⁵ Consider the following scenario: there is a winning package bid of 30 on ABC and three other bids of 29 on AB, AC, and BC. The computed prices for A, B, and C are 10 each. Assuming a bid increment of 3, the minimum acceptable bid for a two-license package in the next round is the sum of two prices plus two bid increments: 26. So the bidders could resubmit their bids of 29, and minimum acceptable bids would again be 26 for each package etc. This cycling behavior can occur whenever a package bid is winning and bidders are bidding on sub-packages. One solution is to increase the bid increment (from 3 to 6 to 9 etc.).

⁶ In the experiment, each license had one bid unit, so the allocation of shortfalls was done by equal division, but in an actual auction these allocations should be done relative to bid units determined by license size (MHz-pop). We do not think the allocations should be done relative to submitted bid amounts, since that would create incentives to keep bids low.

⁷ The exposure problem can be alleviated to some extent by the (limited) bid withdrawal provisions built into the SMR bidding rules under consideration. In this manner a bidder may compete aggressively for a package and then decide to withdraw. In the experiments, bidders who withdraw a winning bid pay a penalty equal to the difference between the withdrawn bid and the final sale price if the final sale price is lower and they pay a penalty equal to 25% of the difference between the withdrawn bid and the final sales price if the license goes unsold. These rules are similar to, but not exactly the same as the FCC rules, which may depend upon sales in future auctions to determine withdrawal payments.

⁸ Consider MPB and suppose there is a package bid of 30 on ABC and three regional bidders submit bids of 2 on A, 12 on B, and 2 on C respectively. The computed prices are 9, 12, and 9. Now suppose in the next round all three bidders bid 12 on A (assuming a bid increment of 3). If only new bids and current winning bids are taken into account, computed prices are now 12, 9, and 9. Next all three regional bidders bid 12 on B. Prices are now 9, 12, and 9. Note that bidders can play 'musical chairs' under the umbrella given by the winning package bid of 30. (For TPB an analogous example can be provided.) The solution is to consider current and past bids. Of course, retaining old bids implies they may become winning at a later stage. To maintain comparability with SMR, where regional bidders could win at most 4 licenses due to activity constraints, we imposed purchase limits of 4 for regional bidders, which were used in the revenue maximization routines in MPB and TPB. These limits are absent when eligibility is determined by the number of licenses bidder register for prior to the start of the auction, in which case revenue maximization follows from the simple recursive structure described above.

IV. Experimental Results: Efficiency, Revenue, and Bidders' Profits

Efficiency

Market efficiency is defined as the ratio of the actual surplus S_{actual} (the value of the allocation obtained in the auction) and the maximum possible surplus $S_{optimal}$ (the value of the best possible allocation):

$$efficiency = \frac{S_{actual}}{S_{optimal}} * 100\%$$

The maximum possible surplus and the actual surplus across auctions formats are shown on the right-side of the data tables in Appendix E. There are five tables corresponding to the five value waves and each table reports data from six auctions. For each auction, the optimal, or surplus-maximizing, allocation is shown together with the observed allocations for the four treatments.

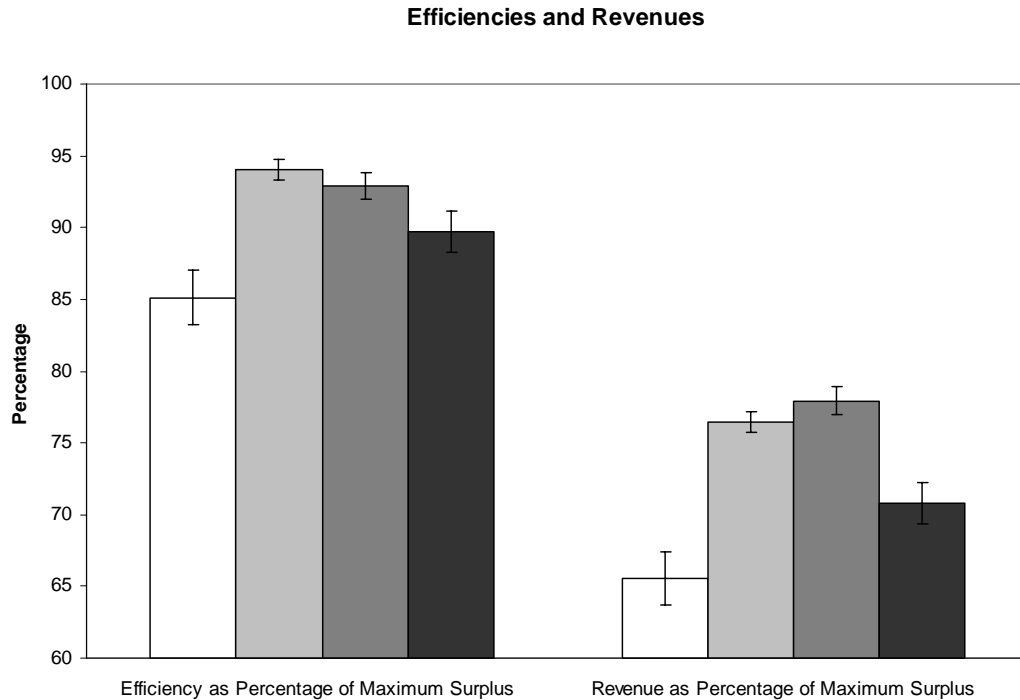


Figure 3. Efficiencies and Revenues by Auction Format
 Key: SMR (white), TPB-odd (light gray), TPB-even (dark gray), MPB (black)

Package bidding is designed to help bidders avoid the “exposure problem” of bidding high for licenses with high complementarities. As expected, switching from SMR to the package auction formats raises efficiency from 85.1% to 89.7% for MPB and to 92.9% and 94.0% for TPB-even and TPB-odd respectively. The left part of Figure 3 shows average efficiencies across formats: the white bar corresponds to SMR, the light-

gray bar to TPB-odd, the dark-gray bar to TPB-even, and the black bar to MPB. The standard deviations are indicated by the bracketed intervals at the top of each bar. Notice that efficiencies are more variable under SMR and MPB than under TPB.

The performance differences suggested by Figure 3 are supported by a statistical analysis. As illustrated by Figure 3, the two TPB environments yield very similar results (in terms of efficiency and revenue) and, hence, we will assume that the observations from these two environments are identically distributed. Below \succ indicates a pair-wise ordering that is not significant, \succ^* indicates significance at the 10% level, \succ^{**} indicates significance at the 5% level, and \succ^{***} indicates significance at the 1% level.

Finding 1: *Efficiencies are ranked $TPB \succ^{**} MPB \succ SMR$.*

Support: See Appendix D for an overview of session averages across auction formats. There are five averages for SMR and MPB, corresponding to the five value waves, and ten averages for TPB after pooling the even and odd environments. The non-parametric test employed is a Wilcoxon matched-pairs sign-rank test. The difference in ranks between MPB and SMR is 9 (five observations), which is not significant. The difference in ranks between TPB and MPB is 49 (ten observations), which is significant ($p = 0.03$). Finally, the difference in ranks between TPB and SMR is 55 (ten observations), which is significant at the one-percent level: $TPB \succ^{***} SMR$.

One cause of efficiency reductions with SMR is the incidence of unsold licenses, which happens at a rate of 2.1 licenses (out of 18) when averaged over all sessions. Likewise, on average 1.0 license is unsold under MPB while there are virtually no unsold licenses with TPB, see Figure 4.

License Distribution

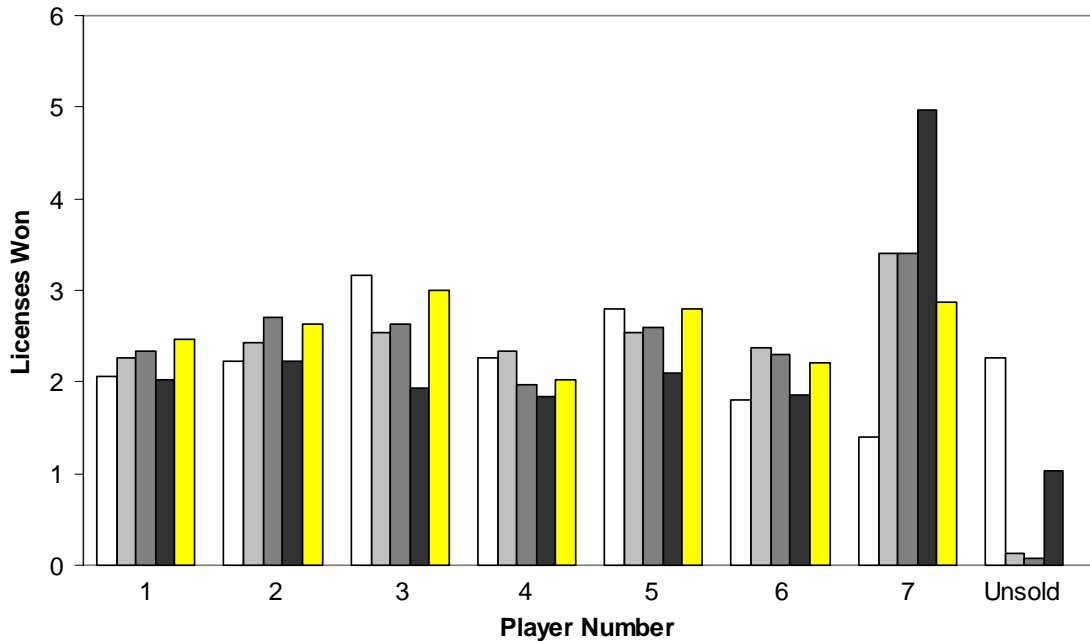


Figure 4. Licenses Acquired by Regional Bidders (1-6), the National Bidder (7), and the Number of Unsold Licenses
Key: SMR (white), TPB-odd (light gray), TPB-even (dark gray), MPB (black), Efficient (Yellow)

Finding 2: *The numbers of unsold licenses are ranked $SMR > MPB >^{***} TPB$.*

Support: The higher rate at which licenses are awarded under TPB is clear from the right-most set of bars in Figure 4. The difference between TPB and MPB in terms of license sales rates is significant with a Wilcoxon matched-pairs signed-ranks test ($p = 0.005$ with ten observations) while the difference between MPB and SMR is only borderline significant ($p = 0.13$ with five observations).

It is important to note that unsold licenses under SMR are due in part to the activity rule that prevents regional bidders from re-entering the auction after the national bidder stops competing for the preferred national package. Likewise, under MPB, unsold licenses are caused by the interplay between activity constraints and mis-coordination among regional bidders.⁹

⁹ As shown in section V, under MPB regional bidders frequently submit overlapping package bids that make it harder for them to compete with a bid on the national package. If, as a result, they turn attention to licenses from the small circle they may not have sufficient activity to compete for enough licenses on the large circle.

Revenues

Revenues are also normalized by the value of the best possible allocation:

$$revenue = \frac{R_{actual}}{S_{optimal}} * 100\%$$

The introduction of package bidding enhances revenues as shown in the right part of Figure 3. Switching from SMR to the package auction formats raises revenue from 65.6% to 70.8% for MPB and to 76.5% and 77.9% for TPB-odd and TPB-even respectively. As before, these comparisons can be evaluated with a Wilcoxon test based on the session averages reported in Appendix D.

Finding 3: *Revenues are ranked TPB \succ^{***} MPB \succ^{**} SMR.*

Support: The difference in ranks between MPB and SMR is 15 (five observations), which is significant ($p = 0.04$). The difference in ranks between TPB and MPB is 55 (ten observations), which is significant at the one-percent level ($p = 0.005$).

The brackets on each of the bars on the right side of Figure 3 indicate the standard deviations of the revenues across formats. Note that SMR and MPB result in more variable revenues compared to TPB.

Bidders' Profits

Consistent with the definitions of revenue and efficiency, bidders' profits are normalized by the value of the best possible allocation.

$$profit = \frac{\sum_i \pi_{actual}^i}{S_{optimal}} * 100\%$$

This profit is the difference between actual surplus and seller's revenue, except for SMR where possible penalties from withdrawing winning bids are recorded separately (see Appendix D). Rather than simply reporting the profits for the bidders as a group it is useful to show them by bidder type since this highlights the impact that package bidding and/or pre-packaging has on different kinds of bidders. Figure 5 displays bidders' profits by treatment and bidder number, using the same color-coding as in Figure 3. Again, the standard deviations are indicated by the brackets at the top of each bar.

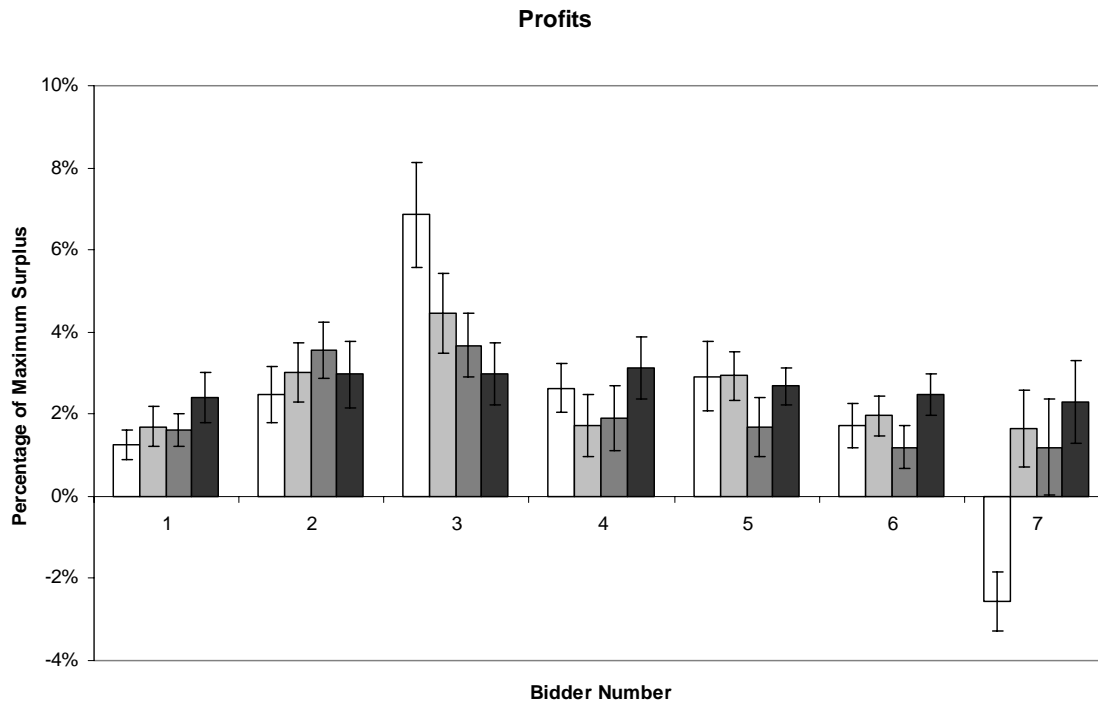


Figure 5. Profits for Regional Bidders (1-6) and the National Bidder (7)
 Key: SMR (white), TPB-odd (light gray), TPB-even (dark gray), MPB (black)

The ability to bid for combinations allows national bidders to bid high on large packages and avoid the exposure problem, resulting in positive profits for the national bidder (7) in the MPB and TPB auctions. In contrast, the national bidder loses money (in all waves, see Appendix D) when the SMR format is used. These losses are not surprising given prior results on SMR experiments; they are the result of a conscious decision to specify complementarities that create a serious exposure problem for the national bidder. The effects of negative earnings on individual behavior were mitigated by the fact that bidder roles were randomly assigned in each auction. The differences in national profits for the national bidder are corroborated by non-parametric tests. In contrast, the differences in profits for regional bidders (slightly higher under SMR than MPB and slightly higher under MPB than TPB) are not significant.

Finding 4: *The national bidder's profit is ranked MPB > TPB > ** SMR. The regional bidders' profits (as a group) are ranked SMR > MPB > TPB.*

Support: For the national bidder's profit the difference in ranks between TPB and SMR is 49 (ten observations), which is significant ($p = 0.03$). The difference in ranks between MPB and TPB is 34 (ten observations), which is not significant. For the regional bidders' profits the difference in ranks between SMR and MPB is 28 (ten observations), which is not significant. The difference in ranks between SMR and MPB is 7 (five observations), which is not significant.

V. Discussion

One reason for the efficiency and revenue advantages conferred by package bidding vis-à-vis SMR is apparent from looking at the outcomes of the first four auctions of wave 5 in which the optimal allocation frequently involved awarding large-circle licenses to the national bidder: 12, 12, 12, and 11 licenses in auctions 1-4 respectively (see Appendix E). Under SMR, the number of licenses actually obtained by the national bidder in these four auctions is 7, 2, 5, and 1, which shows that the national bidder was unable to overcome the exposure problem and obtain large networks even when it was optimal to do so. The national bidder was much more successful for the package bidding auctions with the same value draws; the numbers of licenses obtained by the national bidder were 12, 12, 12, and 12 licenses under MPB and TPB-odd, and 12, 7, 12, 12 under TPB-even.

The awarding of national licenses also provides a perspective on why TPB yields higher revenues and efficiencies than the more flexible package bidding format, MPB. There were several MPB auctions in which the national bidder won many licenses on the large circle when it was not optimal to do so, whereas there are relatively few such cases under TPB. To see how the national bidder was sometimes able to obtain all licenses under fully flexible package bidding (MPB) even when it was not optimal to do so, consider the round-by-round results of auction 2 in wave 1, shown in Table 1. The optimal allocation involved only a single license for the national bidder, but there were only three rounds in which the national bidder was not the provisional winner on all 12 licenses. In each of these three rounds, the regional bidders were not able to coordinate a very strong response in the sense that their provisionally winning bids left numerous unsold licenses (5 out of 12 licenses in rounds 3 and 4, and 2 out of 12 licenses in round 21). With fully flexible bidding, the regional bidders were bidding on “home-made” overlapping packages that did not “fit” in the sense that the revenue maximizing allocation left unsold licenses, which made it easier for the national bidder to regain provisional winner status in the subsequent rounds. In contrast, when TPB was used with the same draws, the regional bidders were able to effectively block the national bidder, and the resulting efficiency was close to 10 percentage points higher: 83% in MPB versus 92% in TPB (efficiency was 95 % in TPB-odd and 89% in TPB-even).

Round	National Circle			Regional Circle	
	National	Regional	Unsold	Regional	Unsold
1	12	0	0	2	4
2	12	0	0	2	4
3	0	7	5	5	1
4	0	7	5	5	1
5	12	0	0	2	4
6	12	0	0	3	3
7	12	0	0	3	3
8	12	0	0	3	3
9	12	0	0	4	2
10	12	0	0	4	2
11	12	0	0	4	2
12	12	0	0	4	2
13	12	0	0	4	2
14	12	0	0	6	0
15	12	0	0	6	0
16	12	0	0	6	0
17	12	0	0	6	0
18	12	0	0	6	0
19	12	0	0	6	0
20	12	0	0	6	0
21	0	10	2	5	1
22	12	0	0	6	0
23	12	0	0	6	0
Optimal	0	12	0	6	0

Table 1. Awards to National and Regional Bidders and Unsold Licenses by Round
Treatment: MPB; Wave: 1; Auction: 2

Motivated by this example, we focused on rounds in which the national bidder wins nothing and counted the number of licenses provisionally won by the regional bidders from the large circle (licenses A—L). The results are shown in Figure 6. Under MPB, the regional bidders are able to coordinate their bids such that they provisionally win all 12 licenses only 10% of the time when the national bidder is not winning any licenses. More than 65% of the time they provisionally win 10 licenses or less (out of 12), resulting in prices for the 12 licenses that can easily be topped by the national bidder. In contrast, under TPB, the regional bidders are able to coordinate and provisionally win 11 or 12 licenses more than 95% of the time.

Coordination among Regional Bidders

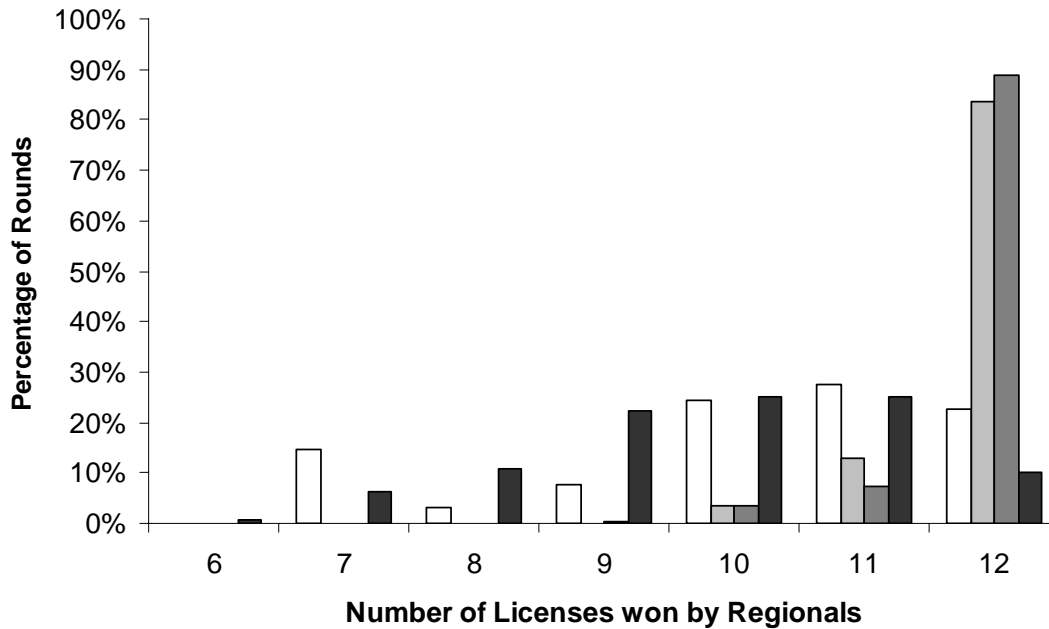


Figure 6. Number of Licenses Regional Bidders Provisionally Win from the Large National Circle (Licenses A—L) in Rounds Where the National Bidder Wins Nothing
 Key: SMR (white), TPB-odd (light gray), TPB-even (dark gray), MPB (black)

Performance differences are also apparent in the awards of blocks of licenses to regional bidders. There were 13 cases where the optimal allocation provided at least 3 of the 4 high-value licenses (E-H) to bidder 3 (see Appendix E), the only bidder who had an interest in all of these and could bid on them as a package in TPB-odd. Package bidding generally does better in these cases, with overall average efficiency of 85% for SMR, as compared with 88% for MPB and 93% for TPB. In both package bidding formats, the EFGH package was sometimes awarded to bidder 3 when it should not have been, but the efficiency consequences were small and certainly smaller than the consequences of not awarding one or more of these high-value licenses at all as happened several times with SMR (a total of six cases out of 13, with three of the four high-value licenses unsold in auction 5 of wave 3).

We are not claiming that TPB will yield better performance in terms of efficiency and revenue in all environments. For instance, if the hierarchical pre-packaging completely mismatches bidders' preferences, the resulting exposure problem that all bidders face would likely reduce bids and revenues. Alternatively, if there is no bidder with an interest in the national package, mis-coordination would be more easily resolved by regional bidders who would be provisional winners in all rounds. The design of our experiment (developed in consultation with FCC staff) was based on the belief that the

FCC would be able to craft economically relevant packages for at least some of the bidders and that there would be one or more bidders interested in a national package.

VI. Conclusions

The results of the laboratory auctions reveal a clear advantage for the tiered package bidding format, TPB, in an environment with value complementarities. The value structure was such that the pre-made packages allowed under tiered bidding did not match the preferred packages for half of the regional bidders in each treatment. TPB yielded significantly higher auction revenues and efficiencies, and lower numbers of unsold licenses. These performance differences were not due to increased fine-tuning over a large number of bidding rounds, since the TPB auctions actually tended to have fewer rounds.

Given the presence of value complementarities, we anticipated the lower efficiencies and revenues observed for the simultaneous multi-round auction format that has been extensively used by the FCC. What came as a surprise was the relative ranking of TPB and the more flexible MPB. One factor that contributed to this difference is that the home-made packages constructed under the flexible MPB format tended to overlap, causing a “fitting problem” that made it difficult for strong regional bidders to unseat a national package bid. Indeed, the number of licenses awarded to the national bidder was much higher than the optimal number under MPB, but not under TPB and SMR. More importantly, in rounds when the national bidder won nothing, regional bidders were unable to coordinate their bids under MPB while their coordination problems were virtually non-existent with hierarchically-structured packages.¹⁰

Pre-packaging has the obvious disadvantage that the chosen packages may not be optimal, but in a non-overlapping hierarchical structure they are chosen to “fit,” which enables bidders to coordinate their bids and avoid threshold problems with positive effects for efficiencies and revenues. In addition, assignment and pricing is transparent and easily verifiable by bidders as the auction proceeds. Of course, one has to be careful in generalizing the relative performance of tiered and flexible package bidding to other environments. But if the number of licenses is increased, thereby increasing the potential complexity of the fitting problem, these results would seem to suggest that the simple TPB procedure would continue to offer advantages over a fully flexible form of package bidding such as MPB.

¹⁰ The inefficiency due to this type of fitting problem is reminiscent of the results for package bidding with the “XOR” rule, which stipulates that bidders can submit many bids on individual licenses and packages but at most one of these bids can be winning. With XOR bidding the fitting problem is even worse in the sense that bidders who desire more than one license must submit package bids even in environments without value complementarities. When the submitted package bids (partially) overlap, some licenses may not be awarded in that round of bidding, resulting in low revenues and low efficiencies (see Goeree, Holt, and Ledyard, 2006).

References

- Brunner, C., J. K. Goeree, C. A. Holt, and J. O. Ledyard (2006) "Combinatorial Auctioneering," unpublished manuscript, Caltech.
- Goeree, J. K. and C. A. Holt (2007) "Simple Combinatorial Pricing," private communication with the FCC.
- Goeree, J. K. and C. A. Holt (2005) "Comparing the FCC's Combinatorial and Non-Combinatorial Simultaneous Multi-Round Auctions: Experimental Design Report," prepared for the Wireless Communications Bureau of the Federal Communications Commission.
- Goeree, J. K., C. A. Holt, and J. O. Ledyard (2006) "An Experimental Comparison of the FCC's Combinatorial and Non-Combinatorial Simultaneous Multiple Round Auctions," prepared for the Wireless Communications Bureau of the Federal Communications Commission.
- Kwasnica, A. M., J. O. Ledyard, D. Porter, and C. DeMartini (2005) "A New and Improved Design for Multi-Object Iterative Auctions," *Management Science*, 51(3), March, 419-434.
- Rassenti, Smith and Bulfin (1982), "A combinatorial auction mechanism for airport slot allocation," *Bell Journal of Economics*, vol. 13, pp. 402-417.
- Rothkopf, M.H., A. Pekeč and R.M. Harstad (1998), "Computationally Manageable Combinatorial Auctions," *Management Science*, 44, pp. 1131-1147.

Appendix A: Rules for the Simultaneous Multi-Round (SMR) Auction

Rounds and Bid Structure: All licenses are put up for bid simultaneously, and participants may only submit bids on individual licenses. The auction consists of successive rounds in which participants may place bids. Following each round, the high bid for each license is posted. These high bids then become the standing bids for the subsequent round.

Acceptable Bids: In the first round, an acceptable bid must be equal to or exceed the initial price of 0 by 3 points (each point equaled 50 cents in the experiment). Subsequently, in order to be acceptable, a bid must exceed the provisionally winning bid for the license by at least 3 points. In other words, multiple bid increments are allowed.

Bid Withdrawal: Each bidder has at most 2 rounds in which they are permitted to withdraw any of their provisionally winning bids. After the withdrawal, the seller becomes the provisionally winning bidder for the withdrawn license and the minimum acceptable bid in the following round equals the second highest bid received on the license, which may be less than or equal to (in the case of tied bids) the amount of the withdrawn bid. A withdrawing bidder pays a penalty equal to the maximum of zero or the difference between the price at which the bidder withdrew its bid and the final sale price in the current auction. If the license goes unsold, the bidder would normally be responsible for paying the difference between the withdrawn bid and the sale price in a subsequent auction, plus an interim payment of 3 - 20 %. In the experiment, there is no subsequent auction, so these penalties for the case of an unsold license were implemented by requiring that the bidder pay a penalty of 25% of the withdrawn bid.

Bidding Eligibility and Activity: Each license in the experiment is assigned one bidding unit. The total number of bidding units available to the bidder establishes the bidder's maximum "eligibility" to bid. The national bidder began each auction with 12 activity units and regional bidders began with 4. In each round, a bidder's activity is calculated as the number of licenses for which that bidder is a provisional winner, plus the number of licenses for which acceptable bids are submitted. If a bidder's activity falls below the bidder's current activity limit, that limit is reduced to equal the bidder's actual activity. There were no activity rule waivers in the experiment, so a reduction in activity would put an upper limit on the bidder's activity for all subsequent rounds of that auction.

End of Round Feedback: At the end of each round, bidders receive information on all provisionally winning bids, withdrawn bids, and the corresponding bidder ID numbers. Bidders also see the sum of their own values for the licenses that they are provisionally winning and prices that would be paid for the licenses if the auction had ended.

Closing Rule: The auction closes after any round in which no new bids were placed and no bids were withdrawn. In this case provisionally winning bids become winning bids that are used to calculate auction earnings. The experiment did not allow for defaults on payments, so gains were added to cumulative earnings and losses were subtracted.

Appendix B: Rules for the Tiered Package Bidding (TPB) Auction

Rounds and Bid Structure: This is a simultaneous, multi-round auction in which participants may submit bids on individual licenses or on pre-defined, non-overlapping packages at each hierarchy level. In the experiment, there are three levels: individual licenses (level 1), regional packages (level 2) and a single national package (level 3). Winning bids are determined by maximizing seller revenue for the round, which is done recursively by finding the highest bid on each individual license, and then comparing the sum of the high bids in each region with the highest regional bid, to determine winning bid amounts at the regional level. Then the sum of the regional winning bid amounts is compared with the highest bid on the national package to determine the final allocation. Bidders can have multiple winning bids on individual licenses or packages.

Acceptable Bids: In the first round, an acceptable bid must be equal to or exceed the minimum opening bid of 0 by 3 points for each license, or by 3 points times the number of licenses in a package. After each subsequent round, license prices are calculated on the basis of bids received in previous rounds. If a bid on a single license is provisionally winning then the price for the license equals that bid. If a bid on a regional package is provisionally winning then the prices for the licenses in that region are scaled up so that their sum equals the winning regional package bid. If a national package bid is winning, then the prices at the individual level are scaled up further so that the sum of all license prices equals the winning national bid. These increases can be thought of as “regional shares” and “national shares” needed to unseat a winning bid. In order to be acceptable, a bid must exceed the price of a license or package at least 3 points times the number of licenses covered by the bid. In other words, multiple bid increments are allowed.

Bid Withdrawal: Withdrawals were not permitted.

Bidding Eligibility and Activity: Each license in the experiment is assigned one bidding unit. The total number of bidding units available to the bidder establishes the bidder’s maximum “eligibility” to bid (4 for regional bidders and 12 for the national bidder). In each round, a bidder’s activity is calculated as the number of different licenses for which that bidder is a provisional winner plus the number of other licenses for which a bid is submitted. As with the other formats, activity has a “use it or lose it feature” and there were no activity rule waivers in the experiment.

End of Round Feedback: At the end of each round, bidders receive information on all provisionally winning bids (for licenses and packages) and the corresponding bidder ID numbers. Bidders also see the prices for all licenses, the sum of their own values for the licenses and packages that they are provisionally winning, and the sum of prices that would be paid for those licenses and packages if the auction had ended.

Closing Rule: The auction closes after any round in which no new bids were placed. In this case provisionally winning bids become winning bids that are used to calculate auction earnings. The experiment did not allow for defaults on payments, so gains were added to cumulative earnings and losses were subtracted.

Details about Pricing Rule: Consider an example with three hierarchy levels as in the experiments. Suppose a single nationwide license is divided into R regions, labeled by $r = 1, \dots, R$, which are subdivided into individual parcels labeled by $i = 1, \dots, I_r$ and r (where I_r is the number of individual licenses in region r), which cover α_{ir} bidding units. Let $\alpha_r = \sum_{i=1}^{I_r} \alpha_{ir}$ denote the total number of bidding units in region r and let $\alpha = \sum_{r=1}^R \alpha_r$ denote the total number of bidding units nationwide. Bidders can submit bids on individual licenses, package bids for an entire region, or a package bid for the nation-wide license. Let b_{ir}^{\max} denote the best bid for license i in region r , b_r^{\max} the best package bid for region r , and b^{\max} the best package bid for the nation-wide license. It is simple to find the best allocation:

1. Pick the larger of b_r^{\max} and $\sum_{i=1}^{I_r} b_{ir}^{\max}$. Denote this Rev_r , the revenue for region r .
2. Pick the larger of b^{\max} and $\sum_{r=1}^R \text{Rev}_r$. Denote this Rev , the national revenue.
3. If $\text{Rev} = b^{\max}$ then the best national bid is winning and all other bids are losing, otherwise all national bids are losing. In the latter case, for each region r , if $\text{Rev}_r = b_r^{\max}$ the best package bid for region r is winning and individual bids are losing, otherwise the individual bids in region r win and all package bids lose.

The price p_{ir} of license i in region r is simply the maximum bid for the license plus possibly a regional share (if the sum of individual bids falls short of the regional profit) plus possibly a national share (if the sum of regional profits falls short of the national profit):

$$p_{ir} = b_{ir}^{\max} + \frac{\alpha_{ir}}{\alpha_r} (\text{Rev}_r - \sum_{i'=1}^{I_r} b_{i'r}^{\max}) + \frac{\alpha_{ir}}{\alpha} (\text{Rev} - \sum_{r'=1}^R \text{Rev}_{r'})$$

Note that the shares, if any, are simply proportional to the number of bidding units and:

1. $p_{ir} = b_{ir}^{\max}$ if an individual license bid wins.
2. $p_{ir} = b_{ir}^{\max} + \mathbf{regional\ share}$ if a regional package bid wins.
3. $p_{ir} = b_{ir}^{\max} + \mathbf{national\ share}$ if a national package bid wins and the sum of individual license bids in region r exceeds the package bid for region r .
4. $p_{ir} = b_{ir}^{\max} + \mathbf{regional\ share} + \mathbf{national\ share}$ if a national package bid wins and the package bid for region r exceeds the sum of the individual bids in region r .

Finally, note that $\sum_{i,r} p_{ir} = \text{Rev}$ and $\sum_i p_{ir} = \text{Rev}_r + (\alpha_r/\alpha) (\text{Rev} - \sum_{r'} \text{Rev}_{r'})$ where the second term on the right side is strictly positive only when a national bid wins. These equalities indicate that TPB pricing is such that license and package prices convey to bidders how high their bids must be to unseat the provisionally winning bids at all hierarchy levels.

Appendix C: Rules for the Modified Flexible Package Bidding (MPB) Auction

Rounds and Bid Structure: This is a simultaneous, multi-round auction in which participants may submit bids on individual licenses or on combinations of licenses (packages). Winning bids are determined by maximizing seller revenue for the round, and bidders could have multiple winning bids on individual licenses or packages.

Acceptable Bids: In the first round, an acceptable bid must be equal to or exceed the minimum opening bid of 0 by 3 points for each license, or by 3 points times the number of licenses in a package. After each subsequent round, “prices” are calculated for each license on the basis of bids received in the previous round. The pricing rule, described in detail below, calculates prices that reflect (as closely as possible) the marginal sales revenue of each license based on bids received. Prices for packages are given by the sum of the prices for each license in the package. In order to be acceptable, a bid must exceed the price of a license or package at least 3 points times the number of licenses covered by the bid. In other words, multiple bid increments are allowed.

Bid Withdrawal: Withdrawals were not permitted.

Bidding Eligibility and Activity: Each license in the experiment is assigned one bidding unit. The total number of bidding units available to the bidder establishes the bidder’s maximum “eligibility” to bid (4 for regional bidders and 12 for the national bidder). In each round, a bidder’s activity is calculated as the number of different licenses for which that bidder is a provisional winner plus the number of other licenses for which a bid is submitted. For example, a regional bidder with a current activity limit of 3 who is provisionally winning license A would be able to bid on packages BC and CD, but not on BCD. If a bidder’s activity falls below the bidder’s current activity limit, that limit is reduced to equal the bidder’s actual activity. There were no activity rule waivers in the experiment, so a reduction in activity would put an upper limit on the bidder’s activity for all subsequent rounds of that auction.

End of Round Feedback: At the end of each round, bidders receive information on all provisionally winning bids (for licenses and packages) and the corresponding bidder ID numbers. Bidders also see the prices for all licenses, the sum of their own values for the licenses and packages that they are provisionally winning, and the sum of prices that would be paid for those licenses and packages if the auction had ended.

Closing Rule: The auction closes after any round in which no new bids were placed. In this case provisionally winning bids become winning bids that are used to calculate auction earnings. The experiment did not allow for defaults on payments, so gains were added to cumulative earnings and losses were subtracted.

Details about Pricing Rule: The current price estimates of the licenses are based on the intuitive idea that every linear optimization problem has a dual problem that provides pricing information. (These “current price estimates” are then used in the next round when calculating minimum acceptable bids as described above.) More precisely, *pseudo-dual prices*¹¹ are used that satisfy two criteria: the sum of the license prices in a package equals the bid amount for the package if the bid is winning, and exceeds the bid amount for the package if the bid is losing. The term “*pseudo*” refers to the fact that the latter constraints cannot always be satisfied (e.g. if complementarities between licenses exist) in which case the pseudo-dual prices are chosen to “get as close as possible” to solving the constraints. In the resource allocation design (RAD) procedure proposed by Kwasnica, Ledyard, Porter, and DeMartini (2005), this is done by relaxing the losing-bid constraints with slack variables, δ_j , and then minimizing the maximum of the δ_j :

$$\begin{aligned}
 & \min \max \left\{ \delta_j \mid b_j \in B^t \setminus (W^t \cup F) \right\} \\
 & \text{s.t. } \sum_{i \in L} a_{ji} p_i + \delta_j \geq b_j, \text{ for all } b_j \in B^t \setminus (W^t \cup F) \\
 & \sum_{i \in L} a_{ji} p_i = b_j, \quad \text{for all } b_j \in W^t \\
 \text{(P1): } & p_i \geq b_j, \quad \text{for all } b_j \in F \setminus (W^t \cap F) \\
 & \quad \quad \quad \text{and } i \text{ is the license index associated with bid } b_j \\
 & \delta_j \geq 0, \quad \text{for all } b_j \in B^t \setminus (W^t \cup F)
 \end{aligned}$$

where $a_{ji} = 1$ if the bid b_j contains license i and 0 otherwise, L is the set of licenses, B^t is the set of all bids in round t , W^t is the set of winning bids in round t , and F is the set of reserve bids.

Multiple solutions can exist to (P1). Suppose, for instance, that there are 3 licenses, A, B, and C, and three bids are submitted: a bid of 30 on the package ABC, a bid of 10 on license A, and a bid of 10 on license B. In this case, the slack variables can be chosen to be 0, e.g. by choosing prices of 10 for each license. However, another solution is to choose prices $p_A = p_B = 12$ and $p_C = 6$. Indeed, a continuum of solutions exists. To resolve this indeterminacy, Kwasnica et al. (2005) propose to run a second optimization problem that involves maximizing the minimum price: $\max \min_{i \in L} p_i$ subject to the constraints in (P1) with the slack variables, δ_j , fixed to the levels that follow from (P1). In the example, this yields equal prices of 10 for licenses A-C.

Kwasnica et al. (2005) also discuss variations of the RAD procedure, e.g. minimizing the sum of slack variables or minimizing the sum of the squared values of the slack variables subject to the constraints of (P1). However, the indeterminacy described in the previous paragraph arises with any variation and, therefore, a second optimization problem is required. An alternative to maximizing the minimum price would be to minimize the sum of squared prices (which would also tend to equalize prices as in the

¹¹ Rassenti, Smith and Bulfin (1982) introduced this terminology.

RAD procedure) or to minimize the sum of squared deviations between computed prices and previous-round prices. This latter variation is part of the SMRPB format that was tested in a previous report (Goeree, Holt, and Ledyard, 2006), but given the performance of SMRPB relative to RAD (Brunner, Goeree, Holt, and Ledyard, 2006) we used the more standard formulation in (P1) in the current study.

There is one additional consideration that must be taken into account in an actual implementation of the pricing mechanism described above. When different licenses cover different population sizes and/or bandwidth (MHz-pop bidding units) it is necessary to adapt the mechanism (and its variations), since otherwise small licenses may be overpriced after an aggressive package bid, pushing smaller bidders out of the auction. Consider again the above example with three licenses A, B, and C, where now licenses A and license C are “small” with MHz-pop weights denoted by $\alpha_A = \alpha_C = 1,000$ while license B is “large” with $\alpha_B = 10,000$. Suppose, as a result, that the values for A and C are somewhere in the $[5, 20]$ range and the value for B is somewhere in the $[50, 100]$ range. There are 3 small bidders interested in a single license: bidder 1 wants A, bidder 2 wants B, and bidder 3 wants C. Bidder 4 is a large bidder who values only the package ABC at 80. Suppose opening bids of the small bidders are 4 on licenses A-C while bidder 4 places a bid of 60 on ABC. Without any correction for bidding units, computed prices are $p_A = p_B = p_C = 20$, which causes bidders 1 and 3 to lose all activity in the next round. In other words, the price-equalizing feature of the pricing algorithm causes small licenses to be over-priced, thereby eliminating small bidders from the auction. The solution is to maximize the minimum price per bidding unit: $\max \min_i (p_i/\alpha_i)$. With this correction, the computed prices would be $p_A = p_C = 5$ and $p_B = 50$.

Appendix D: Data Summary Statistics

	SMR	TPB odd	TPB even	MPB
Efficiency				
Wave 1	88.5%	92.4%	92.4%	88.7%
Wave 2	91.3%	94.6%	91.5%	86.6%
Wave 3	80.2%	94.6%	92.0%	89.0%
Wave 4	81.9%	92.4%	92.3%	86.1%
Wave 5	83.8%	96.0%	96.4%	98.2%
Average	85.1%	94.0%	92.9%	89.7%
Revenue				
Wave 1	67.3%	71.9%	77.2%	70.3%
Wave 2	64.2%	76.8%	81.5%	65.3%
Wave 3	65.5%	77.0%	73.5%	71.9%
Wave 4	62.1%	78.7%	78.7%	71.0%
Wave 5	68.6%	77.9%	78.6%	75.5%
Average	65.6%	76.5%	77.9%	70.8%
Penalties	4.3%	0.0%	0.0%	0.0%
Profit Nationals				
Wave 1	-1.3%	0.8%	0.3%	1.7%
Wave 2	-0.8%	0.0%	-2.7%	0.1%
Wave 3	-4.5%	-0.7%	-0.4%	1.2%
Wave 4	-3.9%	-0.3%	2.3%	0.3%
Wave 5	-2.3%	8.4%	6.5%	8.3%
Average	-2.6%	1.6%	1.2%	2.3%
Profit Regionals				
Wave 1	3.0%	3.3%	2.6%	2.8%
Wave 2	4.4%	3.0%	2.1%	3.6%
Wave 3	2.0%	3.0%	3.1%	2.6%
Wave 4	2.8%	2.3%	1.9%	2.5%
Wave 5	2.6%	1.6%	1.8%	2.4%
Average	3.0%	2.6%	2.3%	2.8%
Unsold Licenses				
Wave 1	2.2	0.2	0.3	1.2
Wave 2	1.5	0.2	0.0	1.8
Wave 3	2.7	0.3	0.0	1.2
Wave 4	2.7	0.0	0.0	0.8
Wave 5	1.5	0.0	0.0	0.2
Average	2.1	0.1	0.1	1.0
# Rounds	17.8	13.2	12.5	15.1

Appendix E: Optimal and Actual Outcomes by Waves

Wave 1

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	Surplus	Rounds	Unsold
Auction 1																					
Optimal	7	7	7	7	7	7	7	7	7	7	7	7	1	2	3	3	5	6	408		
SMR	6	1	0	2	2	2	3	4	4	4	6	6	1	2	3	3	4	6	385	25	1
TPB-odd	7	1	7	7	7	3	4	4	4	4	7	7	1	2	3	3	5	6	345	10	0
TPB-even	7	7	7	7	7	7	7	7	7	7	7	7	1	2	3	4	4	6	395	13	0
MPB	7	7	7	7	7	7	7	7	7	7	7	7	1	1	3	3	5	0	391	13	1
Auction 2																					
Optimal	1	1	7	2	2	2	4	3	5	4	5	5	1	1	2	4	4	5	398		
SMR	0	1	1	2	3	2	4	3	5	4	5	0	1	1	2	3	5	5	359	15	2
TPB-odd	6	6	7	2	2	2	4	3	5	4	5	6	1	1	2	4	5	5	379	8	0
TPB-even	7	7	1	2	2	2	4	7	5	4	7	7	1	1	2	4	5	5	355	12	0
MPB	7	7	7	7	7	7	7	7	7	7	7	7	1	1	2	3	5	5	330	23	0
Auction 3																					
Optimal	1	1	1	2	3	2	3	3	4	4	5	5	1	2	2	4	5	5	478		
SMR	6	1	1	1	3	2	3	3	4	4	5	0	1	2	2	4	5	5	432	24	1
TPB-odd	6	1	1	2	3	3	3	3	4	4	5	5	1	2	2	4	5	5	450	9	0
TPB-even	1	1	1	2	3	2	4	3	4	7	5	5	1	2	2	4	5	5	462	21	0
MPB	7	6	7	2	7	2	7	7	0	4	7	7	6	2	2	4	5	5	382	23	1
Auction 4																					
Optimal	6	1	1	1	3	3	7	3	4	4	6	7	6	1	3	4	4	6	447		
SMR	6	7	2	2	3	3	7	3	4	4	6	7	6	1	3	4	4	6	438	26	0
TPB-odd	7	7	7	7	7	7	7	7	7	7	7	7	6	1	2	4	4	6	399	9	0
TPB-even	6	1	1	2	3	3	0	3	0	5	6	5	6	1	3	4	4	6	379	10	2
MPB	7	7	7	7	7	7	7	3	7	7	7	7	6	1	3	3	0	6	398	28	1
Auction 5																					
Optimal	6	1	2	1	2	3	3	3	4	5	5	6	1	1	3	4	5	5	479		
SMR	6	1	0	1	0	3	3	3	4	5	5	0	1	1	3	4	4	5	416	16	3
TPB-odd	6	1	2	1	3	3	3	3	4	4	5	6	1	1	0	4	4	5	445	11	1
TPB-even	6	1	1	1	3	3	3	4	4	5	5	6	6	1	3	4	5	5	453	7	0
MPB	0	1	0	1	3	3	3	3	4	5	5	0	1	1	0	4	5	5	404	25	4
Auction 6																					
Optimal	7	6	7	2	3	2	3	3	5	5	6	5	6	2	2	3	5	6	422		
SMR	0	0	0	0	3	2	0	3	5	4	5	5	6	0	3	3	5	6	300	14	6
TPB-odd	1	1	1	1	3	2	3	3	5	4	5	5	6	2	2	3	5	6	415	6	0
TPB-even	6	6	1	2	3	2	3	3	5	4	6	5	6	2	3	4	5	5	367	9	0
MPB	7	6	7	2	3	2	3	3	5	5	6	5	6	2	2	3	5	6	422	9	0

Wave 2

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	Surplus	Rounds	Unsold
Auction 1																					
Optimal	7	6	2	2	2	3	7	3	5	5	5	6	6	2	3	3	5	6	433		
SMR	1	1	2	2	2	3	7	3	5	4	6	6	6	2	3	3	4	6	420	12	0
TPB-odd	6	1	1	2	3	3	4	3	5	4	5	6	6	2	2	3	5	6	406	9	0
TPB-even	7	6	2	2	3	3	7	3	4	4	5	6	6	2	2	3	4	6	415	11	0
MPB	1	1	2	2	2	7	7	4	4	4	6	6	6	2	0	4	5	6	302	9	1
Auction 2																					
Optimal	1	6	1	1	2	3	4	3	5	4	6	5	6	2	2	4	4	6	396		
SMR	1	1	1	1	2	3	4	3	5	4	5	5	6	2	3	4	4	6	369	16	0
TPB-odd	1	6	1	1	3	3	4	3	5	4	6	5	6	2	2	4	4	6	388	20	0
TPB-even	1	6	1	1	2	2	4	4	5	5	6	5	6	2	2	4	4	5	367	9	0
MPB	1	0	1	2	2	3	4	3	4	5	6	6	6	2	2	4	4	6	366	16	1
Auction 3																					
Optimal	1	1	1	2	2	2	4	3	5	4	5	5	1	2	3	4	4	5	457		
SMR	1	0	1	0	2	2	4	3	5	4	5	5	1	2	2	3	4	5	419	15	2
TPB-odd	1	7	1	2	2	2	4	3	5	4	5	7	1	1	2	4	4	5	449	17	0
TPB-even	1	6	1	1	2	2	3	3	5	5	5	6	1	2	2	3	5	6	391	18	0
MPB	1	1	1	0	2	2	4	4	5	4	5	5	1	2	2	4	0	5	433	11	2
Auction 4																					
Optimal	6	6	2	1	3	2	3	3	5	5	5	6	1	2	2	3	5	6	485		
SMR	6	6	2	1	3	2	3	3	5	5	5	6	1	1	2	3	5	6	478	23	0
TPB-odd	6	6	2	2	3	2	3	3	5	5	5	5	6	0	2	3	4	6	467	17	1
TPB-even	6	6	2	1	2	2	3	3	5	4	5	6	6	2	3	4	5	5	437	17	0
MPB	1	1	2	2	3	2	3	3	0	5	0	5	1	1	2	3	5	5	393	14	2
Auction 5																					
Optimal	6	1	2	2	3	2	4	3	4	4	6	5	6	2	3	3	4	6	460		
SMR	0	6	2	2	3	2	4	3	4	4	6	5	0	2	3	3	5	5	412	28	2
TPB-odd	6	6	1	2	2	2	4	4	4	4	6	5	1	2	3	3	5	6	444	26	0
TPB-even	1	1	1	2	2	2	4	3	4	4	6	5	6	2	3	3	4	6	436	20	0
MPB	6	0	1	2	2	2	3	3	4	4	6	0	6	2	3	3	0	6	409	20	3
Auction 6																					
Optimal	1	1	7	7	3	7	3	7	5	7	7	5	1	1	3	3	5	5	441		
SMR	1	1	0	0	3	0	3	0	5	0	6	5	1	1	3	3	5	5	343	14	5
TPB-odd	7	7	7	7	7	7	7	7	7	7	7	7	6	1	3	3	5	5	375	11	0
TPB-even	1	1	2	2	3	3	3	4	5	4	6	5	1	1	2	3	5	5	422	8	0
MPB	1	1	1	2	3	3	3	0	5	0	6	5	6	1	2	3	5	5	410	14	2

Wave 3

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	Surplus	Rounds	Unsold
Auction 1																					
Optimal	6	1	1	1	3	3	4	4	5	4	6	6	1	2	3	3	4	6	425		
SMR	6	1	0	1	3	3	4	4	7	0	0	6	1	2	3	4	4	6	362	22	3
TPB-odd	6	6	2	1	2	2	4	4	5	5	5	6	1	1	2	4	4	6	385	9	0
TPB-even	6	1	2	1	3	3	3	3	5	4	6	6	1	2	2	4	4	6	378	19	0
MPB	6	1	0	0	2	2	4	4	5	4	6	6	1	2	2	4	5	6	375	11	2
Auction 2																					
Optimal	6	6	2	2	3	2	3	3	5	5	6	6	1	1	2	3	5	5	426		
SMR	7	7	7	0	3	2	3	3	7	7	7	7	1	2	2	4	5	5	331	12	1
TPB-odd	6	6	1	1	3	3	3	3	5	5	6	6	1	1	0	4	5	5	403	7	1
TPB-even	1	6	2	1	3	2	3	3	5	5	5	6	1	1	2	4	4	5	397	10	0
MPB	1	7	7	1	3	3	3	3	5	7	5	6	6	1	0	4	5	5	376	18	1
Auction 3																					
Optimal	1	6	2	2	3	3	3	3	4	5	5	6	1	1	2	4	5	5	471		
SMR	1	0	0	0	3	3	0	3	4	5	5	0	1	1	3	4	5	5	388	21	5
TPB-odd	1	6	2	2	2	3	3	3	5	4	5	6	1	1	2	3	5	5	460	22	0
TPB-even	1	6	2	2	3	3	3	4	4	5	5	6	1	1	3	4	5	5	454	9	0
MPB	1	6	1	2	2	2	7	4	4	5	5	6	1	1	2	4	5	5	407	15	0
Auction 4																					
Optimal	6	1	1	1	7	3	3	4	4	4	5	6	6	1	3	4	5	6	413		
SMR	6	1	1	2	2	3	3	4	4	5	0	6	6	1	3	3	5	5	342	15	1
TPB-odd	6	1	1	7	7	3	7	3	4	7	5	5	6	1	3	4	5	5	387	14	0
TPB-even	7	7	7	7	7	7	7	7	7	7	7	7	6	1	3	4	5	6	363	8	0
MPB	7	7	7	7	7	7	7	7	7	7	7	7	0	2	2	4	5	5	333	13	1
Auction 5																					
Optimal	1	1	2	2	2	3	3	3	5	4	6	5	1	1	2	3	5	5	447		
SMR	0	1	2	2	0	0	0	4	5	4	5	5	1	2	2	4	5	0	278	10	5
TPB-odd	6	1	2	1	2	3	4	4	4	4	5	5	1	1	3	3	5	5	410	14	0
TPB-even	6	6	2	2	2	2	3	3	5	4	6	6	1	1	3	3	4	5	393	14	0
MPB	7	6	2	2	2	3	4	4	4	4	6	6	0	0	2	3	5	6	405	19	2
Auction 6																					
Optimal	6	6	2	2	2	7	4	4	4	5	6	5	6	2	3	4	5	5	461		
SMR	1	0	2	2	2	7	4	4	4	5	6	5	6	2	3	4	5	5	418	22	1
TPB-odd	6	6	2	2	2	2	4	4	4	5	6	5	6	0	3	4	5	5	459	10	1
TPB-even	1	6	2	2	2	2	4	4	4	5	6	5	6	1	3	4	5	5	451	12	0
MPB	6	6	2	2	2	2	4	4	4	5	6	5	6	0	3	4	5	5	459	16	1

Wave 4

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	Surplus	Rounds	Unsold
Auction 1																					
Optimal	6	6	2	2	3	2	3	4	4	5	5	5	6	2	3	3	5	6	382		
SMR	7	7	2	2	7	2	3	4	4	5	5	5	6	2	3	4	5	6	356	14	0
TPB-odd	6	1	2	1	3	2	3	4	4	5	5	6	1	1	3	3	5	6	347	9	0
TPB-even	7	7	7	7	7	7	7	7	7	7	7	7	6	2	3	4	5	6	373	10	0
MPB	7	7	7	7	7	7	7	7	7	7	7	7	6	1	3	3	5	6	367	21	0
Auction 2																					
Optimal	7	1	2	1	3	3	3	4	4	4	6	5	1	1	3	4	5	5	434		
SMR	0	1	0	1	3	3	3	4	4	4	6	5	1	1	3	4	5	5	413	14	2
TPB-odd	7	6	2	2	3	3	3	3	4	4	6	5	6	2	2	4	5	5	388	12	0
TPB-even	1	1	2	1	3	2	3	3	5	5	6	5	1	2	2	3	5	6	379	16	0
MPB	1	1	2	2	0	2	4	4	4	4	6	5	1	1	2	3	5	5	339	17	1
Auction 3																					
Optimal	6	7	7	2	7	2	7	7	7	7	7	7	6	2	2	4	4	6	365		
SMR	0	7	7	0	3	3	0	3	0	7	0	6	6	2	2	4	4	6	221	25	5
TPB-odd	1	1	2	2	3	3	4	3	4	7	6	7	1	2	2	3	4	6	338	14	0
TPB-even	1	1	2	2	3	3	3	3	5	5	6	6	6	1	2	4	4	6	319	8	0
MPB	7	7	7	7	7	7	7	7	7	7	7	7	1	1	0	4	4	0	294	12	2
Auction 4																					
Optimal	1	1	2	1	2	2	3	3	5	5	7	5	1	2	3	3	5	6	469		
SMR	1	1	0	0	2	2	4	0	0	5	5	0	1	2	3	4	5	5	311	26	5
TPB-odd	1	1	2	1	2	2	4	4	7	4	6	6	6	2	3	4	5	6	422	15	0
TPB-even	1	1	2	1	2	2	3	3	5	5	6	5	1	2	3	3	5	6	467	17	0
MPB	1	1	7	1	2	2	3	3	5	5	7	5	1	2	2	3	5	0	433	17	1
Auction 5																					
Optimal	1	7	1	1	2	3	3	3	5	4	5	5	1	2	2	3	4	5	450		
SMR	1	6	1	1	2	3	3	3	5	0	5	0	1	2	2	0	5	5	390	18	3
TPB-odd	1	6	1	1	2	3	3	3	5	5	5	5	1	2	2	3	4	6	439	16	0
TPB-even	1	7	1	1	3	7	3	7	5	4	5	5	1	2	3	3	4	5	413	11	0
MPB	1	6	2	1	3	3	3	3	4	4	6	5	6	2	2	4	4	6	380	14	0
Auction 6																					
Optimal	6	6	2	2	3	3	3	4	4	4	6	5	1	2	2	3	4	6	499		
SMR	1	6	2	2	3	3	3	0	4	4	6	5	1	2	2	3	5	6	448	20	1
TPB-odd	6	6	2	1	3	3	3	4	4	4	6	5	1	2	2	3	5	6	469	21	0
TPB-even	6	6	2	2	3	3	3	3	5	4	6	5	1	2	2	4	4	6	449	27	0
MPB	1	6	1	2	2	3	3	4	4	4	6	0	1	1	3	3	4	6	425	17	1

Wave 5

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	Surplus	Rounds	Unsold
Auction 1																					
Optimal	7	7	7	7	7	7	7	7	7	7	7	7	1	2	3	3	5	6	403		
SMR	7	7	2	7	7	7	7	4	7	4	6	6	1	2	3	3	5	6	316	12	0
TPB-odd	7	7	7	7	7	7	7	7	7	7	7	7	1	2	2	4	5	6	386	12	0
TPB-even	7	7	7	7	7	7	7	7	7	7	7	7	1	2	3	3	5	6	403	9	0
MPB	7	7	7	7	7	7	7	7	7	7	7	7	1	2	3	3	5	6	403	7	0
Auction 2																					
Optimal	7	7	7	7	7	7	7	7	7	7	7	7	1	1	2	4	5	5	322		
SMR	6	7	7	2	3	3	4	3	5	5	6	5	1	1	2	0	5	0	288	12	2
TPB-odd	7	7	7	7	7	7	7	7	7	7	7	7	1	1	2	4	5	6	316	8	0
TPB-even	1	7	2	7	7	2	4	7	7	7	7	6	1	2	2	3	5	5	267	11	0
MPB	7	7	7	7	7	7	7	7	7	7	7	7	1	1	2	4	5	5	322	12	0
Auction 3																					
Optimal	7	7	7	7	7	7	7	7	7	7	7	7	1	2	2	4	5	5	403		
SMR	7	1	1	0	3	3	3	3	7	7	7	7	1	2	2	4	5	5	350	17	1
TPB-odd	7	7	7	7	7	7	7	7	7	7	7	7	1	2	2	4	5	5	403	15	0
TPB-even	7	7	7	7	7	7	7	7	7	7	7	7	1	2	2	4	5	5	403	11	0
MPB	7	7	7	7	7	7	7	7	7	7	7	7	1	2	2	4	5	5	403	10	0
Auction 4																					
Optimal	7	7	7	7	7	7	7	3	7	7	7	7	6	1	3	4	4	6	395		
SMR	6	0	0	7	3	3	3	3	0	4	5	5	6	1	2	4	4	6	308	15	3
TPB-odd	7	7	7	7	7	7	7	7	7	7	7	7	6	1	3	3	4	6	385	18	0
TPB-even	7	7	7	7	7	7	7	7	7	7	7	7	6	1	3	4	4	6	390	11	0
MPB	7	7	7	7	7	7	7	7	7	7	7	7	6	1	3	4	4	6	390	10	0
Auction 5																					
Optimal	6	1	2	1	3	3	3	3	5	5	5	5	1	1	2	4	4	6	399		
SMR	6	7	0	7	0	3	3	3	4	0	5	7	6	2	3	4	4	5	297	18	3
TPB-odd	6	7	2	2	2	3	3	3	5	5	5	5	1	1	3	4	4	6	373	14	0
TPB-even	6	1	2	2	2	2	3	3	5	5	5	6	1	1	3	4	4	5	381	9	0
MPB	6	1	1	1	3	3	3	3	5	5	5	5	6	1	0	4	4	6	395	9	1
Auction 6																					
Optimal	1	1	1	1	3	2	3	3	5	5	5	5	6	2	2	3	4	6	361		
SMR	7	7	7	7	3	7	3	3	5	5	5	5	6	2	2	3	4	6	346	13	0
TPB-odd	7	7	7	7	7	7	7	7	7	7	7	7	6	2	3	3	5	6	330	13	0
TPB-even	7	7	7	7	7	7	7	7	7	7	7	7	6	2	3	3	5	6	330	9	0
MPB	7	7	7	7	7	7	7	7	7	7	7	7	6	2	3	3	5	6	330	10	0