Modeling Healthcare Expenditures

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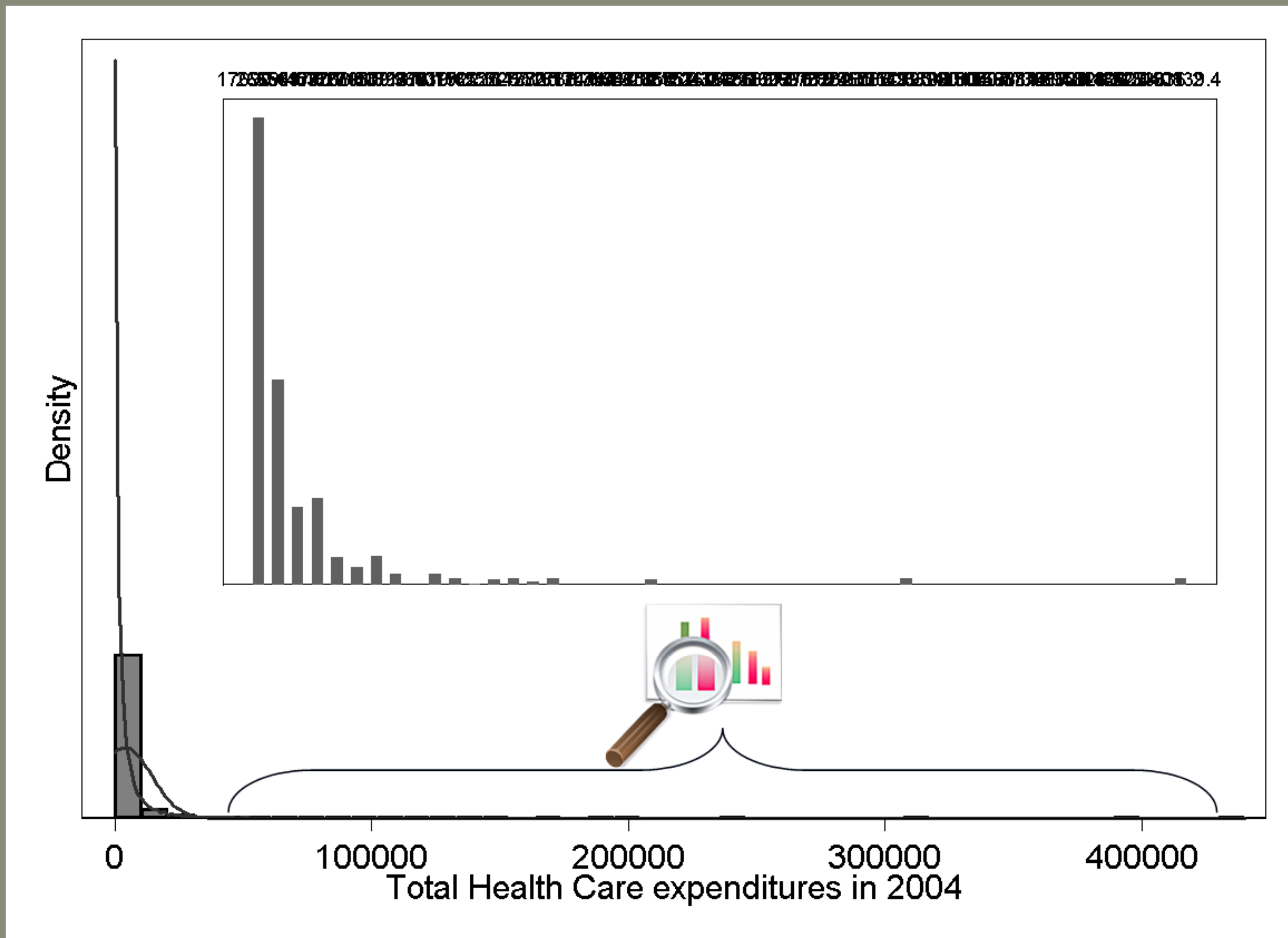
Goals

- Review parametric/semi-parametric statistical models for health care expenditures
- Intent to study how mean costs responds to shifts in covariate levels
- Will assume "selection of observable" throughout
- Main focus is on bias, will look at some efficiency issues too.

Healthcare costs in the U.S.



Slide 2

Healthcare costs in the U.S.

- A substantial fraction of people with no health care costs
- Handful of people with enormous yearly expenditures
- Often top 1% accounts for 25% of the total mean, sometimes the top 10% accounts for half of all costs

- Any attempt to study how the mean of such a distribution responds to a covariate would require careful attention to how different parts of the distribution moves in response to the changes in levels of that covariate

Alternative Methods

- Classes of models:
  - Single equation models
  - Multiple equation models
    - Two-part models
    - Multi-part models
    - Conditional density estimation
  - Mixture models
  - Propensity scores
  - Doubly robust models

Outline

- Single-equation models: relate mean of costs to covariates using a single functional form
  - Linear models
  - Transformation models
    - Log-OLS
    - Box-Cox
  - Generalized Linear models
- Application to datasets of ~1000 to 100,000
- Discuss goodness-of-fit tests

Notation

- Y = Costs
- X = Covariates = (X_T, X_{-T})
- X_T = A binary treatment variable
- β = regression coefficients
- $\mu = E(Y)$; $\mu(x) = E(Y|X=x)$
- Marginal effect: $d\mu(X)/dX$
- Incremental effect: $\mu(X=1) - \mu(X=0)$

An illustrative health-economic study

- Compare the 2-year cost of care under aggressive treatments (radiation or surgery) versus watchful waiting following diagnosis of prostate cancer
- Use SEER-Medicare linked database
- Data: diagnosis between 1995-2002
- Follow-up through 2004.
- No censoring issues
- Ignore issues with death

The parameters of Interest

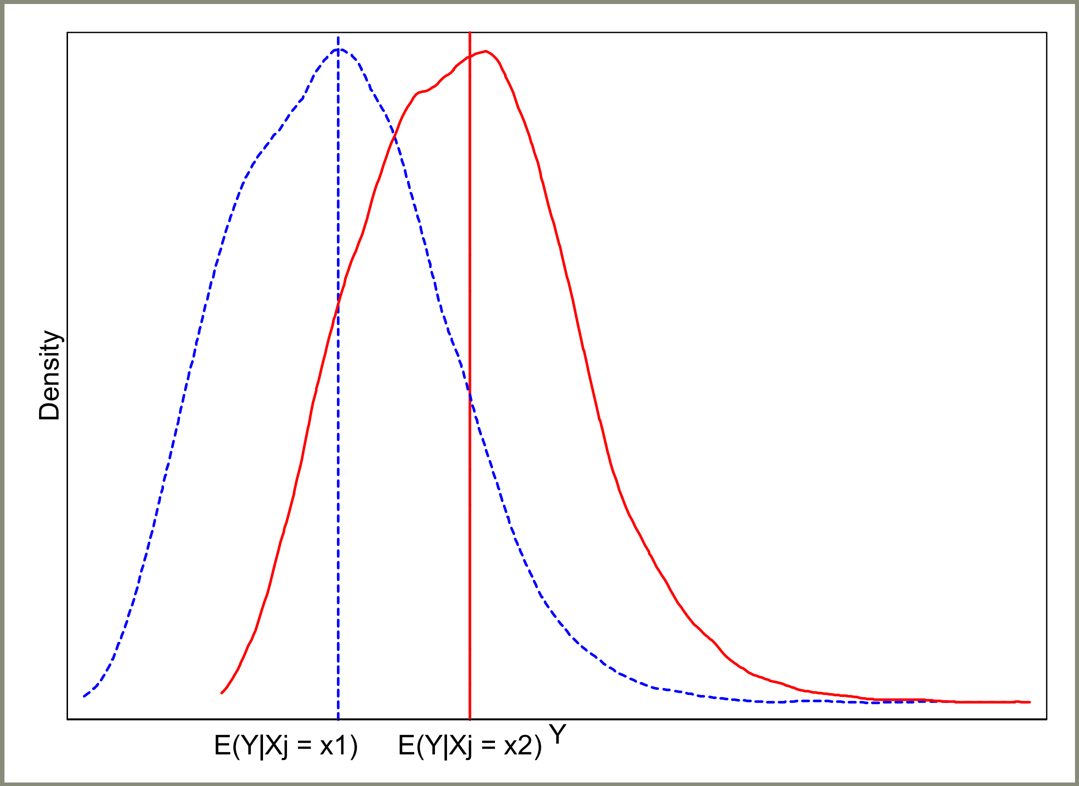
- Main parameters of interest:
  - μ(x) = E(y|x) y = Expenditures
  - Incremental effect of aggressive treatment vs WW on total expenditures (Δ_T)
  - Avg. Costs for TRT – Avg. Costs for WW

$$\Delta_T = E_{(X_{-T}|X_T=1)}\{\ \mu(X_T=1, x_{-T}) - \mu(X_T=0, x_{-T})\ \}$$

  - The interpretation of regression coefficients β is of secondary concern.

Single equation models: Linear model

- $Y = X\beta + \varepsilon$

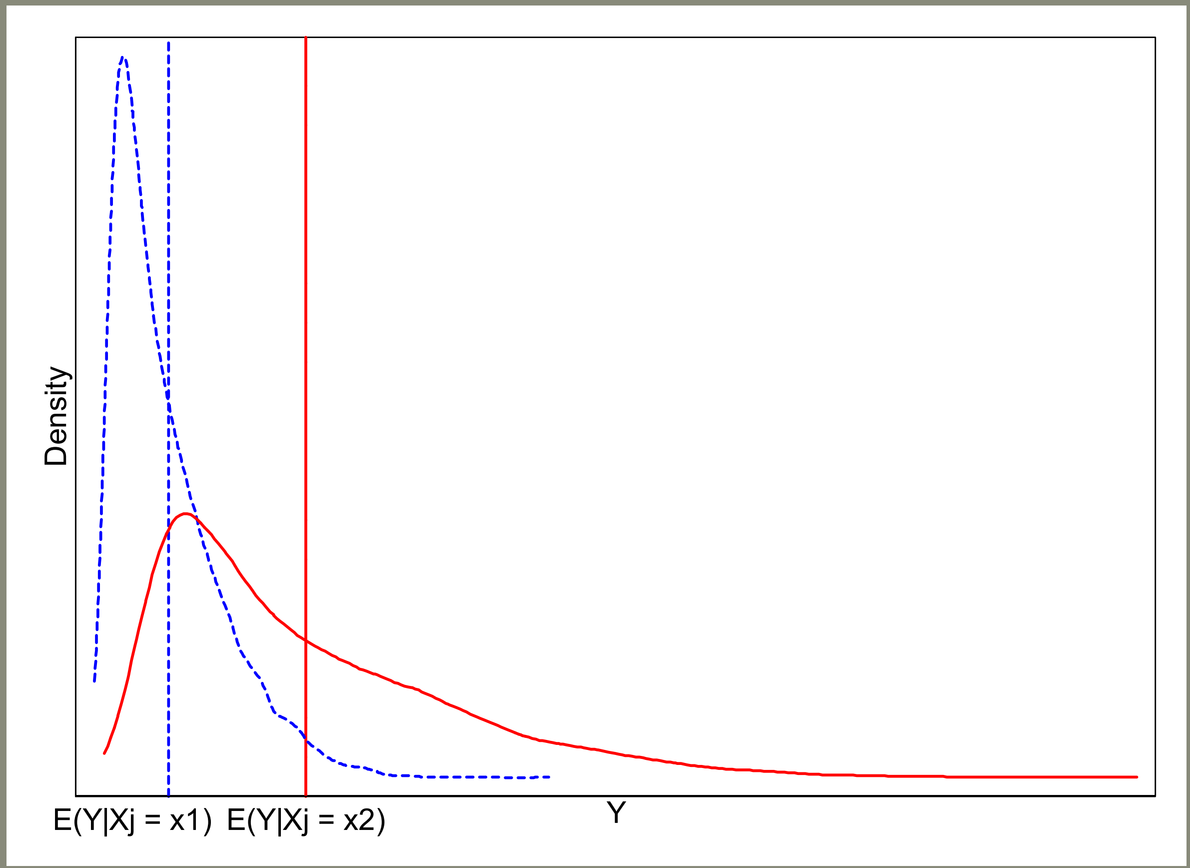


Single equation models: Linear model

- $Y = X\beta + \varepsilon$
- Assumes parallel shift for the entire distribution across levels of any one X, and for all X's
- Biased
  - the non-linearity in the response,
  - the instability caused by skewness and kurtosis,
- Inefficient
  - common failure to deal with the heteroscedasticity (variance increases with the mean)

Single equation models: Transformation models – log OLS

- $Ln(Y) = X\beta + \varepsilon$



Single equation models: Transformation models – log OLS

- $\ln(Y) = X\beta + \varepsilon$
- Assumes proportional shift for the entire distribution across levels of any one X, and for all X's
- Advantages:
  - Usually overcomes skewness issues
  - Allows for additive effects in the log-scale, which translates to non-additive effects in the raw scale
  - Reduces problems with heteroscedasticity & kurtosis

Transformation models

- **Scale of estimation $\neq$ scale of interest**
  - **If modeling $, results in log-$**
- **Model for geometric mean, not arithmetic mean**
  - **Models E{log(y)} and not log(E{y})**

Single equation models: Transformation models – log OLS

- $\ln(Y) = X\beta + \varepsilon$
- Disadvantages:
  - Creates a model for log-$, not $.
  - $E(\ln Y) \neq \ln(E(Y))$
  - Require retransformation, $E(Y) = \exp(X\beta + \varepsilon)$
    $= \exp(X\beta)*s$
  - Duan's smearing estimator provides an estimate for s
  - s-hat = Mean{exp(lnY – X β-hat)}

Single equation models: Transformation models – log OLS

- $Ln(Y) = X\beta + \varepsilon$
- Disadvantages:
  - $E(lnY) = X\beta$
  - $lnE(Y) = X\beta + ln\{s\}$
  - If $s = s(X)$,
    $dlnE(Y)/dX = \beta + dln\{s(X)\}/dX$
  - So, common interpretation of β no longer applies

  - If $\varepsilon \sim Normal(0, \sigma^2)$, $ln(s) = 0.5\ \sigma^2$
  - With log-scale heteroscedasticity,
    $Bias = 0.5 d\sigma^2(X)/dX$

Single equation models: Transformation models – log OLS

Table 1
Univariate statistics on log medical expenditures for users* health insurance experiment

Plan	Mean*	Variance**	Skewness	Kurtosis	Percentile					
					10%	25%	50%	75%	90%	95%
Free	6.349	2.083	0.202	3.385	4.541	5.469	6.289	7.166	8.290	8.976
25%	6.128	2.133	0.456	3.539	4.393	5.191	6.004	6.907	8.117	8.849
50%	6.039	2.003	0.513	4.067	4.265	5.134	5.969	6.766	7.906	8.628
95%	5.998	2.343	0.464	3.226	4.158	4.948	5.873	6.862	8.120	8.892
Individual deductible	6.153	2.384	0.305	2.993	4.237	5.068	6.039	7.093	8.322	8.876

*In 1995 US$, adjusted using the medical care component of the Consumer Price Index.
**Plan differences are significant at $F < 0.001$ for both mean and variance, by F-tests. Tests for variance based on Park Test.

Manning JHE, 1998

% change in costs between Free & 95%:

From log-OLS regression: 6.349 – 5.998 = 0.351 (or 35.1%)

Truth (assuming log-normality) = 6.349 – 5.998 + 0.5*(2.083 – 2.343)

= 0.221 (or 22.1%)

Single equation models: Transformation models – log OLS

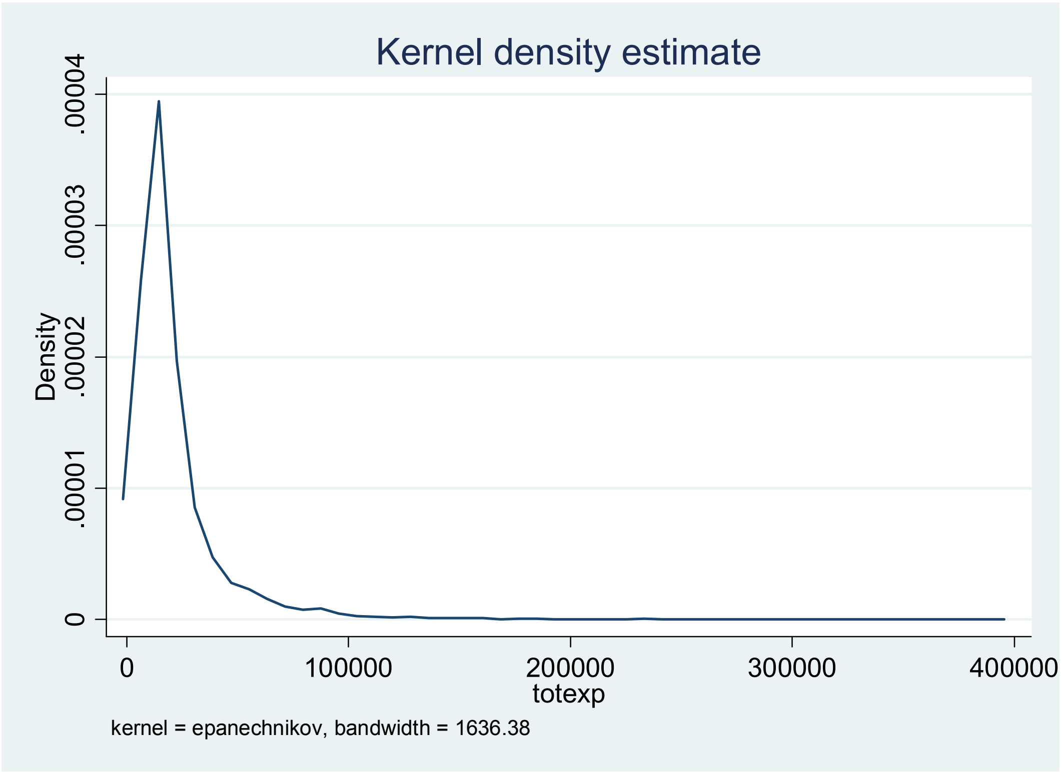
- Plan specific smearing estimate can address bias
- Difficult to come up with appropriate smearing estimator when there are multiple X's that lead to heteroscedasticity
- Park Test – test to determine heteroscedasticity in log-scale
  - Use squared log-scale residuals as an estimator for variance
  - Study if X's predict this estimated variance
  - `reg lny x, robust`
  - `predict r, res`
  - `gen r2= r^2`
  - `glm r2 x, link(log) family(gamma) robust`

Let's try out in Stata

MODELLING HEALTHCARE EXPENDITURES

```
---------------------------------------------------------------------
. use week2_data1, clear
. summ totexp,d

                          totexp
-------------------------------------------------------------
      Percentiles      Smallest
 1%        78.32           2.78
 5%       907.92              3
10%      2702.61           3.63       Obs                7129
25%      8912.18           4.18       Sum of Wgt.        7129

50%     14536.54                      Mean           20265.17
                        Largest       Std. Dev.      22937.29
75%     23375.21       279487.5
90%      41139.5       307933.6       Variance       5.26e+08
95%     59268.49       316789.1       Skewness       4.537538
99%     112434.3       393908.4       Kurtosis       39.71721
```

```
. kdensity totexp, saving(grp1, replace)
```



```
. bysort aggr_trt: summm totexp, d

-> aggr_trt = 0

                              totexp
-------------------------------------------------------------
      Percentiles      Smallest
 1%         50.8           3.63
 5%        648.2           4.18
10%      1423.23           4.57       Obs                2219
25%      4738.09              5       Sum of Wgt.        2219

50%     12317.91                      Mean           19201.76
                        Largest       Std. Dev.      24102.25
75%     23087.59         166289
90%     43853.15       204529.9       Variance       5.81e+08
95%     65831.95       236276.2       Skewness       3.529187
99%     114372.3       316789.1       Kurtosis       24.20243

-------------------------------------------------------------------
```

```
-> aggr_trt = 1

                              totexp
-------------------------------------------------------------
      Percentiles      Smallest
 1%        99.18           2.78
 5%      1172.94              3
10%     5141.575           4.37       Obs                4910
25%        10071           4.55       Sum of Wgt.        4910

50%     15343.07                      Mean           20745.77
                        Largest       Std. Dev.      22376.88
75%     23522.77       263867.3
90%     39927.79       279487.5       Variance       5.01e+08
95%     57092.22       307933.6       Skewness       5.114373
99%     110871.9       393908.4       Kurtosis       48.88715
```

```
. di (20745.77 -  19201.76)/ 19201.76
.08040982
```

```
. /* OLS */

. reg totexp aggr_trt, robust

Linear regression                                      Number of obs =    7129
                                                       F(  1,  7127) =    6.55
                                                       Prob > F      =  0.0105
                                                       R-squared     =  0.0010
                                                       Root MSE      =   22928

------------------------------------------------------------------------------
             |               Robust
     totexp  |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    aggr_trt |   1544.008    603.106     2.56   0.010     361.7408    2726.274
       _cons |   19201.76   511.6135    37.53   0.000     18198.85    20204.67
------------------------------------------------------------------------------
```

. reg totexp aggr_trt agec white black single married well mod stage1 stage2 lnc, robust

totexp	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
aggr_trt	**3490.596**	**636.6122**	**5.48**	**0.000**	**2242.646**	**4738.545**
agec	205.2538	52.4853	3.91	0.000	102.367	308.1406
white	1269.85	1027.947	1.24	0.217	-745.2315	3284.931
black	3706.038	1426.357	2.60	0.009	909.9544	6502.122
single	946.6497	1241.496	0.76	0.446	-1487.051	3380.351
married	-1048.995	696.5203	-1.51	0.132	-2414.382	316.3918
well	-4717.357	1261.522	-3.74	0.000	-7190.314	-2244.4
mod	-2995.753	745.5633	-4.02	0.000	-4457.279	-1534.227
stage1	-304.7167	602.6593	-0.51	0.613	-1486.108	876.6747
stage2	-1803.271	804.8815	-2.24	0.025	-3381.078	-225.4639
lnc	7871.044	502.2525	15.67	0.000	6886.48	8855.608
_cons	15268.15	1349.965	11.31	0.000	12621.82	17914.48

-

.

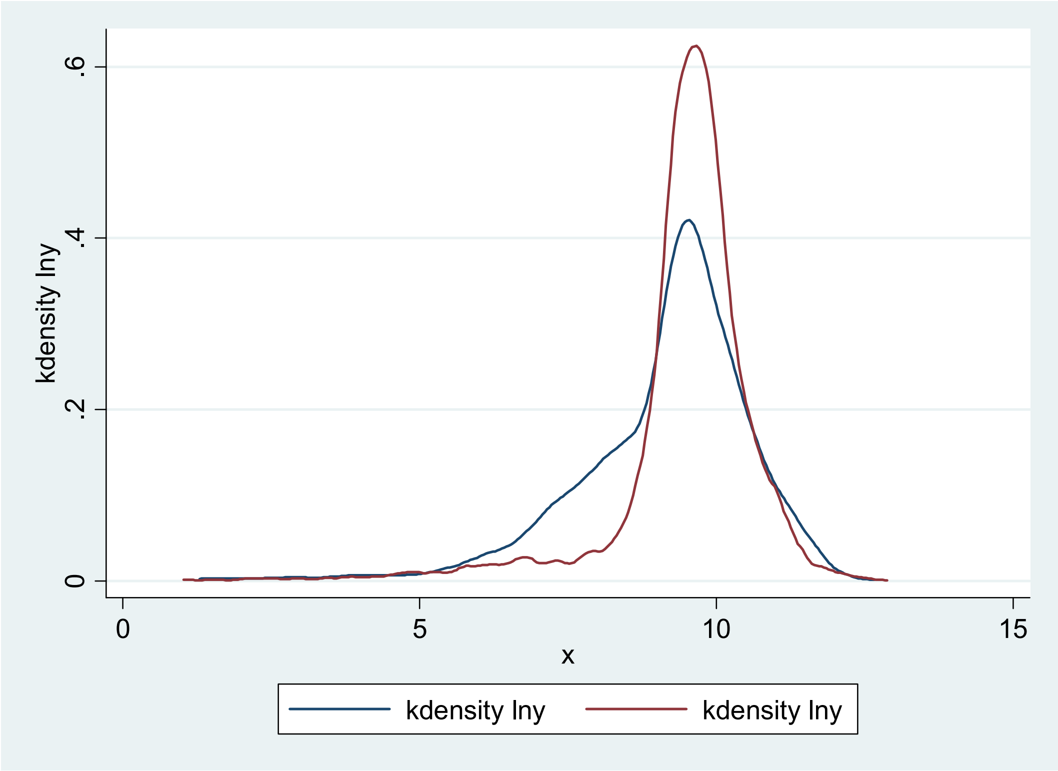
.

```
. /* Log-OLS */
. capture drop lny
. gen lny = ln(totexp)
. reg lny aggr_trt, robust

Linear regression                                      Number of obs =    7129
                                                       F(  1,  7127) =   93.48
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.0150
                                                       Root MSE      =  1.2894

------------------------------------------------------------------------------
             |               Robust
         lny |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    aggr_trt |   .3431855    .035495     9.67   0.000     .2736047    .4127662
       _cons |   9.147605   .0310711   294.41   0.000     9.086697    9.208514
------------------------------------------------------------------------------
. predict xb, xb
. gen mu = exp(xb)
. summ mu totexp

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
          mu |      7129    12040.35    1780.561   9391.919   13237.27
      totexp |      7129    20265.17    22937.29       2.78   393908.4
```

```
. **** Overall smearing
. drop mu
. gen smr = exp(lny - xb)
. summ smr

  Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
         smr |      7129    1.715782    2.016464      .00021   33.72997

. gen smear = r(mean)
. gen  mu = exp(xb)*smear
. summ mu totexp

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
          mu |      7129    20658.62    3055.054   16114.49   22712.27
      totexp |      7129    20265.17    22937.29       2.78   393908.4
```

```
. bysort aggr_trt: summ mu totexp
-----------------------------------------------------------------------------
-> aggr_trt = 0
    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+----------------------------------------------------------
          mu |      2219    16114.49           0   16114.49   16114.49
      totexp |      2219    19201.76    24102.25       3.63   316789.1

-----------------------------------------------------------------------------
-> aggr_trt = 1
    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+----------------------------------------------------------
          mu |      4910    22712.27           0   22712.27   22712.27
      totexp |      4910    20745.77    22376.88       2.78   393908.4

/* Notice difference in log scale Variance */
. bysort aggr_trt: summ lny
-----------------------------------------------------------------------------
-> aggr_trt = 0
    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+----------------------------------------------------------
         lny |      2219    9.147605    1.463769   1.289233   12.66599
----------------------------------------------------------------------------
-> aggr_trt = 1
-------------+----------------------------------------------------------
         lny |      4910    9.490791    1.202411   1.022451   12.88387
```

```
. gr twoway (kdensity lny if aggr_trt==0) (kdensity lny if
aggr_trt==1), saving
```




```
.
```

```
. **** Trt specific smearing
. drop mu
. summ smr if aggr_trt ==0
    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
         smr |      2219    2.044498    2.566275   .0003865   33.72997

. gen smear0=r(mean)
. summ smr if aggr_trt ==1

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
         smr |      4910    1.567224    1.690446     .00021   29.75754

. gen smear1=r(mean)
. gen mu =exp(xb)*smear0 if aggr_trt ==0
. replace mu =exp(xb)*smear1 if aggr_trt ==1

. summ mu totexp

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
          mu |      7129    20265.17    714.9419   19201.76   20745.77
      totexp |      7129    20265.17    22937.29       2.78   393908.4
```

```
. bysort aggr_trt: summ mu totexp

-------------------------------------------------------------------->
aggr_trt = 0

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
          mu |      2219    19201.76           0   19201.76   19201.76
      totexp |      2219    19201.76    24102.25       3.63   316789.1

-----------------------------------------------------------------------
-> aggr_trt = 1

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
          mu |      4910    20745.77           0   20745.77   20745.77
      totexp |      4910    20745.77    22376.88       2.78   393908.4
```

```
. **** ln-OLS regression
. drop mu xb
. reg lny aggr_trt agec white black single married well mod
stage1 stage2 lnc, robust
```

lny	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
aggr_trt	**.4609178**	**.0372838**	**12.36**	**0.000**	**.3878303**	**.5340052**
agec	.0166473	.0027592	6.03	0.000	.0112385	.0220561
white	.2875066	.0703636	4.09	0.000	.1495731	.4254401
black	.239347	.0876897	2.73	0.006	.0674492	.4112448
single	.0702465	.066109	1.06	0.288	-.0593468	.1998397
married	-.0016466	.0381762	-0.04	0.966	-.0764833	.07319
well	-.3312179	.0710317	-4.66	0.000	-.4704613	-.1919746
mod	-.1428405	.0378877	-3.77	0.000	-.2171118	-.0685693
stage1	-.0147308	.0337002	-0.44	0.662	-.0807933	.0513317
stage2	-.0830997	.0480434	-1.73	0.084	-.177279	.0110797
lnc	.5280045	.0233166	22.65	0.000	.4822971	.5737119
_cons	8.627953	.0881616	97.87	0.000	8.455131	8.800776

```
. ** Park test - detect heteroscedasticity in log-scale
. predict xb, xb
. gen res2 = (lny- xb)^2
. glm res2 aggr_trt  agec  white black single married well
mod stage1 stage2 lnc, family(gamma) link(log) robust
------------------------------------------------------------------------------
             |               Robust
        res2 |      Coef.   Std. Err.      z    P>|z|  [95% Conf. Interval]
-------------+----------------------------------------------------------------
    aggr_trt |  -.5202051   .0652508   -7.97   0.000 -.6480943   -.3923159
        agec |  -.0253315   .0054215   -4.67   0.000 -.0359575   -.0147055
       white |  -.4874537   .1051322   -4.64   0.000 -.6935091   -.2813983
       black |  -.1256915   .1388546   -0.91   0.365 -.3978414    .1464585
      single |  -.0352572   .1358728   -0.26   0.795 -.3015629    .2310485
     married |  -.1367799    .073717   -1.86   0.064 -.2812624    .0077027
        well |    .100519   .1393567    0.72   0.471 -.172615     .373653
         mod |  -.0869734   .0785356   -1.11   0.268 -.2409005    .0669536
      stage1 |   -.057802   .0720468   -0.80   0.422 -.1990111    .0834072
      stage2 |  -.0547015   .1094026   -0.50   0.617 -.2691267    .1597236
         lnc |  -.5347174    .046929  -11.39   0.000 -.6266966   -.4427382
       _cons |   1.597288   .1376163   11.61   0.000 1.327565    1.867011
------------------------------------------------------------------------------

. test
           chi2( 11) =  283.67
         Prob > chi2 =    0.0000
```

Single equation models: Transformation models – Box-Cox

$$\left(Y^{\lambda}-1\right)/\lambda = X\beta+\varepsilon$$

- λ →0, Box-Cox → log-OLS
- If λ is known apriori, can use Duan's smearing estimator
- If λ is estimated, use different estimator

$$E(Y|X)=(X\beta\lambda+1)^{1/\lambda}\left\{1+\frac{\sigma^2(1-\lambda)}{2(1+X\beta\lambda)^2}\right\}$$

- All concerns about retransformation persist

Single equation models: Generalized Linear Models

- g{E(Y|X)} = Xβ, g{.} is a link function
- No retransformation problem as link function applies to E(Y) and NOT Y.
- Estimation of GLM
  - Parametric – FIML
  - Semi-parametric – Quasi-likelihood
- FIML:
  - Gamma, Exponential, Inverse Gaussian, Poisson, Neg. Binomial
  - `glm  y x, link(log) family(gamma) robust`

Single equation models: Generalized Linear Models

- Quasi-likelihood approach
  1. Relate $\mu(X)$ to the linear predictor $X\beta$ using a link function. $g(\mu(X)) = X\beta$
  2. $V(Y) \sim h(\mu)$; e.g. $V(Y) = \phi\mu^2$ => Gamma Variance
  3. Does not make any distributional assumption beyond 1st and 2nd moments
  4. Estimate coefficients by solving quasi-score equations

$$\sum_{i=1}^{N} G_{\beta_j}^{i} = \sum_{i=1}^{N} (y_i - \mu_i) V_i^{-1} (\partial\mu_i / \partial\beta_j) = 0$$

- `glm y x, link(log) fam(gamma) robust irls`

Single equation models: Generalized Linear Models

- **Example: Gamma with log link**
  - $\ln(\mu) = X\beta$ or $\mu = \exp(X\beta)$
  - $V = \theta_1 \mu^2$
  - If mean and variance functions are correctly specified, β_1 provides a consistent estimator of $\partial \ln(E\{y|X\}) / \partial x_1$
  - If the empirical distribution of Y is truly Gamma, then FIML will generate the MVUE estimator for β
  - If the empirical distribution of Y is deviates from Gamma, but mean and variance functions are appropriately specified, quasi-likelihood will be a more robust estimator.

To Stata...

```
. /* Generalized Linear Models */
. glm totexp aggr_trt, link(log) family(gamma) robust

Iteration 3:   log pseudolikelihood = -77820.337

Generalized linear models                      No. of obs        =      7129
Optimization     : ML                          Residual df       =      7127
                                               Scale parameter =  1.291687
Deviance         =  7586.035219                1/df) Deviance =  1.064408
Pearson          =  9205.852281                1/df) Pearson  =  1.291687

Variance function: V(u) = u^2                              [Gamma]
Link function    : g(u) = ln(u)                            [Log]

                                               AIC               =  21.83261
Log pseudolikelihood =  -77820.3371            BIC               = -55644.18

------------------------------------------------------------------------------
             |               Robust
      totexp |      Coef.   Std. Err.    z     P>|z|  [95% Conf. Interval]
-------------+----------------------------------------------------------------
    aggr_trt |   .0773403   .0307692   2.51   0.012  .0170338    .1376469
       _cons |   9.862757   .0266422 370.19   0.000  9.810539    9.914975
------------------------------------------------------------------------------
```

```
. predict mu, mu
. summm mu totexp

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
          mu |      7129    20265.17    714.9419   19201.76   20745.77
      totexp |      7129    20265.17    22937.29       2.78   393908.4

. bysort aggr_trt: summm mu totexp
----------------------------------------------------------------------
-> aggr_trt = 0

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
          mu |      2219    19201.76           0   19201.76   19201.76
      totexp |      2219    19201.76    24102.25       3.63   316789.1

----------------------------------------------------------------------
-> aggr_trt = 1

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
          mu |      4910    20745.77           0   20745.77   20745.77
      totexp |      4910    20745.77    22376.88       2.78   393908.4
```

. ** GAMMA FIML

. glm totexp aggr_trt agec white black single married well mod stage1 stage2 lnc, link(log) family(gamma) robust

```
Iteration 3:   log pseudolikelihood = -77563.068

------------------------------------------------------------------------------
             |               Robust
      totexp |      Coef.   Std. Err.       z     P>|z|   [95% Conf. Interval]
-------------+----------------------------------------------------------------
    aggr_trt |   .1794719   .0320742     5.60    0.000   .1166076    .2423362
        agec |    .010508   .0025216     4.17    0.000   .0055658    .0154501
       white |   .0854313   .0532399     1.60    0.109  -.0189169    .1897796
       black |   .1795773    .067756     2.65    0.008   .0467779    .3123767
      single |   .0333528   .0559583     0.60    0.551  -.0763235     .143029
     married |  -.0542054   .0347019    -1.56    0.118  -.1222198     .013809
        well |  -.2484179   .0642366    -3.87    0.000  -.3743194   -.1225165
         mod |  -.1447476   .0333845    -4.34    0.000  -.21018   -.0793151
      stage1 |  -.0276048   .0291643    -0.95    0.344  -.0847657    .0295562
      stage2 |   -.091044   .0429937    -2.12    0.034  -.1753101   -.0067779
         lnc |   .3698705   .0214508    17.24    0.000   .3278276    .4119133
       _cons |   9.628452   .0680018   141.59    0.000  9.495171    9.761733
------------------------------------------------------------------------------
-
```

```
. ** GAMMA QMLE
. glm totexp aggr_trt  agec  white black single married
well mod stage1 stage2 lnc, link(log) family(gamma) robust
irls

Iteration 6:   deviance =  7071.498
------------------------------------------------------------------------------
             |               Semi-Robust
      totexp |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    aggr_trt |   .1794717   .0327351     5.48   0.000     .1153121    .2436313
        agec |    .010508   .0025526     4.12   0.000      .005505     .015511
       white |   .0854311   .0535442     1.60   0.111    -.0195136    .1903758
       black |   .1795774   .0681181     2.64   0.008     .0460683    .3130864
      single |    .033353   .0559614     0.60   0.551    -.0763292    .1430353
     married |  -.0542054   .0349642    -1.55   0.121    -.1227339    .0143232
        well |  -.2484182   .0648256    -3.83   0.000     -.375474   -.1213623
         mod |  -.1447475   .0337717    -4.29   0.000    -.2109388   -.0785563
      stage1 |  -.0276047   .0292326    -0.94   0.345    -.0848996    .0296902
      stage2 |  -.0910438   .0436959    -2.08   0.037    -.1766862   -.0054015
         lnc |   .3698705   .0216682    17.07   0.000     .3274016    .4123394
       _cons |   9.628452   .0676074   142.42   0.000     9.495944     9.76096
------------------------------------------------------------------------------
```

Method of recycled predictions

- In order to calculate effect in the additive scale
- Estimate parameters in

$$\exp(\beta0 + \beta1*TRT + \beta2*X)$$

- Predict after turning on & off the TRT
- $mu1 = \exp(\beta0 + \beta1*1 + \beta2*X)$
- $mu0 = \exp(\beta0 + \beta1*0 + \beta2*X)$
- Summ mu1 , mu0

```
. ** THE METHOD OF RECYCLED PREDICTIONS - TO ESTIMATE EFFECT IN
ADDITIVE-SCALE
. gen trtold=aggr_trt
. replace aggr_trt =1
(2219 real changes made)
. predict mu1, mu

. replace aggr_trt=0
(7129 real changes made)
. predict mu0, mu

. replace aggr_trt = trtold
(4910 real changes made)

. summ mu1 mu0

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
         mu1 |      7129    21447.39    6183.882   12332.27   59634.99
         mu0 |      7129    17923.83    5167.942   10306.23   49837.65
```

Single equation models: Generalized Linear Models

- **Limitations**
  - Inefficient when empirical distribution of Y has fat tails (kurtotic)
  - Log Link is the most common link function used, but assumption that $\log(\mu) = X\beta$ may be violated
  - Appropriate link or variance functions not known a-priori
  - Various tests exist, not always sufficient to overcome these problems

Single equation models: Modified Park Test following GLM

- **Proposed by Manning & Mullahy (2001)**
  - $Var(Y|X) = \theta_1\mu^{\theta 2}$
  - For Gamma, we assume $\theta_1=1$, $\theta_2=2$
  - Test: $\ln(Var(Y|X)) = \ln(\theta_1) + \theta_2 * \ln(\mu)$
  - Use Residual ^2 as an estimate for variance.
  - `glm r2 lnmu, link(log) family(gamma) robust`

```
/* Modified Park test */
. predict mu, mu
. gen lnmu = ln(mu)
. gen r2 = (totexp - mu)^2
. glm r2 lnmu, link(log) family(gamma) robust
------------------------------------------------------------------------------
             |               Robust
          r2 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        lnmu |   1.468981   .2492923     5.89   0.000     .9803774    1.957585
       _cons |   5.419501   2.497738     2.17   0.030     .5240253    10.31498
------------------------------------------------------------------------------

. test lnmu=1
           chi2(  1) =    3.54
         Prob > chi2 =    0.0599

. test lnmu=2
           chi2(  1) =    4.54
         Prob > chi2 =    0.0332

. test lnmu=3
           chi2(  1) =   37.72
         Prob > chi2 =    0.0000
. drop mu lnmu r2
```

Single equation models: Generalized Gamma Models

- Generalized Gamma FIML
- Three parameter distribution
  - Mean is a function of all three parameters
  - Only of them is related to Xβ, implicit log-link
  - Special cases are Gamma, Exponential, Weibull, log-normal
  - Except for log-normal case, no retransformation problem
  - `stset y`
  - `gengam2 x`

```
. /* Generalized Gamma Regression */

. stset totexp
. ****** Download -gengam2- command from
http://home.uchicago.edu/~abasu/index_files/Pagec317.htm  AND place
in C:\ado\Personal\
.
. gengam2 aggr_trt  agec  white black single married well
mod stage1 stage2 lnc, robust
```

_t	Coef.	Robust Std. Err.	z	P>|z|	[95% Conf.	Interval]
aggr_trt	**.193495**	**.0333561**	**5.80**	**0.000**	**.1281183**	**.2588717**
agec	.0104401	.002439	4.28	0.000	.0056598	.0152205
white	.0913009	.0521438	1.75	0.080	-.0108992	.1935009
black	.1798185	.0662471	2.71	0.007	.0499766	.3096605
single	.0355918	.0546193	0.65	0.515	-.07146	.1426436
married	-.0508413	.0332982	-1.53	0.127	-.1161046	.0144221
well	-.2535959	.0625437	-4.05	0.000	-.3761792	-.1310125
mod	-.1453943	.0321754	-4.52	0.000	-.2084568	-.0823318
stage1	-.0261988	.0281348	-0.93	0.352	-.0813419	.0289444
stage2	-.0900821	.0410532	-2.19	0.028	-.1705449	-.0096192
lnc	.3746245	.0203089	18.45	0.000	.3348198	.4144292
_cons	9.578895	.0675292	141.85	0.000	9.44654	9.711249

```
     /ln_sig |   -.053114   .0147078    -3.61   0.000   -.0819408   -.0242872
      /kappa |   .8766324   .0314132    27.91   0.000    .8150637    .9382012
-------------+----------------------------------------------------------------
       sigma |   .9482719    .013947                       .9213265    .9760053
------------------------------------------------------------------------------


Tests for identifying distributions
------------------------------------------------------------------------------

Distributions                       |      chi2       df       Prob>chi2
------------------------------------|-----------------------------------------
Std. Gamma   (kappa = sigma)        |       3.27        1          0.0704
Log Normal   (kappa = 0)            |     778.77        1          0.0000
Weibull      (kappa = 1)            |      15.42        1          0.0001
Exponential  (kappa = sigma = 1)    |      50.89        2          0.0000
------------------------------------------------------------------------------


New Variables for Schoenfield Residuals created: _h*, _c*
```

. drop   smr smear smear0 smear1 res2 _*

Goodness-of-fit tests

- Pearson Correlation
  - Correlation between raw-scale predictions and residuals
- Pregibon's Link Test / Reset Test
  - Run same model with Xβ and Xβ^2 as covariates
- Hosmer-Lemeshow Test
  - Plot mean residuals across deciles of Xβ.
  - Perform joint test that all means are zero
- Copa's Test

```
. /* GOODNESS OF FIT TESTS */
. ** For OLS
. reg totexp aggr_trt agec  white black single married well
mod stage1 stage2 lnc, robust
. predict xb, xb
. gen mu_ols= xb
. gen res_ols = totexp - mu_ols
.           ** Pearson Corr
.           pwcorr res_ols mu_ols, sig
             |  res_ols   mu_ols
-------------+------------------
     res_ols |   1.0000
             |
      mu_ols |  -0.0000   1.0000
             |   1.0000
.           ** Link Test
.           gen xb2=xb^2
.           reg totexp xb xb2, robust
             |               Robust
      totexp |      Coef.   Std. Err.      t    P>|t|   [95% Conf. Interval]
-------------+----------------------------------------------------------------
          xb |   .2645214   .3449794     0.77   0.443  -.4117406    .9407833
         xb2 |   .0000171   8.43e-06     2.03   0.043   5.46e-07    .0000336
       _cons |   7353.987   3357.482     2.19   0.029 772.3251    13935.65
------------------------------------------------------------------------------
```

```
.        ** H-L test
.        xtile xbtile =xb, nq(10)
.        qui tab xbtile, gen(xbt)
.        reg res_ols xbt1 xbt2 xbt3 xbt4 xbt5 xbt6 xbt7
xbt8 xbt9 xbt10, nocons robust
```

res_ols	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
xbt1	857.5334	614.4044	1.40	0.163	-346.8819	2061.949
xbt2	681.1362	674.0399	1.01	0.312	-640.1824	2002.455
xbt3	-525.1787	642.1524	-0.82	0.413	-1783.988	733.631
xbt4	-142.7663	699.025	-0.20	0.838	-1513.063	1227.531
xbt5	938.962	933.5517	1.01	0.315	-891.0768	2769.001
xbt6	-638.9313	716.1985	-0.89	0.372	-2042.893	765.0306
xbt7	-1635.909	696.5204	-2.35	0.019	-3001.296	-270.5221
xbt8	-2250.963	710.2455	-3.17	0.002	-3643.255	-858.6706
xbt9	707.1719	1138.647	0.62	0.535	-1524.915	2939.259
xbt10	2007.334	1231.866	1.63	0.103	-407.489	4422.158

```
.test xbt1 xbt2 xbt3 xbt4 xbt5 xbt6 xbt7 xbt8 xbt9 xbt10

       F( 10,  7119) =    2.41
            Prob > F =    0.0074
. drop xb xb2 xbt* xbtile
```

```
. ** For lnOLS
. reg lny aggr_trt  agec  white black single married well
mod stage1 stage2 lnc, robust
. predict xb, xb
. gen smr = exp(lny- xb)
. qui summ smr
. gen smear = r(mean)
. gen mu_lols= exp(xb)*smear
. gen res_lols = totexp - mu_lols
.          ** Pearson Corr
.          pwcorr res_lols mu_lols, sig
             | res_lols  mu_lols
-------------+------------------
    res_lols |   1.0000
             |
     mu_lols |  -0.1776   1.0000
             |   0.0000
.          ** Link Test
.          gen xb2=xb^2
.          reg lny xb xb2, robust
             |               Robust
         lny |      Coef.   Std. Err.      t    P>|t|   [95% Conf. Interval]
-------------+----------------------------------------------------------------
          xb |   7.586948   1.443073     5.26   0.000   4.758097     10.4158
         xb2 |  -.3492898   .0757058    -4.61   0.000  -.4976956   -.2008839
```

```
      _cons |   -30.9968   6.869053    -4.51   0.000  -44.46218   -17.53141
------------------------------------------------------------------------------
```

```
.        ** H-L test
.        xtile xbtile =xb, nq(10)
.        qui tab xbtile, gen(xbt)
.        reg res_lols xbt1 xbt2 xbt3 xbt4 xbt5 xbt6 xbt7 xbt8 xbt9 xbt10, nocons robust
```

res_lols	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
xbt1	2479.074	635.594	3.90	0.000	1233.121	3725.027
xbt2	2585.675	755.5887	3.42	0.001	1104.497	4066.853
xbt3	608.5663	694.4084	0.88	0.381	-752.6807	1969.813
xbt4	573.4585	733.1452	0.78	0.434	-863.724	2010.641
xbt5	1438.745	775.6945	1.85	0.064	-81.84715	2959.336
xbt6	-215.099	826.1292	-0.26	0.795	-1834.558	1404.36
xbt7	-2386.189	757.4182	-3.15	0.002	-3870.954	-901.4244
xbt8	-3506.212	839.8621	-4.17	0.000	-5152.592	-1859.833
xbt9	-3966.24	1005.915	-3.94	0.000	-5938.133	-1994.347
xbt10	-10547.35	1207.501	-8.73	0.000	-12914.42	-8180.294

```
. test xbt1 xbt2 xbt3 xbt4 xbt5 xbt6 xbt7 xbt8 xbt9 xbt10
       F( 10,  7119) =   15.10
            Prob > F =    0.0000
. drop xb xb2 xbt* xbtile
```

```
. ** For log-GLM
. glm totexp aggr_trt  agec  white black single married
well mod stage1 stage2 lnc, link(log) family(gamma) robust
. predict xb, xb
. gen mu_glm = exp(xb)
. gen res_glm = totexp - mu_glm
.          ** Pearson Corr
.          pwcorr res_glm mu_glm, sig
             |  res_glm   mu_glm
-------------+------------------
     res_glm |   1.0000
             |
      mu_glm |  -0.0062   1.0000
             |   0.6022
.          ** Link Test
.          gen xb2=xb^2
.          glm totexp xb xb2, link(log) family(gamma) robust
------------------------------------------------------------------------------
             |               Robust
      totexp |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          xb |   2.601821    3.09824     0.84   0.401    -3.470618     8.67426
         xb2 |  -.0806185   .1555559    -0.52   0.604    -.3855025    .2242655
       _cons |  -7.950714   15.41964    -0.52   0.606    -38.17266    22.27123
------------------------------------------------------------------------------
```

```
.         ** H-L test
.         xtile xbtile =xb, nq(10)
.         qui tab xbtile, gen(xbt)
.         reg res_glm xbt1 xbt2 xbt3 xbt4 xbt5 xbt6 xbt7
xbt8 xbt9 xbt10, nocons robust
```

res_glm	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
xbt1	-781.7378	554.103	-1.41	0.158	-1867.944	304.4688
xbt2	473.2616	736.601	0.64	0.521	-970.6954	1917.219
xbt3	-323.851	626.755	-0.52	0.605	-1552.477	904.7751
xbt4	49.60152	702.7822	0.07	0.944	-1328.061	1427.264
xbt5	1288.524	797.3756	1.62	0.106	-274.5689	2851.618
xbt6	207.3119	861.6822	0.24	0.810	-1481.841	1896.465
xbt7	-558.6503	690.9381	-0.81	0.419	-1913.094	795.7937
xbt8	-1955.442	718.1396	-2.72	0.006	-3363.209	-547.6744
xbt9	1364.908	1148.222	1.19	0.235	-885.9489	3615.764
xbt10	156.9733	1227.114	0.13	0.898	-2248.535	2562.482

```
. test xbt1 xbt2 xbt3 xbt4 xbt5 xbt6 xbt7 xbt8 xbt9 xbt10
       F( 10,  7119) =    1.48
            Prob > F =    0.1382

. drop xb xb2 xbt* xbtile
```

Single equation models: Generalized Linear Models - EEE

Flexible Mean Model

$$\frac{\left(\mu^{\lambda}-1\right)}{\lambda}=X\beta \quad \text{if } \lambda \neq 0$$

$$\log(\mu)=X\beta \quad \text{if } \lambda = 0$$

Flexible Variance Model

$V(Y) = \phi\mu^{\theta}$

$\theta = 1$ **=> Poisson**

$\theta = 2$ **=> Gamma**

$\theta = 3$ **=> Inv. Gauss**

Single equation models: Generalized Linear Models - EEE

- **Goal: estimate $\gamma = (\beta^T, \lambda, \phi, \theta)^T$**
- **EEE estimator estimates:**

  Mean model parameters (β, λ) and

  Var. model parameters (ϕ, θ)

  simultaneously using estimating equations.
- **Extended Estimating Equations (EEE) ....**

Single equation models: Generalized Linear Models - EEE

MEAN MODEL

$$G^{i}_{\beta_j} = (y_i - \mu_i) V_i^{-1} (\partial \mu_i / \partial \beta_j)$$

$$G^{i}_{\lambda} = (y_i - \mu_i) V_i^{-1} (\partial \mu_i / \partial \lambda)$$

VAR. MODEL

$$G^{i}_{\phi} = \left[(y_i - \mu_i)^2 - V_i \right] V_i^{-2} (\partial V_i / \partial \phi)$$

$$G^{i}_{\theta} = \left[(y_i - \mu_i)^2 - V_i \right] V_i^{-2} (\partial V_i / \partial \theta)$$

Single equation models: Generalized Linear Models - EEE

- Latest Stata code available from http://home.uchicago.edu/~abasu

```
. summarize totcost, meanonly
. global scale=r(mean)
. generate y=totcost/$scale
. pglm y trt x
. pglm y trt x [pw=marsupwt]
. pglmpredict y_hat, mu scale($sc)
. pglmpredict ietrt, ie(trt) scale($sc)
. pglmpredict me_x, me(x) scale($sc)
```

```
. /* EEE - EXTENDED ESTIMATING EQUATIONS */
. ****** Download -pglm- command from
http://home.uchicago.edu/~abasu/index_files/Pagec317.htm  AND place
in C:\ado\Personal\

. qui sum totexp, meanonly
. gen y = totexp/r(mean)
. global scale = r(mean)
. summ y

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
           y |      7129           1    1.131858   .0001372    19.4377
.
. pglm y aggr_trt  agec  white black single married well
mod stage1 stage2 lnc


Extended GEE with Power Variance Function          No of obs       =      7129
Optimization: Fisher's Scoring                     Residual df     =      7114

Variance:      (theta1*mu^theta2)
Link:          (mu^lambda - 1)/lambda
Std Errors:    Robust
------------------------------------------------------------------------------
```

```
             y |      Coef.   Std. Err.      z    P>|z|  [95% Conf. Interval]
---------------+----------------------------------------------------------------
y              |
     aggr_trt |    .176795   .0319912     5.53   0.000   .1140935    .2394965
         agec |    .010301   .0025097     4.10   0.000   .005382      .01522
        white |   .0772856   .0524606     1.47   0.141  -.0255354    .1801065
        black |   .1771791   .0671802     2.64   0.008   .0455084    .3088498
       single |   .0369893   .0563494     0.66   0.512  -.0734535    .1474322
      married |   -.053268   .0341707    -1.56   0.119  -.1202413    .0137054
         well |  -.2421367    .063453    -3.82   0.000  -.3665023   -.1177711
          mod |   -.144254   .0339966    -4.24   0.000  -.2108861    -.077622
       stage1 |  -.0237874   .0289403    -0.82   0.411  -.0805093    .0329345
       stage2 |  -.0913624   .0418396    -2.18   0.029  -.1733666   -.0093583
          lnc |   .3719294   .0221651    16.78   0.000   .3284867    .4153721
        _cons |  -.2769968   .0682525    -4.06   0.000  -.4107693   -.1432244
---------------+----------------------------------------------------------------
lambda         |
        _cons |   .1928841    .303307     0.64   0.525  -.4015866    .7873549
---------------+----------------------------------------------------------------
theta1         |
        _cons |   1.166449   .0810133    14.40   0.000  1.007666    1.325233
---------------+----------------------------------------------------------------
theta2         |
        _cons |   1.457585   .2222653     6.56   0.000  1.021953    1.893217
--------------------------------------------------------------------------------
```

```
. predict xb, xb
. pglmpredict mu_pglm, mu scale($scale)
```

```
. summm mu_pglm totexp
    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
    mu_pglm |      7129    20265.42    5696.584   10770.24   51476.94
     totexp |      7129    20265.17    22937.29       2.78   393908.4

. pglmpredict trteffect, ie(aggr_trt) scale($scale)
. summm trteffect
    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
   trteffect |      7129    3470.224     790.525   2190.088   7521.893

. gen res_pglm = totexp - mu_pglm
```

GOODNESS OF FIT

```
.
.          ** Pearson Corr
.          pwcorr res_pglm mu_pglm, sig

             | res_pglm  mu_pglm
-------------+------------------
    res_pglm |   1.0000
             |
     mu_pglm |  -0.0013   1.0000
             |   0.9158
```

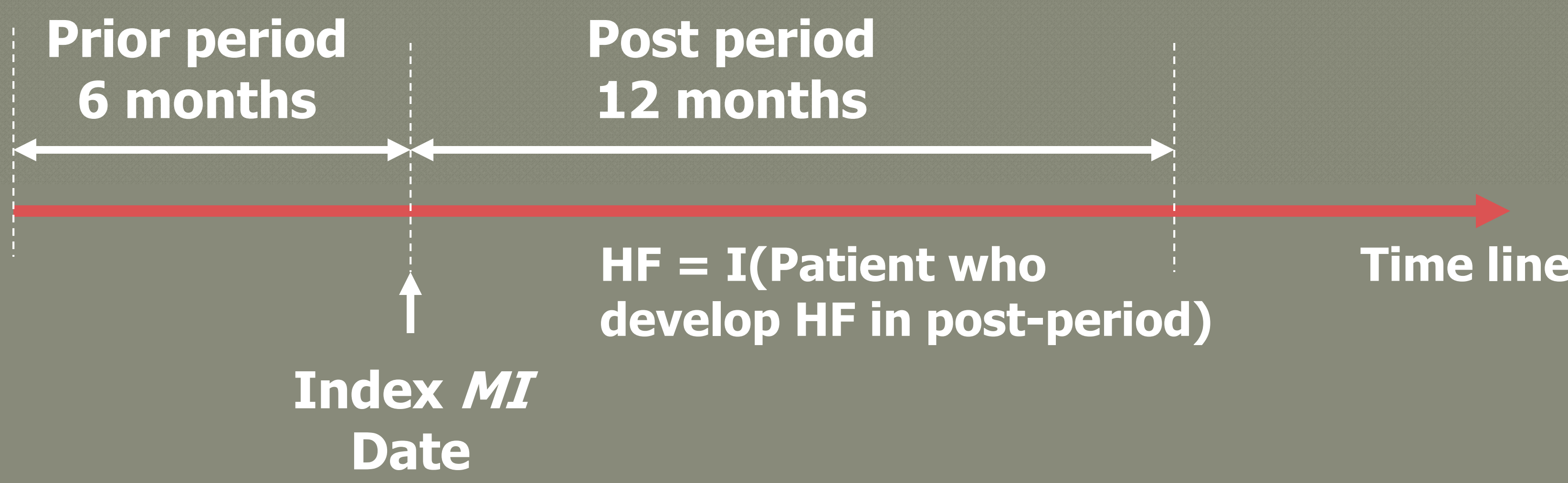
```
.        ** H-L test
.        xtile xbtile =xb, nq(10)
.        qui tab xbtile, gen(xbt)
.        reg res_pglm xbt1 xbt2 xbt3 xbt4 xbt5 xbt6 xbt7
xbt8 xbt9 xbt10, nocons robust
```

res_pglm	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
xbt1	-609.5808	553.8617	-1.10	0.271	-1695.314	476.1529
xbt2	646.0331	742.0791	0.87	0.384	-808.6624	2100.729
xbt3	-471.0719	618.8141	-0.76	0.447	-1684.131	741.9877
xbt4	-84.50918	702.9003	-0.12	0.904	-1462.403	1293.384
xbt5	1116.715	791.1745	1.41	0.158	-434.2219	2667.652
xbt6	365.4751	872.6898	0.42	0.675	-1345.256	2076.207
xbt7	-1006.109	683.5575	-1.47	0.141	-2346.085	333.8666
xbt8	-1908.943	719.4691	-2.65	0.008	-3319.317	-498.5697
xbt9	1258.806	1145.722	1.10	0.272	-987.151	3504.762
xbt10	691.741	1227.973	0.56	0.573	-1715.451	3098.933

```
. test xbt1 xbt2 xbt3 xbt4 xbt5 xbt6 xbt7 xbt8 xbt9 xbt10
       F( 10,  7119) =    1.55
            Prob > F =    0.1164

. drop xb xbt* xbtile
```

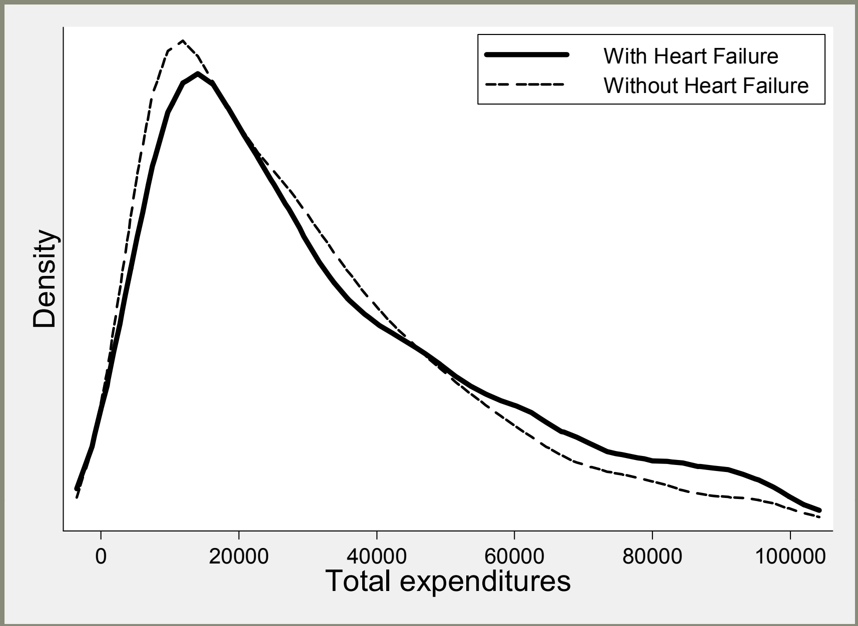
Empirical Example: Cost of Post-MI HF

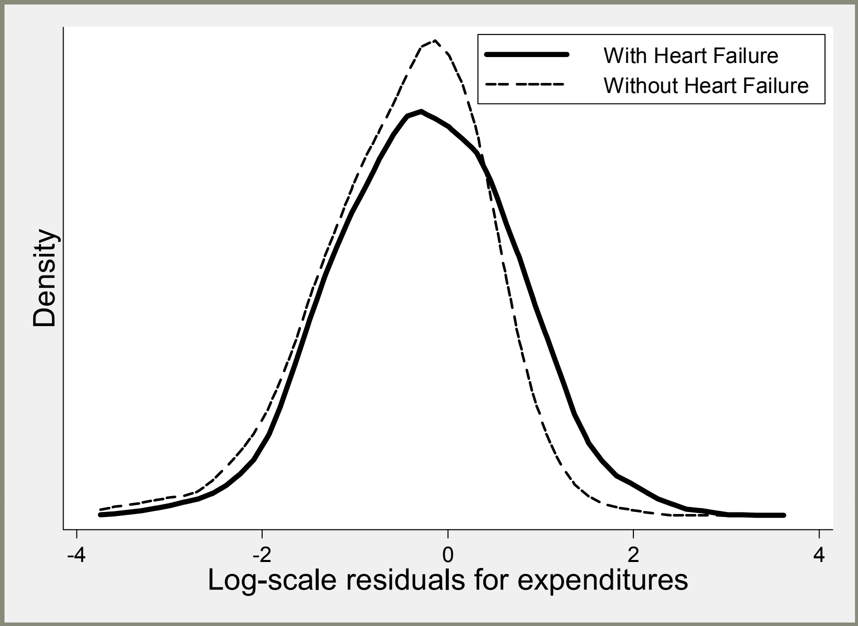
- **Empirical example**
  - Cost attributable to Heart Failure (HF) post myocardial infarction (MI)
  - Use Medstat data, extract all claims between July 1, 1997 and December 31, 2000



Exclusion Criteria		Number of patients
Starting cohort		**22091**
No claims in pre-MI or enrollment information	1279	
Patients with pre-MI HF only	1696	
Patients with pre-MI MI only	806	
Patients with pre-MI HF and MI	269	
Patients with post-MI MI	575	
Post MI HF patients with intermediate coronary syndrome and coronary occlusion without MI	18	
Patients with capitation type insurance plans	2258	
Patients with VSD	16	
Patients with mitral regurgitation	46	
Patients with cost of index hospitalization < $500	9	
Patients with LOS>100 and cost < $10,000	1	
Patients with age<18 years or missing age	3	
Patients with missing plan information	20	
Patient with index date not in (1Dec1998 – 1Dec 2000)	4633	
Eligible cohort		**10462**
Patients without full 1-year follow-up data.	3427	
Observations with missing covariate values	3	
Final sample for this study		**7428**

Empirical Example: Cost of Post-MI HF





With *HF*: N=2259

Mean (sd) = $45,070 (57,366) ;

Sk. = 6.0; Kurt. = 73

Without *HF*: N=5169

Mean (sd) = $31,708 (30,110) ;

Sk. = 4.4; Kurt. = 50

With *HF*: N = 2259

Sk. = 0.13; Kurt. = 3.2

Without *HF*: N = 5169

Sk.= -0.03; Kurt. = 3.4

Empirical Example: Cost of Post-MI HF

- **Final Sample: 7428 patients**
  2259 (30.4%) with *HF*
- **Dependent variable: Total expenditure**
- **Independent variables: HF, age, age-sq, insurance type, Medicare status, procedures performed, year of MI, type of *MI* and indicators for 30 categories of co-morbidities.**

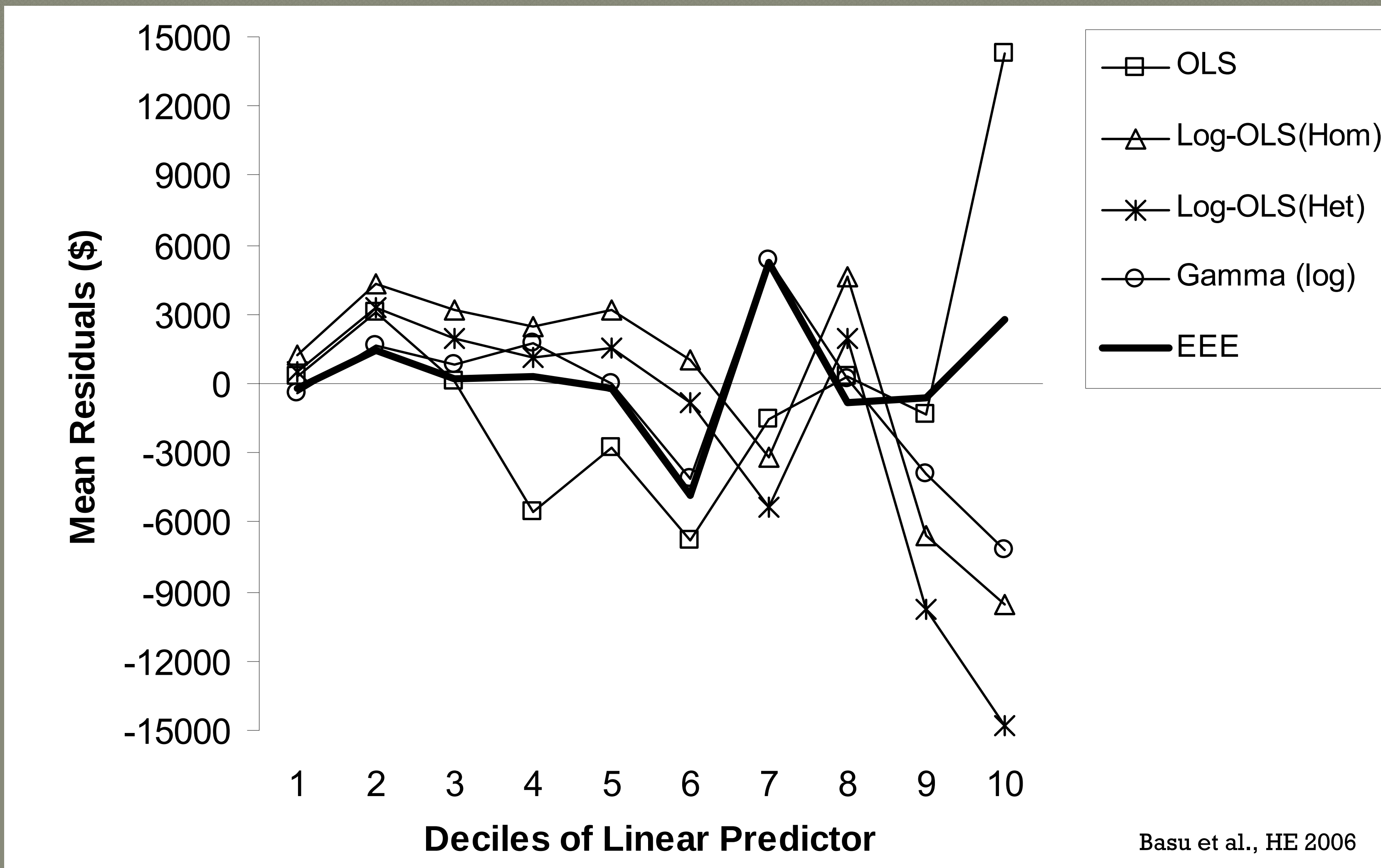
- **Compare: OL*S*, log-OL*S*(with HF-specific smearing), GLM Gamma-Log link.**

Empirical Example: Cost of Post-MI HF

Estimator	$\hat{\pi}$ Mean (std.err.)
OLS	15578 (1172)
[redacted]	[redacted]
Log-OLS (Het.)	17491 (1159)
Gamma (log link)	15890 (1104)
[redacted]	[redacted]

[redacted]

Empirical Example: Cost of Post-MI HF



Empirical Example: Cost of Post-MI HF

- **Results from the EEE Model**
  - **Mean Model (Link parameter)**
  - $\hat{\lambda}$ = **0.201** (95%CI: 0.04, 0.37)
  - **Appropriate link ~ $\mu^{0.2}$ or $\mu^{0.25}$**
  - **Variance Model: $V(y) = \phi\mu^{\theta}$**
  - $\hat{\phi}$ = **0.80** (95% CI: 0.72, 0.89); $\hat{\theta}$ = **1.79** (95% CI: 1.57, 2.01)
  - **Appropriate variance ~ μ^2**
- **Suggest fifth or fourth-root link with Gamma variance.**

Empirical Example: Cost of Post-MI HF

Estimator	$\hat{\pi}$ Mean (std.err.)	$(\hat{\pi}_{Row} - \hat{\pi}_{Col})$ Mean (std.err.)*			
		[redacted]	Log-OLS (Het.)	Gamma (log link)	EEE
OLS	15578 (1172)	[redacted]	**-1913 (627)**	-312 (566)	856 (458)
[redacted]	[redacted]		[redacted]	[redacted]	[redacted]
Log-OLS (Het.)	17491 (1159)			**1601 (239)**	**2769 (560)**
Gamma (log link)	15890 (1104)				**1168 (510)**
EEE	14722 (1135)				

Other models

- Multi-part models
  - Two-part: 1st part model any costs (logit/probit)
    2nd part models costs given any
  - Multi-part: 2-part models for each type of expenditures
  - CDE: 1st part for multinomial categories of costs
    2nd parts for levels within each category
- Mixture models

Comparison of Alternative Estimators

- Comparisons through simulations or empirical applications:
  - Duan et al., JBES 1983; Mullahy, JHE 1998; Lipscomb et al., SMDM 1999; Manning & Mullahy, JHE 2001; Deb & Trivedi, JHE 2002; Deb & Burgess, Hunter College Mimeo 2003; Basu et al., HE 2004; Buntin & Zaslavsky, JHE, 2004; Basu & Rathouz, Biostatistics 2005; Briggs et al., HE 2005; Manning et al., JHE 2006; Basu et al., HE 2006; Hill & Miller, HE 2009;
- Consistently found that no one estimator is appropriate for all expenditure data
- Theory has difficult time predicting signs of covariate effects – provides almost no guidance on functional form

Other issues with cost models

- Longitudinal costs – generalized estimating equations versus random effects model
- Right censoring
- Survival effects versus intensity effects
- Causal estimation – instrumental variables
- Predictions

Basu and Manning, Medical Care 2009