# Limited Dependent Variables

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# Limited Dependent Variables

- 0-1, small number of options, small counts, etc.
- The dependent variable is not continuous, or even close to continuous.

# Outline

- Binary Choice
- Multinomial Choice
- Counts
- Most models in general framework of probability models
  - Prob (event/occurs)

# Basic Problems

- Heteroscedastic error terms
- Predictions not constrained to match actual outcomes, real problem with predicted values being negative when a negative number isn't possible

$$Y_i = \beta_o + \beta X + \varepsilon_i$$

$Y_i$=0 if lived, $Y_i$=1 if died

**Prob ($Y_i$=1) = F(X, β)**

**Prob ($Y_i$=0) = 1 – F(X,β)**

OLS, also called a linear probability model

$\varepsilon_i$ is heteroscedastic, depends on βX

Predictions not constrained to (0,1)

# Binary Outcomes Common in Health Care

- Mortality
- Other outcome
  - Infection
  - Patient safety event
  - Rehospitalization <30 days
- Decision to seek medical care

# Standard Approaches to Binary Choice-1

- Logistic regression

$$prob(Y=1)=\frac{e^{\beta X}}{1+e^{\beta X}}$$

# Advantages of Logistic Regression

- Designed for relatively rare events
- Commonly used in health care; most readers can interpret an odds ratio

# Standard Approaches to Binary Choice-2

- Probit regression (classic example is decision to make a large purchase)

$$y^* = \beta X + \varepsilon$$

$$y=1 \text{ if } y^* > 0$$

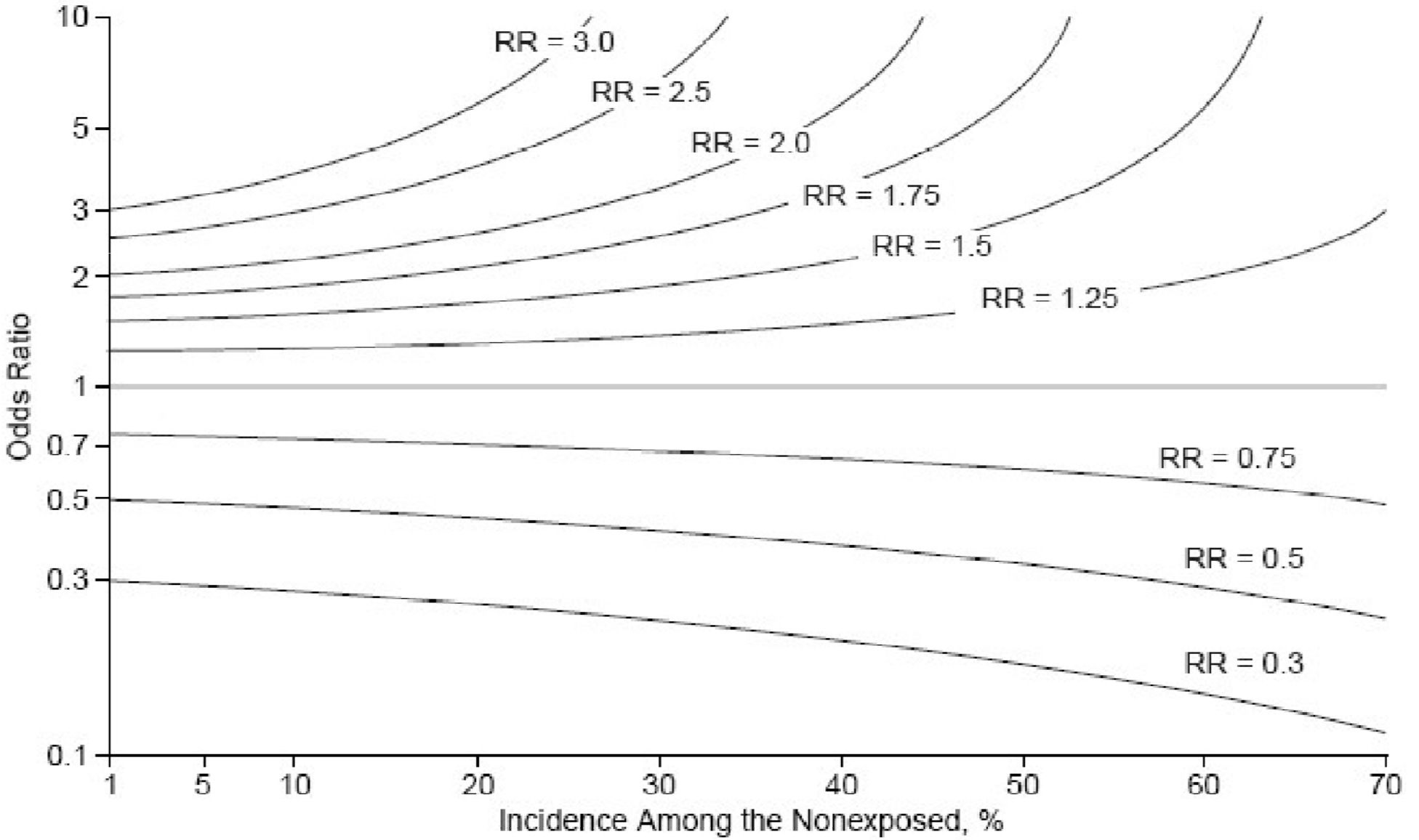
$$y=0 \text{ if } y^* \leq 0$$

# Binary Choice

- There are other methods, using other distributions.
- In general, logistic and probit give about the same answer.
- It used to be a lot easier to calculate marginal effects with probit, not so any more

# Odds Ratios vs. Relative Risks

- Standard method of interpreting logistic regression is odds ratios.
- Convert to % effect, really relative risk
- This approximation starts to break down at 10% outcome incidence




The relationship between risk ratio (RR) and odds ratio by incidence of the outcome.

# Can Convert OR to RR

- Zhang J, Yu KF. What's the Relative Risk? A Method of Correcting the Odds Ratio in Cohort Studies of Common Outcomes. JAMA 1998;280(19):1690-1691.

$$\mathbf{RR = \frac{OR}{(1-P_0) + (P_0 \times OR)}}$$

Where $P_0$ is the sample probability of the outcome

# Effect of Correction for RR

**From Phibbs et al., NEJM 5/24/2007, ≈20% mortality**

| Odds Ratio | Calculated RR |
|---|---|
| 2.72 | 2.08 |
| 2.39 | 1.91 |
| 1.78 | 1.56 |
| 1.51 | 1.38 |
| 1.08 | 1.06 |

# Extensions

- Panel data, can now estimate both random effects and fixed effects models. The Stata manual lists 34 related estimation commands
- All kinds of variations.
  - Panel data
  - Grouped data

# Extensions

- Goodness of fit tests.  Several tests.
- Probably the most commonly reported statistics are:
  - Area under ROC curve, c-statistic in SAS output.  Range 0.50 to 1.0.
  - Hosmer-Lemeshow test
  - NEJM paper, c=0.86, H-L p=0.34

# More on Hosmer-Lemeshow Test

- The H-L test breaks the sample up into n (usually 10, some programs (Stata) let you vary this) equal groups and compares the number of observed and expected events in each group.
- If your model predicts well, the events will be concentrated in the highest risk groups; most can be in the highest risk group.
- Alternate specification, divide the sample so that the events are split into equal groups.

# Estimate Note for Very Large Samples

- If you have very large samples; millions, it takes a lot longer to estimate a maximum likelihood model than OLS
- But, same X matrix, so the p-values for OLS are approximately the same as a logit model. Can use OLS for model development, and only estimate the final models with logit or other maximum likelihood model.

# Multinomial Choice

- What if more than one choice or outcome?
- Options are more limited
  - Multivariable Probit (multiple decisions, each with two alternatives)
  - Two different logit models (single decision, multiple alternatives)

# Logit Models for Multiple Choices

- Conditional Logit Model (McFadden)
  - Unordered choices
- Multinomial Logit Model
  - Choices can be ordered.

# Examples of Health Care Uses for Logit Models for Multiple Choices

- Choice of what hospital to use, among those in market area
- Choice of treatment among several options

# Conditional Logit Model

$$prob(Yi = j) = \frac{e^{\beta X_{ij}}}{\sum_{j=1}^{J} e^{\beta X_{ij}}}$$

# Conditional logit model

- Also known as the random utility model
- Is derived from consumer theory
- How consumers choose from a set of options
- Model driven by the characteristics of the choices.
- Individual characteristics “cancel out” but can be included indirectly. For example, in hospital choice, can interact individual characteristic with distance to hospital
- Can express the results as odds ratios.

# Estimation of McFadden's Model

- Some software packages (e.g. SAS) require that the number of choices be equal across all observations.
- LIMDEP, allows a "NCHOICES" options that lets you set the number of choices for each observation. This is a very useful feature. May be able to do this in Stata (clogit) with "group"

# Example of Conditional Logit Estimates

- Study I did looking at elderly service-connected veterans choice of VA or non-VA hospital

| | | |
|---|---|---|
| Log distance | 0.66 | $p<0.001$ |
| VA | 2.80 | $p<0.001$ |

# Multinomial Logit Model

$$prob(Y = j) = \frac{e^{\beta_j X_i}}{1 + \sum_{j=1}^{J} e^{\beta_j X_i}}$$

$$prob(Y = 0) = \frac{1}{1 + \sum_{j=1}^{J} e^{\beta_j X_i}}$$

# Multinomial Logit Model

- Must identify a reference choice, model yields set of parameter estimates for each of the other choices, relative to the reference choice
- Allows direct estimation of parameters for individual characteristics. Model can (should) also include parameters for choice characteristics

# Independence of Irrelevant Alternatives

- Results should be robust to varying the number of alternative choices
  - Can re-estimate model after deleting some of the choices.
  - McFadden, regression based test. Regression-Based Specification Tests for the Multinomial Logit Model. J Econometrics 1987;34(1/2):63-82.
- If fail IIA, may need to estimate a nested logit model

# Independence of Irrelevant Alternatives - 2

- McFadden test can also be used to test for omitted variables.
- For many health applications, doesn't matter, the models are very robust (e.g. hospital choice models driven by distance).

# Count Data (integers)

- Continuation of the same problem; dependent variable can only assume specific values and can't be <zero
- Problem diminishes as counts increase
- Rule of Thumb. Need to use count data models for counts under 30

# Count Data

- Some examples of where count data models are needed in health care
  - Dependent variable is number of outpatient visits
  - Number of times a prescription of a chronic disease medication is refilled in a year
  - Number of adverse events in a unit (or hospital) over a period of time

# Count Data

- Poisson distribution. A distribution for counts.
  - Problem: very restrictive assumption that mean and variance are equal

$$prob(Yi = y_i) = \frac{e^{-\lambda_i} \lambda_i^{y_i}}{y_i!}$$

# Count Data

- In general, negative binomial is a better choice. Stata (nbreg), test for what distribution is part of the package. Other distributions can also be used.

$$f(y_i|x_i,u_i)=\frac{e^{-\lambda_i u_i}(\lambda_i u_i)^{y_i}}{y_i!}$$

# Interpreting Count Data Models

- lnE(event rate) = Bx
- Incidence Rate Ratio = $e^B$, like on odds ratio, with a similar interpretation
- Example, effect of average RN tenure on unit on infection rate
  - B= -0.262  IRR = 0.770

# Notes for Count Data Models

- More common to see OLS used for counts than for binary or very limited choices
- Real problem with OLS when there are lots of zeros. Will result in reduced statistical significance

# Notes for Count Data Models-2

- 30 is a rule of thumb, but should still consider a count model if most are small counts
- Need to consider distribution and data generating process. If mixed process, may need to split sample

# Example of Mixed Data Generating Processes

- Predicting LOS for newborns
  - Well babies, all with LOS<= 5 days, clearly a count
  - Sick newborns, can have very long LOS
- Solution, separate samples, use count model for well babies and OLS for sick babies

# Other Models

- New models are being introduced all of the time. More and better ways to address the problems of limited dependent variables.
- Includes semi-parametric and non-parameteric methods.

# Reference Texts

- Greene. Econometric Analysis
- Wooldridge. Econometric Analysis of Cross Section and Panel Data
- Maddala. Limited-Dependent and Qualitative Variables in Econometrics

# Journal References


- McFadden D. Specification Tests for the Multinomial Logit Model. J Econometrics 1987;34(1/2):63-82.

- Zhang J, Yu KF. What's the Relative Risk? A Method of Correctingthe Odds Ratio in Cohort Studies of Common Outcomes. JAMA 1998;280(19):1690-1691.

# Next lecture

Right-hand Side Variables

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